Economic Forces Underlying Urban Decentralization Trends: A Structural Model for Density Gradients Applied to Korea
Economic forces underlying urban decentralization trends: a structural model for density gradients applied to Korea

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Abstract. With the rapid pace of urbanization the structure of cities in developing countries is subjected to the very strong pressures of rapidly increasing population, rising incomes, and shifting factor prices. The standard procedure to document shifts in the structure of cities has been the use of density gradients. There is a quasi-universal tendency towards decentralization and the flattening of population-density gradients in all cities of the world. The shifting of density gradients is of great empirical importance to policymakers and we need to know more on how various economic and demographic forces operate to yield decentralization. Using a unique set of structural estimates for the housing and residential-land markets in Korea, this paper shows how an estimable structural model can be used to explore the specific economic forces affecting the shape of density gradients of Korean cities. Crucial estimates of the elasticity of substitution for housing in various Korean cities are presented. The indirect estimates of the shifts in the density gradients derived from the structural model are compared with the trends yielded by directly estimated density gradients. The implications of the analysis for our understanding of density gradients are drawn.

1 Introduction

It is now well known that the pace of urbanization in developing countries is much faster than what was experienced historically by developed countries (Beier et al., 1976). The structure of less developing country (LDC) cities is subjected to very strong pressures because of the rapid increase of their population, rising income, and shifting factor prices. One of the most common procedures for documenting the shifts in the structure of the city has been the estimation of population-density gradients. In a very recent survey of developed and developing countries, Mills and Tang (1978) have reemphasized that the distribution of population within cities exhibits strong regularities and that decentralization is not a postwar North American phenomenon but rather is a quasi-universal tendency of cities over time. Because of the very high densities found in LDC cities, they believe that continued decentralization will be a good thing but warn against expecting converging densities everywhere. The shifting of density gradients is an issue of great empirical significance to government planners who need to have a better perspective on the economic forces generating higher land values. Where will these forces be the strongest? Will the rate of appreciation be uniform across cities or significantly higher at the fringe where the acquisition of land for

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public use will be necessary? What are the economic processes through which urban policies should be taken into account?

It is hardly enough for us to know that all cities exhibit a tendency toward decentralization; we need to know how various economic, demographic, and physical forces operate to yield decentralization. Following the example set by Muth (1969), the analysis which is presented here explores how knowledge of the structural parameters of the housing market in a given country can be used to explain observed shifts in density gradients and to project future changes. From a planning and policy viewpoint we would benefit from knowing the specific forces that yield seemingly comparable results. Microeconomic analysis provides us with a quantifiable structural model to explain the observed reduced-form results of negative-exponential density gradients. This structural model yields hypotheses explaining how two LDC cities might differ within the same country and across countries.

The exercise presented here draws on a study of the structure of housing markets in (South) Korea (Follain et al, forthcoming). Using estimates of the structural parameters of housing demand and supply, we show how it is possible to investigate the changing shape of Korean density gradients and the more exact nature of urbanization in Korean cities. Then we can confront the results of the model with the direct estimation of actual urban density gradients for Korea performed earlier by Mills (1977) and Mills and Song (1977). In their Korean analysis, Mills and Song (1977) make a very instructive comparison of average population-density gradients between the United States, Japan, and Korea. The data

"show a remarkable pattern, comparing urban areas in countries at three very different stages of development. All three countries show the same tendency of urban decentralization. In all three countries the average density gradient flattens rapidly as time passes. Measured by absolute decline in the gradient, Japanese cities decentralized fastest during the period covered .... The average gradient declined 0.0132 points per year in the Japanese cities. The Korean average declined 0.0133 points per year and the U.S. average declined 0.0076 points per year. An annual decrease of 0.0133 points per year in density gradients represents rapid suburbanization indeed by the standards of observations that have been made in the developed world.

The other main characteristic of the data ... is that decentralization has proceeded less far in Korean cities than in Japanese and U.S. cities, and less far in Japanese than U.S. cities” (Mills and Song, 1977, pages 130-131).

The comparison of their direct estimation of shifts in density gradients with our derivation from the structural parameters of the housing market allows us to test the usefulness of the model and the worth of our findings.

A few words concerning the distinguishing features of Korean urbanization are in order before proceeding to the discussion of the model. Analyses of the urban land market in Korea are particularly important because it is a very crowded country of presently 35.9 million people living on 98,800 square kilometers of land. Its gross present population density of 363 people per square kilometer is the highest in the world after Bangladesh. In spite of a very successful family-planning program its population is expected to increase to 45.2 million by 1991, for a population density of 457 people per square kilometer. At the same time its urban population (living in places with population of more than 50,000) is expected to increase from 52 to 75% of the total population over the same period (Korea Development Institute, 1978).

In addition to the effects of population growth, the extremely rapid growth of the economy will place added strains on the land market; even after the oil crises of 1973 the economy has kept growing at well above 10% per year in real terms. Quite
clearly the strong competing demands on land for urbanization, industrialization, and agriculture as well as for recreation and tourism are going to become even sharper. In the overall operations of urban land markets, the residential-land market is a dominant force. In Seoul in 1973 the percentage distribution of land use was as shown in table 1 (see Mills and Song, 1977, table A-7-3). The large proportion of forest land reflects the prevalence of undevelopable mountain land in Korea, a factor adding further urbanization pressure on farm land. Net population densities in Korea are much higher than the gross figures mentioned earlier. Thus the forthcoming analysis applies to urbanization characterized by the moderation of population growth, high population density, high land scarcity, and very rapidly rising income levels.

This investigation of the sources of urban decentralization in Korea is a direct application of the standard model of urban growth developed in separate works by Muth (1969) and Mills (1972). It is not necessary to develop its entire structure because this has already been done by Muth (1969). This model is rather easy to apply provided that the structural parameters of the housing market have been estimated. In a first section we describe only the microeconomic model of changing land-use intensity as determined by the structure of housing supply. In a second section we present our empirical estimates of the elasticity of substitution between land and structure in the production of housing in Korea. In a third section we derive the quantitative implications of the theoretical model for the case of Seoul. We discuss the demand side of the residential-land market and the housing–land-price gradients: this is done briefly because the results of the demand analysis have already been reported elsewhere (Follain et al. forthcoming). Then we compare Seoul with other Korean cities. In the conclusion we draw the lessons of this Korean exercise for our understanding of urbanization patterns in other developing countries.

Table 1. The percentage distribution of land in Seoul.

<table>
<thead>
<tr>
<th></th>
<th>Residential</th>
<th>Arable</th>
<th>Forest</th>
<th>Road</th>
<th>Other</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seoul</td>
<td>21-7</td>
<td>23-5</td>
<td>32-8</td>
<td>4-0</td>
<td>18-0</td>
<td>100-0</td>
</tr>
<tr>
<td>Central business district</td>
<td>58-2</td>
<td>22-6</td>
<td>12-4</td>
<td>6-8</td>
<td>100-0</td>
<td></td>
</tr>
</tbody>
</table>

2 Housing supply and variations in the intensity of land use within cities

The comparative static long-term model of urban structure first explored by Mills (1972) and Muth (1969) is well known. It views the urban area as a set of concentric rings and attempts to explain why population densities tend to decline as one moves from an inner ring to an outer ring. More generally the model seeks to link the general phenomena of declining densities with economic forces. Its central premise is that population density per unit of land is closely and positively linked to the density of housing, or, more directly useful to us, the value of housing per unit of land. By analyzing the economic factors which determine the value of housing per unit of land it is possible to establish more precisely the linkage between observed population densities and the underlying market forces. One can show that the evolution of the population-density gradient for Seoul can be estimated, given knowledge of the land-price gradient for a given year, the elasticity of substitution between land and nonland inputs in the Korean housing production function (σ), and the relative shares of land (K_L) and nonland (K_N) inputs.

As a first step we can derive from the model the sources of variation in the value of housing (or nonland structures) per physical unit of land area in a city. In a given location the value of housing services is related to the combined output of land and nonland inputs. Following the familiar assumptions of a homogeneous production function and competitive factor markets, we can write that the total value of housing
services is

\[ PQ = P_L L + P_N N, \]  

(1)

where

\( P \) is the price per unit of housing services,

\( Q \) is the quantity of housing services,

\( P_L \) is the price per unit of land,

\( P_N \) is the price per unit of nonland input,

\( L \) denotes land inputs in physical terms, and

\( N \) denotes nonland inputs in physical terms.

From this relation we can derive a useful expression of the value of housing per unit of land. First, dividing both sides of equation (1) by \( L \), differentiating the logarithmic form, and making a substitution based on the definition of \( \sigma \) (the elasticity of substitution between land and nonland inputs in the production of housing services), we can write

\[ \frac{d \ln \left( \frac{PQ}{L} \right)}{d \ln P} = \left[ 1 + \left( \frac{K_N}{K_L} \right) \sigma \right] K_L \frac{d \ln P}{P}. \]  

(2)

Furthermore, we can make the very reasonable assumption that variations in the price of nonland inputs are minimal within an urban area. In that case the expression

\[ \frac{d \ln P}{d \ln \left( \frac{PQ}{L} \right)} = K_L \frac{d \ln P_L}{d \ln P} + K_N \frac{d \ln P_N}{d \ln P}, \]  

(3)

simplifies into

\[ \frac{d \ln P}{d \ln \left( \frac{PQ}{L} \right)} = K_L \frac{d \ln P_L}{d \ln P} + 0 \quad \text{because} \quad \frac{d \ln P_N}{d \ln P} = 0, \]

and relation (2) can be written as

\[ \frac{d \ln \left( \frac{PQ}{L} \right)}{d \ln P} = \left[ 1 + \left( \frac{K_N}{K_L} \right) \sigma \right] d \ln P. \]  

(4)

From equation (4) it can be seen that variations in the value of housing per unit of land are positively related to two factors: (1) variations in the price per unit of housing; and (2) the term preceding the price term \( \frac{d \ln P}{P} \), which is the elasticity of the value of housing per unit of land with respect to price changes. Explicitly this elasticity parameter is

\[ \epsilon_S = 1 + \left( \frac{K_N}{K_L} \right) \sigma. \]

As noted earlier, the population-density gradient which has been estimated directly in three countries by Mills (1972), Mills and Ohta (1975), and Mills and Song (1977) is also described in this model by the relative change in the value of housing per unit of land as one moves from the central business district (CBD) to the fringe of the city. If \( u \) is the distance from the CBD, the population-density gradient is represented by the expression

\[ \frac{d}{du} \ln \left( \frac{PQ}{L} \right) = \epsilon_S \frac{dP}{du}, \]  

(5)

or verbally by

population-density gradient

\[ \quad = (\text{elasticity of the value of housing per unit of land}) \times (\text{housing-price gradient}) . \]

The elasticity of the value of housing per unit of land varies with distance from the central city. We can expect relatively more population growth in those parts of the
Economic forces underlying urban decentralization trends

Urban area where the larger values prevail. Since this elasticity varies with distance we can differentiate equation (5) with respect to distance and determine whether it is positive or negative. We have

$$\frac{d\varepsilon_s}{du} = \frac{d}{du} \left[ 1 + \left( \frac{K_N}{K_L} \right)^{1/\sigma} \right] = \sigma(\sigma - 1) \frac{K_N}{K_L} \frac{dP_L}{dP_L} \frac{K_N}{P_L}. \tag{6}$$

or verbally

changes in the elasticity of housing value per unit of land

$$= \text{(elasticity of substitution)} \times \text{(ratio of nonland to land input shares)} \times \text{(land-price gradient)}.$$

Equation (6) shows very clearly the importance of the share of land in the behavior of $\varepsilon_s$ within an urban area, in addition to the influence of the elasticity of substitution.

The sign of elasticity $\varepsilon_s$ is entirely determined by the magnitude of the elasticity of substitution between land and nonland inputs in the production of housing services. As one moves further from the center of the city, producers use more land relative to nonland inputs. If the elasticity of substitution, $\sigma$, is less than unity then expenditures on land drop relative to nonland inputs and $K_N/K_L$ increases. This in turn causes $\varepsilon_s$ to increase and one has sharper differentiation in population growth in various parts of the city. Because the land-price gradient is negative and land prices decline with distance from the central city, with a small elasticity of substitution, $\sigma$ (that is, with fairly marked difficulties in substituting nonland inputs for land inputs), $\varepsilon_s$ increases toward the periphery. We have a differentiated rate of land appreciation, higher at the urban fringe.

3 Estimates of the elasticity of substitution for housing

It is clear from equation (6) that the key parameter determining the sign of changes in the elasticity of the value of housing per unit of land as one moves away from the center of the city is the elasticity of substitution between housing inputs.

For a separate paper dealing with the estimation of the demand for housing in Korea (Follain et al, forthcoming), we have collected the information necessary for the estimation of the value of the elasticity of substitution based on individual observations on housing units in various cities. The estimation procedure for $\sigma$ can take advantage of this unusually detailed data. The method is based on the fact that the elasticity of substitution, $\sigma$, can be defined as the percentage change in the ratio of nonland to land inputs which results from a percentage change in the price of land relative to nonland inputs. That is to say,

$$\sigma = \frac{d \ln \left( \frac{N}{L} \right)}{d \ln \left( \frac{P_L}{P_N} \right)}.$$

From this relation it follows that

$$d \ln \left( \frac{P_L}{P_N} \right) = (1 - \sigma) d \ln \left( \frac{P_L}{P_N} \right). \tag{7}$$

Thus $\sigma$ may be estimated by regressing the ratio of the share of rent attributable to land to the share of rent attributable to nonland inputs against the ratio of land to nonland prices. In the case of our Korean sample, for each observation we do have specific estimates of the values of $K_N$, $K_L$, $P_L$ and $P_N$. The analysis of the exact relationship between the ratios $K_L/K_N$ and $P_L/P_N$, which uses a constant-elasticity-of-substitution (CES) production function, shows that relationship (7) is exactly that upon which the actual regression is based. Since a CES production function has been
found appropriate for a wide variety of situations, the regression estimates of \( \sigma \) should be reasonably precise.

Five estimates of the coefficients of \( \ln(P_L/P_N) \) and the implied value of \( \sigma \) are presented in table 2. These estimates are based on various samples and subsamples of Korean housing units, the largest one having 1293 individual observations. The details of the survey are available in Follain et al (forthcoming).

The estimates for the different groups are rather well clustered and range from a low of 0.33 (group 1) to a high of 0.59 (Seoul). The simple average is 0.41. What is most important to the analysis is that these estimates are always significantly greater than zero and less than unity. The explanatory power of this simple model is satisfactory \( (R^2 \text{ varies from } 0.09 \text{ to } 0.64) \), and all the parameters are highly significant with a confidence level of 1\% or better.

### Table 2. Estimates of the elasticity of substitution.

<table>
<thead>
<tr>
<th></th>
<th>Constant term</th>
<th>( P_L/P_N )</th>
<th>( R^2 )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>National, group 1</td>
<td>0.70</td>
<td>0.67 (0.03)</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>National, group 2</td>
<td>0.68</td>
<td>0.65 (0.04)</td>
<td>0.25</td>
<td>0.35</td>
</tr>
<tr>
<td>National, group 1,</td>
<td>0.82</td>
<td>0.67 (0.06)</td>
<td>0.64</td>
<td>0.33</td>
</tr>
<tr>
<td>averaged by Dong ( ^a )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Seoul</td>
<td>0.49</td>
<td>0.41 (0.06)</td>
<td>0.09</td>
<td>0.59</td>
</tr>
<tr>
<td>Busan/Daegu</td>
<td>0.58</td>
<td>0.56 (0.08)</td>
<td>0.15</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Note: the numbers in parentheses are the standard errors of the estimates of \( P_L/P_N \); \( R \) is the coefficient of multiple correlation.

\( ^a \) Individual observations averaged by Dong (= neighborhood).

4 Sources of urban decentralization in Seoul and other Korean cities

4.1 Structure of the land-price gradient

The estimates of the elasticity of substitution can now be used in conjunction with the previously estimated demand parameters to determine the quantitative impact of various determinants of urban decentralization. With estimates of these parameters Muth (1969) has shown that it is a rather simple matter to trace the differential response of housing output in various parts of the city (the density gradient) to population growth and income growth.

As shown by equation (6), the elasticity of housing value per unit of land— that is, the intensity of residential-land use at various locations in a city—can be derived from knowledge of the elasticity of substitution, the ratio of nonland to land inputs, and the land-price gradient. Because we want to trace the impact of population and income growth on the intensity of land use, it is helpful to spell out explicitly the relationship between the structure of demand (income and price elasticities) and the behavior of the land-price gradient, which is given in equation (6) and for which we have estimates for several Korean cities (see Mills and Song, 1977).

The existence and behavior of housing- and land-price gradients can be derived from what is sometimes described as the Muth condition for housing equilibrium across a city. From the first-order equilibrium condition for a household it is a simple matter to derive the slope of the housing-price gradient as

\[
\frac{dP}{du} = -\frac{1}{Q} T(u) < 0 ,
\]

where \( T(u) \) is the marginal cost of transportation to the CBD from a distance \( u \) away.
This basic relation indicates that if housing expenditures are small or transportation costs are high then the housing-price gradient will be steep. This is seen more clearly by rewriting equation (8) as

$$\frac{1}{P} \frac{dP}{du} = -\frac{1}{PQ} T(u),$$

or verbally as

housing-price gradient

$$= - (\text{inverse of housing expenditures}) \times (\text{marginal cost of transportation}).$$

We do not have for Korea a direct estimate of the housing-price gradient, but we do have an estimate of the land-price gradient. As seen in equation (3), the relationship between the two price gradients is simple. In the case where the price of nonhousing varies little or not at all across the city, the two gradients differ by a factor of proportion which is the inverse of the land share, $K_L$:

$$d\ln P_L = \frac{1}{K_L} d\ln P.$$

From this relationship it is clear that the land-price gradient will always be steeper than the housing-price gradient because the land share is always smaller than one. In Korea we found the land share to be extremely high in Seoul in 1975, about equal to 0.33 (see Follain et al., forthcoming), compared with the value of 0.05 estimated by Muth (1969) for the US in 1960, the country at the other end of the scale in terms of land scarcity. Because $K_L$ is large the land-price gradient is only three times steeper than the housing-price gradient: land values do not decline very rapidly away from the center in Korea.

The relative effects of income growth and changing transportation costs can be clarified from the Muth equilibrium condition in equation (9). Given the fact that the income-elasticity estimate is clearly smaller than one—the best estimate being 0.60—housing expenditures have increased less than proportionately to income and have not contributed strongly to a flatter density gradient. We do not have time-series data on travel and the behavior of marginal transport costs. Because population growth in Seoul has been extremely rapid, with a compounded growth rate of 6.9% per year over 1965–1975, one would expect marginal transportation costs to have risen significantly over the period, when low income levels and strict pricing policies had effectively suppressed the diffusion of car ownership, the motorized modal split being 93% by public modes in Seoul. Thus the rise of income and the transportation situation are presumed to have led to a combined small decrease in the steepness of the price gradient.

We have found that the price elasticity of demand for housing was small, less than one in absolute value (Follain et al., forthcoming). The actual estimate was only -0.2, but cannot be trusted too far because housing-price elasticities are particularly difficult to estimate reliably. Given the price inelasticity of demand in Korea, housing expenditures, which are price times quantity purchased, are insensitive to price, and the price gradient will decline with rising housing prices. The combined effects of very rapidly rising incomes and rising housing prices lead one to expect a significant flattening in the price gradient, particularly in cities where marginal transportation costs have not increased markedly (an element still not quantifiable in Korea).

We do not have to speculate about the actual net effect of changing income, prices, and transportation costs on the price gradient in Korea. Because the Korea Board of Appraisors maintains very detailed records of land values in every city, we have a rare opportunity to see the evolution of price gradients as well as of density gradients.
A set of gradients selected and estimated by Mills and Song (1977, table A-7-3) is reproduced in table 3. One can observe that price gradients have become less steep over time, which supports the view that income effects and price effects have been much stronger than increases in marginal transport costs. Also note that, consistent with equation (3), the land-price gradients are steeper for the smaller cities, for which the values of land's share in the production of housing services are smallest (see also Muth, 1969, pages 49-50). One can note that over the period the price gradients have fallen at a rate of about 7·0% per year, which implies a very high rate of land appreciation at the urban fringe. The special case of Busan is due to the fact that city expansion is blocked by mountains.

Table 3. Land-price and population-density gradients for Korean cities.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>0·201</td>
<td>0·221</td>
<td>0·112</td>
<td>0·134</td>
<td>0·508</td>
<td>0·739</td>
<td>0·975</td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>0·163</td>
<td>0·186</td>
<td>0·094</td>
<td>0·109</td>
<td>0·348</td>
<td>0·666</td>
<td>0·946</td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>0·126</td>
<td>0·191</td>
<td>0·051</td>
<td>0·077</td>
<td>0·136</td>
<td>0·516</td>
<td>0·581</td>
<td></td>
</tr>
<tr>
<td>Average yearly decline (%)</td>
<td>7·5</td>
<td>5·3</td>
<td>3·5</td>
<td>6·2</td>
<td>7·6</td>
<td>3·3</td>
<td>7·0</td>
<td>1·0</td>
</tr>
<tr>
<td>Average household monthly income in 1974 (in thousands of won)</td>
<td>61</td>
<td>54</td>
<td>52</td>
<td>49</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population in 1975 (in thousands)</td>
<td>1888</td>
<td>2454</td>
<td>1311</td>
<td>244</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* The land-price gradients refer to residential land; alternative gradients exist for commercial land.

4.2 Shifts in the density gradient for Seoul

Given values of the land-price gradient and of the average shares of land and nonland inputs, we can obtain estimates of the change per kilometer in the value of $\varepsilon_S$. In turn, knowledge of the value of $\varepsilon$ at different distances from the center of the city permits the explicit calculation of the change in the value of housing per unit of land produced by exogenous price changes. That is to say, we can tell the extent and intensity of decentralization and the degree of change in the population-density gradient.

To present our analysis, we can use the data for the period 1970-1975, when the population of Seoul increased by 24·4% and real household income increased by approximately 35%. The effect of a 35% increase in income does not appear to be crucially dependent upon income distribution in Korea, based upon our earlier finding that the income elasticity of high- and low-income households are approximately equal. By use of a value of 0·6 for income elasticity, $\varepsilon_I$, which we estimated, the demand increase generated by income growth was 21%. Combined with population growth the total demand increase was 45·4%.

An increase in the demand for housing leads to an increase in the price of housing, which will be approximately equal to

$$
\frac{d\ln P}{d\ln} = \frac{\text{demand}}{\varepsilon_S - \varepsilon_P}
$$

where $\varepsilon_P$ is the price elasticity, as shown by Muth (1969, page 317). Our demand analysis has yielded a price-elasticity estimate of -0·2. The value of the average price change for the city can be calculated on the basis of the average value of $\varepsilon_S$. This average value of $\varepsilon_S$ is based on the estimate of $\sigma$ of 0·6 and on the sample
average values of $K_N$ and $K_L$ for Seoul:

$$e = 1 + \frac{K_N}{K_L}, \quad \sigma = 1 + \frac{0.7}{0.3} \cdot 0.6 = 2.4.$$  

This average value of $e$ might be the elasticity of the value of housing per unit of land at, say, five kilometers from the center of Seoul, where the city boundaries are almost exactly contained within a circle of fifteen kilometers radius. The overall housing-price change for the city implied by the total growth in demand was of the order of $18.4\%$.

To arrive at an understanding of the shift of the density gradient we must first calculate the changes in $e_S$ per kilometer from the center of Seoul. Because $\sigma$ is smaller than one, we know already that $e_S$ increases with distance from the center—that is, the density gradient will be flattening out. The third parameter estimate of equation (6), the land-price gradient, has been estimated by Mills and Song (1977) for Seoul in 1973 as being

$$\frac{dP_L}{du} | P_L = -0.126.$$

Thus the change in $e_S$ per kilometer is derived from equation (6) as

$$\frac{de_S}{du} = 0.6 \times (-0.4) \cdot \frac{0.7}{0.3} (-0.126) = 0.071 \text{ per km}.$$

Noting that it is only the rate of change of $e_S$ that matters, we might still wish to see the orders of magnitude of the values of $e_S$ at different locations. If it is reasonable to assume that the average value of $e_S$ is observed at five kilometers from the center, the value at the city center will be $2.4 - (5 \times 0.071) = 2.045$. At the current legal boundary of the city it will be $2.4 + (10 \times 0.07) = 3.11$.

Given the overall $18.4\%$ increase in the price of housing, the value of housing per unit of land will rise by about $35\%$, at the center, and $55\%$ at the periphery.

The associated change in the gradient of the value of housing per unit of land (the flattening of the land gradient between the center and the city limit) can be derived assuming the most common form of density function, the negative-exponential form, which has been used by Mills and Song (1977) for the Korean case. The new gradient is related to the old gradient through the rate of change of the value of housing per unit of land (the population density) calculated for the center and for the fifteen kilometer ring. Since Mills and Song have already estimated the density gradient for the original period of 1973, we have the following relationship between the two gradients, where the letters $A$ and $B$ refer to the values of the gradient ($g$) after and before the increase in demand:

$$\exp(-15g_A) = \frac{(1 + 0.55)D_B(15)}{(1 + 0.35)D_B(0)} \cdot 1.144 \exp(-15g_B),$$

where $D_B(u)$ denotes the population density at a distance $u$ from the center. Thus

$$\exp(-15g_A) \exp(15g_B) = 1.444.$$

and so

$$g_B - g_A = \ln 1.444 = 0.009.$$

The decline in the density gradient due to the housing-supply response is estimated to be $0.009$ for the five-year period. Because the Korean housing demand is not unit-price elastic as assumed by Muth (1969, page 317), we must add to the supply effect
the shift in the price gradient, which is equal to 0.201 - 0.126 - 0.075. Hence the total predicted shift in the density gradient will be the sum of the two effects and will be equal to 0.084. The population-density gradient is expected to decline to the value of 0.721 - 0.084 - 0.137. This value cannot be compared with the actual 1975 gradient estimate which is not available. In terms of yearly changes, the structural model yields a somewhat faster decline of 7.6% per year as against the actual three year rate of 5.3%. An immediate hypothesis for the overprediction is that the long-term static model we have been using assumes perfect fluidity of the population and income growth when density gradients were steeper. This difference may be more marked in Seoul, where both population and income growth have been extraordinarily high during the period studied for a city of such a size.

5 Conclusions
This numerical exercise based on actual estimates of the parameters of housing demand and supply in Korea has clear value for the explanation of shifting population-density gradients and urban decentralization. Although excessive empirical claims should not be made for the rather simple long-term comparative model of urban growth developed by Muth (1969) and Mills (1973), it provides us with explicit hypotheses for the identification of the specific characteristics of urbanization in a given country.

Because the elasticity of substitution between land and housing is less than unity, the increase in total housing demand generated by population and income growth will result in a more decentralized population. The higher the share of land in the production of housing services the fatter will the density gradient tend to be. The smaller land share in the Korean provincial cities rather than the difference in the estimates of the elasticity of substitution explains the much steeper gradients observed in provincial cities. The shifting price gradients observed in every city support our separate finding that the price elasticity of housing is rather small, less than one in absolute value. In Korea, theory and empirical estimates are in agreement and yield population-density gradients which are steeper than land-price gradients.

In addition to the elasticity of substitution between land and nonland inputs, the share of housing value attributed to land is an important element affecting the population-density gradient. In Korea the share of land is extremely high by international standards (we have used a value of 30%), as would be expected from the scarcity of land. To give an idea of the relative scarcity of land, Mills and Song (1977, page 163) have estimated that total land values in Korea in 1975 represented about 1.85 times the value of the GNP. Similarly, in Japan, Goldsmith (1976, page 388) has reported that land represented 3.3 times the value of the GNP in 1970 and was still 2.8 times a much larger GNP in 1975. In comparison with the US estimates for 1966 of a total value of land equal to only 0.7 times the GNP, it can be seen that both in Korea and in Japan the density gradient should be much less sensitive to rapid urban growth than in the US, or, in other words, decentralization much less pronounced.

Theoretical deductions and ordinary intuition converge in this finding. To provide another perspective on the same issue of urban land-use intensity in Korea, we can report the figures estimated by Harratty (1977, page 137). For the ten largest cities in Korea he found that the amount of urban residential land increased from 12.52 to 13.72 pyeong per capita between 1970 and 1974. That is, in 1974 there were about 45.3 square meters of residential land per resident in Korean cities, a very small amount.

The information base available for Korea is exceptionally good for an I DC country, and a similar analytical exercise elsewhere would require extensive preliminary work.
The merit of the exercise for the understanding of decentralization patterns in other countries is to show that it would be wrong to expect identical causes behind seemingly identical shifts in urban population densities. It is necessary to identify the local characteristics of housing production, the degree of land scarcity, the structure of the transportation system, the extent of car ownership, the rates of household-income growth, and the speed of population growth.

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