The Economic Cost of Power Outages: Methodology and Application to Jamaica

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Energy, Water and Telecommunications Department

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Abstract

One of the most important determinants of costs in the design of a power system is the reliability standard to be achieved. The trade-off between the greater security provided by substantial excess capacity and the lower capital requirements of a less reliable system suggests that a cost-benefit approach should be employed to optimize investment levels. However, as is often the case with public utility projects, while the determination of marginal costs is fairly straightforward the measurement of the economic benefits derived from fewer outages is much more difficult.

This paper develops a method for estimating those benefits (or avoided costs) for both residential and business consumers. For the former, outage costs are based upon a calculation of the implied cost of temporarily useless electrical appliances. For the commercial and industrial sectors, loss-of-supply costs are divided into three types: ruined or spoiled product, production lost during restart operations and production lost during the outage itself. These three costs are defined in terms of both sector-specific (i.e., value added production and labor costs) and outage-specific (i.e., annual frequency and duration) variables. Thus the outage cost derived per kWh can be linked directly to the kWh lost corresponding to specific levels of generating reserve in order to isolate the net cost minimizing level of investment.

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The Economic Cost of Power Outages: Methodology and Application to Jamaica

I. Scope of the Problem

The trade-off between the greater security provided by substantial excess generating capacity and the lower capital requirements of a less reliable power system is of major concern to the Bank in its analysis of a developing country's investment needs in the energy sector.

A power system can be designed to be as reliable as desired. However, as the degree of reliability approaches 100 percent the marginal cost of providing additional reliability increases. Thus it is necessary to determine the point where that marginal cost exceeds the marginal benefit gained from outage reduction in order to avoid investments in plant capacity which may be economically unwarranted. For generation capacity, in particular, such mistakes can be costly. For example, in Jamaica, where one of the main issues of the proposed Power II Project is the planned excess capacity in the generating plant, nearly J$140 million1/ is budgeted for additional generating capacity out of a total five year investment program of J$237 million.

The general rule for determining the optimal reserve margin is well known: it should provide the standard of security

"... at which the expected cost at the margin to consumers of kWh not supplied equals the expected cost of supplying those kWh."2/

Considerable technical work has been done investigating the minimum cost combinations of generating unit type and size given the distribution options, inter-connection possibilities, shape of load, and other system-specific parameters.3/ Thus the determination of the expected marginal cost of supplying additional generating capacity is a fairly straight-forward, albeit empirically difficult, calculation. On the other hand, relatively little research has been done on either the theoretical or

1/ US$1.00 = J$0.88.
3/ See, for example, R.X. French "Evaluating System Reserve with Probability Analyses", Electrical World, April 1, 1971.
empirical difficulties involved in estimating the expected marginal cost of a kWh not supplied. The method proposed here incorporates techniques that have been used in Sweden (1969) and Chile (1973) for residential consumers and develops a new approach for estimating outage costs of industrial and commercial users.

This study is limited to the consideration of unanticipated outages. It is almost universally agreed that planned outages are less costly both because, for most firms, some variable costs such as labor can then be saved and because inventory and equipment damage is less likely. By the same token, voltage and frequency reductions have been excluded on the assumption that a kWh lost through an outage is more costly than one lost through power reductions.\(^1\) Outages are also assumed to affect all consumers proportionally since selected outages probably decrease losses.\(^2\) Thus, the maximum marginal cost of a power deficiency is being estimated by focusing on the cost of a kWh lost through an unanticipated, randomly occurring outage.

The other definitional concern relates to the scope of outage costs considered; specifically, how to treat non-market losses. Most previous studies\(^2\) have arbitrarily excluded activities such as unpaid housework and leisure pursuits which do not show up in the GNP accounts because of measurement difficulties or assumed insignificance. However, it is likely that consumers would be willing to pay for higher reliability to avoid "annoyance" costs as well as production costs. In a country such as Jamaica where 30 percent of the publicly supplied energy is sold to residential users\(^3\) it is important to include some

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\(^1\) One business manager contacted disagreed claiming that his firm had suffered more equipment damage through sudden power drops than from outages. However, the JPSC representative who was also present during the interview stated that a voltage drop of the magnitude he claimed to have experienced (17 percent) would have been a very unusual occurrence and probably indicated a distribution problem which would be dealt with as soon as reported.

\(^2\) This assumption should introduce little bias in Jamaican estimates since JPSC claims to have little scope for selective load shedding at the current time. However, in countries with more sophisticated systems the calculation of outage costs should take account of the likely load shedding schedule.


\(^4\) Residential use is also projected to be the fastest growing category so that by 1981 it will account for 39 percent of total JPSC sales.
estimate of their loss-of-supply costs in arriving at a total figure. The problem is that even with an extensive questionnaire survey, the basic information necessary to evaluate losses is very difficult to obtain. The method described below requires a minimal amount of data, and in the limited field time available some sample information was gathered to illustrate its use.

II. Residential Users

The marginal value (or utility) of a kWh to a residential consumer is generated by combining its use with other inputs \((x_1, \ldots, x_n)\) in the total utility function \((U)\). For example, if \(x_e\) represents the electricity input and

\[
U = f(x_1, x_2, \ldots, x_e, \ldots, x_n)
\]

then the specific form of the utility function will include \(x_e\) only in cross-product terms; for example,

\[
U = x_1 + x_2 + \cdots + x_n x_e + x_6 x_e + \cdots + x_n
\]

where perhaps \(x_5\) is a television and \(x_6\) is an electric stove. If these are the only terms which include \(x_e\), then the marginal utility of \(x_e\) would equal

\[
\frac{\partial U}{\partial x_e} = x_5 + x_6,
\]

that is, the usefulness of the electricity compliments.

In order to relate to outage costs, this expression must be measured in terms of dollars per kWh and aggregated over all residential consumers. The calculations shown in Table 1 demonstrate how this could be done for Jamaica in 1975.

JPSC currently has very little information on residential usage of electricity beyond aggregate average consumption figures.\(^1\) The last survey of appliance ownership was carried out almost 20 years ago in order to arrive at some idea of the dispersion of consumption rates the average monthly consumption (over a 10 month period in 1975-76) was tabulated from the computerized billing records of two 20-household samples from a lower income and an upper-middle income area in Kingston. The averages were 112 kWh/month and 536 kWh/month respectively, while the total residential customer average in 1975 was 192 kWh/month.

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### Table 1: Sample Calculation of Implied Residential Electricity Costs

<table>
<thead>
<tr>
<th>Appliances</th>
<th>(1) Percentage of JPSC Customers 1 with Appliance 2</th>
<th>(2) Average Annual Cost 3 (J$1975)</th>
<th>(3) Average Useful Lifetime (years)</th>
<th>(4) Average Lifetime Cost 4 (J$1975)</th>
<th>(5) Average Wattage 5</th>
<th>(6) Hours of Use per month</th>
<th>(7) kWh/year 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lighting, etc. 7/</td>
<td>100</td>
<td>280 8/</td>
<td>30</td>
<td>30</td>
<td>300</td>
<td>150</td>
<td>780</td>
</tr>
<tr>
<td>Radio</td>
<td>95</td>
<td>30</td>
<td>3</td>
<td>12</td>
<td>50</td>
<td>180</td>
<td>108</td>
</tr>
<tr>
<td>Iron</td>
<td>75</td>
<td>25</td>
<td>5</td>
<td>7</td>
<td>800</td>
<td>15</td>
<td>114</td>
</tr>
<tr>
<td>Television</td>
<td>40</td>
<td>250</td>
<td>5</td>
<td>66</td>
<td>200</td>
<td>120</td>
<td>288</td>
</tr>
<tr>
<td>Refrigerator</td>
<td>35</td>
<td>400</td>
<td>20</td>
<td>47</td>
<td>200</td>
<td>240</td>
<td>576</td>
</tr>
<tr>
<td>Water heater</td>
<td>25</td>
<td>125</td>
<td>8</td>
<td>23</td>
<td>1,000</td>
<td>180</td>
<td>2,160</td>
</tr>
<tr>
<td>Stove</td>
<td>10</td>
<td>375</td>
<td>10</td>
<td>61</td>
<td>1,500</td>
<td>120</td>
<td>2,160</td>
</tr>
<tr>
<td>Washing machine</td>
<td>10</td>
<td>200</td>
<td>8</td>
<td>37</td>
<td>250</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>Air-conditioning</td>
<td>5</td>
<td>300</td>
<td>6</td>
<td>62</td>
<td>1,200</td>
<td>120</td>
<td>1,728</td>
</tr>
</tbody>
</table>

Total weighted annual appliance cost 2/  

Average annual electricity consumption per residential connection in 1975  

Implied cost of appliances/kWh  

1/ JPSC serves about one-third of all households in Jamaica so the proportion of total population with each appliance is considerably lower.  
2/ These are very crude estimates. In the case of water heaters and stoves they are based on some annual sales figures obtained from manufacturers and the fact that 55% of all water heaters sold in Jamaica are electric and 10% of stoves sold are electric. (Presumably nearly all electric appliances are sold to JPSC customers.) An aggregate check was made with the average energy consumption per appliance to insure that the product of Columns (1), (5) and (6) summed over all appliances was within 2.5% of the average JPSC residential consumption in 1975 (192 kWh/month).  
3/ Based on an average of prices in two appliance stores in Kingston and Montego Bay.  
4/ Assuming appliance life and cost as shown and a 10% opportunity cost of capita.  
6/ Column (7) = Column (5) x Column (6) x 12 months/year.  
7/ Includes clocks, fans, etc.  
8/ Average installation and connection charges.  
2/ The sum of Column (4) weighted by Column (1).
ago. There is also very little economy-wide information on appliance sales or production so that the usage figures in Table 1 are very rough estimates (and included mainly to illustrate the method of calculation). The average annual equivalent cost of electrical appliances has been calculated based on appliance lifetime, cost and dispersion across customers. This is divided by the average annual power consumption to arrive at the implied appliance cost per kWh.

The result of J$0.05/kWh represents the outage cost imposed by the prior purchase of temporarily useless appliances. The decision to acquire an electrical appliance presumably is made by some explicit or implicit comparison of the value of its use with its annuitized purchase price and operating cost. Once an appliance is bought, however, an unanticipated outage will decrease its usage benefits which are worth at least as much as the sum of the annuitized capital cost (J$0.05) and the marginal operating costs (J$0.07) per kWh. The marginal cost of this outage, however, is only the former component of the benefits since no electricity charges are incurred during the power failure.

It is possible that an unexpected outage could impose additional real costs in terms of spoiled food in process or payments to domestic help who cannot complete jobs for which electricity is required (e.g., ironing, vacuuming, etc.) but who still must be paid as contracted. However, most of the residential consumers contacted in Jamaica agreed that this type of cost was relatively minor, particularly with respect to outages of less than one hour duration. The main point they emphasized was the annoyance factor which can be approximated by the method used in Table 1.

Given the many assumptions involved in the data, conclusions are necessarily tentative. However, the implied appliance cost of J$0.05 per kWh is fairly insensitive to different appliance dispersion assumptions as long as the totals are constrained to yield the actual 1975 average annual consumption. The magnitude of the figure is generally consistent with the typical assumption that the value of electricity to the residential user is fairly low compared to industrial users.

1/ If the consumer has perfect information about future outages, then this will be taken into account in his decision process so that the method outlined here would understated the true outage cost.

2/ This is actually the average revenue received per residential customer by JPS in 1975. A small portion was subsidized by the government during part of the year but the subsidy has now been removed.

3/ Even if expenditures for household help could be freely adjusted, in the short run no alternative employment is likely to be available so the cost to the economy of the idle resources would still be incurred.

4/ Food losses would only occur during certain times of the day, and short outages (such as are most likely in the future) would have little effect on most cooking or chilling.
Even this low figure may contain an upward bias, however, for several reasons. First, if the outage is of fairly short duration the usefulness of some appliances (e.g., refrigerator, water heater) will be unimpaired. Second, some appliances are unaffected by an outage which occurs at a particular time of the day. Lights, for example, may only be used in the evenings, when the iron and washing machine are probably idle. No one outage is likely to affect all appliances simultaneously. While this time factor is partially accounted for in the calculation by scaling energy consumption per appliance by the average hours of usage per month, the result is an average rather than a probability distribution of outage costs over the day. This will be misleading if outages are more likely to occur during the system's period of peak load. Third, to the extent that residential users make up for outages by increasing appliance use after the power resumes (which is likely with the iron, stove, washing machine and, in part, the refrigerator and water heater) this figure will be an overestimate of the true inconvenience, although rescheduling work also involves some cost. Another minor qualification is that the figure for average power consumption in 1975 excludes that lost to outages during the year. While this was fairly low (an average of 2447 minutes over all JPSC customers, including businesses) it theoretically should be accounted for. A more complex estimation procedure could be developed to incorporate these factors but the data requirements were thought too heavy for this brief mission. The advantage of this method is that it does provide an order of magnitude with relatively little investment in data collection.1/

1/ Of course, no aggregated average can represent the marginal value of power to any individual consumer. I spoke with one Jamaican who had purchased a 1.5 KVA standby unit in 1973 for J$1,500. According to the shop owner who sold it, such a unit should last for about 15 years with proper maintenance (averaging around J$150/year) and could supply sufficient energy to assume the total household load during an extended outage. The diesel fuel necessary to operate it costs about J$0.05/hour in Jamaica. Thus, if this customer based his usage expectation on the average minutes lost per customer during 1973 (3,663 = 61 hours = 41.7 kWh for a customer averaging 500 kWh/month), the implied value of electricity to him can be calculated as:

Capital cost/year (assuming an opportunity cost of 10%) J$197
Maintenance per year J$150
Total annual capital and maintenance cost: J$347

Capital and maintenance costs/kWh of use J$8.39
Fuel cost/kWh J$0.03
Total cost/kWh of use J$8.42

It is little wonder that the standby generator salesman reported that fewer than one percent of his customers bought equipment for residential use. A JPSC representative indicated he felt residential standby units were more of a status symbol than a utilitarian purchase in Jamaica.
III. Commercial and Industrial Users

The estimation of loss-of-supply costs for commercial and industrial energy users is theoretically more direct than for residential users since the former's electricity demand stems from efforts to produce a measurable output rather than non-cardinal consumer utility. The marginal value of electricity for sector $i$ is thus equal to its marginal revenue product, $\frac{\partial Q_i}{\partial x_{ei}} \cdot MR$ in that sector. Direct measurement of that derivative for any particular industry is likely to be impossible, however, both because explicit quantitative production functions are rarely found and because even if the equation were available it would likely be discontinuous. The latter problem (translated into cost terms) was mentioned frequently in discussions with Jamaican executives from many different types of industries and led to the step-function approach developed below.

The types of costs incurred during a power outage seemed to fall into three categories: ruined or lost product costs (LPC),$^{1/}$ costs of production lost during restart operations (RSC) and the product production cost (PPC) incurred during the outage itself. In aggregating outage costs over a year (since sectoral production and input cost figures are only available on an annual basis), the relationship between these three types of costs and the frequency and duration of outages must be specified. In general terms, the LPC will be a stepped function of outage duration and a monotonically increasing function of outage frequency. The RSC will take different forms for different industries, ranging from a simple linear multiple of outage frequency to a non-linear function of frequency and duration combinations. The PPC will be independent of frequency but related positively to total outage duration.

(a) Lost Product Costs

Consider first the estimation of the annual LPC. For many industries the lost product cost will be zero (e.g., garment production, wood working, retail trade in non-perishable items), while for others it may be everything in process (e.g., some types of chemicals, pulp, food processing, ceramics), depending upon the length of the outage.

$^{1/}$ Several people mentioned possible equipment damage due to power surges following outages, or carelessness when cleaning spoiled product out of equipment, but no one could quantify either so they are excluded from the following for illustrative simplicity.
For a given industry let us define:

\( a \) = length of time until goods in process are unable to be finished and/or sold

\( b \) = average percentage of annual production in process at any given time

\( Q_s \) = sales value of total annual production

\( n \) = number of outages per year caused by generation failure\(^1\)

\( d \) = average duration of outages caused by generation failure\(^1\)

\( (1) \) Then: \( \text{LPC} = nbQ_s \) if \( d \geq a \)

\( \text{LPC} = 0 \) if \( d < a \)

In order for equation \((1)\) to be an exact expression for the LPC in past periods it should be modified so that \( n \) represents the number of actual outages whose duration exceeded \( a \). If there is reason to believe that \( n \) is not randomly distributed this modification is a crucial one. However, since the eventual purpose in deriving this expression is cost projections for the various \( (n, d) \) combinations implicit in alternative possible levels of future generating plant capacity, the average duration is the only datum which will be available.

It is worth noting that while the LPC may be very high for some industries it does have an upper limit for the economy as a whole (corresponding to a given outage duration), because firms with high product losses from outages will choose to invest in standby generating equipment at the point where those losses exceed the cost of the standby unit. In Jamaica most of the large firms in the mining, sugar refining, rum distilling and cement industries had purchased their own units, which is reflected in the fairly low loss ratios of those industries (see Annex 1, Tables A1 and A2).

\(^1\) For the more general question of total outage costs, all types of outages should be included.
(b) **Restart Costs**

The second type of cost, lost production due to restart activities, will also be zero for some industries where work can resume immediately. For others the recovery period may be a fixed length (e.g., where motors must be manually restarted) or a function of the outage duration (e.g., where furnaces or kilns must be brought back up to specified temperatures or pressures). Thus the annual restart costs for a given industry can be represented as:

\[ RSC = \frac{ns \cdot \left(\frac{pQ_v}{h}\right)}{h} \]

where:
- \( h \) = hours of normal operation per year
- \( s \) = length of restart time (where perhaps \( s = f(d) \))
- \( p \) = percentage of normal output not produced during an outage \( f \) (or restart activities following the outage)
- \( Q_v \) = annual output in value added terms

For industries where \( s = 0 \) the RSC will, of course, also be zero.

(c) **Product Production Costs**

The final type of cost incurred during an outage is the product production cost. Virtually every businessman contacted indicated that at least labor became a fixed cost during an unscheduled outage, so that all input costs except the raw materials and intermediate goods normally used in process (including the electricity charges \( 1 \)) were incurred without the normal output being produced. Thus the loss is equal to the proportion of the entire value added that is affected by the outage:

\[ PPC = \frac{nd \cdot \left(\frac{pQ_v}{h}\right)}{h} \]

---

1/ Note that the value of \( p \) may be derived from psychological as well as technological parameters. Several Jamaicans confirmed that work virtually stopped during past outages even in government and other "paper pushing" offices with adequate natural lighting. Many commercial establishments also closed for fear of increased theft.

2/ A complication arises when electricity billing includes both a maximum demand and a kWh charge since the former will probably not be affected by an outage. In any case, since electricity is an intermediate good (i.e., excluded from the value added of other GNP sectors) no marginal cost from it is incurred during outages.
The total cost accumulated during the year's outages is equal to \((LPC + RSC + PPC)\) as defined. However, there was a general consensus that at least at present in Jamaica there is quite a bit of excess capacity, particularly in the small-scale industrial and commercial sectors, so that some portion of the lost output would be made up during regular working hours (i.e., without using any additional resources); then this made up production (MUR) can be represented as:

\[
MUR = n \left( d + s \right) \left( \frac{p}{h} \right) c
\]

where \(c \leq 1\) since total demand is assumed unchanged. 1/

If MUR is less than the lost production (i.e., if \(c < 1\)) and if the industry being considered is not already operating 24 hours per day, it has an option to make up the remainder by working overtime. Presumably the decision of how much lost production will be made up on overtime will depend upon the relationship between the cost of that lost production and the overtime costs. If the industry chooses not to make up all of the lost production during overtime it must be because the profit maximizing (and therefore net cost minimizing) output level given overtime rates is less than the normal profit maximizing output. Thus the cost of making up for all lost production during overtime will represent the maximum cost of outages (excluding the LPC) for all those industries normally operating less than 24 hours per day. 2/ Where round-the-clock operations make overtime impossible (in the short run) the outage cost will still be \((LPC + RSC + PPC - MUR)\).

In Jamaica legislation requires that labor be paid 1.5 times its hourly wage during overtime, and there are no off-peak power rates to offset this expenditure. While it would be theoretically better to use shadow labor and other input costs rather than actual, the following assumes that there is no wage distortion.

1/ A firm with high inventory levels would presumably have a higher value for \(c\) since a longer time period could be used to make up the lost production without defaulting on orders or disrupting normal processes.

2/ For industries that are price-takers this is probably the typical case. In the long run, outage levels would be a factor in determining the optimal labor-capital mix of production technology.

3/ This analysis does not consider cases of operations where the available hours for overtime are insufficient to restore the lost production on the assumption that the feasible range of annual hours of outage would not encounter them.
If $h_o$ represents the hours of overtime worked; $h$, the regular hours worked; and $L$ is the annual labor bill for normal operation, then the cost of overtime (CO) is:

$$CO = 1.5 \left( \frac{L}{h} \right) h_o + \left( \frac{Q_v - L}{h} \right) h_o$$

(5) or

$$CO = h_o \left( \frac{Q_v + 0.5L}{h} \right)$$

This expression is a general one which assumes that all factors of production which are included in value added become fixed costs during an outage and then must be paid again (at normal rates for all factors except labor) for any overtime use. For some individual firms labor may be the only incremental cost. For the economy as a whole, however, idle capital and buildings during an outage have an opportunity cost just as labor does, albeit not generally at a differential overtime rate since no leisure alternatives exist. Only the costs of intermediate goods and other purchased inputs (such as electricity) are "saved" during an outage and thus available for use later at no extra cost.

We can solve for the maximum $h_o$ by assuming that the total lost production (valued at RSC + PPC) less that made up during regular hours (MUR) is produced on overtime:

$$RSC + PPC - MUR = h_o \left( \frac{Q_v}{h} \right)$$

$$ns \left( \frac{p_Q}{h} \right) + nd \left( \frac{p_Q}{h} \right) - nc \left( d + s \right) \left( \frac{p_Q}{h} \right) = h_o \left( \frac{Q_v}{h} \right)$$

(6) $$h_o = np \left( d + s \right) \left( 1 - c \right)$$

Thus the maximum annual cost of electricity outages (MXC) can be represented as:

$$MXC = LPC + CO$$

(7) $$MXC = LPC + h_o \left( \frac{Q_v + 0.5L}{h} \right)$$

when $h_o$ is defined by Equation (6).

---

1/ A minor qualification is that in periods of very tight demand $L$ may be significantly biased upward by the inclusion of overtime work. This qualification would also affect $Q_v$.

2/ For many individual firms as well, factors other than labor have overtime costs. Equipment rental and maintenance contracts for example, sometimes have penalty clauses for use outside of normal business hours. Buildings must be heated or air conditioned during overtime operations and machinery and property maintenance may increase as well.
While some portions of the economy may find the outage costs of lost production so high that they react by making up everything during overtime (and thereby incurring MOC), other portions may lie on the opposite extreme and choose to work no overtime at all, thereby incurring simply the cost of lost production (MOC).

\[
\text{MOC} = \text{LPC} + \text{RSC} + \text{PPC} - \text{MUR}.
\]

Of course, if MUR is equal to the sum of RSC and PPC (which will be the case if \( c = 1 \)) and if LPC equals zero, then no production would be lost from the outage so that MOC will also equal zero.

Equation (8) can also be expanded into an expression in terms of \( h_0 \) as defined by Equation (6), although the intuitive interpretation is not as clear since MOC applies when no overtime is worked. Thus in the expression below \( h_0 \) should be viewed as the effective/ lost hours of production rather than the actual hours of overtime.

\[
\text{MOC} = \text{LPC} + h_0 (\frac{1}{aF})
\]

The responses of individual firms to outages will vary in accordance with many factors including the relative importance of labor costs, the price elasticity of demand for the firm's product and the size of incremental overtime costs other than labor. In aggregating over the economy (and thereby weighting each firm by its contribution to GNP) these varying responses will result in a total incremental outage cost that also lies between the aggregated MOC and MOC.

IV. Application to Reserve Determination

Now that the range of outage costs to the economy has been defined for both household and business sectors, it is necessary to integrate that information with reserve cost data to determine the net cost minimizing level of generating reserve. The steps involved in finding the optimal level of excess generating capacity are as follows:

(a) Calculate the feasible \((n, d)\) combinations resulting from the various excess reserve levels under consideration using the appropriate computer programs.

1/ Adjusted for the proportion of production possible during an outage and the amount made up later during regular hours.
2/ In a perfectly competitive economy where all firms are producing at their long-run equilibrium positions (and thus earning zero profit above the returns to the factors of production) one would expect the outage cost to be close to MOC since in the short run no one firm could raise its price to cover the additional costs of overtime production.
3/ This analysis could also be extended, of course, to compute total system reserve needs.
(b) Calculate the MOC and MXC resulting from each \((n, d)\) combination and divide by the corresponding kWh lost\(^1\) by commercial and industrial sectors (i.e., weight by their share of the total power consumption).

(c) Add to the scaled MOC and MXC the cost per kWh to residential users (weighted by the proportion of residential to total power consumers) as described in Section II to arrive at total incremental outage costs \((\text{MOC}_T \text{ and MXC}_T)\) per kWh lost.

(d) Calculate the annuitized annual cost (including both capital and operating costs) of each alternative reserve level and scale by the corresponding kWh lost.\(^1\)

The optimizing procedure entails equating the penalty function (the area bounded by MOC\(_T\) and MXC\(_T\)) with the marginal reserve cost as shown in Figure 1 below. If these three equations were continuous (as shown in the diagram), a range of optimal generating capacities\(^2\) would be defined corresponding to the levels of \(K_X\) and \(K_0\) with the optimal point dependent upon the characteristics of the economy which would determine whether MXC\(_T\) or MOC\(_T\) more closely approximated the actual outage response of firms. Any movement toward higher reserve levels would result in spending more for the incremental reserve than the cost of the incremental kWh lost. Similarly, at lower levels a net gain could be achieved by increasing the reserve.

\[\text{MOCT/kWh lost} \quad \text{MOC}_T/kWh lost \quad \text{Cost of reserve/} \quad \text{kWh lost} \quad \text{kWh lost} \quad \text{kWh lost} \]

Figure 1: Determination of Optimal Reserve

\(^1\) Scaling could instead be done by the customer outage minutes if such figures are more readily available from the computer printout.

\(^2\) Note that the horizontal axis, kWh lost, can be mapped directly into megawatts of excess capacity through load duration functions.
In an actual application of this method the MXCT, MOCR and reserve cost curves will be step functions rather than continuous curves since the feasible combination of generating reserve will usually consist of several discrete blocks rather than highly divisible combinations. In addition, each feasible point on the horizontal axis will have an associated (n, d) combination determined by the computer which can be used to define corresponding points on the MXCT and MOCR curves. Thus a more realistic representation of this optimization process might look like Figure 2 where K* clearly corresponds to the optimal reserve choice.

![Figure 2: Stepped Representation of Reserve Determination](image)

\[
\begin{align*}
\text{o} &= \text{MXCT/kWh} \\
\text{x} &= \text{MOCR/kWh} \\
\text{•} &= \text{Cost of reserve}
\end{align*}
\]

V. **Implied Generation Outage Costs in Jamaica in 1975**

While a sample application of this entire process to determine an optimal generating plan is clearly beyond the scope of this paper, this section illustrates how the MOCR and MXCT can be calculated and used in retrospect to find the cost of generation outages to the Jamaican economy in 1975.

1/ This is particularly true of developing countries whose systems are composed of relatively few units.
Annex 1 presents the sample data used to calculate the MOC and MOC corresponding to the recorded generation outage figures for 1975 (see Table 2).1/ JPSC had no information on the average length of generation outages, but indicated that they very rarely lasted longer than one hour. Since the total customer outage minutes (from generation failures) and the number of generation outages were known (86,764,080 and 25 respectively) the average proportion of customers affected could be calculated for the cases where d = 1 hour and d = 0.5 hour. If the average outage duration was one hour, an average 35 percent of JPSC customers was affected per outage.2/ If generation outages lasted an average of one-half hour then 70 percent of all customers would have lost power per outage. Since commercial and industrial consumption accounts for about 60 percent of total energy sales, one such user would be affected by 21 percent and 42 percent of total outages respectively.3/ Therefore, the two cases considered were (n, d) = (5, 1) and (n, d) = (10, 0.5).

Table 2: Generation Outage Costs in Jamaica in 1975

<table>
<thead>
<tr>
<th></th>
<th>Case I</th>
<th>Case II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(n, d) = (5, 1)</td>
<td>(n, d) = (10, 0.5)</td>
</tr>
<tr>
<td>MOC</td>
<td>4,193,400</td>
<td>4,614,700</td>
</tr>
<tr>
<td>MOC/kWh lost by businesses2/</td>
<td>1.40</td>
<td>1.54</td>
</tr>
<tr>
<td>MOC/kWh lost by businesses2/</td>
<td>1.51</td>
<td>1.65</td>
</tr>
<tr>
<td>Residential cost/kWh lost by residences2/</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>MOC/total kWh lost3/</td>
<td>0.87</td>
<td>0.95</td>
</tr>
<tr>
<td>MOC/total kWh lost3/</td>
<td>0.93</td>
<td>1.02</td>
</tr>
</tbody>
</table>

1/ kWh lost was calculated by assuming: kWh lost = customer outage minutes
kWh produced total customer minutes

and that the losses to business and residential consumers are proportional to their respective consumption.

2/ That is, the average customer experienced 35 percent of the total number of outages.

3/ JPSC indicated that they had very little scope for implementing selective outages so that a simple weighted average of residential and non-residential customers should not be systematically biased.
VI. Conclusions

These calculations performed to derive the results shown in Table 2 are mainly illustrative and proceed from data which encompass many assumptions (see footnotes of Table A-1). However, they do permit several practical generalizations:

(a) While the MOC\(_T\) and MXC\(_T\) per kWh lost are not directly comparable to the types of estimates done in previous studies which do not differentiate between various \((n, d)\) combinations and which scale costs by total electricity consumed rather than lost, the range of J$0.87 to J$1.02 is generally consistent with earlier findings.

(b) Both the MOC and MXC are fairly insensitive to the \((n, d)\) combination as long as the duration remains relatively short (i.e., less than that necessary to incur LPC for most industries). Nevertheless, the lost product costs account for over 40 percent of the MOC and MXC even in Case II where \(d = 0.5\). If the average duration is allowed to increase significantly (say, to five hours) as reliability is lowered then outage costs would climb sharply.

(c) The range bounded by the MOC/kWh and the MXC/kWh is a fairly narrow one, in this case only ± 3.8 percent from the mid-point. This seems surprising at first, but is readily explainable upon closer examination. There are four industries in the GNP breakdown which exhibit a large discrepancy between their MOC and MXC: agriculture, construction, transportation, and hotels, restaurants and clubs. As expected, these are all highly labor-intensive industries, but partly because of that labor bias, they are also areas where production is not drastically curtailed during an outage. Therefore, the weightings of their contributions to the total outage costs are low so that the corresponding differences between MOC and MXC are relatively small. From a practical point of view this is fortunate since it greatly reduces the chances for ambiguity in the determination of the optimal reserve investment. From a theoretical point of view it underscores the interdependence of decisions in seemingly unrelated areas: a developing country with a relatively low wage-rental ratio is likely to experience lower outage costs at the same time that its labor bias is reinforced by the low reliability (which is economically optimal) of its power system. Too large an investment in generating reserve would not only be costly in the short run but would also encourage a shift toward more capital intensive means of production in the long run which would be out of line with the country's resource endowment.

To be of maximum use to the project economist or engineer who must make a practical estimate of outage costs with a limited investment of field time, some short-cuts to this method may need to be developed. In homogenous economies or cases where only a few sectors use electricity
it should be possible to obtain estimates for the necessary parameters
directly. In highly diversified economies where power consumption is
widespread, it would be helpful to have mean values available by in-
dustry for the technological parameters. For example, the time until
production is lost (a), the percentage of production in process (b),
and the re-start times (s) for most industries are unlikely to vary
much across countries. Since several of the other variables are taken
directly from GNP accounts, the main ones left to estimate would be
the percentage of output not produced during an outage (p) and the
percentage which could be made up without overtime (c). The former
will depend upon the labor-capital mix of the economy and the preval-
ence of standby generators, while the latter will be a function of
the demand conditions and general degree of capacity utilization
across sectors. Thus, it may be possible to derive crude estimates
of p and c by using a sample unit vector representing inter-industry
relationships multiplied by a constant which describes the relative
labor bias or capacity utilization of the economy as a whole.

Work is currently underway within the Bank both to refine
the theoretical basis of this approach to measuring marginal outage
costs and to expand our practical knowledge of its cross-country
applicability. Once wider experience is gained, it should be possible
to routinely include "penalty functions" quantifying the economic costs
of lower reliability in the programming models which aid the engineer
in power system design, thereby insuring a least cost solution in the
macroeconomic as well as the technological sense.
### Table A-1: Input Values

<table>
<thead>
<tr>
<th>Industry</th>
<th><em>a</em> Time Until Production</th>
<th><em>b</em> Percentage of Process</th>
<th><em>c</em> Percentage of Output Not Made up</th>
<th><em>d</em> Time of Restart Production</th>
<th><em>e</em> Time During Outage</th>
<th><em>f</em> Percentage of Output Not Produced</th>
<th><em>g</em> Percentage of Production Restart</th>
<th><em>h</em> Hours of Normal Operation</th>
<th><em>Q_a</em> Annual Production Value Added (if <em>a</em> 1)</th>
<th><em>L</em> Annual Labor Cost (if <em>a</em> *)</th>
</tr>
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<tbody>
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<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<td>102.4</td>
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<td>178.7</td>
<td>102.4</td>
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<tr>
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<td>102.4</td>
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<td>Hotels, Restaurants &amp; Clubs</td>
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<td>102.4</td>
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<td>178.7</td>
<td>102.4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1/ Captive plants prevalent.
2/ The major firms in this sector have stand-by generating equipment.

Note: Values for *a*, *b*, *c*, *h* are tentative estimates derived from discussions with firms which may or may not be representative of their sectors, and the general observations of a JPSC representative. The value of *b* is probably an overestimate based on the firms contacted, but it was chosen to avoid downward bias. *P* is based partly on the estimates from Mr. Moscote's April 12, 1976 memorandum but modified where the present definition seemed at variance with his undefined "percent sensitivity to JPSC outages" and where stand-by equipment was prevalent. *Q_a* and *L* are from the 1975 Jamaica National Income and Product publication, Tables 1 and 6, respectively. *Q_a* is estimated assuming value added components for food processing (.50), printing (.20), chemicals (.20) and cement (.50).
### Sample Data

#### Table A-2: Cost Calculations

<table>
<thead>
<tr>
<th>Industry</th>
<th>Case I ((n, d) = (5, 1))</th>
<th>Case II ((n, d) = (10, 0.5))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LPC ((\text{J$'000}))</td>
<td>No ((\text{hours}))</td>
</tr>
<tr>
<td>Agriculture, Forestry &amp; Fishing</td>
<td>-</td>
<td>0.1</td>
</tr>
<tr>
<td>Mining and Quarrying</td>
<td>-</td>
<td>0.4</td>
</tr>
<tr>
<td>Manufacture:</td>
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<td></td>
</tr>
<tr>
<td>Food</td>
<td>767</td>
<td>8.0</td>
</tr>
<tr>
<td>Sugar, Molasses &amp; Rum</td>
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</tr>
<tr>
<td>Alcoholic Beverages</td>
<td>-</td>
<td>2.5</td>
</tr>
<tr>
<td>Non-Alcoholic Beverages</td>
<td>-</td>
<td>2.5</td>
</tr>
<tr>
<td>Tobacco &amp; Tobacco Products</td>
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<td>Textiles &amp; Made-up Textiles</td>
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<td>Footwear</td>
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<td>Wood &amp; Wood Products</td>
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<td>Electric &amp; Water</td>
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<td><strong>Total:</strong></td>
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