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Farm Size, Risk Aversion, and the Adoption of New Technology under Uncertainty

FARM SIZE, RISK AVERSION AND THE ADOPTION OF NEW TECHNOLOGY UNDER UNCERTAINTY*

By GERSHON FEDER

The introduction of high yield cultivation techniques in agriculture during the sixties, and the socio-economic impact of these innovations on LDC's agricultural sector have been a subject of considerable interest to economists. At present, a substantial body of literature exists on various micro and macro economic aspects of this so-called green revolution. However, while many works provide detailed description of the experience of different regions, and propose various arguments to explain observed patterns of behavior (which are not the same in all areas), there seems to be a need for a more rigorous analysis. This will enable clarification of the interrelationships between several observed variables, and will help to define in more precise terms the conditions under which certain arguments are valid.

For example, risk and risk-aversion have been used to explain differences in input use and the relative rate of adoption of modern technologies by farmers of different sizes. But different patterns of behavior are observed in different regions, and thus the impact of risk and risk-aversion needs to be examined in relation to other factors and constraints which may exist in the system in certain areas but not in other areas. Furthermore, the notion of risk-aversion should be defined more finely, as will be shown in the present paper.

Using traditional economic terminology, it would seem that the availability of a new production technology (in addition to the old one)—embodied in the use of hybrid seeds or new crops, fertilizers, pesticides and proper timing of production activities—presents the farmer with a typical portfolio selection problem: The choice of an optimal mixture of risky activities differing in both riskness and expected returns. But unlike the simple portfolio problem, the farmer has some degree of control on both the level of risk and the mean return, through the use of inputs such as fertilizer. There are several interesting questions related to this decision problem: How are factor use and output mix affected by attitudes towards risk? Are there differences between farms of different sizes? Are standard results of the theory of the firm still valid and under what conditions? What are the implications of credit constraints on factor use and output mix? How is income distribution affected?

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The present paper derives answers to these questions in the framework of a formal model of production under uncertainty. The model, which is presented in the next section, can apply to a variety of topics involving production uncertainty. The analysis of model implications in the following sections provides possible explanations to observed behavior in various regions which have experienced the introduction of new crops. Following a discussion of the role of a credit constraint, some income distribution aspects are considered.

The model

The farmer is assumed to own a farm of a given size, say $L$ acres, which can be allocated between two crops. The first is a low yield crop, which utilizes traditional techniques, and, in particular, does not require chemical inputs. Another important characteristic of the traditional crop is the lack of uncertainty regarding yields. The second crop, which will be referred to as the "modern crop," is a high yield variety, or a cash crop, utilizing modern inputs such as fertilizers and improved seeds. On the other hand, it is more vulnerable to weather variations so that there is some degree of uncertainty regarding the yield. Additional (and subjective) uncertainty follows from the fact that the farmer is less familiar with the modern crop. Considering this factor, the modern crop may be viewed as more risky even if in reality it is no more susceptible to extreme weather situations than the traditional crop. Indeed, in some cases the modern crop may be more robust than traditional varieties. The case considered here, however, is that in which total uncertainty (from all sources) of the modern crop is higher. There is ample support in the literature to substantiate this point. For instance, Dalrymple (1978) notes that HYV technology requires a well regulated supply of water and thus "the attainment of the full potential of the HYV's without undue risk requires an assured water supply" (p. 8). Similarly, Schutjer and Van der Veen (1977) conclude in their survey that "the adoption of (new) agricultural technology may require the adopter to accept a greater degree of risk and uncertainty... The new wheat and rice varieties introduced in the mid 1960's probably increased the uncertainty facing farmers by ... introducing genetic homogeneity in the variety planted." (p. 23). The role of higher subjective risk is emphasized by Lipton (1978) who notes that "risk is in the eye of the beholder... for the farmer familiar only with traditional varieties, it is the HYVs' subjective risk that counts." Similar views are expressed by O'Mara (1979, p. 9–2). The mean net return per acre of the modern crop is considerably above the (certain) return from the traditional crop.¹

¹ The traditional crop is not necessarily a local variety of the modern crop. For example, beans (traditional) with Maize (modern).
The characterization above is obviously a major simplification. It can be shown, however, that within the framework of the model to be presented below, results are not affected if the traditional crop uses chemical inputs and is subject to uncertainty, as long as its mean yield response to chemicals is lower than that of the modern crop, and its degree of yield uncertainty is lower than that of the modern crop. There is therefore no loss of generality in adopting the simplified characterization above, while the gain in terms of convenience in mathematical manipulations is substantial.

The chemical input referred to in this paper is fertilizer, while pesticides are related to a different type of uncertainty (infestation levels and pest damage), and will not be treated in the present paper.\(^2\)

Considering the impact of fertilizers on the modern crop (assuming a fixed input of land and other factors), it is well known that mean output increases with the input of fertilizers, although at a decreasing rate. Moreover, empirical evidence suggests that the degree of yield variability (i.e., the degree of uncertainty) increases with the level of fertilizers [Day (1965), Fuller (1965) and more recently Just and Pope (1979)]. Ignoring, for simplicity, all other inputs except for land, one may expect that for a given amount of fertilizer, an increase in the scale of production, (i.e., an increase in the area cultivated) will increase both mean output (with a diminishing rate) and the variability of output. A general formulation of a production function exhibiting such properties can be given, following Just and Pope (1978) by

$$Q = Y(L, X) + \varepsilon \cdot H(L, X)$$

where

- \(Q\) — actual (random) output
- \(Y\) — mean output
- \(H\) — a term related to output variability and assumed to be positive (without loss of generality)
- \(\varepsilon\) — a random variable with mean zero
- \(L\) — land input allocated to the modern crop
- \(X\) — fertilizer input

The following properties characterize production

\[
\begin{align*}
    &a) \quad \frac{\partial Y}{\partial L} > 0; \quad b) \quad \frac{\partial^2 Y}{\partial L^2} < 0; \quad c) \quad \frac{\partial Y}{\partial X} > 0; \quad d) \quad \frac{\partial^2 Y}{\partial X^2} < 0; \quad e) \quad Y(L, 0) > 0
    \\
    &a) \quad \frac{\partial H}{\partial L} > 0; \quad b) \quad \frac{\partial H}{\partial X} > 0; \quad c) \quad H(L, 0) > 0
\end{align*}
\]

\(^2\) On the impact of uncertainty on pesticide use see Feder (1979).

\(^3\) The property \(H(L, 0) > 0\) implies that even when fertilizers are not used, the use of hybrid seeds or the planting of a new variety involves uncertainty, since the farmer is not yet familiar with the capabilities of the new crop.
Adopting the assumption of constant returns to scale in the mean output function [which is supported by the discussions of Sidhu (1974), von Blanckenburg (1972) and Shetty (1969)], and assuming in addition that the risk component per acre \((H/L)\) is a function of the fertilizer/land ratio only, the following production function (in per acre terms) is obtained

\[ q = y(x) + \varepsilon \cdot h(x) \]  

\( q = Q/L, \; y = Y/L, \; h = H/L, \; x = X/L, \) and the following properties apply, on the basis of (2) and (3),

\begin{align}
& a) \quad y' \frac{dy}{dx} > 0; \quad b) \quad y'' < 0; \quad c) \quad h' \frac{dh}{dx} > 0; \quad d) \quad y(0) > 0; \\
& e) \quad h(0) > 0 
\end{align}

The production technology for the traditional crop is simply a fixed net (financial) return of \(R\) dollars per acre allocated to that crop. Two additional prices need to be defined in the system, namely, \(P\), which denotes the price per unit of the modern crop, and \(c\), denoting the cost per unit of fertilizer.

Assuming that the farmer's objective is to maximize the expected utility of income, it is reasonable to characterize the utility function as strictly concave, reflecting risk aversion, i.e.,

\[ U = U(\pi); \quad U' > 0; \quad U'' < 0 \]

The objective function is then given by

\[ \text{Max } EU\{ P \cdot L \cdot [y(x) + \varepsilon \cdot h(x)] + R \cdot (\bar{L} - L) - c \cdot x \cdot L \} \]

subject to

\[ L = \bar{L} \]

where \(E\) is the expectations operator and \(\bar{L}\) is farm size.

Three observations may be made at this point: The first relates to the specification of the production function. One standard specification which is common in the literature assumes a multiplicative random effect [Batra (1974)], i.e., \(q = \theta \cdot f(x)\), where \(\theta\) is random with mean \(\bar{\theta}\). But defining \(\varepsilon = \theta - \bar{\theta}\), \(h(x) = f(x)\) and \(y(x) = \bar{\theta} \cdot f(x)\), one can write \(q = y(x) + \varepsilon \cdot h(x)\). Thus the Just–Pope formulation of the production function includes the special case of a multiplicative random variable. In the latter case, however, the elasticities of \(y(x)\) and \(h(x)\) with respect to \(x\) are identical.

Secondly, it is noted that the objective function above belongs in a general class of problems discussed by Feder (1977). The latter paper developed several general properties of the optimal solution which will be of use in the analysis of the present model.
Finally, the relation between the decision variables \((L \text{ and } x)\) and riskiness merits attention. It is obvious from the specification of the production function that increases in \(L\) and \(x\) (or increases in \(L\) and \(X\)) increase output variability. But it can further be shown that higher values of these variables imply increases in riskiness in the Rothschild–Stiglitz sense. That is, if \(L\) or \(x\) are increased while average income is maintained unchanged (through a compensatory lump-sum subsidy or tax), the expected utility level of a risk-averse farmer will decline, and he would reject such a change. A risk-neutral decision maker would be indifferent regarding this change. This point is demonstrated in detail in Appendix A.

Maximization of the objective function above with respect to \(L\) and \(x\) requires the following first order conditions (excluding corner solutions):

\[
\frac{\partial EU}{\partial L} = E\{U' \cdot [P(y + \varepsilon \cdot h) - R - c \cdot x]\} = 0 \tag{7}
\]

Denote 
\[
P(y + \varepsilon \cdot h) - R - c \cdot x = A; \quad P(y' + \varepsilon \cdot h') - c = G.\]

Then, by Feder (1977, Lemma 4)\(^4\)

\[
G = A \cdot (h'/h) \tag{9}
\]

The Hessian matrix (denoted by \(M\)) of equations (7), (8) can be written as

\[
M = \begin{bmatrix}
E(U'' \cdot A^2) & L \cdot (h'/h) \cdot E(U'' \cdot A^2) \\
L \cdot (h'/h) \cdot E(U'' \cdot A^2) & L^2 \cdot (h'/h)^2 \cdot E(U'' \cdot A^2) + P \cdot E(U' \cdot (y'' + \varepsilon \cdot h''))
\end{bmatrix} \tag{10}
\]

Second order conditions for a maximum require that the determinant of \(M(|M|)\) be positive, i.e.,

\[
|M| = L \cdot P \cdot E(U'' \cdot A^2) \cdot [y'' \cdot EU' + h'' \cdot E(U' \cdot \varepsilon)] > 0 \tag{11a}
\]

By the concavity of \(U\), \(E(U'' \cdot A^2) < 0\). Thus (11a) will hold if

\[
y'' < -h'' \cdot [E(U' \cdot \varepsilon)/EU'] \tag{11b}
\]

By Feder (1977, Lemma 1) \([E(U' \cdot \varepsilon)/EU'] < 0\). Condition (11b) can thus possibly be maintained with both concave or convex \(h\). Obviously, \(h'' \geq 0\) is sufficient (but not necessary) for second order conditions to hold. Essentially, (11b) implies a requirement on the relative rates of reduction in the marginal contribution of inputs to mean yield and variability, if a maximum is to be reached. In loose terms, a maximum solution can be obtained if the marginal mean productivity decreases faster than the marginal contribution.

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\(^4\) Essentially, the result is obtained by equating (7) and (8) and rearranging.
to the risk component \((h)\). It will be assumed throughout the paper that condition \((11b)\) holds.

**The optimal input of fertilizer**

We turn now to investigate the implications of equation \((9)\). Substituting for \(A\) and \(G\), equation \((9)\) yields

\[
P \cdot y' \cdot h + (R + c \cdot x - P \cdot y) \cdot h' - c \cdot h = 0
\]

Equation \((12)\) holds only when \(x\) and \(L\) are at their optimal values (say, \(x^*\) and \(L^*\)). It is noted, however, that only \(x^*\) enters in \((12)\), which thus suffices to characterize the optimal solution for the rate of fertilization per acre. To see that equation \((12)\) defines a unique solution for \(x^*\), denote the left hand side of \((12)\) as \(\psi(x)\) and differentiate

\[
\frac{\partial \psi}{\partial x} = P \cdot h \cdot y'' + (R + c \cdot x - P \cdot y) \cdot h''
\]

From equation \((7)\) one can obtain \(P \cdot h \cdot E(U' \cdot e)/EU' = R + c \cdot x - P \cdot y\), and therefore \(\psi_e = P \cdot h \cdot [y'' + h'' \cdot E(U' \cdot e)/EU'] < 0\), the sign established by \((11b)\). Since \(\psi(x)\) is monotonically decreasing, an interior optimal solution for \(x\) is unique.

From this result several important conclusions can be derived:

**Assertion 1:** The optimal level of fertilizer per acre is independent of the degree of risk aversion.

**Assertion 2:** The optimal level of fertilizer per acre is independent of the degree of variability of \(e\).

**Assertion 3:** The optimal level of fertilizer per acre is independent of farm size.

The proofs for these assertions follow simply from the fact that parameters of the utility function, distribution of \(e\) or farm size do not appear in equation \((12)\) which defines \(x^*\).

As will become apparent, these results imply a two-stage decision approach: The farmer decides first how much fertilizer should be applied per acre on the basis of technical (the production function) and price considerations. The marginal optimization condition related to this decision is not affected by risk since the ratio of the marginal risk effects of fertilizers and land is independent of risk. The second stage decision relates to the scale of operations (amount of land in risky production), and in that stage factors pertinent to risk (such as farm size and risk attitudes) have an impact.

It should be noted that a different behavioural model can yield different results. For instance, it can be shown that a farmer operating with a minimax
principle will use less fertilizers per acre than a risk neutral farmer.\textsuperscript{5} The results presented in the text are consistent with an expected utility model only.

The empirical evidence which is of relevance to Assertions (1)-(3) is not conclusive. Barker et al. (1971) and Roumasset (1976 p. 92) report that in the Philippines they found no significant relation between the use of chemical inputs and farm size. Farm size is usually believed to be negatively related to some notion of risk aversion. Thus, both assertions (1) and (3) seem to be validated by these findings\textsuperscript{6}. However, in other areas it was observed that larger farmers applied more fertilizer per acre than smaller farmers [(Cutie (1976), Rao (1975, p. 45)]. One might have been tempted to rationalize the latter case by the hypothesis that smaller farmers are more risk averse, and, since fertilizers increase risk, these farmers would tend to use less of that input per acre (Lipton 1978, p. 323). As is evident from assertions (1)-(3), this argument by itself is not a valid explanation, and different lines of argument (such as credit constraints) should be used.\textsuperscript{6}

Equation (12) can be used to derive the relation between the optimal level of fertilizer per acre and other parameters of the model. Two parameters are of particular interest, since they have been at the center of policy measures designed to promote the adoption of modern crops. These are the price of the modern crop, and the cost of fertilizer. In many developing countries, the modern crop is subsidized, while various measures amount to either direct or indirect subsidy to purchasers of chemical inputs.

Differentiation of equation (12) with respect to \( c \), the cost of fertilizers, yields

\[
\frac{dx^*}{dc} = \frac{(h - x^* \cdot h')}{\psi_x}
\] (14)

Noting that \( h - x^* \cdot h' = \partial H/\partial L > 0 \) (by 3a), it follows that \( \frac{dx^*}{dc} < 0 \), namely, the fertilizer/land ratio is negatively related to the cost of fertilizers, as expected.

The other parameter of interest is \( P \), the price of the modern crop. Differentiation of (12) yields

\[
\frac{dx^*}{dP} = \frac{(y \cdot h' - x^* \cdot y')}{\psi_x \cdot h} = h \cdot y \cdot \frac{[\mu - \eta]}{(\psi_x \cdot x^*)}
\] (15)

where \( \mu = x^* \cdot h'/h; \ \eta = x^* \cdot y'/y \), the respective elasticities of the risk and mean yield components with respect to fertilizer. Recall that in the special case of multiplicative random variable (i.e., the case \( q = \theta f(x) \) discussed earlier) the two elasticities are equal (\( \mu = \eta \)). Thus, in the latter special case

5 This point should be credited to the referee.

6 In extending the results to a more general case of production under uncertainty, it is noted that even if \( x \) were a risk reducing input (such as pesticides) it would still hold that risk aversion, the degree of risk and farm size have no effect on the amount used per acre.
the per acre rate of fertilization is independent of the price of the modern crop. The general case, however, is

**Assertion 4:** The optimal level of fertilizer per acre increases (decreases) with higher prices of the modern crop if the elasticity of the risk response to fertilizer is lower (higher) than the elasticity of the mean yield response to fertilizer.

As the empirical results of Just and Pope (1979) indicate, it seems that "the elasticity for variability is much lower than the elasticity for the mean of yield" (p. 19). It follows then that in practice a higher price for the modern crop will induce a more intensive use of fertilizer per acre. Theoretically, however, if the risk effect of fertilizer dominates its yield increasing effect (i.e., $\mu > \eta$), a higher price will allow the farmer increased profits with a lower per acre fertilizer input, thus reducing the level of risk (ceteris paribus).

**The optimal allocation of land**

The earlier discussion established the point that risk considerations have no effect on the decision regarding optimal input of fertilizer per acre, even though that amount affects the level of risk confronted by the farmer. It must be concluded, therefore, that the impact of risk (and risk aversion) is reflected in the other control variable, namely, the area allocated to the cultivation of the modern crop.

The impact of risk-aversion, for instance, can be studied by assuming a specific form of the utility function. Consider the constant relative risk aversion specification

$$U(\pi) = \sigma^{1-\alpha}/(1-\alpha) ; 0 < \alpha < 1$$

(16)

where $\alpha$ is the Arrow–Pratt measure of relative risk aversion. The larger is $\alpha$, the more risk averse is the farmer. Calculating $U'$ from (16), substituting in (7) and differentiating with respect to $\alpha$, one obtains (noting that, by assertion 1, $x^*$ is unchanged)

$$dL*/d\alpha = E(U' \cdot A \cdot \ln U) / \left[ (1-\alpha) \cdot E(U'' \cdot A^2) \right]$$

(17)

The denominator of (17) is obviously negative. The numerator can be shown to be positive, and therefore $dL*/d\alpha < 0$. A similar result can be

7 The concepts of relative and absolute risk aversion will be used extensively in the analysis. The reader is referred to the seminal articles by Pratt (1964) and Arrow (1971) for a discussion of these measures. In a nutshell, absolute risk aversion (given by $-U''/U'$) measures the insistence of a risk averse individual for more-than-fair odds when faced with a bet whereby he can win or lose a given sum of money. Relative risk aversion (given by $-U''/U'$) measures the same insistence when the bet is such that a given proportion of wealth or income can be won or lost.

8 The sign of the numerator is established as follows: Define $e^*$ such that $A(e^*) = 0$. With given optimal values $(x^*, L^*)$ it holds $U' \cdot A(e) > 0$ for all $e > e^*$ while $(e < e^*) \Rightarrow [U' \cdot A(e) < 0]$. Also, $(e > e^*) \Rightarrow [\ln U(e) > \ln U(e^*)]$ and $(e < e^*) \Rightarrow [\ln U(e) < \ln U(e^*)]$. It follows, then, that $U' \cdot A(e) \cdot \ln U(e) > U' \cdot A(e^*) \cdot \ln U(e^*)$ for all $e \neq e^*$. Taking expectations of both sides of the latter inequality and noting that $U(e^*)$ is constant and $E(U' \cdot A) = 0$ (by equation (7)), it is concluded that $E(U' \cdot A \cdot \ln U) > 0$. 


obtained for a constant absolute risk aversion utility function of the form 
\[ U(\pi) = 1 - e^{-\beta \pi} \] (where \( \beta \) is a parameter measuring absolute risk aversion).
It was thus demonstrated that

**Assertion 5:** The optimal allocation of land for the modern crop declines with higher degrees of risk aversion.

Increased variability of the random factor \( \varepsilon \) (represented by a mean preserving spread of its distribution) will cause a reduction of \( L^* \) (and therefore also a reduction of expected output).\(^9\) It follows that if smaller farmers face higher levels of uncertainty (because of limited access to sources of information or because of inability to secure risk reducing infrastructure such as irrigation) they will plant less of the modern crop, ceteris paribus.

**Assertion 6:** Increased variability of the random variable \( \varepsilon \) induces a lower allocation of land for the modern crop and a lower total expected output.

The results in Assertions (5) and (6) are intuitive once it was established that the yield per acre of the two crops is unaffected by changes in uncertainty or risk-aversion. The farmer faces a simple portfolio problem with two prospects, one of which is risky, and a fixed size of funds (land). Naturally, the higher is risk-aversion or the degree of risk, the smaller will be the amount of the risky prospect acquired by a risk-averse decision maker.

The effect of the holding size \( (L) \) on the allocation of area between the two crops depends on the relation between absolute risk aversion and income. Using the results on Feder (1977, Theorem 3), and given the fact that \( x^* \) is independent of \( \bar{L} \) (Assertion 3 above) one can show that \( L^* \) increases with holding size provided that absolute risk aversion declines with income as argued by Arrow (1971). It is thus expected that land allocation to the modern crop increases with farm size. A variable of interest is then the relative share of modern crop area relative to farm size. Denote the latter by \( \frac{L^*}{\bar{L}} \), and differentiate equation (7) (recalling that \( x^* \) is fixed), obtaining

\[
\frac{dL^*}{d\bar{L}} = \frac{[E(U'' \cdot A \cdot \pi)]}{[-E(U'' \cdot A^2)]} \tag{18}
\]

where \( \pi \) denotes (as before) income. Since the denominator of (18) is positive, the sign is determined by the numerator. The latter can be shown to be positive, zero or negative, depending on whether relative risk aversion is decreasing, constant or increasing with income.\(^{10}\) Arrow (1971, p. 98)

\(^9\) This is verified as follows: By Feder (1977, Theorem 1), a mean preserving spread will cause a reduction of the term \( L^* \cdot h(x^*) \). Since \( x^* \) is not affected by changes in the distribution of \( \varepsilon \) (Assertion 2 above), it must be \( L^* \) which decreases. This result depends on the plausible condition that absolute risk aversion is not increasing with income.

\(^{10}\) See Arrow (1971, pp. 119, 120) for a technique to prove this assertion.
argues, on the basis of both theoretical and empirical grounds, that “It is broadly permissible to assume that relative risk aversion increases with wealth, although theory does not exclude fluctuations.” Accepting this view, we should expect, in general, larger farmers to allocate smaller shares of their land to the modern crop, since the willingness to risk a given proportion of wealth declines at higher levels of wealth. Such behaviour is reported by several studies: Schluter (1971) found an inverse relationship between farm size and share of area planted to modern high yielding varieties (HYV) of several crops in India. Muthia (1971) found that small and medium sized farms in South-India contributed a larger share in total acreage planted to HYV than their share in total cultivated area. Similar evidence is provided by Sharma (1972). However, there is also evidence on areas where the contrary is true, namely, larger farms allocate a larger share of their land to modern crops (e.g., Rao (1975, p. 45)). This can not be attributed, as was shown, to higher absolute risk aversion among smaller farmers, and a different explanation will be provided.

The discussion above is summarized in the following:

Assertion 7: The area allocated to the modern crop increases with farm size if absolute risk aversion is decreasing. The relative share of modern crop area declines if relative risk aversion increases with income.

The optimal behavior pattern established by Assertions (1)-(3) and (5)-(7) indicates a separation between the land allocation and fertilizer intensity decisions: Given the prices in the system and the technology of production, the optimal amount of fertilizer per acre is determined. The optimal number of acres to be allocated to the two crops is then decided, not only on the basis of prices, but also taking in account the degree of risk and the decision makers attitude towards risk (which is usually not independent of farm size).

Since the total output of the modern crop is negatively related to risk and risk-aversion, an immediate policy implication is that better information dissemination regarding the modern crop (through extension agents, radio, etc.), which will reduce the level of subjective uncertainty, will increase the output of that crop. Similar effects will be expected when drainage and irrigation projects are implemented, since these reduce objective uncertainty.

We turn now to discuss the effects of price changes on land allocation. Differentiating equations (7) and (8) with respect to \( c \) (cost of fertilizer) yields

\[
\begin{align*}
\frac{dL^*}{dc} &= x^\star \cdot [EU' + L^\star \cdot E(U'' \cdot A)]/E(U'' \cdot A^2) \\
&\quad - L^\star (h'/h^2) \cdot (EU') \cdot (h - x^\star \cdot h')/[P \cdot EU' \cdot (y'' + \varepsilon \cdot h'')] \tag{19}
\end{align*}
\]

While the sign of (19) cannot be established, the standard result of input
demand theory $dX^*/dc < 0$ can be verified since (by (14) and (19)), one obtains

$$dX^*/dc = h' L^* \cdot (dx^*/dc) + h \cdot (dL^*/dc)$$

$$= h \cdot x^* \cdot [EU' + L^* \cdot E(U'' \cdot A)]/E(U'' \cdot A^2) < 0$$

(20)

That is, the total amount of fertilizers purchased is negatively related to the cost of fertilizers.

The effect of changes in $c$ on total expected output of the modern crop is given (using (14) and (19)) by

$$dY/dc = y(dL^*/dc) + (L^* \cdot y' \cdot (dx^*/dc)$$

$$= [(dX^*/dc) \cdot y/h] + [(dx^*/dc) \cdot (L^* \cdot y/x^*) \cdot (\eta - \mu)]$$

(21)

Given the results of equations (14) and (20), it follows that a sufficient (but not necessary) condition for $dY/dc < 0$ is $\eta > \mu$. It was already argued that empirical evidence suggests $\eta > \mu$, and it would thus seem that for all practical purposes $dY/dc < 0$.

The second parameter of interest is the price of the modern crop, $P$. Differentiating (7) and (8) with respect to $P$ yields

$$dL^*/dP = -(E(U' \cdot q) + L^* \cdot E(U'' \cdot A \cdot q)]/E(U'' \cdot A^2)$$

$$+ (h'/h) \cdot L^* \cdot (\eta - \mu) \cdot (y/x^*) \cdot EU'/[P \cdot E[U' \cdot (y'' + \varepsilon \cdot h'')]$$

$$= -(E(U' \cdot q) + L^* \cdot E(U'' \cdot A \cdot q)]/E(U'' \cdot A^2) - L^* \cdot (dx^*/dP^*) \cdot (h'/h)$$

(22)

where the second step uses equation (15).

Equation (22) implies that in general the sign of $dL^*/dP$ is undetermined. The sign of the first term can be shown to be positive if (but not only if) relative risk aversion is one or less.\textsuperscript{11} It is noted that according to Arrow (1971), relative risk aversion is close to 1, thus this seems to be an acceptable assumption, which is maintained in the discussion below. The second term has the same sign as $-(dx^*/dP)$. Thus, $dx^*/dP \leq 0$ is a sufficient (but not necessary) condition for $dL^*/dP > 0$. Whatever the impact of increases in $P$ on $x^*$ and $L^*$ may be, it is bound to increase total mean output of the modern crop ($Y$), as shown below

$$dY/dP = y \cdot (dL^*/dP) + L^* \cdot y' \cdot (dx^*/dP)$$

$$= -(y \cdot [E(U' \cdot q) + L^* \cdot E(U'' \cdot A \cdot q)]/(EU'' \cdot A^2)$$

$$+ L^* (y/x^*) \cdot (dx^*/dP) \cdot (\eta - \mu)$$

(23)

Since the first term is positive (under the condition that relative risk

\textsuperscript{11} $E(U'q) + L^* \cdot E(U'' \cdot A \cdot q) = E[U' \cdot q[1 + (U''/U') \cdot \pi \cdot (L^* \cdot A)/\pi]]$. It is noted that $\pi = L^* \cdot A + RL$, thus $L^* \cdot A/\pi < 1$. Denoting the measure of relative risk aversion by $\alpha$, the term in brackets is $[1 - \alpha \cdot (L^* \cdot A)/\pi]$ and a sufficient condition for this term to be positive is $\alpha \leq 1$. Given this condition, the expectation of the product $U'q \cdot [1 - \alpha \cdot (L^* \cdot A)/\pi]$ is necessarily positive.
aversion (α) does not exceed 1) and the latter term is non-negative (see equation (15)), it follows dY/dP > 0.

As for the effect of increases in P on the total quantity of fertilizer used (X*), one can show

\[ \frac{dX^*}{dP} = L^* \cdot \left( \frac{dx^*}{dP} + x^* \cdot \frac{dL^*}{dP} \right) = -x^*[E(U' \cdot q) + L^* \cdot E(U''A \cdot q)]/E(U''A^2) + L^* \cdot (1 - \mu) \cdot \frac{dx^*}{dP} \] (24)

Since (η ≥ μ) → (μ < 1) and (μ ≥ 1) → (η < μ), one may conclude (using equation (15)) that sufficient conditions for \( \frac{dX^*}{dP} > 0 \) are either η ≥ μ or μ ≥ 1.

One can also show that \( \frac{d(L^* \cdot h)}{dP} > 0 \), i.e., the overall level of risk the farmer is willing to undertake increases when the price of the modern (risky) crop increases. The increased level of risk is obtained by either increasing x* and/or increasing L*, such that \( L^* \cdot h \) rises.

Using the results obtained in the earlier discussion some explanation may be provided to observed behavior: It is claimed that "...big farmers... generally get a better price than small ones for HYV (high yielding varieties) outputs, and pay less for their extra inputs" (Lipton, 1978, p. 320). In that case, the preceding analysis predicts that smaller farmers will use less fertilizers per acre and will allocate a smaller proportion of their land to the modern crop if \( \eta > \mu \). Since in many areas such is the observed reality, it would seem that differential prices are a factor contributing to low rates of adoption and low intensity of use of chemical inputs by smaller farmers.

The implications of limited credit availability

The purchase of chemical inputs such as fertilizers requires a cash outlay. This is provided from the farmer's own savings and by obtaining credit. Sources of credit vary, and may include monetary institutions (either formal or informal), relatives and friends, rich farmers, etc. To the extent that sufficient credit is available (i.e., the volume of credit is at least equal to the total cost of fertilizers as implied by the preceding analysis), the model presented earlier is valid. The cost of borrowing is included in the parameter for the cost of fertilizer (c) and the impact of higher interest rates is reflected in the comparative static analysis for variations in c. In particular, if smaller farmers pay higher interest rates, the preceding discussion on the impact of differential prices applies.

There are, however, indications that in many developing areas, rural capital markets are not functioning properly [Lipton (1976)], and that at the prevailing institutional interest rates farmers would have liked to obtain more credit and buy more fertilizers. This is obviously the case in areas
where credit is obtained from relatives and friends (usually for a very low interest rate) or from governmental lending agencies which charge a fixed interest, but ration credit. Even commercially oriented lenders may be observed to ration credit at a given interest rate. When such are the circumstances, the model needs to be amended by an effective constraint stating that total cash expenditures cannot exceed farmer's cash availability, the latter being composed of the farmer's own resources plus credit. It is assumed that both own resources and access to credit are proportional to the size of the farm.\(^2\) This is so since one would expect larger farmers to have more savings, and that lenders who are concerned regarding default will accept land as a collateral, thus granting more credit to larger land owners. Denoting the cash available to the farmer from all sources by \(K\), and the factor of proportionality by \(k\), we have

\[ K = kL \]  

(25)

Since it is assumed in the present discussion that the cash constraint is effective, it must also hold

\[ K = c \cdot x \cdot L \]  

(26)

namely, the amount spent on fertilizers equals cash availability. Combining (25) and (26) yields

\[ L = \frac{kL}{(c \cdot x)} \]  

(27)

Equation (27) reflects the fact that, since the total amount of spending is given, the choice of \(x\) determines \(L\). Denoting the interest rate by \(r\), income\(^13\) is now given by

\[ \pi = P \cdot L \cdot [y + \epsilon \cdot h] + R \cdot (\bar{L} - L) - (1 + r) \cdot c \cdot x \cdot L \]  

(28)

The objective function can be written, using equation (27), as\(^14\)

\[
\text{Max } EU\{[P \cdot k \cdot \bar{L}/(c \cdot x)] \cdot (y + \epsilon \cdot h) + R \cdot \bar{L} \cdot [1 - k/(c \cdot x)] - (1 + r) \cdot k \cdot \bar{L}\}
\]

(29)

and the condition for optimum (assuming an internal solution) is

\[ dEU/dx = (k/c \cdot x^2) \cdot \bar{L} \cdot E\{U' \cdot [P(y' + \epsilon \cdot h') \cdot x + R - P \cdot (y + \epsilon \cdot h)]\} = 0 \]  

(30)

\(^{12}\) The implications of deviations from this assumption will be discussed below.

\(^{13}\) Income is defined here as the difference between end of period and beginning of period wealth.

\(^{14}\) It is assumed that the constraint \(L < \bar{L}\) (i.e., \([k/(c \cdot x)] < 1\)) is maintained.
Denote \( P(y' + \epsilon \cdot h') \cdot x + R - P \cdot (y + \epsilon \cdot h) \equiv B \). Second order conditions require (at the point of optimum)

\[
d^2(\text{EU})/dx^2 = [k/c \cdot x^2] \cdot \bar{L}^2 \cdot B(U'' \cdot B^2) + [k\bar{L}/c \cdot x^2] \cdot B[U' \cdot P(y'' + \epsilon \cdot h'')] < 0
\]

(31)

Given assumption (11b), second order conditions are satisfied. Using the optimum condition (30), some of the implications of a credit constraint can be analysed and compared to the results obtained when such a constraint was not binding.

It is observed, first, that while in the earlier analysis (ceteris paribus) increases in the fertilizer/land ratio were risk increasing, this is not the case at present, since the factor multiplying \( \epsilon \) is now \( h(x)/x \) and not \( h(x) \), as before. A differentiation obtains \( d[h(x)/x]/dx = (x \cdot h' - h)/x^2 < 0 \), and thus increases in \( x \) reduce risk (for a given \( \bar{L} \)). In other words, an increase in fertilizer/land ratio (\( x \)) is necessarily related to a reduction the portion of land allocated to the modern crop, since the total expenditure on fertilizers is given by the credit constraint. Thus while \( X \) is constant \( L \) declines and the function \( H(L, X) \) declines, which implies a reduction in risk as the overall risk is given by \( \epsilon \cdot H(L, X) \). This also implies that an increase in \( x \) leads to a decline in expected output \( Y \), since \( X \) remains unchanged while \( L \) declines, thus \( Y(L, X) \) declines.

It is not surprising, then, that the impact of an increase in the level of uncertainty (represented by a mean preserving spread of the distribution of \( \epsilon \)) is as follows: 15

**Assertion 8:** With a binding credit constraint an increase in the degree of uncertainty will:

a) Increase the optimal level of fertilizer per acre.

b) Reduce the optimal allocation of land to the modern crop.

c) Reduce the expected volume of total output of the modern crop.

These results follow simply from the fact that as uncertainty increases the farmer seeks to reduce risk at the margin by changing the level of \( H(L, X) \) through a reduction in \( L \) (land allocated to the risky modern crop) while \( X \) is fixed by the credit constraint. The reduction in \( L \) increases the fertilizer/land ratio but reduces expected output.

Thus, even if a binding credit constraint prevails, a reduction in uncertainty (through extension services and improved irrigation and drainage facilities) will induce higher expected outputs.

15 The assertion can be verified as follows: Part a follows from Feder (1977, Corollary to Theorem 1). Part b utilizes part a and the inverse relation between \( x \) and \( L \) implied by equation (27). Part c follows from the following derivation:

\[
dY = L \cdot y' \cdot dx - (L \cdot y/x) \cdot dx = L \cdot (y' - y/x) \cdot dx,
\]

and since \( y' < y/x \) (by our assumption \( y'' < 0 \)), the sign of \( dY \) is the opposite of the sign of \( dx \) which has been established in part a.
The impact of risk aversion can be studied using a utility function specification as in equation (16) in the first order condition (30) and differentiating with respect to $\alpha$ (the parameter of relative risk aversion):

$$\frac{dx^*/d\alpha} = E(U' \cdot B \cdot \ln \pi)/[-d^2(EU)/dx^2]$$

(32)

While the denominator is positive [see equation (31)], the numerator can be shown to be negative$^{16}$ and thus $dx^*/d\alpha > 0$ and [in view of equation (27)] $dL^*/d\alpha < 0$. This result is expected since an increase in $x$ reduces risk. Using derivation similar to that of Assertion 8, one can also show $dY/d\alpha < 0$. Parallel results are obtained with a utility function which exhibits constant absolute risk aversion. It is thus concluded:

**Assertion 9:** With a binding credit constraint an increase in the degree of risk aversion will:

a) Increase the optimal level of fertilizer per acre.
b) Reduce the optimal allocation of land to the modern crop.
c) Reduce the expected volume of output of the modern crop.

The effect of farm size is of major interest within the present discussion, since size represents here not only income (and thus attitudes towards risk), but also access to credit. Differentiating equation (30) obtains

$$\frac{dx^*/dL} = \frac{(k/c \cdot x^*) \cdot E(U'' \cdot B \cdot \pi)}{[-d^2(EU)/dx^2]}$$

(33)

Since the denominator is positive [equation (31)] the sign of the numerator determines the sign of $dx^*/dL$. However, one can show that $E(U'' \cdot B \cdot \pi)$ is positive if the relative risk aversion is increasing with income.$^{17}$

Thus, the existence of a credit constraint may be one explanation to the fact that in many areas larger farmers are observed to apply more fertilizers per acre than smaller farmers [Cutie (1976)]. This is different from the result obtained earlier (Assertion 3) indicating that in the absence of credit constraints farm size does not affect fertilizer use. The reason for the difference is that with a binding credit constraint the fertilizer/land ratio decision cannot be separated from the land allocation decision. Given the assumption of increasing relative risk aversion, the larger farmers tend to risk a smaller proportion of their income by allocating relatively less land to the risky activity, which implies a higher input of fertilizer per acre. That the

$^{16}$ See the analysis related to equation (17) above, and note that $\partial B/\partial e < 0$ in the present case.

$^{17}$ Assuming $x$ given at its optimal value, define $e^*$ such that $B(e^*) = 0$. Note that $\partial B/\partial e < 0$, and thus $(e > e^*) \Rightarrow B < 0; (e < e^*) \Rightarrow B > 0$. Assuming increasing relative risk aversion, $(-U^\pi/\pi U') > a$ for $e > e^*$, where $a$ is the measure of relative risk aversion and $-U''_\pi U'$ is evaluated at $e$. Similarly, $(e < e^*) \Rightarrow (-U^\pi/\pi U' < a^*)$. Multiplying both sides by $B$ and moving $U'$ to the right hand side of the inequality obtains $-U''_\pi B < a^* \cdot U'. B$. Taking expectations of both sides and noting that $a^*$ is constant while $E(U' \cdot B) = 0$ (by equation (30)), the result is $E(U''_\pi B) > 0$. 

G. FEDER 277
share of land planted to the modern crop \((l^*)\) is negatively related to farm size can be shown by calculating from equation (27).

\[
dl^*/dL = \left[-k/(c \cdot x^2)\right] \cdot (dx^*/dL)
\]  

(34)

The share of the modern crop thus moves in a direction opposite to the intensity of fertilizer use, and in particular, if, relative risk aversion is increasing, the share of the modern crop declines as farm size increases.

The discussion is summarized in the following:

**Assertion 10:** With a binding credit constraint an increase in farm size will increase the fertilizer/land ratio and decrease the share of land planted to the modern crop if relative risk aversion is increasing with income.

The provision of more credit can be shown to induce higher levels of fertilizer per acre as well as increased acreage of the modern crop, given plausible conditions on the attitude towards risk: Differentiation of equation (30) with respect to \(k\) (implying an increase in the supply of credit per acre) yields

\[
dx/dk = E\{U'' \cdot B \cdot [P(y + \varepsilon \cdot h) - R - (1 + r) \cdot c \cdot x]\}/[-c^2 \cdot x^2 \cdot dE^2 U/dx^2]
\]

(35)

The denominator is positive, (by (31)), and the numerator is positive if absolute risk aversion is not increasing and relative risk aversion not decreasing.\(^\text{18}\) Moreover, the elasticity of \(x\) with respect to \(k\) is less than 1 (see Appendix B). Differentiating equation (27) with respect to \(k\) yields

\[
dL/dk = (\bar{L}/cx) - (k\bar{L}/c \cdot x^2)(dx/dk) = [\bar{L}/(c \cdot x)][1 - (dx/dk)(k/x)]
\]

(36)

As argued above the elasticity of \(x\) with respect to \(k\) is less than 1, and thus \(dL/dk > 0\). It immediately follows that increases in \(k\) also increase the total expected output (provided the assumptions regarding attitudes towards risk hold), as asserted below:

**Assertion 11:** With a binding credit constraint, an increase in credit availability will increase the use of fertilizer per acre, total acreage of the modern crop and total expected output, if relative risk aversion is non-decreasing and absolute risk aversion is non-increasing.

**Income distribution effects**

The preceding discussion enables an assessment of income distribution effects related to the introduction of modern crops. Since the situation of all

\(^\text{18}\) Note that \(P \cdot (y + \varepsilon \cdot h) - R - (1 + r) \cdot c \cdot x = (\pi - R \cdot \bar{L})/L\). Therefore, the numerator can be written as \(E(U'' \cdot B \cdot \pi)/L - (R \cdot \bar{L}/L) \cdot E(U'' \cdot B)\). The first term has been shown to be non-negative if relative risk aversion is not decreasing, while \(E(U'' \cdot B)\) is non-positive by Feder (1977, Lemma 2) which requires non increasing absolute risk aversion.
farmers, large or small, is improved, in absolute terms, it would make sense to concentrate on the relative impact. An appropriate index in this context may be the expected income per acre \( E(\pi)/\bar{L} \) and its relation to farm size. If \( E\pi/\bar{L} \) is unaffected by farm size, then income distribution merely reflects land distribution. Obviously, in the framework of the model presented earlier, the situation that prevailed before the introduction of the modern crop was of such a nature, since the income per acre was \( R \), irrespective of farm size. Once the modern crop is introduced, and assuming that credit is not binding, expected income per acre is given by

\[
E\pi/\bar{L} = l^* \cdot (c \cdot x^*) \cdot [P \cdot y - R] / (c \cdot x^*) - (1 + r) + R
\]  

(37)

On the basis of Assertion 3, it follows that all terms on the right hand side of (37), except for \( l^* \), are not affected by farm size. Considering Assertion 7, we may conclude that with constant relative risk aversion, income distribution is unaffected by the introduction of the modern crop (since in that case \( l^* \) is constant). If relative risk aversion increases with income, then expected income per acre declines as farm size increases (since \( dl^*/d\bar{L} < 0 \)), and thus the modern crop improves income distribution.

Similar results are obtained for the case of an effective credit constraint: In that case, expected profit per acre is

\[
E\pi/\bar{L} = k \cdot [(P \cdot y - R) / (c \cdot x^*) - (1 + r)] + R
\]  

(38)

From Assertion 10 it follows that \( E\pi/\bar{L} \) is constant when relative risk aversion is constant, while it declines when relative risk aversion is increasing, since

\[
d(E\pi/\bar{L}) / d\bar{L} = (k/c \cdot x^2)(P \cdot y' \cdot x - P \cdot y + R) \cdot (dx^*/d\bar{L}) < 0
\]  

(39)

where the sign is established using equation (30) and Assertion 10.

Given the assumption of increasing relative risk aversion the conclusion from these derivations is that the introduction of the modern crop improves income distribution even if there is a binding credit constraint. This is not the general experience from the introduction of high yielding varieties, as argued by Lipton (1978, p. 326). It may be implied then, that in fact, in many areas credit is not proportional to farms size, but, rather, it increases more than proportionately with \( \bar{L} \). There are indications that in India this is indeed the case (Rao 1975, p. 138, Parthasarathy and Prasad, 1978, p. 120). Additional explanation for negative income distribution effects of modern crops (in areas where such effects are observed) may be the differential input and output prices faced by small and large farmers. Or, smaller farms may face more uncertainty (if their access to information is more limited), which can be shown to imply lower expected incomes, ceteris paribus.
Concluding remarks

While the present paper deals mainly with the role of risk-aversion and credit constraints in the production decisions of farmers who grow both modern and traditional crops, the results are of relevance in a more general context. The underlying model can apply to a variety of topics involving risky production activities (whether a non-risky alternative activity is available or not). By incorporating a general formulation of a stochastic production function, the importance of inputs' risk vis-a-vis output effects is emphasized and it can be shown that standard results do not necessarily hold.

In the specific case of the modern crop—traditional crop decision model, the analysis clarifies rigorously the effects of risk, risk aversion, farm size and credit constraints on input use, output scale and crop mix decisions. While some of the results confirm intuitive arguments, this is not the case with other results, and deeper understanding of the underlying factors and their interrelations is required. Nonetheless, these results are shown to depend on various behavioral or technical rules which have already been established in empirical studies or which are generally accepted (such as properties of utility functions). The paper thus helps to explain the sometimes conflicting evidence on patterns of production by farms of different sizes, and in some cases refutes or qualifies common beliefs and traditional explanations. One has to recall, however, that not all aspects of the agricultural economic system were considered (such as land tenure, irrigation, labor constraints, etc.). The incorporation of additional elements in the analysis will undoubtedly add to our understanding of the system and may be worthwhile to pursue in detail.

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**APPENDIX A: THE RELATION BETWEEN \( L, x \) AND RISK**

Farmers (stochastic) income is defined as

\[
\pi = P \cdot L \cdot [y(x) + \varepsilon \cdot k(x)] + R \cdot (L - L) - c \cdot x \cdot L
\]

while average income is (in view of the fact that the mean of \( \varepsilon \) is zero)

\[
E(\pi) = P \cdot L \cdot y(x) + R(L - L) - c \cdot x \cdot L
\]

Suppose that a small change in the level of \( x \) is imposed (\( L \) is unchanged), but at the same time a lump-sum cash subsidy (or tax) is being added to (or deducted from) the farmer's income, such that

\[
dE(\pi) = P \cdot L \cdot y' \cdot dx - c \cdot L \cdot dx + dS = 0
\]

where \( dS \) is the subsidy (or tax). This implies that mean income is unchanged, and a risk neutral farmer would thus be indifferent between the original level of \( x \) and the new level. The attitude
of a risk-averse farmer towards this change depends on whether expected utility is increased or decreased. Differentiating \( E(U(\pi)) \) yields

\[
dE(U(\pi)) = E(U' \cdot [P \cdot L \cdot y' \cdot dx + e \cdot dx - c \cdot L \cdot dx + dS])
\]

But by our assumptions mean income is held constant, i.e.,

\[
P \cdot L \cdot y' \cdot dx - c \cdot L \cdot dx + dS = 0,
\]

and thus

\[
dE(U(\pi)) = h' \cdot E(U' \cdot e)
\]

Since \( h' > 0 \) (see equation (Sc)), the direction of change is determined by the sign of \( E(U' \cdot e) \). Now, since \( h(x) \) is positive, and marginal utility declines with income (\( U'' < 0 \)) it must hold that

\[
U'(e) < U'(e = 0) \quad \text{for} \quad e > 0
\]

\[
U'(e) > U'(e = 0) \quad \text{for} \quad e < 0
\]

Multiplying these inequalities by \( e \) yields (except for the point \( e = 0 \))

\[
U'(e) \cdot e < U'(e = 0) \cdot e
\]

Taking expectations of both sides and noting that \( U'(e = 0) \) is non-random obtains

\[
E(U' \cdot e) < U'(e = 0)E(e) = 0
\]

Thus it was established that an increase in \( x \) which is compensated so as to leave mean income unchanged (i.e., a mean-preserving-spread of the distribution of income) reduces expected utility and would be rejected by a risk-averse farmer. This implies that higher levels of \( x \) (ceteris paribus) are risk increasing.

A similar analysis can demonstrate that increases in \( L \) have the same property.

**APPENDIX B: THE ELASTICITY OF FERTILIZER INTENSITY WITH RESPECT TO THE CREDIT CONSTRAINT**

Equation (35) implies

\[
\frac{dx}{dk} = \frac{[\bar{L}/(c \cdot x)] \cdot E[U'' \cdot B \cdot [Pq - R - (1 + r) \cdot c \cdot x]]}{-[L \cdot k/(c \cdot x^2)] \cdot E[U'' \cdot B^2] - PE[U'' \cdot (y'' + e \cdot h'')]}
\]

Therefore

\[
\frac{k \cdot dx}{x \cdot dk} = \frac{E[U'' \cdot B[Pq - R - (1 + r) \cdot c \cdot x]]}{-E(U'' \cdot B^2) - [L \cdot k/(c \cdot x^2)]PE[U'' \cdot (y'' + e \cdot h'')]}
\]

Both terms in the denominator are positive (see (11b)). Note that

\[
P \cdot q - R - (1 + r) \cdot c \cdot x = B + [Pq' - (1 + r) \cdot c] \cdot x,
\]

thus the numerator can be written as

\[-E(U'' \cdot B^2) + x \cdot E[U'' \cdot B \cdot [P \cdot q' - (1 + r) \cdot c] \cdot x] \]

Obviously, \(-E(U'' \cdot B^2) > 0\).

As for the second term, it can be rewritten as

\[
h' \cdot x \cdot EU'' \cdot B \cdot e + x \cdot [P \cdot y'' - (1 + r) \cdot c] \cdot E(U'' \cdot B).
\]

The first item is negative by Feder (1977, Lemma 3). The term \([P \cdot y'' - (1 + r) \cdot c]\) can be shown to be positive by the maximizing expected utility of profit with respect to both \( x \) and \( L \) subject to the constraint \( cxL = kL \). One of the first order conditions then implies \( x[P \cdot y'' - (1 + r) \cdot c] = \)
\[ \lambda \cdot c \cdot x - P \cdot h' \cdot x \cdot E(U' \cdot e') \], where \( \lambda \) is the (positive) shadow price of the constraint. Since \( E(U' \cdot e') < 0 \), the result is \( x \cdot E(U'^* \cdot B \cdot [Pq' - (1 + r) \cdot c]) > G < 0 \)

\[
\frac{k}{x} \frac{dx}{dk} = \frac{-E(U'^* \cdot B^2) + G}{-E(U'^* \cdot B^2) - \left[ \left( L - k(c \cdot x^2) \right) \cdot P \cdot E[U'^* \cdot (y^* + e \cdot h')] \right] < 1 - \frac{G}{E(U'^* \cdot B^2)}
\]

The discussion in the text asserts \( \frac{dx}{dk} > 0 \). But on the other hand \( \{ilf[U'^* \cdot B^2] > 0 \), and thus \( (k/x)(dx/dk) < 1 \).

REFERENCES:


FULLER, W., "Stochastic Fertilizer Production Functions for Continuous Corn," Journal of Farm Economics, 47 (February 1965), 105-119.


LIPTON, M., "Inter-Farm, Inter-Regional and Farm-Non Farm Income Distribution: The Impact of the New Cereal Varieties," World Development 6 (March 1978), 319-337.


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