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International Commodity Markets

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Commodity-Indexed Debt in International Lending

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and
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Initially commodity-contingent debt contracts appear to work best when a group of creditors have co: .rol over the total amount lent, rather than when a single lender acts in isolation. Should a multinational institution take the lead in developing a market for them?

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Superficially, commodity-indexed bonds resemble a combination of a debt and a contract in futures. They are particularly useful in a country dependent on a single commodity for which prices are volatile.

These financial instruments — which explicitly introduce risk management considerations into the credit market — involve a tradeoff between gains in risk-sharing and a deterioration in incentives (or an increased likelihood of default).

The precise costs and benefits of commodity-contingent contracts in international lending depend on the model employed. Commodity indexing seems to work best when:

• The borrower is heavily concentrated in a commodity or set of commodities for which prices are so volatile that income fluctuates greatly.

• A small, well-informed, coordinated set of creditors have control over the total amount lent (a good argument for a public body taking the lead in developing a market for this type of contract).

• Information is fully available about how borrowed funds are used and thus whether conditionality is meaningful.

• The borrower has no control of the index used in the contingent contract.

• There is a low "beta" between returns on the commodity and returns from the rest of the lender's portfolio.

Many of the arguments made for a commodity-dependent borrower may also be made for countries subject to other risks — for example, a country that has borrowed largely in dollars and is thereby exposed to high currency risks.

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Commodity-Indexed Debt in International Lending

by

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I. INTRODUCTION

The idea of tying developing country debt repayments to commodity prices has been discussed in the recent literature on the debt problem (Lessard, 1986, Fischer, 1987 and Priovolos, 1987). However, there has been relatively little work that sets the idea in the context of more general models of debt repayment and considers the implications of such a change on the nature of claims on developing countries.

The object of this paper is to apply a simple model of debt contracts to examine the impact of introducing some form of commodity indexation. When insurance markets and credit markets work perfectly, there is a dichotomy in their functions. The former deal primarily with risk management while the latter deal with inter-temporal resource transfers. Insurance means that borrowers are insulated from bad outcomes; outcomes which are most likely to lead to debt servicing payments not being met and in the worst cases to default.

A premise of the analysis is that insurance markets are incomplete. In view of this, if the return to a borrower's project falls below some level, default will occur. The anticipation of default in certain states in turn has consequences for the performance of credit markets.

When insurance markets are not functioning effectively, credit markets take on a role in risk management as well as in dealing with predictable fluctuations. It is a reasonable conjecture that the cost of malfunctioning insurance markets is diminished in the presence of well functioning credit markets since one can save and dissave to smooth out some of the fluctuations due to the risks.

In fact, reality is even more complicated. When borrowers' behavior is affected by the terms and conditions of loan contracts, there are externalities between insurance and credit markets. Both increased lending and increased insurance coverage lead to a deterioration in the incentives for preventing bad outcomes; yet, if lenders and insurers are distinct, they care only about the impact of their activity on their own payoff--neglecting the impact that it has on the payoffs of others. Hence, once imperfect information is brought into the picture, the impact of more lending and insurance activities becomes ambiguous. There is a favorable effect since insurance supports credit; but there is an unfavorable effect since the consequent increased lending leads to more moral hazard as does the insurance.

Commodity-indexed bonds are a financial instrument which explicitly introduces risk management considerations into the credit market and hence promotes an interaction between dealing with volatility and with risk. The main conclusion from the above discussion that will influence our model design is that there must be careful consideration to the totality of incentives that are available as it may be misleading to analyze the use of such tools in isolation. The main trade-off which we shall elaborate below lies between the gains in risk sharing versus the deterioration in incentives. For those schooled in modern incentive theory, this is hardly surprising. More interesting is the attempt to see how this trade-off bears upon the decision to write a contract.

We first consider this question in the simplest model involving risk-sharing with default explicit. This model serves to show the potential benefits of indexed contracts. In the following sections we complicate matters by introducing firstly an alternative use of funds for the lender and then we allow borrowers to make choices over investment decisions. Each set of assumptions alters the trade-off between benefits and costs and thus alters the optimal form of contract.

Finally, we consider the incentive for lenders to write commodity-indexed debt in a world with many creditors. This, we believe, is the relevant framework for addressing the issues since multiple indebtedness characterizes the dealings of many developing countries in world capital markets. We first examine the incentive for a creditor to write a commodity-indexed bond in isolation. In a world in which incentives are not at issue, we show that there may be a public good problem. The returns to one creditor writing indexed debt may affect other creditors if in doing so it alters the probability of default faced by all creditors. Furthermore, if the returns to indexed debt covary positively with the returns to a creditor's other assets, it may be directly costly to write such debt. We show that there may be conditions under which, despite it being Pareto-improving for all banks to write indexed debt collectively, it may not be worthwhile for any bank to do it unilaterally. Hence, the introduction of commodity-indexed debt may be faced by a collective action problem characteristic of other public goods.

Indexed debt contracts bear a resemblance to contracts written jointly in two markets. For example, they may look like the combination of a debt and a contract in futures. This superficial resemblance may, however, be misleading. Since externalities between markets may be important, there may be very different incentives when contracts are written separately rather than together.

The paper is organized as follows. In section 2 we review the structure of existing models. There are several distinctions that are useful to draw out in respect to assumptions about information availability and control employed. In section 3 we develop the simple model to illustrate the risk-sharing characteristics of commodity-indexed debt. In sections 4 and 5 we introduce two types of costs of indexation. In section 4 we introduce a cost resulting from the effect on the lender's portfolio and in section 5 we introduce an incentive cost due to effects on the behavior of borrowers. In section 6, we develop the argument that indexation has a public good component in a world of many lenders and hence that at a non-cooperative equilibrium there will fail to be optimal provision. Lastly, section 7 provides conclusions and suggests directions for future research.

II. AN OVERVIEW OF THE ISSUES

The theoretical literature on the international credit market includes a considerable number of papers employing models with a variety of control and information structures. Many of the models take the form of the credit contract as a given--normal[1] a fixed interest rate--although the structure of the model may well have strong implications for the optimal contract specification. In this section we critically assess common distinctions made in the literature and attempt to draw out more relevant distinctions to judge when a form of contract corresponding to indexation to a commodity price might be appropriate.

A distinction often made in the literature is that between "ability-to-pay" or "solvency" models and those models that stress the borrower's "willingness-to-pay." For the ease of discussion we shall refer to extreme versions of these two approaches as benchmark cases. However, we will argue that the distinction does not do justice to the complexities of the information and control structures often employed and thus often directs discussion away from important assumptions that may reflect structural features of the international credit market.

An extreme form of the solvency approach assumes that there are very few, if any, control variables open to lender or to the debtor. Thus, the interest rate, the choice of projects (i.e., the use of funds borrowed) and the amount lent are all treated parametrically. Default occurs in an uncertain world when repayments due are greater than returns to investments--where returns are governed by an exogenous stochastic process. Given that there are few, if any, control variables the information structure in this world is of little interest. An extreme version of the willingness-to-pay model would focus entirely on the decision of the borrower to repudiate debt. Repayment only occurs in those states when the borrower's utility from repayment exceeds that of repudiation. In order to explain why borrowers repay at all the borrower must face a penalty on repudiation.

This distinction in model specification has centered discussion on a number of specific issues. The solvency approach tends to focus attention on rather mechanistic relationships between variables, for example, between the level of export growth and the level of interest rates--without a full discussion concerning what variables are endogenous and which under the control of which agents. The willingness-to-pay approach, on the other hand, tends to focus attention on the nature of the penalty faced in the case of repudiation, as this plays such a crucial role in determining other variables, e.g., the amount borrowed, and the default probabilities.

Although the above issues are clearly important we would advocate that there are many other important issues not highlighted by either of these approaches. In particular, a closer consideration of the information and control structures would aid debate in this area. For this purpose we include Figure 1 which illustrates possible control variables and possible non-observabilities in international lending. As noted above, the extreme form of the solvency approach tends to admit very few control variables and as a consequence the non-observabilities are of little importance. The willingness-to-pay approach, for example, may have the amount lent under the control of the lender with the default penalty exogenous. However, this is only one of many possible assumptions on control and information structures; particular assumptions of this nature may be extremely important in determining the results obtained.

<u>Figure 1</u>: A Typology of Lending Contracts

Debtor	Creditor
Control Variables	
Project (ex ante)	Amount Lent
Project (ex post)	Interest Rate
Decision to Default	Default Penalty
Total Amount Borrowed	
Non Observabilities	
	Project (ex ante)
	Project (ex post)
	Total Amount Lent

For instance, in less extreme versions of the solvency approach lenders may have control over the interest rate while borrowers have control over the types of investments pursued and what might be termed the "investment effort." It is this type of approach that we pursue in the next three sections. Many of the results from this analysis are paralleled by work in the area of corporate finance. For instance, if borrowers obtain utility from the consumption of perquisites (a parallel in the international credit market may be capital flight or other non-productive use of funds), the amount invested productively will be reduced to a point where the marginal utility gained from the consumption of the perquisites is equal to that obtained from the productive investment.

A second general result would be that if inability to pay is defined in relation to some minimum utility level (analogous to limited liability in the corporate finance literature), the borrower is in some sense "insured." Borrowers would use funds obtained for riskier projects than creditors might like. Note these results follow because we have made assumptions on the information and control system in line with common assumptions in the corporate finance literature. See, for example, the seminal paper by Jensen and Meckling (1976).

Eaton et al (1986), in an extremely valuable review article, summarize many of the salient points of the literature applying the willingness-to-pay/ability-to-pay distinction. However, other interesting articles contain both elements. Grossman and van Huyck (1985) develop a reputation model that distinguishes between excusable default (inability) and repudiation (unwillingness). Ozler (1984) also has both elements in a model with stochastic income and a stochastic penalty. Kletzer (1984) presents a model largely in the spirit of the willingness-to-pay approach with the important feature that a single borrower exercises choice over the total amount borrowed which is unobservable to each of many creditors. What makes each of these papers of special interest are the different assumptions on control variables and non-observabilities.

Furthermore, as Hellwig (1986) points out, due to the very nature of the information problem, it is often impossible to tell if the current payments problems are the results of inability or unwillingness to pay on the part of debtors. For example, it seems difficult to assess if a country has indeed the ability to tax its citizens further or reduce subsidies on essential goods. Are food riots a signal of legitimate inability to pay or simply one lobby group (say maize consumers) applying pressure to maintain a subsidy? If the latter, what action on the part of developing countries would signal legitimate inability?

While much of the literature takes the form of the contract (normally a fixed interest rate) as a given, many of the assumptions employed have strong implications for the optimal form of the contract. However, some authors have attempted to explain why debt contracts lack forms of contingency. Eaton et al (1986) include a discussion on whether current reschedulings of developing country' debt can be interpreted as a way of introducing a measure of indexation. Gale and Hellwig (1985) show that a fixed interest rate contract is optimal under a model structure that includes the following characteristics; (i) a risk neutral lender, (ii) a single source of uncertainty--project return--and (iii) lenders cannot observe borrowers' ex post returns except in bankruptcy states. Since the borrower may lie in reporting returns to minimize repayments to the lender, it is essential to structure debt contracts in such a way so that this does not happen.

There are other arguments that might explain the lack of commodity contingency in international lending, including: (i) these risks are unimportant to developing countries, (ii) developing countries can insure these risks anyway via commodity futures or options, (iii) the technology to evaluate such contracts is not known and (iv) the costs to a lender of writing such contracts exceeds the benefits to a debtor and thus the market for this type of insurance fails.

To take these points in turn, firstly, we do not believe that these risks are unimportant to a wide class of developing countries and even in cases of very high commodity concentration there has still been little in the way of contingent lending. Commodity futures and options markets are an alternative interesting route for developing countries to manage commodity-related risks but the use of these instruments is also limited. Traded instruments have rather short maturities and although there are non-traded instruments that extend to, say, four to six years, the volume of commodity exports insured in this fashion is believed to be very low. We do not accept that the lack of an appropriate valuation technology for commodity-indexed debt instruments is a reason for their lack of development. The valuation problem is no more complex than, say, the evaluation of convertible bonds--a commonplace activity for many banks' daily research efforts.

The final point (iv) is the starting question for the subsequent sections of this paper. The costs and benefits of writing commodity-indexed debt contracts is dependent on the information and control structure of the model in which they are investigated. The benefits accrue in part due to enhanced risk sharing. Powell and Gilbert (1988) argue that developing countries' liabilities are linked to total indebtedness and either a fixed interest rate, LIBOR or some combination. Revenue, however, for a large range of middle-income developing countries is linked to commodity production and to world commodity prices. World commodity prices and LIBOR rates are not highly correlated and thus contingency in the form of linking debt servicing to commodity prices would yield benefits to developing countries and in many models would lead to a Pareto improvement. Lessard (1986) argues that "comparative advantage" in risk bearing is not being exploited at present given the system of fixed interest rate general obligation lending that dominates international credit markets. It is the risk-sharing benefits that are investigated in the next section.

In a model in which default is explicit there may be further benefits to contingency linked to changes in the default probabilities. However, in a model where the amount borrowed is controlled by the borrower, it is unclear whether this becomes a benefit or a cost. Indeed, the extra insurance may increase the amount the debtor wishes to borrow and could conceivably increase the likelihood of default.

This is only one of a range of possible adverse incentive effects. If the borrower has a portfolio of projects to invest in and obtains some insurance, then this may alter the borrower's optimal portfolio; in particular, the borrower may choose riskier projects than otherwise. This effect will operate whether the borrower has control over the index used to condition the contract or not. Hence, we tend to disagree with those who argue that there is a fundamental difference between contracts that are indexed on a variable under the borrower's control (e.g., project return) and those that may be exogenous to the decisions of the borrower. However, there are clearly more problems associated with writing contracts on earnings than on exogenous prices; once again the relevant trade-offs must be carefully analyzed in the context of assumptions on information and control variables. It is precisely these types of costs that are investigated in section 5.

Contingency may also alter default risks in a willingness-to-pay framework. In the extreme, consider the case of a commodity price being the single source of uncertainty; then clearly, default-free commodity-contingent contracts could be designed. In general, the effects that such contracts will have on incentives depends on the range of choices open to the borrower and on assumptions concerning the information structure.

There are further costs to introducing indexation. There may be a cost from the point of view of the lender's portfolio. Contracts that leave mean returns to the lender unaltered may impose a cost by altering the overall risk of the portfolio. It has been argued (Gemmill and Gordon, 1988) that adding commodity risks to a bank's portfolio should not add considerable risk as commodity returns have a low beta in a capital asset pricing model (CAPM) approach. 1/ This cost is discussed more formally in section 4.

1/ Statistical work by Priovolos (1988) and others does not always verify the assertion that commodity return betas are lower than one.

III. RISK SHARING BENEFITS OF CONTINGENT CONTRACTS

To illustrate the possible benefits of introducing contingent debt servicing, we consider a sequence of models presented in the next four sections. With both contingency and bankruptcy admitted, models become complex and thus we restrict ourselves initially to simplified structures in order to focus on specific issues. The models presented below are in the spirit of the ability-to-pay approach. The lender sets interest rates and the form of the contract and in the first instance the borrower makes no decisions.

First of all a single borrower uses funds to invest in a single project that pays a stochastic return which is a function of the total amount borrowed. A single lender only lends funds to this project and sets interest rates and the amount of funds lent subject to a participation constraint on the part of the borrower. We investigate if it is in the interests of the lender to introduce a measure of contingency and conclude in this case that the answer is unambiguously positive.

The expected utility of the borrower may be expressed as follows:

$$U(0)F(\hat{\theta}) + \int_{\hat{\theta}}^{\infty} U(\theta B - B(r + \rho(\theta - \theta))dF(\theta) \qquad (1)$$

where

$U(\cdot)$ = borrower's utility function, $U'(\cdot) > 0$, $U''(\cdot) < 0$

B = amount borrowed

$\hat{\theta}$ = is the level of θ at which default on the loan is experienced

r = gross interest rate

ρ = contingency parameter

θ = random return from a project whose right continuous cumulative distribution function is given by $F(\theta)$.

As denoted in equation (1) the expected utility of the borrower has two components: utility in the case of default and that where the project is successful enough to prevent default occurring. The set-up which we envisage has the lender observing θ and also having the first claim on returns in the event of default. We shall assume that default occurs with probability one if the net return to the project is less than zero. While we shall assume that costs of bankruptcy are zero here, this is done without loss of generality if any bankruptcy costs are of a one-off fixed variety. Bankruptcy costs which depend upon the extent of the loss would require separate modeling. The value of θ at which bankruptcy occurs is therefore given by

$$\hat{\theta} \equiv \frac{r - \rho\,\bar{\theta}}{1 - \rho} \qquad (2)$$

In the sequel, we will assume that the borrower has a reservation level of utility denoted by U. This represents the utility level which he cannot be driven below while remaining interested in reaching any kind of contract with the lender.

The expected utility of the lender is written as:

$$EV = \int_0^{\hat{\theta}} V(\theta B)dF(\theta) + \int_{\hat{\theta}}^{\infty} V(Br + B\rho(\theta - \bar{\theta}))dF(\theta) \qquad (3)$$

where

$V(\cdot)$ = utility function of lender, $V'(\cdot) > 0$, $V''(\cdot) < 0$.

The expected utility of the lender also has two parts reflecting states with and without default. Built into the first term in (3) is our assumption that the lender has the first claim on any returns to the project in the event of default. Below we shall be lead to question this assumption in the context of dealing with indebtedness of countries. With the contract that we have in mind in this subsection, the lender sets both interest rates and the degree of indexation, ρ. The latter parameter reflects the extent to which the lender is willing to offer insurance by making interest payments depend upon the outcome of the project. Alternatively, it can be thought of as the lender taking an equity stake in the project in question. Note that this type of contract makes sense only if the return to the project represented by θ is public information. Since we envisage here that this variable will be a commodity price, it will be observable in principle. However, the returns to projects depend only in part on commodity prices, they also depend upon unobservables.

If $\bar{\theta}$ is the unconditional mean of θ, then we are restricting ourselves to indexed contracts which are linear in θ and whose mean interest rate is equal to that without contingency. Of course, in the face of indexation the optimal hedge rate and the probability of repayment will not in general remain constant. If the borrower gains when a lender introduces indexation, this results in the borrower being raised above his reservation utility level and implies that the lender can increase the interest rate to a point where the borrower is once again at the reservation utility level.

The optimal debt contract maximizes (3) subject to (1) exceeding U by choice of B and r. The first order condition for the choice of interest rate is:

$$\int_{\hat{\theta}}^{\infty} V'(\cdot)(r + \rho(\theta - \bar{\theta}))dF(\theta) + \lambda\int_{\hat{\theta}}^{\infty} U'(\cdot)(\theta - r - p(\theta - \bar{\theta}))dF(\theta) = 0 \qquad (5)$$

where λ is the Lagrange multiplier associated with the borrower's reservation utility constraint. Consider first the case where $\rho = 0$, i.e., there is no indexation of the debt contract to project returns. Substituting (4) into (5) yields

$$\frac{\int_{\hat{\theta}}^{\infty} V'(\cdot)\theta dF - r\int_{\hat{\theta}}^{\infty} V'(\cdot)dF}{\int_{\hat{\theta}}^{\infty} V'(\cdot)dF} = \frac{\int_{\hat{\theta}}^{\infty} U'(\cdot)(\theta - r)dF}{\int_{\hat{\theta}}^{\infty} U'(\cdot)dF} \qquad (6)$$

From (6), it can be seen that as compared with a world in which the borrower can borrow unlimitedly at interest rate r, there will be credit rationing. To see this, it suffices to note that if B where chosen to maximize the borrower's expected utility (1) it would have to satisfy

$$\int_{\hat{\theta}}^{\infty} U'(\cdot)(\theta - r)dF = 0. \qquad (7)$$

Referring to the left hand side of (6) it is apparent that at the B which satisfies this constraint, the expression on the left hand side of (7) is positive. If the second order condition for (7) is satisfied, this implies that the B which satisfies (7) lies strictly above that which satisfies (6) and hence there is credit rationing. This result has nothing to do with imperfect information. Credit is rationed because with the possibility of bankruptcy which is harmful to the lender yet beneficial to the borrower, there is a tendency for the borrower to borrow "excessively."

Imagine now that the indexation parameter ρ is increased above zero so that some measure of indexation is introduced. Differentiating the lender's objective function with respect to ρ, substituting in (4) solved for λ and evaluating at $\rho = 0$ yields:

$$\frac{dL}{d\rho} = BV'(\cdot)(1 - F(\hat{\theta})) \left\{\frac{\int_{\hat{\theta}}^{\infty} \theta dF(\theta) - \bar{\theta}(1 - F(\hat{\theta}))}{1 - F(\hat{\theta})} - \frac{\int_{\hat{\theta}}^{\infty} u'(\cdot)(\theta - \bar{\theta})dF(\theta)}{\int_{\hat{\theta}}^{\infty} u'(\cdot)dF(\theta)}\right\} \tag{8}$$

If this expression is positive we can conclude that given a present state of zero contingency, it would be optimal for the lender to introduce at least some small measure of indexation into the contract. The term outside the brackets is always positive. If $\bar{\theta}$ is 0, then the first term in parentheses is also positive. However, it is also positive if $\bar{\theta}$ is the unrestricted mean of θ. To see this, note that $\int_{\hat{\theta}}^{\infty} \theta df(\theta)$ is the mean of θ conditional upon θ being greater than $\hat{\theta}$ and hence exceeds $\bar{\theta}$. Since $1 - F(\hat{\theta})$ is less than one, the first term in parentheses is always positive. The denominator of the second term is positive in view of our assumptions on the borrower's utility function. The numerator of the final term is the covariance between an agent's marginal utility of income and the return to the project--conditional upon the project being successful. It is straightforward to see that this is negative since U' is decreasing in θ (by risk aversion) and hence must covary negatively with θ. Hence, the whole expression (5) is unambiguously positive and there are gains to the lender from the introduction of indexed debt in this framework.

Although we have chosen to represent the problem in the form of maximizing the lender's expected utility subject to a participation constraint on the part of borrowers, the same result as that obtained here could be obtained if the problem were "flipped around," i.e., maximizing the borrower's utility subject to a constraint on lenders' expected utility.

IV. THE COSTS OF CONTINGENCY: THE LENDERS PORTFOLIO

We shall now develop the model by allowing the lender to have a further choice: either investing in the project undertaken by the borrower or in a market portfolio. A fixed sum is to be invested but there is a choice between an investment in the commodity-related project or in the market portfolio. There is now a cost to contingency to the extent that the commodity project return is positively correlated with the return on the market portfolio. Thus, contingency may or may not be preferred. A sufficient condition for net benefits to contingency is that the restricted covariance between the returns to the project and the returns to the market are negative, i.e., there is a negative beta in a restricted capital asset pricing model.

The expression for the expected utility of borrowers remains unchanged from equation (1), however, the expected utility of the lender now becomes:

$$EV = \int_{-\infty}^{\infty}\int_{0}^{\hat{\theta}} V(\theta\beta + (1 - \beta)\omega)f(\theta,\omega)d\theta d\omega$$

$$+ \int_{-\infty}^{\infty}\int_{\hat{\theta}}^{\infty} V(\beta r + \beta\rho(\theta - \bar{\theta}) + (1 - \beta)\omega)f(\theta,\omega)d\theta d\omega \qquad (9)$$

where

ω = random market return

$f(\theta,\omega)$ = joint distribution function for θ and ω

β refers to the proportion of the bank's assets invested in the project where we have normalized the bank's assets at unity.

The condition for the optimal determination of interest rates now becomes:

$$\int_0^\infty \int_0^{\hat{\theta}} V'(\cdot) f(\theta,w)\, d\theta\, dw = \lambda \int_{\hat{\theta}}^\infty u'(\cdot)\, dF(\theta) \tag{10}$$

where λ is the Lagrange multiplier associated with the borrower's participation constraint. We ask once again the question of the last section of whether raising ρ from zero raises the lender's expected utility. Differentiating the lender's objective function with respect to ρ , and using equation (2) we obtain the result that the marginal gain to a small increase in indexation for the lender is:

$$\frac{dL}{d\rho} = B \int_{-\infty}^{\infty} \int_0^{\hat{\theta}} V'(\cdot) f(\theta,\omega)\, d\theta\, d\omega \left[\frac{\int_{-\infty}^{\infty} \int_{\hat{\theta}}^{\infty} V'(\cdot)(\theta - \bar{\theta}) f(\theta,\omega)\, d\theta\, d\omega}{\int_{-\infty}^{\infty} \int_{\hat{\theta}}^{\infty} V'(\cdot) f(\theta,\omega)\, d\theta\, d\omega} - \frac{\int_{\hat{\theta}}^{\infty} u'(\cdot)(\theta - \bar{\theta})\, d\theta\, d\omega}{\int_{\hat{\theta}}^{\infty} u'(\cdot)\, dF(\theta)} \right] \tag{11}$$

Note that the first term inside the square bracket is positive or negative depending on the covariance between the lender's marginal utility of income and ω , conditional upon θ being a non-default state for the borrower (i.e., $\theta > \hat{\theta}$). The first term represents the cost of adding an "asset," whose return is linked to that of the borrower's project, to the lender's portfolio. The sign of this term (and hence whether the new asset is directly desirable) depends upon the sign of this covariance which in turn is determined by how θ and ω covary. If the project return and market return are positively correlated, there will be a cost to taking on this new risk and hence the first term in parentheses in (11) is negative in contrast to what we found in the last subsection. The second term in the parentheses represents the benefit to the borrower of indexation. As we argued above this will accrue to the lender once other elements of the credit contract are adjusted to "push" the borrower back down to his reservation utility constraint. Since this term is positive (guaranteed by repeating the argument that we gave above) the lender gains from indexation covary with this term. Overall, there is now an ambiguity as to whether indexation is desirable to the lender and whether indexation improves the position of the lender--depending on which of the effects that we have described is dominant. A sufficient condition for a small amount of indexation to be desirable for the lender is that the first covariance term in parentheses on the right hand side of (11) is non-positive.

Note that the result obtained here can be phrased in terms of the β's traditionally used in CAPM models. Our model yields the result that if commodities have positive "betas," then it is less likely that lenders will find it desirable to use them as the basis of indexed contracts.

V. THE COSTS OF CONTINGENCY: INCENTIVE EFFECTS

So far, we have not examined any consequences of the fact that indexation of debt may affect the incentives of agents. The main reason why indexation creates potentially serious incentive problems is that borrowers make choices which cannot be controlled by lenders, such as how to use the funds which they borrow. Such actions are beyond the control of the lender both because of unobservability and because they may constitute sovereign actions of borrowers. While loan contracts can in principle be made contingent upon the behavior of borrowers, in practice there are severe limitations upon performance of such contingencies set by the fact that monitoring is costly. Furthermore, political constraints may put controls beyond reach.

Indexation of debt to commodity prices has a two-edged effect upon incentives. First, if it means that there is more indebtedness (this will depend on the lenders' rationing of credit), it enhances the scope of borrowers to invest in risky projects. Second, since indexation offers a kind of insurance, it encourages investment in riskier projects for a given level of indebtedness. Either of these effects is potentially important in assessing the consequences of commodity-indexed debt. A corollary of both is that it is uncertain whether there will be a reduction in the probability of default when there is indexation. While there is an immediate effect with indexation lowering the probability of default, the adverse incentive effects may more than outweigh the gains.

In this section, we shall look in a very stylized fashion at how incentive considerations affect the arguments which we presented earlier. We shall try to place the trade-off which we have just elucidated in sharp relief. In particular, we consider the case when a borrower has the choice between investment in a risky commodity-related project, with uncertain return θ, or in a safe project with certain return s, both expressed as returns per unit level of investment. Our model could alternatively be interpreted as investment in a commodity-related risky project versus investment in the same project but simultaneously obtaining price insurance through the medium of a futures contract. Abstracting from quantity uncertainty, this latter course of action would yield a safe return equivalent in form to the investment in a safe project.

We follow a similar strategy to that employed above to consider the effects of introducing a small measure of indexation. Our aim ultimately is to consider the effect on the lender's utility of introducing a small measure of indexation starting, for simplicity, from a position of no contingency. From the above, we know the potential benefits of such a strategy. In this section, we wish to highlight the costs in terms capturing the adverse incentive effects. We begin by considering the change in the borrower's behavior from the introduction of contingency. In this stylized model, borrowers simply choose the proportion of funds to invest in a risky project under the assumption that all other funds are invested in a safe project. Hence, the borrower's problem may be written:

$$\operatorname{Max}_{\gamma}\ U(0)F(\hat{\theta}) + \int_{\hat{\theta}}^{\infty} U(B(\gamma\theta + (1 - \gamma)s) - B(r + \rho(\theta - \bar{\theta}))\, dF(\theta) \quad (12)$$

where γ is the proportion of funds to be invested in the risky asset and $\bar{\theta}$ is the unconditional mean of the distribution of θ. The other variables are defined as in the previous sections, in particular $\hat{\theta}$ is that value of θ at which default just occurs. It is straightforward to show that:

$$\hat{\theta} = \frac{r - (1 - \gamma)s - \rho\bar{\theta}}{\gamma - \rho} . \tag{13}$$

The borrower's first order condition may be written;

$$\int_{\hat{\theta}}^{\infty} U'(\cdot)\theta dF(\theta) = s - \int_{\hat{\theta}}^{\infty} U'(\cdot)dF(\theta) . \tag{14}$$

This simply states that the marginal utility integrated over all non-default outcomes from investing in the safe project must equal that of investing in the risky project and thus represents the typical portfolio condition to characterize an optimal investment choice.

If γ^* is the optimum γ , then using the first order condition above, we can investigate the way in which γ^* responds to a change in ρ, the contingency parameter. It may be shown using conventional techniques that:

$$\text{sign} \left| \frac{d\gamma^*}{d\rho} \right| = \text{sign} \left| - U'(\cdot)(\hat{\theta} - s)\frac{\partial\hat{\theta}}{\partial\rho} - \int_{\hat{\theta}}^{\infty} U''(\theta - \bar{\theta}) (\theta - s)dF(\theta) \right| \tag{15}$$

We consider each of the terms in the square brackets on the right hand side in turn. The first term represents the effect of the change in the probability of default given a change in the contingency parameter, ρ. It may be shown from (13) that if we assume $\bar{\theta}$ is strictly greater than s (which seems reasonable if risk averse agents are to be willing to invest in the risky project at all) then s is strictly greater than $\hat{\theta}$. Thus, the first term takes the sign of $\partial\hat{\theta}/\partial\rho$, i.e., the sign of the first term depends on whether the probability of default increases or decreases on the introduction of contingency, all else remaining constant. It is straightforward from (13) to show that:

$$\left.\frac{\partial\hat{\theta}}{\partial\rho}\right|\rho = 0 = \frac{(s - \bar{\theta})}{\gamma} + \frac{r - s}{\gamma^2} \qquad (16)$$

As argued above it seems reasonable to assume that $\bar{\theta}$ exceeds s and also it seems reasonable that s must exceed r, otherwise there would be little point in borrowing money at a cost of funds r to obtain a known certain return that did not match the interest cost. Hence, there would appear to be a convincing argument that signs the first term in (15) as negative. Thus, the reduction in the probability of default will tend to lead to less money invested in the risky project. Although this conclusion may appear counter-intuitive at first sight, this effect arises because a contingent debt repayment system means that default is less likely and thus increases the number of states with low utility payoffs for the borrower. This type of effect highlights the importance of the precise assumptions about what occurs on default. Our assumptions follow those from the previous sections, i.e., that there is a minimum utility level and that when default occurs the lender gets the proceeds, such as they are, from both the risky and the safe project and the borrower gets the minimum utility level.

The second term in (15) may be rewritten for convenience as follows;

$$-\int_{\hat{\theta}}^{\infty} U''(\theta - \bar{\theta})(\theta - s)dF(\theta) = -\int_{\hat{\theta}}^{\infty} U''(\theta - \bar{\theta})^2 dF(\theta) -$$

$$(\bar{\theta} - s) \int_{\hat{\theta}}^{\infty} U''(\theta - \bar{\theta})dF(\theta) \qquad (17)$$

It follows that if U''' > 0, then the first term is positive and the second term is negative, hence the sign of the second term in (13) is ambiguous. It is well known that U''' > 0 is necessary for decreasing absolute risk aversion although not sufficient. Note that (17) is more likely to be positive overall the greater is the conditional variance of θ (then the first term is more positive) and the lower the expected return of the risky project (then the less negative is the second term). It is difficult to be more concrete on signing this term without making precise assumptions on the form of the distribution functions or utility functions. However, the sign of this term is related to the risk-return character of the risky project.

The first term is related to the riskiness of the project, while the second relates to the mean return. Note that the contradictory signs would remain even if the utility function were quadratic (i.e., U'' = 0). The introduction of contingency is most likely to raise γ when the riskiness of the project is large and the difference between the mean return of the safe and of the risky project is small.

In summary the sign of (15) remains ambiguous. A decrease in the default probability will tend to decrease the extent of funds devoted to the risky project but through an insurance effect we would expect the proportion of funds in the risky project to increase. We regard the "normal" case as having (15) positive.

We now turn to analyze the effects of the borrower's behavior on the lender. The expected utility of the lender may be expressed as;

$$EV = \int_0^{\hat{\theta}} V((\gamma\theta + (1 - \gamma)s)B)dF(\theta) + (1 - F(\hat{\theta}))V(rB) \qquad (18)$$

Note that the argument of V(.) in the first term is dependent on θ but the argument of V(.) in the second term is independent of θ and thus independent of γ. Thus, the effect of a change of γ on the lender's expected utility is described by the following:

$$\frac{\partial EV}{\partial \gamma} = B \int_0^{\hat{\theta}} V'(.)(\theta - s)dF(\theta) \qquad (19)$$

Given risk aversion on the part of the lender, $V'' < 0$, $\partial EV/\partial \gamma$ is negative. Hence, we can surmise that any increase in the investment in the risky project at the expense of the safe project is a cost to the lender. This will appear as an extra term in equation (5). We can write this symbolically as follows: 1/

$$\frac{\partial EV}{\partial \rho} = [\text{equation } 8] + \frac{\partial EV}{\partial \gamma}\frac{\partial \gamma}{\partial \rho} \qquad (20)$$

The final term thus represents the incentive effect due to the contingent element of the loan contract. If this term is negative, it represents a cost to the lender. As discussed above, there will indeed be a cost given that the introduction of contingency causes borrowers to increase their investment in more risky projects.

We have considered the case of introducing contingency to a fixed interest rate loan contract. Because we have evaluated the effect of introducing ρ around the optimal choices of B and r, we have abstracted from the consequences on the optimal level of the interest rate or the optimal amount lent. Even under these conditions it is a result from the above that

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1/ If B and r are set recognizing the dependence of α on r and B, this formulation is not strictly true. Is is the analogue of equation (8) in this case which actually matters.

the probability of default may increase or decrease as a result of introducing contingency. We can see this from the following:

$$\frac{d\hat{\theta}}{d\rho} = \frac{\partial\hat{\theta}}{\partial\rho} + \frac{\partial\hat{\theta}}{\partial\gamma}\frac{d\gamma}{d\rho} \quad . \tag{21}$$

We know that $\partial\hat{\theta}/\partial\rho$ is negative but the second term will be positive if there is increased investment in the risky project. If we also admit terms to represent optimal changes in the level of the interest rate or the level of borrowing, the situation would become even more complex.

In this section we have investigated the incentive effects on the use of funds borrowed from the introduction of a contingent contract. It should be appreciated that this is only one of many possible incentive effects, the precise nature of which depend on the range of variables under the control of the borrower and which are impossible or costly for the lender to monitor. Clearly, each of these incentive effects might represent a cost that could conceivably exceed the benefits of the introduction of such a contract. The precise nature of the incentive problems may also have strong implications for the design of the optimal contract. In the next section we present a further argument why such contracts might not have appeared in international lending the result of a collective-action problem.

VI. MANY LENDERS AND A SINGLE BORROWER

We turn now to a model in which there are many lenders and one borrower. If a single lender introduces contingency, again there are associated costs and benefits--both related to the amount the single lender invests in the project. If, however, all lenders simultaneously introduce contingency, then the cost to each individual lender is related to the amount that lender has invested while the benefits are related to the total amount invested in the project. Thus, it may be rational for no lender to introduce a contingent contract although the optimal contract form for all lenders may be one with positive contingency. In view of the structure of international indebtedness, this specification seems entirely reasonable, if not necessary, to consider many of the issues which arise. We shall need to introduce some extra notation. Let r_i = interest rate of the ith bank, B_i = amount of funds extended by all other banks. We assume in this model that each individual bank makes a decision on whether or not to introduce indexation in isolation, i.e., assuming that no other banks will follow. Hence, we rule out coordinated provision of indexation. The borrower's expected utility is now as follows:

$$EU = U(0)F(\hat{\theta}) + \int_{\hat{\theta}}^{\infty} U(\theta B - B_{-i}r_{-i} - B_i r_i - B_i \rho(\theta - \bar{\theta}))dF(\theta) \qquad (22)$$

where it will be evident that we are disallowing selective default, i.e., default upon the loans of some subset of creditors only and $-i$ denotes the relevant variables of banks other than i. We continue to use $\hat{\theta}$ to denote the default return to the borrower's project. The expected utility of lender i is expressed as:

$$EV = \int_0^{\hat{\theta}} V(\theta B_i) f(\theta) d\theta + \int_{\hat{\theta}}^{\overline{\theta}} V(B_i r_i + B_i \rho(\theta - \bar{\theta}) f(\theta) d\theta \qquad (23)$$

We envisage a Nash equilibrium between the banks, i.e., each takes the lending strategies of the others as given in choosing its own. Under this assumption, the optimal lending pair (r_i, B_i) will solve first order conditions which are formally identical with those in section III above. We can now reiterate one of the questions posed above, i.e., whether a small amount of indexation pays. We shall do this from the perspective of a particular bank i. Differentiating bank i's Lagrangean with respect to ρ, we obtain

$$\frac{dLi}{d_\rho} = B_i\{\int V_i'(\theta - \bar{\theta}) f(\theta) d\theta + \lambda \int U'(\theta - \bar{\theta}) f(\theta) d\theta\}$$

$$- \lambda_i\{\int U'(\theta - r_{-i}) f(\theta) d\theta \frac{dB_{-i}}{d_\rho} + B_{-i} \int U' f(\theta) d\theta \frac{dr_{-i}}{d_\rho}\} \qquad (24)$$

where we have assumed that equilibrium between creditors other than i is symmetric.

The term in parentheses multiplying B_i is the by now familiar term representing the direct risk-sharing gain from introducing some indexation. H is the latter expression which is the source of the difference when there are many lenders. It represents the response of the other lenders to the indexation of bank i. This is a kind of "externality" created by the non-coordination of bank lending strategies. Using (5) and assuming that equilibrium is symmetric, it is easy to check that the bank i will lose from indexation if

$$\frac{dB_{-i}}{d_\rho} \text{ and } \frac{dr_{-i}}{d_\rho} > 0 \tag{25}$$

i.e., other banks increase their lending and interest rates charged to the borrower. It is possible to calculate exactly what the expressions for the terms in (25) are; however, these are complicated and the basic insight is in any case clear. It seems likely that other banks which also benefit from bank i's indexation will raise their lending to the borrower, which increases the probability of default and leads to a smaller appropriation of the benefits of improved risk sharing by bank i. It is, therefore, less likely that indexation introduced by <u>one</u> bank in a world of many lenders will be privately beneficial.

Interestingly, this result may not hold if there is coordinated provision of indexation, for two reasons. First, the coordination may involve explicit attempts to mitigate any temptations for banks to raise lending, although there may be enforcement problems which usually attend attempts to enforce collusive equilibria. Second, the order of the risk sharing gain to bank i is enhanced directly. The first of the two expressions in (24) in this case becomes

$$B_i \int V_i' \, (\theta - \bar{\theta}) f(\theta) d\theta + \lambda B \int U'(\theta - \bar{\theta}) f(\theta) d\theta \tag{26}$$

The direct gains to the lender accruing from a rise in the borrower's expected utility is of order B rather than B_i, i.e., the whole market rather than the ith bank's share of it. Overall, this rather brief analysis reveals the possible importance of coordination problems and externalities in markets with many lenders and the possibility of unilateral provision of indexation failing to be desirable.

This result feeds upon the fact that the introduction of commodity indexed bonds has a public good element. Lenders other than the one actually introducing indexation may benefit from a contingency since indexation results in gains to all lenders by raising the borrower above his reservation utility constraint. As with the standard public good results, there is some presumption therefore in favor of the view that there will be an insufficient supply of the good (in this case contingent lending) relative to the optimum in which all actors coordinate their actions.

VII. CONCLUSIONS

The aim of this paper has been to investigate certain issues surrounding the introduction of commodity contingent contracts in international lending. We have argued that the precise costs and benefits depend critically on the model structure employed. The benefits are derived mainly from enhanced risk sharing although there may also be a beneficial effect on the probability of default. We have investigated two types of costs; the first stemming from the effects on the lender's portfolio and the second due to incentive effects and thus due to changes in the behavior of the borrower. It remains a puzzle why markets in contingent debt instruments have not developed in great depth. One reason may be that these costs outweigh the benefits so that the market fails to provide such products. We present two reasons for this in a model of a single lender.

We also present a further argument why these markets might fail. If a single bank introduces a measure of contingency, the benefits accrue not just to that bank but to all other lenders. Thus, there is a collective action problem and as is well known from the literature on public goods this will, in general, result in an under-supply of the good in question. If this indeed characterizes the market for international lending, there may be a potent role for a public body in taking the lead in developing a market in these types of contracts. This action may take various forms from simply fostering the coordinated action by other lenders to reach the socially-optimal level of provision to actually writing loan agreements itself in this form.

Our research naturally points to a set of characteristics that would enhance the value of commodity indexation and that would make it more likely that the benefits would exceed the costs. These characteristics include the following: (i) a high commodity concentration on the part of the borrower in a commodity or set of commodities with high price volatility such that this volatility forms a significant element in income variability, (ii) a small, well informed, integrated set of creditors with control over the total amount lent, (iii) good information on the uses of funds borrowed and thus the possibility of meaningful conditionality (iv) no control by the borrower on the index used in the contingent contract and (v) a low "beta" between the returns on the commodity and the returns from the rest of the lender's portfolio.

We believe that there are many country, commodity, lender combinations that might fit these characteristics. Furthermore, the detailed characteristics of the creditor and the borrower may well have implications for the actual contingent structure employed. Although our benchmark case is a commodity-dependent borrower, many of the arguments made would also be relevant for countries subject to other risks; for instance, a country that has borrowed largely in dollars and is thereby exposed to high currency risks. There is a significant amount of work to be done to consider these questions in more detail and to determine in particular cases the optimal provision of indexed contracts.

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