The Economics of Rail Line Closure

May 22, 1970

The paper discusses how to determine which sections of a rail system to close and when. These decisions are analyzed for both the economic (net social benefits) and financial (railway company's gains) objectives, when either exogenous traffic projections or traffic demand curves are available. The issue of government subsidization of rail service is also discussed. Examples and procedures for computing costs and optimum timing are included among the annexes.

The approach of the paper is essentially theoretical. Further work is envisaged on actual case studies to develop pragmatic solutions for the line closure problem.

The author, associate professor of economics at the State University of New York College at Oneonta, is grateful for the comments, criticisms and constructive suggestions of Mr. Jan de Weille and for the comments of other colleagues. He wishes to thank Messrs. de Weille and H.G. van der Tak for making his work at the Bank highly interesting and enjoyable. He also appreciates the assistance of Miss S. Snell, who edited the paper, and of Mrs. B. Easter and Miss J. Nasseir, who typed it.

Sector and Projects Studies Division
Prepared by: Ranjit K. Sau (Consultant)
# The Economics of Rail Line Closure

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I. INTRODUCTION: THEME, SCOPE, AND OUTLINE

1. An adequate transport system is usually a prerequisite if not a guarantee of economic growth. Since transport is highly expensive, frequently accounting for as much as 15 to 30 percent of public investment, it is of considerable importance that the resources engaged are optimally utilized. This requires a continuous review of the transport network. Should a new railway line be built? Should it use diesel or electric power? Should an existing railway line be replaced by a road improvement project? Should a port be further developed? These and related questions need periodically to be posed, analyzed, and resolved.

2. We are concerned in this paper with one aspect of this broad issue: for any existing rail line carrying passenger and freight traffic, is the service it provides justified? If so, for how long? When and under what conditions would it be more economical to partially or fully replace the line with an alternative mode of transport (a road)? If the decision involves a budget deficit, should the government provide a subsidy? These are the basic themes of this paper.

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2/ Throughout this paper we shall assume that the railway line under review carries both passenger and freight traffic. If in a certain case, it carries only one kind of traffic -- only passenger, or freight -- the situation is much simpler and the logic of our analysis still holds.
3/ For convenience, we shall take road as the only alternative to rail.
4/ These questions are usually raised in connection with so-called "branch lines". But, the label of "branch line" is often applied to sections which are, economically speaking, no different from "mainline" sections. This is clear in the cases of Talora-Kigoma and Tabona-Mwanza sections of the East African Railways, the former being classed as the official mainline and the latter as a branch line despite its heavier traffic densities. See: The Economist Intelligence Unit, and Freeman, Fox, Wilbur Smith and Associates, East Africa Transport Study: Draft Report, London, 1969, Vol. V, p. 285. Terminology should not obscure the true nature of the problem. These questions should be raised for any railway section whose viability is in doubt, no matter whether or not it is officially called a "branch line".
3. Why does an existing railway line, presumably thought to be viable during its construction, lose its viability over the course of time? The reasons include loss of rail traffic to road, construction of lines in areas which subsequently failed to develop, construction of lines to develop resources which were eventually exhausted, and substandard construction of lines. In Australia, the decline of ports outside the capital cities is an additional reason.

4. A common feature of uneconomic railway lines is a relatively light volume of traffic. Rail is characteristically suited to handle a large volume of traffic and long-distance traffic; when traffic density is low, the viability of the line is open to question whether it is officially a branch line or not. One rule of thumb to identify uneconomic railway sections is to consider, first, the annual traffic density in gross tons-kms.per rail km, covering both passenger and freight services. Since this index may occasionally underestimate the importance of the section's relation to the rail system as a whole, the second index is the annual figure of traffic originating in the section. However, it is impossible to suggest any precise numerical cutoff points which would be valid for all countries and at all times.

1/ See ibid., p. 473. "In order to keep construction costs to a minimum the State Government Railways sometimes were forced to build branch lines with only lightweight rails and a minimum of ballast. Although these lines generally served their purpose, in later years their substandard condition usually gave rise to high maintenance costs and imposed serious operating restrictions on trains using the lines."


3/ In Argentina, for example, any railway section which did not have a gross ton density of at least one million tons per year (excluding the weight of locomotives) or which did not show originated traffic equal to at least 500 tons per km. of line was considered subject to study. As a result, 40 percent of Argentina's railway lines as of 1958-59 was brought under a critical examination. Ministry of Public Works and Services, Transportation Planning Group, A Long-Range Transportation Plan for Argentina, Buenos Aires, 1962, Appendix III, pp. 29-30.
5. In countries such as Australia, Britain, Canada, Finland, Ireland, Japan, Sweden and the U.S.A., the withdrawal of railway services in favor of road services has been confined mainly to lines with light traffic density. The chief test of railway line closure in most of these countries has been "that by so doing there would be a financial betterment to the Administration itself." Because traffic flow may be very unevenly distributed over the transport network, reduction in service on lightly-traveled lines can produce such savings. The Beeching Report, for instance, points out that in 1963 one-third of the route mileage of the British Railways carried only one percent of the total passenger miles; one-third of the mileage carried only one percent of the freight ton-miles. Similarly, one-third of the railway stations produced less than one percent of the total passenger revenue; one half of the stations produces only two percent. The Report finds that the withdrawal of services from part of the light-traffic routes would result in a net saving of more than £ 18 million per year.


4/ This, however, does not necessarily mean that the closure of that one-third of the route mileage would affect only one percent of passenger traffic, or freight traffic of the railway system.

6. In some instances, investigations of rail line closure go beyond the relatively narrow limits of the railway's direct financial benefit. In a transportation study in Argentina, for example, where the relative costs of the substitute highway service were also taken into account, it was found that about one-third of the railway network was uneconomic and should be abandoned, and that an additional one-tenth needed further study with an eye to possible later abandonment. 

7. The magnitude of the problem of uneconomic railway lines can be of considerable importance, as in the countries mentioned so far, while in other countries it may be relatively insignificant, as in Korea. 

8. In this paper, we shall study the problem with reference to two alternative objectives, namely, profit maximisation (or loss minimisation) within the railway system, and the maximisation of net social benefits. For the first objective, only those costs and revenues directly relevant to the railway system itself are included, and profit is defined as the difference between revenue and cost. Costs are reckoned using the market prices of the inputs and revenues at their nominal value. For the second objective, all costs and all benefits to society as a whole are relevant; costs are calculated using shadow prices of the inputs and benefits are measured on an economy-wide basis. These two objectives are sometimes referred to as the financial objective and the economic objective, respectively.

1/ Adler, op. cit., p. 7.
3/ See Adler, op. cit., pp. 33-60; and also J.M.D. Little and James A. Mirrlees, Manual on Industrial Project Analysis in Developing Countries, Vol. II, Paris: O.E.C.D., 1969. The mechanics of calculation are the same for both objectives. Costs and revenues (benefits) occurring at different points of time are made comparable by discounting (present value calculation).
4. We shall distinguish between two types of cases in our analysis. In one case, the total (all modes) traffic volume is exogenously projected on the basis of expected economic growth and the pattern of industrial location. The problem then boils down to minimizing transportation costs. In the other case, the traffic volume is not given but is derived from the known demand function for transport services. Under such circumstances, the problem is no longer one of simple cost minimization, but of profit or net social benefit maximization.

10. Replacement of rail by highway service is not always a simple matter of economics. It may involve change in the pattern of community life in the affected area; it may require redistribution of authority among government ministries, and it may cause some inconvenience in the transition from rail to road. In short, it is both a socio-political and economic matter. Let it be clearly understood that we are presently concerned with only the economic aspect of the problem, and excluding fiscal questions at that. We shall not deal with financing of possible railway deficits (local vs. general taxation).

11. Our plan of study is as follows. Chapter II emphasizes careful examination of all possible cost reductions and improvements of rail service, and careful analysis of all the rail and substitute highway service alternatives. This chapter also introduces the cost concepts to be used subsequently.

1/ For example, some believe that the closure of branch rail lines in the prairies of Canada would reduce most prairie communities to "ghost towns". For an opposing view supported by factor analysis and multiple regression analysis, see Gerald Hodge, "Branch Line Abandonment: Death Knell for Prairie Towns," Canadian Journal of Agricultural Economics, February 1968, pp. 54-70.

12. Chapters III and IV are concerned with the analysis of the problem when the objective is to maximize net social benefits or social surplus (economic analysis). Chapter III examines the case in which the traffic volume is exogenously given; the following chapter, the case in which demand curves for transportation are given.

13. Chapter V presents a financial analysis of the two cases of Chapters III and IV. In Chapter V, the objective is to maximize railway profit (minimize loss). Chapter VI briefly notes the importance of pricing policy in relation to optimal investment planning. In the main, it deals with the question of government subsidy of railway deficit. Chapter VII presents a summary of the text, and its main conclusions. The annexes are devoted to mathematical analysis, elaboration of some concepts used in the text, examples, and a discussion of the marginal cost pricing rule and the theory of second best.

14. The method of optimization developed in Chapter III and Annex III minimizes estimated costs, while that in the beginning of Chapter V minimizes loss (maximizes profits) under certain circumstances. We believe these two methods are fairly operational, considering the data usually available to a project evaluator. The methods presented elsewhere in this study are basically applications of cost-benefit analysis, and are as practical as other cost-benefit techniques.
II. PROJECT ALTERNATIVES AND COSTS

1. This chapter explores the possibilities of reducing railway operating costs to improve the viability of service offered. All feasible rationalization of rail service needs to be examined and all alternatives prepared before raising the issue of substitution by road service. Options other than continuing present service and closure are open: it may be worthwhile, for example, to operate at a certain scale, for only certain types of traffic, on a certain schedule, and to reorganize the personnel. Considerable economies may be achieved from such operational rationalization. The next step is to estimate costs for each alternative, a topic also covered in this chapter.

A. Determination of Alternatives

1. Rail Operating Alternatives

2. A fundamental principle of efficient resource allocation is that output should be produced at minimum total costs. However, the valuation of output for transportation service is problematic because transport users consider speed, comfort, reliability, and scheduling in addition to price. Any measure which affects these elements is likely to influence both the demand for and the costs of transportation. In principle, therefore, closure of even the tiniest station in a light traffic line requires

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1/ On the light-traffic Central Wales line, for instance, seven stations were used by fewer than five passengers per week. It was suggested that operating economies could be achieved by closing some stations, reducing the number of calls made by trains at others, and reducing the number of journeys per day in each direction from four to three. See G. Clayton and G.H. Rees, The Economic Problems of Rural Transport in Wales, Cardiff: University of Wales Press, 1967; and also P.K. Else and M. Howe, "Cost-Benefit Analysis and the Withdrawal of Railway Services," Journal of Transport Economics and Policy, May 1969, pp. 186-187. In some Western European countries, a number of rail stations have been closed in order to improve the financial position of the railways, from 20 to 50 percent during 1955-65. See D. Herlihy, op.cit., p. 1133.
an elaborate cost-benefit analysis. In practice, however, such issues can be settled with simpler analyses, considering the order of magnitude of the costs and benefits involved.

3. Most operating alternatives can be broadly classified into two groups: those which produce more or less the same benefits to transport users but involve different time patterns of costs for the railway, and those which have different user benefits and different time patterns of railway costs. The choice between dieselisation or electrification of railway is one example of the former. Transport users may be indifferent as long as they get the same kind of service. But these two types of projects require different capital expenditures, and maintenance and operating expenses. For the technique of finding an optimal solution in such a case, see Annex I, "Choice Between Dieselisation and Electrification."

4. A more complex example would be a proposal to switch from scheduled freight service between points A and B to on-demand service because the traffic is so light that trains usually carry small, uneconomic loads. We assume that the probability distribution of arrival of goods at each station to be carried to the other is given and is independent of train scheduling, that costs of waiting time at the originating station are known, and that running time between A and B is independent of train size but operating costs (fuel, wagons, maintenance, etc.) vary with size. Under such circumstances, queuing theory can be used to find a schedule that minimizes total costs -- users' costs and railway costs.\(^1\)

5. The problem becomes more complex when the probability distribution of originating traffic is a function of the frequency and dependability of

\(^1\) For an introduction to queuing models, and their application to scheduling problem, see C.W. Churchman, R.L. Ackoff and E.L. Arnoff, Introduction to Operations Research, New York: John Wiley, 1957, Chapters 11, 15 and 16. Our problem is more complicated than most textbook applications of the queuing theory since it involves two-way traffic.
The uncertainty may divert part of the traffic to an alternative mode. To some extent, this can be incorporated into the model by introducing customer impatience. If some traffic dries up as a result of the switch from the existing schedule to on-demand service, there is a loss of benefit which is difficult to predict. If traffic does not dry up but is simply diverted from rail to another mode, the on-demand schedule can in principle be worked out by minimizing the total costs of rail and diverted traffic. If this total cost is less than that of the existing service, it is worthwhile to switch to on-demand service. In effect, one schedule is being replaced by another to reduce costs.

3. The second kind of operating alternatives, involving changes in both user benefits and railway costs, arises more frequently. For example, faster and more comfortable rail service could draw more traffic, but it may also raise costs. Under these circumstances, the correct method is to line up all feasible, mutually exclusive rail service alternatives and then choose the one with the largest net benefits, i.e., the highest present value of benefits minus costs.

2. Substitute Service Alternatives

7. The rail service alternatives should be compared with the substitute road service alternatives. If a highway network parallel to the rail line already exists, alternatives such as bus service for passengers are possible. However, if a highway must be built to replace the rail service,
broader alternatives exist. The route has to be chosen; various types of highway will be possible (two-lane, three-lane, four-lane, divided) each with different standards of surface, gradient, alignment, and so on. The planning authority should have enough experience to weed out clearly unsuitable plans, such as a four-lane divided highway with a low-grade surface (if the volume of traffic justifies the one it must rule out the other). The problem is therefore to choose the best of several practicable plans.

B. Types of Costs and the Principle of Avoidable Costs

8. For subsequent analysis we shall need information on three types of costs, namely, long-run average cost (Chapters III and V) and short-run and long-run marginal costs (Chapters IV and VI). By long run, we mean a period of time long enough to permit variation in input; correspondingly, the short run means a span of time during which, broadly speaking, the plant is fixed and production can be increased only by incurring operating expenses. In the short-run, major investment is ruled out for technical or other reasons.

9. In practice, it may be difficult to identify the costs attributable to a particular line or a certain section of the railway system. To avoid difficulty, we use the following principle: reckon all costs which can be avoided by not carrying traffic on the railway section under review, whether these costs are incurred on this line or elsewhere in the system. This principle holds for estimating the costs of highway service as well.

For example, suppose that the present value of the investment, maintenance, and operating costs of producing one unit of output per year over an extended period is $900. This is equivalent to the present value of an infinite stream of $90 per year at an interest rate of 10 percent. Though the actual cost stream may fluctuate, it is equivalent to a constant stream of $90 per year (See Annex II, "Computation of Costs"). Similarly, suppose that the cost streams of producing two units annually is equivalent to a constant stream of $112 per year, $138 for three units, and so forth (see Table 2.1, columns (1) and (2)). The long-run average cost is defined as the ratio of long-run total cost per period to the level of output per period (Table 2.1, column (4)). The marginal cost is, by definition, the additional cost of producing one more unit of output. It is simply the difference between the cost of n units and that of (n-1) units (Col. 3).

Because short- and long-run total costs may be different, short- and long-run marginal costs may be different.

Table 2.1: Long-Run Costs. An Illustration

<table>
<thead>
<tr>
<th>Output in Physical Units</th>
<th>Long-Run Costs ($)</th>
<th>Total</th>
<th>Marginal</th>
<th>Average (2)/(1)</th>
</tr>
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<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td></td>
<td>(4)</td>
</tr>
<tr>
<td>1</td>
<td>90</td>
<td>90</td>
<td></td>
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<td>2</td>
<td>112</td>
<td>22</td>
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<td>56</td>
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<td>3</td>
<td>138</td>
<td>26</td>
<td>52</td>
<td>46</td>
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<td>4</td>
<td>168</td>
<td>30</td>
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<td>5</td>
<td>202</td>
<td>34</td>
<td>84</td>
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<td>6</td>
<td>240</td>
<td>38</td>
<td>76</td>
<td>40</td>
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<td>7</td>
<td>282</td>
<td>42</td>
<td>84</td>
<td>40.3</td>
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<td>328</td>
<td>46</td>
<td>92</td>
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</tr>
<tr>
<td>9</td>
<td>378</td>
<td>50</td>
<td>100</td>
<td>42</td>
</tr>
<tr>
<td>10</td>
<td>432</td>
<td>54</td>
<td>108</td>
<td>43.4</td>
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</table>

10. The calculation of costs is complicated when more than one product is produced. The railway provides two services, passenger and freight. Cost of inputs which are used for both (railway tracks) may not be avoidable even if the output of one product is discontinued. In this case, the allocation of joint costs between services is irrelevant and unnecessary. We may note that all joint costs are non-avoidable costs, if only one of the activities is suspended. However, not all non-avoidable costs are joint costs. The pension payment to a retired freight train operator, for example, is a non-avoidable cost, but not a joint cost of freight and passenger services.

11. It is important to distinguish between operating costs given joint service and those of one service operating alone. Estimated operating expenses of freight and passenger traffic in four railway systems of Denmark are given in Table 2.2. For instance, in system A, the total operating expenses are 483 thousand crowns, while the savings of such costs by discontinuing either passenger or freight service individually are given by 203.6 and 136.7 thousand crowns, respectively. These are, in essence, avoidable costs.

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1/ The Economist Intelligence Unit et al., op. cit., has distributed joint costs arbitrarily between freight and passenger traffic. Such cost estimates are not valid for the purpose of deciding about partial railway line closure.

2/ This table considers only operating expenses and does not include any avoidable capital or maintenance costs.

But if either the passenger or freight service is operated alone, the operating costs would be 326.3 and 279.4 thousand crowns respectively. This basic asymmetry has to be kept in mind.

### Table 2.2: Expenses in Four Private Danish Railways

<table>
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<tr>
<th>Railway</th>
<th>Savings in Operating Costs by Discontinuing Service</th>
<th>Operating Costs When the Railway is Operated Only for Passenger Service</th>
<th>Total Operating Costs (2) + (5) = (3) + (4)</th>
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<tbody>
<tr>
<td>A</td>
<td>203.6</td>
<td>156.7</td>
<td>326.3</td>
</tr>
<tr>
<td>B</td>
<td>142.8</td>
<td>129.5</td>
<td>262.6</td>
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<td>C</td>
<td>95.5</td>
<td>113.7</td>
<td>170.6</td>
</tr>
<tr>
<td>D</td>
<td>55.6</td>
<td>147.6</td>
<td>136.1</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>497.5</strong></td>
<td><strong>447.5</strong></td>
<td><strong>945.6</strong></td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>124.4</strong></td>
<td><strong>111.9</strong></td>
<td><strong>233.3</strong></td>
</tr>
</tbody>
</table>

Source: Sjoberg, op. cit., pp. 21-22.

12. The nature of avoidable costs also depends on the time horizon. A comparison of avoidable costs with the estimated revenues (benefits) in the short-run (say, five years) or in the long-run (say, fifty years) is not in itself necessarily a good basis for closure decisions. As an illustration we take two cases from the British Railways. In 1951, the British Transport Commission recorded that costs would include the actual expenditures incurred, or to be incurred in the long run, to maintain and operate the apparatus required for a given level of services and traffic. In keeping with this approach, the cost calculations presented at the Lewes-East Grinstead inquiry included renewal charges for all capital assets involved, though some would not have needed replacement until many years after the inquiry. The other extreme -- very short time horizon -- is illustrated...
by the St. Ives-Kettering case, where in estimating avoidable costs, a bridge
with a technical life of possibly 100 years was written off in five years. 1/
These two cases pursued the wrong question. The appropriate question for
the profit maximization (or loss minimization) criterion is not whether a
railway service will cover its costs if run for a specific number of years,
say, five or fifty, but whether there is any period of time for which the
avoidable costs can be covered by the estimated extra revenue.

Therefore, all costs have to be calculated keeping in view
the time horizon for alternative combinations of the railway services. We
need to know, for example, the costs of passenger traffic when freight traffic
is to remain at a certain given level per unit of time; the costs of
passenger traffic when freight traffic is to be discontinued; the
costs of freight traffic when passenger traffic remains at a certain given
level per unit of time; the costs of freight traffic when passenger traffic
is to be terminated, and the costs of selected items of freight traffic under
other assumptions about other traffic, etc. The highway costs also have to
be computed under similar assumptions. Thus, a number of cost estimates are
necessary if partial closure of the rail service is under consideration. If
only complete closure is being considered, we compute the aggregate costs of
freight and passenger traffic.

C. Summary

The railway plant which has been inherited from the past may or
may not be optimal in view of expected future traffic flow. There may
exist scope for rationalization of the railway operation. The first step is
to review the present position and prepare practical alternatives to preser·
service. They should be compared with similar alternatives for substitute

1/ See M. Howe and G. Mills, "The Withdrawal of Railway Services, "Economic Jour-
nal, June 1960; British Transport Commission, Annual Report for 1951, and
Chapter V of this paper.
road service. For each alternative, three kinds of costs have to be calculated: long-run average cost, and short- and long-run marginal costs, as defined above.
III. Economic Analysis, Traffic Volume Given

A. The General Method

1. The principle of selection among alternatives with equivalent benefits and benefit patterns is to choose the alternative with the least (discounted) cost. In this chapter we shall apply this principle to determine the optimal timing of replacing rail by highway service under the assumption that the volume of traffic is given. When rail and highway services are perfectly equivalent (transport users are indifferent between the two if the same price is charged for traffic between any points A and B on the line), and either the price is given or the demand curve is perfectly inelastic, then the total volume of rail and road traffic is known, regardless of whether and when the railway line is replaced by the highway. A perfectly inelastic demand curve implies an infinitely large consumers' surplus at any finite price; thus, at any cost short of infinity, the transport project is economically justifiable. This assumption, although admittedly rather heroic, is frequently made in transportation studies. In the next chapter, we deal with the optimal timing of rail replacement in the case when the assumption of perfectly inelastic demand does not hold.

1/ In the Argentina transportation study, for example, the traffic forecast is made on the basis of the anticipated economic growth and industrial location; the question of replacing certain railway lines is solved by minimizing the transport cost of the projected traffic. Essentially the same procedure has been followed in the Spanish Railway Study: for each of three alternative growth rates for future traffic, the costs of the railway operation vis-a-vis those of the substitute highway services are examined. See Ministry of Public Works and Services, Transportation Planning Group, A Long Range Transportation Plan for Argentina, Buenos Aires, 1962, Appendix I; and Ralph E. Rechel, Spanish State Railway Study Program for Discontinuance of Services on Light Traffic Railway Line: Interim Report Illustrating Appropriate Methods of Economic and Financial Analysis, Washington, D.C.: August 1966, p. 8.
2. When the railway line is replaced by highway, two types of highway costs are incurred, (i) capital expenditure for construction or improvement of the highway, and (ii) maintenance and operating expenses on account of the traffic now diverted from rail to road. For simplicity, let us assume that the required capital expenditure for the highway remains unchanged by timing, i.e., no matter in which year the construction or improvement is initiated. For each additional year the railway service is allowed to operate, there is a "gain" consisting in savings on the capital charge for the postponed capital expenditure of the highway and on operating and maintenance costs of road service. On the other hand, there is a "loss" consisting in the costs incurred to operate and maintain the railway service for a year. Eventually the loss in postponement of the highway program by one more year will exceed the gain. This is the optimal time to replace rail service by highway service.

3. The determination of the optimal time to replace the railway service can be broken into two steps. Firstly, compare the long-run average cost of the railway service with that of the substitute highway service. If the former is less than the latter, the railway service

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1/ If, for example, the required capital expenditure for the highway is $10,000, the highway operating and maintenance costs per year, $100, and the railway operating and maintenance cost per year, $250, then if the railway service continues for one more year, the cost is $250. By the same token, we have postponed a capital expenditure of $10,000, which would earn $500 in a year at an interest rate of 5 percent. Thus we gain $500 in capital charges and $100 in road maintenance and operating expenses. So the "loss" is $250 but the "gain" is $600, as a result of postponing the replacement of the railway by a highway for a year.

2/ Strictly speaking, this formulation is only valid for a constant flow of traffic year in, year out, and does not allow for (dis)economies of scale. More generally, one should compare the aggregate present value of total long-run costs (i.e. operating, maintenance, and investment costs) required to serve future traffic demands permanently by either rail or road.
should continue. If the former is larger, sooner or later, highway service should replace the existing railway service. Secondly, to determine the precise optimum timing, determine at what point the "loss" in the postponement of the highway program by one more year tends to exceed the "gain" therefrom. Three formulas for computing this time are given in Annex III; examples from Argentina and Spain are given in Annex IV.

B. The Boiteux Effect

Investment policy, operating and maintenance policy, and the optimal timing of replacing rail service are interrelated. As Marcel Boiteux writes, in connection with certain local railroad lines in France:

"Until a decision is reached concerning the retirement date of an unprofitable line, the line will be maintained normally and its elimination will never seem justified. After all, a line kept in proper order can carry on for quite some time under its old momentum, and can operate at a profit for several years at minimum expense (often for less than the total cost of operating a highway transport service). Let us suppose, on the other hand, that it has been decided some fifteen years ago to set the dates at which deficit lines would be successively abandoned, and that these lines had been managed accordingly. We should find in 1952 that a line scheduled for retirement in 1954 can still be operated for two or three years without prohibitive costs, but that, beyond that time, its maintenance would involve such expenditures that nobody would deny for a moment the advantage of replacing it forthwith with a highway service." 1/

The point is that operating and maintenance expenses depend largely on retirement policy, and vice versa. In view of this interdependence, sometimes referred to as the Boiteux effect, the general procedure of dynamic

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2/ Ibid., pp. 49-59.
cost minimisation is as follows. Designate an arbitrary time period $\theta$ when the railway service would be replaced by road service. Observing the quality and safety requirements of railway operation, determine the time pattern of investment and maintenance expenses entailed by continuing service such that the present value of such costs less the salvage value of the disposable assets at period $\theta$ is a minimum. To this, add capital, maintenance, and operating expenses of highway service from period $\theta$ to a finite or infinite time horizon, as desired. The minimum total discounted costs will vary with the choice of $\theta$. The optimum timing is that $\theta$ for which total discounted costs are minimised.

C. Summary

5. If the volume of traffic per unit of time is given from projections and the problem is to minimise the transport cost, compare the long-run average cost of carrying the traffic on the existing railway line with the corresponding long-run average cost of highway service. If the former is less, the railway service need not be replaced by a highway service; if greater, it should be replaced.

6. To determine the optimum timing of replacement, take an arbitrary time period $\theta$ when the railway service is assumed to be discontinued. Compute the minimum costs of carrying the traffic by rail up to period $\theta$ with the knowledge that rail service will then be terminated; add the highway transport costs from period $\theta$ to the time horizon. The total cost thus computed is a function of $\theta$. Choose $\theta$ such that these total costs are minimised with reference to $\theta$. The computation of the optimal $\theta$ is much

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1/ We assume that highway width, curvature, gradient, pavement, and other design components have been chosen to minimize cost and that the resulting highway cost streams are given.

2/ See also footnote 2 on page 17.
simplified if it is further assumed that rail operating and maintenance costs are independent of the time at which the rail service is terminated; this amounts to ignoring the so-called Boiteux effect. Formulas for calculating optimum $b$ for this simplification are given in Annex III.
In the previous chapter the optimum timing problem came down to minimizing total transport costs given the total volume of rail and road traffic. The approach of this chapter is more general, since the volume of traffic is not taken as given but as a function of price and of whether rail or highway service is being used. Furthermore, the objective is not just to minimize transport costs, but to maximize social surplus, defined as the difference between the social benefits and social costs in the familiar language of cost-benefit analysis.

A. The General Method

1. The Concept of Social Surplus

2. Benefits received by transport users are fully reflected in the demand curve for transportation. If in Fig. 4.1 the price of rail journey per mile is set at $P$ and the volume of traffic is $X$, then the $X$th marginal unit of traffic is deriving benefits from the railway to the extent of $P$ dollars; benefits to intra-marginal traffic (at each point from 0 to $X$) are higher (from $A$ to $P$). The total benefits are given by the area under the demand curve from the origin to the point $X$, or the area $OABX$ in Fig. 4.1.
Since the price per unit is \( P \), the total amount paid by traffic \( X \) is \( OPX \). Area \( APE \), or \( OAPX \) minus \( OPX \), is called consumers' surplus.

3. Parallel to the notion of consumers' surplus is that of producers' surplus. If for traffic output \( OX \) the average cost of production is \( CX \), the producer earns a surplus to the extent of \( BC \) per unit, or, a total surplus of \( BCDP \).\(^1\) Alternatively, if the average cost is \( C'X \), the producers' surplus is negative, and amounts to area \( BC'D'P \).

\(^1\) Note that if \( ML \) is the marginal cost curve, the total cost of output \( OX \) is given by the area \( OMNX \). Thus \( ODCX = OMNX \); and \( BCDP = EMMP \).
Whether producers' surplus is positive or negative, the relevant quantity is consumers' surplus plus producers' surplus, called the social surplus. With a decreasing demand curve, as in Fig. 4.1, consumers' surplus is always positive within the relevant price range, but producers' surplus may be positive or negative depending upon the price and the average cost of production. In any case, a project is justified if the sum of the consumers' and producers' surpluses is positive, because this implies that the total social benefits received from the project exceed total costs. In principle, the recipients of benefits could compensate those to whom costs accrue and still be better off. A corollary to this is that the price should be equal to the short run marginal cost of production.  

2. Some Typical Cases
5. Initially, we assume that only rail service exists and that a new highway would have to be constructed to provide alternative service. Under these circumstances, the first case we consider is based on the assumption that the railway service and the alternative highway service are perfect substitutes, in the sense that the demand curve for transportation remains the same irrespective of whether rail service or road service is available. The quantity demanded -- the actual volume of traffic -- depends upon the price charged. In the second case, the rail and highway services are not perfect substitutes and can no longer be treated as identical commodities. Then we examine the case in which road and rail coexist and the issue is road improvement rather than road construction. After the discussion on cost-benefit analysis, we will briefly present methods of project selection.

1/ See Chapter VI and Annex VI.
Our strategy is to reduce the problem to choosing among a set of mutually exclusive projects. Although it is not analytically difficult to include complex projects involving both rail and road (for example, faster train service with fewer stops for long-distance traffic combined with bus service for short-distance traffic), for the sake of simplicity of presentation, we limit ourselves to simple projects involving either rail or road. In view of the discussion in Chapter II, we first line up the alternative projects, beginning with passenger service alone. Let present rail passenger service be denoted by $V_0$, and let two alternative rail passenger services be denoted by $V_1$ and $V_2$, respectively. Similarly, let two substitute road services be called $V_3$ and $V_4$. We want to choose among these five mutually exclusive projects.

The procedure we follow is to examine four proposed passenger service projects, with reference to their respective social surpluses per period in the long run. Whichever has the largest social surplus is then compared to $V_0$, the present rail service. A similar exercise is carried out for freight traffic alone. This constitutes a review of all partial closure alternatives. Then we examine all the proposed alternatives of rail and road services for passenger and freight traffic together, covering the complete closure possibilities. The project selected is the one with the maximum of social surplus. It may involve partial or complete closure of the railway services, with or without the substitute road service. If all rail and road alternatives have negative social surpluses, the rail services should be terminated without providing any substitute road service.
8. The next issue is to determine the optimum timing of the switch-over. The optimization method developed in detail below assumes that some variant of the substitute road service has been found to be the best of the alternatives, for passenger service alone, freight alone, or for passenger and freight together, as the case may be. This road project is then compared with the existing rail project.

B. Case A: Rail vs. New Highway, Perfect Substitutability

9. In Fig. 4.2, DD' is the demand curve for transport per unit of time, whether it is rail or road service. For instance, if the price charged is OC per ton, 00 tons of traffic move between two given points per period. IMCR is the long-run marginal cost curve for railway service, while LNC is that for the highway service. The social surplus is maximized if price is equated to the marginal cost. So the social surplus with the railway service is given by area DAF per period; and that with the highway service is given by area DBE per period. The termination of rail service should be considered if and only if area DAF is less than area DBE; otherwise, rail service should continue.1/

Figure 4.2: Demand For Transport and Long-Run Marginal Cost

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1/ See also footnote 2, page 17. Strictly speaking, the comparison should be between the aggregate present value of the sequence of (annual) social surpluses if future traffic demands are served permanently either by
10. Unless the demand curve is perfectly inelastic (a vertical line) or the cost curves are perfectly elastic (horizontal lines), then the long-run marginal costs of rail and highway services at any specific volume of output, taken alone, are irrelevant to the closure decision. If the rail service is in operation, the equilibrium volume of traffic in Fig. 4.2 would be OC, at which FH is the marginal cost. At this traffic level, the marginal cost of alternative road service is given by HJ, which is greater than the marginal cost of rail service by FJ. Alternatively, if the road service is in operation, the marginal cost of road service at the equilibrium traffic level OC is still more than that of the alternative rail service. These facts do not necessarily mean that rail service should not be replaced. The social surplus generated in equilibrium by a transport service is the sole criterion; the marginal cost alone is not an indicator of that amount.

11. To continue with our example, if the social surplus with rail is less than that with the highway, then the latter should replace the railway service. \(^1\) How, then, is the replacement time determined? In general, if the highway program is postponed for an additional year, the "loss" is given by area DEE. The gain is the social surplus generated by continuing the railway service for one more year. Thus for every successive year of postponement, the corresponding marginal gains and marginal losses should be compared. The time for replacement is at hand when the two are equal.

12. More specifically, once the demand and cost curves show that rail service will be discontinued sooner or later, major railway investment is ruled out. The Boiteux effect states that operating, maintenance, and

\(^1\) See also footnote 1, page 25.
equipment policy depends upon the anticipated date of retirement of the service. Therefore, we designate an arbitrary time period $\phi$ for rail service closure and work out the implied operating, maintenance, and equipment policy. For any given $\phi$, we can compute the short-run marginal cost curves of the rail service for periods $T-1, \ldots, \phi$ denoted respectively by $SMCR_1, SMCR_2, \ldots, SMCR_\phi$ in Fig. 4.3. The short-run marginal cost curves presumably shift upward over time, since, in the absence of any major investment, production is based on progressively older and more deteriorated plant and equipment. Since price is equal to short-run marginal cost in every period, the price also tends to rise, traffic to fall, and the social surplus to shrink.

13. If the social surplus from the railway at the designated $\phi$ is equal to area $DBE$, then that is the optimum $\phi$ to discontinue rail service: $SMCR_{\phi + 1}$ will be greater than $SMCR_\phi$ and the social surplus from the rail service in $\phi + 1$ will be less than that from the alternative highway service.

**Figure 4.3: Demand for Transport and Short-Run Marginal Costs**

![Diagram showing demand for transport and short-run marginal costs](image-url)
If, however, the social surplus from the railway service at the designated \( \phi \) is greater or less than area DBE, the exercise must be repeated using a larger or smaller value for \( \phi \), respectively. Through successive iterations, the optimum \( \phi \) can be found. This is the algorithm of finding the optimum when the objective is to maximize social surplus.

14. The validity of this procedure is contingent upon the assumption that price is actually equal to the short-run marginal cost; if a different pricing policy is followed, the analysis loses some of its significance. However, given any pricing policy and its objective, a similar analytical process can be worked out. For example, if price is equated to average cost in order to maximize only consumer surplus while keeping the transport budget in balance, our model needs only one modification: replace marginal cost curves with corresponding average cost curves. If the objective is to maximize profit, the exercise can be carried out with marginal revenue and marginal cost curves.

C. **Case B: Rail vs. New Highway, Imperfect Substitutability**

15. Now we examine the case of non-identical demand curves for rail and road services. Let DD' be the demand curve for rail service and dd' the demand curve for the alternative highway service in Fig. 4.4. Again, IMCN and IMCH

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1/ For a forceful reasoning in favor of short-run marginal cost pricing in transport, see A.A. Walters, *The Economics of Road User Charges*, World Bank Staff Occasional Papers No. 5, 1968.


3/ See Chapter V of this paper.
are the two long-run marginal cost curves for railway and highway respectively; hence DAF and dBE are the social surpluses generated by rail and road.

Figure 4.4: Demand for Existing Rail and New Highway Services and Long-Run Marginal Costs

As before, rail service should ultimately be withdrawn if and only if area DAF is less than area dBE, and the optimum timing of withdrawal is determined by following a similar procedure as described in connection with Fig. 4.4. If area DAF is not less than area dBE, the railway service should continue.

D. Case C: Rail vs. Highway Improvement

16. In cases A and B, we assumed that a substitute highway would have to be built. If, however, a highway adjacent to the railway line already exists, rail line closure may require only improvement of the highway to carry the higher volume of traffic. The basic principles of our analysis still hold, but the method of computation is somewhat different. If, for

\[1/\] See also footnote 1, page 25.
example, the existing equilibrium is as in Fig. 4.5, the total social surplus in equilibrium is given by \( \text{DAP} + \text{dEE} \). Note that when both rail and road service are available, consumers reveal a preference for rail service to the extent of OR. If rail service is withdrawn and only existing road service is supplied, consumers may be worse off. In any case, we emphasize again that we are seeking to maximize social surplus.

17. As mentioned in Chapter II, a number of road service alternatives may exist, differing in speed, comfort, scheduling, and other respects from existing road service. Let \( V_x \) in Fig. 4.6 be the road service with the greatest social surplus of all road and rail alternatives being considered as replacements for the existing road and rail services in Fig. 4.4. In Fig. 4.6, \( HH' \) is the demand curve for \( V_x \), \( \text{DMCL} \) its long-run marginal cost curve; thus, the social surplus generated by \( V_x \) is given by \( HH' \). Both the existing rail service and the existing road service should be terminated in favor of road alternative \( V_x \) if and only if \( HH' \) in Fig. 4.6 is greater than the sum of \( \text{DAP} \) and \( \text{dEE} \) in Fig. 4.5. If this is the case, the optimum timing of withdrawal has to be worked out following a procedure analogous to that described in connection with Fig. 4.3.

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1/ When road and rail service are taken as close but not perfect substitutes, their demand curves are interdependent and the measurement of consumers' surplus for both together is somewhat difficult. It can be shown that the demand curves in Fig. 4.5 slightly underestimate the true consumers' surplus derived jointly from road and rail combined. To simplify the exposition, the discussion in the text assumes that the underestimation is negligible. See Annex V, "Consumers' Surplus Measurement for Close Substitutes."

2/ For example, the Villacanas-Quintanar section of the Spanish railway system has an adjacent, well-developed highway. At least two alternative variants of the passenger service on the existing highway have been considered as substitutes for the rail service. See Annex IV, "The Optimum Timing of Substitution: Case Studies From Argentina and Spain."

3/ See also footnote 1, page 25.
Figure 4.5: Demand for Existing Rail and Existing Road Services and Long-Run Marginal Costs

Figure 4.6: Demand for Road Service Alternative $V_x$ and Long-Run Marginal Cost
18. The analysis can get somewhat more complicated if we recognize that demand curves may not remain stable over time but may shift, depending upon the price charged for and the economic development induced by the new transportation service. Also, if the case for a highway stands on its merit as a stimulus to rapid economic development, the optimum time to replace rail service by road should be advanced accordingly.

E. The Role of Cost-Benefit Analysis

19. In cost-benefit terms, a project is worthwhile if the benefits, to whoever they may accrue, are in excess of the estimated costs. As we have seen above, the area under the demand curve measures the total benefits received by the consumers of a transport service, while the area under the marginal cost curve represents the estimated total costs. We call the difference between these two areas the social surplus. The social surplus is therefore a cost-benefit criterion. Why then have we so far used the social surplus rather than the more usual cost-benefit? The fundamental reason is that the former underlines the importance of pricing policy. To quote Prest and Turvey,

"The pricing policy adopted will affect project outputs, and hence project costs. Tolls on a motorway, for instance, will affect the volume of traffic, and this may affect the appropriate width at which it should be constructed. Thus, benefits and costs are not independent of pricing policy." 2/


Once pricing policy is given, the size of the transport project and the volume of traffic are determined. Once the volume of traffic is thus determined, it does not matter whether the analysis is carried out in terms of social surplus or cost-benefit; they are identical.

20. This observation should be qualified. If the planning authority of a country has a multiplicity of objectives such as "better" income distribution and balanced regional development, benefits may not all be equally valuable. Different weights may have to be attached to benefits accruing to certain sections of the population in certain regions. Cost-benefit analysis is evidently more flexible and operationally more suitable than the straightforward social surplus analysis as outlined above in accounting for such factors. With cost-benefit the timing of the railway service withdrawal can be determined to maximize net social benefit. A set of cost-benefit analyses has to be done for rail service replacement by the optimum highway service in year t = 1, 2, 3, .... The optimum year is the year in which the net social benefits are maximized from the present to the end of the time horizon. An example of cost-benefit analysis for rail service withdrawal is given in Annex VII.

F. Project Selection and Optimum Timing

1. So far we have been concerned primarily with a single railway section. When a number of railway sections are subjected to analysis, we may come up with a list of investment projects, some involving investment in railway service and others in road service. Since investible resources are limited, priorities have to be assigned to these projects as in all economic planning. We mention two attempts to cope with this problem. Peter Steiner and

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Stephen Marglin have developed programming models for choosing among alternative public investment projects, subject to a budget constraint.

Marglin's model is dynamic in the sense that it takes into account budget constraints over a number of years, and determines accordingly the optimum time sequence of projects. But these models require an amount of data which may not be readily available to an evaluator of rail and road projects.

22. Another method of selecting projects at the sector level which are consistent with national planning has been suggested by A.K. Sen. It calls for three sets of data, namely, (a) the objectives and their relative weights, (b) the rate of discount, and (c) the shadow prices of major inputs such as labor and foreign exchange, to be provided to the project evaluator at the sector level by the national economic planners. The project evaluator, using social surplus (cost-benefit) analysis, draws up a list of projects and their time sequence. The national planner now compares the total demand for resources in the country with the availability of resources. If the demand is higher, for instance, he would raise the discount rate and shadow prices. Next the project evaluator at the sector level would repeat his calculations using the new data, producing a new list of projects in time sequence. The national planner again compares the resource requirements of these projects with the available supply. It can be proved that under certain assumptions this iterative process

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converges on an optimum solution. It is therefore suggested that the project evaluator prepare lists of projects and their time sequence corresponding to a set of reasonable alternative discount rates and shadow prices of labor and foreign exchange, etc. The national planner then would select that list which is consistent with the overall resource availability now and in the years ahead.

0. Summary and Conclusions

When the traffic volume is not given as a datum, but the demand curves for the railway service and for a reasonably comparable highway service are given, the question as to whether and when the rail service should be withdrawn in favor of road service is relatively complex to resolve. Assuming that the objective is to maximize the social surplus, defined as the sum of consumers' and producers' surpluses, from which it follows that price should be equated to marginal cost of production, the problem at hand can be resolved in the following way. First, from the demand curves and long-run marginal cost curves, determine the social surplus per period generated by the railway service and by the alternative highway service. If the long-run social surplus of rail service per period is less than that of highway, the rail service should be withdrawn in due course. Optimal timing of withdrawal is to be determined by comparing the marginal gain of postponement with the corresponding marginal loss. Conversely, if the long-run surplus of railway per period is greater than that of highway, the rail service should be allowed to continue.

Footnotes:
2/See also footnote 1, page 25.
24. Cost-benefit analysis is essentially the same as social surplus analysis. However, if there is a multiplicity of objectives, the former is operationally more flexible for taking them into account.

25. In order to promote consistency with overall national planning, the project evaluator at the sector level should evaluate the projects and determine their time sequence for reasonable alternative sets of discount rate and shadow prices of major inputs such as labor and foreign exchange.
V. FINANCIAL ANALYSIS

1. In the preceding two chapters, the basic objective was maximization of social surplus for a given volume of traffic and for given demand curves. Now we shall consider the financial objective, namely, maximization of the railway profit (minimization of the railway loss). This change in objective is reflected in estimation of costs. With the financial objective, costs are reckoned at market prices (including taxes and subsidies); only those costs are taken into account which are incurred by the railway itself. Secondly, for computing profit, only the revenue earned by the railway is relevant. We shall distinguish two cases: (i) volume of traffic and expected future revenue are assumed to be known, (ii) the demand curve for transportation is known.

A. Traffic Volume and Revenue Given

If in a certain railway section the expected volume of traffic in the years ahead is known, on the basis of past experience or otherwise, and the rate to be charged is also known, then one can calculate the discounted present value of total earnings corresponding to the closure of the railway service in year $\theta = 0, 1, 2, 3, \ldots$. In Fig. 5.1, curve $OR$ represents these discounted present values. If the railway service is closed in year $\theta = T'$, for instance, the present value of total earnings attributable to this section from now to year $T'$ is given by $PT'$. The curve $OR$ rises at a decreasing rate because annual revenue does not rise fast enough to overcome the effect of the discount. One may also calculate the present value of the total costs from now to year $\theta$ if the railway service were to be closed in $\theta$. In Fig. 5.1, $CC'$ is a curve of such costs. If the service is continued only up to period $T'$, for instance, and then terminated, the present value of the total costs is given by $QT'$. Thus
Figure 5.1: Railway Revenue and Cost Over Time
with the revenue curve OR and cost curve CC', the present value of the profit earned by operating the service up to year $T^2$ and then closing it amounts to PQ. Similarly, the profit for $\delta = T$ is UV.

3. With these two curves, the rail service is unprofitable up to year t. It would be profitable to close the service some time between years t and $t'$. The optimum time to discontinue the service is the time at which present value of profit is at a maximum, or year T. In year T, the marginal gain of postponing the discontinuance by one year is equal to the marginal loss in the sense that the slope of curve OR is equal to that of CC'.

4. If the cost curve never crosses below the revenue curve, the termination of rail service would never yield a positive profit, as for cost curve EE' and revenue curve OR. If rail service is abandoned immediately, the loss would amount to OE. Loss could be minimized by continuing the service to year $T'$, when the tangents to EE' and OR are parallel. Alternatively, the cost curve may lie below the revenue curve beyond some point in time. In this case it may be profitable to continue service indefinitely beyond that point, even though it is unprofitable up to then.

5. The curves show us that even though a rail service is commercially unprofitable now and is likely to be so in the next few years, it does not necessarily follow that profit can be maximized or loss minimized by terminating the service immediately. We must consider the alternatives of closing it now, next year, and many years into the future and calculate present values of revenue, and costs, and profit as summarized in Table 4.1. If the P's in column (4) are all negative, the optimum year
Table 5.1: Railway Closure: Year-by-Year
Revenue, Cost and Profit

<table>
<thead>
<tr>
<th>Year of Closing</th>
<th>Present Value of</th>
<th>Cumulative Revenues</th>
<th>Cumulative Costs</th>
<th>Net Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td></td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>0 (Now)</td>
<td></td>
<td>B_0 = 0</td>
<td>C_0</td>
<td>P_0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>B_1</td>
<td>C_1</td>
<td>P_1</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>B_2</td>
<td>C_2</td>
<td>P_2</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>B_3</td>
<td>C_3</td>
<td>P_3</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>B_4</td>
<td>C_4</td>
<td>P_4</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>B_5</td>
<td>C_5</td>
<td>P_5</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

of discontinuing the service is that year when $P$ is lowest. If some of the $P_i$'s are positive, it may be profitable to continue service for some years, although it may be unprofitable at present and in the near future. The optimum year for closure is that year in which $P$ has the highest value.

B. Demand for Transportation Given

6. When the demand curve is given, the analysis is somewhat more complicated. The procedure is formally very similar to that given in the preceding chapter using social surplus maximization. First, the question: should the line be discontinued? In Fig. 5.2 let $RR'$ be the curve of marginal revenue, defined as the increment in total revenue resulting from production and sale of one additional unit. $LMCR$ is the long-run marginal revenue.

---

Figure 5.2: Railway Marginal Revenue and Long-Run Marginal Cost

Figure 5.3: Railway Marginal Revenue and Short-Run Marginal Cost
cost curve. Profit is maximized at the output at which the marginal cost curve intersects the marginal revenue curve. Thus, area $\text{AB}$ is the profit per period in the long run, equilibrium output being $\text{OL}$. After computing the profit for each alternative rail service, choose the alternative yielding the highest profit. If no alternative, including the present one, shows profit in the long run, present rail service should be discontinued.

Second, the question: if the line is to be closed, when?

Suppose rail service is tentatively to be discontinued in an arbitrarily chosen year $\text{O}$. No further major rail investment will be made. Compute the short-run marginal cost curves: $\text{SMCR}_1$, $\text{SMCR}_2$, $\text{SMCR}_3$, ..., $\text{SMCR}_\text{O}$ for years 1, 2, 3, ..., $\text{O}$, respectively. Since production is using progressively older and more deteriorated plant and equipment, maintenance and operating expenses will increase over time. The short-run marginal cost curve will therefore shift upward as shown in Fig. 3.1. Accordingly, the amount of profit per period falls over time. If the pre-determined $\text{O}$ is such that profit in year $\text{O}$ is exactly zero, then this is the optimum $\text{O}$ and the rail service should be closed in that year. If profit in year $\text{O}$ turns out to be negative, repeat the exercise with a smaller value of $\text{O}$; alternatively, if profit in year $\text{O}$ is positive, try a higher value of $\text{O}$. This iterative procedure will converge on the optimum $\text{O}$. 
The objective is to maximize railway profit (minimize loss). Costs are to be reckoned at market prices (including taxes and subsidies); only those costs are taken into account which are incurred by the railway in providing the rail service under review. For computing profit, only the revenue earned by the railway system is relevant.

If the volume of traffic and the corresponding revenue per year expected in future are known, it is relatively easy to determine whether and when the rail service should be discontinued. Calculate the present value of total profit derived from continuing the service up to year \( t \), \( t = 0, 1, 2, 3, \ldots \). The result is a list of profits to be earned, and/or losses to be incurred. That value of \( t \) is optimum for which profit is highest, or the loss lowest.

If, instead, the demand curve is given, the analysis is somewhat more complicated. Equilibrium output is determined by the intersection of the marginal cost and marginal revenue curve; the area between these two curves measures the profit per period. The rail service alternative for which profit is greatest is the optimum. If no variant, including present service, yields profit in the long run, the present service should be terminated. The method of determining the optimal time for the termination is formally similar to the one presented in the preceding chapter but uses the marginal revenue curve instead of the transportation demand curve.
VI. PRICING, DEFICIT, AND SUBSIDY

1. In a classic paper, Foster and Beesley estimate a positive discounted present value of the proposed Victoria Line in London. At a 5 percent discount rate, social benefits minus all social costs would amount to £65 million; at 6 percent to £39 million; and at 8 percent £19 million. Then they examine the implications of alternative methods of financing the investment. In their words, "There is a serious problem now: ... to finance the Victoria Line in the traditional manner would cause serious social losses... A high proportion of social benefits (up to between 41 and 61 per cent) would be lost if the Victoria Line were financed in the traditional way by a flat-rate percentage increase in fares." 1/

This example clearly underlines the relationship between pricing and optimal investment. This is a highly complex subject, dealt with extensively in the literature. We do not propose to go into it here in a comprehensive way and shall only touch on certain aspects.

2. In course of examining the unprofitable railway lines of the British Railways, the Beeching Report considered the merits of reducing the fare to attract more traffic and of increasing the fare. In theory, the amount of price change required to make the railway service financially viable may not in fact be feasible. Let P stand for price and Q for quantity demanded. The elasticity of demand, e, is defined as: 
\[ e = \frac{d \log Q}{d \log P} \]

Therefore, 
\[ d \log Q = e \cdot d \log P \]

Now, total revenue is P.Q. So 
\[ d \log P \cdot Q + d \log Q = (1 + e) d \log P \]

If the elasticity of demand is -1.5, a 10 percent fall in

2/ See, for example, R. Turvey (ed.), Public Enterprise, Penguin, 1968.
price is required to increase revenue by only half as much, while it induces a 2 percent rise in traffic. If the elasticity of demand is -2, a 10 percent reduction of price results in an increase of revenue by the same percent, but traffic volume rises by twice as much. Such relationships between price change and total revenue are given in Table 6.1 for three values of the elasticity of demand.

Table 6.1: Price Change Required to Achieve Change in Total Revenue

<table>
<thead>
<tr>
<th>Change in Revenue</th>
<th>Change in Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e = -0.8 )</td>
<td>( e = -1.5 )</td>
</tr>
<tr>
<td>+ 6</td>
<td>+ 25</td>
</tr>
<tr>
<td>+ 10</td>
<td>+ 50</td>
</tr>
<tr>
<td>+ 50</td>
<td>+ 250</td>
</tr>
</tbody>
</table>

The more fundamental question is whether railway pricing policy should be designed solely to improve the financial position of the railway itself. In the case of the Victoria Line, price change significantly affects social benefits.

If we take as given the marginal cost pricing rule, the railway may face budget deficit for the following reasons.

(i) If the railway exhibits increasing returns to scale, marginal cost would be less than average cost, \( 1^\) .

(ii) Since the economic costs are calculated in terms of the shadow prices of capital, labor and foreign exchanges, the financial costs may be greater than economic costs.

\[footnote{1}\] Parenthetically, we mention here that French economists Allais, Boiteux, and Demois have suggested that all prices should be proportional to marginal costs irrespective of demand elasticities. The factor of proportionality would be so chosen as to bring about a break-even situation. See Droze, op. cit., pp. 27-44. Foster suggests average cost pricing which would consequently maximize consumer surplus subject to the condition of balanced budget. Foster, op. cit., Chapter IV.
(iii) For political or other reasons, the government may decide to operate certain railway services which, on economic grounds, should be closed.

If deficit arises for the third reason, the government will pay for the deficit, one way or another. But if a deficit is expected for either of the first two reasons, should the government subsidize the service?

5. While the resources at the disposal of the government are limited, claims on them are practically unlimited. When the government raises revenue by taxation, it displaces private investment to some extent. Whether government funds thus collected should be spent on meeting deficits of a certain railway sector raises complex questions. As long as the government budget is limited and there is an opportunity cost of public funds, the issue of subsidizing a rail deficit cannot, in principle, be resolved in isolation by the project evaluator at the sector level. What the evaluator can and should do is use sensitivity analysis to prepare alternative lists of projects corresponding to reasonable, alternative sets of shadow prices of capital, labor, and foreign exchange. Then the national planner, keeping in view the overall resource availability and requirements, would determine which list of projects is consistent with the national plan. Any projects on the list requiring subsidy should be funded.

However, if the objectives of national economic policy and the shadow prices of capital, labor, and foreign exchange are given and the net social benefits of the railway service turn out to be positive, the service should be subsidized, or the project should not be undertaken at all.

1/ The British Government's White Paper on transport policy, for instance, states that, in deciding whether to provide financial assistance to socially desirable but unremunerative railway services, "the Minister will weigh the cost of retaining the service - in terms of the amount which will have to be paid against the social and economic benefits it will bring". See Ministry of Transport and British Railways Board, Railway Policy, Cmd. 3439, London: H.M. Stationery Office, November 1967, para. 8.

2/ See Chapter IV, Section F, "Project Selection and Optimum Timing," in this paper.
VII. SUMMARY AND CONCLUSIONS

1. We begin with a rail service which is apparently unprofitable or uneconomic compared to alternative transport service such as road. Should rail service be replaced by a highway service? If so, when? If it is not to be replaced, should any deficit which arises be subsidized by the government?

2. We consider two alternative objectives, namely, social surplus maximization, and profit maximization or loss minimization of the railways.

3. Closure or continuation of present service are not the only two options available to the railway line. The line may be partially closed, or closed only to a certain kind of traffic. It may be operated on a different scale, on a different schedule, and with a different operational system. In other words, it may be rationalized in a number of ways, several of which should be worked out with an eye to future traffic. These operating variants of the rail service should be matched with alternatives for the substitute road service. The method of comparison is given below.

4. Consider first the case where total future traffic is given. The problem is simply to minimize transport costs and compare the long-run average cost of rail service with that of road service. If the former is larger, the rail service should be replaced at some point, if not, it should carry on. If the rail service should be terminated, the optimum timing can be worked out as follows. For an arbitrary date of closure, $\varnothing$, optimize maintenance and operating policy. Now calculate the present value of the costs up to $\varnothing$, and those of highway service thereafter. Obviously the total costs, thus computed, is a function of $\varnothing$. The optimum $\varnothing$ is that for which total cost is minimized with reference to $\varnothing$. This is the general principle of dynamic cost minimization. The computation of the optimum $\varnothing$
is much easier under certain simplifying assumptions; three simple formulas for \( \delta \) have been derived (See Annex III).

5. Next, consider a more general but more complex case where the traffic volume is not given but the demand curve for transport (rail and road) service is. For the moment, assume that the rail and road services are perfect substitutes in the sense that the demand curve is independent of whether and when rail service is replaced by road service. (We also examine the case when rail and road are not perfect substitutes). Also assume that the objective is to maximise the social surplus, defined as the sum of consumers' surpluses and producers' surpluses.

6. Now, take the demand curve and the long-run marginal cost curve of railway. The social surplus is maximised when the price is equated to the marginal cost. The long-run social surplus generated by the rail service per period has to be compared with that generated by the alternative road service. If the former is greater than the latter, the rail service should continue. If the former is less, the rail service should be terminated sooner or later in favor of its substitute. If so, when? Again, take an arbitrary time period \( \delta \) when the rail service is assumed to be replaced. Given the demand curve and the short-run marginal cost curves for every period up to \( \delta \), the social surplus generated by the rail service can be computed. The short-run marginal cost curves for rail would shift up over time because, in the absence of major investment, the operating and maintenance costs would increase as the existing plant and equipment ages. If the social surplus in period \( \delta \) generated by the rail service is more than the social surplus derived from the demand curve and the long-run marginal cost curve of the road service,
take a higher value of $\phi$ and repeat the above process. The optimum $\phi$ is reached when the social surplus of the rail service in period $\phi$ is approximately equal to that of the road service.

7. Nowadays, cost-benefit analysis is a familiar exercise in transport. This analysis is equivalent to the social surplus analysis as given above under the assumption that there is only one objective, namely, maximisation of the net benefits -- social benefits minus social costs -- without regard to whom they accrue. However, cost-benefit analysis is a more powerful tool in the sense that a multiplicity of objectives, such as more equitable income distribution, and a multiplicity of constraints, such as savings and balance-of-payments, can be directly taken into account.

8. In any event, since resources are limited, there is a need to assign priorities to the investments in the railway sections of a system which emerge from the above-mentioned social surplus analysis. With this end in view, the project evaluator should conduct a sensitivity analysis by taking reasonable, alternative sets of shadow prices of capital, labor and foreign exchange. The highest priority is assigned to that project which both has the largest social surplus and also survives all or most of the sensitivity analysis.

9. Next we consider the objective of improving the financial position of the railway system alone. If the volume of traffic and the expected future revenue are known, the optimum year of discontinuing the service has to be worked out which maximizes the railway profit or minimizes the loss. This
method is relatively simple. If the traffic depends upon the price charged and the demand curve is known, the profit maximizing volume of traffic in the long run is determined by equating the marginal revenue and the long-run marginal cost. The profit for each of the alternative variants of the rail service can thus be computed. And that variant is chosen which yields the largest amount of profit. If no variant is profitable, the service should be discontinued. The algorithm for finding the optimal time is analogous to that used in the economic analysis.

10. Finally, we ask whether the government should subsidize railway deficits. In case the government decides for political or other reasons to retain a railway service against economic considerations, it is in effect taking the responsibility of subsidizing it in some form or other. However, deficit may arise also because long-run marginal cost is less than long-run average cost. As long as the government budget is limited in quantity and there is an opportunity cost of raising government funds, the question of subsidizing the deficit of a railway service cannot be resolved in isolation. It requires a broader analytical framework. However, given the objectives of national economic policy and their relative weights, and the shadow prices of capital, labor and foreign exchange, if the railway project has a positive net present value in terms of the cost-benefit analysis the government should subsidize the deficit if any; otherwise the project should not be undertaken at all and the subsidy issue does not arise.
1. The principle of choice between dieselization and electrification of a railway service is to accept the alternative with the minimum present value of total costs, from a class of alternatives equivalent in the sense that the magnitudes and time patterns of their benefits are similar. Let:

- \( K, K' \) = capital expenditures for dieselization and electrification, respectively;
- \( A, A' \) = annual operating and maintenance expenses for the two projects, respectively;
- \( T, T' \) = probable life-spans of plants and equipment for the two projects, respectively; and
- \( i \) = the constant rate of discount.

The present value of the costs of dieselization appears to be:

\[
C = K + A \int_{0}^{T} e^{-it} \, dt
\]

Likewise, the present value of the costs of electrification is:

\[
C' = K' + A' \int_{0}^{T'} e^{-it} \, dt
\]

2. It is necessary to correct for the unequal life-spans of plant and equipment. We simply suppose each is indefinitely renewed at the end of its economic life. Thus, for dieselization we get:

---

\[ C = K \int_0^T e^{-it} dt + Ke^{-iT} \int_T^{2T} e^{-it} dt + Ke^{-2T} \int_{2T}^{3T} e^{-it} dt + \ldots \]

(original plant and equipment) (first replacement) (second replacement)

or,

\[ C = K \int_0^\infty e^{-it} dt + A \int_0^\infty e^{-it} dt \]

where \( a \) represents the rate of replacement annuity at compound interest \( i \) for \( T \) years. This expression can be rewritten as follows:

\[(3) \quad C = K + \frac{(aK + A)}{i} \]

Similarly, the total discounted costs of the electrification project is given by:

\[(4) \quad C' = K' + \frac{(a'K' + A')}{i} \]

where \( a' \) is the rate of replacement annuity at compound interest \( i \) for \( T' \) years. The choice depends on whether \( C \) or \( C' \) is greater.

3. If we wish to calculate the cost per period from (3) and (4), the procedure is as follows: Let \( Y \) denote the cost of dieselization

\[ C = Y \int_0^\infty e^{-it} dt = \frac{Y}{i} \]

or, in view of (3), we get:

\[(5) \quad Y = (1 + a) K + A \]

Similarly, the cost of electrification per period, denoted by \( Y' \), is given by:

\[(6) \quad Y' = (1 + a') K' + A' \]
4. If, however, instead of indefinite renewal, a finite time horizon is envisaged, further corrections are necessary in the total cost calculation to take into account the salvage value of the plant and equipment at the end of the specified time period. Also, in the above exercise, we assume that the economic life of plant and equipment and its maintenance expenses are fixed. There may exist scope for choice: the life of the equipment may depend upon the maintenance and operating policy.

5. This example of choice between two kinds of equivalent projects is intended to establish one principle: If the benefits are the same but cost streams are different, choose the alternative which has the lowest discounted costs.

1/ For a discussion of a range of other issues relative to replacement, see D. W. Jorgenson, J. J. McCall, and R. Radner, Optimal Replacement Policy, Chicago: Rand McNally, 1967. See also Masse, op. cit., Ch. II.
ANNEX II

COMPUTATION OF COSTS

A. Long-Run Average Cost.

1. For any given volume of traffic, the time stream of costs is uneven; it is particularly high when a major capital expenditure takes place. Let C denote the expenditure at time period t, under the assumption that a certain level and quality of transport service per unit of time must be provided indefinitely. The present value of the cost stream at the discount rate i, denoted by C, is as follows:

\[ C = \int_0^\infty C_t e^{-it} dt \]

From the total C, we need to derive the amount of costs per period, denoted by Y. By definition,

\[ C = Y \int_0^\infty e^{-it} dt = Y/i \]

or, in view of (1),

\[ (2) \quad Y = i \int_0^\infty C_t e^{-it} dt \]

If the volume of traffic per period is Q, the average cost of traffic per period is simply Y/Q.

2. It is easier to understand (2) if it is assumed that in order to sustain indefinitely the given volume of traffic we have to incur now a capital expenditure of K, with an economic life of T years, and operating and maintenance expenses of A per period. Then it becomes clear that Y of (2) here is formally identical with Y of (5) in Annex I.

B. Short-Run Marginal Cost.

3. The estimation of short-run marginal cost of rail service has given rise to an amusing story known as the "Passenger to Calais Paradox" of Gabriel Dessus.1/

Suppose that a passenger gets to the Station Paris-Nord just in time to board the Calais-bound train. If unoccupied seats are available, the cost of transporting this passenger is, in practice, nil. But if it is necessary to put an additional coach, run an extra train, lay a new track, or widen the Chantilly bridge, the cost of moving this last passenger mounts with astonishing rapidity. How, in a situation like this, do we estimate marginal cost of transport? The indivisibility of inputs of railway service and the unforeseen nature of passenger demand are at the root of this paradox.

Let us assume that the number of Calais passengers who travel each day from Paris-Nord remains unchanged, and let us focus our attention on the discontinuity of the coach factor. Suppose that a coach has a seating capacity of eighty passengers, and that as long as a coach is not full, the transport cost of an extra passenger is nil. When the coach is full, the cost of an additional passenger is indeterminate, since the possibility of accommodating an eighty-first traveller has been ruled out ex hypothesi.

For a given number of coaches, a passenger's marginal cost relative to coaches will be nil or indeterminate, depending on whether the coaches are full or not. However, the number of coaches to be utilized is itself a variable to be determined by marginal considerations: it must be such that the proceeds from fares remunerate the coaches in service at their marginal cost of production. Consequently, the problem of determining the number of coaches to be run has to be considered not in terms of one additional passenger per day, but in terms of a group of eighty additional passengers per day. It is in this manner that the indeterminacy of marginal transport cost relative to coaches is circumvented: its magnitude is equal to the eightieth part of the marginal coach's daily running cost.

However, it is possible that the demand for passenger service is random in nature and does not necessarily come in multiples of eighty, so the last coach will not be full and the cost of transporting a further passenger will be nil. Under such circumstances, the railway's planning has to be based on a certain number of expected passengers, with provision of "spare capacity" to accommodate unforeseen passengers.

The short run, however, is defined as that span of time within which no major investment in plant and equipment is feasible, technically or otherwise. In the shortrun, therefore, the capacity of plant is by definition more or less fixed. Whether minor replacement of equipment, which is allowed, is worthwhile depends on the length of the period.

8. Assume that the duration of the short run is precisely given, and that production is scheduled to increase by \(dQ\) from its present level. If \(dC_t\) is the additional amount of expenses (for maintenance, operation, and replacement if called for) in period \(t\), then the present value of the additional costs, denoted by \(dC\), is given by:

\[
dC = \int_0^\infty (dC_t) e^{-\delta t} dt
\]

where \(\infty\) is the period marking the end of the short run. From \(dC\) we can calculate the marginal cost per period, denoted by \(y\). By definition,

\[
dC = y \int_0^\infty e^{-\delta t} dt
\]

or, in view of (3) and (4),

\[
y = \int_0^\infty (dC_t) e^{-\delta t} dt (1 - e^{-\delta \infty})
\]

The \(y\) of (5) is the short-run marginal cost per period; it represents the avoidable additional costs reckoned on a per period basis.

C. Long-Run Marginal Cost.

9. The definition of the long-run marginal cost is similar to that of short-run marginal cost, except that \(dC\) now includes capital expenditures as well, and \(\infty\) is replaced by \(\infty\), in equations (3), (4), and (5).

10. While the empirical estimation of railway costs has a long history, no method has yet been fully developed and tested for estimating relevant transport costs in the context of railway closure decision. There are, broadly speaking, three methods of estimating long-run marginal cost, accounting, statistical

---


2/ These methods are also applicable for estimating short-run marginal cost; see Lesourne, op. cit., pp. 292-313.
analysis, and direct technical analysis. The first method is used for cost estimation in the Spanish railways study and the East Africa Transport Study, while the second method has been used in connection with the U.S. railway system.

11. The East Africa Transport Study computes essentially the average cost which differs from marginal cost in nature and magnitude. We have already seen in Chapter II that the basic framework of cost estimation this study uses is somewhat irrelevant to the problem we have posed. The method used in the Spanish Railways study is much more reasonable, since it considers only avoidable costs; nonetheless, it does not calculate the long-term marginal cost of transport as we have defined above. This calculation would be possible using the cost data given in the report, since the sets of cost data are related to alternative levels of traffic (high, medium and low forecasts).

12. In the U.S. railway study, the method of statistical analysis for estimating marginal cost of railway transport is quite sophisticated, but again rather irrelevant to our purpose. The results, summarized in Table 11-1, have been derived by the method of least squares multiple regression, with cross-section data (1947-50 averages and 1952-55 averages) from all Class I United States railway systems with over 3,000 miles of total track, except the New York Central and the Pennsylvania Railroad. For example, 1947-50 maintenance of way and structure expenses have been computed by:

\[ E_w = 3.540.801 + 7.11 S_t + 0.000281 Q_f + 0.000772 Q_p \]

\[ R^2 = 0.9321 \]

(372) (0.000069) (0.000415)

where \( E \) represents an expense account, \( S \) a size measure, and \( Q \) a traffic variable, and the subscripts denote the following sub-categories:

- \( w \) = maintenance-of-way expenses;
- \( t \) = track mileage;
- \( f \) = gross ton miles of freight traffic;
- \( p \) = gross ton miles of passenger traffic.

1/ For an approximate example of the direct technical analysis for railway marginal cost, see Sjoberg, *op. cit.*, pp. 42-55.

2/ Nechel, *op. cit.*

3/ The Economist Intelligence Unit *et. al.*, *op. cit.*


Tie figures in parentheses below the parameters are standard errors of estimates; if a probability model is appropriate, the parameters are considered significant if the parameters exceed their standard errors by a factor of 1.6 to 2.0.

13. A major statistical objection against this kind of estimation of long-run marginal cost is that the presence of multi-collinearity between freight and passenger traffic tends to reduce the economic significance of individual parameters. Besides, in view of possible multicollinearity, the high correlation may be spurious. For purposes of predicting total $E_s$, these phenomena are not harmful, as long as the spurious relationship is uniform. But they invalidate the method for measuring marginal costs of freight traffic and passenger traffic separately. In any case, for our purposes, the parameters do not signify avoidable costs, in the sense that (6) may not hold at all if one of the services is closed altogether. Furthermore, one has to be prepared for getting absurd results. For example, the equation for 1947-50 joint equipment repair costs turned out to be:

\[
E_{i} = \$265,600 + \$0.000113 Q_f - \$0.000109 Q_p \\
(0.0000236) (0.0000951)
\]

where the subscript $i$ on the expense variable indicates that joint equipment repair costs are being analyzed. The parameter of the passenger variable has the wrong sign. With that variable eliminated, the cost function was modified to the following:

\[
E_{i} = \$351,100 + \$0.0000889 Q_f \\
(0.0000108)
\]

where $Q_f$ is the freight variable. Obviously, the elimination of the passenger service makes very little difference to the analysis; neither the multiple correlation nor the freight variable coefficient are modified substantially.

14. This kind of analysis of marginal cost, therefore, is not useful in the context of railway line appraisal, because the individual coefficients do not have the kind of economic significance which is relevant here. The authors of the U. S. study, after all, had a different purpose in mind. How, then, should the long run marginal cost be computed? The planning authority must

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10/ Ibid, pp. 299-300.
Table II-1: Summary Estimates of Long-Run Marginal Costs in the U.S. Railways

(Per gross-ton-mile; U.S. cents at current prices)

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Train, station, yard, traffic and general expenses</td>
<td>0.872</td>
<td>1.2330</td>
<td>1.700</td>
<td>1.50</td>
</tr>
<tr>
<td>(2)</td>
<td>User cost portion of maintenance cost</td>
<td>0.690</td>
<td>1.4100</td>
<td>0.910</td>
<td>1.53</td>
</tr>
<tr>
<td>(3)</td>
<td>Variable portion of depreciation expenses</td>
<td>0.212</td>
<td>0.9760</td>
<td>0.216</td>
<td>1.00</td>
</tr>
<tr>
<td>(4)</td>
<td>Marginal operating costs: (1) + (2) + (3)</td>
<td>1.7741</td>
<td>6.6190</td>
<td>2.326</td>
<td>7.03</td>
</tr>
<tr>
<td>(5)</td>
<td>Variable portion of capital costs</td>
<td>0.4011</td>
<td>1.4079</td>
<td>0.445</td>
<td>1.48</td>
</tr>
<tr>
<td>(6)</td>
<td>Total long run marginal operating and capital costs: (4) + (5)</td>
<td>2.1752</td>
<td>8.0269</td>
<td>3.271</td>
<td>8.51</td>
</tr>
</tbody>
</table>

* At 6% interest rate.

Note: For pricing decisions, these cost figures need several modifications. First, some sort of conversion between gross ton miles and revenue ton miles is needed. Secondly, these figures should be price corrected to bring them up to current levels. Thirdly, these estimates are "typical" figures applying to freight and passenger movements of an average or ordinary kind. Adjustments should be made for quality differences in the service rendered. See Meyer et al., pp. 51-53.

Source: Meyer et al., op. cit., p. 62.
make estimates of the time stream of avoidable capital expenditures and of maintenance and operating expenses for alternative volumes of traffic, and then calculate the marginal cost from an equation such as (5), replacing $\$ by \infty$. The validity of such an estimate of the long-run marginal cost, of course, depends entirely on the validity of the original estimates of expenditures, expenses, and the discount rate.
ANNEX III.

DYNAMIC COST MINIMIZATION AND THE OPTIMUM TIMING OF SUBSTITUTION

1. If the volume of traffic is given and rail and road services are equivalent, the relevant indicator is the long-run average cost. If it is lower for rail than for road, the question of replacing rail service does not arise. If long-run average cost for road is lower, rail service should be replaced. Then the question of timing arises.

2. For existing railway facilities, slated to be replaced, let \( m \) be the annual operating and maintenance expenses of the railway service in the base year. Major railway investment now or in future is ruled out. Under these circumstances, suppose that \( m \) rises at the rate of \( g \) per year. Further, let \( S \) be the amount of costs which would be saved in the railway system as a whole, plus the current salvage value of the equipment of the railway service under review, and suppose \( S \) falls at the rate of \( f \) per period of time as long as the closure decision is postponed.

3. Also, let \( K_H \) be the required capital expenditure for the highway, to be incurred in the period when the railway service is replaced and repeated every \( J \) years thereafter. Thus, \( J \) is the lifespan of highway plant. Let \( m_H \) be the annual operating and maintenance expenses for the highway service.

Then, the present value of the cost of operating the rail service up to, and not beyond, year \( \phi \), and replacing it by road service in that year is given by:

\[
C = m_R \int_0^{\phi} e^{(g-1)t} dt - S e^{-(f+1)\phi} + m_H \int_{\phi}^{\infty} e^{-it} dt
\]

or

\[
C = m_R \left[ \frac{e^{(g-1)\phi}}{g-1} - 1 \right] - S e^{-(f+1)\phi} + m_H e^{-i\phi}
\]

where \( a \) is the constant replacement annuity at compound interest \( i \), the lifespan of \( K_H \) being \( J \) years. Obviously, \( C \) in (1) is a function of \( \phi \).

\[\text{1/ Thus we are assuming there is no Boiteux Effect, described in Chapter III.}\]
5. Now, choose $\phi$ such that $C$ is minimized. This requires the following:

\[(2) \quad m e = m + K (1 + a) - (f + i) S e \]

The solution for $\phi$ is given in the figure below.

6. If $S$ is negligible, (2) reduces to:

\[(3) \quad \phi = \frac{1}{e} \log_{e} \frac{m_{H} + K}{m_{R}} (1 + a) \]

If $S$ is ignored when in fact it is not a negligible magnitude, the optimal time $\phi^{o}$ would be overestimated by (3). Statement (3) also implies that if $m_{R}$ is greater than $m_{H} + K (1 + a)$, the optimal $\phi$ is negative, i.e., the replacement of the railway service by road service is overdue.2/

---

2/ Such instances can be found in certain railway sections of Argentina, under certain assumptions. See Annex IV.
7. If \( f \) is zero, i.e., if the salvage value of the railway equipment does not fall with the passage of time, (2) becomes:

\[
\varphi = \frac{1}{2} \log_e \frac{m + k (1 + a) - i S_R}{m_R}
\]

8. In summary, if it turns out that the long-run average cost of railway service is more than that of the alternative highway service for a given volume of projected traffic, the former service should give way to the latter. Under certain simplifying assumptions the optimal timing of the switch-over can be calculated with (2), (3), or (4).
ANNEX IV

THE OPTIMUM TIMING OF SUBSTITUTION: EXAMPLES FROM ARGENTINA AND SPAIN

1. This annex relates to the economic analysis when traffic volume is given (Chapter III) and to Annex III, "Dynamic Cost Minimization and the Optimal Timing of Substitution." In principle, costs should be calculated at shadow prices of the inputs; but in the following cases market prices have been used.

FIFTEEN EXAMPLES FROM THE ARGENTINA TRANSPORTATION STUDY

2. Table IV-1 gives relevant data for fifteen selected railway sectors of the Argentina railway system. Traffic on these lines has been projected on the basis of the anticipated pattern of economic growth and industrial location. The problem is to minimize transport costs, given the traffic forecast. The substitute highway is assumed to be equivalent to the railway in the sense that the volume of traffic is not expected to be sensitive to the mode of transport. The required capital expenditure and operating and maintenance expenses of the highway service are also given in Table IV-1.

3. The fifteen railway sections of Table IV-1 fall into five categories, defined by the relations between annual operating and maintenance expenses of rail and highway ($m_R$ and $m_H$, respectively) and between the capital expenditures ($K_R + K_H^H$).

Sections (1) - (5): $m_H < m_R$, $K > K_R = 0$

(6) - (10): $m_H < m_R$, $K_R > K_H > 0$

(11) - (12): $m_H < m_R$, $0 < K_R < K_H$

(13): $m_H > m_R$, $K_R > K_H > 0$

(14) - (15): $m_H > m_R$, $0 < K_R < K_H$

The first five sections are examples of how economies of operating and maintenance costs can be achieved by switching to road service requiring no additional capital expenditure. In such cases, rail service should be replaced by highway service immediately.

4. For the next five sections, both operating and maintenance costs and capital expenditures are higher for rail than for road. However, the economic life of railway investment is assumed to be 30 years while that of highway investment is 25 years. Since \( K \) is the capital expenditure, the annual operating and maintenance expenses, and \( a \), the constant replacement annuity at compound interest \( i \) for \( K \) with a life span of 30 years, the long-run total cost per period is \( (m_R + (i + a_R) K_R) \), which is the total of operating and maintenance costs, and interest and capital charges per unit of time. If this amount is divided by the volume of traffic, we get long-run average cost of railway service. Correspondingly, the long-run total cost per period for highway service is \( (m + (i + a) K) \).

5. A comparison of Columns (4) and (5) for sections (6) - (10) reveals that in all cases the long-run average cost of railway is higher than the corresponding highway cost. These railway sectors, therefore, should be replaced by highway. What should the timing be? While the Argentina report suggests a time-table for the replacement of railway services by highway services, it seems to be arbitrary. If it is based on the highway cost-benefit analysis, the methodology is incorrect, since it does not take into account relative costs of the two transport modes.

6. Applying our method, let us suppose that the railway operating and maintenance costs in 1960 as given in column (8) rises thereafter at a certain rate. Ignoring the possible salvage value of the railway plant, we can use equation (4) for sections (6) - (10) leads to the conclusion that the optimum \( \theta \) is negative; in other words, the replacement of the railway service by a highway service is overdue. As for sections (11) and (12) columns (4) and (5) show that the long-run average rail cost is higher than the long-run average road cost. By the same reasoning as above, the railway line should have been discontinued long ago in favor of equivalent road service. The last three

---

1/ Ministry of Public Works and Services, op. cit., Appendix II, p. 68.
2/ See op. cit., Appendix III, p. 95.
3/ Reported in Appendix II, pp. 67-70.
railway lines should not be discontinued at all because the long-run average cost of rail is lower than that of road.

A Case Study from the Spanish Railway System

7. The fifteen railway sectors of the Argentine railways were considered for complete closure. A distinguishing feature of the analysis of Spanish railway sections is that the sections are being considered for partial closure, namely, discontinuance of passenger service keeping freight service intact.

8. The Villacanas-Quintanar rail line in Spain is a stub end branch line, 25 km. in length, extending east from a junction point 121 km. south of Madrid on the main line to Andalucia (see figure). Villacanas is the junction station. Both the branch line and the main line are paralleled by paved highways over their entire length, and Quintanar, the terminus of the line, lies on National Highway 301. Quintanar and two interim stations on the line receive regularly scheduled bus service direct to Madrid, whereas rail passengers going to Madrid must change trains at Villacanas.

---


2/ Rachel, op. cit., p. 3.
9. Two alternative plans of highway passenger service to replace the Villacanas-Quintanar rail service are being considered. Alternative A is a straight substitution of bus for the present rail passenger service between Villacanas and Quintanar; all passengers going to or coming from points beyond Villacanas would transfer to or from mainline trains as at present. Alternative B, on the other hand, provides a separate and continuous through bus service from Quintanar to Madrid and other points north of Villacanas in addition to the services within the study line. The costs of the Alternative B through bus service to Madrid have been allocated between the within-the-line portion of the trip and the Villacanas-Madrid portion. This allows comparisons of the total costs of the services moving within the line, and identification of the incremental costs of introducing through service from Villacanas to Madrid.

10. The estimated annual operating costs and required capital investment for the Quintanar to Villacanas railway passenger service from 1965 through 1990 are given in Table IV-2. The table also gives the corresponding highway costs for Alternative A alone. This is a very clear case: rail costs dominate road costs in every year through 1990, with two insignificant exceptions (1975 and 1985). There is no possibility of any gain by postponing the switchover from rail to road; the optimal time is already overdue.

11. In order to decide between Alternatives A and B of the road service, their respective cost streams would have to be compared, which we have not done. In any event, the road services between Quintanar and Villacanas as proposed in Alternatives A and B are qualitatively different and may have different demand curves. This is a separate matter which has been treated in Chapter IV above.
### TABLE IV-2: Spain:

**Estimated Costs for Railway and Highway Passenger Services**

**Between Quintanar and Villacanas**

(million pesetas)

<table>
<thead>
<tr>
<th>Year</th>
<th>Rail Operating Costs</th>
<th>Rail Capital Investment</th>
<th>Rail Total Outlay</th>
<th>Highway Operating Costs</th>
<th>Highway Capital Investment</th>
<th>Highway Total Outlay</th>
</tr>
</thead>
<tbody>
<tr>
<td>1965</td>
<td>4.598</td>
<td>4.598</td>
<td>1.570</td>
<td>2.710</td>
<td>4.280</td>
<td></td>
</tr>
<tr>
<td>1968</td>
<td>4.677</td>
<td>4.677</td>
<td>1.932</td>
<td>1.355</td>
<td>3.287</td>
<td></td>
</tr>
<tr>
<td>1969</td>
<td>4.745</td>
<td>4.745</td>
<td>2.163</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1970</td>
<td>4.745</td>
<td>4.745</td>
<td>2.248</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1971</td>
<td>4.821</td>
<td>4.821</td>
<td>2.248</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1972</td>
<td>4.821</td>
<td>7.650</td>
<td>12.471</td>
<td>2.479</td>
<td>2.479</td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>4.821</td>
<td>4.821</td>
<td>2.479</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>5.180</td>
<td>5.180</td>
<td>2.609</td>
<td>2.710</td>
<td>5.319</td>
<td></td>
</tr>
<tr>
<td>1976</td>
<td>5.180</td>
<td>5.180</td>
<td>2.810</td>
<td>1.355</td>
<td>4.195</td>
<td></td>
</tr>
<tr>
<td>1977</td>
<td>5.180</td>
<td>5.180</td>
<td>2.810</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1978</td>
<td>5.180</td>
<td>5.180</td>
<td>2.810</td>
<td>1.355</td>
<td>4.195</td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td>5.256</td>
<td>5.256</td>
<td>2.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>5.256</td>
<td>5.256</td>
<td>2.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1981</td>
<td>5.256</td>
<td>5.256</td>
<td>2.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1982</td>
<td>5.256</td>
<td>5.256</td>
<td>2.927</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1983</td>
<td>5.256</td>
<td>5.256</td>
<td>3.013</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>5.256</td>
<td>5.256</td>
<td>3.013</td>
<td>1.355</td>
<td>4.360</td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>5.256</td>
<td>5.256</td>
<td>3.013</td>
<td>2.710</td>
<td>5.723</td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>5.256</td>
<td>5.256</td>
<td>3.013</td>
<td>1.355</td>
<td>4.368</td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>5.333</td>
<td>5.333</td>
<td>3.013</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>5.333</td>
<td>5.333</td>
<td>3.288</td>
<td>2.710</td>
<td>5.993</td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>5.333</td>
<td>5.333</td>
<td>3.288</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>5.333</td>
<td>5.333</td>
<td>3.288</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>127.261</strong></td>
<td><strong>15.300</strong></td>
<td><strong>142.561</strong></td>
<td><strong>67.528</strong></td>
<td><strong>17.625</strong></td>
<td><strong>85.143</strong></td>
</tr>
</tbody>
</table>

Note: These figures relate to the "high" traffic forecast for Alternative A, which is a straight substitution of bus for the present rail service, as distinct from Alternative B, in which provision is made for a through service from Quintanar to Madrid and other points north of Villacanas, as well as the required service within the study line.

Source: Rochel, op. cit.
ANNEX V

CONSUMERS’ SURPLUS MEASUREMENT FOR CLOSE SUBSTITUTES

1. Let there be n consumers \((j = 1, 2, \ldots, n)\) who buy two substitute goods, say, road \((x)\) and rail \((y)\). Consider consumer \(j\). His utility function is: \(u_j = u_j(x_j, y_j, z_j)\) where \(z_j\) is the generalized commodity "money" which stands for all other commodities. (Hereafter, for convenience we shall omit subscript \(j\).) The prices are respectively \(p\), \(q\), and 1. Consumer \(j\) maximizes his utility function subject to the budget constraint. By the Kuhn-Tucker theorem, the necessary conditions are:

\[
\begin{align*}
(1) \quad & \left( \frac{\partial u}{\partial x} \right)/p \leq \lambda, \quad \text{for } x > 0 \\
(2) \quad & \left( \frac{\partial u}{\partial y} \right)/q \leq \lambda, \quad \text{for } y > 0 \\
(3) \quad & \left( \frac{\partial u}{\partial z} \right) = \lambda. \quad z > 0 \text{ by assumption}
\end{align*}
\]

where \(\lambda\) is the Lagrange multiplier.

2. Elementary theory of consumer's behavior assumes that the consumer buys a positive amount of each and every commodity, mainly because the usual method of calculus cannot cope with corner solutions. However, following the contributions of Kuhn, Tucker, and Arrow in 1950, the theory has now been generalized.

3. For every consumer, conditions (1)-(3) hold. Accordingly, we can construct the interdependent market demand curves for road and rail by lateral summation of individual demand curves. Let road and rail services be supplied at their respective marginal costs. While this is not essential, it only makes the road-rail model determinate. Referring to (1)-(3), three cases can be distinguished.

4. Case A. All consumers can be divided into two exclusive groups. The consumers in one group consume road only, but no rail \((x > 0; \ y = 0)\); while in the other they consume rail only, but no road \((x = 0; \ y > 0)\). The market demand curves are given in Fig. V-1, where \(p^*\) and \(q^*\) are the

---

equilibrium prices. In other words, \( DD' \) is drawn on the assumption that rail price is \( q \); similarly, \( dd' \) is drawn assuming that road price is \( p \).

In this sense, \( DD' \) and \( dd' \) will be referred to as demand curve "in equilibrium". Consumers' surplus from road is \( Dp^e E \), and that from rail is \( dq^e E \). The total consumers' surplus from road and rail together for consumers as a group is given by \( Dp^e E + dq^e E \).

5. **Case B.** In this case both \( x \) and \( y \) are positive for each and every consumer. That is, everyone consumes both road and rail (see Fig. V-2). The consumers' surplus from road only is \( Dp^e E \) and that from rail only is \( dq^e E \). To find total consumers' surplus, suppose that road is withdrawn and the consumers are paid in money to the extent of \( Dp^e E \) as compensation for their lost consumers' surplus. Since road is withdrawn, the marginal utility of rail will rise; accordingly, \( dd' \) shifts up to \( aa' \). Now the consumers' surplus from rail is \( aa'eq^e \), which is larger than \( dq^e E \). So the total consumers' surplus from road and rail together is \( Dp^e E + aa'eq^e \).

6. Alternatively, we could suppose that the rail service is withdrawn and the consumers are compensated in money to the extent of \( dq^e E \). In the absence of rail, the marginal utility of road rises; \( DE \) shifts up to \( AA' \). So the total consumer surplus is given by \( dq^e E + AA'E \). It can be shown that \( Dp^e E + aa'eq^e = dq^e E + AA'E \).

7. By contrast, note that in Case A, if road service is withdrawn the marginal utility of rail does not shift up (Fig. V-1a), because the rail consumers do not buy road in the first place. Similarly, the marginal utility of road, \( DE \), is unaffected if rail is withdrawn, because road consumers do not buy rail anyway. Under the conditions of case A, therefore, \( Dp^e E + dq^e E \) is the correct measure of total consumers' surplus of road and rail together.

8. **Case C.** This case lies in between the two above extremes. Here some consumers buy road only, but no rail; some buy rail only, but no road; the remaining consumers buy both road and rail (see Fig. V-3). Let \( DB \) be the demand curve for road of those consumers who buy road only, and let \( DC \) be that for rail of those who buy road and rail. Demand curve \( DD' \) is derived by lateral summation of \( DB \) and \( DC \). Similarly, \( db \) is the demand curve of those consumers who buy rail only, while \( dc \) is that of those who buy both rail and road. As in Case A, the total consumers' surplus of pure road-users and rail-users is given by \( Dp^e F + dq^e F \). Next, employing the method of Case B, we can find the total consumers' surplus from road and rail for those consumers who buy both; it is given by \( Dp^e G + hig^g \), which is equivalent to \( dq^g G + HIG^g \). So in Case C, the total consumers' surplus from road and rail for consumers as a group is given by \( Dp^e E + dq^e E + hig^g \), or, \( Dp^e E + dq^e E + HIG^g \).

9. In summary, the measurement of the consumers' surplus of two substitute goods with many consumers is complicated. In the simplest case, when every consumer buys either rail or road and none buys both, total consumers' surplus
Figure V-1: Demand for Road and Rail, Case A

Figure V-2: Demand for Road and Rail, Case B

Figure V-3: Demand for Road and Rail, Case C
from both road and rail is simply the addition of the Marshall triangles under the demand curves "in equilibrium". However, whether this case, or Case B or C as presented, is more realistic is an empirical question and cannot be settled a priori. In any event, if some consumers buy road as well as rail, the sum of the above-mentioned Marshall triangles underestimates total consumers' surplus from road and rail together. To simplify the exposition, the discussion in Chapter IV assumes that the extent of such underestimation is negligible. This assumption need not be always correct; whenever necessary, total consumers' surplus should be measured as shown in Cases B and C.
ANNEX VI
MARGINAL COST PRICING AND THE THEORY OF SECOND BEST

1. In a frictionless competitive economy, the Pareto optimum requires that the price of every product be equated to its short-run marginal cost. The same rule has been recommended for the transport sector. But real economies are not frictionless competitive systems; there are distortions, such as external economies, monopoly elements, lack of certainty, etc. If in most sectors of the economy, the price-equals-marginal-cost rule is not observed, should it be followed by rail or road transport? This is the problem of second best.

2. The factors which may justify pricing transport services above or below marginal cost in the interest of efficient resource allocation are as follows. The reasons for pricing in excess over marginal cost are:
   a) important close substitutes sell at prices significantly above marginal cost, or generate large external economies;
   b) products for which transport is a major input sell at prices significantly below marginal cost or involve large external diseconomies;
   c) important close complements sell at prices significantly below marginal cost or generate large external diseconomies; and
   d) major inputs of the transport industry are bought at prices significantly below marginal cost or involve large external diseconomies.

The reasons for pricing transport service below marginal cost can be inferred from the above.

1/ See Turvey (ed.), Public Enterprise.
2/ See Maurice Allais and others, Options in Transport Tariff Policy, Brussels: EEC, 1965. Also cf.: "Efficiency in the allocation of resources and economic activity requires that the road authority levy charges at marginal cost", quoted from Walters, op. cit., p. 97.
ANNEX VI
Page 2

3. While it may be desirable to adjust transport pricing to allow for these non-optimalities, it would, in many cases, be better to tackle them directly. It is better to aim for the best rather than adjusting to the second best. This approach has the subsidiary merit of simplicity, since it requires less data from those responsible for transport pricing.\(^1\)

Of course, where the above-mentioned distortions are not directly alterable, departures from marginal-cost pricing in transport are called for. Since such departures may take various forms, they are not covered by this general discussion.

\(^1\) Ibid. Also cf., "The most noticeable feature of the results, for any one who is concerned with trying to formulate practical pricing policies, is their complexities, and the fact that their informational requirements are unlikely to be met." R. Rees, "Second-Best Rules for Public Enterprise Pricing," Economics, August 1968, p. 270.
ANNEX VII

COST-BENEFIT ANALYSIS: AN EXAMPLE

1. In this annex, we present an example of cost-benefit analysis as applied to the withdrawal of railway services.¹

2. The Sheffield-Barnsley route of the British Railways is 15 miles long and is provided with local passenger service approximately every hour in each direction. The places on this route are fairly well supplied with alternative transport facilities; the railways play a relatively small part in providing local transport in the area. It is estimated that, of commuters from outside Sheffield arriving in the city before 9 A.M., approximately 3 percent travel by train, 45 percent by bus, and 52 percent by car. Nonetheless, the Sheffield-Barnsley railway service carries an appreciable volume of passengers, including commuters into Sheffield, and it appears to be the busiest local service in the area.²

3. The main social costs arising from the withdrawal of the passenger service have been divided into three categories:

   (a) those arising from journeys diverted to other forms of transport, namely, bus and private car;
   (b) those arising from journeys no longer made; and
   (c) those affecting the rest of the community.

Three alternative assumptions are made about the distribution of the displaced rail traffic:

<table>
<thead>
<tr>
<th>Proportions of Journey (percent)</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
</tr>
<tr>
<td>1. Diverted to bus</td>
<td>33.3</td>
</tr>
<tr>
<td>2. Diverted to private car</td>
<td>33.3</td>
</tr>
<tr>
<td>3. No longer made</td>
<td>33.3</td>
</tr>
<tr>
<td></td>
<td>100.0</td>
</tr>
</tbody>
</table>

4. Whether the traffic diverted to bus would call for additional investment in bus and highway capacity depends on how much of the diverted traffic takes place during peak hours and on existing capacity limits. Accordingly, annual

² Else and Howe, op. cit., p. 180.
capital cost of additional vehicles (and highway expenditure, if necessary), and their operating costs are to be taken into account. In addition, the extra time which passengers have to spend on bus journey must be reckoned with. Beasley suggests accounting this time at 10 percent of an estimated average earning rate for the persons concerned.

5. For traffic diverted to private car, it is assumed that the cars used would already be owned by the rail passengers concerned. Hence, only additional running cost of cars, as well as the extra time of journey required during peak hours, is to be taken into account.

6. Journeys no longer made represent a social loss of consumers' surplus. In the locality under study, bus and train fares are more or less comparable. If a journey is no longer made, the reason must be that bus takes more time; this additional time can be valued in monetary terms. Assuming that the relevant part of the demand curve is approximately linear, it can be shown that the average social loss of consumers' surplus equals half the additional cost (i.e. half the value of additional journey time) of bus travel for each journey.

7. Social costs will also be increased by the cost of the additional congestion during peak hours to vehicles already on the road.

8. These four sources of social costs are quantified in Table VII-1. The benefits are considered to be mainly the resources presently used in operating the service which would be saved if the service is withdrawn. These do not include any savings in track and signalling costs, since it is assumed that the tracks would still be required for the area's considerable freight traffic.

9. Table VII-1 indicates that the railway service should be withdrawn under assumptions I and II, but retained under assumption III. Suppose assumption III is accepted. Does it follow then that the railway service should continue? If so, then should the government subsidize the service in case it involves financial loss? Under assumption III the social cost of withdrawal of the railway service is more than the corresponding social benefit. If the cost and benefit have been properly measured, withdrawal of service would not be an improvement, and the service justifies government subsidy should it incur financial loss.

2/ Else and Howe, op. cit., p. 186. Note that this viewpoint is in sharp contrast with that of the East African Transport Study.
ANNEX VII

TABLE VII-1: Cost-Benefit Analysis of Withdrawal of Sheffield-Barnsley Service

(In thousand pounds sterling per year, based on 1966 volume of traffic)

<table>
<thead>
<tr>
<th></th>
<th>Assumption I</th>
<th>Assumption II</th>
<th>Assumption III</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Traffic diverted to buses:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) cost of additional bus journey</td>
<td>6.510</td>
<td>9.810</td>
<td>19.620</td>
</tr>
<tr>
<td>(b) value of additional journey time</td>
<td>15.750</td>
<td>21.330</td>
<td>28.990</td>
</tr>
<tr>
<td>Total</td>
<td>22.300</td>
<td>31.140</td>
<td>48.610</td>
</tr>
<tr>
<td>2. Traffic diverted to private car:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a) Running cost of vehicles</td>
<td>25.040</td>
<td>25.040</td>
<td>25.040</td>
</tr>
<tr>
<td>(b) Value of additional journey time</td>
<td>1.590</td>
<td>1.050</td>
<td>1.050</td>
</tr>
<tr>
<td>Total</td>
<td>26.630</td>
<td>26.090</td>
<td>26.090</td>
</tr>
<tr>
<td>3. Traffic lost:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value of benefits foregone</td>
<td>11.150</td>
<td>11.140</td>
<td>5.5140</td>
</tr>
<tr>
<td>4. Congestion Cost of other road users</td>
<td>8.070</td>
<td>8.920</td>
<td>11.420</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total social costs:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>68.150</td>
<td>77.290</td>
<td>91.960</td>
</tr>
<tr>
<td>Total social benefits (excluding any savings in track and signalling cost) $</td>
<td>80,000</td>
<td>80,000</td>
<td>80,000</td>
</tr>
</tbody>
</table>

Source: Elsc and Howe, op. cit., p. 183.
10. The proper valuation of the costs and benefits of a project, especially in a developing economy, requires that at least three sets of basic data be supplied to the project evaluator by the top-level policy-makers: (a) the objectives of the national economic policy and their relative weights, (b) the rate of discount, and (c) the shadow prices of labor and foreign exchange.  \(^7\)