Incorporating Uncertainty into Planning of Industrialization Strategies for Developing Countries

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Explicit incorporation of uncertainty into policy analysis is crucially important for many issues. For example, development planners have long recognized that the typical five-year plan quickly becomes outdated as unforeseen events arise, but they have so far lacked tools to reflect this adequately in either planning or implementation. In an era of increased uncertainty and greater fluctuations in international prices and supplies, it is particularly warranted to develop analytical and empirical tools for contingency planning. More specifically, planners need mechanisms to formulate adjustment rules that are consistent with the desired trajectory of the economy and which specify how the economy can best adapt to unforeseen departures from forecast trends.

This paper provides a survey of the existing literature on planning under uncertainty, focusing on issues of international trade and investment allocation. It discusses various ways of incorporating uncertainty in target-planning models and outlines proposals for possible empirical applications. In these proposed applications, the emphasis is on formulating adjustment rules that can be used to ensure that the economy stays as close as possible to a planned target path given random shocks to exogenous variables, but assuming that the underlying structure of the economy is known with certainty. Such models can be used to analyze policy questions such as: (1) does the stabilization of an economy embarked on an import-substitution development strategy require more or less active intervention than the stabilization of one embarked on an export-led strategy? and (2) given a particular development strategy, does the need to adjust to random shocks require a significantly different allocation of investment than if there were no such uncertainties?
1. Introduction

Most of the theoretical and practical planning work to date has dealt with planning under the assumption of certainty. Projections of likely paths of exogenous variables are made and then the plan is formulated to fulfill some objectives given the assumed structure of the economy and the projected paths of the exogenous variables. The plan is solved for a few alternative scenarios and for a few combinations of "high, medium, low" forecasts of the most important exogenous variables. The plan is then based on the "medium" variant, and the information on how the investment strategy differs under the alternative variants is used to educate the intuition of the Planning Office to cope with different policy regimes.

It is known a priori that the exogenous variables and the behavioral representation of the economy will deviate from their assumed values by some random magnitudes; but this information is not taken into account systematically in the initial preparation of the plan. Therefore, when during plan implementation unforeseen contingencies arise, the original plan is usually hastily adjusted to the contingency or even completely scrapped. In principle, it would be possible to engage in rolling planning at this juncture; but in practice the annual recomputation of the entire sectoral plan under the different realizations of the forecast variables is never carried out. By this time, the original planning furor has usually dispersed; the plan has served its political function (and, hence, the attention of the chief
policymakers has shifted from it); and short-term stabilization considerations, arising at least partly out of the unforeseen contingencies, take over the center of the stage in economic policy analysis. Instead of reformulating the plan, it is hastily patched up. If the disturbance imparted by the unforeseen event(s) is large, the national plan is abandoned entirely.

Therefore, what is needed to make technical planning have more lasting relevance for the actual management of the economy in this era of increased uncertainty and greater fluctuations in international prices and supplies is a set of adjustment principles or rules formulated at the same time as the original plan itself and in a manner that is consistent with the desired trajectory of the economy, specifying how the economy could best accommodate to unforeseen departures from forecast trends.

The absence of good contingency planning—namely, planning that allows for subsequent adaptation to random events—is a major reason why formal planning is perceived by most policymakers and heads of planning offices to be an essentially irrelevant exercise, useful mostly for window dressing and political mobilization. Under current planning practice, it is as if a flight engineer were taught how to fly an airplane only under ideal wind and visibility conditions and given no training in how to adjust to turbulence.

The purpose of this paper is to propose a methodology, based on the ideas of stochastic control theory, that can be applied in an operational manner for devising a set of adjustment rules for coping with actual departures from the forecast conditions under which the plan is formulated. Such a set of rules, formulated in conjunction with the original plan and aimed toward keeping the economy on its plan trajectory, would greatly enhance the practical relevance and lasting power of a technical plan as a document for guiding the process of
resource allocation and policy formulation in an economy. In addition, an analysis of characteristics of these adjustment rules could also increase our insights into the robustness of the original development strategy underlying the basic plan. For example, it may reveal that some strategies may require a more active (interventionist) policy stance than others. (This is the intuitive argument currently used by neoimport-substitution advocates.) It may indicate that some instrument variables offer more potent controls for keeping the economy on target than others. (This is one of the arguments between free traders and their opponents.) This type of information would greatly enhance the understanding of development strategy and policy analysis in developing countries under the current international regime and offer a basis for better policy advice to developing countries.

There are two different classes of uncertainty which an economy faces. One class consists of stochastic fluctuations of a recurrent nature, such as yearly swings in international prices of raw materials or domestic agricultural yields. The second class consists of very unlikely events of potentially catastrophic consequences, such as an embargo on exports or imports or an earthquake. The distinction between these two classes of uncertainties is important because they require different conceptual treatments. A recurring event can be dealt with by means of a feedback type of adjustment rule, while a highly unlikely but major event usually gives no prior signals which one can use as an early warning system for changing immediate plans prior to the realization of the catastrophe.

The class of uncertainties upon which we intend to focus in this paper is the first. We feel that the problem of adapting medium-term plans to unforeseen, relatively small random variations about a reasonable forecasted
trend is one of the major problems that must be solved in order to increase the relevance of technical economic planning for development planning. Currently, technical five- or seven-year plans are usually scrapped within the first or, at most, the second year of the implementation period as the accumulation of unplanned contingencies makes them increasingly irrelevant to current realities and planning teams are not continuously available to redo the planning exercise. If, on the other hand, guidelines for continuous adaptation were provided at the beginning of the plan, it would not become so quickly outmoded.

The focus of the paper on recurrent events is not to belittle the importance of catastrophic shocks to an economy. However, such events usually change the underlying structure of the process, and there is almost no a priori way to foresee the structural shifts so as to accommodate them in a planning exercise. This point brings out the issue of "learning" and the related issue of "ignorance" versus "uncertainty." This paper focuses on the consequences on an economy of external or internal "uncertain" events where future values are not known for sure but for which some subjective or objective probability distributions can be specified, given that the underlying structural model for the economy is known with certainty. It is, of course, true that no empirical model is an exact representation of reality and, furthermore, that the underlying economic structure might be changing over time. While the issue of identifying and estimating the correct structural model, as well as detecting structural shifts over time, is a very important one, our interest and focus lie in adaptation of plans to uncertainties caused by economic forces—not by modeling inadequacies. For instance, we know that an unexpected rise in the international price of a country's major export will have a short-run impact
on the balance of payments and the availability of investment funds, although we might not be able to specify the exact magnitude of the impact. The relevant question in this case from our point of view is how the plan should allocate the extra savings among sectors and not whether the extra savings would be, for instance, $1 versus $10 billion.

It is true that a structural shift in the economy would change the adjustment rules. It would, however, also change the underlying model on which the initial plan was based hence necessitating the repetition of the whole initial planning exercise as well. Detecting structural shifts is a very difficult problem which we shall not deal with in this paper. We shall be concerned mainly with adjustment rules assuming that a target path for the economy has already been devised. Uncertainty, i.e., the possibility that the future values of exogenous variables will be different from some "mean," has implications both for the selection of the optimal target path and for the adjustment rules. While much of the theoretical literature that will be reviewed is concerned with the selection of the optimal target path under uncertainty, the empirical determination of such paths is a very difficult and as yet unsolved problem (as will be illustrated in section 3). The problem, however, of adjusting the economy to withstand the impact of uncertain shocks and stay close to some (albeit, not necessarily optimal) target plan is relatively easier to solve (and is discussed in section 4).

The remainder of the paper is organized as follows. First, in section 2, we review the literature on trade and economy-wide production and investment planning under uncertainty. Then, in section 3, we discuss some ways of incorporating risk in target-planning models. In section 4, we outline a
methodology for computing sequential investment adjustment rules in a planning context. Section 5 contains the conclusions and caveats from the analysis.

2. Previous Literature on Trade and Investment Planning Under Uncertainty

Before we embark on a discussion of the different problems tackled in the previous literature, it is useful to restate the problem which motivates our survey. The problem is one of allocating production and investment resources in different sectors of an open economy under conditions of recurrent uncertain future events. The investment-planning problem under uncertainty is different from that under certainty because decisions must be taken ex ante, that is, before the future is revealed. In practical planning exercises, decisions are also designed to be taken before the future is known. The standard practice is to ignore all possibilities of future policy adaptations and, furthermore, to set all uncertain variables equal to their mean values and deal with planning models under certainty (see, e.g., Blitzer, Clark, and Taylor [10] for a thorough survey of current planning models in which uncertainty is neglected). It is becoming appreciated, however, that such a practice has severe implications, such as the increased frequency of choice of irreversible investments (see Henry [21]). Typically, when uncertain states of nature reveal themselves, some adjustments are necessary to keep an economy on an even keel. It is the possibility of these ex post adjustments that needs to be taken into account in ex ante planning exercises.

In our review, we concentrate mainly on a survey of the literature of trade under uncertainty. There are several reasons for this emphasis. First, it is primarily through trade that the economy can radically alter its structure of production; without trade, the economy is constrained to a structure
of production which matches the structure of demand and, therefore, only fairly marginal production adjustments are possible. Second, much of the uncertainty—at least as perceived by national policymakers and planners—originates in international commodity, foreign exchange, and capital markets. Third, it is mainly the trade-under-uncertainty literature that has attempted to link adjustments in the national economywide production profile to uncertainty.

The only other literature at the economywide level on planning under uncertainty relates to short-run macroeconomic stabilization issues. This literature addresses quite different questions from the ones relevant to our problem—namely, ones of resource allocation—and uses very different policy instruments than would be relevant for the analysis of investment policy under uncertainty. Therefore, it is not reviewed here (for a useful review, see Johansen [24]).

As will be seen below, all of the literature on trade under uncertainty concerns itself with the ex ante selection of an optimal production plan for an economy facing uncertain terms of trade or uncertain production. As such, it is highly relevant to target-path planning under uncertainty rather than to the planning of adaptive adjustment rules for maintaining the target path. One would expect this literature to yield some intuitive principles upon which a planner could determine, in the absence of more detailed empirical calculations, how he should shift his investment plan given that some industries present more uncertainties than others. Unfortunately, the theoretical literature does not yield such principles because, as will be seen below, it is formulated in such a way that its results depend more on the characteristics of demand than on those of production. In addition, the major effort
in this literature has been in discovering conditions under which the time-
honored Heckscher-Ohlin (HO), Stolper-Samuelson (SS), Factor-Price Equiliza-
tion (FPE), and Rybczynski (R) theorems of international trade under certainty
hold or fail under conditions of price and production uncertainty. Because
these propositions are only marginally relevant to our major theme, we will be
highly selective in our review and shall emphasize only the aspects relevant
to our problem (for other reviews of this literature, see Pomery [35],
Helpman and Razin [20], and Cheng [15]).

For the effects of uncertainty to be subject to analysis, there are two
conditions that must be met. The first is that the set of uncertain events or
states of nature must be specified. As an example, it is futile to blame bad
planning in less-developed countries (LDCs) for their inadequate grain stocks
in 1973 when the relevant event (that the huge U. S. grain stocks would become
unavailable overnight) was not even considered to be within the realm of the
feasible. In other words, it was not even thought to be in the universe of
possible events. Enumeration of the possibilities that can arise in the un-
certain future is not a trivial task, but it is the basis for any kind of
analysis. Should a real price of oil of $1,000 a barrel be considered within
the set of feasible events in the next 10 years? If it is not and if a real
price of $1,000 a barrel becomes a reality, then all a planner can hope for is
that his economy has enough flexibility or "reserves" of resources to cope
with the unexpected event.

The second condition that must be met in order to make the analysis of an
uncertainty problem possible is that there are probabilities attached to the
possible events or states of nature. These probabilities might be objective
or subjective, but they must, nevertheless, be given. For the remainder of the paper, we assume that these two conditions have been met.

The theoretical literature suggests that the results depend quite critically on the particular assumptions one makes as to which policy instruments adjust \textit{ex ante} and which adjust \textit{ex post}; we divide our discussion accordingly.

2.1. \textbf{Ex Ante Trade Models}

The general structure of the \textit{ex ante} trade models for a single price-taking country is the following. If $x_1$ and $x_2$ are the \textit{ex ante} quantities of goods 1 and 2 that are committed as exports and $p_1$, $p_2$ are the prices of the goods, then by the \textit{ex post} balance-of-payments equilibrium condition, the physical imports of goods 1 and 2 in the second period will be equal to $p(a) x_2$ and $x_1/p(a)$, respectively, where $p = p_2/p_1$, and $a$ is the state of nature in the second period on which uncertain terms of trade depend. Assume the existence of a social welfare function $U(c_1, c_2)$ where $c_1, c_2$ are the quantities of the two goods that are consumed in the second period. Let $q_1$ and $q_2$ be the quantities of the goods produced in the second period. The production and trading decisions, $q_1, q_2, x_1, x_2$, are assumed to be determined in the first period, i.e., \textit{ex ante}. The problem to be solved is:

$$\max_{x_1, x_2, q_1, q_2} E \left[ U(q_1 - x_1 + p(a) x_2, q_2 - x_2 + \frac{x_1}{p(a)}) \right]$$ \hfill (1)

subject to $T(q_1, q_2) = 0$ where $T$ is the country's transformation frontier and $E$ denotes the expectation operator.
The major results of this line of research that includes contributions by Brainard and Cooper [13], Bardhan [4], Batra and Russell [8], and Ruffin [38] are twofold. First, the so-called nonautarchy theorem is derived (Ruffin [38]), where zero trade (i.e., \( x_1 = x_2 = 0 \)) is never an optimal solution. The second result obtained is that more price uncertainty leads to less production of the export good than under certainty.

While this line of inquiry is relevant to investment planning, the institutional framework it posits is unrealistic. In practice, the sectoral allocation of investment is planned usually for a minimum of five years ahead, while trade commitments are not made until much later. The analysis is applicable only to the case of investment coupled with long-term export contracts. What the theory suggests in this case is that, if the contracts do not specify price, the country should invest less in the sector producing the randomly priced export good than if the expected price were to hold with certainty; furthermore, a nonzero a\text{ priori} contract to trade is preferable to no trade at all.

2.2. Ex Post Trade Models

The ex post trade literature permits all of the trade decisions to be undertaken after the resolution of external price uncertainty while all production decisions must be taken ex ante. This framework comes a little closer to the spirit of the problem at hand.

The general structure of these models is as follows: Suppose there are two goods, each produced by two factors denoted by K and L according to some well-behaved production function. Suppose that, in the second period after the resolution of uncertainty, \( a \), production of these two goods is known and is \( q_1(a) \) and \( q_2(a) \). Also, the externally fixed terms of trade,
\[ p(a) = p_2(a)/p_1(a), \] are revealed. Then the \textit{ex post} problem is to find appropriate amounts, \( c_1 \) and \( c_2 \), of the goods to be consumed so that some index of social welfare \( U(c_1, c_2) \) is maximized.

Formally, the \textit{ex post} problem is to choose the consumptions \( c_1 \) and \( c_2 \) so as to

\[
\max_{c_1, c_2} U(c_1, c_2)
\]

subject to the constraint that real \textit{ex post} expenditure has to be equal to the real \textit{ex post} income

\[
c_1 + p(a)c_2 = q_1(a) + p(a)q_2(a) = I(a)
\]

where by \( I(a) \) we denote the amount of \textit{ex post} real income achieved by an appropriate \textit{ex ante} allocation of the fixed total amount of the factors \( K \) and \( L \) among the two sectors. Problem (2) is a standard deterministic one since \textit{ex post} \( a \) is known. Denote the optimizing solutions by \( c_i[p(a), I(a)] \), \( i = 1, 2 \). If one substitutes these into the welfare function, one obtains the indirect utility in period (2)

\[
U\left[ c_1[p(a), I(a)], c_2[p(a), I(a)] \right] = V[p(a), I(a)].
\]

The first-period \textit{(ex ante)} problem is then to choose the factor allocations among the sectors so as to maximize the expected value of the indirect utility, \( V \), in (3). The fact that \( a \) is included in the expressions for \( q_1 \) and \( q_2 \) in (2) means that technological or other production uncertainty is allowed. Simplifications of the model are possible if, for instance, one of
the factors is eliminated, if production uncertainty is absent, or if price uncertainty is absent.

Earlier models of this sort were one-factor models (Kemp and Liviatan [27], Ruffin [39], Turnovsky [41], and Kemp [26]) with labor as the only factor of production and with no technological uncertainty. One of the main results derived from them is that the country could choose to be diversified in production under uncertainty, while under certainty it would be specialized in the production of only one good. The intuitive reason for this is the following. If the external terms of trade (which are not a function of quantity traded under the small-country assumption) hold with certainty, then generally the external price of one good in terms of the other will always be higher than the domestic rate of transformation leading to specialization. If external prices fluctuate, however, while the domestic rate of transformation is nonstochastic, there might be some states of nature which, if they were to hold with certainty, would imply specialization in the other good. Depending on how frequently these states of nature occur and depending on the curvature of the country's welfare function, the country should avoid having all of its production in one good so as to be able to take advantage of profitable trade opportunities when "abnormal" states of nature occur.

Similar reasoning also leads to the possibility of a "reversal of specialization" where, under certainty of external terms of trade, a country specializes in the production of one good while, under uncertainty, it might well be optimal for it to specialize in the production of the other.

Ex post trade models with two factors of production both chosen ex ante, as opposed to one factor in the Ricardian models reviewed above, include those of Batra [6], [7]; Bardhan [4], [5]; Das [17]; Mayer [34]; and Anderson and
Riley [2]. With the exception, however, of the Anderson and Riley (herein abbreviated as AR) model, the rest deal with the conditions under which the various standard positive theorems of international trade, such as the HO, SS, FPE, and R, hold under uncertainty. Conditions on the sources and distributions of uncertainties, as well as on the utilities of the participants in the economy, are derived that make the theorems hold or fail to hold under uncertainty. Since the validity of these theorems has little to do with the subject matter, those ex post models are not discussed any further.

Anderson and Riley's model represents the closest that the theoretical literature comes to our problem and, thus, we will analyze it more closely. They use a standard concave to the origin, nonstochastic production transformation frontier for two goods as opposed to a linear one in the one-factor models and an ex ante production, ex post trade model like the one exhibited above in equations (2) and (3), with no production uncertainties. They postulate the existence of a well-defined social welfare function. They show that uncertainty in the foreign terms of trade \( p_2/p_1 \) (represented by a higher variance of \( p_2 \) relative to that of \( p_1 \)) can induce a lower (higher) optimal production of good 2 as compared to a situation where terms of trade are at their mean with certainty, according to whether the expression

\[
E \equiv n_2 - (1 - S_2) R
\]

is positive (negative) for all feasible international prices. In (4), \( n_2 \) denotes the income elasticity of demand for good 2, \( S_2 \) the self-sufficiency ratio, \( q_2/c_2 \) (where \( q_2 \) is the amount of good 2 produced and \( c_2 \) the
amount of good 2 consumed), and \( R \) denotes the dimensionless (and positive) coefficient of relative income risk aversion [equal to

\[
- \frac{I a^{2V/aI^2}}{aV/aI}
\]

using the notation of equation (3)].

Condition (4), which we have cast in a format slightly different from that of AR for easy interpretation (and which has also been derived by Ruffin [39]), can be utilized for some interesting thought experiments to test one's intuition. If, for instance, a country is mainly a raw-material exporter, then \( S_2 > 1 \) and \( E > 0 \). Hence, increased uncertainty in the international price of raw materials would lead the country to reduce its relative production of these goods and shift toward production of the other (imported) good. This accords with intuition, namely, price uncertainty should shift production toward more price-certain goods. If a poor country is an importer of food-stuffs, then \( S_2 < 1 \) and the optimal production shift under terms-of-trade uncertainty depends on the relative magnitude of the income elasticity of demand for food and the relative income risk aversion of the country. If risk aversion is very large, then \( E \) could be negative, justifying a policy of food self-sufficiency under uncertainty. This also accords with intuition. If, however, the country is so poor that \( n_2 \) is very large, then \( E > 0 \) and the model prescribes lower production of foodstuffs for the food importer which seems somewhat counterintuitive. The result can be rationalized if it is realized that the model requires \textit{ex ante} production decisions that are fixed \textit{ex post}. In the face of low risk and high-income elasticity of demand for importables, increased \textit{ex ante} production of exportables (the good with the
relatively more stable international price) leads to higher expected \textit{ex post} income and, hence, higher expected \textit{ex post} imports and consumption of the needed importable.

Notice that condition (4) depends mostly on demand characteristics. This is because no \textit{ex post} production substitution is allowed in any of the \textit{ex ante} production models. Hence, the characteristics of the demand and the utility function tend to dominate the conditions that are derived with such models. This, of course, is a far cry from the planning reality where some factors are always mobile \textit{ex post} except in the extreme short run. A meaningful planning exercise should take these \textit{ex post} production flexibilities into account and the methodology discussed later does so.

Let us define the production flexibility of an economy as the ease of substitutability in the economy's production of alternative goods. An inflexible production frontier would be represented by a single point in commodity space, whereas a flexible one would allow for substitution among goods. In Figure 1, we illustrate the concept. Production frontiers AA, BB, and CC all produce at point R when the terms of trade are \( p \). When the terms of trade change to \( p' \), however, the inflexible frontier AA implies no change in the quantities of any of the produced goods. Among other frontiers, the more flexible one, CC, induces a larger absolute change in the production of both goods than does BB (\( H_c R \) and \( F_c H_c \) versus \( H_B R \) and \( F_B H_B \)).

As early as 1969, Berry and Hymer [9]—in a paper largely neglected in the subsequent literature—examined the problem of an economy's choice of a flexible versus a rigid \textit{ex post} production frontier. In other words, the problem there is not merely to choose \textit{ex ante} the \textit{ex post} fixed quantities of the
Figure 1. Production Structures With Different Flexibilities.
goods to be produced but to choose *ex ante* a whole production possibility frontier that is rigid or flexible *ex post*. What Berry and Hymer showed was that the *a priori* reasonable intuition that a flexible, structured economy with lower output under nonstochastic terms of trade, e.g., \( p \) (see BB in Figure 1) would suffer less from export price fluctuations than one with a rigid production structure but higher output at \( p \) (A'R'A' in Figure 1) is not necessarily true. The reason is that external price fluctuations lead to income mean variance trade-offs under the two production structures that cannot be ranked unambiguously. Ambiguous ranking occurs because one frontier does not lie strictly inside the other. If, for instance, frontier BB is compared with frontier ARA under uncertain terms of trade, then BB (the more flexible one) is always preferred to the rigid one.

Cheng [15] made a step in the right direction by considering an *ex post* production possibility frontier that is flexible (with the only restriction that it lies strictly inside the *ex ante* one). Cheng showed that the possible threat of a foreign market foreclosure (an embargo for a country's exports) is sufficient to move the country toward the autarchic point. Cheng viewed this result as a rationalization for recent self-sufficiency policies pursued by many LDCs, but the type of uncertainty that Cheng deals with in showing this result is not of the recurring type that concerns us here. In a later part of his thesis, however, Cheng--using a rather elegant and general theoretical framework--also showed that an increase in international price uncertainty would lead a country to adopt a more flexible production structure as expressed by the ease of transforming one machine from the production of one good to the production of another.
It would be desirable in future research to relate the *ex post* production flexibility more directly to the underlying technological relationships and relative factor mobilities in order to obtain more intuitively operational guiding principles for planning. Consider, for instance, a putty-clay production technology with the overall fixed quantity of labor freely movable *ex post* among industries while the allocation of capital among sectors is free *ex ante* but unchangeable *ex post*. Then two of the possible *ex post* partially flexible production frontiers are shown in Figure 2 as BB and B'B' while AA represents the envelope of all *ex post* frontiers. The problem would then be to choose among *ex post* frontiers of the type BB. Unfortunately, this problem, which would come closest to the subject of this paper, has not been dealt with in the literature. In section 3 we indicate the empirical difficulties of trying to solve this problem in an operational multisectoral dynamic context.

2.3. **Other Relevant Models**

In this subsection we review some other theoretical and empirical models that are related to our problem of investment planning under uncertainty and use different objectives and methodologies than the trade-theoretic ones reviewed above.

The earliest study of production and trade planning under uncertainty was the one by Brainard and Cooper [13]. Their approach was cast in a portfolio model of production (following the ideas of Markowitz [33]), and their objective was to maximize a linear function of the mean and variance of consumption. Using this function, they showed that production diversification was better than specialization. This result was, of course, to be expected given
Figure 2. Production Frontiers With Limited ex-post Flexibility.
our theoretical discussion in this section. Their approach, which was de-
signed for empirical analysis, has not been followed up in later works which
generalized the utility functions but sacrificed applicability.

Another strand of planning literature concerned mostly with closed, cen-
trally planned economies has considered the optimal allocation of investment
among sectors producing intermediate and final goods. Weitzman [43] shows in
a very simplified model with uncertainty in the input-output coefficients that
the proper intertemporal allocation of resources between production of inter-
mediate and final goods depends on the degree of uncertainty about the future
input requirements if the economy is not to experience shortages or excessive
reserves. He shows, furthermore, that the current level of material resources
points the direction of the plan proportions of the next period. His analysis
and observations are quite useful when applied to a closed economy; it would
be interesting to investigate his results when intermediate and raw material
imports are allowed should shortages occur and exports are possible when there
is overproduction of intermediate goods.

The question of whether improper output of a critical industry might lead
to fluctuation in economywide output in a closed economy and the related ques-
tions of the allocation of investment among inventories or capacity expansion
are the subjects of a paper by Brada [11]. The observation made above about
opening the country to foreign trade holds here also as the possibility of
substitution between domestic and foreign production in periods of supply-
demand imbalances might render the results inapplicable to LDC investment
strategies. The fact that Brada's results suggest that a larger ratio of
investment in inventories to investment in capacity expansion is warranted
under uncertainty is dependent on his closed economy assumption.1
A third strand of the literature on planning under uncertainty is at the one-sector level for developed countries dealing mostly with investment in nonrenewable resources and with corporate planning. We shall not review this literature here (for a typical example, see Gilbert [19]) but, instead, will concentrate on particular topics we feel are potentially relevant for empirical LDC planning.

One of the first empirical treatments of sectoral investment planning under uncertainty was by Manne [31]. In his stochastic model, demand for one product grew randomly; and the problem was the timing and size of fixed capacity plants. In the most general model of the paper, "backlogs" (possibilities for capacity not satisfying demand) are considered; and it is shown empirically that higher demand variance does not necessarily imply larger production cost if the size of the plant and the amount of backlog are adjusted optimally.

An immediate extension of Manne's model for one sector would be the possibility of satisfying the backlog by imports at uncertain world prices. Another possibility would be to extend his framework to two sectors with fixed total investment to be allocated between them. A further extension would be to consider the flexibility of the plant; namely, the trade-off between fixed and variable costs in the design of the capacity. This would constitute a stochastic extension of the deterministic choice-of-technique problem. Apparently, Manne's stochastic model has not been pursued or extended. Later work (see, e.g., Manne [32] and Westphal [44]) has concentrated on deterministic models.

Jabara and Thompson [22] recently attempted to incorporate uncertainty into a static linear programming model for Senegal's agriculture. Their
approach to dealing with uncertainty was rudimentary because they imposed a set of domestic prices that is equal to the mean of international prices (which are used under certainty) adjusted by a factor that is linear in the standard deviation. In other words, uncertainty enters in a purely mechanical way by essentially altering the equivalent external prices in an otherwise completely deterministic model.

Another partial-equilibrium attempt to incorporate uncertainty into development planning was made by Kornai [29] who used an objective function that depended on the expected value and variance of profits of the investment in one industry. However, his framework was strictly partial equilibrium (used basically to determine the optimal output of one sector) and could not be generalized to the more complex economy-wide setting.

The mean-variance objective function used by Kornai is one useful possibility in static planning problems. Other operational objectives, however, have been specified in what is known as the risk-programming literature.

Roumasset [36] gives a useful overview of the rules that are usually used in risk programming. One is the "safety principle," namely, minimizing the probability, $\alpha$, that some stochastic objective function, $R$, falls below some prespecified disaster level, $\bar{d}$

$$\text{Min } \alpha = \text{Prob}(R < \bar{d}). \quad (5)$$

Another rule is the "strict safety first principle" which is equivalent to chance-constrained programming CCP (see Charnes and Cooper [14]). The objective function, $R$, is minimized subject to a chance constraint of the type
\[ \text{Prob} \ (R < d) \leq \overline{\alpha} \]  

where \( \overline{\alpha} \) is now an exogenously specified probability level.

Finally, the "safety-fixed principle" introduced by Kataoka [25] implies maximizing the minimum return that can be attained with a fixed confidence level \( \overline{\alpha} \), i.e., maximize \( d \) subject to

\[ \text{Prob} \ (R < d) \leq \overline{\alpha}. \]  

Roumasset shows that other criteria used in the literature can be written as combinations of these rules. For instance, the confidence limit criterion that involves maximizing the expected value, \( u_R \), of the objective function, \( R \), subject to a constraint of the form

\[ u_R - k\sigma_R \geq d \]

where \( k \geq 0 \) and \( \sigma_R \) is the standard deviation of \( R \) is a special case of the strict safety-first principle.

There exists a well-developed literature on methodologies for solving such programming problems (see, for example, Sengupta [40] and Vajda [42]). Agricultural economists have already made use of these ideas and techniques (Roumasset [36]; Roumasset, Boussard, and Singh [37]), and there is no reason that national planners should not follow suit.

2.4. Conclusions

From the above review, it should be clear that all previous models for analyzing uncertainty are one-shot decision models with the interesting single decision usually to be taken \textit{ex anto}. As such, they do not really deal with
dynamic adjustment-planning rules. In the ex ante production ex post trade models of the AR type, for example, after the uncertainty is revealed, no adjustment is envisioned and the only constraint imposed on the model is the one of balance of payments. Translated into a multiperiod format, this model envisions that, if one imposed some ex post balance-of-payments constraint in every period, then the problem would be to choose ex ante the whole multiperiod production and investment structure with no possibility for ex post revision of investment and production plans. This is unrealistic. The approach, as reviewed, could be used to incorporate future uncertainty into the design of nonstochastic target paths but not for the ex ante design of ex post adjustment rules. No existing model analyzes the issue of planning with an uncertain structure as well as the related issue of learning while planning.

In the next section, we take up the general dynamic planning problem under uncertainty; in section 4, we consider the problem of designing (in the beginning of the plan) ex post adjustment rules, given a desired planned trajectory for the economy.

3. A General Dynamic Target-Planning Model Under Uncertainty

In this subsection we examine the structure of a very general, dynamic, stochastic investment-planning problem and illustrate the difficulties inherent in incorporating uncertainty in target planning. The formulation of the problem requires two things: an objective function which meaningfully incorporates uncertainty and a model describing the structure of the economy in a way that permits one to analyze the most prevalent sources of recurring uncertainty. We deal with the second problem first.
To summarize the structure of the economy, the simplest description would be that offered by an input-output framework. This approach, however, would not permit incorporating endogenous market-price uncertainties. It would allow one to deal only with foreign price fluctuations and, if the A matrix is taken to be stochastic, with production uncertainty. To incorporate domestic price uncertainty as well requires using a Computable General Equilibrium (CGE) model (examples can be found in Adelman and Robinson [1] and Dervis, de Melo, and Robinson [18]) instead of a Leontief model, although the computational requirements of such a generalization might be very large. We shall, therefore, assume that the structure of the economy within one single period, t (which can be one or more years), can be adequately described by a CGE model of the general form

\[ \tilde{g}_t(K_{t-1}, d_t, P_t, Z_t, U_t) = 0 \]  

(8)

where \( \tilde{g}_t \) is a vector-valued function of excess demands for the goods of the various sectors of the economy; \( K_{t-1} \) is the vector of sector-specific capacities in place at the end of the previous period (assumed to stay fixed for production purposes within the current period); \( d_t \) is the vector of capacity augmentations in period t, i.e., the "control vector"; \( P_t \) is the vector of endogenous prices; \( Z_t \) is a vector of exogenous variables and policy variables that are assumed fixed for the period t; i.e., not under control; and \( U_t \) is a vector of exogenously specified disturbances. If a component of \( Z_t \), say, \( z_{it} \), is a policy variable, it could be a function of \( U_t \). For instance, an operating buffer stock policy for grains could specify that current year net additions to government stocks are a known function of current year grain production. Since the functional forms, \( z_{it}(u_t) \), for all i would be known, they could be substituted in (8) to obtain a new CGE model of the form
In the sequel it will be assumed that all substitutions of this form have been done.

The dynamic equations of the problem involve "updating" the sector capacities via the following equation:

\[ K_t = K_{t-1} + f(d_t, K_{t-1}) \]  \hspace{1cm} (10)

where \( f \) would generally be a nonlinear set of functions involving absorptive considerations (for an example of these, see Kendrick and Taylor [28]).

Assume an objective function for period \( t \) that could depend on levels of consumption, incomes of specific income groups, etc. We can write a general scalar-valued objective function as

\[ U_t(K_{t-1}, d_t, p_t, u_t). \]  \hspace{1cm} (11)

The general optimization problem then consists of

\[ \max_{d_1, d_2, \ldots, d_T} \prod_{t=1}^{T} \mathbb{E} \sum_{t=1}^{T} \rho_t U_t(K_{t-1}, d_t, p_t, u_t) \]  \hspace{1cm} (12)

subject to (9) and (10). In (12), \( \rho_t \) is a generalized discount factor and the expectation is taken over all uncertainty vectors, \( u_t \), \( t = 1, \ldots, T \).

Note that, in the description of the planning problem via (10)-(12), we have designated the control variables to be the sectoral capacity augmentations \( d_t \). If the planning authorities cannot control \( d_t \) directly, which would be the case if there is a large private sector in the economy, then the controls might be indirect ones such as sectoral subsidy rates. This presents no problem in the formulation above, as one could make \( d_t \) functions of the subsidy rates (using some theory of private investment) and proceed with these rates as the controlled variables. Notice that in this case a CGE model would be the appropriate tool via which one could model the sectoral investment changes arising from variations in relative prices.
Notice that (12) represents a very general optimization criterion that includes as special cases, for instance, the mean-variance criterion. If the constraints, in addition to the model equation (8), include stochastic inequalities of the type shown in equation (5), (6), or (7), then this criterion can also be used for multiperiod risk-programming exercises.

Since $p_t$ can be solved from (9) in terms of the other variables (usually numerically), the problem, in principle, reduces to one in which $p_t$ is eliminated from the argument of $U_t$ in (11). It is well known from the theory of stochastic control (see, e.g., Kushner [30]) that problems such as the one stated above, namely, exhibiting separable and additive utility over time, have feedback solutions of the general form

$$d_t = h_t(K_{t-1}, U_t).$$

(13)

The general stochastic control problem is then one of finding the form of the functions $h_t$. If this problem could be solved empirically for the general case, it would provide a very powerful tool for designing target plans under uncertainty. By evaluating the $h_t$ functions at the mean values of $U_t^0$, one could obtain a deterministic rule in feedback form of the type

$$d_t^0 = h_t(K_{t-1}^0, U_t^0)$$

(14)

which could easily be translated by combination with (10) into a set of optimal investment paths,

$$d_1^0, d_2^0, \ldots, d_T^0.$$
These investment paths can be considered the "target paths" for a medium-term plan that incorporates uncertainty.

How would the investment plans above be different from the ones that could be computed by setting $u_t$ equal to $u_t^0$ in the original problem and solving it by deterministic optimal control methods? If we ignore all uncertainties by setting all the $u_t$ equal to their expected values $u_t^0$, the associated deterministic problem is the following:

$$\max_{d_1, d_2, \ldots, d_T} \sum_{t=1}^{T} a_t U_t \left( K_{t-1}, d_t, p_t, u_t^0 \right)$$

subject to

$$K_t = K_{t-1} + f(d_t, K_{t-1})$$

and

$$g_t\left(K_{t-1}, d_t, p_t, u_t^0\right) = 0.$$  

Notice that the solution of the presumably simpler deterministic problem in equations (15) through (17) is still quite complex. The complexity arises from the fact that the solution for $p_t$ from equation (17) (namely, the CGE solution) is usually not an analytical one; hence, one cannot just substitute $p_t$ in (15) and then apply a numerical version of Pontryagin's maximum principle. This problem, of course, arises because a price-endogenous model of the economy is utilized. This difficulty was avoided in the pioneering numerical application of the deterministic maximum principle to economywide planning by Kendrick and Taylor [28] who totally neglected yearly prices. Quite often, however, it is through prices that uncertainties in one sector are transmitted to others necessitating their explicit incorporation in the model.
The difference between the deterministic and the target stochastic plans would lie in the fact that the \( h_t \) functions in (14) would be different from the ones computed by solving the problem in equations (15) through (17). This is because the computations of \( h_t \) in the general stochastic control problem would depend on all of the higher moments of the distributions of \( u_t \), while in the deterministic problem they would depend only on the means of \( u_t \).

It would be very useful to have computational techniques for solving the general stochastic control problems defined by equations (9), (10), and (12) because then one could investigate directly the extent to which uncertainty would shift the optimal investment target plan. Unfortunately, such techniques are not currently available.

The next question is whether one can use some approximation technique to obtain a better idea of how a target plan is affected by uncertainty. Linearizing the functions, \( h_t \), in (13) about \( u_t^0 \), we obtain

\[
\hat{d}_t = h_t(K_{t-1}^0, u_t^0) + A(K_{t-1}^0, u_t^0)(u_t - u_t^0) + B(K_{t-1}^0, u_t^0)(K_{t-1}^0 - K_t^0).
\]

In (18) the matrices \( A \) and \( B \) have as rows the gradient vectors of the components \( h_{i_t} \) of \( h_t \), evaluated at \( K_{t-1}^0, u_t^0 \). It can be seen that in (18) the first term represents the target plan that incorporates the higher moments of the distribution of the random variables, while the second and third terms represent a linear approximation to an adjustment rule.

Notice that when \( u_t = u_t^0, \hat{d}_t = d_t = h_t(K_{t-1}^0, u_t^0) \) and \( K_t = K_t^0 \) and, furthermore,
Equation (19) suggests the following procedure for approximating $h_t(K_{t-1}, u_t^0)$ $t = 1, \ldots, T$. First, set all $u_t$ equal to $u_t^0$ and solve the deterministic problem in equations (15) through (17). As already discussed, if the deterministic solution can be found, it might be cast in feedback form as

$$
E \hat{d}_t = h_t(K^0_{t-1}, u_t^0).
$$

(19)

The idea is now to replace $u_t^0$ by $u_t$ in (20) and, by stochastic simulation, to evaluate several sample paths, $\tilde{d}_t(t = 1, \ldots, T)$, and then to take the average of those to obtain an estimate of $h_t(K_{t-1}^0, u_t^0)$. In other words, we would be evaluating numerically the expression

$$
E \tilde{h}_t(K_{t-1}, u_t).
$$

(21)

Comparing (19) and (21), it is clear that, since $h_t$ and $\tilde{h}_t$ are in general different, there is no reason that the expectation of $\tilde{h}_t(K_{t-1}, u_t)$ should be close to $h_t(K_{t-1}^0, u_t^0)$. Furthermore, stochastic simulations with CGE models are bound to be an extremely expensive proposition given current computing technology. Since, however, the general solution of the stochastic control problem is impossible to compute, exercises, as illustrated above, might be useful as qualitative indicators of how seriously uncertainty affects the optimal multisectoral choice of an economywide investment plan.
The incorporation of uncertainty in the computation of a medium-term, economywide dynamic multisectoral investment plan seems like a very difficult exercise. While it is evident that differential uncertainty in different sectors has some implications about the optimal allocation of investment among industries, there is no clear-cut intuitive or computational rule that can be used to design an investment plan that takes into account uncertainty about the future. For instance, a simple rule such as "invest more in industries whose international prices are less volatile" breaks down when the interdependence among sectors is taken into account.

The discussion in this section, while concentrating on target plans under uncertainty, should not overshadow the importance of adjustment rules. Notice from equation (14), for example, that, even if the component functions of \( h_t \) had been accurately computed, the implementation of only the deterministic investment sequence \( \left( d_t^0, \ldots, d_t^0 \right) \) could drive the economy increasingly away from the prespecified planned trajectory. This would be because successive random shocks would probably lead to haphazard reactions of the planning authorities that would be, in general, inconsistent with the overall plan and, hence, potentially destabilizing. Such reactions in many cases will reinforce and multiply the consequences of the original shocks. It is only through proper ex post adjustment rules, which are designed to be consistent with the target plan, that a plan can last its intended duration. It is to the computation of these rules that we now turn.

4. A Methodology for Computing Adjustment Rules for Investment Plans

In this section we outline a method for approximating the second and third terms of equation (18), namely, the adjustment rules for the medium-term investment plan.
The way in which these adjustment rules should be conceived is the following. Suppose a medium-term target plan has been designed for the economy (by optimization, fiat, political decision, simulation, or whatever other method is deemed feasible or desirable). This plan would typically consist of a deterministic "blueprint" for the economy of the type, "invest so many millions in this sector in each year of the plan." Uncertainty about the future means that the investable funds actually available during the implementation of the plan would be different from what was used in designing the target investments. The adjustment rules should be regarded as the blueprint that dictates how the excess or the shortage in investment funds is to be allocated among the different sectors. Notice that, while these adjustment blueprints are computed ex ante—namely, at the same time as the target path is estimated—their actual use and implementation would be delayed until the time of the resolution of the future uncertainty. In other words, the adjustment blueprints might take the form of a set of rules for each year of the plan.

Since the general nonlinear feedback rules, $h_t$, are not estimable, the computation of the "best" adjustment matrices $A$ and $B$ in (18), by deriving the gradients of the components of $h_t$, is not feasible. Hence, some other technique must be given to find an approximation for the adjustment rules. Such a methodology is now proposed.

Denote, as before, the target paths by $d_t^0$, $p_t^0$, $K_t^0$ ($t = 1, \ldots, T$). As already mentioned, these could be optimizing solutions of a deterministic control problem or a set of consistent investment programs for the different sectors of the economy.
The next step is to linearize equations (9) and (10) about these nominal paths. The dynamic system in the deviations would now look as follows:

\[ G_{1t} (p_t - p_t^0) + G_{2t} (k_{t-1} - k_t^0) + G_{3t} (d_t - d_t^0) + G_{4t} (u_t - u_t^0) = 0 \]  

\[ k_t - k_t^0 = k_{t-1} - k_{t-1}^0 + F_{1t} (d_t - d_t^0) + F_{2t} (k_{t-1} - k_{t-1}^0) \]

where the matrices \( G_{it} \) (i = 1, . . . , 4) and \( F_{it} \) (i = 1, 2) are the Jacobian matrices of the excess demand and capacity equations, respectively, evaluated at the nominal planning paths.

If the problem has been cast in a consistency mold, the planner could specify the objective as trying to keep the economy close to the nominal targets and postulate a penalty function that is quadratic in the deviations of the actual paths from their nominal ones. If, on the other hand, the original investment problem is cast in normative terms, then one could expand the \( U_t \) in (12) about the nominal paths up to and including the second-order terms. In either case, the objective function \( (W_t) \) replacing equation (11) is bound to look like the following:

\[ W_t = (x_t - x_t^0)^' Q_t (x_t - x_t^0) + (x_t - x_t^0)^' R_t \]

where, for economy of notation, we have defined a new vector \( x_t \) by
and primes denote transposition of vectors or matrices. If the problem specifies an objective function, $U_t$, then in (24) $Q_t$ would commonly be a positive semidefinite matrix consisting of the Hessians of $U_t$ evaluated at the deterministic target paths. Otherwise, $Q_t$ would consist of some judgmental penalty weights. Denote the deviations of variables from their nominal paths by $\Delta$'s so that $\Delta Y_t \equiv Y_t - Y^0_t$.

The stochastic control problem at this stage would then be the following:

$$\max_{\Delta d_1, \ldots, \Delta d_t} \quad \mathbb{E} \left[ \sum_{t=1}^{T} \rho_t \left( \Delta \x_t' Q_t \Delta \x_t + \Delta \x_t' R_t \right) \right]$$

subject to

$$G_{1t} \Delta p_t + G_{2t} \Delta K_{t-1} + G_{3t} \Delta d_t + G_{4t} \Delta u_t = 0$$

$$\Delta K_t = \Delta K_{t-1} + F_{1t} \Delta d_t + F_{2t} \Delta K_{t-1}.$$  \hspace{1cm} (28)

The problem as cast in equations (26)-(28) is a typical linear quadratic stochastic control problem which could be solved by stochastic dynamic programming (see Chow [16] and Athans [3]). The general solution would be of the form
$$\Delta d_t = H_{1t} \Delta K_{t-1} + H_{2t} \Delta u_t$$  \hspace{1cm} (29)$$

where the $H_{1t}$ and $H_{2t}$ would be appropriately computed matrices. The rules represented by (29) are what would constitute the \textit{ex post} adjustment blueprints to the already prespecified nominal targets $d_t^0$. Notice that what would be computed \textit{ex ante}, i.e., in the beginning of the plan, would be the quantities, $d_t^0$, and the matrices, $H_{1t}$, $H_{2t}$, for $t$ spanning the duration of the plan. The actual values of the adjustments, $\Delta d_t$, would be computed \textit{ex post}, i.e., after $\Delta u_t$ and $\Delta K_{t-1}$ have been observed. However, the key difference between this approach and the typical planning practice would be that the planners and policymakers would know what rules (in the form of $H_{1t}$ and $H_{2t}$) are necessary at a later stage to accommodate unforeseen shocks to the economy. Furthermore, these rules and later adjustments would be consistent with the overall initial plan.

How would the $H_{it}$ ($i = 1, 2$) matrices be computed, and on what would they depend? If one envisions solving (27) for $\Delta p_t$, substituting the resulting expression for $\Delta p_t$ in (26) and collecting terms, then it could be seen that the general $t$th term of the criterion in (26) would contain terms that are quadratic in $\Delta K_{t-1}$, $\Delta d_t$, and $\Delta u_t$ as well as terms that are products of $\Delta K_{t-1}$ with $\Delta d_t$ and $\Delta u_t$; products of $\Delta d_t$ with $\Delta u_t$; and, finally, terms that are linear in $\Delta K_{t-1}$, $\Delta d_t$, and $u_t$. Applying stochastic dynamic programming to the resulting system would result in a set of nonlinear backward recursive equations for $H_{it}$ ($i = 1, 2$) of the general type

$$H_{it} = L_{it} (H_{1t,t+1}, H_{2t,t+1}) \hspace{1cm} i = 1, 2$$  \hspace{1cm} (30)$$

with boundary conditions at time $T$, the end of the plan (the matrices $H_{1T}$, $H_{2T}$ will in general depend upon $Q_T$ and $R_T$; cf. equation 26).
The functions $L_{1t}$ and $L_{2t}$ would generally depend on the matrices $Q_t$, $R_t$, $G_{1t}$, $G_{2t}$, $G_{3t}$, $G_{4t}$, $F_{1t}$, $F_{2t}$ ($\tau = t, t+1, \ldots, T$) of equations (26) through (28) and the expected values of $\frac{u_t}{\sigma_t}$ ($\tau = t, t+1, \ldots, T$) but not on the covariance matrices of $u_t$ for $T \geq \tau \geq t$. The matrices $Q_t$, $R_t$, $G_{1t}$, $F_{1t}$, however, depend on the particular target path that was used to linearize the CGE model. In other words, different deterministic investment strategies will imply different adjustment rules as well. For instance, the rules in equation (30) might dictate that an export-promotion type of investment strategy would necessitate that shortfalls in investment funds should be used to reduce the year's allocation of investment to the import-competing industries and maintain planned investment in the export sectors. These conclusions, however, cannot be obtained analytically and must be evaluated empirically for each economy and each different investment strategy although, perhaps, some regularities might emerge for classes of economies and classes of development strategies. Notice that the overall problem has been cast in terms of minimizing the overall expected departure of the economy from some desired plan. Hence, it is natural that the adjustment rules would depend on the particular investment strategy as well.

It might seem surprising that the functions $L_{1t}$ and $L_{2t}$ would not depend on the covariance matrices of the random variables. In other words, if a target plan is specified and if the variances of all random variables are doubled, this would not affect the adjustment rules. This, however, is not to say that the actual values of the ex post adjustment, $\Delta t$, would not depend on the distribution of $u_t$. In fact, a higher variance for $u_t$ would necessitate a larger variance of $\Delta t$ as is evident from (29). In other words, a more
active **ex post** adjustment policy would be required so as to keep the economy close to the desired path.

The reason for this lack of dependence of the adjustment rules on the higher moments of the random variables can be understood if one looks back to equation (18). In that equation the ideal adjustment rules—namely, the matrices $A$ and $B$—depend on the higher order moments of the distribution of $u_t$ because these matrices are obtained by the gradients of the functions $h_t$ whose shapes depend on the higher order moments of $u_t$.

Suppose, for instance, we consider a scalar element of the matrix $A$ or $B$. This element will, in general, be a real valued function $\phi(\cdot)$ of the vectors $K^0_{t-1}$ and $u^0_t$. The shape of the function will depend on the higher order moments of $u_t$. The shape of the scalar-valued functions that constitute the elements of our adjustment rules in (30), on the other hand, do not depend on the higher order moments of the distribution of $u_t$. The reason for this is that, when one uses a quadratic criterion or any second-order approximation to a criterion function, one loses all higher order terms such as $\Delta u' \cdot \Delta u \cdot \Delta d_t$ that would yield terms of $\Delta d_t$ multiplied by higher order moments of the distribution of $u_t$.

This discussion then highlights the fact that the reason for the non-dependence of the adjustment rules on the higher order moments of the probability distribution of $u_t$ lies in the necessity to use approximations to solve an otherwise unmanageable nonlinear problem. The error committed in the approximation of the matrices, $A$ and $B$, will generally be second-order small.

The extent to which these approximate adjustment rules successfully manage to keep an economy growing in a manner not far off the planned path depends on the actual nonlinearities of the underlying economic model and the magnitude
of the uncertainties. In other words, the success of the approximation depends on whether or not the actual behavior of the economy violates the assumptions under which the approximations are valid.

It must be emphasized, however, that these feedback-type adjustment rules will constitute a large improvement in the ability of planners to keep the economy reasonably close to the planned path. In the terminology of control theory, the economy would be operating in a "closed loop" fashion—namely, regulating itself under various external shocks—as opposed to an "open loop" fashion, i.e., without feedback. The absence of regulating feedback, as has already been discussed, will, in the presence of shocks, most likely lead the economy increasingly away from the planned path.

One might argue that, in planning practice, yearly unpredicted shocks that move the economy away from target are dealt with in some fashion, whether by emergency cabinet meetings, rushed meetings of the planning board, or other haphazard means. The usual result in such situations is that the overall objectives of the plan are scrapped in an effort to cope with the short-run problems. The methodology proposed here would provide insights and guidance as to how the future adjustments could be made to be consistent with the overall plan.

The CGE models that we have in mind in the application of this technique are not in the standard control-theoretic or macroeconometric format. Hence, it seems useful to go through a fairly complete exercise in order to point out the subtleties of linearization and the economic assumptions underlying it. Such an exercise is performed with a simple CGE model in the Appendix to this paper.
5. Conclusions and Discussion

In this paper we have dealt with the problems of (1) modifying a medium-term plan to accommodate uncertainty within the plan and (2) devising adjustment rules that would be used during the plan to accommodate deviations in the actual values of exogenous variables from their forecasted means and keep the economy evolving in a path close to the originally planned one.

From the discussion of section 3, it is seen that the general nonlinear stochastic, dynamic planning problem is not solvable practically. Furthermore, the current state of the art, apart from stochastic simulations, does not allow the easy computation of modifications that should be made in a deterministic plan to account for uncertain future events. This is because the mathematical theory behind extending Pontryagin's principle and multiperiod nonlinear programming models to accommodate price-endogenous economic models does not seem to have been developed as yet. Section 4 then analyzed a methodology by which one could compute adjustment rules that could be used to "correct" the original plan for deviations in exogenous variables such as international prices and production uncertainties.

The rules which the proposed methodology helps to derive answer the question, "To what sectors should unforeseen excesses or deficits in investment funds that become available in year t go so as to make the economy not deviate too much from the original plan?" The adjustment rules are not very difficult to compute, given a CGE model of an economy and a particular planning path, and could easily be made part of every medium-term planning exercise. Changes in the underlying objective functions or structural changes in the system would certainly necessitate new "target paths" for the plan as well as new adjustment rules. However, under conditions of no huge external or internal
shocks, we feel that the technique of estimating and applying feedback adjustment rules could free the practical planners from the onerous task of continuously adjusting \textit{ex ante} rigid plans to small \textit{ex post} fluctuations and would allow them to concentrate on the more fundamental problems of detecting and accommodating significant structural shifts in the economy.

How would the approach here compare with one where the plan is just rolled every year, i.e., when, say, a new five-year plan is computed in every period? This practice, if carried every year, would itself constitute a feedback strategy because new information (including the values of the uncertain variables) would be incorporated in each year and used in designing a new set of investment programs.

Assume that the original target set of $T$-period investment programs was \( \{d_1, d_2, \ldots, d_T\} \) and that the new rolled $T$-period program computed in year 2 is \( \{\tilde{d}_2, \tilde{d}_3, \ldots, \tilde{d}_T + 1\} \). If the original plan had been followed instead along with some adjustments based on rules computed, for example by the method above, then the actual implemented investment vector of period 2 would be equal to \( d_2 = d_1^1 + \Delta d_2 \) where \( \Delta d_2 \) would be the value of the actual adjustment calculated via the adjustment rule for period 2 and consistent with the original plan. How would this value of \( d_2 \) compare with \( d_2^2 \)? It is not clear \textit{a priori} which technique would give a better economic performance. If the structure of the underlying economic model is expected to change greatly from period to period, and if the information that becomes available in every period is sufficient to detect the structural shifts, then a rolling plan might be more useful, even though it would be much more cumbersome and expensive in terms of manpower, because a new structure would violate the model upon which the original plan was based. Since, however, the adjustment rules can be
easily computed, there is no reason why the new rolled plan could not be supplemented with adjustment rules. If, then, the structure of the economy does not change significantly, no new rolled plan would be necessary in the periods ahead. In short, we argue that every medium-term plan should be supplemented by adjustment rules. When changing circumstances require a modification of the underlying plan, a modification of the adjustment rules could also be computed. In other words, we advocate that a new target plan is necessitated only when the economic structure changes or if the planning objective is significantly modified. If neither of these conditions is met, then the adjustment rules should provide all of the necessary short-term modifications to the original plans that are needed to keep the economy close to the planned path.

One might ask whether the possibility of future learning about the structure could somehow be incorporated in the computations of the adjustment rules so that the whole planning exercise would not have to be repeated every time some parameters of the model change values. The answer, in principle, is yes. The types of rules one would have to compute, however, would not have the simple linear form of equation (29) but, instead, would be given by implicit nonlinear equations. Their solution would involve more approximation and computation. In fact, the size of the computations would grow prohibitively if one wanted to build into the plan *ex post* learning about more than one or two parameters. Given the number of potentially incompletely known parameters in a CGE model, incorporating *ex post* learning in the computation of the adjustment rules becomes impractical. In fact, it probably would be much cheaper to run the whole planning exercise computing new target paths and new adjustment rules with the new "learned" values of parameters than to build in *a priori* the
possibility of learning. Furthermore, it has been invariably shown in the adaptive control literature that the improvement in performance that one obtains by incorporating learning with all of the new approximations is hardly worth the tremendous additional computational cost.

How does traditional sensitivity analysis relate to our proposed methodology for computing and implementing adjustment rules? In our context, sensitivity analysis pertains to the computation of deviations of economic variables from their planned paths due to unforeseen fluctuations of the exogenous variables. As such, it could be used in the design of the overall nominal target path as already pointed out in section 3. Furthermore, sensitivity analysis could also be used to compare the performance of the economy with and without the feedback adjustment rules and, hence, to evaluate our proposed "closed loop" planning methodology as compared with current "open loop" practices.

The computation of adjustment rules could further development policy by answering questions such as (1) given a particular development strategy (for instance, export promotion or import substitution), do subsequent external and internal shocks imply that short-term adjustments should channel investment toward the same sectors originally favored by the plan or away from them? and (2) does the stabilization of an economy embarked upon an import-substitution strategy require more or less active intervention than does that of an economy pursuing an export-led growth strategy? An analysis of the structure of the $H_{it}$ matrices used to compute the adjustment rules in equation (29) and their comparison for alternative target paths could shed some light on the desirability of the target paths themselves for an economy faced with uncertainty.
In this paper we were concerned with adjusting medium-term investment plans to yearly fluctuations and assumed [see the discussion preceding equation (9) in section 3] that the rules for adjusting other policy variables, such as tariffs and indirect taxes, to fluctuations were known. Nevertheless, the methodology proposed in section 4 could readily accommodate the computation of adjustment rules for these variables as well. Notice that, if we allow, for example, all tariffs and all production taxes to be controlled, then we could directly control the amount of external or internal fluctuation faced by each producing sector. The feedback rules would then yield the optimal yearly adjustments to all of these control variables necessary to keep the economy close to the target path. By enlarging the set of controllable variables, we can achieve closer adherence to the target plan. It is possible, however, that many controls would be redundant; and this would also come out of the computation of adjustment rules.

One could envision an experiment by which, if a previously fixed variable such as a tariff were to become a control variable, the potential gain in economic performance from making that variable a controlled one could be ascertained. Hence, one could find, for example, which tariffs (if any) were the ones most crucial for balancing the economy. A comparison of alternative policy instruments would, thus, also be possible through these means.

In summary, although the methodology proposed cannot be used to answer completely the problem of choosing among alternative medium term plans under uncertainty, it can nevertheless contribute substantially toward a deeper understanding of the sensitivity and behavior under random shocks of different development policies. As such, it should provide another set of information to be used in the plan evaluation process.
APPENDIX

In this Appendix we shall illustrate the approach to computing adjustment rules proposed in the paper with an extremely simplified but easy-to-handle CGE model that, nevertheless, maintains many of the features of more complicated models.

Consider an economy with only two producing sectors, of which only one is producing tradeable products. Denote the physical output of the two sectors in period t as $X_1$ and $X_2$ (the subscript, t, is suppressed except when needed). Assuming for simplicity that there are no interindustry intermediate flows (this assumption has no effect on the functional form of the adjustment rules), we can write the simplified material-balance equations in the model as follows:

\[ X_1 = C_1 + I + T \]  \hspace{1cm} (A1)
\[ X_2 = C_2 + G \]  \hspace{1cm} (A2)

where $C_i$ ($i = 1, 2$) is the final consumption of the $i$th product; $I$ is total investment by origin; $T = E - M$ (exports - imports) is net trade of the tradeable good; and $G$ is government spending, all in period t.\(^2\)

The production of good 1 uses noncompetitive imports, $M_0$, according to the fixed coefficient relation

\[ M_0 = mX_1. \]  \hspace{1cm} (A3)

The country is assumed to be a price taker in international trade so that domestic prices of the traded goods are given by

\[ p_i = \pi_i (1 + \tau_i) e \hspace{1cm} i = 0, 1 \]  \hspace{1cm} (A4)
where $\pi_i$ are international prices, $\tau_i$ are border taxes (tariffs or subsides), and $e$ is the exchange rate.

Production in the two sectors occurs according to the following putty-clay production functions:

$$X_i = A_i L_i K_{i-1}^{\alpha_i}$$  \hspace{1cm} i = 1, 2 \hspace{1cm} (A5)

where $A_i$, $\alpha_i$, and $\beta_i$ are constants, $K_{i-1}$ denotes the capacity in place at the end of the previous period that is unchanged during the current production period, and $L_i$ is the current period labor input. Labor is mobile between sectors.

The net prices of the two sectors are defined as

$$\bar{p}_1 = p_1 - m p_0 - \theta_1 p_1$$  \hspace{1cm} (A6)

$$\bar{p}_2 = p_2 - \theta_2 p_2$$  \hspace{1cm} (A7)

where $\theta_i (i = 1, 2)$ is the indirect tax (subsidy) rate on the production of the two sectors.

If the wages are common to both sectors and fixed at $w$ and if the producing units maximize short-run profits, the labor input in each sector is

$$L_i = \left[ A_i \left( \frac{\bar{p}_i}{\alpha_i} \right) K_{i-1}^{\beta_i} \right] \frac{1}{1-\alpha_i}$$  \hspace{1cm} (A8)

and the sector outputs are
Total employment is $L_1 + L_2$, and gross domestic product (GDP) at factor cost generated in the economy is equal to

$$Y = \tilde{p}_1 X_1 + \tilde{p}_2 X_2.$$  \hfill (A10)

Consumable income is equal to

$$Y^d = Y(1 - z)(1 - s)$$  \hfill (A11)

where $z$ is the direct tax rate and $s$ is the constant savings rate. Income distribution is obviously slighted here. To introduce it at this point would complicate notation, but it should be borne in mind that fluctuations in income shares might be one of the objectives of policy control.

Consumption of the two goods is governed by consumption relations of the sort

$$C_i = g_i(p_1, p_2; Y^d) \quad i = 1, 2.$$  \hfill (A12)

Equations (A1) through (A12) describe the equilibrium part of the model. From these we can derive the savings-investment identity of macroeconomic balance.
\[ p_1 I = \left( \pi_0 \cdot X_1 - \pi_1 T \right) e + \left[ s(1 - z)Y \right] + \left[ \sum_{i=0}^{2} \left( M_i - E_i \right) \pi_i \cdot \tau_i \cdot e + zY + \sum_{i=1}^{2} p_i \cdot \theta_i \cdot X_i - p_2 G \right]. \]  

In (A13) the left-hand side denotes funds available for investment whereas, on the right-hand side, the brackets denote the sources of these funds—namely, foreign savings, domestic savings, and government savings from fiscal operations.

Dynamics can be introduced in a rudimentary fashion by augmenting the capital stocks of the next period via allocation of the investment, \( I \), among the two producing sectors as follows:

\[ K_i = K_{i-1} + d_i \quad i = 1, 2 \quad (A14) \]

where

\[ d_1 + d_2 = I. \quad (A15) \]

The general problem is how to allocate the \( d \)'s in each period and how to incorporate uncertainty in this structure.

Before we get to this problem, however, observe that there are too many variables and too few equations to determine them in the model. If, for instance, domestic trade and government fiscal policies are exogenously fixed, then there are three prices that must be determined (\( p_2, w, \) and \( e \)), while there is only one balancing equation [the home-good market clearing condition (A2)]. Since money is not included in the model, we need two closure rules. Assume, for the exposition, that one of the rules is that \( w = 1. \)
The second closure rule could come from the foreign sector. One possibility, for example, would be that the exchange rate adjusts so as to always zero the balance of trade via the equation

$$\pi_0 mX_1 - \pi_1 T = 0 \quad \text{all } t.$$  \hspace{1cm} (A16)

Assume that this is indeed the case so that the two equations, (A2) and (A16) plus \( w = 1 \), completely determine the system.

Uncertainty could be introduced in a model such as this, for instance, from the foreign-trade side by making the external price, \( \pi_1 \), random; in the sequel this is what we assume. If the government policy is known (in other words, if \( z, \pi_i, \phi_i, G_i \) are governed by fixed rules that depend on time and, possibly, on the yearly random variables), then the variations of \( \pi_1 \) would readily induce fluctuations in all of the endogenous variables of the system such as prices, consumption levels, and available real investment funds.

If some objective function, \( U_t(c_1, c_2) \) is defined,\(^4\) then the dynamic stochastic problem of investment allocation could be defined as

$$\max_{d_1t, d_2t} \mathbb{E} \rho_t U_t(c_{1t}, c_{2t})$$  \hspace{1cm} (A17)

subject to

$$K_{it} = K_{i(1-t)} + d_{it} \quad i = 1, 2$$  \hspace{1cm} (A18)

and the equations of the CGE model (A1)-(A13). Notice that the CGE helps to determine yearly prices and through them income and consumption.
Assume that this problem has been solved for the deterministic case where the foreign price, \( \pi_1 \), is set equal to its mean value, \( \pi_1^0 \), and where all government policies, \( z, \theta_i, \tau_i, G_i \), are assumed to be known (we shall assume in what follows that \( \tau_i = \theta_i = 0 \), while \( z \) is a fixed parameter and \( G \) is adjusted to balance the budget to simplify the exposition). Denote the nominal target paths by a superscript zero (e.g., \( d_i^0 \)). If an objective welfare function has not been specified, the nominal paths could be considered as some sort of desired consistency solutions that are to be planned.

Consider now some period \( t \) (whose subscript is suppressed) and compute linear approximations to the deviations of the model solutions from nominal paths in that period that arise out of the external price fluctuations. We use a technique pioneered by Johansen [23]. The deviations of the new equilibrium solutions from the nominal paths are computed by log differentiation so as to bring into the analysis traditional economic concepts such as elasticities. Denote the log derivative of a variable \( x \) by \( x' \).

\[
x' \equiv \frac{d \log x}{dx} = \frac{dx}{x} = \frac{x_t - x_t^0}{x_t^0}. \tag{A19}
\]

Log differentiating the production functions from the nominal path and keeping in mind our convention that \( w = 1 \), we obtain the percent deviations in some period \( t \) (of course, we suppress the \( t \)'s).

\[
x_i = \frac{a_i}{1 - \alpha_i} \pi_i' + \frac{\beta_i}{1 - \alpha_i} \pi_{i-1}'. \tag{A20}
\]
The price equations (A4) yield

\[ p'_1 = \pi'_1 + e' \]  
\[ p'_0 = e'. \]

(A21)

Here we assume, for simplicity, that only the international price, \( \pi_1 \), of tradeable goods can fluctuate, while intermediate external import price, \( \pi_0 \), does not vary from its mean.

Equations (A6) and (A7) yield after log differentiation

\[ \tilde{p}'_1 p'_1 = p'_1 p'_1 - m p'_0 p'_0 \]  
\[ \tilde{p}'_2 = p'_2. \]

(A22)

(A23)

The log deviation in GDP from its nominal path is

\[ Y' = \tilde{p}'_1 (\tilde{p}'_1 + \chi'_1) + \tilde{p}'_2 (\tilde{p}'_2 + \chi'_2) \]

(A24)

where \( \phi^0_i = p^0_i x^0_i / Y^0 \) are the nominal shares of sectoral value added in year \( t \) (as usual the subscript \( t \) has been suppressed).

Since by (A11) and our assumption that the aggregate tax rate stays constant and consumable income is a constant fraction of \( Y \), we have

\[ Y^{d'} = Y' \]

(A25)
and the consumption relations (A12) yield

$$C_i^t = n_{i1} p_1^t + n_{i2} p_2^t + \epsilon_i Y^{d'}$$  \hfill (A26)

where $n_{ij}$ is the price elasticity of consumption of the $i$th good consumption with respect to the price of the $j$th good, the $\epsilon_i$ are the income elasticities of consumption, and avoidance of money illusion requires that

$$n_{i1} + n_{i2} + \epsilon_i = 0 \quad i = 1, 2. \hfill (A27)$$

Equation (A14) yields

$$\Delta K_i = \Delta K_{i-1} + \Delta d_i \quad i = 1, 2. \hfill (A28)$$

The savings investment identity (A13) with the simplifying assumptions of this model (remember the balanced-trade condition) becomes in every period $t$

$$p_1 I = s(1 - z) Y + zY - p_2 G. \hfill (A29)$$

Notice that, if we had assumed that $G$ was constant, the government budget would not, in general, be balanced in any actual period because $zY - p_2 G$, in general, will not be equal to zero. Since, however, we have imposed this balanced-budget property for the plan, this automatically determines ex post the value of $G$. In other words, the ex post rule for $G$ is

$$G = \frac{zY}{p_2}, \hfill (A30)$$

and this is an example of what we have referred to earlier in section 3.3 as a priori known ex post government feedback rules. From (A30) we can obtain by log differentiation
Referring now to the savings investment relation (A29), we obtain

\[ p_1 I = s(1 - z) Y \]  

(A32)

so that

\[ \Delta I = I^0 I' = I^0(Y' - p'_1). \]  

(A33)

Equation (A33) gives the ex post available excess real savings that can be used to finance ex post excess investment. Notice that, depending on the particular fluctuation, excess savings could be negative necessitating a downward adjustment to planned investment. In other words, if we consider deviations from nominal paths in equation (A15),

\[ \Delta d_1 + \Delta d_2 = \Delta I = I_0(Y' - p'_1). \]  

(A34)

Notice that, in this simplified model, we could use (A34) to solve for one of the control variables explicitly in terms of the other. In larger models the savings investment identity would be much more complicated, but it still could be used to solve for one of the control variables in terms of the other.

The equations of this model that must balance ex post are the home-goods balance equation, (A1), and the balance of payments one, (A16).

In log-deviation form, these conditions become

\[ x_0^0 x_2' = c_2^0 c_2' + g^0 g'. \]  

(A35)

\[ x_1' = \pi_1' + t'. \]  

(A36)
If we use equations (A19)-(A25) as well as (A31), we can reduce (A35) and (A36) to two linear equations involving $p^i_2$ and $e'$ as unknowns of the form

$$y_{11} p^i_2 + y_{12} e' = \delta_{11} + \delta_{12} \pi^i_1 + \delta_{13} \Delta I$$

(A37)

$$y_{21} p^i_2 + y_{22} e' = \delta_{21} + \delta_{22} \pi^i_1 + \delta_{23} \Delta I$$

where the $y_{ij}$ and $\delta_{ij}$ are functions of nominal variables only. Notice that the solution for $p^i_2$ and $e'$ requires specification of $\Delta I$, the investment deviations by origin. While, in this model, there is only one sector that provides investment goods and, hence, $\Delta I$ can be readily eliminated via equation (A33), in realistic models this would not be true. In general, if the investments by destination are related to investments by origin via some Leontief capital coefficients, then, as equation (A37) shows, the endogenous ex post solutions for the deviations of prices would depend linearly on the control variables, $\Delta d_{-i}$. Note that (A37) and (A28) are special cases of the general equations (26) and (27) of section 4 that are applicable to this simple model.

If, now, the objective is to minimize a function of the squares of the expected deviations of some variables from their nominal paths or to maximize the expected squared deviations of the objective function from its nominal path subject to (A37) and (A28), then the ex post adjustment rules could be found by stochastic dynamic programming (see [16] for a discussion of the technique).
Footnotes

*The authors are assistant professor and professor, respectively, in the Department of Agricultural and Resource Economics, University of California, Berkeley. We would like to thank Sherman Robinson and Larry Westphal for useful comments on an earlier draft. The usual disclaimer, however, about errors and omissions applies.

1/ In a later paper, Brada [12] considered the structure of production and trade in a centrally planned economy under foreign price uncertainty. His model belongs to the class of two-period \textit{ex ante} production and \textit{ex ante} trade models that were discussed earlier; hence, his results—not surprisingly—are almost identical to those of Brainard and Cooper [13] and Ruffin [39].

2/ The conventions that I, T, and G are part of only one sector are made for simplicity only so that we may deal with scalars rather than vectors. The results are readily generalized.

3/ \( K_{i-1} \) is a short notation for \( K_{i,t-1} \). Remember, we drop the subscript \( t \) for ease of exposition.

4/ The objective function could, of course, contain many more variables.
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