EQUILIBRIUM AND PRICES IN MULTISECTOR MODELS

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Abstract

Multisector economy-wide models have long been used to analyze policy problems and the characteristics of growth in developing countries. Many of these models have attempted to incorporate "structuralist" features such as rigidities, limited substitution possibilities, imperfect markets, and disequilibrium adjustment mechanisms. Two different modelling methodologies have been widely used: (1) linear and nonlinear programming models, which involve optimization at the economy-wide level, and (2) computable general equilibrium (CGE) models which simulate the workings of a system of interdependent markets. It has long been recognized that, under strong assumptions, there are formal similarities between the market prices determined in a CGE model and the shadow prices generated in certain programming models. However, incorporating structuralist features raises a number of theoretical difficulties in interpreting the shadow prices generated by programming models, and in reconciling the two different modelling approaches. In this paper, we discuss the relationships between the two modelling approaches under a variety of assumptions about the way in which an economy operates and, second, consider approaches to extending the framework to represent more realistic but theoretically more difficult systems. We consider, in particular, issues of macroeconomic equilibrium.
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Equilibrium and Prices in Multisector Models *

1. Introduction

Advances in economics—indeed, in any science—occur through an interplay among different strands or styles of analysis. First, a complex reality must be simplified and "stylized" in order to isolate the most important features relevant for a particular problem. Second, theories must be developed that explain the relationships among the various important factors. Finally, empirical models are needed to quantify the effects under study and to test the validity of both the stylization and the theory as representations of the real world. There is a natural tension among the three strands of analysis, but any real understanding of how economic systems function and evolve must be based on a successful integration. In development economics, both the need for integration and the tension are widely perceived, and the difficulties in achieving a satisfactory integration are also especially great.

In his career, Hollis Chenery is rare in that he has made major contributions in all three strands of analysis. His work on "patterns of development" has been very important in determining the stylized facts of long run growth and structural change. He has had a major impact on the theoretical development and application of multisector models, and he has also

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1/ The current shambles in macroeconomics is a good example of what happens when theory, empirical models, and stylized facts diverge.
been a persistent critic of the naive application of "standard" theory to analyze problems of developing countries. He has argued for a "structuralist" approach to theorizing and model building that emphasizes rigidities, limited substitution possibilities, imperfect markets, and disequilibrium adjustment mechanisms.2/ Multisector models in this "Chenerian" tradition, which include most models applied to developing countries, are characterized by attempts to capture such structuralist features, often by imposing various special inequality constraints in the context of linear or nonlinear programming models.3/

More recently, economywide simulation models have been developed which seek to capture the role of prices and the workings of the market system. These empirical general equilibrium models operate by specifying the behavioral rules for the various economic actors (e.g., producers and consumers) and explicitly solving the resulting highly nonlinear model to yield market-clearing prices and equilibrium values for all variables. While they have been widely used to simulate perfectly competitive markets, there are also many examples of multisector general equilibrium models which embody "structuralist" elements such as behavioral rigidities or constraints on the operation of important markets (especially the factor and foreign exchange markets).4/

Incorporating structuralist features into multisector models has led to a certain amount of tension because empirical practice has often gotten

2/ See, for example, Chenery (1975) and Chenery (1979), Chapter 2.

3/ For some examples, see Chenery (1971, 1979); Blitzer, Clark and Taylor (1975); and Adelman and Thorbecke (1966).

4/ For a survey of development models, see Dervis, de Melo, and Robinson (1982).
ahead of the available theory. There are essentially two sources of problems. First, many of the constraints imposed in both programming and simulation models are based on macroeconomic theory—for example, problems of savings-investment equilibrium or foreign exchange constraints. However, multisector models are fundamentally based on neoclassical, multi-market, general equilibrium theory that is essentially Walrasian, with little room for problems of macroeconomic "imbalance." Any incorporation of such macroeconomic features into empirical models has an ad hoc flavor that arises from the fact that there is as yet no acceptable theoretical reconciliation between the two branches of theory.

For example, there are many macro models in which nominal wages and/or the exchange rate are assumed to be fixed, or to vary to achieve some notion of macro "balance." Capturing the implications of such a specification may require the inclusion of various kinds of assets (including money) and asset markets into the model, which raises serious theoretical problems in integrating real and monetary variables in a general equilibrium model. However, it is often possible to capture macro "stories" in multimarket general equilibrium models in which only relative prices matter. While the resulting models are no longer strictly Walrasian, they nonetheless do not require the explicit consideration of money or other assets.

A second source of tension arises from the fact that, even within the

5/ As Dervis, de Melo and Robinson (1982) put it (p. 6): "Walras rather than Keynes is the patron saint of multisector analysis."

6/ See, for example, Weintraub (1979) and Hahn (1977).

7/ Below, we restrict our attention to such models. However, we will often refer to "nominal" variables and magnitudes. Clearly, all "nominal" flows in these models must be interpreted in terms of some numeraire, but need not involve money explicitly.
Walrasian framework, there are difficulties in interpreting programming models as representing the functioning of market systems. It has long been recognized that there are formal similarities between the market prices determined in a general equilibrium system and the shadow prices that are generated from certain planning models. The equivalence of "market" and "shadow" prices generally depends on very strong theoretical assumptions about the way agents behave and markets work. Since actual economies never satisfy such assumptions, builders of programming models have developed ingenious ways of relaxing the stringent assumptions, while trying to preserve the links between the various price systems. Chenery was deeply involved in seminal work in this area which sought to extend the linear programming framework to include non-linear specifications and more careful treatment of shadow prices.8/ While such optimizing models came close to an explicit consideration of market-clearing prices, unresolved theoretical problems are evident in most actual applications.

This paper has two main objectives. First, we seek to clarify the relationship between programming models, which involve optimization at the system-wide level, and empirical general equilibrium models which involve simulating a system of interdependent markets. Second, we consider approaches to extending empirical general equilibrium models to provide a framework for modeling more realistic but theoretically more complex systems, especially involving problems of macroeconomic equilibrium. We consider two such macro issues: foreign exchange constraints and savings-investment equilibrium. Chenery’s "two-gap" model is perhaps the most famous example which

incorporates problems of both foreign exchange constraints and savings-investment equilibrium, and has long provided a major focus of discussion of these problems in developing countries.\footnote{See Chenery and Strout \cite{1966}.} We also consider more recent models of savings-investment equilibrium in the "structuralist" school.

The paper is organized as follows: In Section 2, we define the basic ingredients of economywide models, described in terms of "optimization" or "market simulation." In Section 3, we show how these two approaches reflect the same economic framework if we restrict our attention to purely competitive market systems. Section 4 extends the theoretical discussion to consider different approaches to specifying and solving empirical models. Section 5 goes beyond the Walrasian competitive framework and considers how multisector market equilibrium models have been extended to incorporate different concepts of macroeconomic equilibrium.

2. Optimization and Market Equilibrium

It is convenient to describe a general equilibrium model in terms of its: (1) actors, (2) behavioral rules, (3) signals, (4) institutional structure, (5) system constraints and (6) maximand (in an optimizing model).

The economic "actors" in the economy are the agents whose behavior we seek to analyze and/or whose welfare matters to policy makers (for example, firms, consumers, the government, and the rest of the world). In terms of data, we must provide the economic accounts of each of these agents.\footnote{For the entire model, these accounts can be summarized in a social accounting matrix (SAM) whose rows and columns give the receipt and expenditure flows for all agents. One advantage of the SAM framework is that it depicts in one table all the nominal flows among the agents in the economy.} These agents operate according to behavioral rules reflecting their basic
motivation. For example, firms are typically assumed to maximize profits and consumers to maximize utility, subject to various constraints. Agents make their decisions based on signals which are variables generated in the economy. For example, in a competitive economy, the only important signals are prices. In other settings, signals such as quantities might be relevant.

The institutional structure of the model economy delineates the "rules of the game" according to which the various agents interact. If one assumes perfect competition, then each agent is a price taker, markets are assumed to work perfectly, and all prices are flexible. One can specify various imperfections or constraints in the institutional structure that will affect the operation of the model; for example, a fixed wage. One might then assume that firms will set their labor demands given the fixed wage and that any excess supply of labor is simply involuntarily unemployed. In this example, the labor market "clears" with demanders satisfied and suppliers off their supply curves.

Embodied in the institutional specification are assumptions about the signals which the actors consider in making decisions. For example, under perfect competition actors need only know prices. Alternatively, if some market is monopolistic, then one must specify that the monopolist makes supply decisions based on information about the demand functions of the demanders, not just the market price. In a model with some fixed prices, agents will also be subject to rationing. The nature of the required signals and the ability (or inability) to decompose the operation of the economy into decentralized activities by separate agents are crucial features of the model.

With the specification of the agents, their motivation, and the institutional constraints under which they interact, a general equilibrium model is still not completely determined. There are still various constraints
that must be satisfied, but that are not taken into account by any individual agent in making his decisions. These are "system constraints" which essentially determine the characteristics of an "equilibrium" in the economy. Indeed, an equilibrium can be defined formally as a set of signals such that the decisions of all agents jointly satisfy the system constraints. For example, in a competitive equilibrium model, the assumption that all markets clear with excess demands of zero is a system constraint that defines the nature of a market equilibrium.

The notion of system constraints is fundamental to an understanding of how a model operates and can include much more than standard models of market equilibrium. For example, in the literature on applied general equilibrium models, the term "model closure" has been used to refer to how an economywide model achieves balance between savings and investment. The term is confusing in that any fully specified general equilibrium model satisfying Walras' Law is closed in the sense that there are no variables left indeterminate. Macro "closure" is really better seen as a system constraint defining macroeconomic equilibrium—an approach that will be developed further below.

The use of system constraints to define an "equilibrium" can be seen as a useful simplification and a substitute for writing out a complete description of dynamic adjustment processes. For example, instead of specifying that market excess demands must equal zero during a period, one could write down, as part of the model, dynamic "disequilibrium" price adjustment rules which describe how prices are determined period by period. Such a specification is theoretically quite difficult to implement and

11/ See, for example, Taylor (1979) and Dervis, de Melo, and Robinson (1982), Chapters 5 and 12.
completely unnecessary if one is willing to accept the market-clearing system
constraints as a reasonable description of the final result of such a process
within the time period described by the model. There are, however, times when
such market-clearing assumptions are not reasonable. In applied general
equilibrium models of developed countries, it is usually assumed that capital
is mobile across sectors and is allocated so as to equate sectoral rental
rates—an equilibrium condition that is consistent with an assumption of
perfect capital markets.12/ In models of developing countries, such an
assumption is rarely if ever reasonable, and so modelers have had to specify
explicitly how the sectoral allocation of investment is determined from period
to period.

Finally, for a programming model, one must specify the form of the
maximand. In planning models, this is typically some measure of utility or,
alternatively, aggregate consumption. It is also possible to specify some
sort of "planner's preference function" which need not be based on welfare
theory. The maximand is, of course, crucial in determining and interpreting
the shadow price system. We will, in this paper, restrict ourselves to
programming problems whose maximands reflect utility functions of consumers.

In terms of the features described above, we will now provide formal
definitions of "market equilibrium" and "programming" models, and then examine
the assumptions under which they have the same solution. We assume that there
is a finite number of commodities and an associated price vector, \( p \). The only
agents are consumers and producers. There are \( m \) consumers (with subscript \( i \))
each of whom has an initial endowment of commodities, \( w_i \). Following standard
practice in the general equilibrium literature, "commodities" include factors

12/ For some examples, see Scarf and Shoven (1983).
of production such as labor and capital as well as consumer goods and intermediate goods. The consumption bundle of each consumer is given by the vector $x_i \in X_i$, where $X_i$ is the set of possible consumption bundles available to consumer $i$. There are $r$ producers (with subscript $j$) each of whom chooses a production point $y_j \in Y_j$, where $Y_j$ is the set of feasible production points for producer $j$ determined by the available technology. The production point $y_j$ is a vector with positive elements denoting outputs and negative elements denoting inputs. We also assume that agents respond to prices (in a way to be defined later) in making decisions, and so we can write $x_i(p)$ and $y_j(p)$ to represent the vectors of demand curves for consumer $i$ and supply curves of producer $j$. Aggregate demand, supply, and endowments are given by the vectors:

$$x(p) = \sum x_i(p), \quad y(p) = \sum y_j(p), \quad \omega = \sum \omega_i.$$  

**Definition 2.1: Market Equilibrium.** The allocation $x_i(p)$ ($i=1, \ldots, m$) and $y_j(p)$ ($j=1, \ldots, n$), supported by the market price vector $p$, constitutes a market equilibrium if:

(a) no commodity is in excess demand,

$$e(p) = x(p) - y(p) - \omega < 0$$

(b) consumers satisfy their budget constraints,

$$p \cdot x_i - \kappa_i \text{ for all } i,$$

where income, $R_i$, is defined in Assumption 3.2 below.

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13/ Note that the condition $x_i \in X_i$ does not include the budget constraint facing consumers which must be considered separately.

14/ These "curves" may be functions or correspondences, depending on the assumptions of the model.
(c) additional system constraints are satisfied,
\[ \phi(p, x_1, \ldots, x_m, y_1, \ldots, y_n, \omega_1, \ldots, \omega_m) < 0 \]
(d) individual consumption and production plans are feasible,
\[ x_i(p) \in X_i \text{ and } y_j(p) \in Y_j \text{ for all } i \text{ and } j. \]

Assuming that each consumer is on his budget constraint (for properly defined \( R_i \)), then the system as a whole must satisfy Walras' Law: \( p.e(p) = 0 \).

The additional system constraints \( \phi(-) < 0 \) will be used to represent deviations from, or extensions to, the standard neoclassical assumptions of a competitive market equilibrium. The price vector \( p \) is included in the list of arguments because such functions are often conceptually defined in nominal terms. For example, macro issues can be treated by imposing as constraints on the model functions that determine variables such as aggregate savings, investment, public income, and/or expenditure. The \( \phi(-) \) can also be used, for example, to represent the balance of trade and/or "second best" constraints such as price distortions.

The second approach defines a programming model used by a planner to determine an optimum allocation or plan. Assume that the planner has a welfare function \( V(x_1, \ldots, x_m, y_1, \ldots, y_n) \) which he seeks to maximize.

**Definition 2.2: A Plan.** The allocation \( x_i \) (\( i=1, \ldots, m \)) and \( y_j \) (\( j=1, \ldots, n \)), constitutes a plan if it is a solution of the following mathematical program:

\[
\begin{align*}
\text{max} \quad & (x_1, \ldots, x_m, y_1, \ldots, y_n) \\
\text{subject to} \quad & \phi(p, \ldots) < 0 \\
& x - y - \omega < 0 \\
& x_i \in X_i \text{ and } y_j \in Y_j \text{ for all } i \text{ and } j.
\end{align*}
\]
Denote by \( \pi \) the shadow prices associated with the excess demand or material balance constraints, \( x - y - \omega < 0 \), which "support" the primal solution. Then, the planning authority can use the shadow prices \( \pi \) generated by the plan to decentralize decisions. With the \( \pi \)'s as signals, agents will respond with demands \( x_i(\pi) \) and supplies \( y_i(\pi) \) which will satisfy the plan and also individual budget constraints. In general, there is no reason to expect the solution of a plan to be a market equilibrium with prices equal to \( \pi \) and there is no guarantee in the plan that individuals will satisfy their budget constraints (evaluated at shadow prices \( \pi \)).

We will consider below the conditions under which the solution of a plan including the shadow prices \( \pi \) constitutes a market equilibrium with \( p = \pi = \tilde{p} \). The \( \Phi(\tilde{p}, \ldots) < 0 \) system constraints have been discussed above. Note that here \( \tilde{p} \) is a vector of prices that are either exogenously given or endogenous. For example, they might represent fixed world prices in a model where \( \Phi(\cdot) \) is the balance of trade. However, it is often desirable to specify such constraints defined with "solution" prices. For example, an aggregate savings constraint should conceptually be defined in nominal terms. Such a specification causes computational problems since we then require that \( \tilde{p} = \pi \), and shadow prices then appear as part of the constraints in the primal problem. Although various modelers have been quite ingenious in their attempts to specify theoretically justifiable additional system constraints with fixed prices, such an approach is rarely satisfactory. In the next section, we will restrict our attention to competitive market equilibria in which the additional \( \Phi(\cdot) < 0 \) constraints do not appear, and then later move on to models that incorporate such constraints.
3. Competitive Equilibrium Models

We first specify how producers and consumers behave and then discuss two approaches to specifying a competitive equilibrium model.

Assumption 3.1: Producer Behavior. Producer \( j \) takes prices \( p \) as given and chooses his production point so as to maximize his profits \( p.y_j \), subject to his technological constraints, \( y_j \in Y_j \). \( Y_j \) represents the production technology available to firm \( j \).

Assumption 3.2: Consumer Behavior. Consumer \( i \)'s preferences can be represented by a utility function \( U_i(x_i) \). He takes prices \( p \) as given and chooses his consumption bundle so as to maximize \( U_i(x_i) \) subject to \( x_i \in X_i \) and to his budget constraint \( p.x_i \leq R_i \). \( R_i \) is his income which consists of the value of his endowment \( p.w_i \) and the profits distributed by firms to him according to the share parameters \( \theta_{ij} \) (where \( \sum_i \theta_{ij} = 1 \)). Thus, \( R_i = p.w_i + \sum_j \theta_{ij}p.y_j \)

Definition 3.1: Competitive Equilibrium. The allocation \( x_i(p) \) and \( y_j(p) \), supported by the price vector \( p \), is a competitive equilibrium if Assumptions 3.1 and 3.2 are satisfied and no commodity is in excess demand:

\[
e(p) = x(p) - y(p) - \omega < 0
\]

Clearly, a competitive equilibrium is a special case of a market equilibrium (and will satisfy Walras' Law). We now define a special case of a plan which yields solutions which are "Pareto Optima."

Definition 3.2: Pareto Optimal Plan. The allocation \( x_i(\pi) \) and \( y_j(\pi) \), supported by the shadow price vector \( \pi \) associated with the constraints

\[
x - y = p
\]

15/ Note that in models with constant returns to scale production functions, profits are zero at a competitive equilibrium. Then \( p.y_j(p) = 0 \) and \( R_i = p.w_i \). The distribution of income is determined by endowments and prices, independently of the \( \theta \) parameters.
\( \omega < 0 \) is a Pareto Optimal Plan if it is a solution of the following mathematical program:\(^{16/}\)

\[
\max \sum \alpha_i u_i(x_i)
\]

with respect to \( x \) and \( y \),

subject to

\[
x - y - \omega < 0
\]

\( x_i \in X_i; \quad y_j \in Y_j. \)

The \( \alpha_i \)'s are called "welfare weights" and are strictly positive.

We are now ready to state two major existence theorems which bridge Definitions 3.1 and 3.2. The first is the Arrow-Debreu theorem that under suitable assumptions there exists a vector of prices supporting a competitive equilibrium. The second, due to Negishi (1960), is that there exist certain plans that yield competitive equilibria.

**Theorem 3.1: Existence of Equilibrium Prices.** Under suitable assumptions,\(^{17/}\) there exists a vector of market prices \( \bar{p} \) such that the conditions defining a competitive equilibrium (Definition 3.1) are satisfied. Assumptions 3.1 and 3.2 specifying producer and consumer behavior are satisfied and no commodity is in excess demand:

\[
e(\bar{p}) = x(\bar{p}) - y(\bar{p}) - \omega < 0.
\]

This is the classic Arrow-Debreu equilibrium result: there exists a set of prices such that profit maximizing producers and utility maximizing consumers, subject to their budget constraints, will generate production and consumption decisions such that excess demands are non-positive.

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\(^{16/}\) Given the maximand, the solution is clearly a Pareto Optimum since it would always increase the maximand to make one person better off without making someone else worse off.

\(^{17/}\) The assumptions are technical and somewhat more general for Theorem 3.1. See Arrow and Hahn (1971) and Negishi (1960). In most empirical models, they are strengthened so as to assure strictly positive prices and zero excess demands.
Theorem 3.2: Existence of Equilibrium Welfare Weights. Under suitable assumptions, there exists a vector of non-negative welfare weights \( \alpha_i \) such that the solutions of the Pareto Optimal Plan (Definition 3.2), with associated shadow prices \( \pi \), are competitive equilibria with \( \bar{p} = \bar{\pi} \).

The essence of the Negishi Theorem is that there exists at least one set of welfare weights such that the allocations \( x_i \) and \( y_j \) and the shadow prices from the solution of the programming problem will satisfy the individual budget constraints (evaluated at \( p = \pi \)). Thus, if consumers and producers behave as stated in Assumptions 3.1 and 3.2, there exists a planning model the solutions of which are competitive equilibria.

In the definition of a competitive equilibrium, the budget constraints are an explicit part of the model, while in the programming problem in the Negishi Theorem the budget constraints do not appear explicitly. Ginsburgh and Waelbroeck (1981) present a program which adds the budget constraints explicitly and show that this Master Program includes the models in Theorems 3.1 and 3.2 as special cases.

Definition 3.3: A Master Program

\[
\max \sum \alpha_i U_i(x_i)
\]

with respect to \( x \) and \( y \)

subject to

\[
p x_i - p \omega_i - \sum_j \theta_{ij} p y_j < 0
\]

\[
\bar{p} - y - \omega < 0
\]

\[
x_i \in X_i; \ y_j \in Y_j
\]

If one drops the material balance constraints \( (x - y - \omega < 0) \), the remaining program embodies the behavioral specification underlying Theorem 3.1 (for a given set of fixed prices). This can be seen intuitively as follows (and can be proved without difficulty). Given \( p \), the various budget
constraints can be loosened by increasing profits of any producer \((p.y_j)\). Hence, at the solution, profits must be maximized. Given \(a_i > 0\), the overall maximand is a weighted sum of individual utilities. Given \(p\) and profit maximizing production plan \(y_j\), the overall problem can be decomposed into \(m\) subproblems. Thus the overall maximum is achieved when each consumer maximizes his utility subject to his budget constraint. Note that this solution is independent of the choice of welfare weights (no lump sum transfers are allowed). The program for the Negishi theorem is obtained from the master program by dropping the individual budget constraints. In the Negishi theorem, of course, the welfare weights are important because the shadow prices of the material balance constraints depend on them. At equilibrium, in fact, the welfare weights are equal to the inverses of the marginal utility of income for the consumers.

Ginsburgh and Waelbroeck (1981) prove an existence theorem which essentially links Theorems 3.1 and 3.2 and emphasizes the symmetry between the competitive model and certain planning models.

**Theorem 3.3: Competitive Equilibrium in a Master Program.**

Let \(\pi\) be the vector of shadow prices associated with the \(x - y - \omega < 0\) constraints of the Master Program. Any solution in which \(\pi^* = p\) is a competitive equilibrium (where \(k\) is a positive scalar).

In any Walrasian general equilibrium model, it is well known that only relative prices "matter" and that one is free to choose a numeraire to set the absolute price level. In Theorem 3.1 (Arrow-Debreu), the implication is that some normalization rule can be imposed on the solution price vector \(\bar{p}\), for example \(\bar{p}_1 = 1\) (assuming \(p_1\) is non-zero). In Theorem 3.2 (Negishi), a normalization can be imposed on the welfare weights, say \(\bar{a}_1 = 1\). In Theorem 3.3 (Ginsburgh-Waelbroeck), the vector of shadow prices \((\pi)\) at equilibrium
does not depend on the choice of welfare weights. One is free to normalize prices as in the Arrow-Debreu model, for example by dropping the first material balance constraint and setting $p_1 = 1$.

4. Model Specification and Empirical Solution

A variety of empirical general equilibrium models have been built based on the different underlying approaches implicit in these existence theorems. We will distinguish two broad families of models: (1) Computable General Equilibrium (CGE) models (the term introduced by Adelman and Robinson) and (2) Activity Analysis General Equilibrium (AGE) models introduced by Ginsburgh and Waelbroeck (1981). CGE models are based on the model underlying Theorem 3.1, and simulate the behavior of producers and consumers to generate numerically the set of excess demand equations. For example, if utility functions and production functions are "well-behaved" neoclassical functions, then it is possible to write out the various first-order conditions explicitly and generate a set of consumer demand functions and producer supply functions. Assuming all prices are strictly positive, the solution problem then reduces to finding a set of prices which makes all excess demands equal zero. The model involves a lot of non-linear mathematics, but no inequality constraints. AGE models, on the other hand, are characterized by inequality constraints.

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18/ For a proof, see Ginsburgh and Waelbroeck (1981), pp. 69-70. To see this intuitively, suppose you start from equilibrium $p$ prices, and solve the master program without the material balance constraints. The primal solution is independent of the choice of $\alpha$'s, and satisfies, by definition the budget and the balance constraints. Clearly the addition of the material balance constraints will not change the primal solution. However, varying the $\alpha$'s will change the multipliers associated with the budget constraints.

19/ See Adelman and Robinson (1978); Dervis, de Melo and Robinson (1982); and Scarf and Shoven (1983) for examples of CGE models. For AGE models, in addition to Ginsburgh and Waelbroeck (1981), see Manne et al. (1980), who independently developed a similar approach, and Dixon (1975) who empirically implemented the Negishi approach.
constraints and are always cast in the format of a programming problem of the type specified in Theorems 3.2 or 3.3.

We will discuss the differences between the two approaches in terms of the different strategies required to solve the models. Such an algorithmic focus has two advantages. First, ease of solution is a major criterion for choosing between different model formulations and, second, the different solution strategies highlight the major differences in structure between the two families, and among different models within the AGE family.

Any method of solving a general equilibrium model empirically can be seen as a general algorithmic procedure consisting of two steps. The first step is a "function evaluation" which consists of solving a mathematical program or a set of equations in which a certain number of parameters are fixed. The second step is that of "parameter revision." The solution of the function evaluation step is examined to test whether certain equilibrium conditions are satisfied. If they are, stop the procedure. If not, change some parameters and go to a new function evaluation step. In terms of this general two-step procedure, there are at least three strategies for solving CGE and AGE models based on Theorems 3.1, 3.2 and 3.3.

Strategy 1: based on Theorem 3.1. The function evaluation step involves evaluating, for given prices $p$, the consumer demand and producer supply equations, $x(p)$ and $y(p)$. The solutions determine the aggregate excess-demand equations. If all such excess demands are zero, then $p$ is an equilibrium price vector. If not, the price vector has to be modified and a new function evaluation step is started.

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20/ Some technical issues concerning the nature of these relationships are being finessed here. In fact, for empirical models, these equations are virtually always well-behaved functions.
Strategy 2: based on Theorem 3.2. For a given vector of welfare weights $\alpha$, solve the mathematical program. Using the solution, check whether every consumer is on his budget constraint. If so, $\alpha$ is an equilibrium vector of welfare weights and the solution, including the shadow prices, represents a competitive equilibrium. If not, revise the $\alpha$'s and start a new function evaluation.

Strategy 3: based on Theorem 3.3. For a given vector of prices $p$, solve the Master Program and check whether the dual prices $\pi$ associated with the excess-demand constraints are equal to $p$. If so, stop; the solution is an equilibrium. If not, change the price vector $p$ and start a new function evaluation$^{21}$/

To summarize, in Strategy 1, prices are revised until excess demands are all non-positive, $e(p) < 0$. For Strategy 2, welfare weights are revised until "excess-budget" equations all equal zero, $b_i(\alpha) = \pi(\alpha) x_i(\alpha) - R_i(\alpha) = 0$. For Strategy 3, prices are revised until $\pi(p) - \bar{p} = 0$. $^{22}$/ Parameter revision involves some kind of algorithm for solving systems of non-linear equations. A variety of such algorithms are now available, and the choice among them is largely a technical issue of cost and convenience. $^{23}$/

In contrast, the function evaluation phase involves a choice solution strategy based on alternative existence theorems and on different ways of formulating the model. For example, if it is possible to specify the model as a CGE model by writing out the excess-demand equations explicitly, it is

$^{21}$/ Since, as noted above, the shadow prices $\pi$ are independent of the choice of $\alpha$'s, the $\alpha$'s can be set arbitrarily to any strictly positive numbers.

$^{22}$/ More generally, $k\pi(p) - \bar{p} = 0$ where $k$ is a positive scalar.

probably efficient to do so since it greatly simplifies the function evaluation phase.\textsuperscript{24} In general, however, one cannot always solve the individual consumer and producer problems to yield explicit expressions for supply and demand. There are a number of reasons why such a formulation might not be possible or desirable.

First, even when all the constraints take the form of equalities, it may be impossible to get explicit functions from the first-order conditions for a maximum. One is then forced numerically to solve a system of non-linear equations (first-order conditions) either for every agent or for the economy as a whole (using Theorem 3.1). In such a case, it might well be easier to deal with the mathematical program directly in the AGE format and thus avoid having to write out the first-order conditions explicitly.

Second, one may well wish to specify inequality constraints, which then requires an AGE approach. In choosing the CGE formulation, a modeler must have faith in the smoothness and continuity properties of neoclassical utility and production functions. While such assumptions may be reasonable for the demand side -- at least at the level of aggregation of most models -- they are often felt to be unreasonable for producers. One interesting possibility would be to construct a mixed AGE-CGE model in which the demand side is represented by explicit demand functions while the production side is represented by one or several linear or non-linear activity analysis models.

5. Prices in Planning Models

The early programming models were rarely specified as true AGE models, although some of the planning models came close.\textsuperscript{25} Empirical model

\textsuperscript{24} Indeed, the same procedure could be used with the Negishi theorem, leading to a CGE model with explicit excess-budget equations.

\textsuperscript{25} Blitzer et al. (1975) provides a good survey. See also Dixon (1975).
builders have run into problems with programming models for a number of reasons. First, limitations on the ability to solve programming models have led to simplifications that yield unrealistic behavior. For example, all the early models used linear programming, which is prone to corner solutions and extreme specialization. Modelers then imposed various ad hoc bounds on production, consumption, investment, export, and import activities in order to achieve "realistic" behavior, but which resulted in distortions in the shadow price system.

Second, even when not constrained by solution technology, modelers have often specified empirical models whose underlying theoretical structure has led to unrealistic results. Perhaps the best example is the specification of trade which has long provided a major focus for applied programming models. Problems arise because most modelers have introduced international trade using the small-country assumption and also assuming that domestically produced and imported goods are perfect substitutes. The excess-demand constraints $x \leq y + \omega$ are replaced by $x + e \leq y + m + \omega$ (where $e$ and $m$ are export and import vectors), and a balance of payments constraint is added, $p^m_m - p^e_e < 0$. The prices $p^m$ and $p^e$ are assumed fixed (defining the "small-country" assumption), which implies infinitely elastic export demand and import supply equations. This specification leads to extreme specialization even in the non-linear case and modelers have reacted by imposing a number of additional ad hoc constraints to make the solution more realistic. Such constraints pick up shadow prices and distort the price system.

26/ For examples, see some of the case studies in Chenery (1971) and Adelman and Thorbecke (1966). Evans (1972) probably has the most sophisticated treatment of trade in a linear programming model. See also the comments on Evan's model by Dixon and Butlin (1977) who criticize it within the framework of general equilibrium theory.
The introduction of price distorting ad hoc constraints has been much discussed in the planning literature (see, for example, Taylor (1975)). If such constraints can be justified as additional system constraints that define a reasonable notion of economic equilibrium, then there is no theoretical problem.\(^\text{27}\) For example, including a balance-of-trade constraint is perfectly legitimate to define the notion of equilibrium in the market for "foreign exchange." If the added constraints can be seen as a legitimate part of the constraints facing an individual agent, then again there is no theoretical problem. For example, if an upper bound on output in a sector can be defended as part of the technical constraints facing a producer (and hence part of the definition of his production possibility set \(Y_j\)), then its inclusion raises no problems of interpretation of the shadow prices. When added constraints cannot be defended either as part of the definition of overall equilibrium or as part of the constraints facing individual agents, then it becomes impossible to interpret the solution as reflecting the operation of a market system. Any survey of existing models reveals that such problems were very common.

Empirical planning models have often been used in situations where "macroeconomic" issues such as balance-of-trade and/or savings-investment "imbalances" are important. Some linear programming models were extended to include such "macroeconomic" constraints, but often expressed in real terms (i.e., without reference to solution prices) and so hard to justify as representing theoretically valid system constraints. For example, Bruno (1966) incorporates "Keynesian" aggregate savings functions that do not reflect the behavior of individual agents. They are instead justified as reflecting macroeconomic equilibrium conditions, but with no explicit

\(^{27}\) These are the constraints \(\phi(-) < 0\) defined in section 2.
consideration of the nominal flow of savings and investment in the system. The more recent literature on model "closure"--how a model determines aggregate savings and investment--has also raised the issue of macroeconomic equilibrium in the context of CGE models. We turn to these issues in the next section.

6. Macroeconomic Equilibrium

In the competitive model, the nominal flows among agents are very straightforward. Producers pay out their receipts to households who, in turn, spend all their income on goods. The focus is on the "real" system, which is equilibrated by flexible prices, and no additional assumptions are needed to ensure that the various nominal income and expenditure flows are consistent or in equilibrium. In Table 1, the competitive equilibrium model is extended somewhat and presented in a form that will facilitate a discussion of its macroeconomic features. The presentation is in the framework of a CGE model, but could easily be done in that of an /GE model. The equations describing the product and factor markets are presented separately (separating the negative factor input elements in the $y_j$ vectors from the positive output elements). The price vector is also split into two vectors, with $p$ now denoting only product prices and $w$ denoting factor prices. The consumer income variables $R_i$ in the models are now replaced by "institutional" nominal income and expenditures, $R_k$ and $E_k$. These institutions (indexed by $k$) would include not only households, but also other actors such as the government, an aggregate "bank" which collects savings and buys investment goods, and the rest of the world. Each institution has a nominal expenditure $E_k$ and an

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28/ See Bruno (1979) who introduces a symposium dealing with such issues, largely in the context of models of income distribution.

29/ See, for example, Ginsburgh and Waelbroeck (1981), Chapter 4.
Table 1
Product Markets, Factor Markets and Flow of Funds

<table>
<thead>
<tr>
<th>Real Flows:</th>
<th>Nominal Flows:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product and Factor Markets</strong></td>
<td><strong>Nominal Income and Expenditure</strong></td>
</tr>
<tr>
<td>(1) Product supply $X^S(p,w)$</td>
<td>(7) Institutional income $\bar{R}(w,F^S,\phi)$</td>
</tr>
<tr>
<td>(2) Product demand $X^D(p,\bar{E})$</td>
<td>(8) Institutional expenditure $\bar{E}(\bar{R},\phi)$</td>
</tr>
<tr>
<td>(3) Factor supply $F^S(w,p)$</td>
<td>(9) Macro balances $\bar{R} - \bar{E} = 0$</td>
</tr>
<tr>
<td>(4) Factor demand $F^D(w,p)$</td>
<td>(10) Price normalization $f(p,w) = \bar{F}$</td>
</tr>
</tbody>
</table>

**System Constraints**
(5) Product markets $X^D - X^S = 0$
(6) Factor markets $F^D - F^S = 0$

**Nominal Flow Identities**
(11) $w.F^D \equiv p.X^S$ Factor income equals total sales.
(12) $\sum_k \bar{R}_k \equiv w.F^S$ Institutional income equals factor income.
(13) $p.X^D \equiv \sum_k \bar{E}_k$ Total demand equals institutional expenditure.

**Equilibrating variables**
- $\rho$ = vector of product prices,
- $w$ = vector of factor prices, and
- $\phi$ = vector of additional equilibrating variables.
associated commodity expenditure function. The system constraints, equations (5) and (6), define equilibrium in the factor and product markets, exactly as in the models above.

The nominal flows in the system are given by equations (7), (8), and (9). Equation (7) maps factor incomes into institutional incomes, with all factor supplies being paid \( w \) (corresponding to \( p \cdot w \) in the earlier models). Equation (8) is new and allows for the possibility that \textit{ex ante} expenditure plans of institutions need not match their income—a possibility not allowed in the earlier models. New system constraints—equation (9)—define flow-of-funds equilibrium and new equilibrating variables (\( \phi \) discussed below) are introduced. Equation (10) is the price normalization equation that defines the numeraire price and sets it to the exogenous constant \( F \). This equation is listed as a nominal system constraint since it sets the absolute price level and hence affects all nominal magnitudes.\(^{30/}\)

Equations (11) - (13) present various nominal flow identities implicit in the equations. They reflect fundamental attributes of the "circular flow" in the economy. In terms of a model, the requirement is that all funds must be accounted for and that every real transaction must generate a corresponding nominal flow. Adding (11) - (13) indicates that the three system constraints (5), (6) and (9) taken together satisfy Walras' Law:

\[
p.X^D + w.F^D + \sum_k \hat{R}_k = p.X^S + w.F^S + \sum_k \hat{E}_k.
\]

Some examples will clarify the relationships between macroeconomic and market equilibria. Consider a model that includes among its institutions

\(^{30/}\) In models which are not homogeneous, the absolute price level "matters" and issues of neutrality become important. The aggregate price equation then no longer simply serves to define the numeraire. Hansen (1970) provides a clear exposition of such issues in the context of Walrasian models.
the "rest of the world," which buys exports and sells imports, and a "bank" or aggregate capital account which collects savings and buys investment goods. The associated system constraints are the equations specifying balance-of-payments equilibrium and savings-investment equilibrium, which we discuss in more detail below.\(^3\)

As a benchmark, one should start with the classical model which tries to capture such macroeconomic phenomena by simply adding some new markets. The balance of payments is modeled by adding a market for foreign exchange in which supply is generated by selling exports and demand by buying imports. A new price, the exchange rate, is introduced as the equilibrating variable. Savings-investment is modeled by adding a market for "loanable funds" in which supply is given by savings and demand by investment. Both supply and demand are assumed to be sensitive to the interest rate, which is the new equilibrating variable. In terms of the framework in Table 1, this classical approach is really an attempt to keep macroeconomics in the Walrasian framework. No special treatment of nominal flows is required. One simply defines new commodities, markets, and prices which are added to those on the left side of Table 1. While appealing, few economists would accept this approach without extensive qualifications. These new "markets" are clearly special and the "commodities" are quite different from factors and produced goods. They really do not fit the simple Walrasian paradigm of a barter economy in which only relative prices matter.

The interactions between the real and nominal sides of Table 1 have provided much fuel for economic controversy. At one extreme, in a world of

\(^3\) In our discussion, we will focus on the system constraints and the choice of equilibrating variables, and neglect any detailed discussion of the behavioral rules of the institutional actors.
competitive markets and neutral money, the real economy is insulated from "disturbances" emanating from the nominal side. Multisector planning models have usually focused exclusively on the real sphere, with a medium to long term horizon, and have ignored links between the real and nominal sides. In a competitive economy, in which there is full employment of all factors, such a focus is clearly warranted. Both theory and experience with empirical models indicate that while disturbances in the nominal side of the model may affect the composition of production and demand, they have little effect on aggregate output and growth in a competitive neoclassical world. Unfortunately, the real world does not appear to be so flexible and exhibits phenomena such as prolonged periods of unemployment, wide variations in growth rates, and strong links from macroeconomic adjustment mechanisms to the real economy. Modelers have long sought to capture such phenomena in models applied to both developed and less developed economies. We will discuss two examples from the development literature that focus on savings-investment equilibrium: the Chenery two-gap model and the Latin American distributional or structuralist models.

The two-gap model starts from the basic premise that, in a developing country, real investment and production require imports of capital goods and intermediate inputs. The existence of such "non-competitive" imports of crucial inputs is a characteristic feature of many development planning models with which Chenery is associated.\(^32\) It is one of the major features of what he has called the "structuralist" approach to development policy and provides a strong potential link between the balance of payments and the real economy.\(^33\) We can justifiably describe the resulting potential impact of foreign exchange

\(^{32}\) See Chenery (1980), Chapters 4, 8, 9, and 10, and Adelman and Chenery (1966).

\(^{33}\) See Chenery (1975) and Chenery (1979), Chapter 2.
shortages on production and growth in a model as the "Chenery effect."

The two-gap model, with its rigidities and assumption of non-competitive, complementary imports, highlights the impact of the balance of payments on the real economy. However, the Chenery effect remains important even in a model in which more substitutability is assumed. The essence of the Chenery effect and of the two-gap model remains, even in models which are far more neoclassical than any of the early programming models in which the ideas were first embodied.

In the two-gap model, foreign exchange and savings provide the focus of macroeconomic equilibrium. Another strand of work in the development literature focuses on the links between the distribution of income, aggregate savings, and hence macroeconomic equilibrium. The nature of the links depends on the way in which savings-investment equilibrium is achieved. According to what has been called the Latin American "structuralist" school, relative price mechanisms—including a flexible exchange rate—cannot achieve macro equilibrium because of structural rigidities in certain markets. A different mechanism that works through changes in the distribution of income is specified. The fundamental assumption is that recipients of capital and

34/ Michalopoulos (1975) discusses a two-gap model in which imported and domestic capital goods are substitutable according to a CES function. Dervis, de Melo, and Robinson (1982) develop the implications of imperfect substitutability in some detail in the context of CGE models.

35/ See, for example, Dervis and Robinson (1982) who use a CGE model to analyze the causes and impact of a foreign exchange crisis in Turkey. Findlay (1973), Chapter 10, makes the same theoretical point and argues that the two-gap model can be seen as a special case of a neoclassical model.

36/ See, for example, Diamand (1978).
wage income have different savings rates and hence changes in the distribution of income will affect overall savings. This "Kaldor effect" provides a major link between the real economy and the nominal side.37/

Taylor (1979) discusses a number of models that not only incorporate the Kaldor effect, but also make two assumptions about the institutional structure of the economy. First, aggregate investment is fixed exogenously and, second, the wage is assumed not to clear the labor market. The first can be seen as an additional system constraint defining macroeconomic equilibrium in the nominal side.38/ The second presents a more difficult problem of interpretation.

In terms of the framework in Table 1, the assumption that the labor market does not clear implies that one should drop the corresponding system constraint in equation (6). Some assumption must also be made about how the excess supply is rationed in the system—what happens to the unemployed? The simplest treatment is to assume that firms are always on their demand curves for labor. In equation (7), $F^S$ is replaced by $F^D$, and the unemployed are essentially assumed to drop out of the economy. They receive no income and generate no effective demand.39/ The labor supply function in equation (3) becomes a side equation to compute the amount of unemployment, but has no real effect on the model economy. Note that this treatment ensures that the remaining system constraint equations still satisfy Walras' Law.

Given that the system constraint for the labor market is dropped, L

37/ See Kaldor (1955).

38/ The assumption of fixed aggregate investment is usually specified in real terms, but can be just as easily handled in nominal terms.

39/ The unemployed could alternatively be assumed to receive income through transfers without changing the essential nature of the model.
what then is the role of the corresponding wage in the new model? In these models, the real wage becomes the equilibrating variable to achieve savings-investment equilibrium on the nominal side. The wage adjusts to achieve a distribution (and level) of income that generates the necessary savings to validate the exogenous level of investment, and there is an "equilibrium" real wage that achieves macro balance. As with the two-gap approach, the extreme version of the model serves to focus attention on an important mechanism. Both foreign-exchange-production links and distribution-macro links appear to be very important institutional characteristics that should be incorporated into models of developing economies, even at the price of compromising the purity of the Arrow-Debreu model.

6. Conclusion

Multisector models have come a long way from the early static input-output model. Throughout their development, there has always been a tension between empirical practice and available theory. Recently, with the rapid advances in solution algorithms, the gap between the development of new theoretical models and the ability to implement them empirically has narrowed considerably. Indeed, recent models are operating on the border between macro and micro theory, where there are many theoretical inconsistencies and no widely accepted reconciliation. The field is very active, with a number of different approaches being pursued simultaneously. Our purpose has been to sort out some of the links among the different approaches and to indicate the lineage that relates the current models to past empirical planning models, as well as to theoretically founded general equilibrium models.
References


