Concession Contract Renegotiations

Some Efficiency versus Equity Dilemmas

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If having firm-driven renegotiations of contracts for infrastructure services is a major concern, efficiency should not be the only consideration in selecting an operator. Indeed, consumers may want to award the concession to a less efficient firm if that would reduce the probability of renegotiation.
Summary findings

Estache and Quesada analyze the possibility of tradeoffs between efficiency and equity as well as the possibility of distributional conflicts in the context of renegotiation of infrastructure contracts in developing countries.

To do so, they present a model in which contracts are awarded by auctioning the right to operate an infrastructure service to a private monopoly, and consider the possibility of renegotiation. To identify the potential sources of tradeoffs, they track the possible outcomes of different renegotiation strategies for the monopoly running the concession and for the two groups of consumers—rich and poor—who alternate in power according to a majority voting rule.

Among the model's most important policy implications is this: if having firm-driven renegotiations is a major concern, efficiency should not be the only consideration in selecting an operator. Indeed, consumers may want to award the concession to a less efficient firm if that would reduce the probability of renegotiation, since a lower probability of firm-driven renegotiations (due to demand shocks, for example) is associated with higher welfare for all service users.
Concession Contract Renegotiations:
Some Efficiency versus Equity Dilemmas*

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This paper was started while the second author was visiting at the World Bank during the summer of 2000 and is an input in a series of courses organized for developing countries policymakers. We are indebted to Luis Guasch, Jean-Jacques Laffont, and Mathias Dewatripont for useful discussions, comments, and suggestions

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1. Introduction

During the 1990s, investment in utility projects associated with some form of privatization added up to about US$470 billion in developing countries. This figure reflects investment commitments made through almost 1500 contracts between governments and private operators in the context of major infrastructure restructuring programs. The contracts, most of them concession contracts, were generally awarded through auctions to make the most of competition for the market ex ante and minimize the need for discretionary regulatory decisions ex post (Crampes and Estache (1998)). An ongoing analysis of a sample of 1000 utilities and transport contracts signed during the 1990s (Guasch (2001)) suggests that renegotiation happens in around 50% of the cases. Moreover, it suggests that the odds of renegotiations are the highest when the auction criteria is driven by the desire to minimize the average tariff to be paid by users of the services bided out. More generally, the difficulty is often related to a poor initial effort to assess the sources of demand fluctuations, often related to a limited ability to pay of a good share of the population.

These stylized facts define the policy issues this paper is concerned with. If awarding the contract to the bidder promising to cut the most the average tariff leads to renegotiations, it seems fair to be concerned with the possibility that renegotiation will end up increasing tariffs and penalizing the poorest. The main purpose of this paper is to check for some of the conditions under which there is an efficiency-equity trade-off in the context of contract renegotiation and to identify the winners and the losers of various policy options in a changing political environment.

Section 2 highlights the main results of the literature on contract incompleteness and renegotiation and shows that the equity dimension has essentially been left out so far from the discussion. Section 3 presents a model of contract award through an auction. In view of the experience of the last decade, the possibility of renegotiation is explicitly recognized as part of the initial contract preparation. Renegotiation can be driven either by the firm, in which case it will happen if the firm's profits are negative, or by the government, in which case it will occur if a new group is in power at the renegotiation stage. To also take into account the possible influence of distributional issues in the context of renegotiation, we assume that the economy is composed by two groups of consumers, rich and poor, who alternate in power according to a majority-voting rule. In Section 4 we solve the model
and we obtain the characteristics of the winning firm and those of the renegotiated price. Section 5 analyzes the results in terms of efficiency and equity trade-offs. Section 6 concludes.

2. Renegotiation and contract incompleteness

Renegotiation and contract incompleteness are quite related in the literature. The single possibility of renegotiation implies some incompleteness, because the parties are not able to commit not to renegotiate. But even this kind of incompleteness is not enough to explain why renegotiation occurs in the real world. Indeed, the renegotiation-proof principle states that any outcome that a principal can get can be achieved through a contract that leaves no room for renegotiation.

Unfortunately, there is no unique framework under which contract incompleteness can be analyzed. Tirole (1999) summarizes the main reasons that could generate incomplete contracts. He mentions basically three reasons. First, some contingencies that may arise during the execution of the contract cannot be foreseen when the contract is signed. Second, even if the parties could anticipate all the possible contingencies that should be included in the contract, they might be so numerous that the costs of describing them all can be prohibitive. In that case, the parties have to trade off the benefit of having a more comprehensive contract with the cost of writing new clauses. Finally, the contract can only be contingent on variables that could be verified by a third party, usually an arbitration commission or a Regulatory Agency in the context of infrastructure “privatization” contracts. If this were not the case, the contract would not be enforceable.

A seminal paper on renegotiation and incomplete information is Hart and Moore (1988). In their model they assume that actions and future contingencies are all ex post observable, but they are not verifiable by a third party. They show that in that case the parties cannot achieve an ex ante optimal level of investment and renegotiation is used to achieve ex post efficiency.

On the other hand, Aghion et al. (1994) argues that the assumption of unverifiability is not enough to explain under-investment. Very often, this problem can be overcome with an appropriate ex ante design of the renegotiation process itself. In their model, playing with the default option and the bargaining power at the renegotiation stage restores efficiency even when actions are not verifiable.
Closer to the model presented here are the papers of Jeon and Laffont (1999) and Kartacheva and Quesada (2000). Jeon and Laffont (1999) modeled a government regulating a firm under some uncertainty conditions on the demand faced by the firm and its fixed cost. They assume that both the government and the firm are myopic and are not able to anticipate future renegotiations. Ex post, if the profit of the firm is negative, the regulatory contract is renegotiated. In this context, renegotiation occurs more often when the firm is inefficient, which is not always the case in our model.

Finally, Kartacheva and Quesada (2000) assume a government auctioning the concession of some public utility. Firms anticipate that renegotiation will occur if demand is low and take this fact into account at the auction stage. Firms know that if profits are negative the government will prefer to renegotiate rather than to stop production and they will face no competition at that point. As a consequence, the announced bid will be biased downward given that a firm expects renegotiation if it makes losses. Furthermore, different firms can have different degrees of bargaining power at the renegotiation stage. Both effects give incentives for firms to announce a bid lower than the one that would have been announced without the possibility of renegotiation. This implies also that the ex post probability of renegotiation is higher.

This paper differs from the previous ones in that we allow here to have renegotiation both driven by the firm and by the government. Of course, the outcome of renegotiation will be different in one case or in the other. Moreover, we allow in our model to have two different types of consumers and, therefore, we are able to analyze how renegotiation affects differently each of these groups.

3. The model

The government wants to concession a public utility. To choose the firm to operate the concession, it runs an auction and chooses the firm that bids the lowest price. The chosen firm will produce for two periods. We are not looking for the optimal auction from the government's perspective and consider the mechanism as given. Indeed, we will assume that both the government and the firm are naïve in game-theoretical sense, meaning that future stages are not taken into account when a decision or action is undertaken. In this particular case, it means that the government does not consider or does not care about the possibility of renegotiation when designing the auction and that neither do the firms when deciding on the bid. This limited rationality on the agents' behavior
is what allows renegotiation to happen in equilibrium. Indeed, without such naïve agents, renegotiation would not be an issue. This behavior may be explained in several ways. It may be the case that the agents overestimate their commitment power. More realistically, from the viewpoint of the government, what may matter more is that the political cycles are much shorter than the typical concession contract duration and more often than not, the political turnover is such that the “privatizing team” of a government is no longer around when renegotiation takes place and hence has little incentive to be too concerned about it. From the viewpoint of the operator, being naïve may simply reflect overconfidence in their ability to get anything they want from a weak government or regulator and is hence perceived as a no risk attitude towards the market.

On the supply side, each firm $i$ is characterized by two parameters. The first parameter is its marginal cost of production $\theta_i \in [\tilde{\theta}, \bar{\theta}]$ and is firm $i$’s private information. The second parameter is the bargaining power the firm would have whenever renegotiation occurs, $\alpha_i \in [0,1]$ and is ex ante unknown to anyone but would become common knowledge at the renegotiation stage. To simplify, we assume that all $\theta_i$ are independent and identically distributed with a cumulative distribution function $F(\theta)$ and probability density function $f(\theta)$. The profit of firm $i$ is $\pi_i = (p - \theta_i) q(p) - I$, where $p$ is the price of the public utility, $q(p)$ is the demand function and $I$ is a fixed cost.

We consider an increasing return to scale production technology with a constant marginal cost. The assumption of a constant marginal cost is made just to simplify computations and all our results will still be valid with a more general cost function of the form $C(q(p), \theta)$ with $\frac{\partial C}{\partial \theta} < 0$. At the auction stage, the only difference across firms is the marginal cost. Therefore, a firm with a low value of $\theta$ (a low marginal cost) is unambiguously more efficient than another firm with a larger $\theta$. On the other hand, the bargaining power, $\alpha_i$, is statistically independent of the marginal cost of production, $\theta_i$, and is independently drawn from a cumulative distribution function $G(\alpha)$ with probability density function $g(\alpha)$. Alternatively, one could assume that each firm knows its own parameter $\alpha_i$, but this would complicate the equilibrium of the auction procedure, without adding much to the analysis.

On the demand side, we assume that there are two groups of consumers. The first group (indexed by 1), the rich consumers, appropriates all the rents of the firm, whereas the second group (indexed by 2), the poor consumers, does not share the profits of the firm. The proportion of rich consumers is $\beta \in [0,1]$. 

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Both groups of consumers alternate in government depending on whether $\beta^*$ is larger or smaller than 1/2. That is, if $\beta^* > 1/2$ then rich consumers are a majority and they are in power. On the other hand, if $\beta^* < 1/2$, poor consumers are majority. We will call $S(q)$ the gross consumer utility and $u(q) = S(q) - pq$ the net utility. The demand function is then determined by utility maximization, which implies that, if $p$ is the price of the good, $q(p)$ is such that $S'(q(p)) = p$.

The utility function is stochastic in such a way that $q(p) = \bar{q}(p) + \tilde{\epsilon}$, where $\tilde{\epsilon} \sim U[-\epsilon, \epsilon]$ is a demand shock and $\bar{q}(p)$ is the mean demand function. We also assume that once the demand function is realized it remains the same in both periods. That is, at the end of the first period, when renegotiation can happen, the demand for the second period is perfectly known.

A rich majority will want to maximize its own surplus, $W_1$. They receive a proportion $\beta^*$ of the total net utility and all the firm’s profits, therefore $W_1 = \beta^*[S(q(p)) - pq(p)] + \pi = \beta^*S(q(p)) + (1 - \beta^*)pq(p) - \theta q(p) - I$. On the other hand, a poor majority will maximize $W_2 = (1 - \beta^*)[S(q(p)) - pq(p)]$, because they get only a proportion of the total net utility equal to $(1 - \beta^*)$. Any group has to take into account the constraint that the profits of the firm are non-negative for the firm to be willing to produce.

Finally, we assume that after production, the profits of the firm are perfectly observable by the government and, therefore, renegotiation occurs under complete information. Both the profit and the demand are observed by the government and therefore the marginal cost, $\theta$, can be inferred.

The timing of the game is as follows:

1. Each firm $i$ learns its private information, $\theta_i$.

2. A first-price sealed-bid auction is run and each firm chooses a price to sell the good, $p_i$. The firm with the lowest bid wins the auction and is selected to produce the good for 2 periods.

3. The actual demand is realized and is observed by the government and the firm.

4. The first period of production ends and the firm and the government observe the first period profit of the firm.

5. If the profit is negative the firm asks the government to renegotiate the contract. If there is a new majority and the price that can be obtained when
renegotiating is smaller than the actual price, the government asks for renegotiation. We assume that at this stage both the government and the firm are locked in the relationship. We model renegotiation as a Nash bargaining game with bargaining powers $\alpha_i$ for the firm and $1 - \alpha_i$ for the government.

6. If an agreement is reached, the firm runs the concession in the second period with a new price, $p^*$. Both the government and the firm will get 0 if there is no agreement.

7. If there is no renegotiation, the original contract prevails.

Note that we assume that the firm only asks for renegotiations when profits are negative. The reason is the following. Suppose that the firm asks for renegotiations when it is making positive profits because it expects to increase the price. If the government rejects the offer, the firm can either leave the market and get 0 (its outside option) or stay and get some positive profit. So the government knows that the firm will always stay and, therefore, will reject any renegotiation that increases the price.

On the other hand, if the majority does not change in the second period, there is no renegotiation driven by the government. So we are assuming that there is some limited commitment on the government side. A given group can commit not to renegotiate as long as it is in power and the firm is not making losses, but is not able neither to force a new group not to renegotiate nor to oblige a firm to stay in the market when profits are negative. So, in terms of commitment power, our model lies in between the stylized cases presented in Laffont and Tirole (1993), namely, full-commitment, one period-commitment and commitment only when no renegotiation is mutually beneficial.

4. Optimal bidding and renegotiation

In this section we will look for the solution of the auction game and its implications on the renegotiation process. Because we assume that firms are myopic in the sense that they do not consider the possibility of renegotiation at the auction stage, we start by solving the equilibrium of the first-price sealed-bid auction, considering that a firm, if it wins, will produce for two periods with the same price.
4.1 First stage: solution of the first-price sealed-bid auction

We look for a symmetric Bayesian Nash equilibrium of the auction in which each firm bids according to a function $p_i = p(\theta_i)$, increasing in $\theta_i$.

So, the objective function of firm $i$ can be written as

$$\max_{p_i} \gamma(p_i) 2E_z \left[ (p_i - \theta_i) \tilde{q}(p_i) + \tilde{\varepsilon} \right] - 1$$

where $\gamma(p_i)$ is the probability of winning the auction. Therefore, $\gamma(p_i) = \Pr(p_i < p_j, \forall j \neq i)$. Because we want to find the Nash equilibrium we assume that all the firms but $i$ are playing the equilibrium strategy, and we look for the best response of firm $i$. After that, we will check that, indeed, there is a symmetric equilibrium and the function $p(\theta)$ is increasing.

Because all the other firms are bidding according to the function $p(\theta_j), \forall j \neq i$, we have that $\gamma(p_i) = \Pr(p_i < p_j, \forall j \neq i)$. Now, because $p(\theta)$ is increasing it can be inverted so that $\theta_j = p^{-1}(p_i)$. So firm $i$ will win the auction if, $\forall j \neq i, \theta_j > \theta_i = p^{-1}(p_i)$. Thus, $\gamma(p_i) = \Pr(p^{-1}(p_i) < \theta_j, \forall j \neq i)$. Using the assumption of independence of $\theta_j, \gamma(p_i) = \left[ 1 - F(p^{-1}(p_i)) \right]^{N-1}$, with $N$ being the number of firms participating in the auction.

Thus, firm $i$'s objective function writes:

$$\max_{p_i} \left[ 1 - F(p^{-1}(p_i)) \right]^{N-1} 2E_z \left[ (p_i - \theta_i) \tilde{q}(p_i) + \tilde{\varepsilon} \right] - 1$$

(1)

**Proposition 1** The winning firm is the most efficient one in the pool. The expected profit of the winning firm is strictly positive at the symmetric equilibrium for any finite $N$ and for $N \geq 2$, the winning bid is lower than the monopoly price.

**Proof.** See appendix.

Because the firm does not take into account that an ex post renegotiation with the government could increase the price in the second period the first-price sealed-bid auction succeeds in selecting the most efficient firm to produce the good. The intuition is that given the price of the good, a more efficient firm has always higher profits, and therefore is willing to reduce a little bit its bid in

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1 But not necessarily the most efficient in an absolute sense.
order to increase the probability of winning the auction. This will always be true as long as firms are symmetric in the sense that all marginal costs are drawn from the same distribution function. More generally, if the firms' marginal costs are drawn from different distribution functions or if they were correlated with a known $\alpha_i$, the concession may be awarded to a firm whose marginal cost is not the lowest one in the sample.

On the other hand, competition at this stage implies that the price of the good is smaller than the monopoly price, $p^M(\theta, \tilde{\varepsilon})$, but larger than the Ramsey price, $p^P(\theta, \tilde{\varepsilon})$ defined such that $\left(p^P(\theta, \tilde{\varepsilon}) - \theta_i\right)\tilde{\varepsilon} = 1$ computed at $\tilde{\varepsilon} = 0$. Remember that our assumption of increasing returns to scale implies that competition in the market is not desirable. Moreover, the fact that total costs are not observable ex-ante by the government makes regulation by average cost pricing (or Ramsey pricing) impossible. Therefore, the best way to avoid \textit{ex ante} monopoly pricing is to introduce competition for the market. The benefits are illustrated in Proposition 1: In order to increase the probability of winning the auction firms prefer to charge a price lower than the monopoly price. Furthermore, as the number of firms participating in the auction increases, the price converges to the Ramsey price.

4.2 Renegotiation

There are two cases in which renegotiation can happen in this model. In the first case, the firm triggers renegotiation if the demand shock is such that the profit becomes negative. In the second case, the government may trigger renegotiation. A new government majority will ask for renegotiations if these can reduce the price.

4.2.1 Figuring out the changes in prices from renegotiation.

Since the probability of renegotiation is influenced by the resulting price, we first need to see how the renegotiated price depends on the bargaining power of the firm. Of course, this renegotiated price will depend on the majority at the renegotiation stage. The appendix shows how the problem is solved analytically. In a nutshell, the optimal renegotiation for each majority case will result from the maximization by the new ruling majority of its own welfare subject to a positive constraint on profits and taking into account the strength of
its bargaining power. The first order conditions of this optimization problem yield the following result:

**Proposition 2** (a) If the rich win the election for the second period, (i.e. $\beta_2^* > 1/2$), the renegotiated price is given by:

$$
\frac{p_1^r - \theta_i}{p_1^r} = \frac{1}{\eta(p_1^r)} \left[ \alpha W_1(p_1^r) + \left(1 - \beta_1^*\right)\alpha_1 \pi(p_1^r) \right] \forall \alpha_i \in (0,1]
$$

(b) If the poor win the election (i.e. $\beta_2^* < 1/2$), the renegotiated price is given by:

$$
\frac{p_2^r - \theta_i}{p_2^r} = \frac{1}{\eta(p_2^r)} \left[ \alpha W_2(p_2^r) + \left(1 - \beta_2^*\right)\alpha_1 \pi(p_2^r) \right] \forall \alpha_i \in (0,1]
$$

where $p_1^*(\theta, \varepsilon)$ is such that

$$
\frac{p_1^*(\theta, \varepsilon) - \theta_i}{p_1^*(\theta, \varepsilon)} = \left(1 - \beta_2^*\right)\frac{1}{\eta(p_1^*(\theta, \varepsilon))}
$$

and $\eta(p) = -\frac{\partial \pi(p)}{\partial p} > 0$ is the demand elasticity.

(b) If the poor win the election (i.e. $\beta_2^* < 1/2$), the renegotiated price is given by:

$$
\frac{p_2^r - \theta_i}{p_2^r} = \frac{1}{\eta(p_2^r)} \left[ \alpha W_2(p_2^r) + \left(1 - \beta_2^*\right)\alpha_1 \pi(p_2^r) \right] \forall \alpha_i \in (0,1]
$$

Proof. See appendix.

Proposition 2 is quite intuitive and reasonable. It means that the renegotiated price is a weighted average of the government’s most preferred price and the firm’s most preferred price, and the weights are given by each party’s bargaining power.

The extreme solutions of this Nash bargaining game are actually quite interesting as well. If the firm has all the bargaining power ($\alpha_i = 1$), the renegotiated price is equal to the monopoly price. This is true whatever the group in power. On the other extreme, when the government has all the bargaining power ($\alpha_i = 0$), the renegotiated price will be equal to the Ramsey
price (equal to the average cost)\(^2\). These extremes define the upper and lower bound for the renegotiated price prevailing under intermediate values of \(\alpha\).

The derivative of the first order conditions to the optimization problem yielding (2) and (3)—see appendix for a formal derivation—provides useful additional information on how the effects of the changes in each one of the "policy" parameters. The most interesting results are summarized in:

**Proposition 3.** The comparative static results suggest that:

- \(p^R_h\) is increasing in \(I, \theta_h, \text{ and } \alpha_h\) for \(h = 1,2\), meaning that, ceteris paribus, the renegotiated price will be higher for higher levels of costs (both fixed and marginal) and for higher levels of bargaining power for the firm; keeping fixed the cost of the firm, the larger the bargaining power, the closer the price to the corresponding monopoly price.

- \(p^R_2\) is constant in \(\beta^*_2\), meaning that, whether \(\beta^*_2\) is slightly lower than 1/2 or the whole society belongs to the poor category, the resulting price will be the same for given values of \(I, \theta, \text{ and } \alpha\). So, things won't get any better for the poor after they renegotiate since under permanent ruling by the poor, prices are likely to be as low as possible from the beginning.

- \(p^R_1\) is decreasing in \(\beta^*_2\), meaning that if there is a rich majority in the second period, the renegotiated price will be smaller if the whole society is rich than if \(\beta^*_2\) is slightly higher than 1/2.

- For given values of the parameters, and for any \(\beta^*_2\), \(p^R_2 \leq p^R_1\), with equality when \(\alpha = 1\), meaning that changing majority from the rich to the poor will result in lower tariff.

**Proof.** See appendix.

Figure 1 summarizes the main results and illustrates how the renegotiated price changes with the proportion of rich consumers.

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\(^2\) If there is a majority of rich consumers and \(\beta^*_2\) is close to 1/2 it could be the case that the renegotiated price for \(\alpha = 0\) is larger than the Ramsey price, if the Ramsey price is smaller than \(p^*_1\). But, of course, it will never be lower than the Ramsey price.
4.2.2 Figuring out who will initiate the renegotiation and when

Using the results of Proposition 2, we compute the probability of renegotiation as a function of the parameters of the model. Define \( \varphi = \Pr(\beta_2^* > 1/2 / \beta_1^* < 1/2) = \Pr(\beta_2^* < 1/2 / \beta_1^* > 1/2) \) the probability of having a new majority in period 2. For simplicity we assume that this probability is the same, whether in the first period there is a rich majority or a poor majority.

The total probability of renegotiation if there is initially a majority \( k \) (\( \Pr(R / k) \)) is then the sum of the probability of renegotiation driven by the firm (\( \Pr(R_f) \)) and the probability of renegotiation driven by the government, given that in the first period group \( k \) is in power (\( \Pr(R_g / k) \)):

\[
\Pr(R / k) = \Pr(R_f) + \Pr(R_g / k)
\]

Consider now separately the cases in which the firm and the government initiate the renegotiation.

As explained before, the firm will ask for renegotiations when its profits are negative which means:
\[ \Pr(R_f) = \Pr(\pi(p) < 0) \]
\[ = \Pr((p(p) - \theta)(\bar{q}(p(p)) + \bar{\varepsilon}) < 1) \]
\[ = \Pr(\bar{\varepsilon} < \frac{1}{p(p)} - \bar{q}(p(p))) \]  

(4)

Define \( \varepsilon^*(p(p), \theta) \in [-\varepsilon, 0) \) such that

\[ \varepsilon^*(p(p), \theta) = \begin{cases} -\varepsilon & \text{if} \quad \frac{\bar{q}(p(p))}{p(p)} - \bar{q}(p(p)) < -\varepsilon \\ \frac{\bar{q}(p(p))}{p(p)} - \bar{q}(p(p)) & \text{if} \quad -\varepsilon \leq \frac{\bar{q}(p(p))}{p(p)} - \bar{q}(p(p)) \end{cases} \]

So \( \varepsilon^* \) gives the minimum demand shock that will allow the firm to make positive profits and therefore the firm will not initiate renegotiations if \( \bar{\varepsilon} \geq \varepsilon^* \).

With this definition, \( \Pr(R_f) = \Pr(\bar{\varepsilon} < \varepsilon^*(p(p), \theta)) \), because we have assumed a uniform distribution for the demand shock. The price bid in the first period is higher than the Ramsey price for \( \bar{\varepsilon} = 0 \), so \( \varepsilon^* < 0 \) and \( \Pr(R_f) < 1/2 \). Therefore, only negative demand shocks can trigger renegotiation by the firm.

The government will ask for renegotiations if there is a new majority, \( h \neq k \), which occurs with probability \( \varphi \), and only when the price can be decreased, which means both that the profit of the firm is positive, so the firm can accept to produce with a lower price, and that its bargaining power is small enough, because the renegotiated price increases with \( \alpha \). Thus,

\[ \Pr(R_{R_f} / k) = \Pr(\text{Majority } h) \Pr(\pi \geq 0) \Pr(\alpha_i < \mathcal{P}_h(\theta_i, p(\theta_i))) \]
\[ = \varphi \left( 1 - \frac{\varepsilon^*(p(p), \theta)}{2\varepsilon} \right) G(\mathcal{P}_h(\theta_i, p(\theta_i))) \]

(5)

where \( \mathcal{P}_h(\theta_i, p(\theta_i)) \) is defined such that

\[ p^R_h(\mathcal{P}_h(\theta_i, p(\theta_i))) = p(\theta_i) \]

So the renegotiated price will be lower than the initial price if and only if \( \alpha_i < \mathcal{P}_h(\theta_i, p(\theta_i)) \).
From Proposition 2 we can find an expression for the maximum bargaining power of the firm that will induce a new majority to ask for renegotiations. Indeed, replacing \( p_1^R \) and \( p_2^R \) by \( p(\theta) \) in 2 and 3, and solving for \( \alpha_i \), we obtain:

\[ \frac{\eta(p_i)^{\alpha_i - \theta_i}}{(\eta(p_i)^{\alpha_i - \theta_i} - (1 - \beta_i^*))(\tau(p_i) + (1 - \eta(p_i)^{\alpha_i - \theta_i})W_1(p_i))} \]

where \( p_i = p(\theta) \). So, \( \phi_1 \) is increasing in \( p_i \) and decreasing in \( \theta_i \).

Because the renegotiated price is always lower when there is a poor majority, it can be shown that, given \( (p(\theta), \theta) \), \( \phi_1 < \phi_2 \), meaning that, ceteris paribus, \( \Pr(R_{g/1}) > \Pr(R_{g/2}) \), that is, the probability of renegotiation is larger if in the first period the rich consumers were in power as illustrated by Figure 2.

Intuitively, if the group of rich consumers is in power in period 1, then the government can ask for renegotiations only if in period 2 the group of poor consumers becomes a majority. And whatever the bargaining power and the marginal cost of the firm, bargaining with the group of poor consumers will achieve a lower price than bargaining with a group of rich consumers, as stated in Proposition 3. Therefore, the renegotiated price will be lower than the old price for a larger range of values of \( \alpha_i \) when the new majority is a poor one.

Analytically, using (4) and (5), the probability of renegotiation when group \( k \) is in power in period 1 is equal to:

\[ \Pr(R_{g/k}) = \frac{1}{2} + \frac{\epsilon(p(\theta), \theta)}{2\epsilon} + \phi \left( \frac{1}{2} - \frac{\epsilon(p(\theta), \theta)}{2\epsilon} \right) G(\phi_k(\theta, p(\theta))) \]

4.2.3 Figuring out how the initial efficiency level drives renegotiation odds

Since regulators tend to be mainly concerned about efficiency, it seems important to be able to assess the importance of the initial efficiency levels obtained through the auction for the probability of having a renegotiation under any type of majority. It also helps in assessing the impact of regulatory decision aiming at changing these efficiency levels. The formula for \( \Pr(R_{g/k}) \) allows us
to address this concern and to test for the impact of efficiency changes for both sources of renegotiation. Indeed:

\[
\frac{d \text{Pr}(R/k)}{d \theta_i} = \frac{d \text{Pr}(R_f)}{d \theta_i} + \frac{d \text{Pr}(R_g/k)}{d \theta_i}
\]

\[
\frac{d \text{Pr}(R_f)}{d \theta_i} = \frac{1}{2 \epsilon} \frac{\partial \epsilon^*}{\partial \theta_i} + \frac{1}{2 \epsilon} \frac{\partial \epsilon^*}{\partial \theta_i} p'(\theta_i)
\]

\[
\frac{d \text{Pr}(R_g/k)}{d \theta_i} = \frac{1}{2 \epsilon} \frac{\partial \epsilon^*}{\partial \theta_i} \left( (\epsilon - \epsilon^*) \left( \frac{\partial \delta}{\partial \theta_i} \right) - G(\delta) \frac{\partial \epsilon^*}{\partial \theta_i} \right)
\]

Notice that the efficiency parameter of the firm affects both the probability of renegotiation driven by the firm, which is determined by \( \epsilon^* \) and the probability of renegotiation driven by the government, determined both by \( \epsilon^* \) and \( \delta_h \).

Starting with the probability of having the firms requesting a renegotiation, the result allows us to investigate, in particular, whether a more efficient firm (which is the firm that is going to be selected by the auction procedure) is always preferred by the two groups of consumers. It suggests that changes in the efficiency of the firm has two opposite effects on the probability of renegotiation driven by the firm:

(i) the odds of renegotiation decreases with the firm's efficiency (i.e. increases with \( \theta_i \)) because a more efficient firm has lower costs and this reduces the range of shocks for which the profit is negative;

(ii) the odds of renegotiation increases with its efficiency because a more efficient firm bids a lower price and this increases the range of shocks for which the profit is negative.

So, a priori, we cannot know whether the probability of renegotiation driven by the firm increases or decreases with \( \theta_i \).
The same is true for the probability of renegotiation initiated by the government. On the one hand, this probability decreases with the firm’s efficiency because for a more efficient firm:
1. the renegotiated price is smaller and, therefore, the maximum bargaining power that triggers renegotiation decreases;
2. the price in the first period is smaller and therefore the range of demand shocks for which the firm’s profits are positive decreases.
On the other hand, the probability of renegotiation driven by the government increases with the firm’s efficiency because for a more efficient firm:
1. the maximum bargaining power that triggers renegotiation increases;
2. costs are lower and therefore the range of demand shocks for which the firm’s profits are positive increases.

5. Effects of renegotiation on the efficiency-equity trade-off

In this section we discuss how the model developed so far answers questions such as: Is there a conflict between efficiency and equity? How does renegotiation affect rich and poor consumers? Is there any conflict between the two groups of consumers? What is the role played by the bargaining power of the firm in this context?

5.1 Why users may prefer to start with an inefficient firm

One source of trade-off between efficiency and equity arises when users have a collective incentive not to prefer to have the most efficient firm winning the auction knowing that a renegotiation is likely. If renegotiation were not possible, in other words, if both parties could perfectly commit to the concession contract signed at the auction stage, there would be no possible conflict between efficiency and equity. Indeed, the auction allows the government to choose the most efficient firm within the candidates and this firm charges the lowest possible price, which benefit the poor consumers in a larger proportion. And given that the efficiency level is ex ante unknown, there is no better way to select the firm.

When the possibility of renegotiation is opened a conflict between efficiency and equity may appear. For instance, we know that the welfare of the poor consumers decreases when the firm initiates renegotiation, because the price increases. On the other hand, we also know that this welfare increases when the government initiates renegotiation, because in this case the price decreases. In
the previous Section we showed that the probability of renegotiation might increase or decrease with the efficiency parameter of the firm. We want to know how the poor consumers’ welfare changes with the efficiency parameter of the firm. So suppose that \( \frac{dP(\theta)}{d\theta} < 0 \) and \( \frac{dP(\theta)}{d\theta} > 0 \), that is, an improvement in efficiency (a decrease in \( \theta \)) improves the chances of a renegotiation by the firm—which the poor dislike—and reduces those requested by the government—which the poor like. This suggests that poor consumers may prefer to have a less efficient firm winning the auction to provide opportunities for government-driven renegotiations which will lead to realistic tariff reductions requests. But of course, they have to take into account that a less efficient firm will operate with a higher price in the first period, and also in the second period if there is no renegotiation.

To see how the two effects work together, consider the following very simple (but very particular too) example. Assume first that \( \phi = 0 \), so the government never asks for renegotiations and, second, that the probability of renegotiation driven by the firm decreases with \( \theta \), that is

\[
\frac{dP(\theta)}{d\theta} = \frac{\partial P}{\partial \theta} (\theta) + \frac{\partial P}{\partial \theta} < 0
\]  

(8)

This implies that a more efficient firm will more often have negative profits in the first period, and, therefore, will ask for renegotiations more often. But remember also that when the firm has negative profits, the renegotiation will always increase the price. Now, the magnitude of the increase in price will depend on the bargaining power of the firm at the renegotiation stage, which is ex ante unknown. So, consider for instance two firms, characterized by \( \theta_1 \) and \( \theta_2 = \theta_1 + d\theta, d\theta > 0 \), so firm 1 is more efficient than firm 2 and, because of (8), the probability of renegotiation is larger if firm 1 is selected.

The ex ante expected welfare for the two groups of consumers as a function of \( \theta \) is the sum of the expected welfare in period 1 given the price level charged initially by the firm and of the expected welfare resulting from the price in the second period which is determined by the existence or absence of renegotiation:

\[17\]

\( ^3 \) If the only source of renegotiations is negative profits, then if the probability of renegotiation driven by the firm increases with \( \theta \), all consumers will always prefer the most efficient firm. Indeed, a more efficient firm will charge a smaller price in the first period, which is good, and will trigger renegotiation less often, which is also good.
\[ EW_1(\theta) = \int_{-\varepsilon}^{\varepsilon} W_1(p(\theta), \theta, \tilde{\varepsilon}) \frac{d\tilde{\varepsilon}}{2\varepsilon} + \int_{\varepsilon}^{\varepsilon} W_1(p(\theta), \theta, \tilde{\varepsilon}) \frac{d\tilde{\varepsilon}}{2\varepsilon} 
\]
\[ + \int_0^{1/\varepsilon} \int_{-\varepsilon}^{\varepsilon} W_1(p_h^*(\theta, \tilde{\varepsilon}, \alpha), \theta, \tilde{\varepsilon}) g(\alpha) \frac{d\tilde{\varepsilon}}{2\varepsilon} d\alpha \]

\[ EW_2(\theta) = \int_{-\varepsilon}^{\varepsilon} W_2(p(\theta), \theta, \tilde{\varepsilon}) \frac{d\tilde{\varepsilon}}{2\varepsilon} + \int_{\varepsilon}^{\varepsilon} W_2(p(\theta), \theta, \tilde{\varepsilon}) \frac{d\tilde{\varepsilon}}{2\varepsilon} 
\]
\[ + \int_0^{1/\varepsilon} \int_{-\varepsilon}^{\varepsilon} W_2(p_h^*(\theta, \tilde{\varepsilon}, \alpha), \theta, \tilde{\varepsilon}) g(\alpha) \frac{d\tilde{\varepsilon}}{2\varepsilon} d\alpha \]

where \( p_h^* \) is an increasing function of \( \alpha \), and \( q(p) \) is an increasing function of \( \tilde{\varepsilon} \).

The first term is the expected welfare in the first period given that the firm charges a price equal to \( p(\theta) \). The second term is the expected welfare in the second period if there is no renegotiation, so the price is still equal to \( p(\theta) \). Finally, the third term is the expected welfare in the second period if there is renegotiation, in which case the price will be equal to \( p_h^* \) and \( \alpha \) is ex ante unknown.

We are interested in how the expected welfare would change if there is a small decrease in the efficiency level (a small increase in \( \theta \), the marginal cost).

\[
\frac{dEW_1}{d\theta} = \int_{-\varepsilon}^{\varepsilon} \left[ \frac{\partial W_1}{\partial p}(p(\theta), \theta, \tilde{\varepsilon}) \frac{dp}{d\theta} + \frac{\partial W_1}{\partial \theta}(p(\theta), \theta, \tilde{\varepsilon}) \right] \frac{d\tilde{\varepsilon}}{2\varepsilon} 
\]
\[ + \int_{\varepsilon}^{\varepsilon} \left[ \frac{\partial W_1}{\partial p}(p(\theta), \theta, \tilde{\varepsilon}) \frac{dp}{d\theta} + \frac{\partial W_1}{\partial \theta}(p(\theta), \theta, \tilde{\varepsilon}) \right] \frac{d\tilde{\varepsilon}}{2\varepsilon} 
\]
\[ + \int_0^{1/\varepsilon} \int_{-\varepsilon}^{\varepsilon} \left[ \frac{\partial W_1}{\partial p}(p_h^*, \theta, \tilde{\varepsilon}) \frac{dp_h^*}{d\theta} + \frac{\partial W_1}{\partial \theta}(p_h^*, \theta, \tilde{\varepsilon}) \right] g(\alpha) \frac{d\tilde{\varepsilon}}{2\varepsilon} d\alpha 
\]
\[ - \frac{d\varepsilon}{d\theta} \int_0^{1/\varepsilon} \int_{-\varepsilon}^{\varepsilon} \left[ W_1(p(\theta), \theta, \varepsilon') - W_1(p_h^*, \theta, \varepsilon') \right] g(\alpha) \frac{d\tilde{\varepsilon}}{2\varepsilon} d\alpha \]
Four effects of $\theta$ on the expected welfare can be distinguished. The first three are the classical effects: an increase in $\theta$ reduces welfare in each period, whether there is renegotiation or not, both because the firm's costs are higher and because the price of the good in any case is also higher$^4$. These effects are represented by the first 3 terms, which are negative, implying that, with the same probability of renegotiation, both groups of consumers would have preferred firm 1 over firm 2.

The fourth effect is given by the fact that when $\theta$ increases the probability of renegotiation, given the assumption made in (8), decreases, decreasing the range of demand shocks for which the price is high (equal to the renegotiated price) and, therefore, welfare increases, meaning that both groups of consumers prefer firm 2 over firm 1.$^5$

Therefore, the net effect of $\theta$ over the expected welfare can be either negative, in which case efficiency is always better, or positive, in which case a less efficient firm might be preferable, if the fourth effect is large enough.

The fourth effect will be large, and hence a less efficient firm will be preferred, when the bargaining power distribution function $g(\alpha)$ is biased

\[ \frac{dE W_2}{d\theta} = \frac{\varepsilon}{\varepsilon'} \frac{\partial W_2}{\partial p} (p, \theta, \varepsilon) \frac{dp}{d\theta} \frac{d\varepsilon}{d\theta} \frac{2\varepsilon}{\varepsilon'} 
+ \frac{\varepsilon}{\varepsilon'} \frac{\partial W_2}{\partial p} (p, \theta, \varepsilon) \frac{dp}{d\theta} \frac{d\varepsilon}{d\theta} \frac{2\varepsilon}{\varepsilon'} 
+ \int_{0}^{\varepsilon} \int_{-\varepsilon}^{\varepsilon} \frac{\partial W_2}{\partial p} (p^R, \theta, \varepsilon) \frac{dp}{d\theta} g(\alpha) \frac{d\varepsilon}{d\theta} \frac{2\varepsilon}{\varepsilon'} d\alpha 
- \frac{d\varepsilon}{d\theta} \int_{0}^{\varepsilon} \left[ W_2 (p, \theta, \varepsilon') - W_2 (p^R, \theta, \varepsilon') \right] g(\alpha) \frac{d\varepsilon}{d\theta} \frac{2\varepsilon}{\varepsilon'} d\alpha \]

$^4$ In the case of the rich consumers it is not always true that an increase in price will reduce welfare. Indeed, if the price is lower than $p^*_R$, which is the price that maximizes rich consumers' welfare, a small increase in price will increase welfare. Indeed, if $\beta^*_R$ is close to 0, $p^*_R$ is close to the monopoly price and the price without renegotiation is smaller than $p^*_R$. We will assume that in those cases, the direct decrease in welfare due to an increase in $\theta$ completely offsets the increase due to an increase in the price, so the net effect is negative.

$^5$ Of course, if the probability of renegotiation decreases with the efficiency of the firm ($\frac{d\theta}{d\varepsilon} > 0$), this fourth effect would go in the same direction as the first three and more efficiency would be always better for consumers.
towards large values of $\alpha$, because more weight is put on the cases for which the increase in the price is the largest. In practice, this bias exists when competition is limited on the supply side (as is the case in the water sector and to a lesser extent in the power, port and railways sectors, for instance where the number of global players is such that competition for the market seldom results in no more than 2-3 bidders).\footnote{Any time we can expect a close relationship between the operators and the government (or a politically driven regulator), collusion is a risk; the fewer the alternatives, the stronger the firm's bargaining power and hence the stronger the risks of collusion.}

In addition, to the equity efficiency trade-off just described, there is an underlying distributional conflict between rich and poor. For $\beta^*_{2}$ small enough (i.e. when the poor rule significantly)\footnote{The exact condition is that $\beta^*_{2} < \frac{1}{2} + \frac{n \cdot \theta}{P} \frac{n(p) \Phi^p}{d\theta}$. In particular, the condition is satisfied when poor consumers are majority.}, we have that $\frac{d\theta}{d\theta} > \frac{d\theta^*$}{d\theta}$, and, therefore, there are cases in which the poor consumers prefer firm 2 over firm 1 while the reverse is true for the rich consumers. The efficiency-equity trade-off continues to hold as well since the auction procedure will always select firm 1.

The bottom line is that we can conclude that the efficiency level is not the unique relevant parameter driving the selection of a firm to operate a concession, equity issues must be taken into account as well and may in fact have to prevail over efficiency consideration. When renegotiation driven by the firm is an issue (when the uncertainty about market conditions is high) a more efficient firm can end-up operating with a quite high price after renegotiations.

5.2 When rich and poor consumers don’t see eye to eye

The distributional conflict just eluded suggests that it is also important to understand more generally the differences in efficiency outcomes since they may result in conflicts between the two groups of consumers. In particular, we should be interested in trying to classify the situations in which rich consumers and poor consumers will agree or disagree about the situation they prefer the most.

The possible situations are as follows:

1. When at the auction stage there is a rich majority ($\beta^*_{1} > 1/2$) and at the end of period 1:
(a) The winning firm $i$, has negative profits, so renegotiation occurs whatever the majority in period 2 and the price of the good increases.

(b) The firm has positive profits and in period 2 there is again a rich majority, so no renegotiation can happen, whatever the value of $\alpha$.

(c) The firm has positive profits but in period 2 there is a poor majority, so renegotiation occurs if the firm's bargaining power is low enough ($\alpha_i < \delta_i$) and the price of the good decreases.

2. When at the auction stage there is a poor majority ($\beta_1^* < 1/2$) and, at the end of period 1:

(a) The firm has negative profits, so renegotiation occurs whatever the majority in period 2 and the price of the good increases.

(b) The firm has positive profits but in period 2 there is a rich majority, so renegotiation occurs if the firm's bargaining power is low enough ($\alpha_i < \delta_i$) and the price of the good decreases.

(c) The firm has positive profits and in period 2 there is again a poor majority, so no renegotiation can happen, whatever the value of $\alpha$.

Cases 1a and 2a show that if the firm has negative profits in the first period, both groups of consumers are better off when poor consumers win the election and rule during the second period. This is because the magnitude of the price increase—and the price will increase after renegotiations under any type of majority—is always lower when the poor are in charge in the second period. Indeed, according to Proposition 3, the renegotiated price will be smaller if $\beta_i^* < 1/2$ (see Figure 1). Furthermore, the probability of being in one of these two cases is the same because the price charged in the first period is independent of $\beta_1^*$.

Cases 1b and 2c show that if the firm has positive profits and there is no change in majority, whatever the majority, no renegotiation happens whatever the bargaining power of the firm. Moreover, given our assumption that the probability of preserving the majority is independent of $\beta_1^*$, both cases also arise with the same probability.

The most interesting cases are 1c and 2b when the firm has positive profits but there is a change in majority. In these two cases, renegotiation occurs if the
bargaining power of the firm is small enough. According to Figure 2, $\bar{a}_1 < \bar{a}_2$ and therefore $Pr(\alpha_i < \bar{a}_1) < Pr(\alpha_i < \bar{a}_2)$ and renegotiation occurs more often when there is a poor majority in the second period. Furthermore, even when $\alpha_i < \bar{a}_1 < \bar{a}_2$, so renegotiation occurs under any majority, the renegotiated price will be lower in case 1c than in case 2b, because poor consumers are more concerned with a decrease in price. Without ambiguity, poor consumers will prefer case 1c to case 2b. Rich consumers will also prefer case 1c as long as $p_2^R(\alpha_i, \bar{\theta}) \geq p_1^*(\bar{\theta})$. On the other hand, if $\bar{a}_1 < \alpha_i < \bar{a}_2$, renegotiation will occur in case 1c, while it would not in case 2b.

The conflict between rich and poor consumers arises as follows. When the rich win the election, for $\beta_2^*$ small enough $p_1^*(\bar{\theta})$ is larger than the renegotiated price obtained with a poor majority because as $\beta_2^*$ goes to 0, $p_1^*(\bar{\theta})$ approaches the monopoly price. If this were the case, the group of rich consumers prefers not to renegotiate and maintain a price closer to the monopoly price. This behavior hurts the poor, which would have bargained a much lower price if they had won the election and is thus the main source of conflict in the second period.

A final source of distributitional conflict is that the loss incurred by poor consumers when the price is higher than the Ramsey price is always larger than the one incurred by rich consumers because the latter group receives the increased profits of the firm.

6. Concluding remarks

In this paper we analyze the possible sources of contract renegotiation and their impact on the efficiency-equity trade-off in non-competitive markets in which regulation by average cost pricing cannot be implemented because there is incomplete information about the firm's costs.

To do this, we presented a model of incomplete contracting for concession award in which neither the government nor the firm consider ex ante the possibility of renegotiation, yet renegotiation happens. Competition for the contract through an auction ensures that the consumers benefit by a price that is lower than the monopoly price and allows the government to select the firm with the lowest costs. We assume that once the firm has started to produce, the marginal cost of the firm can be inferred, so at the renegotiation stage the government and the firm share the same information. The government still cannot always force the firm to charge a price equal to the average cost because
at this stage the firm has acquired some bargaining power and therefore the government is not able to extract all the rents from it.

The model provides possible explanations for the various types of renegotiations that have been observed in the last decade and offers testable results. First, the probability of election-driven renegotiations inducing reductions in tariffs is larger if: (i) in the first period the “rich” consumers (i.e. the co-owners of the operating firms) are in power, (ii) the operators are making profits, (iii) the “poor” take over after a change in government and/or (iv) the government expects to have a strong bargaining power. Second, whatever the bargaining power and the marginal cost of the firm, bargaining with the group of poor consumers will achieve a lower price than bargaining with a group of rich consumers. Third, the smaller the group of rich in the population, the higher the chances of having high prices because they receive the whole increase in the profit of the firm while they lose a small proportion of the reduction in the consumer surplus. This implies that conflicts between rich and poor consumers are more likely to occur the larger the relative power of the poor at the time of renegotiation. Fourth, the expected direction of the effects of changes in the firm’s marginal cost on the probability of renegotiation depends on a set of factors with opposing sign and is hence an empirical matter reflecting the relative strength of these factors. Fifth, the perverse effects of excessive bargaining power by the firm tend to penalize the poor relatively more because they pay a higher price without benefiting by the increase in the firm’s profits.

In terms of policy advise, the model suggests that if the fear of renegotiation is a major concern, efficiency is not the only variable that matters when selecting a firm to operate a concession. Indeed, consumers may want to award the concession to a less efficient firm in order to reduce the probability of renegotiation since lower probabilities of firm driven renegotiations (dues to demand shocks for instance) are associated with higher welfare levels. Second, the model suggests also that a “benevolent, welfare maximizing government” should make every possible effort to balance the bargaining power of its regulators with that of the operators to ensure a fair treatment of all users. One of the actions to take, and certainly not the least important, is to minimize the possibility of collusion/corruption between the firm and the government units in charge of the renegotiations and who may not have the same degree of benevolence. Reducing bargaining power will also reduce strategic bids and low-balling by firms convinced that they can win any renegotiation and get closer to monopoly prices ex-post. Third, any policy instrument that reduces
demand uncertainty will be in the interest of all users but in particular the poor. Indeed, the implicit insurance paid by selecting a somewhat inefficient firm at the auction reduces renegotiations due to negative demand shocks, which in turn leaves the poor better off.

In order to refine some of the results, additional research could explore the following ideas. First, we have considered the bargaining power of the firm as an exogenous parameter. In practice, it depends on several variables such as the owners of the firm, the political interference with the choice of bidders resulting from tied bilateral aid, the level of profits or of sunk costs, etc. A more careful study of the potential effects of each one of these factors seems appropriate. Second, there is a reputation effect that is not captured in the model. In order to avoid the kind of strategic behavior aforementioned, the government can build a reputation of being tough regarding renegotiations. This would have a dynamic positive effect on welfare, by avoiding underbidding and, therefore, ex post non-desired increases in price. A final additional direction for improvement could be the modeling of the distributional payoffs of insurance mechanisms to protect from demand uncertainty.
7. Appendix

Proof of Proposition 1. We are looking for a subgame perfect Nash equilibrium of the game. Each firm bids according to the first order condition:

\[ q(p_i) + (p_i - \theta_i) \frac{dq}{dp}(p_i) - (N - 1) \frac{f(p^{-1}(p_i))}{1 - F(p^{-1}(p_i))} \frac{(p_i - \theta_i) q(p_i) - 1}{p'(p^{-1}(p_i))} = 0 \]  \hspace{1cm} (9)

Now we need to verify that a solution of equation 9 satisfies \( \frac{dq}{dp} > 0 \) at the symmetric equilibrium, which means that the lowest price will be offered by the firm with the lowest marginal cost.

At the symmetric equilibrium \( p_i = p(\theta) \) and \( p^{-1}(p_i) = \theta_i \), so condition 9 writes:

\[ (N - 1) \frac{f(\theta_i)}{1 - F(\theta_i)} \frac{(p(\theta_i) - \theta_i) q(p(\theta_i)) - 1}{p'(\theta_i)} = \]
\[ q(p(\theta_i)) + (p(\theta_i) - \theta_i) \frac{dq}{dp}(p(\theta_i)) > 0 \]  \hspace{1cm} (10)

where the inequality holds because the price is lower than the monopoly price whenever \( N > 1 \). If the price were higher than the monopoly price, the firm could increase both the expected profit and the probability of renegotiation by decreasing the price.

This also implies that the expected profit of the firm is strictly positive for any \( \theta_i \), so the price is lower than the Ramsey price. If this were not the case, the expected profit of the firm would be negative, and therefore, it would win by bidding the Ramsey price.

Proof of Proposition 2. a) Suppose first that \( \beta_2^* > 1/2 \) so at the renegotiation stage there is a rich majority. Because of Nash bargaining, we know that the optimal renegotiation price will solve the following problem:
\[
\begin{aligned}
\max_p & \left[ W_1(p) \right]^{1-\alpha_i} \left[ \pi(p) \right]^{\alpha_i} \\
\text{subject to} & \quad \eta(p) \geq 0
\end{aligned}
\]

So, using the definitions of \( W_1 \) and \( \pi \) and remembering that \( S'(q(p)) = p \) we obtain the first order condition, \( FOC_1(p) = \frac{d \left[ W_1(p) \right]^{1-\alpha_i} \left[ \pi(p) \right]^{\alpha_i}}{dp} = 0 \):

\[
\begin{aligned}
&- (1-\alpha_i) \left[ (1-\beta_i^*) \eta(p) + (p - \theta_i) \frac{dq}{dp} (p) \right] \\
&= \alpha_i \frac{W_1(p)}{\pi(p)} \left[ (p - \theta_i) \frac{dq}{dp} (p) + q(p) \right]
\end{aligned}
\]

Rearranging terms, equation 2 can be found. The constraint is binding for \( \alpha_i = 0 \) if \( p^*(\theta_i, \tilde{\varepsilon}) > p_i^*(\theta_i, \tilde{\varepsilon}) \) and is not binding for any \( \alpha_i \in (0,1] \).

b) Now suppose that \( \beta_i^* < 1/2 \) so at the renegotiation stage there is a poor majority. The optimal renegotiation price will solve the following problem:

\[
\begin{aligned}
\max_p & \left[ W_2(p) \right]^{1-\alpha_i} \left[ \pi(p) \right]^{\alpha_i} \\
\text{subject to} & \quad \eta(p) \geq 0
\end{aligned}
\]

So, using the definitions of \( W_2 \) and \( \pi \) and remembering that \( S'(q(p)) = p \) we obtain the first order condition, \( FOC_2(p) = \frac{d \left[ W_2(p) \right]^{1-\alpha_i} \left[ \pi(p) \right]^{\alpha_i}}{dp} = 0 \):

\[
\begin{aligned}
&\frac{1}{1-\alpha_i} \left[ (1-\beta_i^*) \eta(p) \right] \\
&= \frac{\alpha_i}{\pi(p)} \left[ (p - \theta_i) \frac{dq}{dp} (p) + q(p) \right]
\end{aligned}
\]
Rearranging terms, equation 3 can be found. The constraint is always binding for \( \alpha_i = 0 \) and is not binding for any \( \alpha_i \in (0,1] \).

**Proof of Proposition 3.** For comparative static, we compute the derivative of the first order condition with respect to the parameter that changes. To simplify notations, we will call \( q_i^R = q(p_i^R) \). Thus, \( \text{sign}(\frac{\partial FOC_1}{\partial \alpha_i}) = \text{sign}(\frac{\partial FOC_1}{\partial \alpha_i}(p_i^R)) \) for any parameter \( a \).

\[
\frac{\partial FOC_1}{\partial \theta_i} = -\frac{dq_i^R}{dp} \left( 1 - \alpha_i + \alpha_i \frac{W_1(p_i^R)}{\pi(p_i^R)} \right) + \left( p_i^R - \theta_i \right) \frac{dq_i^R}{dp} + q_i^R \left[ \frac{W_1(p_i^R) - \pi(p_i^R)}{\pi(p_i^R)} \right] > 0
\]

\[
\frac{\partial FOC_1}{\partial \alpha_i} = \beta_i q_i^R + \left( p_i^R - \theta_i \right) \frac{dq_i^R}{dp} + q_i^R \left[ \frac{W_1(p_i^R) - \pi(p_i^R)}{\pi(p_i^R)} \right] > 0
\]

\[
\frac{\partial FOC_1}{\partial \beta_i} = -\frac{1 - \alpha_i}{W_1(p_i^R)} \left( p_i^R - \theta_i \right) \frac{dq_i^R}{dp} + q_i^R \left[ \frac{W_1(p_i^R) - \pi(p_i^R)}{\beta_i} \right] < 0
\]

\[
\frac{\partial FOC_2}{\partial \theta_i} = -\alpha_i \frac{W_2(p_2^R)}{\pi(p_2^R)} \frac{dq_2^R}{dp} + \left( p_2^R - \theta_i \right) \frac{dq_2^R}{dp} + q_2^R \left[ \frac{W_2(p_2^R) - \pi(p_2^R)}{\pi(p_2^R)} \right] > 0
\]
\[
\frac{\partial FOC_2}{\partial l} = \alpha \left[ (p_2^R - \theta_i) \frac{dq_2^R}{dp} + q^R_2 \right] \frac{W_2(p_2^R)}{\pi(p_2^R)} > 0
\]

\[
\frac{\partial FOC_2}{\partial \alpha_i} = (1 - \beta_2^*) q_2^R + \left[ (p_2^R - \theta_i) \frac{dq_2^R}{dp} + q^R_2 \right] \frac{W_2(p_2^R)}{(1 - \beta_2^*) \pi(p_2^R)} > 0
\]

\[
\frac{\partial FOC_1}{\partial \beta_2^*} = -\alpha \left[ (p_2^R - \theta_i) \frac{dq_2^R}{dp} + q^R_2 \right] \frac{W_2(p_2^R)}{(1 - \beta_2^*) \pi(p_2^R)} + (1 - \alpha_i) q^R_2 = 0
\]

Which gives our result.

To show that \( p_2^R \leq p_1^R \) it suffices to show that the first order condition when there is a poor majority evaluated at \( p_1^R \) is negative, meaning that, decreasing the price will increase the objective function. That is, we will show that \( FOC_2(p_1^R) < 0 \).

\[
FOC_2(p_1^R) = -(1 - \alpha_i) q_1^R + \alpha_i \frac{S(q_1^R) - p_1^R q_1^R}{\pi(p_1^R)} \left[ (p_1^R - \theta_i) \frac{dq_1^R}{dp} + q_1^R \right]
\]

Now, taking into account that, by definition of \( p_1^R \), \( FOC_1(p_1^R) = 0 \), we can replace and we obtain:

\[
FOC_2(p_1^R) = -(1 - \alpha_i) q_1^R - (1 - \alpha_i) \frac{S(q_1^R) - p_1^R q_1^R}{W_1(p_1^R)} \left[ (p_1^R - \theta_i) \frac{dq_1^R}{dp} + (1 - \beta_2^*) q_1^R \right]
\]

Finally, using the definition of \( W_1(p_1^R) \), we have

\[
FOC_2(p_1^R) = -\frac{(1 - \alpha_i)}{W_1(p_1^R)} \left[ q_1^R \pi(p_1^R) + (S(q_1^R) - p_1^R q_1^R) \left( (p_1^R - \theta_i) \frac{dq_1^R}{dp} + q_1^R \right) \right]
\]

(*)

28
So, $FOC_2(p_1^R) < 0$, meaning that $p_2^R \leq p_1^R$.

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