Alternative Mechanisms for Financing Social Security

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Alternative Mechanisms for Financing Social Security

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This paper provides a review, clarification, and extension of the theoretical literature concerning the effect of social security on capital accumulation and labor supply. Within a framework of overlapping generations, the paper reviews the theoretical results regarding the impact of both pay-as-you-go financed and fully-funded schemes. Relevant empirical studies using U.S. data are also reviewed. The paper also analyzes the characteristics of optimal social security systems.

A major part of the paper is concerned with the neutrality or non-neutrality of social security's impact on capital accumulation and labor supply. While it is generally agreed that a fully-funded system is neutral, views differ significantly on the issue of neutrality under a pay-as-you-go system. The analysis presented in the paper demonstrates that these differences arise from two assumptions. The first requires that, in providing bequests, those in the older generation must focus on net (i.e., net of all other inter-generational flows including those arising from the social security system) bequests rather than gross bequests. The second requires that the representative individual be aware of the government's budget constraint. If these two assumptions are met, a pay-as-you-go social security system is neutral. If either assumption is unfulfilled, the system will affect behavior. In particular, if the second assumption is not met, social security will affect capital accumulation (a dynamic effect) and will also introduce a distortion into the labor market (a static effect).

The second major part of the paper is concerned with normative issues. It contrasts the standard procedure of equating optimality to the maximization of a representative individual's utility function with an approach which rationalizes the need for social security by reference to a paternalistic concern for the welfare of the retired population. It is shown that with a paternalistic motivation, a fully-funded system can be used to move the economy towards its social optimum whereas a pay-as-you-go system moves the economy away from its optimal path. These results reverse the conclusions in the literature where the focus has been exclusively on private utility maximization.

The paper provides a starting point for anyone interested in assessing the relevance of the existing theoretical literature for social security issues in developing countries. These issues are already important in Latin America and are also becoming important in some Asian countries. At present, most of the theoretical literature captures characteristics more akin to those of developed countries. Nevertheless, the existing approaches have the advantage of capturing both static and dynamic effects, a very important consideration in the analysis of an instrument that can be expected to influence static resource allocation through the labor market and the dynamics of growth through capital accumulation.
ABSTRAIT

Le document ci-après vise à analyser, préciser et développer les travaux théoriques antérieurs concernant l'effet de l'assurance vieillesse sur l'accumulation de capital et l'offre de main-d'œuvre. Au moyen d'un modèle à générations multiples, on étudie les effets théoriques des systèmes fonctionnant selon le principe de la répartition d'une part, et des systèmes à capitalisation intégrale, d'autre part. On examine également des études empiriques pertinentes utilisant des données des États-Unis. On analyse aussi les caractéristiques des régimes d'assurance vieillesse optimaux.

Une grande partie du document est consacrée à la neutralité ou à la non-neutralité des effets de l'assurance vieillesse sur l'accumulation de capital et l'offre de main-d'œuvre. On s'accorde généralement à penser qu'un régime à capitalisation intégrale est neutre, mais les avis sont très partagés pour ce qui est du système de la répartition. L'analyse présentée démontre que ces différences découlent de deux hypothèses de travail. Selon la première, en accordant des legs, la génération la plus ancienne doit considérer les legs nets (c'est-à-dire nets de tous autres flux intergénérations, y compris les flux provenant du régime d'assurance vieillesse) plutôt que sur les legs bruts. Selon la deuxième hypothèse, l'individu représentatif doit avoir conscience des contraintes du budget de l'État. Si ces deux hypothèses sont vérifiées, les régimes à répartition sont neutres. Si l'une ou l'autre des deux hypothèses ne l'est pas, le régime influe sur le comportement. En particulier, si la deuxième hypothèse n'est pas vérifiée, l'assurance vieillesse influe sur l'accumulation de capital (effet dynamique) et introduit une distorsion sur le marché du travail (effet statique).

La deuxième grande partie du document est consacrée à des questions normatives. Elle oppose la méthode habituelle consistant à définir l'optimalité comme étant la maximisation de la fonction d'utilité d'un individu représentatif à une approche qui fonde la nécessité de la sécurité sociale sur le souci paternaliste du bien-être de la population retraitée. Il est démontré que dans une perspective paternaliste, la capitalisation intégrale peut contribuer à rapprocher l'économie de son optimum collectif, tandis que la répartition l'en éloigne. Ces résultats vont à l'encontre des conclusions auxquelles parviennent les chercheurs qui se concentrent exclusivement sur la maximisation de l'utilité individuelle.

Le document ci-après peut fournir un point de départ à ceux qui cherchent à évaluer dans quelle mesure les travaux de recherche théoriques qui ont été faits sur les questions d'assurance vieillesse sont applicables dans les pays en développement. Ces questions sont déjà importantes en Amérique latine et commencent à se poser dans certains pays d'Asie. À l'heure actuelle, la plupart des travaux traitent plutôt de caractéristiques apparentées à celles qu'on observe dans les pays développés. Néanmoins, les méthodes actuelles présentent l'avantage de saisir à la fois les effets statiques et dynamiques, ce qui n'est pas à dédaigner lorsqu'on analyse un instrument qui a des chances d'influencer l'affectation statique des ressources par l'intermédiaire du marché du travail, et sur la dynamique de la croissance, par le biais de l'accumulation de capital.
En este documento se examina, clarifica y amplía la literatura teórica relativa a los efectos de los planes de seguridad social en la acumulación de capital y la disponibilidad de mano de obra. Dentro de un marco de generaciones superpuestas, se examinan en él los resultados teóricos relativos al efecto de los sistemas de transferencia inmediata (pay-as-you-go) y de financiamiento pleno (fully-funded). También se examinan estudios empíricos pertinentes que utilizan datos relativos a los Estados Unidos. Además se analizan las características de los sistemas óptimos de seguridad social.

Una parte importante del documento se refiere a la neutralidad o falta de neutralidad de los efectos del seguro social en la acumulación de capital y la disponibilidad de mano de obra. Si bien en general hay acuerdo en que un sistema de financiamiento pleno es neutro, las opiniones difieren ampliamente con respecto a la neutralidad en el de transferencia inmediata. El análisis que aquí se presenta demuestra que estas diferencias surgen de dos supuestos. El primero requiere que al proporcionar donaciones las personas mayores se concentren en las donaciones netas (es decir, deducidas todas las demás corrientes intergeneracionales que incluyen las que surgen del sistema de seguridad social) más bien que en donaciones brutas. El segundo supuesto requiere que el individuo representativo esté consciente de las limitaciones presupuestarias del gobierno. Si los dos supuestos se cumplen, el sistema de seguridad social de transferencia inmediata es neutro. Si no se cumple uno de ellos, el sistema afectará al comportamiento. En especial, si no se cumple el segundo supuesto, el seguro social afectará a la acumulación de capital (efecto dinámico) y también introducirá distorsiones en el mercado laboral (efecto estático).

La segunda parte principal del documento se refiere a cuestiones normativas. Se contrasta el procedimiento estándar de igualar lo óptimo con la maximización de la función de utilidad de un individuo representativo empleando un enfoque en el que se racionaliza la necesidad de la seguridad social con referencia a una preocupación paternalista por el bienestar de la población jubilada. Se demuestra que, con una motivación paternalista, un sistema de financiamiento pleno puede utilizarse para llevar la economía hacia su grado social óptimo, en tanto que el sistema de transferencia inmediata aleja a la economía de su curso óptimo. Estos resultados invierten las conclusiones de la literatura en que el punto central ha sido exclusivamente el acrecentamiento máximo del provecho privado.

El documento sirve de punto de partida para toda persona interesada en evaluar la pertinencia de la literatura teórica existente sobre los problemas de la seguridad social en los países en desarrollo, que ya son importantes en América Latina y empiezan a adquirir importancia en algunos países de Asia. En la actualidad la mayor parte de la literatura teórica capta características más parecidas a las de los países desarrollados. Sin embargo, los enfoques existentes tienen la ventaja de presentar los efectos tanto estáticos como dinámicos, consideración muy importante en el análisis de un instrumento que se puede prever influirá en la asignación estática de recursos a través del mercado laboral y en la dinámica del crecimiento mediante la acumulación de capital.
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Introduction

The objective of this paper is two-fold. First, it clarifies the role of different assumptions made by previous authors in the determination of results regarding the neutrality or non-neutrality of social security's impact on capital accumulation and labor supply. To this end, it analyses the differential effects of alternative financing mechanisms - fully-funded and pay-as-you-go schemes - within the framework of a multiperiod model with overlapping generations. Second, the paper addresses the question of the optimal size of social security. It contrasts the standard procedure of equating optimality to the maximization of a representative individual's utility function with an approach which rationalizes the need for social security by reference to a paternalistic concern for the welfare of the retired population.

Section I describes the central model and the main issues that are addressed in the following sections. Section II investigates "positive" concerns. Clarifying the alternative financing mechanisms within our framework, it traces their differential effects on capital accumulation and labor supply, and provides pertinent empirical evidence from the U.S. in support of our findings. Section III is concerned with normative issues of optimality in both an individualistic and a paternalistic framework. It examines the ability of alternative social security mechanisms to achieve the rate of capital accumulation associated with the Golden Rule and explores the effect of social security on the optimality of the Golden Path. A concluding section summarizes the main results and an appendix elaborates a more complicated version of the simple normative model developed in Section III.
I. ENUNCIATION OF MODEL AND CENTRAL ISSUES

I.1 Overlapping Generations Models

Overlapping generations models are so called because at any point in time they include at least two distinct generations. Most models, and all of those reviewed here, identify only two generations - a working generation and a retired generation, or, more accurately, a young generation for whom retirement is not a feasible choice and an old generation for whom retirement is a possibility. To clarify the generational structure of these models and to introduce our notation, the distribution of consumption between groups in each of two periods is described by:

<table>
<thead>
<tr>
<th></th>
<th>Period t</th>
<th>Period t + 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young Generation</td>
<td>c₁ₜ</td>
<td>c₁ₜ + 1</td>
</tr>
<tr>
<td>Old Generation</td>
<td>c₂ₜ</td>
<td>c₂ₜ + 1</td>
</tr>
</tbody>
</table>

The table shows the complete, lifetime consumption of only one generation - c₁ₜ and c₂ₜ + 1 are the levels of consumption of the same generation when young in period t and when old in period t + 1.

Consumer Maximization

In the simplest models (Diamond (1965) and Samuelson (1975)), the only choice to be made by the consumer is that of determining his intertemporal pattern of consumption. In particular, the individual must choose how much to save while working in order to finance his consumption when retired. Accordingly, the consumer is assumed to:

\[
\text{max } U = U(c₁ₜ, c₂ₜ + 1) \quad u_i > 0, u_{ii} < 0 \tag{1}
\]

\[
\text{s.t. } \frac{c₁ₜ + c₂ₜ + 1}{(1 + r)^{-1}} = w \tag{2}
\]
where \( r \) is the rate of interest and \( w \) is the wage rate for one period of work. Consumer equilibrium requires that:

\[
\frac{\delta u}{\delta c_1} / \frac{\delta u}{\delta c_2} - 1
\]

which signifies the standard relationship between the marginal rate of substitution between consumption in periods 1 and 2 and the interest rate. Equations 2 and 3 can be solved for \( c_1 \) and \( c_2 \) as functions of the two prices, \( r \) and \( w \).

**Producer Maximization**

Technology is characterized by a constant returns to scale production function which may be written as:

\[
f = f (\ell)
\]

where \( f \) is output per unit of capital and \( \ell \) is labor input per unit of capital. Profit maximisation implies that

\[
w = f'
\]

and

\[
r = f - \ell f'
\]

or

\[
r = \psi(w) \quad \psi = -\ell < 0 \quad \psi'' = -1/f'' > 0
\]

which is the factor-price frontier. Equations 5 and 6 can be used to express the factor prices as a function of the labor: capital ratio.

**Factor Supply**

Total population \((P)\) is assumed to grow at a rate of \( g \) percent a year. Accordingly, labor supply \((L)\) in period \( t \) is:

\[
L_t = P_t (1 - \theta)
\]
where $\theta = 1/(2 + g)$ is the proportion of the population in the second period of their life and who, in this simple model, are fully retired.

Since the capital stock ($K$) in period $t$ is determined by the savings decisions of those who are in the young age group in period $t-1$, we have

$$K_t = P_t^\theta k_{t-1}$$

(9)

where $k_{t-1}$ is the per capita supply of capital provided by the older generation's savings from the previous period. Or,

$$K_t = S_{t-1} = (w_{t-1} - c_{t-1}^1) L_{t-1}^1$$

(9')

And since, as noted above, $c_{t-1}^1$ is a function of $w_{t-1}$ and $r_{t-1}$, the supply of capital is determined exogenously as far as decisions in period $t$ are concerned. This completes the system of equations and implies that consumption per capita in period $t$ is $w_t - s_t$ (consumed by the young generation) plus $(1 + r) s_{t-1}$ (consumed by the old generation) and that total investment is $S_t - S_{t-1}$. In the following, we will be exploring more complicated models but the basic structure remains the same.

I.2 The Golden Rule

An important part of the literature has investigated, not just the positive issue of how a perfectly competitive economy functions, but the normative question of what is the highest level of steady-state consumption per capita that can be achieved when steady-state is characterized primarily by a constant value for the labor: capital ratio. In particular, authors have explored the choice among alternative steady-states in the context of a centrally planned economy. This has involved the maximisation of equation 1 subject to the production technology (equation 4) and subject to the
exogenously given growth in the labor supply (equation 8). Unlike the preceding analysis, however, in this version the capital stock (and hence the labor: capital ratio) is chosen by the planner to maximise the steady-state consumption path, whereas previously it was determined by individual preferences between consumption and saving.

The constraint faced by the planner may be formulated as:

\[ c^1_t L^1_t + c^2_t L^2_t = K_t f_t - I_t \]

that is, total consumption must equal total output less investment. Since in steady-state the labor: capital ratio is constant, the capital stock must grow at the same rate \( g \) as the labor force. It follows that:

\[ I_t = gK_t \]

so that dividing through by \( L^1_t \), the constraint may be expressed as

\[ c^1_t + c^2_t/(1+g) = (f_t - g)/\bar{\lambda} \]

(10)

where \( \bar{\lambda} \) is the constant labor: capital ratio. The planner maximizes equation 1 by choosing the values of \( c^1, c^2, \) and \( \bar{\lambda} \) subject to equation 10. This yields:

\[ g = \frac{\delta u}{\delta c_1} / \frac{\delta u}{\delta c_2} - 1 \]  

(3\text{'}

and

\[ g = f - \ell f' \]  

(6\text{'}

Equations 3\text{'}, 6\text{'} and 10 define the maximum level of steady-state consumption in a planned economy. From equations 2, 3, 5 and 6, it is clear that the first-order conditions defining equilibrium in a perfectly competitive economy are identical to those in the planned economy with the sole exception that in the latter "the rate of interest" is exactly equal to the rate of population growth. This extra condition arises from the planner's ability to treat the labor: capital ratio as a policy variable whereas in the
perfectly competitive economy it is the result of private decision-making. Except by chance, there is no reason to suppose that in a competitive economy the equilibrium rate of interest will equal the population growth rate, and hence there is no reason to suppose that a competitive economy will achieve the steady-state level of consumption associated with the Golden Rule.

I.3 Central Issues

These simple models are useful in indicating the major issues that have arisen in the analysis of social security systems. In general, we can identify three questions that have been central to the social security literature:

1) Can the rate of capital accumulation be influenced by the introduction of a social security system? Within our simple model, this would involve investigating whether or not savings \( S_t \) are a function of tax payments and pension benefits. The question becomes somewhat broader if other motives - beside the life-cycle motive - for saving are introduced. The bequest motive is one that has received considerable attention (Barro, 1974).

ii) If labor supply is endogenous, does the presence of a social security system influence labor supply decisions? This turns attention away from the dynamic behavior of the economy and towards issues of static resource allocation. In terms of our model, one would want to investigate whether the payment of taxes and the receipt of pensions drives a wedge between labor's supply price and its marginal revenue product (Hu, 1979).

iii) The questions so far have been positive in character. If, indeed, private behavior can be influenced by social security, then
important normative questions arise. In particular, one would want to know if social security can enable a competitive economy to obtain the steady-state associated with the Golden Rule of Samuelson. More fundamentally, one can question whether or not the presence of social security causes the social optimum to depart from the Golden Rule. These questions form the point of departure for the remainder of the paper. Before exploring them, however, we comment briefly on the main financing mechanisms of social security and their relative importance in a selection of countries.

II. **POSITIVE ISSUES**

II.1 **Social Security Systems**

The theoretical literature has focussed on two different types of funding that are usually known as the fully-funded system (FF) and the pay-as-you-go system (PAYG). The former involves an intertemporal transfer in which workers are taxed in period $t$ and then receive in period $t+1$ their tax payments $(t)$ plus interest in the form of pension benefits $(z)$. For this system, therefore, we have:

$$L_{t+1}^2 z = L_t^1 t(1 + r)$$

In most systems in the world, $t$ is a payroll tax and hence raises the possibility of a distortion in the labor market. In several models in the literature, however, labor supply is exogenously determined and the tax is assumed to be of the lump-sum variety. This is an important point at which the models are at variance with reality and we return to it in Section II.3. Note that in this system, although the government's account is obviously balanced through time, it will not generally be balanced at any point in time.
(i.e., in a cash-flow sense). Thus, in period \( t \), total receipts are \( tL_t^1 \) and total expenses are \( zL_t^2 = t(l + r)L_t^2 \). If the tax remains constant through time, and if the population growth rate is \( g \), then total receipts exceed total expenses if \( g > r \) and vice versa if \( r > g \).

By way of contrast, it is a characteristic of the PAYG system that the government's account is balanced at each point in time:

\[
zL_t^2 = tL_t^1
\]

that is, the benefits paid to the retired population exactly equal the tax receipts from the working population. The PAYG system, therefore, involves an intergenerational transfer rather than an intertemporal one.

Table 1 groups selected developed and developing countries, which have sizeable social security institutions, according to broad financing mechanisms of the old age pension component of their social security systems. The first group of countries, Australia, New Zealand and the U.K., finance their benefit payments from general revenues; hence there is no surplus or deficit in their pension schemes by definition. Nor is there any earmarking; social security payments are not linked in any way to a specific tax such as a payroll tax. Most of the analysis presented here does not, therefore, apply to these countries.

The second group of countries, which run their old age pension schemes primarily on a funded basis, usually run sizeable surpluses in their pension accounts, ranging between 50 percent (Japan) and 80 percent (Malaysia, Sri Lanka) of total receipts. As explained, this is because in a funded system, the social security budget need not be balanced at any point in time. Instead, it is balanced across time for every contributor because the discounted value of the returns he receives from the system should match his contributions.
The third group of countries are those which rely primarily on a PAYG system of social security. Here we expect, as in our theoretical analysis, the social security budget to balance every year. Indeed, while we see that the surplus (or deficit) in this system is relatively small when compared to that of a FF system, in reality, however, the budget cannot be expected to balance exactly. Even in a PAYG system, depending on the prevalent social security tax laws, a national old age pensions scheme can be expected to yield small surpluses or deficits. Thus, in 1977 and subsequently, both the U.S. and West Germany yielded deficits, necessitating a revision upwards in social security taxes.

The vintage of a PAYG system affects the social security budget. Relatively young PAYG systems, for example most of the post-War European and the U.S. systems, yielded annual surpluses since every old age pensioner was supported by a bigger pool of workers, compared to later years. As these schemes matured and the demographic compositions of these societies changed, a lower pool of workers had to support every old age pensioner. As a result, with the progress of time, the surpluses vanished, the systems then arrived at a balanced position, and finally ran into deficits unless compensated by higher taxes. This was evinced by Sweden which, with a heavy reliance on high tax rates, is still generating a considerable surplus in its PAYG system of old age pension. Thus while it is theoretically correct to expect a PAYG system to be balanced every year, in practice, one may perceive deviations from this balance.
Table 1: FINANCING OF OLD AGE PENSIONS
In Selected Countries 1977 1/

<table>
<thead>
<tr>
<th>Country</th>
<th>Receipts x 100 GDP</th>
<th>Total Receipts (=100)</th>
<th>Surplus x 100 Total Receipts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>3.9</td>
<td>0 0 100 0 0</td>
<td>0</td>
</tr>
<tr>
<td>New Zealand</td>
<td>5.5</td>
<td>0 0 100 0 0</td>
<td>0</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>4.0</td>
<td>0 0 100 0 0</td>
<td>0</td>
</tr>
<tr>
<td>India</td>
<td>1.2</td>
<td>40 40 1 19</td>
<td>72</td>
</tr>
<tr>
<td>Japan</td>
<td>3.4</td>
<td>32 28 19 21</td>
<td>53</td>
</tr>
<tr>
<td>Malaysia</td>
<td>2.8</td>
<td>29 29 0 42</td>
<td>79</td>
</tr>
<tr>
<td>Singapore</td>
<td>8.7</td>
<td>38 42 0 20</td>
<td>63</td>
</tr>
<tr>
<td>Sri Lanka</td>
<td>1.3</td>
<td>27 40 0 33</td>
<td>80</td>
</tr>
<tr>
<td>Turkey</td>
<td>2.3</td>
<td>35 40 0 25</td>
<td>62</td>
</tr>
<tr>
<td>Argentina</td>
<td>4.0</td>
<td>40 50 0 10</td>
<td>26</td>
</tr>
<tr>
<td>Belgium</td>
<td>5.4</td>
<td>33 43 17 7</td>
<td>6</td>
</tr>
<tr>
<td>Canada</td>
<td>3.7</td>
<td>14 14 58 14</td>
<td>28</td>
</tr>
<tr>
<td>Chile</td>
<td>3.6</td>
<td>27 48 10 15</td>
<td>26</td>
</tr>
<tr>
<td>Germany</td>
<td>11.2</td>
<td>34 33 19 14</td>
<td>-7</td>
</tr>
<tr>
<td>Sweden</td>
<td>12.9</td>
<td>3 63 13 21</td>
<td>32</td>
</tr>
<tr>
<td>Uruguay</td>
<td>5.6</td>
<td>31 44 8 17</td>
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</tr>
<tr>
<td>United States</td>
<td>3.8</td>
<td>50 46 1 3</td>
<td>-2</td>
</tr>
</tbody>
</table>

1/ These figures exclude old age pension schemes for public sector employees.

II.2 Capital Accumulation

Fully-Funded System

Much of the analysis of this and the next section can be conducted by means of a careful investigation of the effect of social security on the individual's budget constraint. For example, consider the introduction of a FF system into an economy characterized by equations 1, 2, 4 and 8. The only equation affected by the SS system is the lifetime budget constraint. In particular, we can write:

\[ c_t^1 + c_{t+1}^2 / (1+r) = w - t + z (1+r) \]

in place of equation 2. Recall, however, that in a FF system benefits equal tax payments plus accrued interest - that is, from equation 11, \( z = t(l+r) \) and hence equation 2 collapses to equation 2 and the economy is totally unaffected by the presence of the FF system.

This result is well-known and is generally accepted in the literature (Samuelson, 1975). For future reference, however, it is worth noting that it depends on the individual's awareness of the relation between taxes and benefits and on the equivalence of the rate of return on public savings and the interest rate facing the private sector. Provided these conditions are fulfilled, total capital accumulation remains the same but its distribution between the public and private sectors depends on the rate of taxation. If, however, these conditions are not fulfilled, the rate of capital accumulation will be affected. For example, assume that the rate of return on public saving is less than that on private savings. In this event, the individual suffers a real income loss and, since consumption in both periods is assumed to be normal, the reduction in private savings will be more
than social security payments and the overall level of capital accumulation will suffer.

The significance of different rates of return to private and public savings emerges clearly from the Sheshinski and Weiss (1981) model. In their model, the length of life and hence the length of retirement is uncertain. Accordingly, their FF social security system offers an annuity. The return on private savings is \((1 + r)\). Assuming that the social security system is actuarially fair, the return on an annuity is \(\phi(l+r)/\phi\), where \(\phi\) is the fraction of the individual's potential retirement that is actually realized, and \(\phi\) is the expected duration of retirement in the population. Clearly, in this case the presence of a social security system can be expected to influence private behavior. Equally clearly, however, had the individual had access to a private annuities market that offered a return of \(\phi(l+r)/\phi\), the introduction of a social security system would only imply a reallocation of the individual's annuities from the private market to the public sector without any consequence for real prices or capital accumulation.

**Pay-as-you-go System**

The effect of a PAYG system on capital accumulation is considerably more controversial. Just as private saving offers a means of offsetting the intertemporal transfers associated with a FF system, so, according to Barro, private bequests offer a means of offsetting the intergenerational transfers associated with a PAYG system. Views on this matter, however, diverge sharply. Consider these quotes from two recent contributions:

"If social security is financed on a pay-as-you-go basis, i.e., if taxes on the currently working population are used to finance benefits to the retired population, it is a perfect substitute for private bequests. Hence, a forced increase in social security will reduce bequests by an equal amount. Consumption, private savings, and aggregate savings will be unaffected. In such models the optimal level of social security is clearly undetermined." (Sheshinski and Weiss, 1981, pp. 189-190.)
"Without further complications and great loss of generality, let us assume that the only role played by the government is to provide Social Security. Moreover, it finances Social Security by a "pay-as-you-go" system under which the government does not accumulate capital and it merely transfers payments by workers to retirees. It is further shown that an appropriate Social Security system can increase the long-run well being of the economy by causing the rate of return on capital to converge to the Golden Rule level." (Hu, 1979, p. 274 and p. 278.)

The authors quoted above arrive at fundamentally different conclusions regarding the effect of a social security system financed on a PAYG basis. Sheshinski and Weiss reproduce Barro's original result whereby changes in private decisions with respect to bequests completely offset social security tax payments leaving all real variables unchanged. Hu, on the other hand, demonstrates that a PAYG system can influence the economy's steady-state rate of capital accumulation. The purpose of this section is to pinpoint the causes of these different conclusions.

To explore this issue we need to introduce a bequest motive into the model captured by equations 1, 2, 4 and 8. In particular, we adopt the procedure followed by Hu and write:

\[ U = u(c^1) + v(c^2, B) \]  

which implies that the marginal rate of substitution between second-period consumption and bequests \((B)\) is independent of first-period consumption. Several authors have used a more general version of the utility function. For example, Barro (1974) writes:

\[ U = u(c^1, c^2, U^*) \]  

where \(U^*\) is the maximum level of utility attainable by the individual's heir. However, since \(U^*\) is itself a function of any bequest, the first-order conditions derived from equations (1') and (1'') both involve \(\partial u/\partial B\). From equation (1') \(\partial u/\partial B = \partial v/\partial B\), whereas from equation (1'') it is expressed as \(\partial u/\partial U^* - \partial U^*/\partial B\). That is, apart from the additive characteristics of
equation 1', the two formulations are the same.

An advantage of equation 1", however is that the effect of a bequest on utility can be divided into its two component parts. The first - $\partial u/\partial u^*$ - represents the individual's evaluation of his own utility relative to that of his heir. Presumably, the former receives more weight than the latter so that $\partial u/\partial u^*$ may be interpreted as the rate at which an individual discounts the utility of his heir. For a general utility function, Drazen (1978) demonstrates that bequests will be positive only if this rate of discount is less than the market rate of interest. This condition ensures that the loss of consumption to the donor entailed by a bequest is, despite the discount factor, more than offset by the value to him of the consumption - $(1+r)$ times the bequest - enjoyed by his heir. The second component - $\partial u^*/\partial \beta^*$ - is simply the effect of a bequest on the heir's maximum attainable level of utility. This advantage notwithstanding, we work with equation 1' throughout most of the paper because of its notational convenience.

In the context of a model with a growing population, some authors (Drazen (1978), Carmichael (1982), Burbidge (1983)) have explored whether $\beta$ should represent the bequest per heir or total bequests. If one focusses on the bequest per heir and the population growth rate is $g$, then, in addition to the discount factor discussed above, the value of a bequest to the donor will depend on the number of heirs. The population growth rate will operate, therefore, as a second discount factor. We, however, will follow prevalent opinion (see Drazen (1978) and Burbidge (1983)) as well as personal preference, and focus exclusively on total bequest rather than bequest per heir. Accordingly, $\beta$ should be interpreted as the total amount bequeathed so that each heir receives $\beta/(1+g)$ where $g$ is the population growth rate.
With these clarifications of equation (1) examine, first, the consequences of a PAYG system for a person in the second period of his life. Unlike the earlier models in which second-period consumption was determined by first-period savings, in this model, given the level of first-period savings, the individual still has a choice between consumption and leaving a bequest. Accordingly, the individual is assumed to:

$$\max \ v = v(c^2, \beta)$$

subject to:

$$c^2 + B = (1+r)(s_{t-1} + h_{t-1}) + z$$

where $\beta$ is the individual's measure of the value of his bequest, and $B$ is the monetary cost of that bequest (see below). The individual's income comprises savings $(s_{t-1})$ from the last period plus any inheritance $(h_{t-1})$ from the last period and his pension $(z)$.

If the individual focuses on the net transfer of resources from him to his children, then:

$$\beta = B - t(1+g)$$

where $t$ is the tax on the beneficiary generation, and, assuming that each individual has the same number of children, $(1+g)$ is the number of heirs. In addition, if the individual knows that the government's budget for social security must be balanced in every period, then, from equation 12:

$$z = t(1+g)$$

On substituting equations 15 and 16 into the second-period budget constraint, the social security variables disappear, indicating that the individual's behavior is not influenced by the introduction of a PAYG system. Moreover, provided members of the younger generation recognize that their inheritance, received at the end of the first period, will be the same regardless of the
social security system, then, even though they pay taxes, their behavior will also be unaffected.

Assumptions 15 and 16 guarantee the Barro result obtained by Sheshinski and Weiss. If, however, either or both of the conditions represented by equations 15 and 16 are not fulfilled, the Barro result no longer holds. For example, Hu assumes that:

\[ \beta = B \]  

(15)

implying that the individual derives utility from the gross level of the bequest rather than from the net intergenerational transfer of resources. For an individual in period two of his life, the budget constraint would be:

\[ c^2 + \beta = (1+r) \left( h_{t-1} - h_{t-1}^+ \right) + t(1+g) \]  

(14)

In this case, the receipt of a pension \([t(1+g)]\) would be perceived as a net increase in wealth, utility being unaffected by the fact that the generation receiving the bequest \((\beta)\) will now be poorer because of their tax payments. Since both \(c^2\) and \(\beta\) are assumed to be normal, the introduction of a PAYG system will increase the level of bequests.

Turning our attention to the young generation, their budget constraint is now:

\[ c_t^1 + c_{t+1}^2/(1+r) + \beta_{t+1}/(1+r) = w_t - t + \beta_t/(1+g) + t(1+g)/(1+r) \]  

(2"

We have already seen that the bequest \((h_t = \beta_t/(1+g))\) to be received by this generation will be larger as a result of PAYG system. It follows that if \(g > r\), lifetime income is unambiguously increased and, again assuming that \(c^2\) and \(\beta\) are normal, saving will be increased. Since in this model, total capital accumulation is:

\[ K_{t+1} = \beta_t L_t^2 + s_t L_t \]
it is clear that the introduction of a PAYG system will affect capital accumulation (at least in a partial equilibrium sense) and, if the population growth rate exceeds the interest rate, that effect will be unambiguously positive. Equation 15, an exclusive focus on gross bequests, is sufficient, therefore, to generate conclusions similar to those of Hu.

This analysis bears on the debate between Barro and Feldstein regarding the effect of social security in a growing economy. Barro had established his original result in the context of a zero-growth economy. Feldstein (1976) contended that the result was unlikely to hold in a growing economy. Our formulation of the problem demonstrates clearly that, regardless of the growth rate of the economy, the Barro focus on net transfers is sufficient to ensure the neutrality of a PAYG system, provided the individual is aware of the government's budget constraint (equation 16), a result also shown by Carmichael (1982).

Sheshinski and Weiss (1981) introduce uncertainty into the Barro model. The length of life, and hence the duration of retirement, is assumed to be uncertain. If $\phi$ is the fraction of the potential retirement period that is actually realized, the second-period budget constraint may be written as:

$$\phi c^2 + B = (1+r)(s+h) + z\phi$$  \hfill (14')

Sheshinski and Weiss assume that the social security system offers annuities so that the budget is balanced when:

$$\bar{\phi}z = (1+g) \tau$$  \hfill (16')

where $\bar{\phi}$ is the expected length of retirement in the population. This implies that even if the individual is aware of the government's budget constraint and focusses on net bequests (equation 15), second-period income is still changed
by an amount equal to \( t(1+\phi)(\phi-\bar{\phi})/\bar{\phi} \). The PAYG system, therefore, modifies private sector behavior. Sheshinski and Weiss, however, do not allow for a private annuities market. Assuming the existence of such a market, an individual in the second (uncertain) period of his life could buy an annuity instead of providing a lump-sum bequest. The individual would then be able to bequeath \( B\phi/\bar{\phi} \) and, accepting equation 16 together with a focus on net bequests, the social security variables disappear from the budget constraint once again.

II.3 **Endogenous Labor Supply**

Labor supply can be made endogenous without affecting any of the earlier results provided the payment of taxes and the receipt of benefits is not linked to the retirement decision, that is, provided the tax and the benefit retain their lump-sum characteristic. This can be verified by including in the utility function, the proportion \( (\phi) \) of the second period that is allocated to retirement, and by adding \((1-\phi)w\) to the second period budget constraint. This will now yield a new first-order condition:

\[
\frac{dv}{da} - \frac{dv}{dc} = w
\]

which equates the marginal rate of substitution between leisure and second-period consumption to the wage rate. Apart from these changes **everything else remains the same**. That is, if equations 11 and 15 hold for a FF system and if equation 15 and 16 hold for a PAYG system, then social security has no effect on capital accumulation or labor supply. And, in the event that these conditions are not fulfilled, social security will influence total lifetime income and hence will affect both capital accumulation and labor supply.

Now let us explore what happens if it is assumed that, instead of
lump-sum taxes and pensions, the individual is taxed and receives benefits at a fixed rate per unit of time. For a FF system, the government's budget will be balanced if:

\[
L_{t+1}^2 Za = L_t^1 t(l+r) + L_{t+1}^2 t(1-\omega)
\]

that is, if total benefits paid for the duration of retirement in period two equal the tax paid in period one plus accrued interest plus the tax paid while working in period two. If the individual is aware of equation 11', then, as before, capital accumulation and labor supply are unaffected. If, however, the individual is unaware of equation 11' and acts as though \( z\alpha = t(l+r) + t(l-\omega) \), then behavior is altered, not only by an income effect, but also by a price effect. To verify this, note that the lifetime budget constraint in this case would be:

\[
c^1 + [c^2 + B]/(1+r) = w - t + h + [(w-t)(1-\omega) + \alpha z]/(1+r)
\]

or  

\[
c^1 + [c^2 + B + \alpha(w-t-z)]/(1+r) = w - t + h + (w-t)/(1+r) \quad (2''')
\]

From this constraint, it is clear that differentiation of the utility function with respect to \( \alpha \) will lead to a first-order condition of the form:

\[
\frac{dv}{c^2} = w - t - z
\]

relating the marginal rate of substitution between second-period consumption and retirement to the net wage. Thus, in this version, if the individual is unaware of the link between tax payments and pension benefits, behavior is affected not only by a change in lifetime income but also by a change in the effective wage rate facing the individual. Moreover, since there is no reason to suppose that the (gross) wage is not equal to labor's marginal revenue product, this result implies a distortion in the labor market in that the
marginal rate of substitution between consumption and leisure is no longer equal to labor's marginal revenue product. Thus, while in general it is impossible to predict the overall effect on savings, bequests, and retirement, it is clear that in this case the introduction of social security distorts the labor market.

A similar point arises with a PAYG system. For example, let us accept the Barro focus on net bequests (i.e., \( \beta = B - t(l+g) \)) but now assume that, as far as the individual is concerned, taxes and pensions are perceived to be independent: the individual responds to a change in taxes without anticipating any associated change in pensions. This is the assumption adopted by Hu. In exploring the general equilibrium consequences of social security, Hu does, of course, introduce the government's budget constraint. The point remains, however, that in the Hu model the individual behaves as though taxes and pensions can be varied independently. Accordingly, for an individual in the second-period of life the budget constraint becomes:

\[
2Wc + 8 + a(w-t-z) = (1+r)(\bar{w}+\bar{h}) + (w-t) - t(l+g) \tag{14''}
\]

indicating both a price effect (\( w \) is now \( w-t-z \)) and an income effect (second period income is decreased by \( t + t(l+g) \)). Again this will imply a labor market distortion and, of course, capital accumulation will be affected. If, however, the individual is aware of the government's budget constraint, that is, if he knows that:

\[
z\alpha L_t^2 = tL_t^1 + t(1-\alpha)L_t^2 \tag{12''}
\]

then, insertion into equation 14'' reveals clearly that the relative price of consumption to retirement is once again given by the gross wage \( (w) \) thereby eliminating the source of the distortion in the labor market.

It appears, therefore, that the individual's awareness of the
overall government budget constraint is of some importance. Even if payroll taxes and social security benefits infringe directly on the retirement decision, provided the individual is aware of the government's budget constraint, individual labor supply decisions will not be affected and there is no reason to suppose that this method—payroll taxation—of raising revenue will distort the labor market. More generally, this point arises in the context of any earmarked tax where the link between a particular payment and the receipt of a particular transfer or service can be easily identified in a separate budget account. Because then, even if the tax payment is related to a particular activity—such as consumption of a given commodity or supply of labor—provided the individual believes that he will ultimately receive a benefit of equivalent value, the imposition of a tax need not be distortionary. Moreover, if this could be verified for any given tax—say, payroll taxation—it would be a strong argument for recommending a more general use of earmarking as a means of raising revenue in a nondistortionary manner.

The behavior implied by the use of equation 11 for a FF system and equation 12 for a PAYG system is, of course, unusual in that it requires an assumption by each individual that all other individuals will behave as he does. For example, consider what happens if a tax at the rate of \( t \) is levied on earnings in the second period. If the individual does not change his behavior—that is, if, as far as this individual is concerned, the tax does not distort labor market behavior—then a certain amount of revenue will be generated. If the individual could guarantee that an equivalent amount of money would be returned to him then, indeed, he would have no motive to change his behavior. This would be true if the government balanced its social security budget by balancing each individual's contributions and receipts. Of
course, the government does not do this: it is interested solely in the overall balance. Accordingly, if everybody else reduces their labor supply, some of the revenue contributed by the individual who maintains his pre-tax labor supply will be transferred to the other members of society to ensure that the rate at which pensions are paid is the same for everybody and to finance the additional time that everybody else is retired. In short, it only makes sense for the individual to maintain his labor supply in the face of an increase in tax rates if he believes that everyone else will behave in the same way. If, on the other hand, he sees a population of free riders, then it would be prudent to reduce labor supply implying the usual distortion for the labor market.

This analysis of the two assumptions—that relating to bequests and that relating to knowledge of the government's budget constraint—yields the following conclusions. First, if the individual is unaware of the government's overall budget constraint, then a social security system that links payment of taxes and receipt of benefits to the length of working life will distort the labor market. It will also reduce lifetime income. This holds for both FF and PAYG although for PAYG the magnitude of the reduction depends on one's assumptions regarding bequests. Second, if the individual is aware of the government's budget constraint, then a FF system has no effect on behavior. In this case, the issue of net or gross benefits is irrelevant, since there are no intergenerational tax payments. Third, if the individual is aware of the government's budget constraint, then a PAYG system will not distort the labor market but it may affect lifetime income. Fourth, if the individual focuses on the net intergenerational transfer, the income effect is zero and PAYG has no effect on behavior. And fifth, regardless of the assumption regarding the individual's awareness of the government's budget
constraint, a focus on gross rather than net intergenerational transfers
implies a change in lifetime income that is positive or negative depending on
whether the population growth rate exceeds the interest rate or not.

II.4 Empirical Evidence on the U.S.

Evidence is accumulating to suggest that the primary rationale of
private saving is the provision of bequests to one's progeny rather than the
reallocation of consumption within one's lifetime. Mirer (1979, p. 435), for
example, finds that the "aged tend to increase their wealth over time" and
Kotlikoff and Summers (1981, p. 729) conclude that, "Intergenerational
transfers appear to be the major element determining wealth accumulation in
the United States". Assuming that bequests are an important component of
total saving behavior, it remains to discover whether individuals focus on the
gross or the net intergenerational transfer of resources and whether they are
aware of the government's budget constraint.

If those in the old age group are concerned about the net transfer
(equation 15) and are aware that the government must balance (approximately)
its budget (equation 16), then any transfer of resources from the young to the
old through the social security system would be returned via an increase in
bequests. It follows that if the accumulated savings of the aged are
regressed on the transfer from the young to the old induced by social
security, the coefficient on the independent variable should approximate
one. In a regression to explain the accumulated wealth of those at retirement
age, Kotlikoff (1979) includes among his independent variables the lifetime
wealth increment (LWI) associated with participation in the social security
system. Kotlikoff (1979, p. 397) notes that "the vast majority of Social
Security beneficiaries up to the present have enjoyed positive lifetime wealth
increments from the system due on the one hand to high real benefit levels and, on the other hand, to their escape from taxation when young. It follows that LWI approximates the transfer of resources from the currently young to the currently old. Kotlikoff, however, finds that the coefficient on LWI is 0.237 and is not significantly different from zero. In a similar exercise, David and Menchik (1981) regress accumulated wealth at death on LWI. In several different formulations the coefficient on LWI is either insignificant or significant and negative. This evidence, therefore, is clearly inconsistent with the Barro hypothesis and suggests that either individuals are primarily concerned with the gross measure of bequests (equation 15') or they are unaware of the method of financing the transfers arising from the social security system.

Empirical evidence concerning labor supply response can also be used to shed light on the choice of assumptions. Recall that the price of second-period consumption relative to leisure changes from $w$ to $w \cdot t \cdot z$ if individuals are unaware of the overall budget constraint for the social security system or if they perceive a population of free-riders who will reduce their labor supply in response to a tax increase. Recent experiments have indicated that, while free-riding behavior is observed, it is not as extensive as one might imagine. For example, Schneider and Pommerehne (1981, p. 702) conclude "that there is only modest evidence for free-riding as compared with the importance attributed to it in the literature. The individuals [in their experimental study] did systematically behave as free riders in line with the different incentives offered by the experiment's three stages, but the extent to which free riding occurred was not great." The behavior implied by equations 11' and 12' may not, therefore, be so improbable.

Nevertheless, direct estimates of labor supply functions suggest
that labor supply behavior is influenced significantly by the introduction of PAYG social security. From equation 17', it is clear that the retirement decision will be a function of $z$. In particular, assuming that the substitution effect dominated, the duration of retirement should be positively related to $z$. On the other hand, if individuals are aware of the social security budget constraint and believe that others will behave as they behave, then the price faced by the individual will once again be $w$ and, assuming individuals focus on net bequests, the retirement decision should be independent of $z$. Alternatively, even if the price faced by the individual is $w$, a focus on gross, rather than net, bequests would result in a positive income effect (see equation 14'). If either of the basic assumptions for neutrality is violated, we should, therefore, be able to identify a positive relationship between the length of retirement and $z$. Both Boskin (1977) and Boskin and Hurd (1978) demonstrate that the probability of retirement is related positively and significantly to the potential benefits ($z$) to be derived from social security. Boskin (1977, p. 131) remarks that "An increase in social security benefits from $3,000 to $4,000 per year per couple raises the annual probability of retirement from 7.5 percent to 16 percent." Similarly, Boskin and Hurd (1978, p. 373) conclude that "a thousand dollar increase in benefits leads to an increase in the probability of retirement of about eight percentage points over the two years of the sample. This is a large increase, almost equal to our estimated probability of 0.115". It appears, therefore, that either individuals attach considerable weight to the importance of free-riding behavior in this particular instance or they focus on the gross intergenerational transfer. It also suggests the possibility that a PAYG social security system will distort the labor market.

On the basis of admittedly inadequate empirical evidence, therefore,
it appears that at least, one, and possibly both of Hu's assumptions may be more appropriate. 1/ That is, individuals focus on the gross intergenerational transfers and perceive no link between the payment of taxes and the receipt of pensions. Accepting these propositions, PAYG social security will affect the economy-wide rate of capital accumulation and will distort the labor market. Recent results by Feldstein (1982) and Leimer and Lesnoy (1982) can be interpreted in this light. The former finds that, between 1929 and 1971, social security reduced personal saving by 44 percent in the U.S., while the latter find no significant relationship between the two. The basic difference in their results can be related to the divergent ways in which they calculate SSW, or the social security wealth variable.

Feldstein attached little importance to the "changing perceptions" of contributors to the system regarding potential benefits that they will receive, assuming, instead, that they expect the ratio of benefits per beneficiary to per capita disposable income to remain primarily the same. This implies, in terms of our analysis, that participants in the U.S. social security system are not necessarily aware of the government's budget constraint, even as the demographic composition changes.

In their calculation of SSW, Leimer and Lesnoy, on the other hand, focus on, and correct for, the changing expectations regarding the benefit: income ratio with decreasing mortality rates, increasing ratios of retirees to dependent beneficiaries among women, and so on. Inasmuch as these expectations reflect the actual benefit structure and revenue potential of the

1/ Blinder, Gordon and Wise (1980), for example, have identified another aspect of social security provisions - the automatic benefit recomputation - which, if fully recognized by participants in the system, casts some doubt on existing econometric results.
system at any point in time, one may interpret their individual as being aware of the government's social security budget constraint. Also since Leimer and Lesnoy's results carry over to the Barro model of net bequests (see their Footnote 16), one can say that their individual's interest lies in leaving a constant net bequest.

It is then quite apparent why Feldstein's result is so different from that of Leimer and Lesnoy. Feldstein's individual is not aware of the government's budget constraint: his social security system, therefore, affects capital accumulation. Leimer and Lesnoy's individual focuses on net bequests and recognizes the government's budget constraint: their pay-as-you-go system, therefore, is neutral in its effects on personal saving. If, as we argued above, Hu's assumptions are right, i.e. individuals focus on gross bequests and perceive no link between social security taxes and benefits, then Feldstein's calculation of SSW and, therefore, his results, are more convincing than those of Leimer and Lesnoy. If, on the other hand, individuals do become increasingly aware of a tax-benefit link, then Leimer and Lesnoy's assumption of "changing perceptions" becomes realistic.

III. **NORMATIVE ISSUES**

So far we have addressed positive issues within the context of intertemporal models of overlapping generations. We have shown that, in certain circumstances, private sector behavior is modified by the social security system. In those models where the social security system is shown to influence private behavior, most authors have usually extended their analysis to address normative issues. That is, they have investigated whether or not a competitive economy can be moved towards the path associated with the Golden Rule of capital accumulation. In one of the most comprehensive models, for
example, Hu (1979) demonstrates that, when allowance is made for general equilibrium effects on the wage rate and the rate of interest, a PAYG system creates a distortion in the labor market - the marginal rate of substitution between consumption and retirement does not equal labor's marginal product - but can ensure a steady-state, long-run equilibrium of the economy that is closer to the path defined by the Golden Rule of capital accumulation. In the model, the individual is assumed to focus on gross bequests and to be unaware of the overall government budget constraint. For a FF system, however, provided the individual expects his pension to exactly equal his tax payments plus accrued interest, all real prices and quantities will remain unchanged, although the distribution of capital between the public and private sectors will be altered. This, of course, parallels Samuelson's result (1975).

The models referred to above define "optimality" by reference to the maximum steady-state value of a representative individual's utility. Social security, however, is usually rationalized on paternalistic rather than individualistic grounds. For example, Musgrave and Musgrave (1980, p. 729) remark that "the compulsory approach [to social security] may be viewed as a paternalistic decision by society to protect the imprudent against starvation in old age." Similarly, Samuelson (1975, p. 543), although basing his own analysis on individual utility maximization, notes:

Many social security systems, like the New Deal U.S. System, may be deemed most valuable precisely because the myopia ignored by the present models does in fact prevail. People live miserably in old age because they do not realize when young what are the consequences of their private saving habits. So by democratic fiat, they paternalistically impose on themselves a within-life pattern of consumption that favors old age at the expense of the young. Precisely because of the myopia that makes paternalism optimal, once citizens are subject to social security taxation and benefits they do not see clearly how they can undo by private-saving offsets what the
mandatory system is doing to them. So, both because of themselves and yet despite themselves, they contrive social security that makes them better off.

Accordingly, in this paper, the relative merits of a FF system and a PAYG system are re-examined in a paternalistic version of the individualistic model presented above. It is shown that, whereas Hu's conclusions with respect to the PAYG system remain unchanged, a FF system can now also be used to influence the long-run, steady-state of the economy even if the individual is assumed to be fully aware of the overall government budget constraint.

III.1. Retirement Exogenous, No Bequests:

In this Section, we derive the relationship between the equilibrium rate of interest and the level of taxation in both PAYG and FF systems, using the simple overlapping generations model introduced in Section II.1. Labor market equilibrium is secured if the demand for labor emerging from equation 5 matches the supply implied by equation 8. Using equations 8 and 9, equilibrium in the labor market requires that:

\[ w(t) = f'(\frac{1-\theta}{\theta k(t-1)}) \]  

(18)

Equations 7 and 18 can now be solved to obtain the equilibrium values of \( w \) and \( r \) given the supply of capital from the previous period.

Pay-as-you-go System:

In PAYG, the government never accumulates investible funds through the social security system as explained in Section II.2. The supply of capital is, therefore, determined solely by private savings. In a two period model, savings will equal the difference between disposable income and consumption in the first period. That is:
where lifetime income is:

\[ \text{y} = w_{t-1} - t_{t-1} + z/(1+r), \]

t is the payroll tax and \( c \) is first period consumption written as a function of the rate of interest, \( r \), and lifetime income, \( y \). The latter is defined as disposable income in the first period plus the discounted value of the pension, \( z \), received in the second period. Eliminating subscripts, if \( s \) is written as:

\[ s = s(w-t, r, z) \]

then \( 1 > s_1 > 0, s_2 > 0, \) and \( s_3 < 0 \). That is, assuming that consumption in each period is a normal good, an increase in the net wage will increase savings less than proportionately. The effect of an increase in the rate of interest, however, is indeterminate. It definitely increases savings by reducing the present value of lifetime income and hence consumption in period one. But it also reduces the relative price \((1/(1+r))\) of second-period consumption. Assuming that the substitution effect dominates, then \( s_2 \) is unambiguously positive. And finally, an increase in the pension level has a positive income effect on period-one consumption and thus reduces savings.

For a long-run, steady state equilibrium, in which \( k \) remains constant, equations 7 (the factor-price frontier), 12 (the government budget constraint), 18 and 19 can be solved for the four endogenous variables, \( w, r, k \) and \( z \), as functions of \( t \). Totally differentiating this system of equations
and expressing in elasticity notation, yields

\[
\frac{d\ln r}{d\ln t} = \frac{\xi (\epsilon_1 t - \epsilon_3 w)}{\eta r - \epsilon_1 r + \xi_2 w} \\
(20)
\]

where \( \epsilon_1 = s_1 w/s > 0 \)
\( \epsilon_2 = s_2 r/s \geq 0 \)
\( \epsilon_3 = s_3 z/s < 0 \)

and \( \eta = -w\theta/\phi'(1-\Theta) > 0 \) and is the elasticity of labor demand.

Equation 20 depicts the change in \( r \) as \( t \) is changed and the economy shifts to a new long-run, steady-state equilibrium. (It says nothing about the process of transition between states). The numerator of equation 20 is unambiguously positive. Although the denominator appears indeterminate, Hu (1979, p. 280) argues that it must be assumed positive in consideration of the stability of the long-run equilibrium, in which case changes in \( r \) and \( t \) are positively related.

Hu argues that PAYG can move the steady-state equilibrium of the economy closer to the Golden Path where the return to capital equals the population growth rate. If payroll taxation and the rate of interest are related positively, however, this can only occur along a curve such as \( AA' \) in Figure 1 which requires that in the absence of social security the economy's rate of capital accumulation must exceed that associated with the Golden Path and the rate of return to capital must lie below the population growth rate. If, on the other hand, the rate of return exceeds the population growth rate in the absence of social security, then PAYG will tend to move the economy further away from the Golden Path on a curve such as \( AA' \) in Figure 1.
In FF, the pension received by a participant equals exactly the future value of his tax payments compounded at the market rate of interest, the relationship between the social security tax and pension being given by equation 11. Moreover, the total supply of capital by the older generation is now equal to the sum of their savings (private investment) and their tax payments (public investment), i.e. $k_t = s_{t-1} + t_{t-1}$. Substituting equation 11 into the definition of lifetime income, and ignoring subscripts, $k$, the supply of capital per member of the older generation, may be expressed as:

$$k = w - c(r,w) \quad (9'')$$

If, as before, we solve for $k$, $r$, and $w$ in a steady-state equilibrium, it is clear from an examination of equations 7, 18 and $9''$ that the resulting solutions are independent of the level of payroll taxation. As Samuelson (1975, p. 541) notes, "Any increase in 'fully-funded' social security merely displaces as much private capital as the public capital it
brings into being", the representative individual being indifferent between the two forms of ownership as long as each yields exactly the same return. It follows that the relationship between the steady-state, equilibrium rate of interest and the level of payroll taxation can be represented by horizontal lines such as AB and A'B' in Figure 1. It may be concluded that a fully-funded social security system has no capability whatsoever of moving the economy towards the steady-state associated with the Golden Rule.

This result holds as long as the level of taxation is constrained to be less than, or equal to, the individual's preferred level of savings had there been no social security. If the optimal social security system is defined by means of individual utility maximization, then, clearly, the optimal payroll tax will never be large enough to force private savings to zero because this would imply a (private) distortion between the interest rate and the marginal rate of substitution between consumption in period one and consumption in period two. Maximization of a paternalistic welfare function, however, is perfectly consistent with zero private savings. In particular, assume that society considers the privately chosen level of consumption in the second period to be inadequate. In this case, maximization of a paternalistic welfare function would yield a level of taxation in excess of the individualistically determined rate of capital accumulation. Private savings would, therefore, be zero and k would equal t. This information is sufficient to allow the solution of equations 7 and 18 for the equilibrium interest rate and wage rate as functions of t, the level of payroll taxation. Moreover, totally differentiating the resulting solution yields:

\[ d\pi = - \frac{n}{\lambda} dt < 0 \]  

(21)
where \( \pi \) is the ratio of labor income to capital income.

Unlike equation 2\(^{2}\), which depicted the relationship between changes in \( r \) and changes in \( t \) for PAYG, equation 21, which depicts the same relationship for FF, is unambiguously negative. It follows that if private savings are insufficient to guarantee a socially acceptable level of consumption during old age, a paternalistically motivated, fully-funded system can move the economy towards its preferred steady-state equilibrium. Moreover, if society selects the consumption path associated with the Golden Rule, it can be obtained by setting payroll taxation at the appropriate level. Returning to Figure 1, if capital accumulation is below the rate associated with the Golden Rule, and if \( B \) is the point at which private savings become zero, then increasing \( t \) beyond \( B \) will move the steady-state economy along a path such as BC towards the Golden Rule equilibrium. Alternatively, if the economy is accumulating too much capital, then a FF system would move the economy even further away from the Golden Rule equilibrium along a path such as \( BC' \).

This analysis is open to the same type of criticism leveled against the Sheshinski and Weiss model (see section II.2). Optimality in their model rests on two basic assumptions - an uncertain lifetime and the absence of a private annuities market. In Barro, where there is no uncertainty, any increase in the social security tax is simply offset by decreasing private bequests. In such a model, "the optimal level of social security is clearly undetermined" (Sheshinski and Weiss, p. 190). In addition to making lifetime uncertain, however, Sheshinski and Weiss also assume that "all annuities are supplied by the public sector through a social security system" (p. 192). Obviously, if a private market for annuities was incorporated in their model, individuals would be able to offset an increase in social security taxes by
reducing their participation in the private annuities market and exactly the same results, as in Barro, would follow without any implication for the optimal size of social security. The discussion of an "optimal" social security system by Sheshinski and Weiss, therefore, rests heavily on the assumption of exclusive government provision of annuities. Inasmuch as a private annuities market does exist, their "optimality" results must be qualified. In our model of a paternalistically motivated FF system, each individual is taxed by the social security institution to the point where his private saving is driven to zero. Then, were a private annuities market available to him, it could be argued that he would turn to it for loanable funds, and borrow against his future social security benefits and/or his future labor income. In this event, the concept of optimality would once again be undefined.

There is considerable doubt, however, whether individuals can actually borrow against such future income. As Evans (1982) points out, "capital markets are such that it is not possible, or only minimally possible, to borrow against future labor income to finance current consumption." Similarly, it is equally questionable when "we simply do not know how (even) individuals perceive their social wealth" (Leimer and Lesnoy, p. 619), to expect the private capital market to lend against future social security benefits. Thus, in our treatment of the paternalistic case of FF, it is not unrealistic to assume away the possibility of individuals turning to the private annuities market for loanable funds when their private savings has been driven to zero by the social security institution. Similarly, it would be possible to construct a paternalistic version of the Sheshinski and Weiss PAYG system in which the individual is obliged by the social security authorities to hold more annuities than he would have purchased in the private
market. In this case, the existence of a private annuities market is not sufficient to ensure that individuals can offset the effects of social security. It is also necessary to assume (unrealistically) that individuals can borrow against anticipated social security benefits.

III.2 Long-Run Welfare Implications of Endogenous Retirement and Bequests

The paternalistic approach developed above can be extended to incorporate retirement and a bequest motive. In this section, we explore the optimal steady-state solutions for these more general models within a paternalistic framework. We start, therefore, with a social welfare function of the form:

\[ W = \omega(c^1) + \nu(c^2, \alpha, \beta) \]  

(32)

where, within the relevant range, \( \frac{\partial \omega}{\partial c^1} < \frac{\partial \omega}{\partial c^2} \) implying that, compared with individual preferences, society attaches more importance to future consumption relative to present consumption.

**Fully-Funded System**

Totally differentiating equation 27 with respect to a change in payroll taxation and assuming that the individual is aware of the government’s budget constraint yields:

\[ \frac{dW}{dt} = \frac{\partial \omega}{\partial c^1} \frac{dc^1}{dt} + \frac{1}{(1+r)} \left( \frac{dc^2}{dt} + \frac{w}{\partial c^2} \frac{dc^2}{dt} \right) \]  

(23)

where \( (1+r) = \frac{\partial \omega}{\partial c^2} \) and reflects society’s preference for current relative to future consumption. Assuming that first-period savings have been driven to zero by social security, the second period income constraint is:
\[ c^2 + \beta + \omega w = w + (1+r)(h+t) \]  

(24)

where \( h = \beta/(1+g) \) is his bequest from the previous period. Since savings are zero, we know that \( k = h+t \) and that \( c^1 = w-t \). Finally, noting that from equation 7 we have:

\[
\frac{dr}{dt} = -\left( \frac{1+g + 1-\alpha}{k} \right) \frac{dw}{dt}
\]

we can derive the welfare maximizing level of \( t \) by setting \( dW/dt = 0 \):

\[
\frac{d\omega}{dt} (r-g) + (r-t) = -\frac{d\beta}{dt} \frac{(1+r)}{(1+r)(1+g)}
\]

(25)

Hence, it is clear that a paternalistic FF system will yield its maximum benefit by moving the economy to the steady-state path defined by the Golden Rule, only in the absence of a bequest motive. On this path, both society's marginal rate of substitution between present and future consumption and the marginal return to capital will equal the population growth rate.

In this more complicated model, it is no longer possible to derive a simple, unambiguous relationship between the rate of interest and the rate of taxation (See Appendix). In general, however, if the private individual's labor supply and bequests are relatively unresponsive to changes in the wage, the interest rate, and taxation, then the likelihood of a negative relationship between the rate of interest and taxation is increased. In the extreme, when these response coefficients are zero, the relationship is that given by equation 21.
It is unlikely that a social security system would be implemented in such a way that all private savings are driven to zero. Nevertheless, it is highly likely that, if the objective is to prevent the consumption of a particular group in society from falling below socially preferred levels during retirement, a social security system would be implemented such that the savings of the target group were driven to zero. Provided the system is financed on a fully-funded basis, total savings—the sum of private savings and payroll taxation—by the non-target group would, of course, remain unchanged. This approach would thus achieve the objective of ensuring minimum consumption standards for everyone during old age, without affecting the savings behavior of the non-target group.

**Pay-as-you-go System**

Lastly, we demonstrate the consequences of our paternalistic assumption for Hu's model of a PAYG system. Corresponding to equation 23 we have:

\[
\frac{dw}{dt} = \frac{\partial \omega}{\partial c} \left[ \frac{dc}{dt} + \frac{1}{1+r} \left( \frac{dc}{dt} + (w-t-z) \frac{da}{dt} + \frac{d\delta}{dt} \right) \right]
\]

where the price of retirement—\(w\)—in FF is replaced by the corresponding price—\(w-t-z\)—for PAYG. Recall that this arises because in the Hu model, the individual is unaware of the government's budget constraint (see section II.3). Making use of the budget constraints for lifetime income and for second period income (see equation 2° − 7) and using the relationship between the equilibrium interest rate and wage rate (equation 7), we can derive the optimal value of \(t\) by setting equation 26 equal to zero. This yields:

\[
\frac{dw}{dt} \left( r - \beta \right) + \frac{dk}{dt} \left( r_0 - \bar{r} \right) = - \frac{d\delta}{dt} \frac{1+r}{1+g} + (1-\bar{r}) + (1-\beta) - \alpha \frac{dz}{dt}
\]
Unlike equation 25, equation 27 only indicates the optimality of the Golden Rule if both the bequest motive and the retirement decision are absent. The former, of course, implies \( \frac{d\beta}{dt} = 0 \). The latter may be seen by examining \( \frac{dz}{dt} \). In PAYG with retirement endogenous, the government's budget is balanced when:

\[
z^\omega = (1 - \theta) t
\]

Although in the model the individual behaves as though he is unaware of this constraint, the model's equilibrium does, of course, impose this constraint. It follows that:

\[
\frac{dz}{dt} = \left( \frac{1 - \theta}{\theta} \right) - \left( \frac{t + z}{\theta} \right) \frac{d\alpha}{dt}
\]

If labor supply is fixed, \( \frac{d\alpha}{dt} = 0 \). Recalling that \( \theta = 1/(2+g) \), with no bequest motive and retirement exogenous, equation 27 collapses to:

\[
\frac{dw}{dt} (r - g) + \frac{dk}{dt} (r - \bar{r}) = 0 \tag{28}
\]

Thus, in the absence of a bequest motive, PAYG would indicate the optimality of the Golden Rule only if labor supply is fixed. This result for the paternalistic model parallels that obtained by Hu (1979) in his individualistic model. It differs from the result for the FF system because of the difference in assumption regarding the individual's awareness of the government's budget constraint.

As with the FF case, the relationship between the rate of interest and the rate of payroll taxation is ambiguous [see equations 33, 34 and 35 of Hu (1979)]. Nevertheless, the basic point remains clear: in choosing between
a (paternalistic) FF system and a PAYG system, one needs to know where the economy is in relation to the Golden Path and whether or not the introduction of a social security system modifies the Golden Path. Consider the following possibility. Omitting the bequest motive, assume that, in the absence of any form of social security, the rate of interest is observed to lie above the rate of population growth. That is, the rate of accumulation in the economy is less than that associated with the Golden Path. Assume further that the government investigates the relationship between the rate of interest and the rate of payroll taxation in a PAYG system and finds that it is negative. Is this sufficient information on which to base a recommendation to institute a PAYG system? The answer is no, because the introduction of a PAYG system may itself modify the Golden Path. If the behavior of individuals is such that the PAYG system causes a distortion in the labor market, the optimum long-run equilibrium for the economy is no longer characterized by the equality of the rates of interest and population growth (see equation 27). In fact, the PAYG system may move the economy towards the (now irrelevant) Golden Path and away from the modified Golden Path. Thus, where social security is presumed to influence total capital accumulation, we conclude that an investigation of optimality requires information on the relationship between the rate of interest and the rate of population growth and on the relationship between the modified Golden Path and original Golden Path.

Conclusion

This paper has analyzed the major results in the social security literature of recent years, clarifying the underlying assumptions responsible for their diversity. It has extended the analysis of both "positive" issues - the effects of social security on capital accumulation and labor supply - and
normative issues—whether social security can move the economy to its optimal steady-state growth path. In the process, the paper introduced and analyzed a paternalistic motivation for social security. In the analysis of both positive and normative issues, the results are shown to depend on whether the system is fully funded (FF) or financed on a pay-as-you-go (PAYG) basis.

With respect to the positive issues, the effect of social security on capital accumulation is neutral if one common condition is fulfilled for both FF and PAYG systems and if one other system-specific condition is met. The common condition is that the representative individual recognize the budget constraint faced by the social security system. For a FF system, the second condition is that the rates of return in the private and public sectors be the same, and for a PAYG system, it is that the representative individual focus on the net, rather than the gross, value of bequests. Provided these conditions hold, neutrality is assured whether or not labor supply is endogenous. For each system, if either or both of the conditions are not fulfilled, capital accumulation will be affected.

When labor supply is made endogenous, the importance of the common condition—that the individual recognize the social security’s budget constraint—is increased. If this condition is not fulfilled when labor supply is endogenous then, not only is capital accumulation affected, but also the presence of a social security system introduces a distortion into the labor market. Moreover, with labor supply endogenous, the likelihood that the common condition be fulfilled is reduced since the fulfilment of that condition now requires an expectation on the part of the representative individual that the labor supply behavior of all other individuals matches his own.
Our focus on both capital accumulation and the labor market allows us to draw on empirical evidence for savings behavior and labor supply behavior in the U.S. to shed light on the two assumptions required for neutrality of a PAYG system. Although scant, both sources of empirical evidence are consistent in rejecting the joint fulfilment of the two critical assumptions. The results do not therefore, support the neutrality hypothesis of Barro but suggest as in the Hu model the possibility of both a non-neutral effect on capital accumulation and the creation of a labor market distortion.

With respect to normative issues, the paper contrasted the accepted approach of defining optimality by reference to individual utility maximization with a paternalistic definition of optimality. In particular, the paper explored Samuelson's observation that society may adopt a social security system that forces individuals to "save" more than they otherwise would thereby ensuring that the myopic vision of the individual does not lead to socially unacceptable levels of consumption when old. It is shown that a paternalistically-motivated FF system that drives private savings to zero can influence capital accumulation even if the two critical conditions for neutrality are fulfilled. Paternalism of this kind, however, does not affect a PAYG social security. Since all social security transfers are intergenerational rather than intertemporal in this case, it follows that a PAYG system cannot be used to offset individual myopia. For a PAYG system, paternalism would have to involve some attempt to limit the amount individuals bequeath, a within-period decision that is not affected by myopia. Since this may prove administratively difficult within the context of a social security system, we conclude that, unlike a FF system, a PAYG system will only influence capital accumulation if one or both of the basic neutrality conditions are violated.
We also show that the effect on capital accumulation is system-specific. For example, in a simple model that excludes bequests and labor supply decisions, a paternalistic FF system will move an economy closer to the steady-state path associated with the Golden Rule only if the rate of capital accumulation is below that characterizing the Golden Path. A PAYG system, on the other hand, will move the economy towards the Golden Path only when the rate of capital accumulation on the existing path is above that associated with the Golden Rule. Thus, in choosing between systems, it is important to take account of their effects on capital accumulation and of the economy's present location relative to the Golden Path.

Finally, in more complicated models in which both bequests and labor supply are endogenous, the introduction of a social security system implies that the optimal steady-state path is no longer that described by the Golden Rule. The optimum will depart from the Golden Path both because bequests provide a return—an increase in the utility of the donor in excess of the rate of interest—and because of the possibility of static welfare losses in the labor market. Unlike the former cause—the bequest motive—of a departure from the Golden Rule, the latter cause—a labor market distortion—is a direct consequence of the social security system. As a result, the introduction or modification of a social security system may move the economy towards the now-irrelevant Golden Path (appropriately modified to allow for bequests) and away from the new steady-state optimum (which must also accommodate the static welfare loss in the labor market). This raises at least a theoretical possibility that, even though a social security system may influence capital accumulation, the optimal steady-state path with positive taxes and benefits may be less preferable than the existing path on which taxes and benefits are zero.
Appendix to Section III.2

Fully-Funded System

Since the individual's savings in period one are zero, \( c^1 = w - t \). In addition, since the individual is fully employed during the first period of his life, his period-one labor supply is also uniquely determined. The individual retains choice, however, over the distribution of his income among second-period consumption, retirement, and his bequest. Thus, the individual in period 2 is assumed to:

\[
\max v(c^2, \alpha, \beta)
\]

subject to \( c^2 + \beta + \omega \alpha = w + (1+r)(h+t) \)

where we have used the government's budget constraint - \( z\alpha = t(l+r) + t(l-\phi) \) - to eliminate the pension \( (z\phi) \) and second-period taxes \( (t(1-\phi)) \).

The first order conditions can be solved for \( c^2, \alpha \) and \( \beta \). In particular, since \( h + t \) is the available capital stock \( (k) \), we have:

\[
\alpha = \alpha(w, r, k)
\]

\[
\alpha_1 > 0, \quad \alpha_2 > 0, \quad \alpha_3 > 0
\]

and \( \beta = \beta(w, r, k) \)

\[
\beta_1 > 0, \quad \beta_2 > 0, \quad \beta_3 > 0
\]

Provided the negative effect of a higher price for retirement dominates the positive effect of a larger income, an increase in the wage will reduce the length of retirement. Both an increase in \( r \) and \( k \), however, will increase retirement as a result of positive income effects. Similarly, bequests can be assumed to increase as a result of increases in \( w, r, \) and \( k \) because of positive income effects.
Total labor supply is now given by the sum of \((1-\theta)\) from period-one individuals and \(\theta(1-\alpha)\) from period-two individuals. To ensure equilibrium in the labor market, therefore, we require:

\[
w = f\left(\frac{L}{K}\right) = f\left(\frac{1-\theta \alpha}{\theta \alpha}\right)
\]

We can now derive the relationship between \(t\) and \(r\). Using the condition for labor market equilibrium, the factor price frontier (equation 7 in the text), the fact that \(k = t + \beta\), and the behavioral functions for \(\alpha\) and \(\beta\), we have five equations to solve for the five unknowns - \(w, r, k, \alpha\) and \(\beta\). Taking the total differential of this system and expressing in elasticity notation, we obtain:

\[
d\ln r = \frac{\pi (1-\alpha \omega)(1-m)}{m(1-\alpha \omega)(\zeta_w - \zeta_r) - (1 - \zeta_k m)(\eta \alpha - \sigma_r)} \, d\ln t
\]

where

- \(\Pi = w L / r K\)
- \(m = \beta / k < 1\) - private ownership of capital
- \(\sigma_w = -\theta \alpha w / (1-\theta \omega) > 0\) - partial elasticities of labor
- \(\sigma_r = -\theta \alpha r / (1-\theta \omega) < 0\) - supply
- \(\sigma_k = -\theta \alpha k / (1-\theta \omega) < 0\)

- \(\zeta_w = \beta_1 w / \beta > 0\) - partial elasticities of supply of bequests
- \(\zeta_r = \beta_2 r / \beta > 0\)
- \(\zeta_k = \beta_3 k / \beta > 0\)

and \(\eta = -w k \theta / f''(1-\theta \omega) > 0\) - partial elasticity of labor demand.
The relationship between \( r \) and \( t \) is indeterminate. Relatively simple conditions will, however, ensure a negative relationship. For example, if
\[
\zeta_m < 1 \quad \text{and} \quad \zeta_w - \zeta_r^m < 0,
\]
r and \( t \) are negatively related. If labor supply is fixed and there are no bequests, the relationship collapses to that reported as equation 21 in the text.

**Pay-as-you-go System**

For a PAYG system in which the individual focusses on gross bequests and is unaware of the government's budget constraint, the relationship between \( r \) and \( t \) can be derived immediately from Hu's results since the individualistic and paternalistic models coincide. The three critical behavioral functions in this model are those for bequests, savings and labor supply. Recall also that because the individual is unaware of the government's budget constraint, the social security variables affect private behavior. Accordingly, we introduce the following definitions. In addition to the partial elasticities of labor supply defined above, we also have:

\[
\sigma_t = \Theta \alpha_1/(1-\Theta \omega) < 0
\]
and
\[
\sigma_z = -\Theta \alpha_4/(1-\Theta \omega) < 0
\]
where \( \alpha_4 = \partial \omega/\partial z \) (see Hu (1979) p. 277). The partial elasticities for the supply of capital (i.e., savings and bequests) are given by:

\[
\zeta_q = (s_1 + \beta_1) w/k
\]
\[
\zeta_r = (s_2 + \beta_2) r/k
\]
\[
\zeta_k = \beta_3 (s_3 + 1)
\]
\[
\zeta_t = -(s_1 + \beta_1) t/k
\]
and
\[
\zeta_z = (s_4 + \beta_4) z/k
\]
where \( s_4 \) and \( \beta_4 \) are \( \partial s/\partial z \) and \( \partial \beta/\partial z \) respectively (see Hu (1979) p. 280).
With these definitions, and using equations 34 and 35 in Hu (1979) p. 280, and the equilibrium relationship between \( w \) and \( r \), we obtain:

\[
\frac{d\ln r}{d\ln t} = \frac{\Pi[a_1(\zeta_w - \zeta_f) + a_2(\sigma_w - \sigma_f) + a_3(\sigma_f - \sigma_w, \zeta_f)]}{\Pi[a_1(\zeta_f - \zeta_t) - a_2(\sigma_f - \sigma_t) + a_3(\sigma_f - \sigma_t, \zeta_f)]}
\]

where \( a_1 = t(1-\sigma_k) \), \( a_2 = t(1-\zeta_k) \), and \( a_3 = (t+\zeta) \).

Although complicated, this expression can be analyzed by exploring the consequences of making labor supply and bequests exogenous. If labor supply is fixed, then \( q_i = 0 \) for all \( i \). And if bequests are zero, the \( \zeta \) elasticities collapse to the \( \epsilon \) elasticities of the text, and the relationship between \( r \) and \( t \) is captured by equation 20 of the text. Thus, as the responsiveness of labor supply and bequests becomes less, the likelihood that \( r \) and \( t \) are positively related increases.
References


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