PUBLIC DEFICIT FINANCE AND PRIVATE FOREIGN ASSETS

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Abstract

Government cannot ignore the external effects of their actions on private sector behavior. In many developing countries, government foreign borrowing has been associated with private sector capital outflows. This paper analyzes the effect of public spending, tax and deficit-financing strategies on economy-wide foreign borrowing and investment. It addresses such issues as (i) what is the effect of government spending and tax change on the foreign asset holdings of the private sector, (ii) how does the mix of government deficit finance between domestic and foreign borrowing affect the private (and hence national) foreign asset position, and (iii) under what circumstances does increased government borrowing, either domestically or abroad, result in a private capital outflow.

The analysis utilizes a neoclassical framework in which private individuals have finite horizons. Intertemporal constraints are imposed on government behavior. Thus, current spending increases or tax cuts must result in future spending reductions or tax increases. The main findings are that: an increase in government spending results in a higher national debt level. If the spending is financed by public external borrowing, however, there is a smaller build-up of national debt than if it is financed by domestic bond financing. Similarly, if government foreign borrowing is associated with lower tax revenues, private capital flight may increase in some cases.
I. Introduction

Most of the attention devoted to the debt problems of developing countries has focussed on the ability of borrowing country governments to directly acquire and repay foreign debt. Less attention has been given to how the policies of these governments affect the behavior of local private sectors in international capital markets. That governments cannot ignore the external effects of their actions on the private sector is apparent from the recent experience of many countries. In Argentina and Mexico, for example, increases in total foreign debt between 1978 and 1982 were accompanied by large private capital outflows.¹

This paper analyzes the effect of government spending, tax, and deficit-finance policies on the foreign borrowing and asset accumulation behavior of the private sector in a small, open economy. It addresses such issues as (i) what is the effect of government spending and tax change on the foreign asset holdings of the private sector, (ii) how does the mix of government deficit finance between domestic and foreign borrowing affect the private and national foreign asset position, and (iii) under what circumstances does increased government borrowing, either domestically or abroad, result in an outflow of private capital and accumulation of foreign assets.

The analysis utilizes the framework developed by Blanchard (1985) and adapted by Frenkel and Razin (1984a, 1984b) and Buitert (1984) according to which individuals facing a given probability of death have finite horizons. This framework permits the relatively simple specification of aggregate private behavior within a setting in which individuals respond rationally to government policies. With finite horizons individuals do not take full account
of any tax liabilities associated with the need to service the debt created by current budget deficits.

Section II of the paper characterizes the consumption and asset accumulation behavior of individuals with finite horizons. It also specifies the nature of the public-sector budget constraint. The steady-state and dynamic behavior of the economy is then described. The effects of permanent and temporary balanced-budget spending and tax policies on private foreign asset holdings are also described. It is shown that a permanent balanced-budget increase results in a long-run decline in private consumption and foreign assets. A temporary increase causes consumption and foreign assets to decline in the short run, but to converge back to the initial steady state once the budget increase is reversed.

Section III explores the effects of spending and tax policies that result in budget deficits that are financed by the issuance of domestic bonds to the local private sector. Since the tax structure in many developing countries is very rudimentary and relies strongly on taxes that are not easily changed over time, the level of taxes is treated as exogenously determined. To insure the satisfaction of the government budget constraint it is therefore assumed that a portion of government spending adjusts endogenously over time in response to the rate of change and level of government debt. This specification contrasts with models such as Blanchard's which assume that tax levels adjust endogenously, but more realistically captures the relative flexibility of government spending and tax levels within developing countries. It implies that budget deficits created by tax cuts or initial spending increases financed by public debt must in the long run be accompanied by spending cuts. Such policies are shown to result in a long-run increase in government domestic debt and decrease in private foreign assets.

In Section V the model is extended to incorporate the effects of the
finance of public deficits by foreign debt. There it is seen that the greater
the extent of foreign financing of budget deficits the less the decline in
private foreign asset holdings in response to an increase in government
spending. In the case of a tax cut it is possible that capital outflows may
occur and private foreign assets actually rise. The national foreign asset
position is found to always decline.

Section VI considers the effects of an interest rate increase on budget
deficits and the effects of different government means of responding to this
disturbance. Section VII contains concluding remarks.

II. Private and Public Spending Behavior

Consider a small, open economy facing a given world interest rate \( r \) at
which individuals and the domestic government can freely borrow and lend.
Aggregate consumption and asset accumulation behavior of the private sector
within this economy is described first. Thereafter the public sector budget
constraint is specified. The formulation utilized follows closely that of
Blanchard (1985). Mathematical details are presented in the appendix.

Private Sector

The central assumptions of Blanchard's model are that (i) all individuals
face through life a given probability of death between one period and the next,
and (ii) individuals make no bequests and contract with life insurance
companies to fulfill outstanding debt obligations in the event of their death.
Denoting the probability of death by \( p \), the first assumption implies that all
individuals have an expected lifespan of \( 1/p \).\(^2\) As \( p \) goes to zero, \( 1/p \) goes to
infinity, implying individuals have infinite horizons. Because of the uncertain
probability of survival from one period to the next, the second assumption
implies that guarantees of debt repayment require a premium to the life
insurance company in addition to regular interest payments. Perfect competition among these companies implies that the insurance premium rate equals the probability of death. The effective discount rate for individuals is thus \( r + p \).

Under the further assumption that instantaneous private consumption is logarithmic in consumption, it may be shown that aggregate private consumption \( (C) \) can be expressed as a linear function of aggregate human \( (H) \) and financial \( (W) \) wealth: \(^4\)

\[
C = (p+d)(H + W)
\]

(1)

where the aggregate marginal propensity to consume depends on \( p \), the probability of death, and \( d \), the rate of time preference for consumption. Either an increased probability of death or an increased preference for current over future consumption results in increased current consumption out of human and financial wealth.

Aggregate human wealth is defined as the present value of future disposable non-interest income accruing to all individuals currently alive and grows at the rate \( r + p \), the effective discount rate for individuals. Its evolution is described by the following equation:

\[
\dot{H} = (r+p)H - (Y - T),
\]

(2)

where \( Y \) is non-interest income and \( T \) are lump-sum taxes. \( Y \) is assumed constant throughout the analysis. Financial wealth is net of financial assets and liabilities held by the private sector and evolves as

\[
\dot{W} = rW + Y - T - C
\]

(3)

Note that aggregate financial wealth grows at the rate \( r \), while individual financial wealth grows at the higher rate \( r + p \). For those who remain alive the
amount \( pW \) is a transfer from those who die through the life insurance companies and is not therefore an addition to aggregate wealth.

Domestic private residents hold domestic assets issued by the government \( B_d \) and foreign assets \( F \). Thus \( W = B_d + F \). \( F \) is positive or negative depending upon whether the private sector is a net holder of foreign assets or liabilities. The private sector is precluded from financing an ever increasing level of spending through the unlimited issuance of debt by the constraint that \( \lim_{t \to \infty} F e^{-rt} = 0 \); i.e. the present value of future private sector debt must ultimately go to zero.

**Public Sector**

The government spends on domestic goods and finances this spending either by taxes or by the issuance of debt.\(^6\) The government's total stock of debt is denoted by \( B \). This debt may consist of issues to the domestic private sector \( B_d \) or to foreigners \( B^f \). The government dynamic budget constraint is thus

\[
B = rB + G - T,
\]

(4)

where \( G \) is government spending. Note that since the government has an infinite horizon and knows it can tax both existing and yet-to-be born individuals its discount rate is the interest rate \( r \).\(^7\)

The government solvency constraint requires that \( \lim_{t \to \infty} B e^{-rt} = 0 \), i.e. the present value of future government debt must ultimately go to zero. This precludes the government from financing an ever increasing level of spending through the unlimited issuance of debt. It also implies that over the government's infinite horizon the present value of future government spending and the servicing of existing debt must equal the present value of future taxes.
II. Dynamics and Steady State without Budget Deficits

In this section the characteristics of the steady-state are analyzed assuming no budget imbalances and no government debt, i.e. \( G = T \) and \( B = B^d = B^f = 0 \).

First differentiate (1) with respect to time and use (2) and (3) to substitute out for \( H \) and \( F \) and obtain

\[
\dot{C} = (r-d)C - p(p+d)F
\]

(5)

Note that since \( B \) is constant (and equal to zero), the dynamics of \( W \) given by (3) also describe the dynamics of \( F \):

\[
\dot{F} = rF - C + Y - T
\]

(6)

It is shown in the appendix that the dynamic system in \( C \) and \( F \) represented by (5) and (6) has the characteristic roots \( r-d-p \) and \( r+p \). Saddlepoint stability is assured if \( r < d+p \). This condition precludes the possibility that individual wealth and consumption grow at a rate larger than the death rate and hence that aggregate wealth and consumption increase forever.

The system is depicted in the \( C-F \) phase diagram in Figure 1. The slope of the \( \dot{C} = 0 \) locus may be positive or negative depending on whether \( r \) is greater or less than \( d \); the first case is presented in Figure 1. The slope of the \( \dot{F} = 0 \) locus is positive, but flatter than the \( \dot{C} = 0 \) locus. Observe that, if the initial stock of foreign assets is below the steady-state equilibrium level, the private sector increases consumption and reduces its foreign borrowing and accumulates foreign assets over time along the saddle path to the equilibrium.

The steady-state values of \( C \) and \( F \), \( \dot{C} \) and \( \dot{F} \), respectively, may be determined algebraically by setting \( \dot{C} \) and \( \dot{F} \) equal to zero and solving (5) and (6) simultaneously.\(^8\)
\[ \overline{C} = \frac{p(p+d)(Y-T)}{(p+d-r)(r+p)} \]  \hspace{2cm} (7)

\[ \overline{F} = \frac{(r-d)(Y-T)}{(p+d-r)(r+p)} \]  \hspace{2cm} (8)

Since \( B \) is assumed zero, the wealth constraint implies \( \overline{W} = \overline{F} \).

Steady-state consumption is clearly a positive function of disposable (non-interest) income \( Y-T \). Whether the private sector (and the economy as a whole) is a net international creditor or borrower in the steady state depends on the difference between the interest rate and the rate of time preference. If \( r > d \), the return to investing in foreign assets exceeds the rate of time preference for current consumption, private residents accumulate assets and increase consumption (see equation (5)) over time, and the steady-state level of foreign assets is positive. If \( r < d \) then the private sector decumulates assets over time, \( \overline{F} \) is negative, and the country is a net foreign debtor in the steady state. Throughout the remainder of the analysis it generally will be assumed \( r > d \), though mention will be given where results are sensitive to this assumption.

The effects of (unanticipated) balanced-budget policies are illustrated in Figure 2. Panel a depicts a permanent balanced-budget increase: \( dT = dG > 0 \). The \( C = 0 \) locus is unaffected, but the \( F = 0 \) locus shifts down. While such a policy clearly induces no budget imbalance nor change in the stock of public debt, an immediate fall in consumption occurs upon announcement of the policy, which is followed by a gradual decline in foreign-asset holdings and further decreases in consumption. The reason for this is that since private non-interest income (\( Y \)) is assumed constant and unaffected by government spending changes, the balanced-budget increase amounts to a decline in private disposable income.
In the case of a temporary balanced-budget spending and tax increase, depicted in panel b, consumption falls on impact, but not by as much as in the case of a permanent increase. From there the system follows a southwest path determined with reference to the \( E_1 \) equilibrium. The economy moves along this path so that at the time the spending and tax increase is ended, the economy is in position to follow the saddle path back to the original equilibrium \( E_0 \). Thus while the budget increase is in effect private foreign assets and consumption fall. When the budget change is reversed, foreign assets and consumption return to their initial steady-state levels.

An increase in the interest rate \( r \) shifts the \( C = 0 \) locus to the right and the \( F = 0 \) locus upward (not shown), implying an increase in private foreign asset holdings. An increase in the probability of death \( p \) and thus a decrease in private sector horizons shifts the \( C = 0 \) locus to the left, implying a decrease in steady-state foreign-asset holdings.

III. Dynamics and Steady State with Domestic Public Debt Finance

The analysis turns next to the short-run and long-run effects of spending and tax changes that result in (temporary) government budget deficits. In this section it is assumed that any debt issued by the government to finance these deficits is in the form of domestic debt acquired by the domestic private sector, hence \( B^d = B \). In Section IV the analysis is extended to include the more general case in which budget deficits may be financed by the issuance of public foreign debt as well.

In considering these policies it is important to assure that they do not imply any violation of the government budget constraint. To capture the relative inflexibility of tax levels over time in many developing countries, taxes are regarded as insensitive to changes in government debt. They are
assumed exogenously determined and fixed at the level \( T_0 \). Government spending is treated as the relatively more sensitive fiscal policy tool necessary to finance budget deficits over the long run. More specifically, total government spending \( G \) is assumed to consist of an exogenous component as well as components related to both the change and level of government debt as described by the following relation:

\[
G = G_0 + g_1B - g_2B; \quad g_1, g_2 > 0
\]  

(9)

Equation (9) says that the level of spending at any period in time consists of a pure lump-sum component \( G_0 \) and of amounts related positively to the change and negatively to the level of the government's stock of debt.\(^9\) This is consistent with the perception that, given the level of taxes, new borrowing is used to finance additional spending, while the debt burden associated with the entire stock of debt outstanding has a contractionary effect on spending. It will be seen below that satisfaction of the government budget constraint requires restrictions on the sensitivity of the endogenous spending components.

Substitution of (9) in (4) implies

\[
B = \frac{(r-g_2)B}{1-g_1} + \frac{G_0 - T_0}{1-g_1}
\]  

(10)

A necessary and sufficient condition for convergence of the level of public debt to a steady state and satisfaction of the government budget constraint is that \( (r-g_2)/(1-g_1) < 0 \); i.e. the rate of increase in government debt falls the higher the existing stock of debt. To ensure the satisfaction of this condition it is assumed that \( g_2 > r \) and \( g_1 < 1 \). The latter assumption implies that an initial increase in exogenous spending (rise in \( G_0 \)) leads to an increase in the budget deficit (though by a less than proportionate amount since \( 1/(1-g_1) < 1 \)). The former assumption implies that an increase in the stock of debt causes
total government spending to fall by an amount that is sufficient to cover the
interest costs associated with servicing the debt.

Inserting (10) into (9) gives

\[ G = \frac{G_0}{1 - g_1} - \frac{g_1 T_0}{1 - g_1} - \frac{(g_2 - r_1)B}{1 - g_1} \]  

(11)

Expression (11) implies that an exogenous increase in spending or decrease in
taxes leads to a (proportionately smaller) increase in total spending, while an
increase in the stock of government debt induces a decline in G (since the
coefficient on B is positive).

Since the focus here is on the interrelation of government debt and
private assets, it is convenient to solve the model in terms of the variables B
and F. The first dynamic equation of the system is given by (10) which involves
B. A second equation is obtained by first noting that the definition of H as
the present value of future disposable non-interest income implies that, for
constant levels over time of non-interest income \( Y \) and lump-sum taxes \( T \), \( H = \frac{Y - T_0}{r + p} \). Substituting in (1) for H and in (3) for C with the resulting
expression gives a dynamic expression for W. Noting that \( F = W - B \) and using
(10) to substitute for B then gives the following dynamic expression involving
F:

\[ F = (r - p - d)F + \frac{g_2 - r}{1 - g_1} + \frac{(r - d)(Y - T_0)}{r + p} - \frac{G_0 - T_0}{1 - g_1} \]  

(12)

It is shown in the appendix that the system represented by equations (10) and
(12) has the characteristic roots \( (r - g_2)/(1 - g_1) \) and \( r - p - d \) and is globally
stable if both are negative, as is assumed.

The system is depicted in the B–F diagram in Figure 3. The slope of the
\( F = 0 \) locus may be positive or negative depending on the sign of the coefficient of \( B \) in (12). This coefficient is negative if an increase in \( B \) causes the rate of financial wealth accumulation to fall by more than the rate of public debt creation. If, as shall be assumed here, this is so, the locus is negatively sloped. The \( B = 0 \) locus is independent of \( F \) and thus appears as a horizontal line. The phase arrows of the diagram indicate that the system is globally stable.

Solving (10) and (12) simultaneously (actually the solution is recursive since (10) is independent of \( F \)) gives the steady-state levels of \( F \) and \( B \):

\[
\begin{align*}
\bar{B} &= \frac{G_0 - T_0}{g_2 - r} \\
\bar{F} &= \frac{(r-d)(Y-T_0)}{(p+d-r)(r+p)} - \frac{G_0 - T_0}{g_2 - r}
\end{align*}
\]  

(13)  
(14)

Observe that if exogenous government spending is financed out of taxes \( (G_0 = T_0) \), then the steady-state level of government debt is zero. For \( G_0 > T_0 \), \( \bar{B} \) is greater, the larger is \( g_2 \), i.e. the more sensitive is government spending to the stock of public debt. Intuitively, the more spending falls in response to increases in debt, the more rapidly are budget deficits eliminated. Note also that \( \bar{B} \) does not depend on \( g_1 \). This parameter affects the rate at which government debt is accumulated, but not its equilibrium level.

The financial wealth constraint \( W = B + F \) together with (13) and (14) imply

\[
\bar{W} = \frac{(r-d)(Y-T_0)}{(p+d-r)(r+p)}
\]

(15)

while the steady-state level of human wealth (which as noted above is constant
over time for given values of \( Y \) and \( T_0 \) is given by

\[
\bar{H} = \frac{Y-T_0}{r+p}
\]  

(16)

\( \bar{G} \) remains as given by (7), reproduced below:

\[
\bar{G} = \frac{p(p+d)(Y-T)}{(p+d-r)(r+p)}
\]  

(7)

Substituting (13) for \( B \) in (11) gives the steady-state level of total government spending:

\[
\bar{G} = \frac{g_2 T_0 - rG_0}{g_2 - r}
\]  

(17)

Note that if \( G_0 > T_0 \) then \( G < G_0 \). This implies that an exogenous increase in government spending not fully financed by taxes requires a decline in total spending in the long run. This is necessary in order to finance the interest payments associated with the addition to the government stock of debt. The spending reduction is assured by the government spending relation (9) that implies that spending declines as the outstanding stock of government debt increases.

Consider now the following spending/tax change fiscal policies: (i) a permanent spending increase and (ii) a permanent tax cut. In both of these cases it is assumed that government spending is adjusted endogenously according to (9) and that public debt changes to finance any resulting budget imbalances. The steady-state comparative statics multipliers are presented in the appendix.

**Government Spending Increase**

Consider first the effects of a permanent increase in the exogenous component of spending (rise in \( G_0 \)). From (10) and (12) observe, as illustrated
in Figure 4, the \( B = 0 \) schedule shifts up and the \( F = 0 \) schedule shifts left. The exogenous spending increase initially causes total spending to increase as well and generates a budget deficit and an increase in government debt issued to the private sector (see equations (10) and (11)). Since the spending increase leaves disposable income unaffected, private consumption remains constant. As private individuals add government bonds to their portfolios they correspondingly reduce their foreign-asset holdings. The increase in \( B \) and fall in \( F \) continue until the new steady-state is reached at \( E_1 \).

By inspection of (7) and (13)-(16) it may be determined that in the new steady-state the stock of government debt is higher and the stock of foreign assets is lower, while financial and human wealth and consumption are unchanged. Thus in the long run the increase in government debt generated by a spending increase results in a one-for-one decrease in private foreign asset holdings. Moreover, this one-for-one substitution of domestic for foreign assets prevails along the entire path to the steady state as well. For the spending increase, as long as it is financed without any change in taxes, has no effect on disposable income nor on human or financial wealth. Thus any additions to private portfolio holdings of government assets require an equivalent reduction in foreign asset holdings. In sum, government spending-induced budget deficits financed by domestic debt lead to matching capital account surpluses by the private sector.\(^{12}\)

**Tax Decrease**

Consider now the effects of a permanent tax decrease (fall in \( T_0 \)). For the purposes of comparison with the previous case it is assumed that \(-dT_0 = dG_0 > 0\). It will be seen that the tax cut causes government debt to increase as in the previous case, but that foreign asset holdings do not necessarily fall by the same amount.
Figure 4 can also be used to depict the effects of a tax cut. The $B = 0$ locus shifts up and the $F = 0$ locus shifts down, though the latter shifts by a lesser amount than in the case of the government spending increase. The reduction in taxes requires additional government debt which endogenously induces more government spending initially (as long as $g_1 > 0$). From (11) observe that (since $g_1/(1-g_1) > 1$) the tax decrease dominates this increase in spending implying that the public sector initially runs a budget deficit, and the stock of government bonds marketed to the private sector rises. The private sector also experiences increases in disposable income and human wealth which cause an immediate jump up in consumption. Thereafter foreign assets are gradually decumulated. (Whether consumption continues to rise or begins to fall depends on the nature of the financial wealth effect discussed below.)

As in the case of a government spending increase, $B$ is higher and $F$ is lower in the steady state. Given the assumption that the increase in $G_o$ and decrease in $T_o$ are equal in magnitude, the rise in $B$ is the same in the two cases. However, private foreign asset holdings do not necessarily decline by the same absolute amount. The relative change in the steady-state levels of $B$ and $F$ may be determined from (13) and (14):

$$\frac{dF}{dT_o} = -1 + \frac{(r-d)(g_2-r)}{(p+d-r)(r+p)} > -1 \text{ as } r > d \quad (18)$$

Thus while the increase in government debt that accompanies the permanent tax increase results in the crowding out of private foreign assets, the extent of this crowding out depends on the relative magnitudes of the interest rate and rate of time preference. The crowding out is one-for-one only if the two parameters are equal. If $r$ exceeds $d$ (as assumed in Figure 4) then the stock of foreign assets falls by less than the increase in public debt. The reason
for this dampening in the crowding out effect is that when \( r > d \) the tax cut generates an increase in total financial wealth (see equation (15)) and a correspondingly greater demand for foreign assets abroad. (While steady-state consumption is in any event higher as a result of the tax cut, the increase in financial wealth also implies that, following its initial jump up, consumption subsequently rises, rather than falls, along the adjustment path to its new equilibrium level.) Thus, if \( r > d \), the budget deficit induced by a tax cut exceeds the magnitude of the capital account surplus run by the private sector.

It is interesting to compare the impact changes in \( B \) and \( F \) with their long-run adjustment. Expressions (10) and (12) imply that for given initial levels of \( B \) and \( F \), the impact multipliers for a change in \( T_0 \) are

\[
-dB/dT_0, t=0 = 1/(1-g_1) > 0
\]

\[
-dF/dT_0, t=0 = (r-d)/(r+p) - 1/(1-g_1) < 0
\]

From (13) and (14) the corresponding steady-state multipliers are

\[
-d\bar{B}/dT_0 = 1/(g_2-r) > 0
\]

\[
-d\bar{F}/dT_0 = -1/(g_2-r) + (r-d)/[(p+d-r)(r+p)] < 0
\]

Comparison of these expressions reveals that a sufficient condition for both \( B \) and \( F \) to overshoot their long-run levels in the short run is \( g_2 + g_1 > 1 + r \). The intuition behind this condition is straightforward. The higher is the value of \( g_1 \) the more sensitive is total government spending to additional government borrowing and hence the larger is the initial budget deficit and increase in \( B \) that arises from the tax cut. The higher is the value of \( g_2 \) the greater is the the contractionary dampening effect of the stock of government debt on total government spending and hence the less is the long-run increase in \( B \) before the budget is again balanced.\(^14\)
IV. Dynamics and Steady State with Domestic and Foreign Public Debt Finance

The effect of the issuance of government debt to foreigners is now incorporated into the model. Recall the definition of $B^f$ as the stock of government debt held by foreigners, and of $B^d$ as the stock of government debt held by the domestic private sector. The total stock of debt $B$ equals $B^f + B^d$. The small country assumption implies that the interest rates on domestic and foreign public debt are equal. To simplify the analysis it is assumed that government debt issued abroad remains over time a constant fraction $b$ of the government's total debt: $B^f = bB$.

The introduction of foreign debt requires modification of the government spending rule (9) to include the effects of the level and changes in foreign debt on total government spending:

$$G = G_0 + g_1 \dot{B}^d - g_2 B^d + h_1 \dot{B}^f - h_2 B^f; \quad g_1, g_2, h_1, h_2 > 0$$

(19)

Thus total government spending depends positively on both changes in public domestic and foreign debt and negatively on both the levels of domestic and foreign debt. In general, the relative sensitivity of spending to these variables varies. One plausible relationship is that spending out of new domestic borrowing exceeds that out of new foreign borrowing ($g_1 > h_1$), while the stock of foreign debt has a more contractionary effect on spending than does the stock of domestic debt ($h_2 > g_2$). (Since interest rates on domestic and foreign debt are equal, the stronger dampening effect of foreign debt on spending must be attributed to other factors such as a relative aversion to being indebted to foreigners).

Note that since $B^f = bB$ and hence $B^d = (1-b)B$, then $\dot{B}^f = \dot{b}B$ and $\dot{B}^d = (1-b)\dot{B}$. Substituting out in (18) for all variables in terms of $B$ and $\dot{B}$, inserting the resulting expression into (4), and solving in terms of $B$ gives the analogue
dynamic expression for (10):

\[ B = \frac{(r-k_2)B}{1-k_1} + \frac{G_0-T_0}{1-k_1}, \]  
(20)

where \( k_1 = (1-b)g_1 + bh_1 \) and \( k_2 = (1-b)g_2 + bh_2 \) define the sensitivities of total government spending to the change and stock of total public debt as weighted averages of those for domestic and foreign debt. To ensure satisfaction of the government budget constraint it is assumed that \( k_2 > r \) and \( 1 > k_1 \). The interpretation of these conditions is similar to that given in Section III for \( g_2 \) and \( g_1 \).

By analogy to the derivation of (12), the following expression may be derived for dynamic movements in \( F \):

\[ F = (r-p-d)F + \left( \frac{k_2-r}{1-k_1} + r-p-d \right)(1-b)B + \frac{(r-d)(Y-T_0)}{r+p} - \frac{(1-b)(G_0-T_0)}{1-k_1}. \]  
(21)

Note that if \( b = 0 \), all government deficits are purely financed by domestic debt held by the private sector and the system simplifies to that utilized in the previous section.

Figure 3 remains relevant for depicting the steady-state equilibrium for the general case in which \( 0 < b < 1 \). The \( B = 0 \) locus is horizontal and the \( F = 0 \) locus is negatively sloped as before (assuming the coefficient term on \( B \) in (21) is negative). Note now, however, that the \( F = 0 \) locus becomes steeper the larger is \( b \) and the more budget deficits are financed by foreign debt. In the extreme case that \( b = 1 \) and all deficits are financed by government foreign borrowing, the \( F = 0 \) locus is vertical (at the steady-state level of \( B \)).

The steady-state levels of \( B \) and \( F \) obtained by setting \( \dot{B} \) and \( \dot{F} \) equal to zero and solving (20) and (21) are
\[ B = \frac{G_0 - T_0}{k_2 - r} \]  (22)

\[ F = \frac{(r-d)(Y-T_0)}{(p+d-r)(r+p)} - \frac{(1-b)(G_0 - T_0)}{k_2 - r} \]  (23)

Observe that \( B \) only depends on \( b \) through its effect on the weighted sensitivity of government spending to total public debt. \( F \) depends on \( b \) as well through its effect on private sector holdings of domestic bonds and the resulting displacement of foreign assets in private portfolios.

\( \bar{C}, \bar{W}, \) and \( \bar{H} \) remain as given by (7), (15), and (16), respectively. \( \bar{G} \) can be found to be given by

\[ \bar{G} = \frac{k_2 T_0 - r G_0}{k_2 - r} \]  (24)

The existence of public foreign assets implies that the country's net capital movements do not depend solely on movements of private asset holdings. Defining \( F^n = F - B^2 \) as the net national foreign asset position, (22) and (23) give the steady-state level of \( F^n \):

\[ F^n = F - bB \]

\[ = \frac{(r-d)(Y-T_0)}{(p+d-r)(r+p)} - \frac{G_0 - T_0}{k_2 - r} \]  (25)

Consider now how the effects of exogenous changes in government spending and taxes depend on the mix of financing between domestic and foreign public debt. It is assumed that \( k_1 \) is a negative and \( k_2 \) is a positive function of \( b \), i.e. \( g_1 > h_1 \) and \( g_2 < h_2 \). The steady-state multipliers are presented in the appendix.
Government Spending Increase

For $0 < b < 1$, an increase in $G_0$ causes the $B = 0$ locus to shift up and the $F = 0$ locus to shift left (not shown), but by lesser amounts than in the case in which all public debt is domestically financed. Thus the higher is $b$ and the more that the government borrowing necessary to finance the spending increase takes the form of foreign debt, the less is the resulting increase in $B$ and decrease in $F$. $\bar{C}$, $\bar{W}$, and $\bar{H}$ remain unchanged.

Intuitively, if the government finances its deficit with foreign debt which induces less new spending and has a more contractionary effect on spending in the long run, then the cumulative budget deficit is less in the long run. Accordingly, the increase in public debt necessary to finance the deficit is smaller. Note that this result is reversed if the government spends more out of new foreign borrowing ($g_1 < h_1$) and total spending is less sensitive to the stock of foreign debt ($g_2 > h_2$).

As found in Section III, inspection of (22) and (23) indicates that private domestic bond holdings displace private foreign asset holdings one-for-one. Since interest rates and private financial wealth remain constant, the less the government finances its deficits by issuing domestic debt, the less is the effect on foreign asset holdings in the portfolios of the private sector. In the extreme case that $b = 1$ and the increase in government spending is financed totally by public debt issued to foreigners there is no displacement and the level of private foreign assets remains constant.

What happens to the country's net national foreign asset position as a result of the spending increase? Clearly the fall in $F$ (except when $b = 1$) and rise in $B^f$ (except when $b = 0$) observed above imply that $F^n$ falls and the net national foreign asset position deteriorates (see (25)). A comparison of (22) and (25) reveals that $F^n$ falls by the same magnitude as total public debt rises. Thus government spending-induced budget deficits lead to a matching capital account surplus by
the combined private and public sector irrespective of the debt finance mix. Interestingly, the greater the relative degree of foreign public borrowing, the less is the decline in the national foreign asset position. The assumptions that $h_2 > g_2$ and $h_1 < g_1$ again play a role in explaining this result: since a smaller cumulative budget deficit results from the initial spending increase, a smaller displacement of private foreign assets occurs.\(^\text{15}\)

**Tax Cut**

A decrease in $T_o$ causes the $B = 0$ locus to shift up. The shift in the $F = 0$ locus is ambiguous. The greater is $b$ and the relative mix of foreign borrowing, the greater the possibility that it shifts to the right (assuming $r > d$). The impact of foreign borrowing on the steady-state response of $B$ and $F$ to a tax cut are analogous to those discussed in the case of a spending increase: the greater is $b$, the less the resulting increase in $B$ and decrease in $F$.

The increase in total public debt is dampened since the more public debt is financed by foreign borrowing, the greater the long-run reduction in total government spending, and the less the cumulative budget deficit. The fall in $F$ is dampened because the less the issuance of domestic public debt, the less the crowding out of foreign assets in private sector portfolios.

It is in fact possible that the tax cut may lead to an *increase* in private foreign asset holdings if $b$ is large enough (i.e. close enough to 1). Such a situation may be interpreted as a manifestation of capital flight: government foreign borrowing is accompanied by greater private asset holdings. The actual relation between changes in $F$ and $B$ induced by the decrease in $T$ can be obtained from (22) and (23):

\[
\frac{dF}{dT_o} = \frac{(r-d)(k_2-r)}{(p+d-r)(r+p)} - (1-b)
\]  
\[
\frac{dB}{dT_o} = \frac{(r-d)(k_2-r)}{(p+d-r)(r+p)}
\]  
\[
\text{Eq. (26)}
\]
The reason why private capital outflows may arise in response to a tax decrease, in contrast to the increased spending case, is that a tax cut generates a wealth effect (if \( r > d \)) on demand for all assets, which, as \( b \) increases, is more likely to dominate the crowding-out effect of any domestic debt on foreign assets.

A permanent tax cut also results in a long-run decline in net foreign assets; while government foreign liabilities rise and the direction of change of private assets is in general ambiguous, on balance it may be determined that net foreign assets fall. Thus the possible capital flight phenomenon is never great enough to exceed the government's increase in foreign liabilities. If the government borrows abroad to finance a tax-cut induced budget deficit the international investment position always declines. Moreover, (22) and (23) imply (note the similarity to (18)):

\[
\frac{dF^*_n/dT}{dB/dT} = -1 + \frac{(r-d)(k_2-r)}{(p+d-r)(r+p)} > -1 \text{ as } r > d
\]

Thus for \( r > d \), the total budget deficit induced by the tax cut exceeds the magnitude of the combined capital account surplus run by the private and public sectors.

V. Interest Rate Increase

This section briefly considers the effect of a disturbance to the economy such as an interest rate increase and discusses the implications of different government policy responses for the stocks of public debt and private assets.

Observe first that an increase in \( r \) must be accommodated by the government in a way which satisfies its short and long-run budget constraints. Consider first that the government does so by leaving \( G_0 \) and \( T_0 \) unchanged. Note from
(20) and (22) that, with given levels of $G_0$ and $T_0$ (and assuming $G_0 > T_0$), the government must initially accommodate the effects of the disturbance on the interest costs of existing debt by running a budget deficit and issuing additional debt. In the long run the increase in the stock of debt endogenously results in appropriately lower government spending levels so that sufficient revenue is obtained to cover the higher interest charges on the initial debt as well as charges on the additional debt.

This case is depicted in Figure 5. The increase in $r$ causes the $B = 0$ locus to shift up. The shift of the $F = 0$ locus, however, is ambiguous and depends on the relative weight of three factors: for any given level of $B$, on the one hand, foreign asset holdings increase because of (i) a financial wealth effect generated by the greater return earned on outstanding assets, and (ii) a decline in consumption induced by a fall in the present value of noninterest disposable income and hence in human wealth, while, on the other hand, foreign assets decline (iii) to the extent that deficit finance by domestic debt displaces foreign private assets. On balance, for $b$ sufficiently great, the former effects can be expected to dominate, implying that the $F = 0$ locus shifts to the right as shown in Figure 5. Clearly, the steady-state stock of public debt rises. The long run change in $F$ is unclear. As $b$ approaches 1 the likelihood that $F$ rises increases.

It is interesting to consider how $F$ is affected if for some reason the government is unable to accommodate the interest rate increase by a rise in $B_f$, because of, for example, a ceiling imposed by foreign lenders. In this instance, in order to satisfy its budget constraint, the government will be compelled to make some offsetting change in $b$, $T_0$, or $G_0$. More specifically, in response to the increase in $r$ it is necessary either for the government to lower $b$, i.e. raise the share of its debt burden that is financed by domestic debt, while at the same time allowing the total stock of debt to grow; or raise
taxes and/or lower spending in order to reduce the overall level of public debt.

The former option presumes additional debt can be marketed to the domestic private sector which dampens any accumulation of private foreign assets. If this is not possible, then the latter option must be adopted which implies that the greater interest charges on debt service must come at the expense of spending or in taxes. The requisite decrease in spending or increase in taxes unambiguously induces an accumulation of foreign assets by the private sector. Thus an interest rate increase totally accommodated without any increase in government debt results in an outflow of private capital.

VI. Conclusions

This paper has analyzed the effects of government spending, tax, and deficit-finance policies on the foreign borrowing and asset accumulation behavior of the private sector in a small, open economy. Spending increases and tax cuts inducing budget deficits are assumed to result in a long-run decline of government spending so that the budget is eventually balanced. The mix of government financing between domestic and foreign public debt of deficits induced by spending or tax level changes has significant effects on the foreign asset holdings of the private sector. In particular, the greater the extent of foreign financing of budget deficits, the less the decline in private foreign asset holdings in response to an increase in government spending. In the case of a tax cut it is possible that private foreign assets may actually rise.
Footnotes

1. The total external debt of Argentina and Mexico grew by $27 and $35 billion, respectively, over the period 1978–82. Dornbusch (1984) estimates that over this period private capital flight into foreign real and financial assets amounted to as much as $23 billion from Argentina and $36 billion from Mexico.

2. Since \( p \), the probability of death is constant, the expected remaining life of any individual is given by

\[
\int_0^\infty t e^{-pt} \, dt = \frac{1}{p}.
\]

3. The assumption of a large number of identical individuals implies life insurance companies can offer these guarantees risklessly.

4. Time subscripts are ignored where convenient throughout the analysis.

5. Since this is a real model without any monetary and price variables, the exchange rate is assumed constant and domestic and foreign variables are expressed in the same currency.

6. These purchases are assumed not to affect the marginal utility of private consumption.

7. If the government has a finite horizon then the results of the analysis would depend on the relative survival probabilities of the public and private sectors.

8. As pointed out by Blanchard (1985) and Buitert (1984), in the infinite horizon case where \( p = 0 \), a steady state value of \( C \) exists only if \( r = d \). The value of \( \bar{F} \) is in this case indeterminate.

9. Ideally such a relation should reflect the optimizing behavior of the government. Blanchard adopts a similar specification for the endogenous determination of total taxes and assumes the analogue coefficient to \( g_1 \) is zero. Buitert (1984) works with a relationship like (9), but concentrates on the case in which \( g_2 = 0 \).

10. Note that a positive intercept of the horizontal axis is assured for the \( F = 0 \) locus if \( r > d \) and \( G_0 > T_0 \).

11. Another policy experiment often considered in models like this is a short-run tax decrease financed by a long-run tax increase, while leaving government spending unchanged. It can be shown that as long as the private sector has a finite horizon, individuals will not take full account of the prospective tax liabilities associated with the need to service the public debt used to finance the temporary budget deficits. Accordingly, they treat government debt as net wealth and will increase current consumption and foreign assets.
12. An increase in \( r \) increases the sensitivity of changes in \( F \) and \( B \) to the increase in \( G_0 \). A change in \( p \) has no effect on result.

13. As noted in footnote 9, Buitert also considers the effects of a permanent tax cut in a model in which \( g_2 = 0 \) and \( g_1 > 1 \). These assumptions imply that the initial effect of the tax cut is the creation of a budget surplus.

14. A rise in \( r \) magnifies the increase in \( B \) resulting from a tax cut. The effect on \( F \) is unclear. On the one hand, the accompanying rise in \( W \) dampens the fall in \( F \). On the other hand, the greater rise in \( B \) causes the private sector to reduce \( F \) by more. A fall in \( p \) and lengthening in private sector horizons, increases the sensitivity of \( F \) (if \( r > d \)) to tax changes and magnifies the fall in \( F \). \( B \) is not sensitive to \( p \).

15. It can be demonstrated that \( F_n \), \( F \), and \( B \) may overshoot their new steady-state levels in the short run under a condition similar to that discussed in Section III.

16. As long as the private sector has positive net wealth the interest rate increase raises \( W \); any adverse effects of higher charges on foreign borrowing are offset by a greater return on holdings of domestic public or foreign private assets.
References


Appendix

I. Derivation of Aggregate Private Behavioral Relations

In this appendix the equations for aggregate private consumption and wealth accumulation are derived from individual maximizing behavior.

Denote by $c_{st}, y_{st}, w_{st},$ and $h_{st}$ consumption, noninterest disposable income, financial wealth, and nonhuman wealth in period $t$ of an individual born in period $s$. (Taxes are ignored here implying that $y$ equivalently represents noninterest income before taxes as well.)

Let the utility function in period $t$ of an individual born in period $s$ be

$$U_t = \int_t^\infty \log[c_{sv}]e^{d(t-v)}dv$$

where $d$ denotes the rate of time preference. Since the time of death is the only source of uncertainty and the probability, as of period $t$, that the individual will remain alive in period $v$ is $e^{p(t-v)}$, his expected utility is

$$E_t[U_t] = \int_t^\infty \log[c_{sv}]e^{(d+p)(t-v)}dv.$$  

(1.2)

Individuals receive (pay) a total rate of return of $r+p$ on their financial assets (liabilities), $r$ in the form of interest and $p$ from (to) the insurance company. The dynamic budget constraint in period $t$ for an individual born in period $s$ is thus

$$w_{st} = (r+p)w_{st} + y_{st} - c_{st}$$

(1.3)

The transversality condition implies that if the individual is still alive at time $v$

$$\lim_{v \to \infty} e^{(r+p)(t-v)}w_{sv} = 0,$$

(1.4)

which together with the dynamic budget constraint implies the lifetime budget constraint.
\[ \int_{t}^{\infty} c_{st} e^{(r+p)(t-v)} dv = w_{st} + h_{st}, \quad (1.5) \]

where
\[ h_{st} = \int_{t}^{\infty} y_{st} e^{(r+p)(t-v)} dv. \quad (1.6) \]

Maximization of (1.2) subject to (1.5) implies
\[ c_{st} = (p+d)(w_{st} + h_{st}). \quad (1.7) \]

Population is normalized so that every cohort born in period \( s \) has the size \( p \). The size of the cohort declines through time as the result of death and in period \( t \) has the size \( p e^{d(s-t)} \). (The law of large numbers ensures that the relative shrinkage in the size of a given cohort from one period to the next equals the probability of death.) The size of the population, summed across cohorts, at any time \( t \) is therefore
\[ \int_{-\infty}^{t} p e^{d(s-t)} ds = 1. \]

For any individual variable \( x_{st} \), the corresponding aggregate variable \( X_t \) is defined as
\[ X_t = \int_{-\infty}^{t} x_{st} p e^{d(s-t)} ds. \quad (1.8) \]

Denote by \( C_t, Y_t, W_t, \) and \( H_t \) aggregate consumption, noninterest disposable income, financial wealth, and nonhuman wealth, respectively.

From (1.7) the aggregate consumption function may be expressed as
\[ C_t = (p+d)(H_t + W_t) \quad (1.9) \]

which is equation (1) in the text.

Assuming that all individuals have the same, constant level of noninterest disposable income over time, (1.6) implies aggregate human wealth can be expressed as
\[ H_t = \int_{-\infty}^{t} s p e^{d(s-t)} ds = \int_{t}^{\infty} Y_ve^{-(r+p)(v-t)} dv \quad (1.10) \]

or in differential equation form
\[ \dot{H}_t = (r+p)H_t - Y_t, \quad (1.11) \]

which is equation (2) in the text.
The evolution of aggregate financial wealth may be obtained by substituting equation (1.3) into the aggregate definition of wealth implied by (1.8):

\[ W_t = \int_{-\infty}^{t} W_{st} Pe^p(s-t) ds. \]  

(1.12)

The differential form of (1.12) gives equation (3) in the text:

\[ W_t = \frac{d}{dt} W_{tt} - pW_t + \int_{-\infty}^{t} W_{st} Pe^p(s-t) ds \]

\[ = 0 - pW_t + (r+p)W_t + Y_t - C_t \]

\[ = rW_t + Y_t - C_t \]

where \( W_{tt} \), the financial wealth of newly born individuals at time \( t \), is zero, and \(-pW_t\), the wealth of those who die at time \( t \) which is transferred to the life insurance company, cancels with the premium payments of those still alive.
II. Stability of Dynamic Systems

A. Equations (5) and (6) expressed in matrix form give

\[
\begin{bmatrix}
  C \\
  F
\end{bmatrix} = \begin{bmatrix}
  r-d & -p(p+d) & 0 \\
  -1 & r & 0
\end{bmatrix} \begin{bmatrix}
  C \\
  F
\end{bmatrix} + \begin{bmatrix}
  0 \\
  Y - T
\end{bmatrix}
\]

Factorization of the determinant of the system \((r-d)r-p(p+d)\) into the roots \(r+p\) and \(r-p-d\) implies the system is characterized by saddle-point stability if \(r+p > 0\) and \(p+d > r\).

B. Equations (10) and (12) expressed in matrix form give:

\[
\begin{bmatrix}
  \dot{\mathbf{B}} \\
  \dot{\mathbf{P}}
\end{bmatrix} = \begin{bmatrix}
  \frac{-(k_2-r)}{1-k_1} & 0 \\
  \frac{k_2-r}{1-k_1} + \frac{r-d-p(1-b)}{1-k_1} & \frac{r-p-d}{1-k_1}
\end{bmatrix} \begin{bmatrix}
  \mathbf{B} \\
  \mathbf{F}
\end{bmatrix} + \begin{bmatrix}
  \frac{G_0-Y_0}{1-k_1} \\
  \frac{(r-d)(Y-T_0) - (1-b)(G_0-Y_0)}{r+p-1-k_1}
\end{bmatrix}
\]

The system is globally stable under the conditions \(-\frac{(k_2-r)}{(1-k_1)} < 0\), \(r < p + d\), and the determinant, \((k_2-r)(p+d-r)/(1-k_1)\), is positive.
III. Comparative Steady-State Multipliers

<table>
<thead>
<tr>
<th>Increase in $G_0$</th>
<th>Decrease in $T_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{B}$</td>
<td>$1/(k_2-r) &gt; 0$</td>
</tr>
<tr>
<td>$\bar{B}_P$</td>
<td>$(1-b)/(k_2-r) &gt; 0$</td>
</tr>
<tr>
<td>$\bar{F}$</td>
<td>$-(1-b)/(k_2-r) &lt; 0$</td>
</tr>
<tr>
<td>$\bar{F}_n$</td>
<td>$-1/(k_2-r) &lt; 0$</td>
</tr>
<tr>
<td>$\bar{C}$</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{W}$</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{H}$</td>
<td>0</td>
</tr>
<tr>
<td>$\bar{G}$</td>
<td>$-r/(k_2-r) &lt; 0$</td>
</tr>
</tbody>
</table>

where $z_1 = (p+d-r)(r+p) > 0$ and $r-d > 0$

$k_1 = (1-b)g_1 + bh_1$

$k_2 = (1-b)g_2 + bh_2$

Note that the multipliers for full domestic debt finance can be obtained by setting $b = 0$. 
Figure 1