Rules of Thumb for External Borrowing

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Abstract

During the past three years (1983-85), most indebted LDCs undertook major adjustment efforts in order to achieve trade balance surpluses aimed at "repaying their debt". In Latin America (on which we focus in our numerical applications), the results have been, on average, remarkably successful. Cumulated over the last three years, the ten major Latin American countries have achieved a resource balance surplus in the vicinity of $100 billion. In many important cases, we observed both the decline of the debt-to-export ratios and the replenishment of reserves. If the same pattern of trade balance surpluses was to be maintained over the next decade, this could well yield a reduction by half of her debt-to-export ratio. We may now have reached a point when it is worth asking: Is the adjustment period of the last three years only transitory, or do we expect that it will be continued for the next five, maybe ten, years? From the strict point of view of the creditworthiness of the countries, what should be the guiding strategy?

It is the purpose of this paper to examine these questions. We first set up an intertemporal framework of analysis in which we deal with the issue of creditworthiness; we then move to derive some rules of thumb to guide the long-run strategy of an indebted nation.
I. INTRODUCTION AND OVERVIEW OF THE PAPER

During the past three years (1983-85), most indebted LDCs undertook major adjustment efforts in order to achieve trade balance surpluses aimed at "repaying their debt". In Latin America (on which we focus in our numerical applications), the results have been, on average, remarkably successful. Cumulated over the last three years, the ten major Latin American countries have achieved a resource balance surplus in the vicinity of $100 billion. In many important cases, we observed both the decline of the debt-to-export ratios and the replenishment of reserves. For instance, Brazil has achieved a net transfer of resources (a non-interest current account surplus) equal to 37.1 percent and 31.3 percent of her exports in 1984 and 1985, respectively. Her net debt-to-export ratio has declined from a peak of 4.0 in 1982 to 3.1 in 1985. If the same pattern of trade balance surpluses was to be maintained over the next decade, this could well yield a reduction by half of her debt-to-export ratio. We may now have reached a point when it is worth asking: Is the adjustment period of the last three years only transitory, or do we expect that it will be continued for the next five, maybe ten, years? From the strict point of view of the creditworthiness of the countries, what should be the guiding strategy? It is the purpose of this paper to examine these questions. We first set up an intertemporal framework of analysis in which we deal with the issue of creditworthiness; we then move to derive some rules of thumb to guide the long-run strategy of an indebted nation.
The framework of analysis draws upon my previous work with J. Sachs (1985). Following the work originated by Eaton and Gersovitz (1981), we measure creditworthiness as follows: it is the maximum amount of debt that a country would rather service than repudiate. "Maximum" and "service" are two related notions. "Service" means: generate a sequence of trade balance surpluses whose present value (discounted at the world rate of interest) equals the face value of the debt. Servicing the debt therefore involves a refinancing scheme and the "maximum" amount that a country is willing to repay depends upon the future credit lines that it expects. The equilibrium strategy which the lenders could safely follow is therefore the solution of a complex intertemporal problem. The purpose of our proposed "rules of thumb" is to define lending strategies which maximize the welfare of the indebted nation without ever threatening its ability and willingness to repay.

In the framework I analyze, the equilibrium can be characterized by a ceiling on the debt-to-resource ratio (resource to be defined below). I analyze how this credit ceiling fluctuates along with the world interest rate and the rate of growth of the country and show how transitory movements of interest and growth rates should be accommodated. In a stationary environment, I show that the equilibrium is characterized by the simple rule: keep the debt-to-resource ratio constant, (i.e., let the debt grow along with resources).

What definition of "resources" should be used? Both exports and GDP are inadequate. In effect, consider the following "moral hazard" problem: if credit ceilings were related to GDP, then the
country would have an incentive to over-appreciate its currency in order to raise the available amount of lending. Conversely, if exports were taken as the measure of resources, then the country may be led to excessive depreciation of its currency. As one sees, both measures create a wrong incentive. However, these disincentives are of opposite signs, so there is a hope that an intermediate measure may provide a correct answer. In a simple extension of our initial framework, we do show that an "invariant measure of wealth" can be found. More work is clearly needed before the generality of the result can be granted, but the direction of research is certainly promising.

Having discussed these qualitative aspects, we go on analyzing the quantitative issues. We propose three rules of thumb. The simplest, yet maybe the most important for any long-run exercise, is: keep the debt-to-export ratio a constant. We insist on the following important point: there is no need to project a long-run decline of this ratio, nor to force the country into a permanent current account surplus in order to declare the country solvent. All that is needed is that the debt grow, on average, strictly less rapidly than the rate of interest and not faster than the resources of the country. Aside from short-term considerations, one cannot both argue (from the strict point of view of creditworthiness) that the debt-to-export is "too high" and ask to bring it down: if it is "too high", it means that the country would rather default than bring it down. If it can be brought down, it could as well be kept constant.
Now, exports may be too pessimistic a measure of resources when they are not inelastically supplied, so we then move to give an empirical content to an "invariant" measure of wealth. This yields a rule of thumb #2: keep the debt-over-resource ratio constant. I illustrate the definition of "resources" in the case of Brazil. The "invariant measure of wealth" is shown to be equal to 90 percent of exports plus 10 percent of GDP. Since exports are roughly 10 percent of GDP, this yields the following (simple !) rule-of-thumb for Brazil: Let the debt (in dollar terms) grow at half the rate of growth of exports plus half the rate of growth of GDP (in dollar terms).

Both rules 1 and 2 apply to the medium and long term. In the short run, we want to allow the debt-over-resource ratio to fluctuate counter-cyclically to the difference between interest and growth rates. This provides a rule of thumb #3. I show that this rule of thumb #3 yields the following result: stretch out the repayment of the debt by repaying a fixed fraction of the exports to the creditors. Drawing from my earlier work (1985), I obtain that this fixed fraction is roughly: five percent of the debt-to-export ratio. For Brazil, for example, where the debt-to-export ratio is 3, it means repaying 15 percent of her exports to the creditors.

Section II sets up the model; Section III calculates the rules of thumb; the Conclusion takes a look at the last three years in view of our analysis.
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II. A FRAMEWORK OF ANALYSIS

In this section, I would like to set up a simple framework that will provide a rationale for the rules of thumb which are presented in the next section. I assume that growth and world interest rates are exogenous and deterministic. These are severe restrictions but they will keep the model analytically tractable. (In the Appendix, an extension to a stochastic framework is proposed. See our previous work with J. Sachs (1985) for a model in which growth is endogenous.)

To start our analysis we shall first assume that the measurement of wealth and consumption is unambiguously defined by a good which serves as a numeraire and is traded internationally. In the last paragraph of this section, we shall explicitly deal with a model in which exports and GDP provide two alternative measures of a country's wealth and we shall provide, in a simple framework, an "invariant measure of wealth" out of these two indices.

A. A One-Good Model

Let us assume, here, that the resources of the country can be unambiguously measured by a good \( \Omega_t \) which is exogenously endowed to the country according to a law of motion:

\[
\dot{\Omega}_t = n_t \Omega_t
\]  

(1)

The country has access to a world market on which this good is traded. A world financial market whose rates of interest are \( \{r_t\}_{t > 0} \) allows
the country to spread its consumption intertemporally. Let us assume that the country's representative consumer (or, equivalently, its social planner) has the following objective:

$$\text{maximize } \int_0^\infty e^{-\delta t} \log C_t \, dt$$

(2)

in which $C_t$ is consumption (of the numeraire) at time $t$.

B. **Assuming No Threat of Debt Repudiation**

In this paragraph, let us first assume that the country can never repudiate its debt. The only constraint it faces is therefore an intertemporal budget constraint: Intertemporally, all that it consumes must match all that it earns, with an intertemporal discount factor equal to the world rate of interest. Mathematically, this condition is written:

$$\int_0^\infty e^{-R(t)} C_t \, dt = \int_0^\infty e^{-R(t)} G_t \, dt$$

(3)

with $R(t) = \int_t^\infty r_u \, du$.

Maximizing the intertemporal criterion (2) subject to the intertemporal budget set (3) yields the following optimal pattern of consumption:

$$C_t = \delta(Z_t - D_t)$$

(4)
in which \( Z_t - D_t \) is the wealth of the country. It is defined by \( Z_t \), the sum of all future endowments (discounted at the world rate of interest), minus the outstanding debt at time \( t \). \( Z_t \) is therefore defined as:

\[
Z_t = \int_t^\infty (\exp - \int_u^s r_u \, du) \, \Omega_s \, ds
\]

Since \( \Omega_s = n_s \Omega_s \), \( Z_t \) can also be conveniently defined as

\[
Z_t = \frac{1}{\Theta^p_t} \, \Omega_t
\]

in which

\[
\Theta^p_t = \left( \int_t^\infty [\exp - \int_u^s (r_u - n_u) \, du] \, ds \right)^{-1}
\]  \( (5) \)

\( \Theta^p_t \) will play a crucial role in our analysis. Call

\[
\Theta_t = r_t - n_t
\]  \( (6) \)

In the simple case when \( \Theta_t \) is a constant after time \( t \), we see from \( (5) \) that \( \Theta^p_t \) can be interpreted as a "permanent" value of \( \Theta_t \) after time \( t \), in the following sense: \( \Theta^p_t \) is the constant number which would yield the same definition of wealth as the original sequence \( (\Theta_s)_s > t \). From the strict point of view of its wealth, a country is therefore indifferent between \( \Theta^p_t \) all the time, or the original fluctuating sequence \( (\Theta) \).
Let us now describe the law of motion of external debt. Call \{D_t\}_{t \geq 0} the pattern of external borrowing; one has:

\[ D_t = r_t D_t + C_t - \Omega_t \quad (7) \]

\( r_t D_t \) is the autonomous movement of debt due to interest rates; \( C_t - \Omega_t \) is the trade balance deficit (the difference between expenditures and resources).

Let us now define the debt-to-resource ratio as:

\[ h_t = \frac{D_t}{\Omega_t} \quad (8) \]

From (7) and (4) we get the following law of motion of \( h_t \):

\[ h_t = (\theta_t - \delta) h_t + \frac{1}{\delta_t} (\delta - \delta_t^p) \quad (9) \]

At initial time, we have \( h_0 = 0 \) (the country has not borrowed yet). Therefore, we see from (9) that the initial decision to borrow \( (h_0 > 0) \) or to lend \( (h_0 < 0) \) will depend upon the comparison between \( \delta \), the internal discount factor, and \( \delta_t^p \), the initial value of \( \delta_t \). When \( \delta > \delta_t^p \), the country's initial decision is to borrow. (However, when time passes, the decision to borrow may be reversed if subsequent values of \( \delta_t^p \) become large enough to make \( h_t < 0 \) in Equation (9)). In the simple case where \( \theta_t = \theta_t^p = \theta \), for all \( t \), the country will become a net borrower forever if and only if \( \delta > \theta \).
that case, the long-run debt-to-resource ratio is \( h_\infty = \frac{1}{b} \). One sees from (4) that this case yields a nil value of \( C/\Omega \), the long-run consumption over resource ratio. All resources are channelled to the repayment of the debt, and the country is driven to asymptotic starvation.

C. The Repudiation of the Debt

In order to cope with this extreme case where the country would repay its debt out of starvation, the recent literature on external borrowing (originated by Eaton and Gersovitz, 1981) has introduced the possibility that the country might repudiate its debt if the cost of repaying it becomes "too" heavy. The threshold which this "excessive" burden is compared to is an "autarkic" utility level which measures what the country has access to, after it has defaulted on its debt. Here, we shall define the autarky utility threshold as follows. Take a country which defaults at time \( t^* \). We shall assume that its resources are reduced by a factor \( \lambda_{s-t^*} \) at time \( s \), and that it is forced to financial autarky after time \( t^* \). In other words, a country defaulting at time \( t^* \) only receives the endowments \( (1 - \lambda_{s-t^*})\Omega \) at time \( s \), and cannot spread out its intertemporal pattern of consumption. A defaulting country therefore necessarily consumes:

\[
C_{t+s}^a = (1-\lambda_s) \Omega_{s+t^*}, \quad s > 0
\]  

(10)
\{\lambda_s\}_{s>0} \text{ measures the cost of debt repudiation. In our framework of analysis, we need not assume that it is a constant. For instance, we may assume that } \{\lambda_s\}_{s>0} \text{ is a decreasing function of time as in Figure 1.}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{cost_to_default.png}
\caption{The Cost to Default}
\end{figure}

This would represent a situation in which the cost to repudiate the debt is heavier just after default than later on. This would come, say, from trade disruption, the sudden interruption of trade credits, which the country would take some time to accommodate.

Given this option to repudiate the debt, the lenders will limit their exposure on the country so that the repayment never becomes so heavy as to make debt repudiation a superior alternative. Associated with this equilibrium lending strategy, the borrower can reach a utility level which we can write $U(D_t, \Omega_t, F_t)$ in which $F_t = \{r_s, n_s\}_{s>t}$ measures the prospects of future interest rates and future rates of...
growth. The lenders select their lending strategy so that the country never finds it profitable to repudiate its debt. Call \( U_a(\Omega_t, F_t) \) the autarky utility level. From our assumption in Equation (10), \( U_a \) can be written:

\[
U_a(\Omega_t, F_t) = \int_0^\infty e^{-\delta(s-t)} \log \left[ (1 - \lambda_{s-t}) \Omega_s \right] ds \quad (10')
\]

the equilibrium lending strategy is designed so as to generate:

\[
U(D_t, \Omega_t, F_t) > U_a(\Omega_t, F_t), \text{ for all } t > 0. \quad (11)
\]

It is straightforward to check that (11) implies

\[
U(zD_t, z\Omega_t, F_t) > U_a(z\Omega_t, F_t), \text{ for all } z > 0 \quad (11')
\]

(If you can consume \( \{C_s\}_{s \geq t} \) with a wealth \( \{\Omega_s\}_{s \geq t} \) and an initial debt \( D_t \) without ever defaulting, you can as well consume \( \{zC_s\}_{s \geq t} \) with a wealth \( \{z\Omega_s\}_{s \geq t} \) and an initial debt \( zD_t \) without ever choosing to default).

Therefore, by taking \( z = \frac{1}{D_t} \), we see that Equation (11) can be written:

\[
\frac{D_t}{\Omega_t} < h^*(F_t) \quad (12)
\]
In other words, the condition which the lenders have to impose in order to keep a country from defaulting is simply to impose a ceiling to the debt-to-resource ratio, a ceiling which only depends upon $F_t$, the prospects of future growth and interest rates.

How is $h^*(F_t)$ selected, how does it vary with $F_t$? In order to answer these questions, we first show:

**Proposition 1.** On any interval of time during which the constraint (12) is binding, the country repays its creditor a fixed fraction of its resources.

The proof is left to a footnote. What proposition 1 tells is that there exists a fixed scalar $\beta$ such that the country must obtain a trade balance surplus equal to $\beta \Omega_t$ whenever (12) is binding. In other words, whenever the country hits the constraint (12) the country's external debt follows a law of motion:

$$\delta_t = r_t D_t - \beta \Omega_t$$

(13)

In footnote 1 we show that $\beta$ is a scalar which solves:

$$\log (1-\beta) = \delta \int_0^\infty \log (1-\lambda_s) e^{-\delta s} ds.$$ 

In the simple case where $\lambda_s$ is a constant $\lambda$, $\beta$ is exactly equal to $\lambda$. In that case, the interpretation of proposition 1 is straightforward: the lenders restrict their lending so that the country reaches
a trade balance surplus which drives the country's consumption to what it would get by defaulting. However, it is important to notice that this only needs to hold locally: it may well be that later on the constraint stops binding so that the country stops consuming the autarkic pattern of consumption. (We return to this aspect below.)

On any interval on which the constraint (12) is binding, Equation (13) therefore implies that the optimum debt-to-resource ratio is a solution to:

\[ h^*(t) = \theta_t h^*(t) - \beta \]  \hspace{1cm} (14)

From which level of \( h^*(t) \) this differential equation is started depends upon the cyclical pattern of \( \theta^p_t \). Let us first assume that

\[ \theta^p_t < \delta, \text{ for all } t > 0. \]  \hspace{1cm} (15)

In that case, it is easy to show that the constraint (12), once it has started to bind, will bind forever. Assume that (12) starts to bind at \( t_0 \), when the value of the debt is \( D_{t_0} \). Since the country will only repay \( \beta \Omega_{t_0}^{t+s} \) later on, it must be that the discounted value of all \( \beta \Omega_{t_0}^{t+s} \) will exactly match the face value of the debt. In other words, one must have:

\[ \int_{t_0}^{t+s} \beta \Omega_{t_0}^{t+s} \exp(-\int_{t_0}^{u} \theta_t du) \, ds = D_{t_0} \]
This condition can also conveniently be written (given the definition of $\theta^P_t$ in Equation (5)):

$$\beta = \theta^P_t \cdot h_t$$  \hspace{1cm} (16)

Plugging this definition of $\beta$ into the law of motion of the debt-to-resource ratio in Equation (14), one finds the following crucial relation:

$$\frac{\dot{h}^*(t)}{h^*(t)} = \theta_t - \theta^P_t$$  \hspace{1cm} (17)

Equation (17) has the following important content: if the debt ceiling is to bind forever after, the debt-to-resource ratio increases, decreases or stays constant depending upon the current value of $\theta_t (= r_t - n_t)$ to be above, below or equal to its permanent value.

In a stationary environment, when $\theta^P_t = \theta_t$, we see that the debt-to-resource ratio is simply kept to a constant value. Otherwise, when $\theta_t$ oscillates around its permanent value $\theta^P_t$, the debt-to-resource ratio may be allowed to oscillate contra-cyclically.

Equation (17) was obtained by assuming that the constraint on external borrowing was binding forever after it has initially binded. Let us now investigate what happens if this is not so. First, when will it be the case that the constraint on external borrowing will stop binding? This will only be the case when the country willingly undertakes to reduce its debt-to-resource ratio below the ceiling imposed by the
banks. For instance, in the case when the future has become stationary
\((F_t \text{ is a constant})\), this can be shown to happen when \(\theta > \delta\)

that is, when the rates of interest are sufficiently large vis-a-vis the
rate of growth so as to induce the country to lend rather than to bor-
row. (Remember that \(\theta^p_0 < \delta\) was the condition for initial borrowing.)
In any case, if the debt constraint stops binding on some intervals, it
means that the country is willing to repay more than what the minimum
repayment scheme \(\beta_0\) forever would imply. In other words, one has

\[
\int_{t_0}^{\infty} \left( \exp - \int_{t_0}^{u} r_u \, du \right) \beta_0 \delta_0 \, ds < D_t \, \eta_0
\]

\((t_0\) is the time when the constraint on the debt initially binds).
Equivalently, one has:

\[
\beta < \frac{\theta^p_0}{t_0} h(t_0)
\]  
(18)

instead of the equality prevailing in Equation (16).

Therefore, the maximum debt-to-resource ratio is now allowed to
follow a rate of growth which satisfies

\[
\frac{h^*(t)}{\theta^p_t} > \theta_t - \frac{\theta^p_t}{\theta_t}
\]  
(19)
The constraint on the debt is now looser than under the assumption underlying Equation (17): the debt-to-resource ratio may grow more rapidly than suggested by Equation (17).

To summarize this paragraph, we can say

**Proposition 2.** When a country hits a constraint of creditworthiness, the constraint will bind temporarily if the country is expected to willingly reduce its debt-to-resource ratio later on. This will happen if the difference between the rate of interest and the rate of growth is expected to increase substantially. If the constraint is expected to bind forever, the debt-to-resource ratio is set to a maximum value which oscillates contracyclically to the difference between interest rates and growth. If the constraint is expected to bind temporarily, the debt-to-resource ratio can grow more rapidly.

D. An Invariant Measure of Wealth

Thus far, we have assumed that the resources of the country could be unambiguously measured by a numéraire \( R_t \). We now want to provide a framework in which \( R_t \) is not observable, while exports or GDP are.

Assume that the country produces two goods. A home good, and an export good. The consumers only consume the home good, but they need to buy some imports in order to produce it, so that they also need to
export. Call sector 1 the home good sector, sector 2 the export good sector and assume the following technology:

\[
Q_1 = M^{1-a} \Omega^a \\
Q_2 = \Omega_2 \quad \Omega_1 + \Omega_2 < \Omega
\]

\(\Omega\) is the endowment of the country, \(M\) is the imports which enter into the production of good 1 (sector 2 production may be viewed as a net export, \(\text{Max}\{M_2^{1-a} \Omega_2^a - M_2\}\), production). Imports are the numeraire. Call \(p_2\) the term of the trade so that

\[M = p_2 Q_2 - P\]

where \(P\) is the trade balance surplus. Assume that the cost to default is a penalty \(\lambda_s\) imposed on the exports. A country which chooses to default would therefore be entitled to import \(M_{t+s} = p_2 (1 - \lambda_s) Q_{2t+s}\) in exchange for its exports \(Q_{2t+s}\), so that the cost to default is equivalent to a reduction of the terms of trade. The lending strategy of the lender must be such that \(U(D, \Omega, F) > U_a(\Omega, F)\) and as before we can check that \(U(zD, z\Omega, F) > U_n(z\Omega, F)\) holds for all \(z\) whenever \(U(D, \Omega, F) > U_n(\Omega, F)\) holds.

The optimal strategy should therefore be set as previously:

\[
\frac{D_t}{\Omega_t} < h^*_t \tag{20}
\]
The question which we would like now to raise is the following. Assume that \( \Omega_t \) is not observable, and that only GDP and exports are known. Which of these two values should proxy the resources of the country in Equation (20).

Let us first assume that the economy is correctly priced (taking the world price of imports of the numeraire). The price of \( Q_1 \) is

\[
p_1 = \frac{1}{a^\alpha(1-a)^{1-a}} p_2^a
\]

and \( GDP = p_1 Q_1 + p_2 Q_2 - M \) is simply:

\[
GDP = p_2 \bar{\Omega}
\]

while exports are:

\[
X = p_2 (1-a) \bar{\Omega} + aF.
\]

We see that GDP provides an immediate measure of \( \bar{\Omega} \), and \( X \) an indirect one (out of which \( \bar{\Omega} \) can be easily reconstructed: \((1-a)\) is the share of imports bought by sector 1).

Now, the lenders are faced with the following "moral hazard" problem. If they base their lending strategy on GDP, this will induce an incentive for the country to over-value its currency so as to artificially inflate the measure of its income. On the other hand, if the lenders base their calculations on exports, this will induce the borrower to depreciate its currency so as to increase its exports. As one sees, measuring wealth by exports or GDP induces a bias of opposite
signs. This gives some hope that maybe an intermediate measurement will be neutral. In our simple model, such an invariant measure does exist.

Assume that the country adopts a rationing or a subsidiary scheme on imports or on exports, respectively, which we shall measure by the shadow price, $\gamma$, of imports. When $\gamma = 1$ imports are priced at their world level, and we are back to the equilibrium pricing examined above. When $\gamma > 1$, imports are rationed and their shadow price (the black market premium) is above world price. When $\gamma < 1$, on the other hand, exports are subsidized above their world level. Given this measurement of price distortion, GDP and exports become

$$\text{GDP} = \frac{1}{1+\alpha(\gamma-1)} \left[ \gamma p_2 \bar{w} - (1-\alpha)(\gamma-1)p \right]$$

$$x = \frac{1}{1+\alpha(\gamma-1)} \left[ (1-\alpha)p_2 \bar{w} + \alpha p \right]$$

(21)

We see that GDP increases with $\gamma$: by rationing its imports and over-appreciating its currency, the country artificially inflates its GDP. On the other hand, for the opposite reasons, exports are a decreasing function of $\gamma$. Now consider the following measure:

$$w_t = \frac{\alpha}{1+\alpha} \frac{\text{GDP}}{t} + \frac{1}{1+\alpha} \frac{x}{t}$$

(22)

$w_t$ is designed so as to be independent of $\gamma$. By setting their lending strategy to rely upon $w_t$, the lenders keep the country from dis-
torting their price structure. With this definition, \( W_t \) is an "invariant" measure of wealth.

Because of our various "Cobb-Douglas linearities", the definition of \( W_t \) is extremely simple. If we had added some curvatures to the production possibility frontiers, the coefficient \( \alpha \) would be consequently changed. For instance, assume that the resource constraint is written \( \Omega_1 + \phi \left( \frac{-2}{\Omega_1} \right) \Omega_2 < \bar{\Omega} \) in which \( \phi \) is a decreasing function. \( \phi \) measures the cost to shift resources from sector 1 to sector 2. (Note that it keeps the key linearities which we need to set the lending strategy under the form of Equation (12).) Now, the coefficient \( \alpha \) should be shifted to \( \alpha^1 = \frac{\alpha}{\rho} \), in which \( \rho > 1 \) is a measure of the curvature of the production possibility frontier. In the extreme case when \( \rho = \infty \), \( \phi = 1 \) if \( \frac{\Omega_2}{\Omega_1} = * \), 0 otherwise) exports become the sole measure of wealth. Otherwise, the more difficult it will be to shift resources, the more weight will be given exports in the measurement of wealth.

III. RULES OF THUMB

In this section, I would like to derive a number of rules of thumb based upon the previous analysis. I restrict my attention to the issue of creditworthiness. Assuming that a country has reached a point where voluntary lending has stopped, I ask: what rules of thumb should guide its forthcoming borrowing strategy?
A. Rule of Thumb #1: Keep the Debt-to-Export Ratio a Constant

In view of our previous analysis, this is a rule which will yield the exact optimizing strategy under two circumstances:

(a) exports are a good measure of wealth,

(b) the future is stationary.

Exports are a good measure of wealth when all production is bounded by the available amount of imports ($\alpha = 0$ in our analysis of the invariant measure of wealth). This would fit a situation where all exports are inelastically supplied or demand-determined by world trade.

We would like to stress the importance of this rule of thumb for any long-run exercise (for which transitory disturbances are usually ignored): There is no rationality to target a long-run decline of the debt-to-export ratio in name of a further resumption of "voluntary lending". If the country can reduce its debt-to-export ratio without ever choosing to default, then it can certainly keep it constant without defaulting either. It is certainly not optimal to force a country to reduce its borrowing in order to allow it later on to resume its borrowing. As our previous analysis shows, it can happen that a country willingly decides to reduce its debt even after it has hit a constraint on its external debt. However, this will only happen if future large rates of interest make it profitable to repay the debt faster than what the constraint on the debt would impose. From the strict point of view of the creditworthiness of the country, a constant debt-to-export ratio forever is sufficient to repay the debt. 3/
In the Appendix, we qualify this rule of thumb in a framework in which the environment is stochastic. We discuss two examples. One in which the growth trend is deterministic while stochastic transitory deviation may occur. In that case, the consumption stream is stochastic, but the debt-to-export ratio is kept within a deterministic bound. We also discuss another example which may be more relevant for the analysis of the current debt situation. In our model, the world can go into two states of nature. A 1970-like state of nature in which the difference between the rate of interest and the rate of growth is low, and a 1980-like state of nature in which it is large. We assume that the transition matrix between the two states is stationary. Two equilibria might occur. One in which the country borrows and repays its debt in state-'70, but defaults in state-'80. Another one in which the country defaults in neither state. In the former case, the lenders ask for a risk premium which corresponds to the probability to go from state-'70 to state-'80. In the latter case, the lenders pick up a maximum debt-to-export ratio which applies to both the '70s and the '80s.

We show the pattern of the debt-over-export ratio in the Appendix. We see that they all went down but in Chile and Venezuela. Does this pattern fit the cyclical path which we described in Proposition 2? We return to this in rule-of-thumb #3.

B. Rule of Thumb #2: Keep the Debt-to-Resources Ratio Constant

I now suggest to use an index based upon the previous analysis of the measurement of wealth. To do so, I look for a scalar \( \kappa \) such that \( W_t = \{ GDP_t + (1-\kappa)X_t \} \) fails to depend upon the real exchange rate.
The analysis is conducted in the case of Brazil. For this country, exports are shown to depend upon real exchange rate as follows:

\[
\log X(t) = 5.75 + 0.08 \text{ time} + 0.88 \log z(t-1) \\
(4.6) \quad (11.9) \quad (2.6) \\
R^2 = 0.97 \quad DW = 1.3 \quad (t\text{-statistics in parenthesis})
\]

with \( z(t-1) \) the lag value of the real exchange rate, and \( X(t) \) the constant dollar value of exports. We do find a significant responsiveness of export to real depreciation (the volume elasticity is 1.88). Now, on the other hand, the constant dollar value of GDP\(_t\) is shown to depend upon real exchange rate as:

\[
\log GDP_t = 14.6 + 0.06 \text{ time} - 0.78 \log z(t-1) \\
(11.5) \quad (9.9) \quad (-2.3) \\
R^2 = 0.93 \quad DW = 0.97.
\]

As expected, we find a negative relationship between the real exchange rate and the dollar value of GDP. We then look for a value \( \kappa \) which makes \( \log W_t = \log[\kappa GDP_t + (1-\kappa) X(t)] \) independent of \( z(t-1) \). We find that the sign of the relationship between \( \log W_t \) and \( \log z(t-1) \) changes for \( \kappa \) between 0.095 and 0.1. Therefore,

\[
W(t) = 0.1 \text{ GDP}(t) + 0.9 X(t)
\]
yields the "invariant measure of wealth" we were after. Since exports are near ten percent of GDP (except for the last two years when it rose to 13 percent), this says that keeping \( D(t)/W(t) \) constant is like following the rule:

**Rule of thumb #2 for Brazil:** Let the debt grow as half the rate of growth of exports plus half the rate of growth of GDP (both in dollar terms).

In Figure 1, the debt-over-resource ratio so defined is shown for the Brazilian case. We see that it is smoother than both the debt-to-GDP or the debt-to-export ratio (in Figures 2 and 3).

C. **Rule of Thumb #3: Keep the Debt-to-Export Ratio at a "Permanent" Level**

If one believes in the prediction of the analysis above, the debt-to-export ratio (which we take, in this section, to proxy the debt-to-resource) should not be imposed to stay a constant, but should be allowed to fluctuate along with the cyclical pattern of the difference between interest rates and growth. (In order to warrant this analysis, we needed the assumption that the constraint on the debt was binding forever after. Otherwise, keeping the debt-to-export ratio to fluctuate along with \( \theta^* \) would be too tight a measure.) Before we go on analyzing what should be the repayment scheme associated with this rule, it may be interesting to calculate what is the implicit value of \( \theta^* \), which would warrant the observed pattern of borrowing. To do so, we simply use Equation (16) and calculate:

\[
\theta^*_t = \frac{1}{h(t)} \beta^*_t
\]

(in which \( \beta^*_t \) is the observed value of \( \beta \)).
Figure 2:

Brazil: $\frac{\text{Net Debt}}{(\text{GDP} \times 1) + (\text{Exports} \times 0.9)}$ 1970-1985
Figure 2:

Brazil: Net Debt/Exports 1970-1985
Figure 3:

BRAZIL: NET DEBT/GDP 1970-1985
We give the value of $\theta_t^*$ in Table 1.

<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>%</th>
<th>Country</th>
<th>Year</th>
<th>%</th>
</tr>
</thead>
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<td>2.9</td>
<td>Ecuador</td>
<td>1983</td>
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<tr>
<td></td>
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<tr>
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<td>10.3</td>
<td></td>
<td>1985</td>
<td>3.8</td>
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<td>1985</td>
<td>-2.3</td>
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<td>1985</td>
<td>24.7</td>
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</table>
We see that for such large debtor countries as Brazil, Mexico and Venezuela, the value of $r_t^p$, which would warrant the observed pattern of adjustment is extremely high, above 10 percent. (The very important issue of capital flight is left aside in this analysis and would certainly modify the analysis for countries such as Argentina and Venezuela). In other words, a 10 percent expected “permanent” difference between interest and growth rates is what would rationalize the adjustment programs in the past two years.

Now, we need not believe that 10 percent is an accurate (even pessimistic) prediction. If 5 percent is a permanent difference which seems more realistic, then we would get:

$$\frac{\dot{h}_t}{h_t} = (r_t - n_t) - 0.05 \quad (23)$$

This rule can be used as follows. We know, from Proposition 1, that the country should spread out the servicing of its debt by repaying a fixed fraction of its resources every period. This fixed fraction $\beta$ is simply $\beta = \frac{r_t^p}{h_t}$. If we take the view that $r_t^p$ is worth 5 percent, we get the following rule of thumb:

**Rule of thumb #3:** The country should repay a fixed fraction of its exports every period. The value of this fraction is: 5 percent of the debt-to-export ratio.

The results associated with this rule are shown in Table 2. (The rule is consistent with my previous study (1985). If 5 percent seems too optimistic or pessimistic, we can change the last part of the
rule by \[ \text{[...]} \text{ this fraction is: } \theta^p \text{ percent of the debt-to-export ratio} \], with \( \theta^p \) your favorite predictor of the permanent difference between interest and growth rates.

<table>
<thead>
<tr>
<th>Country</th>
<th>%</th>
<th>Country</th>
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In Table 3, we give the observed value of the fraction of exports devoted to servicing the debt. In the case of Brazil, for example, it is twice as large as the rule \#3 would predict.
<table>
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<tr>
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<td>1985</td>
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Important Remark. It must be emphasized that the correct lending strategy must be written as in Equation (23); it is a target debt-to-export ratio which must be set by the lenders, not the repayment scheme itself. Rule #3 only indicates how this can be achieved by the country. If the lending strategy was set by imposing the repayment scheme indicated by rule 3, then the country would go into the following "moral hazard" problem: reduce exports so as to reduce the repayments.

CONCLUSION

We have indicated three rules of thumb, all equivalent to maintaining a constant long-run debt-to-resource ratio. We allowed some cyclical variations of this ratio to take place along with possible fluctuations of the difference between current interest and growth rates. We have shown that many important instances of adjustment efforts could only be rationalized by making the assumption of extremely pessimistic forecast such as a 10 percent difference between interest and growth rates. This could reveal rather extreme risk aversion from the lenders or simply that they did not pursue the optimal strategy of lending which we designed in the text. This latter alternative would tell that lenders have asked for rapid repayment schemes (not "too" rapid however so as to keep the country from defaulting) in order to reduce their exposure on these countries. If this is so, it is not so much the creditworthiness of the countries which triggered the debt
crisis but rather the behavior of the banks themselves. One would then be left with the following set of questions: did the banks "panic" and over-adjust to the 1982 crisis?. Is the 1983-1985 period simply a transitory period aimed at "verifying" the willingness of the countries to undertake an adjustment program (this line of reasoning could follow some of the conclusions of the asymmetric information literature)? Is it related to internal financial constraints faced by the lenders themselves? This is a wide array of questions which we have to leave on the agenda of future research.
FOOTNOTES

1/ Assume that, on some time interval, \([t_0, T]\), the constraint (12) is binding. Call \(\beta_s\) the trade balance surplus-over-resources which is achieved by the country. We have for all \(t \in [t_0, T]\):

\[
\int_t^\infty e^{-\delta(s-t)} \log [(1-\beta_s) Q_s] = \int_t^\infty e^{-\delta(s-t)} \log [(1-\gamma_{s-t}) Q_s] \, ds
\]

\[
\int_0^\infty e^{-\delta u} \log (1-\beta_{s+u}) \, du = \int_0^\infty e^{-\delta u} \log (1-\gamma u) \, du
\]

Differentiating both sides with respect to \(t\) shows that \(\beta\) is a constant which solves

\[
\log (1-\beta) = \delta \int_0^\infty e^{-\delta u} \log (1-\gamma u) \, du
\]

2/ Assume that the constraint ceases to bind at a time when \(\beta_t\) has reached a stationary value \(\theta\). The unconstrained law of motion of the debt is written:

\[
h_t = (\theta - \delta) h_t + (\frac{\delta}{\theta} - 1).
\]

It will imply a faster decline than the maximum credit ceiling imposed by the banks if:
\[(\theta - \delta)h + \left(\frac{\delta}{\theta} - 1\right) < 0,\]

an inequality which can only happen if \(\theta > \delta\) (remember that \(h < \frac{1}{\delta}\), because the debt cannot exceed the wealth).

If the debt-over-export ratio is kept constant, the long-run discounted value of the debt is written:

\[V = \lim_{t \to \infty} \frac{D(t)}{E_t} = D \lim_{t \to \infty} \frac{(1+n)^t}{(1+r)^t}\]

with \(n\) the long-run value of the rate of growth of the exports. Assuming \(n < r\), we see that \(V\) is zero: the debt is eventually repaid (if \(n > r\), the country is infinitely wealthy and there is no solvency problem to care about, see our paper (1985)).

This is the approach that we took in our previous study (1984, 1985). We forecasted some pessimistic growth and interest rates for all countries reporting to the debt reporting system of the World Bank. We then calculated the value of the fixed fraction of export which would be sufficient to repay the debt. As a first approximation our results fit the rule of thumb #3.
REFERENCES

Cohen, D. (1985), "How to Evaluate the Solvency of Indebted Nations" Economic Policy, 1


Continuous Stochastic Disturbances

Let us first consider a model in which the growth of resources follows a Wiener stochastic process:

\[ \frac{d\Omega_t}{\Omega_t} = ndt + cdz \]

the growth trend is deterministic, but stochastic deviations may occur.

Assuming the world rate of interest to be a constant, \( r \), the country maximizes the expected value:

\[ V(D_0, Q_0) = E_0 \int_0^\infty e^{-\delta t} u(C_t) \, dt \]

subject to

\[ dD_t = [RD_t - C_t] \, dt. \]

Consumption is now stochastic. Assume that \( u(t) = \frac{C_t^\gamma}{\gamma} \). A defaulting country has access to a minimum level of welfare (take \( \lambda_s = \lambda \) for all \( s \)):

\[ V_a(\Omega_0) = E_0 \int_0^\infty e^{-\delta t} [\Omega_t(1-\lambda)]^{\alpha - \frac{1}{\alpha}} \, dt. \]

Exactly as in the simpler case examined in the text, it is straightforward to check that a condition such that
$V(D_t, Q_t) > V_a(\Omega_t)$

must also yield a constraint:

$V(zD_t, zD_t) > V_a(z\Omega_t) = \frac{1}{\alpha} z^\alpha V_a(\Omega_t)$

so that a condition

$\frac{D_t}{\Omega_t} < h^*$

is again the equilibrium lending strategy.

**Discontinuous Stochastic Disturbances**

The Brownian movement examined previously was a continuous stochastic process. Let us now move to the other extreme and analyze a situation in which the parameter $\theta = r - n$ only takes two values, in two states of nature $S_1$ and $S_2$.

- $S_1$: $\theta = \underline{\theta}$
- $S_2$: $\theta = \bar{\theta}$ ; $\bar{\theta} > \underline{\theta}$

In state 1, the difference between the rate of interest and the rate of growth is low; in state 2, the difference is large. Let us assume a stationary transition matrix: Call $(p_1, 1-p_1)$ the probability to go on to state 1 and 2 when you are in state 1; $(p_2, 1-p_2)$ the probability to go on to state 2 and 1, respectively, when you are in state 2.
At each time \( t \), the country solves one of the two following problems:

(a) If it is in state 1:

\[
J_1(D_t, \Omega, F_t) = \max_{C_t} \{ u(C_t) + \beta_1 \int J_1 [(1+\tau_{t+1})D_t + C_t - \Omega_t, \Omega_{t+1}, F_{t+1}] \} \\
+ \beta (1-p_1) J_2 [(1+\tau_{t+1})D_t + C_t - \Omega_t, \Omega_{t+1}, F_{t+1}] 
\]

(b) If it is in state 2:

\[
J_2(D_t, \Omega, F_t) = \max_{C_t} \{ u(C_t) + \beta_2 \int J_2 [(1+\tau_{t+1})D_t + C_t - \Omega_t, \Omega_{t+1}, F_{t+1}] \} \\
+ \beta (1-p_2) J_2 [(1+\tau_{t+1})D_t + C_t - \Omega_t, \Omega_{t+1}, F_{t+1}] 
\]

Now, one can see that there are two possible equilibria. One in which a country in state 1 will be imposed:

1. \( J_1(D, \Omega, F) > J_1^a(D, \Omega, F) \)

Another one where the constraint is

2. \( J_2(D, \Omega, F) > J_2^a(D, \Omega, F) \)

If the constraint (1) applies, it means that the country will not default when the economy is (stays) in the good state 1, but that it will
default when it goes to state 2. In that case, the interest rate on the debt includes a risk premium which corresponds to the probability $1 - p_1$ to go from state 1 on to state 2. On the other hand, the country might prefer not to pay this risk premium and instead be constrained by a ceiling corresponding to inequality 2. In that case, the same credit ceiling applies in both states 1 and 2.