GOVERNMENT DEFICITS, PRIVATE INVESTMENT AND THE CURRENT ACCOUNT;
AN INTERTEMPORAL DISEQUILIBRIUM ANALYSIS

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SUMMARY

We use a model with full intertemporal optimization and Fischer-Gray type short run real wage rigidities to demonstrate effects of deficit spending in different employment regimes. In an departure from the standard disequilibrium literature, we allow for upward price flexibility although prices, once set at the beginning of the period, will be rigid downward until the beginning of the next period. We show that under Keynesian unemployment (conditional on a plausible assumption about public and private sector discount rates) deficit spending reduces unemployment, improves the future terms of trade and therefore leads to an increase in private investment (crowding in) and deteriorates the Current Account.

Under classical unemployment (where, in our definition, goods markets clear but unemployment persists because of contract based real wage rigidity), fiscal expansion goes partly into prices (terms of trade improvement) and only partly into quantities, to the extent that contract based real consumption wage rigidity coupled with a real appreciation allows a lower real product wage. A temporary increase in government expenditure in classical unemployment leads to a bigger terms of trade improvement today than tomorrow, so both income and substitution effects lead to a current account improvement. The cost of capital increases more than the value of future output so investment falls (crowding out). This effect on investment also improves the period one current account. The direct effect of increased first period government expenditure (for given wages and prices) at least partially offsets this surprising positive effect on the first period CA.

Permanent increases in government expenditure have ambiguous affects on the CA but will be negative if a sufficiently large part of the first period expansion goes into prices rather than quantities. A permanent increase also leads to a bigger period 2 terms of trade improvement than obtains in period one, essentially because of the moderating effect of the first period fall in real domestic product wages allowed by the first period real appreciation coupled with real consumption wage-indexation. This in turn implies that tomorrow’s value marginal product of capital goes up more than today’s cost of capital, so investment once again improves: a permanent increase under this regime leads to crowding in, contrary to a temporary increase.
Introduction

The emergence of substantial government deficits in the late seventies and early eighties has brought government deficits and their effects on the economy back to the forefront of policy debate. Large deficits in the '80s have played a part in the remarkably fast recovery in 1983 and 1984, but, to many observers claim, at the costs of high real interest rates and external deficits on current account. A cautious increase in investment has begun, those high real interest rates notwithstanding. In Western Europe however increasing deficits have not prevented increasing unemployment, while private investment has remained extremely weak. An extreme example is Holland where large government deficits have gone together with ever increasing unemployment, collapsing private investment and surprisingly enough substantial current account surpluses.

The existing literature does not give us too much guidance. Two strands can be distinguished. The first can be found in every textbook and goes back to Keynes (1936), Hicks (1939) and Haavelmo (1945). The focus here is exclusively on short run aggregate demand effects; a traditional set up of static consumption functions and demand driven supply is used. Explicit incorporation of the implied disequilibrium in labour and goods markets is not given, while the intertemporal aspects (less taxes today means more taxes tomorrow) are typically ignored.

The other strand in the literature focuses almost exclusively on these intertemporal aspects. One of the best known examples of this part of the literature is Barro (1974). He and later participants in the discussion of the so-called Ricardo equivalence theorem typically base private behavior on explicit intertemporal optimization, which allows them to satisfactorily
analyze changes in the intertemporal pattern of taxes (an ambitious open economy example of this literature is Frenkel and Razin (1984)). Their use of full employment, market clearing models precludes however a meaningful discussion of the stabilization aspects of fiscal policy.

In this paper we will attempt to bring the two strands together. We analyse fiscal policy in the context of a model with full intertemporal optimization underlying private behavior, which allows us to deal with the intertemporal aspects; but we also explicitly incorporate the possibility of (short run) labour and goods market disequilibrium caused by Fischer (1977)-Gray (1978) type contract based real wage rigidities and within period downward price inflexibility. Prices are however assumed to be flexible upward, in a departure of the standard disequilibrium literature.

This potential for labor and good market disequilibrium allows us to address stabilization aspects of fiscal policy (impact on aggregate output and employment). Cuddington and vinals (1984a,b) also incorporate temporary disequilibrium in an intertemporal optimization model in their discussion of fiscal policy but they focus on monetary aspects and moreover maintain complete within period price rigidity, as opposed to our asymmetric price adjustment (flexible upwards but rigid downward). Their more traditional approach implies the existence of widespread goods market rationing in the classical unemployment regime, which is clearly counter factual. They moreover ignore investment and have to rely on very restrictive functional forms. Other disequilibrium models incorporating rational optimizing savings and investment behaviour can be found in Bruno (1982), Neary and Stiglitz (1983) and van Wijnbergen (1982).

In section 2 the basic model is presented. We use a two period-full optimization set up where wages and prices are set at the beginning of each
period using all available information. This is done in such a way that all markets will be in equilibrium if no unanticipated events occur after the contracts have been concluded. If such events do occur, disequilibrium in labour and goods markets can take place. The implications of that are analysed in the tradition of Barro-Grossman (1976) and Malinvaud (1977), while maintaining an explicit intertemporal optimization framework and allowing for upward price flexibility, contrary to much of the existing disequilibrium literature. Fiscal policy effects under Keynesian unemployment are analyzed in section 3, while section 4 looks at the case of classical unemployment. Section 5 concludes.

2. An Intertemporal, Contract-Based Disequilibrium Model

2.1 An equilibrium version of the model

Consider a two-period, two-commodity world. The economy specializes completely in the production of good 1 while foreigners specialize in the production of good 2. Firms use the beginning of period capital stock and labour as factors of production, and determine output based on the size of the capital stock and the level of the real product wage. This process can be described by using a revenue function (a good exposition of this and other duality tools can be found in Dixit and Norman (1980)).

\[ R = R(P ; K,L) \]
\[ = P X(K,L) \]  

where \( P \) is the price of home goods in period 1. Capital letters indicate first period variables, and lower case symbols second period variables. Foreign goods are chosen as numeraire, so we can interpret \( P \) and \( p \) as the first and second period terms of trade. Labour demand is given by the requirement that the marginal value product equals the wage:
(2)

We can invert (3) to get a labour demand function $L = L(W/P, K)$. If we insert that back into (1) we get:

$$R = P X(K, L(W/P))$$

(3)

with output supply equal to

$$X = \frac{2R}{\delta P} = R_P$$

(6)

Second period output $x$ is predetermined for any given value of the second period capital stock because of our assumption of full employment in period 2:

$$r = r(p, k)$$

(5)

$$= p x(k)$$

and

$$x = \frac{\partial r}{\partial p}$$

(6)

We will ignore depreciation, so $k = K + I$ with $I$ first period investment. Investment is determined by value maximizing firms, equalizing the discounted value of future marginal revenue to the production cost of capital:

$$\delta p \frac{\partial x}{\partial k}(K + I) = P$$

(7)

$\delta$ is the discount factor $1/(1 + r)$ with $r$ the world rate of interest.

(7) yields an investment demand function:

$$I = I(\delta p/P), I' > 0$$

(7a)
I > 0 because \( \frac{3x^2}{\delta} < 0 \), \( \delta_n = \frac{\delta p}{P} \) is the home goods discount factor, related to the own rate of interest in terms of home goods, \( r_n = (\delta p/P)^{-1} \).

Consumer demand can be derived from the expenditure function, which gives the minimum discounted value of expenditure needed to reach utility level \( U \) given prices today and tomorrow (again see Dixit and Norman (1980) for an exposition of expenditure functions):

\[
E = E(\Pi(P,1), \delta \pi(P,1), U) \tag{8}
\]

where \( \Pi \) and \( \pi \) are unit expenditure functions, they can also be interpreted as the exact aggregate price indices corresponding to the utility structure \( U \). \(^1\) By property of expenditure functions (cf Dixit and Norman (1980)) we know that domestic consumer demand for home goods equals the derivative of \( E \) with respect to corresponding price:

\[
C = E_p = E_p \tag{9}
\]

Also \( \Pi \) and \( \pi \) are aggregate price indices so we can define real expenditure in each period:

\[
A = E_{\Pi}, \quad a = E_{\pi} \tag{10}
\]

Since \( \Pi \) and \( \pi \) are also unit expenditure functions, \( \Pi_P = C_D/A \) and \( \pi_P = c_D/a \), so (9) and (10) are indeed mutually consistent. Furthermore we can derive similar expressions for foreigners, for whom we will use starred variables \( (E^*, C^*_D, A^* \text{ etc.}) \).

\(^1\) We assume \( U \) to be Weakly Identically Homothetically Separable, which allows us to write \( E \) as a function of \( U \) and the within period unit expenditure functions \( \Pi \) and \( \pi \). See Razin and Svensson (1983) and van Wijnbergen (1984a) for a detailed discussion.
Consider the expression for real expenditure in period 1:

\[ A = A(P, 1), \delta \pi(p, 1), U) \]  \hspace{1cm} (19)

Due to homogeneity of degree zero of Hicksian demand functions we can rewrite (19) as

\[ A = A(1, \delta \pi(p, 1)/(P, 1), U) \]  \hspace{1cm} (19a)

\[ \delta \pi(p, 1)/(P, 1) \] represents the consumption discount factor \( \delta \) or one over one plus the Consumption Rate of Interest (CRI). The CRI represents the terms at which future consumption can be traded against current consumption.

(1) - (10) can be used to derive goods market equilibrium conditions. For first periods goods markets we get:

\[ R_p = C_d + C_d^* + I + G \]  \hspace{1cm} (11a)

\[ = E_p + E_p^* + I(\delta p/P) + G \]

where \( G \) is first period government expenditure. Similarly, for period 2:

\[ r_p = E_p + E_p^* + g \]  \hspace{1cm} (11b)

Labour market clearing in period 1 is given by the inverse of equation 3:

\[ L = L(W/P) \]  \hspace{1cm} (12)

Finally the private sector budget constraint:

\[ \kappa + \delta r = PI - \tilde{T} = E \]  \hspace{1cm} (13)
where $\bar{T}$ equals the discounted value of current and future taxes:

$$\bar{T} = T + \delta t$$

with obvious definitions of $T$ and $t$. Under the assumption of identical government and private discount rates, $\bar{T} = P G + \delta pg$ via the government budget constraint. More on that in Section 2.5.

Differentiating $E$ and inserting $\bar{T} = PG + \delta pg$ yields an expression for private welfare $U$:

$$E dP + \delta (r_p - E - g) dp - P dG - \delta pg = E_u dU$$

which can be rewritten using the goods market equilibrium expressions:

$$E_p^* dP + E_p^* \delta dp - P dG - \delta pg = E_u dU$$

A similar expression holds for foreigners:

$$-E_p^* dP + E_p^* \delta dp = E_u^* dU^*$$

(11a,b), (12) and (14b,c) constitute an equilibrium version of the model, with variables $P$, $p$, $W/P$, $U$ and $U^*$. We will first give a diagrammatical representation of this equilibrium version, because that will facilitate introduction of the consequences of first period disequilibrium.

Inserting (14a, b) in (11a) to substitute out $U$ and $U^*$ gives us a locus describing goods market equilibrium in period 1, represented by locus GM1 in fig. 1:

$$\frac{d(W/P)}{dP} \bigg|_{GM1} = \frac{E_{pp} - I - \delta p/P^2}{X_L L}$$

(15)
with $\Sigma_{PP} = E_{PP} + E_{PP} + (C_{DE} - C_{DE}) E_{P}^*$, the world substitution matrix plus income effects of an increase in $P$. $E_{PP} < 0$ for normal goods. $C_{DE}$ is ("our") marginal propensity to spend on our first period goods. $C_{DE}^*$ is the foreign marginal propensity to spend on our first period goods. GM1 is upward sloping: a higher (relative) price of our goods reduces world demand for our goods, leading to excess supply; a higher real product wage however will reduce aggregate supply, so getting us back in goods market equilibrium (cf fig. 1).

Fig. 1 Diagrammatic Representation of the market clearing model.

Labour market equilibrium is represented by (12), a horizontal line in the $W/P - P$ plane:

$$\frac{d W/P}{dP} = 0 / L^* = 0$$

(16)
Since labour is the only variable factor in this model, there is only one market clearing real product wage for any configuration of output prices in period one.

The second period goods market equilibrium locus GM2 slopes upward (cf. fig. 1 and keep in mind that \( p \) increases from 0 downwards):

\[
\frac{dp}{dP} \bigg|_{GM2} = - \frac{(\Sigma_P + \frac{r}{pk} \frac{1}{\delta p/P} \cdot \delta p/P^2)}{(\Sigma_P - \frac{r}{pk} \frac{1}{\delta P} \cdot \delta P)} > 0
\]

(17)

where \( \Sigma_P \) and \( \Sigma_P \) are the relevant elements of the world substitution matrix plus income effects \( \Sigma_P = E_P + E_P + (c_{DE} - c_{DE}^*)E_P, \) etc; \( \Sigma_P > 0 \) and \( \Sigma_P < 0 \) if income effects do not dominate substitution effects. A sufficient but by no means necessary condition for that is \( c_{DE} = c_{DE}^* \) etc. (identical expenditure patterns in both countries).

There are basically two channels linking \( p \) and \( P \) via the second period goods market. The first runs via \( \delta p/P \) (here equal to one over one plus the accounting rate of interest, to use a phrase from the project analysis literature (see Little and Mirrlees (1974)). For given \( p \), a higher \( P \) raises the production cost of capital which will lead to fall in investment. That in turn reduces aggregate supply in period two, leading to upward pressure on \( p \).

The second channel also leads to a positive relation, and runs via the Consumption Rate of Interest \( \delta_c \). Higher prices today for given \( p \) will decrease \( \delta_c \) (remember that \( \Pi_p = C_D/A > 0 \) or, equivalently, increase the CRI. This will lead to a shift of expenditure towards the future because of pure substitution effects, some of which will fall on period 2 home goods. This also puts upward pressure on their relative price \( p \), like the effect
via the first channel, so there is an unambiguously positive link between \( p \) and \( P \) along GM2 (cf. fig. 1).

Before we introduce disequilibrium, one final point: the diagram in Fig. 1 involves a fudge, necessary to allow diagrammatical representation. The same channels linking \( p \) and \( P \) via period two goods market clearing also work backwards: future prices do influence first period goods markets, via the value of capital in period 2 (and so via first period investment), and via the impact of the CRI on private savings. This of course implies that we cannot really represent GM1 in \( \frac{W}{P} - P \) space. The algebraic derivation of all results incorporates this extra link, but we will ignore it in the diagrammatical representation. It is left to the interested reader to demonstrate than an increase in \( p \) shifts GM1 to the right.

2.2 Consequences of wage-price rigidity

Consider now the introduction of the possibility of disequilibrium. Assume that at the beginning of the period wage contracts are concluded that index real wages in terms of the cost of living, and relative prices are announced. All this is done using all information available at that time, and in such a way that labour and goods markets will clear if no unanticipated shocks will occur during the period. We will not consider period 2 shocks unanticipated at the beginning of period 2, so in that period (the "long run") the economy will always be in Walrasian equilibrium. Green and Laffont (1981) in their discussion of similar pricing behaviour, call this "rational anticipatory pricing."

Such temporary real wage rigidity of course has a firm basis in modern contract theory. On the other hand the foundation for price rigidity is less solid. The disequilibrium literature has always assumed complete price rigidity. Although there is empirical evidence that relative prices
move gradually rather than instantaneously in response to goods market disequilibrium (see especially Alogoskopoulis and Pissarides (1983)), there are at least two major problems with that assumption. First of all it of course does not make sense in any but the short run. We incorporate that by assuming second period ("long-run") price flexibility.

The second problem is that the implication of wholesale nationwide goods market rationing in excess demand situations is clearly counterfactual, although it may take place on a lower level of aggregation. Moreover there is theoretical support for asymmetric price adjustment, with prices more flexible upwards than downwards (Reagan (1983), Reagan and Weitzman (1983)). We therefore adopt an extreme form of such asymmetric behavior: we assume complete price flexibility in excess demand situations (upward flexibility), but within-period downward rigidity when unanticipated (at the time prices were set) shocks cause excess supply of goods.

Consider now the modifications to our model that arise because of potential disequilibrium and the associated spillover effects. The expression describing goods market equilibrium under excess supply of labour itself does not change as firms are under no constraint in this situation. However consumers' intertemporal budget constraint is affected because now employment and therefore income is variable and will depend on the real product wage. Substituting out $U$ via the modified budget constraint gives:

\[
\frac{\partial W/P}{\partial P} \bigg|_{\text{GML}} = \frac{\Sigma P - I' \delta p/P^2}{(1 - C_{DE})X_L} > 0
\]

Comparison of (20) with (15) shows that this segment of GML will still be upward sloping, and in fact will be steeper than that in the equilibrium version, because of the term $-C_{DE}X_L$ in the denominator. The
reason is that now higher wages will not only cut aggregate supply, but also via their negative impact on employment reduce total income and therefore demand for home goods, reducing its impact on excess supply of goods. This means that a smaller price increase is needed to rebalance goods markets after a given increase in real product wages W/P, or, in other words, GM1 is steeper (cf fig. 2).

Figure 2  First period disequilibrium regions and second periods goods market clearing locus
As is by now well known the loci describing goods market equilibrium under excess demand for labour and labour market equilibrium under excess supply of goods collapse into one locus in this type of model without inventories and complete specialization of production, eliminating the so-called "under-consumptionist" regime. That combined locus has a negative slope but since it plays no role in the analysis to come we will not elaborate further. In the K-region to the NE of A there is unemployment because effective demand is insufficient (at given wages and prices) to sustain full employment, with the marginal productivity of workers at going wages and prices in excess of the real product wage if there would have been sufficient demand for their output.

In the areas above LL and to the left of GM1 prices and wages are such that labour is in excess supply and domestic goods in excess demand. However because of our assumption of upward price rigidity the economy will never be in that region: prices will increase until the economy is on the GM1 locus, with goods markets in equilibrium, but real wages too high for labour market equilibrium.

Accordingly one unemployment regime is the K-region to the right of GM1, characterized by Keynesian unemployment: here output is demand determined because prices are set too high for all the supply to be absorbed. The second regime is along the GM1 locus from A upwards, with the goods market in equilibrium but real wages too high for labour market clearing. We will call this regime classical unemployment, although it is different from what is commonly called classical unemployment in the disequilibrium literature because our "C region" is not characterized by goods market rationing.
The G142 locus will now change and will depend on the regime prevailing in period 1. It is still sloped as in fig. 1 in all three cases. So we can use the diagram in fig. 2 as long as no regime switches are considered.

2.3 The model in the Keynesian Unemployment regime.

The behavioural equations of course change discontinuously across regimes. Consider first the K-region. First period output in that region is demand determined:

\[ X = E_p + E_p^* + I + G \]  

(21)

where \( R_p > X \). Second period goods market is similar to 11b:

\[ r_p = E_p + E_p^* + g \]  

(22)

but the private intertemporal budget constraint is different:

\[ PX + \delta r - PI - T = E \]  

(23)

where \( PX \) replaces \( R \) (note that \( R > PX \) in this regime !).

2.4 The model in the classical unemployment regime.

Under classical unemployment, first period output is supply determined and prices (\( P \)) will adjust until goods markets clear. Accordingly the following goods market clearing condition holds:

\[ R_p = E_p + E_p^* + I + G \]  

(24)
Second period goods market equilibrium, as before, is represented by:

\[ r_p(p,K + I) = E_p + E^*_p + g \]  \hspace{1cm} (25)

The difference with the equilibrium model of section 2.2 lies in the labour market: wages are fixed in terms of the cost of living, at a level that would have led to labour market clearing if no unanticipated shock had occurred after the conclusion of wage contracts. We have an exact measure of the first period the cost of living via our unit expenditure function \( \Pi \), so the indexation gives us:

\[ \frac{W}{\Pi} = T \]  \hspace{1cm} (26)

with \( T \) the negotiated level of the real consumption wage.

Finally the intertemporal budget constraint:

\[ R(P,W/P) + \delta r(p,K+I) - \tilde{T} - P I = E \]  \hspace{1cm} (27)

Employment in this regime is of course demand determined: by inverting equation (3) we get:

\[ L = L(W/P) \]  \hspace{1cm} (28)

Note that prices are flexible upwards while there is real consumption wage rigidity. Accordingly the real product wage \( W/P \) is not rigid.

The model applying to the Repressed Inflation regime is left to the interested reader to explore since we will not be concerned with that regime in this paper.
2.5 The Government Budget Constraint.

Before we turn to an analysis of fiscal policy, a short discussion of the government-budget constraint is in order. The benchmark case is equal discount rates for public and private sector. This implies

\[ PG + \delta pg = T + \delta t (= \bar{T}) \]  

(29a)

There are however a variety of reasons to expect that the appropriate rate for the public sector to discount future income is different from the private discount rate. A straight-forward argument would be that governments can borrow at more favorable terms in international capital markets than are available to the private sector. A different and rather ingenuous argument has been presented by Blanchard (1983) who points out that uncertain lifetime coupled with absence of private bequest motive, will lead to a higher private discount rate. (cf Frenkel and Razin (1984) for an exploration of what the Blanchard hypothesis implies for interest rate and current account effects of fiscal policy in an market clearing full employment model).

We think these arguments carry sufficient force to consider the effects of fiscal policy when such a wedge exists. A caveat is in order however. Since the reason for that wedge is not modelled explicitly, we cannot really do explicit welfare analysis, a necessary precondition for sound policy advice. Accordingly the aim of this paper is positive rather than normative, we try to explain the consequences of fiscal policy, but we do not take the next step, say discussion of optimal pattern and size of deficits, public debt etcetera. Such an analysis requires explicit modelling of the market imperfections that make it an interesting issue to begin with.
The positive consequences of a lower public sector discount rate (higher discount factor) can be taken on board however; in that case (29) becomes

\[ \tilde{G} = PG + \delta_{t} T > T + \delta t \]  

(30)

or an expenditure plan adequately covered by current and future taxation using public sector discount rates will imply a net financing gap when evaluated at private sector discount rates:

\[ \tilde{G} - T - \delta_{t} T = 0 \Rightarrow \tilde{G} - T - \delta t > 0 \]  

(31)

3. Fiscal Policy with Keynesian Unemployment

The benchmark case is quite straightforward and does not require much formal analysis. Since there are perfect capital markets, a shift of taxation towards the future \( dt = -\delta^{-1} dT > 0 \) does not affect private wealth nor therefore private expenditure. As can be seen from equ. 23, all that matters in the benchmark case is the discounted value of taxes, not their pattern. Accordingly the period 1 government deficit covered by period 2 taxes has no real economic effects, the existence of Keynesian unemployment notwithstanding. Future taxes are discounted back to today and incorporated in expenditure plans, so the tax cut, while increasing disposable income in period 1, will be accompanied by a one for one increase in private savings, exactly offsetting government dissaving. No effect therefore on unemployment, current account or investment.

Similar results obtain for a deficit financed increase in government expenditure; we can see from (21) that \( dX/dG = 1 \) as a first round effect; the private sector however internalizes the government budget constraint so that since \( d\tilde{T} = PdX \), again nothing happens to private wealth and expenditure and no
multiplier effects occur. Accordingly, private investment and the CA do not change, and deficit financed government expenditure has no further multiplier effects on employment and income, the existence of Keynesian unemployment notwithstanding (a formal proof will be given below). Clearly Keynesian disequilibrium and wage-price rigidities are not sufficient for a multiplier larger than one, contrary to what is often claimed. What is needed of course, is some capital market imperfection leading to incomplete internalizing of the government budget constraint, as argued before in the context of equilibrium optimizing models by Tobin and Buiter (1980). Haavelmo’s balanced budget multiplier of one also applies to deficit spending if capital markets are perfect. There is moreover no direct crowding out (cf section 4) because there is excess supply on the goods markets. All this of course assumes that \( dG \) is not so big as to actually lead to a regime switch.

One such capital market imperfection would be the existence of a group in society (wage earners ?) that is liquidity constrained; government expenditure today financed by taxes tomorrow would effectively allow them to borrow against future income leading to a multiplier larger than one. An alternative with similar effects is the one mentioned in section 2, a wedge between public and private sector discount rates, for example because of different terms available to them in international financial markets, or because of Blanchard type uncertain lifetime effects coupled with absence of bequests (Blanchard (1983)). Assuming \( \delta_g > \delta \) is not fundamentally different from a flat private sector liquidity constraint: one way of modelling such a constraint would be to set \( \delta = 0 \) (i.e. an infinite private discount rate), which obviously implies \( \delta_g > \delta \).

The discount factor wedge implies that an increase in government expenditure financed by taxes tomorrow (i.e. a bond issue today) will not be
considered neutral by the private sector; the government budget constraint implies that

$$PdG = \delta_g \cdot dt$$

so.

$$PdG - d\bar{T} = PdG - \delta dt = (\delta_g - \delta)dt > 0$$

Total differentiation of the Keynesian system (21)-(23) yields

$$\frac{dU}{dG} = \frac{\Delta}{\Delta_g} (\delta - \delta) > 0$$

where

$$\Delta = \left( \Sigma_{PP} + \Sigma_{PK} \right) P_{DE} - \Sigma_{DE} (\Sigma_{PP} - \Sigma_{PK} \right) + P + E^* P_{DE} > 0;$$

and the Jacobian of (21)-(23),

$$\Delta = c_{DE} P \left( \Sigma_{PP} + \Sigma_{PK} \right) - (1 - PC_{DE}) \left( \Sigma_{PP} - \Sigma_{PK} \right) > 0$$

(33) establishes that a temporary bond financed expansion in government expenditure will unambiguously increase private welfare in Keynesian unemployment (if $\delta_g > \delta$ of course). It is straightforward to show that a current-tax financed temporary fiscal expansion would not do that.

We can use (34) and (37) to derive from (33) the impact welfare effect directly due to (32) but without incorporating the induced welfare gains due to second round multiplier effects in excess of one and induced second period terms of trade effects:

$$E_u \left. \frac{dU}{pdG} \right|_I = \frac{(\delta - \delta)}{\frac{\delta}{g}}$$

This expression will be useful later on.
But higher private welfare implies higher private expenditure, some of which will fall on tomorrow's home goods, shifting out the GM2 curve (cf fig. 3; since we start in the K-region, the economy is initially at a point like A).

![Graph](image)

**Fig. 3** Effects of a bond financed increase in $G$ when $\delta_g > \delta$.

That in turn leads to a second period terms of trade improvement:

$$\frac{dp}{PdG} \frac{C_{DE} \cdot (\delta - \delta)}{\Delta \delta_g} > 0.$$  \hspace{1cm} (34)
Of course no such future terms of trade improvement will occur if private and social discount rates are the same; the expression in (34) then equals zero.

(34) lead to an interesting result on the investment response to deficit spending under Keynesian unemployment:

\[
\frac{dI}{dG} = \frac{I'\delta}{P} \frac{dp}{dG} \quad (35)
\]

\[
= \frac{I'\delta c_{DE}}{\delta \Delta} \cdot (\delta - \delta) > 0
\]

or higher government spending today financed by taxes tomorrow will lead to more rather than less private investment today under temporary Keynesian unemployment conditions. The mechanism is quite straightforward: such an expansionary fiscal policy raises welfare (if \( \delta_g > \delta \)) and therefore first and second period expenditure. This pushes up the future terms of trade \( p \), which in turn increases the value of the marginal product of capital in period 2: the goods produced with that capital now have a higher value. Tobin's "q" goes up and so does private investment. This result is the opposite of the crowding out hypothesis: in fact under temporary Keynesian unemployment there will be crowding in, private investment responds positively to the expansion induced future terms of trade improvement.

Moreover part of the increase in private expenditure will fall on today's goods, the equivalent of second and higher rounds of induced spending familiar from standard macro-economic textbooks, so now we do get a multiplier in excess of one:

\[
\frac{dX}{dG} = 1 + \left( \frac{\tau_{pk} I'\delta}{P} - \Sigma_{pp} \right) \frac{c_{DE}}{\Delta \delta \Delta} \cdot (\delta_g - \delta) \quad (36)
\]

(A)
\[
\begin{align*}
&\left(\Sigma_{pp} + I^{\delta/P}Pc_{DE}\right) \frac{\partial G}{\partial \delta G} \cdot (\delta - \delta) \\
&+ \frac{\partial G}{\partial \delta G} \cdot (\delta - \delta)
\end{align*}
\]

So the (excess over one of) the multiplier can be broken down in two components, one familiar (A) and one unique to the intertemporal framework used here (B). (A) represents induced expenditure effects on output and corresponds to the standard multiplier mechanism explained in all macro-textbooks; (B) however will only arise in an explicit intertemporal framework. An increase in \(p\) increases the Consumption Discount Factor \(\delta_C\) (= \(\delta\pi(p,1)/\Pi(p,1)\), equal to one over one plus the Consumption Rate of Interest) measuring the terms at which future consumption goods can be traded for current consumption goods. This increase in \(\delta_C\) (or, equivalently, fall in the CRI) has a pure substitution effect on private expenditure leading to

\[\frac{dX}{dG} = 1 + \left(\Sigma_{pp} I^{\delta/P} \Sigma_{pp}\right)PC_{DE} E u \frac{du}{dG} I\]

\[+ \left(\Sigma_{pp} + I^{\delta/P}\right) \frac{dp}{dG}\]

(36) can be rewritten as 1/:

\[
\frac{dX}{dG} = \left(\Sigma_{pp} I^{\delta/P} \Sigma_{pp}\right)PC_{DE} E u \frac{du}{dG} I
\]

\[+ \left(\Sigma_{pp} + I^{\delta/P}\right) \frac{dp}{dG}\]

We use \(\frac{dX}{dG}\) etc. as shorthand for \(\frac{\partial X}{\partial G} + \frac{\partial X}{\partial t} \bigg|_{dG = \delta G \ dt}\)
more expenditure today and less tomorrow adding further terms to the standard expression of the multiplier. This term will not arise in more conventional static models.

Finally, the effect of deficit spending on the current account. This is straightforward since we have seen, first of all, that private first period consumption expenditure will increase, so that private saving will not offset the decrease in government savings one for one, leading to a decline in aggregate savings, and second, that private investment will increase. So aggregate savings fall and investment increases in period 1. The net effect on the CA (savings minus investment!) is therefore negative. A formal proof is straightforward and is left to the interested reader.

It should be noted that a permanent increase in government expenditure (PdG = δpdg > 0) would add further upward pressure on tomorrow's terms of trade, and so on investment, and furthermore on consumption via income effects and substitution effects via the CRI in period one. This would magnify the negative impact on the first period current account deficit. The formal analysis is straightforward and left to the interested reader.

4. Effects of temporary and permanent increases in government expenditure under classical unemployment.

In this section we will assume δg = δ. Our earlier assumption, δg > δ, does not make for qualitative changes in the results here (it only leads to larger wealth effects), contrary to the Keynesian case. We therefore revert to δg = δ for simplicity's sake.

Assume then that for one reason or another the country has ended up in a classical unemployment regime (say at point A in Figure 4) and the government attempts to get out via expansionary fiscal policy. We will for analytical convenience assume that the change in government expenditure is
small enough not to cause a regime switch. Since temporary and permanent increases in government expenditure have rather different effects in this regime, we consider them in turn. The diagram in fig. 4 can be used to get a feel for the results. Full analytical treatment is relegated to appendix 1.

A temporary fiscal expansion of course has the same effect on the period one equilibrium loci as before: no direct impact on the labor market given wages and prices (so LL does not shift), but an outward shift of GM1 due to the increase in demand for our goods in period one (cf Figure 4).

Figure 4. Effects of a fiscal expansion under classical unemployment.
At the end of section 2.2 I pointed out that the expression for the second period goods market equilibrium, GM2, depends on the first period regime. That dependence is demonstrated now, the fiscal expansion with first period classical unemployment induces a GM2 shift in a manner different from what happens when the first period is characterized by Keynesian unemployment. In fact now there will be a negative second period effect: the governments increased use of first period resources implies they are not available for the private sector, with a welfare loss as a result. \(^1\)

This spills over into period two, reducing "our" expenditure (ceteris paribus) and therefore causing an ex ante excess supply of home goods. Accordingly their relative price will have to fall to maintain period two goods market equilibrium:

\[
\frac{dp}{PdG} \Bigg| _{GM2} = \frac{C_{DE}}{\Sigma_{PP}} < 0 \quad (38)
\]

or GM2 shifts down (i.e. to the origin in Fig. 4). \(\Sigma_{PP}\) is defined in the appendix.

For a given value of \(W/P\) however increased first period government expenditure on our goods \((dG > 0)\) is, at given wages and prices, inconsistent with first period goods market clearing, since in this regime firms are on their aggregate supply curve. Accordingly for given real product wage, the terms of trade will have to improve to accommodate the increased government expenditure, or GM1 shifts to the right (fig. 4):

\[
\frac{dp}{dG} \Bigg| _{GM1} = \frac{(1 - PC_{DE})}{\Sigma_{PP} - \Sigma_{PP} \delta_P/P^2} > 0 \quad (39)
\]

\(^1\) The results discussed in this section are sensitive to a relaxation of the assumption made here (as in most of the macro literature), that \(G\) has no direct impact on private welfare.
Before proceeding we need to incorporate one further constraint, the fact that $G$ goes up after real wage contracts have been concluded at the beginning of period one. They can be renegotiated at the beginning of the second period, but during period one the real consumption wage $W/P(1)$ is fixed. This gives another relation between $W/P$ and $P$ that needs to be satisfied:

$$\dot{W} - \dot{P} = \ddot{W} - \psi \ddot{P} = 0$$

where $\psi = P_{P_1}/P$ is the expenditure share of home goods in first period expenditure. This can be rearranged to give a relation between changes in $W/P$ and $P$:

$$\dot{W} - \dot{P} = (1-\psi) \ddot{P}$$

We have drawn (40a) in Fig. 4, labeling it Wage Indexation Line (WIL), going through the starting point $A$ with slope $- (1-\psi)$. (Note that WIL represents a restriction on changes in $W/P$ and $P$).

So assume that for whatever reason the economy is at $A$ (cf. Fig. 4), in what we call classical unemployment, with goods market clearing (i.e. we are on GM1) but labour in excess supply because wage contracting prevents within period labour market clearing.

A temporary increase in $G$ will shift GM1 as indicated in Fig. 4. That means that the starting point $A$ now is in a region characterized by excess demand for goods. However upward price flexibility means that that is not sustainable, the real exchange rate will appreciate today (terms of trade $P$ improve) until the goods market is back in equilibrium.

1/ The financing (taxes versus bonds) does not matter since we have reverted to $\delta = \delta_g$, which gives Ricardo equivalence.
If that were all, the economy would move horizontally from A to B in Fig. 4, with a substantial appreciation but no output response since the real domestic product wage at B equals the one at A. However the wage contract stipulates a real consumption wage, indexing is on the CPI. With the real exchange rate appreciating a fall in the real product wage is possible while maintaining the real consumption wage (cf equ. 40a), so instead of going from A to B the economy moves along WIL from A to C, with an appreciated real exchange rate and a lower real product wage.

This first result, a real appreciation (terms of trade improvement) in response to an increase in G is of course not surprising, although it does not take place in standard disequilibrium models (cf Cuddington and Vinals (1984a,b)) or in the K-region analyzed in section 3. However we already mentioned, and can also see in Fig. 4, that the appreciation combined with a fixed real consumption wage allows lower real product wages in period 1. Since output in this regime is constrained by the real product wage and not by effective demand, there will in fact be some increase in output, although less than in the K-region. In the C-region the effect of an increase in fiscal expenditure \((dG > 0)\) goes partly into prices \((dP > 0)\), partly into quantities. In the K-region it goes only into quantities.

It is of interest to see what determines how the value increase is split up over quantities and prices. Clearly the flatter WIL, the more the effect consists of a real appreciation and the less it consists of a real product wage cut induced output increase. Now the slope of WIL \((1-\psi)\) can be considered a measure of openness of the economy. A very flat line implies \(\psi\) is close to one, imports do not play much of a role and therefore any given real appreciation buys only a small decrease in the domestic real product wage. This leads to the interesting result that in the C-region (GM1 axis
north of $E$) more openness means a larger output response and a smaller price response to fiscal expansion. This contrasts with the K-region where a higher import component in expenditure leads to more dissipation of effective demand and smaller output effects.

The effects on the second period terms of trade are ambiguous in the case of a temporary increase in $G$. We already discussed why in that case the GM2 locus shifts back to the zero $p$-axis (fig. 4), since the impact welfare effect of $G$ on $U$ for given wages and prices is negative, leading to a fall in private expenditure (compare $F$ and $G$ in fig. 4).

On the other hand the real appreciation in period one causes intertemporal substitution effects on consumption, increasing second period demand for our goods; moreover since $dP > 0$ means the production cost of capital increases, first period investment falls (there is crowding out in the C-region), reducing the supply of period 2 home goods. Both factors work toward a real appreciation (a move along GM2 from $D$ to $F$). The net effect is ambiguous. It is shown in the appendix however that even if there is a second period real appreciation $dp > 0$, it will be smaller than the first period one, so the consumption discount factor $\delta_c = 1/(1 + CRI)$ and the capital discount factor $\delta_h = \delta p/P$ decrease unambiguously. In other words the Consumption Rate of Interest and the Accounting Rate of Interest go up unambiguously.

This leads to a rather surprising possibility of a positive current account response to increased first period fiscal expenditure, even when it is deficit financed (i.e. by bonds rather than taxation). There are a variety of channels influencing the CA response. The induced relative price changes influence both savings and investment via their impact on CRI and ARI, if foreign and domestic spending patterns differ, income effects come into play,
higher output because of a lower real product wage and the direct effect of the fiscal expansion also influence the CA.

Consider the formal expression, where for convenience we look at $CA_2$ rather than $CA_1$, (since $CA_1 + \delta CA_2 = 0$ one can take either one):

$$dCA_2 = d(r-E_{\pi-\delta}pg)$$

\[ (41) \]

\[ r \frac{I'}{I} \delta p \left( \frac{dp}{P} - \frac{dP}{P} \right) \]  
\[ + \frac{E_{\pi\delta}}{E_{\delta}} \delta c \left( \frac{dp}{P} - \frac{dP}{P} \right) \]  
\[ + E_p (C_{IE}^* - C_{IE}) dp + E_p (C_{IE}^* - C_{IE}) dP \]  
\[ + C_{IE} \frac{R_{PW}}{(1-\psi)} W/P^2 dP \]  
\[ + C_{IE} PdG - C_{IE} \delta pdg \]  

$C_{IE}$ is the marginal propensity to spend in period one (two).

Consider the five effects in turn for a temporary increase in government expenditure, $PdG > 0$, $pdg = 0$. We already saw that in that case $\frac{dP}{P} > \frac{dp}{P}$.

Channel (A) indicates that such an increase in the cost of capital in excess of future value of marginal product gains will lead to a fall in first period investment, therefore to a fall in second period capital and output and so to a decline in $CA_2$ (increase in $CA_1$).

Similarly, $\frac{dP}{P} > \frac{dp}{P}$ raises the $CKI$ and has positive pure substitution effects on first period savings which deteriorates $CA_2$ (improves $CA_1$). If spending patterns here and abroad are similar ($C_{IE} = C_{IE}'$, etc.), terms of trade change induced income effects will not influence the CA, if we spend more today and less tomorrow than foreigners do ($C_{IE} > C_{IE}', C_{IE} < C_{IE}'$), the higher terms of trade gain today associated with a temporary increase in fiscal expenditure will improve $CA_1$ (deteriorate $CA_2$). This is channel C.
Furthermore, the first period real appreciation coupled with real consumption wage indexation allows a fall in real product wages and an increase in first period output. This improves CA₁ (deteriorates CA₂), as captured by channel D.

Finally, the direct effect of increased fiscal expenditure deteriorates the CA, but this effect is moderated because it leads to a fall in welfare which in turn leads to an equal present value drop in private expenditure. For a temporary increase, consumption smoothing explains why this private cut only partially offsets the first period direct current account effects. So this channel (E) leads to a deterioration in CA₁ and an improvement in CA₂.

Summing up, (A), (B), (C), and (D) are positive influences on the first period CA, while (E) has a negative impact on CA₁. A positive first period CA response to a temporary increase in government expenditure is therefore a distinct possibility.

Finally, the effects of a permanent increase in government expenditure in the classical unemployment regime. This merits separate treatment, since the results are strikingly different. The clue to the difference lies in differential price effects.

Consider first terms of trade effects of a permanent increase in government expenditure for given investment level. In that case the only difference between today and tomorrow is the fact that first period physical output can increase somewhat because the real appreciation coupled with real consumption wage indexation allows a fall in the first period real product wage. This does not play a role in period two since there was and will be full employment in that period. But that in turn implies a larger supply response today than tomorrow and therefore a smaller terms of trade
improvement today than tomorrow, contrary to the results obtained in the case of a temporary increase in expenditure.

Investment response will moderate but not reverse that result as can easily be seen. A bigger increase in terms of trade tomorrow than today implies a larger increase in tomorrow's value marginal product of capital than today's cost of capital, so investment will go up. This puts upward pressure on today's terms of trade but increases tomorrow's supply of home goods, thus moderating tomorrow's terms of trade improvement. It is clear that these moderating effects of the increase in investment cannot reverse the fact that tomorrow's terms of trade improve more than today's: in that case investment would not go up to begin with. A formal proof of all this is in the appendix.

This differential increase in the terms of trade in favor of tomorrow also establishes that the cost of capital $P$ goes up less than the value marginal product of capital tomorrow (which goes up in proportion to $p$), so investment increases, there is once again crowding in of investment, contrary to the case where the increase in government expenditure is temporary.

We can use (41) for an analysis of the CA effects. Since $\frac{dp}{p} > \frac{dp}{P}$ now, the terms (A) and (B) are positive, indicating a negative first period CA response. This should not be a surprise: higher investment means a lower CA, while $\frac{dp}{p} > \frac{dp}{P}$ implies a falling CRI and therefore negative pure substitution effects on savings.

On the other hand lower real product wages in period one mean temporarily higher income in period one so a positive influence on $CA_1$ (term (D) is still negative.)

Under symmetric expenditure patterns over time terms (C) and (D) will equal zero, so we are left with ambiguity: investment, and pure substitution
effects on savings work towards a negative first period current account effect, but the positive income effect associated with the fall in the first period real product wage and the resulting increase in output go in the other direction. If (D) is small enough, i.e., if the first period demand expansion goes mainly into prices rather than quantities, (A) and (B) will dominate and the first period current account effects will be negative.

5. Conclusion

In the introduction we noticed that the existing literature on the links between government deficits and private savings and investment behavior either focus on short run aggregate demand within a static framework, or on the intertemporal aspects introduced by the intertemporal government budget constraint in an optimizing, forward looking but full employment framework.

The first part misses out on intertemporal aspects (less taxes now means more taxes tomorrow), furthermore although the exclusive preoccupation with Keynesian (insufficient effective demand) unemployment of that literature is understandable given the time in which it was written, it is surely less satisfactory or at least incomplete now that real wage resistance problems in the seventies have led to a reemergence of classical unemployment and the associated concerns about profitability. The second part of the literature misses out on the stabilization policy aspects of fiscal policy.

In this paper we have attempted to incorporate both aspects in a framework that (A) includes the possibility of short run disequilibrium of labour and goods markets due to Fischer (1977) - Gray (1978) type wage-price rigidities and (B) explicitly adopts the intertemporal optimizing framework necessary for a satisfactory analysis of the intertemporal aspects of tax cuts, private savings and investment behavior. We furthermore depart from the traditional disequilibrium literature by allowing for upward price
flexibility, eliminating the counterfactual goods market rationing regime that characterizes that literature.

We used this model to demonstrate the effects of deficit spending (fully incorporating the intertemporal government budget constraint) in different employment regimes, and show that the reason for employment, if any exists, is a crucial determinant of the effects of fiscal policy.

When lack of effective demand is causing unemployment (a "Keynesian" regime), deficit spending is shown to lead to more output and employment; if furthermore a plausible assumption about government and private discount rates is introduced, we obtain an interesting result on the private investment response. If private and social discount rates diverge (with social rates below the private ones), private expenditure will increase both today and tomorrow in response to the increase in public expenditure, the anticipation of increased future tax liabilities notwithstanding. Increased future consumption expenditure will lead to a fully anticipated future terms of trade improvement (the price of home goods in terms of foreign goods goes up), which increases the value of the marginal product of capital given its (first period) production costs. This in turn will trigger an increase in private investment in period 1: deficit spending under Keynesian unemployment will lead to crowding in rather than crowding out. Similarly, the increase in private consumption expenditure today coupled with higher government deficits and increased private investment lead to an unambiguously negative link between deficit spending and the current account in this unemployment regime.

The results are very different when unemployment is caused by real wages staying above their market clearing level, leading to low output and employment because of insufficient profitability. Temporary increases in government expenditure in this classical unemployment regime where prices are
upwardly flexible are shown to lead to larger terms of trade improvements today than tomorrow, with as a result both income and substitution effects leading to a CA improvement. Investment will decline (crowding out), further improving the current account. The direct (i.e. for given wages and prices) effect of increased first period government expenditure tends to at least partially offset these surprising positive effects on the CA; the net effect is ambiguous. It is of interest however to note at least the possibility of a first period CA improvement in response to a temporary increase in government expenditure.

If one accepts the proposition that unemployment in the US is Keynesian but unemployment in Western Europe classical (suggested in van Wijnbergen (1982)), the results of this paper could provide one explanation why deficits in the US are associated with CA deficits while countries like Holland run large budget deficits together with substantial current account surpluses.

A permanent increase in government expenditure is show to lead to a larger appreciation tomorrow than today so that income and substitution effects lead to a shift of consumer expenditure towards today and so that investment in fact increases (a permanent fiscal expansion also leads to crowding in!). Both factors tend to deteriorate the first period current account. On the other hand, the real appreciation in period 1 coupled with contract based real consumption wage rigidity allows a fall in the first period real product wage and therefore to an increase in period one output. This at least particularly offsets these negative CA effects of a permanent fiscal expansion.
References


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Fiscal Expenditure Effects in Classical Unemployment: Analytics

1. The Model

Goods market, period one:

\[ R_p(P, W/P) = E_p + E_p^* + I(\delta P/P) + G \]  \hspace{1cm} A.1

Goods market, period two:

\[ r_p(p, K+I) = E_p + E_p^* + g \]  \hspace{1cm} A.2

Intertemporal Budget constraint

\[ R + \delta r - PI - T = E(\Pi, \delta \pi, U) \]  \hspace{1cm} A.3

Wage Indexation

\[ W/\Pi(P, 1) = T \]  \hspace{1cm} A.4

2. Solution

- Differentiate A.1-4, substitute out \( U, W \) via A.3-4. This results in:

\[ \begin{cases} \{ -\Sigma^C_{PP} \Sigma^C_{PP} \} dP & (1-\Pi_{DE})dG - \delta \Sigma_{DE} dG \\ \{ -\Sigma^C_{PP} \Sigma^C_{PP} \} dp & -c_{DE} dG + (1-\delta c_{DE}) dG \end{cases} \]  \hspace{1cm} A.5

CJW/Sweder/appendix/9-10-84
where \( \Sigma_{PP}^c = E_{pP} + E_{pP}^* - R_{pP} - \delta p/P \)

\[ + E_{p}^* (C_{DE} - C_{DE}^*) + R_{pW} (1-\psi) (1-PC_{DE}) W/P^2 < 0 \]

\[ \Sigma_{pp}^c = E_{pp} + E_{pp}^* - r_{pp} - \delta p/P \]

\[ + E_{p}^* \delta (c_{DE} - c_{DE}^*) < 0 \]

\[ \Sigma_{PP}^c = E_{pp} + E_{pp}^* - r_{pk} \delta p/P^2 \]

\[ + E_{p}^* \delta (c_{DE} - c_{DE}^*) - (1-\psi) R_{pW} c_{DE} W/P^2 > 0 \]

\[ \Sigma_{PP}^c = E_{pp} + E_{pp}^* \delta p/P + E_{p}^* \delta (C_{DE} - C_{DE}^*) > 0 \]

Note that symmetric expenditure patterns across countries and, more importantly, time periods, implies

\[ \Sigma_{PP}^c < \Sigma_{pp}^c, \Sigma_{PP}^c > \Sigma_{PP}^c \]  \hspace{1cm} A.6

\[ \Sigma_{PP} + \Sigma_{pp} < 0, \Sigma_{pp} + \Sigma_{PP} < 0 \]

These symmetry assumptions (i.e. \( \Pi_{pP}/\Pi = \pi_P p/\pi \) etc.) are made throughout.

3. **Terms of Trade Effects of a Transitory Increase in Government Expenditure**

\[ dG > 0, dG = 0. \]

\[ \frac{dP}{dG} = \frac{1}{A} \left\{ - \Sigma_{pp}^c (1-PC_{DE}) - \Sigma_{PP}^c (1-\delta PC_{DE}) \right\} > 0 \]  \hspace{1cm} A.7

\[ \frac{dP}{dG} = \frac{1}{A} \left\{ \Sigma_{PP}^c (1-PC_{DE}) + \Sigma_{pp}^c c_{DE} P \right\} > 0 \]  \hspace{1cm} A.8

The inequalities in A.7 and A.8 can be obtained by applying A.6.

\[ \Delta = \Sigma_{PP}^c \Sigma_{pp}^c - \Sigma_{pp}^c \Sigma_{PP}^c > 0 \]
also, \[
\frac{dP}{dG} - \frac{dp}{dG} = \frac{1}{\Delta} \left\{ - z^c_{pp} (1 - PC_{DE}) + \Sigma^c_{pp} PC_{DE} \right. \\
- \left. z^c_{pp} (1 - PC_{DE}) - \Sigma^c_{pp} c_{DE} p \right\}
\]

It is straightforward to see that this expression is positive (note that all terms involving \( R_{PW} \) cancel out!).

4. **Terms of Trade Effects of a Permanent Increase in Government Expenditure**

\[
PdG = \delta pdg > 0.
\]

Define \[
\frac{dP}{PdG} = \left( \frac{dp}{PdG} + \frac{dP}{\delta pdg} \right) \bigg|_{PdG = p\delta dg}
\]

\[
\frac{dP}{dG} = \frac{1}{\Delta} \left\{ - z^c_{pp} (1 - PC_{DE}) + \Sigma^c_{pp} (1 - \delta pc_{DE}) \right. \\
+ \Sigma^c_{pp} PC_{DE} - \left. \Sigma^c_{pp} \delta pc_{DE} \right\}
\]

\[
\frac{dp}{dG} = \frac{1}{\Delta} \left\{ - z^c_{pp} (1 - \delta pc_{DE}) + \Sigma^c_{pp} (1 - PC_{DE}) \right. \\
+ \Sigma^c_{pp} \delta pc_{DE} - \left. \Sigma^c_{pp} PC_{DE} \right\}
\]

Since symmetry over time implies \( PC_{DE} = \delta pc_{DE} \),

\[
1 - PC_{DE} - PC_{DE} = 1 - \delta pc_{DE} - \delta pc_{DE} > 0; \text{ therefore}
\]

\[
\frac{dp}{dG}, \frac{dp}{dG} > 0.
\]

Finally straightforward application of A.6 to A.9-10 shows that

\[
\frac{dp}{dG} < \frac{dp}{dG}.
\]