A Valuation Model for Developing-Country Debt with Endogenous Rescheduling

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Creditors have little recourse if a sovereign state repudiates its external debt obligations. They can, however, threaten to impose penalties if such action occurs which results in deadweight losses to the system as a whole. A preferred alternative for both borrower and lender is to recontract debt obligations. Reschedulings are a device that creditors can use to structure the incentives faced by borrowers such that repudiation is never a rational action.

This article develops a numerical method of valuing the option to reschedule. The model shows why fees are preferred to higher interest spreads during a rescheduling exercise; why maturities get shorter prior to a debt crisis but are lengthened in the rescheduling; why tougher supervision by regulatory authorities could be damaging to renewed voluntary loans; and why little has been done to attempt to seize the assets of countries that have not repaid any interest or principal for extended periods of time.

The model shows that lenders are willing to commit greater amounts if reschedulings are possible than if they are not, and that precommitment to provide additional funds at rescheduling can raise the market value of existing debt and should not be construed as concessions by commercial lenders. Alternately, the model can be used to improve systems for ranking country creditworthiness, to assess the degree of adjustment required to spark a full resumption of spontaneous lending, or to estimate by how much interest rates would have to fall to restore a country’s creditworthiness.

In a period of financial crisis for heavily indebted developing countries, creditor guidelines are needed for optimal choices between rescheduling outstanding debt and demanding full repayment. The dilemma for the creditor is that the latter may precipitate a repudiation of debt by the borrowing country, because of an inability or unwillingness to bear the full burden of debt servicing in the short term. These decisions and the terms of rescheduling depend on the estimated valuation of a country’s foreign exchange earning capacity, which is a determinant of the market value of its debt. Debt value is also a function of the variance in foreign exchange earnings, front-end fees, the interbank “prime” rate and the interest rate spread, new lending available to the country, debt maturity, and the

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amount of assets which a creditor could seize upon default. Reaching agreement on repayment terms of developing-country debt for this range of variables is understandably complex. Recent international debt negotiations reflect the need for further comprehensive analysis of debt valuation.

A predominant share of the international commercial debt of developing countries is in the form of syndicated credits. These credits are held by a relatively small number of banks. Absence of a widespread secondary market ensures that developing-country debt remains in the hands of a few known agents. A critical feature of such an arrangement is that it permits a negotiated rescheduling of obligations. Indeed, fourteen developing countries rescheduled commercial debt in 1983, in the midst of a severe downturn in the global economy, and there were a further twenty reschedulings in 1984. These reschedulings did not involve write-downs of principal or concessional interest rates. By contrast, during the last wave of reschedulings in 1932-33, during the Great Depression, international creditors lost up to 90 percent of the face value of their developing-country debt. These loans were in the form of bonds which automatically fell into default once coupon payments were missed.

The likelihood of external shocks triggering debt repudiation has been emphasized by Eaton and Gersovitz (1981) and Sachs and Cohen (1982). These papers suggest that repudiations stem from strategic behavior by debtors. When global conditions change, so do the relative payoffs from continued debt service and repudiation, and the latter may become an attractive option for the debtor. Kulatilaka and Marcus (1985) discuss the optimal timing of the repudiation decision. The analysis illuminates important features of developing-country debt: the existence of endogenous credit rationing, the need for self-enforcing contracts, the role of uncertainty, and the advantages of precommitment by a borrower. It is, however, couched in the context of bond financing, which limits strategic behavior by creditors. In addition, it predicts that reschedulings would involve concessional terms or debt write-downs. This does not seem to be an accurate description of recent events.

In this article we provide a more realistic framework in which a creditor is able to avoid the occurrence of a strategic repudiation by agreeing to reschedule debt payments under conditions of financial distress. We assume that a repudiation or default results in a deadweight loss. It is then in the best interests of all parties to design a flexible loan contract. A country would prefer to reschedule rather than repudiate its debt because it then retains the option to repudiate at a later stage. The lender prefers rescheduling because there is then some probability that favorable future developments will allow it to recover the full value of its debt.

This article describes a methodology for valuing debt claims on developing countries held by commercial banks. The value of the claim is given by the

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1. Repudiation refers to a debtor's action to unilaterally abrogate a debt agreement. Default refers to a creditor action to terminate a debt contract when a borrower fails to meet the contract obligations.
discounted stream of expected future debt service payments (the contractual payments weighted by the probability that a payment will be made). A lender’s behavior is motivated by the desire to maximize the value of its claim, which it does by rescheduling and by choosing the maturity, spread, and fees on its loans. The value of the debt claim (the “market” price of the debt) will differ from its nominal face value because of uncertainty about the evolution of the economy’s productive assets, which generate the revenue to service the debt. This contingent claims aspect of sovereign debt is discussed in section 1.

The outcome of the valuation process is an estimate of the market price of the debt, the price that would prevail if secondary markets existed. This price is useful for several reasons. First, it provides a cardinal measure of creditworthiness. If the price of debt in one country is higher than in another, it can be called more creditworthy in the sense that less adjustment would be required to restore full creditworthiness. Second, the price provides information about when and whether to expect new money to be forthcoming. Spontaneous lending from new creditors will occur only if the price of a debt claim is greater than or equal to its nominal face value. Lending from existing creditors may occur in other circumstances, however, as it could raise the value of the existing stock of creditors’ debt. Third, the approach allows the effects of exogenous variables, such as changes in the interest rate, to be quantified in a systematic way by analyzing their impact on the price of debt. Fourth, some light is shed on rescheduling policies and the role of alternative financial instruments, such as bonds. In this respect, a distinction is made between a rescheduling to overcome liquidity problems and one designed to alleviate insolvency.

Section II presents a numerical valuation model for developing-country debt, based on a Black and Scholes (1973) option pricing scheme in which the value of the debt varies with the value of the underlying assets. The novelty here is that sovereign debt never need be actually paid off as long as it is serviced. A standard option pricing formulation, however, only considers a finite maturity. Section III presents some simulation results for various parameters. We also illustrate the impact of optimal rescheduling on the value of debt. This supports the conjecture (Sachs 1983) that the development of syndicated Eurocredits was an important innovation that promoted the sizable growth of external liabilities of developing countries. In the same framework, we are able to explain certain characteristics of the developing-country loan market—the shift to short-term maturities as the stock of debt grows; the existence of credit rationing and the associated cutoff of new lending at certain times; the process of reschedulings involving high front-end fees but only minor changes in interest rates; and the absence of any attempts by creditors to push for tough legislation to increase their ability to seize country assets in the event of a default.

2. Fledgling markets do exist for developing-country debt, but information on prices and quantities traded is very scanty. The markets are more in the nature of bilateral swaps than of competitive trades.
I. Contingent Claims Aspects of Sovereign Debt

International lending to developing countries takes place in an institutional framework from which collateral, seniority provisions, and the seizure of assets upon bankruptcy or default are absent. This distinguishes sovereign debt contracts from corporate debt. Freed from the obligation to perform or pay damage measures upon unilateral breach of a contract, parties will adhere to contract terms only if the gains from doing so exceed the costs. When a firm goes bankrupt, creditors receive ownership of the real assets; by contrast, creditors of sovereign countries are able to recover only a small fraction of a debtor country's tradable wealth. Creditors can, however, inflict deadweight losses on a country that repudiates its debt by restricting its future access to financial markets and by creating general impediments to its international trade. The existence of these penalties allows some international lending to take place until outstanding debt reaches a ceiling (Eaton and Gersovitz 1981; Sachs and Cohen 1982).

As with any financial claim, the value of debt to creditors is determined by the present value of future repayments. The occurrence of a repayment, however, depends not only on the country's ability to repay but also on its willingness to repay. If creditors were able to distinguish between these two behaviors, they could choose not to apply deadweight penalties when repudiation results from an inability to repay. It is reasonable to think, however, that creditors cannot discriminate among borrowers in this way. Furthermore, enforcement of penalties in a particular case would be important to maintain the credibility of the threat in other cases. Since neither party benefits from such punitive measures, it is in their best interests to reduce ex ante the likelihood of unilateral repudiation. This can be done by designing flexible contracts that allow some sharing of the risk of unforeseen events without precipitating a repudiation or default.

Suppose a country has borrowed up to its credit ceiling. Following a large adverse shock, it may decide that repudiation is preferable to the further resource outflow involved in meeting a debt repayment obligation. It is then in the best interest of the creditor to negotiate and reschedule part or all of the required payment. In the latter case, the country's incentive to repudiate is removed: it does not suffer the immediate penalty associated with repudiation, and it retains the option to repudiate in the future. Similarly, the bank is better off: an immediate payment would not have been made, regardless, but rescheduling increases the likelihood of future payments. Indeed, the recent debt crisis shows the default is generally avoided by a combination of short- and long-term rescheduling of both interest and principal repayments.

Rescheduling of interest obligations leads to a buildup of debt. Over time, if unfavorable states of nature persist, this creates a situation in which the country cannot repay. If, conversely, favorable shocks occur, the country would be able

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3. See Sadeq (1985) for a full exposition of the concepts developed here.
4. A "state of nature" is the complete characterization of economic conditions.
and willing to repay the full amount of the debt obligation. In our analysis, uncertainty rests with the value of future foreign exchange earnings discounted to the present. For example, unexpected changes in the real interest rate would affect the value of such earnings considerably. By explicitly incorporating the possibility of rescheduling into sovereign debt contracts, our analysis can focus on the long-run ability of the country to repay its debt.

In the following paragraphs, we will first define a measure of the assets underlying the claim on a sovereign country and then discuss the effect of alternative rescheduling policies on the debtor’s behavior and consequently on the market value of debt. Finally, we will show that sovereign debt can be viewed as a contingent claim on the value of the country’s current reserves and potential future trade surpluses, its “tradable value.”

The Underlying Assets

In the case of a corporation, debt is essentially a prior claim on corporate assets. This stems from the legal rights of creditors to appropriate all assets in the event of bankruptcy. The “assets” of a country, however, are not transferable in any meaningful sense. For our purposes, country assets have three aspects. First, they represent the capacity of the economy to earn foreign exchange to service the debt. The total value of the assets is the amount that a foreign investor would pay in exchange for the rights to all the country’s future net foreign earnings. It is given by the present discounted value of potential future trade surpluses. Second, they represent the base against which creditors, or their governments, can impose deadweight penalties in the event of repudiation. Following Sachs and Cohen (1982), it is convenient to think of the penalty as an embargo on future trade. Then, the higher the assets, the higher the penalty. Third, they include some appropriable elements, such as a country’s holding of foreign exchange reserves abroad, which can be seized by a creditor upon default.

The asset value is not readily observable by banks but must be derived by applying standard valuation techniques to the stream of anticipated foreign exchange cash flows. The key determinants are the level, expected growth rate, and risk characteristics of the net foreign exchange flows. Precise derivation of the asset value requires an in-depth analysis of trade prospects and is a key variable in the creditor’s decisionmaking. Note that the asset value also incorporates current reserves and therefore fully characterizes the foreign wealth of the debtor. A short-cut measure to determine this value is applied to twenty high-debt developing countries in the appendix.

The creditor’s valuation is the market price of the assets, \( X \), which is, in general, different from the value of the assets to the sovereign country, \( X' \). The preferences of the country (such as risk aversion), the extent of credit rationing, and the correlation of earnings with domestic (nontradable) production will all clearly play a role in determining what value the country places on its assets.
This, in turn, will affect how and when the country chooses to repudiate its external debt. We will show below that it is reasonable to assume that the country will find it is in its best interest to accept the repayment condition chosen by the creditor if the latter adopts a flexible rescheduling policy. We are thus able to abstract from the important issues discussed in the preceding section and look only at the value of the claim to creditors.  

By construction, the asset value at any time reflects all the information then available to creditors. Consequently, the value changes only as a result of unexpected events, and it can be viewed as a random variable. For example, when exchange earnings are unexpectedly high or when the expectation of future earnings rises as a result of exogenous events (for example, a major oil discovery) the asset value rises. Note that, although the expected dollar return is always positive, changes in the asset value can be negative. That is, the asset value can fall from one period to the next, and imports can exceed exports (the trade balance can be negative). The total value of assets at any point in time, however, is always positive.  

We make two technical assumptions about the stochastic process which characterizes the asset value. First, the returns on the asset are intertemporally uncorrelated. Note that we do not require foreign exchange earnings to be intertemporally uncorrelated. Since expected earnings are a determinant of the asset value, we are only assuming that unexpected changes ("shocks") in the earnings stream are not forecastable from past expected changes. Second, the variance of the asset return is constant through time. This means that the variance of the change in value is proportional to the asset value: the larger the value, the larger the amplitude of the change.  

5. We do not consider issues of moral hazard, which may make the lender's assessment of the asset value a function of a bargaining equilibrium between borrower and lender (see Gennotte 1986).  
6. A positive $X$ is an essential requirement for a net debtor country, or else the intertemporal budget constraint will be violated.  
7. These assumptions are summarized by viewing the stochastic process governing the asset price as a lognormal process. The asset value evolves as:

$$dX = X\mu dt + X\sigma dz$$

where

$X = \text{instantaneous asset value as perceived by the lender}$

$\mu = \text{expected instantaneous rate of return}$

$dt = \text{time (instant)}$

$\sigma = \text{constant instant mean standard deviation of asset returns}$

$dz = \text{standard Wiener process}$

$X$ is assumed to have a continuous sample path. The value of the asset at time $t + \tau$ is lognormally distributed,

$$X_{t+\tau} = \exp [(\mu - 1/2 \sigma^2) \tau + \sigma \sqrt{\tau} dz]$$

with expected value $X_{t+\tau}$:

$$E(X_{t+\tau}) = X_{t+\tau}.$$
Rescheduling Policies

The value of debt depends on the magnitude and risk characteristics of promised payments. Consequently, it depends on the rescheduling policy chosen by the lender. The optimal rescheduling policy, from the lender's point of view, is the one which maximizes the value of its claim. The lender's choice, however, is constrained because the sovereign country always has the option to repudiate unilaterally. In addition, the lender is constrained by external and internal regulations. If there is little likelihood of any payment being received because a country is unable to repay, these regulations force the lender to declare a default and to write off its loans. This forces creditors to set a limit to the country's debt relative to its asset value, above which default occurs.\(^8\)

Debtor Behavior. The sovereign country owns its current reserves and a stream of future exchange revenues, which are valued at \(X_t\), but has contracted a liability in the form of the promised debt payments. The strategy for the borrower is thus to maximize the value of its residual claim on the assets. Because of the risk of repudiation or default, there is a difference between the market value of the debt, \(C_t\), and the face value, \(D_t\).

At any time \(t\), the value of the country's net external or tradable wealth is given by \(X_t - C_t\), the asset value less the value of the debt. If the country repudiates its debt obligations, it retains the asset value \(X_t\), but suffers a penalty, with present value \(P_t\), imposed by lenders. The country would only have an incentive to repudiate if \(P_t < C_t\).

If lenders will reschedule a country's debt only if the penalty they are able to impose is larger than the country's valuation of its debt, then the country will never voluntarily choose to repudiate. The choice to repudiate arises at each repayment date; there is clearly no benefit to repudiation between payments. On repudiation, the sovereign country does not make the current payment, repudiates future payments, and thus incurs the associated penalties. If the creditor chooses to reschedule all payments, however, the country is always better off because it keeps the option to repudiate later. The decision is thus not whether to repudiate but rather whether to repudiate today or in the future. By repudiating now, the debtor forgoes its options to repudiate or to make debt payments later.

Creditor Behavior. Commercial lenders to developing countries, with access to efficient worldwide capital markets, are assumed, in contrast with the sovereign country, to be able to borrow and lend risklessly at the same rate. Their objective is to maximize the market value of their debt, subject to the constraints

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\(^8\) The limit can be thought of as imposed either by external regulators or by internal rules. For example, several banks have recently cleared up their Bolivian debt (at 10 cents on the U.S. dollar) to eliminate the administrative costs of continuing to hold the debt on these books.
imposed by the risk of repudiation and the risk that a country will be unable to repay its debt. A complete rescheduling policy is the choice, for any state of nature, between rescheduling, declaring a creditor to be in default, and obtaining complete payment.

In what follows, we assume that banks can impose a penalty on a country in the event of repudiation in proportion to the value of the country's gross tradable wealth, \( P = \lambda X_r \). A portion of this penalty is a deadweight loss, however, so that the return to lenders is not equal to the size of this penalty, but only to the seizable portion of total assets, \( \epsilon X_r \), where \( \lambda > \epsilon \). Lenders may be better off, therefore, if they avoid repudiation through a rescheduling policy rather than permit repudiation to occur.

Consider the following example of a rescheduling policy designed to eliminate the risk of repudiation. The incentive to repudiate varies with the country's tradable wealth and the amount of the promised payment. Suppose that the yearly required payment is a given fraction of outstanding debt. When the country is heavily indebted relative to its wealth, the required payment is large relative to repudiation costs (a fraction of tradable wealth), and incentives to repudiate are strong. The bank would jeopardize future payments by requiring immediate payment and thus prefers to reschedule. If the tradable value of the country increases relative to its debt because of good fortune, the repudiation incentive diminishes, and the creditor can safely require repayment of the debt obligations. Consequently, if the bank adopts a rescheduling policy which postpones payments until the country's tradable wealth recovers from very low levels, strategic repudiation will be avoided.

This proposition does not imply that the country's ability to repudiate the debt contract unilaterally is irrelevant. In fact, if there were no penalty that lenders could impose in retaliation for repudiation, the value of debt to lenders would always be zero. We only claim that lenders are able to remove the incentive for strategic repudiation by adopting a flexible rescheduling policy. They are then able to take advantage of the wedge between the penalty suffered by the country on repudiation and the amount that they directly recover. Avoiding the deadweight costs of repudiation renders the debt contract self-enforcing for both the borrower and the lender.

A similar logic applies to a rescheduling policy designed to minimize the risk of bankruptcy. As the value of tradable assets relative to debt approaches zero, the value of debt to the lender tends to zero as well because the country will be unable to repay. The lender does not benefit from calling the country in default since it can appropriate only a small fraction of assets, an amount which may be less than the total debt outstanding. Thus, when a country is unable to service debt, lenders would prefer to reschedule rather than call a default. However, our analysis requires some assumption about the termination of the debt contract in case of adverse economic events. We model this in terms of the constraints on exposure faced by international banks because of internal and external regula-
Such limitations prevent the bank from granting complete reschedulings perpetually. They have the same effect on the lenders' choice of rescheduling policy as the risk of voluntary repudiation by the borrower. In addition, this "automatic" bankruptcy serves the bank in the sense that it represents a credible threat to the country (especially if bankruptcy is accompanied by punitive measures) and provides an incentive for the country to comply with the creditors' rescheduling policy. Given the deadweight costs of bankruptcy, it is in the mutual interest of both borrowers and creditors to adopt a debt repayment strategy that minimizes the risk of bankruptcy actually occurring. Following this principle, the lender will agree to reschedule only part of the required payment (say, the principal portion) for lower debt-asset ratios. Finally, when the level of debt is very small relative to wealth, the lender is increasingly confident that demands for full payment can be met, and debt becomes virtually riskless.

The value of debt is thus determined as a function of the value of the underlying assets and the rescheduling policy. When the asset-debt ratio, $x_t$, is high, both the ability and willingness of the country to repay are high, and the debt value approaches the stream of promised payments discounted at the risk-free interest rate. At the other limit, when $x_t$ is very low, both the ability and willingness of the country to repay are very low. Default is called to comply with regulatory authorities, and the value of debt is equal to the value of appropriable assets.

For intermediate levels of $x_t$, the repayment is determined by the rescheduling policy, which is designed to be self-enforcing. In essence, the strategic alternatives of the sovereign country are controlled by the lender's choice of rescheduling policy.

**Debt as a Contingent Claim**

In the preceding paragraphs, we have shown that sovereign debt can be viewed as a claim on the tradable wealth of the country, the payoffs of which are determined at each repayment date by the current debt level. This allows us to model sovereign debt as a contingent claim held by the lender on the borrowing country. In their seminal articles on contingent claims pricing, Black and Scholes (1973) and Merton (1973) noted that the equity and debt of a corporation can be viewed as claims, the payoffs of which are contingent on the value of the corporation. We summarize below some concepts of the contingent claims pricing literature which provide powerful tools to price sovereign debt in our framework.

Let us first think of straight corporate debt: the firm promises to repay the
amount borrowed at a specified maturity date, T. If, on that date, the firm is unable to meet its obligations (that is, the worth of the firm is less than the debt), the bondholders take control of the firm. In the absence of additional features to the contract (such as indenture provisions or interest repayments), the owners of the firm have exchanged the value of the firm against the amount borrowed and a claim which allows them to repurchase the firm on the specified maturity date at a price equal to the face value of the debt. Such a claim is equivalent to a call option on the stock of an unleveraged company.

Call options and a variety of contingent claims are actively traded on all major stock exchanges, and extensive research has been conducted on their pricing. Black and Scholes's and Merton's contributions have been to derive a theoretical model for contingent claims pricing which accurately fits the observed prices of most actively traded options. The key insight of their analysis was to establish that the short-term variations in the option price can be perfectly replicated by a portfolio of the underlying stock and riskless bonds. By continuously rebalancing the amounts invested in the two securities, investors can perfectly replicate the terminal payoffs of any option at maturity. The composition of the equivalent portfolio, and hence the contingent claim price, depend on the stock price, on its variability through time, on the interest rate, and on the promised payoffs. The promised payoffs are also called boundary conditions because the value of the claim at any date, t, is the solution of a differential equation which is equal to the promised payments at maturity date T.

For a given rescheduling policy, similar reasoning will lead to the pricing of sovereign debt developed below. The inherent difficulty of debt pricing when reschedulings are considered is that the maturity of debt is itself a stochastic variable. Indeed, rescheduling may be repeated indefinitely, and the claim potentially has an infinite time to maturity. This is in agreement with observation of actual international debt contracts, which have de facto become revolving credit agreements. As is extensively developed in the next section, we solve this problem by viewing debt pricing as a contract which is periodic in the sense that the same set of repayment decisions is faced in every future period. Once the value of debt to the lender is determined, contingent on a rescheduling policy, the derivation of the optimal rescheduling policy and of the associated value of debt reduces to optimizing the value of debt over possible rescheduling policies.

II. The Debt Valuation Model

In this section, we first outline the assumptions of the analysis and discuss their economic meaning. Next, we present the model of international lending with endogenous reschedulings. This requires a specification of the boundary conditions and of a valuation process for an infinite maturity claim.10

10. The method for deriving a numerical solution for the value of the debt and the optimal rescheduling policy is outlined in the appendix.
Assumptions

A sovereign country has an initial stock of debt and an associated stream of repayment obligations comprised of interest and principal. Both are assumed to be a constant fraction of the stock of debt and are denoted $I$ and $A_d$, respectively. Debt payments come from the foreign exchange earnings of the country. The value of these is summarized in terms of a hypothetical asset. Debt is a claim held by the lender on this asset. The debt contract specifies the rule that determines the sharing of the earnings from the asset between creditor and debtor.

In the initial situation, the country has just made a debt payment. Prior to the next payment, the asset value changes according to an exogenous stochastic process. The creditor then makes a decision whether to demand full repayment of the obligations, interest only, or no repayment. The country then chooses whether or not to make the requested payment. As discussed above, because the lender is assumed to know the country’s willingness and ability to repay, and thus sets the rescheduling conditions accordingly, accepting the rescheduling offer is always a dominant strategy for the country. Because both the incentive to repudiate and the capacity to repay depend on the same argument, the asset-debt ratio, $x$, only one will be binding on the rescheduling policy. In the development of the model below, we assume that it is the latter. This sequence continues indefinitely; it is halted if the lender is forced, by its regulatory authorities, to call the country in default. If the cash flow from the country’s asset is insufficient to service the debt, we assume that the country is able to “sell off” its asset to meet the required payment; this can be conceived as running down foreign exchange reserves or as obtaining financing from official creditors (the International Monetary Fund or World Bank) with claims on the country’s assets senior to those of the commercial lender.

A Model of International Debt with Reschedulings

In the following paragraphs, we describe our formulation of the valuation problem, which can only be solved numerically. We rely on important results of the contingent claims literature (Black and Scholes 1973; Merton 1973; Cox, Ross, and Rubinstein 1979). Essentially, the instantaneous variation in the value of the debt is a function of the variations in the value of the asset. A continuous-time differential equation specifies this relationship. Our assumption on the set of possible rescheduling policies determines the boundary conditions of the numerical problem. The equation is integrated numerically.

Exogenous Boundary Conditions. Let $D$ be the face value of debt, the amount borrowed by the sovereign country, and $C$ the market value. There are two cases in which $C$ is perfectly determined: (i) when the country has such a

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11. In theory, the amount rescheduled could be anything between zero and the full amount of the obligations due. For simplicity, we assume that the decision is limited to rescheduling both interest plus principal or principal alone.
high asset-debt ratio, \( x \), that debt becomes riskless, and (ii) when it has such a low asset-debt ratio that the bank is forced to declare default.

As \( x \to \infty \), debt becomes riskless. This will translate into an arbitrary numerical (\( \hat{x} \)) limit beyond which it is assumed that the country will always be required to (and be able to) pay off its debt in full. \( C \) then attains its maximum, which is equal to the value of all promised payments on \( D \) discounted at the riskless interest rate. \( C_{\text{max}}(D) \) is exogenously specified in this model. Since repayments are a linear function of \( D \) and the present value operator is linear,

\[
C_{\text{max}}(D) = \alpha D
\]

where \( \alpha \) depends on both the risk-free rate and the rate charged to the country (that is, the rate including the margin over the London Interbank Offer Rate).

The bank faces a regulatory lending constraint for exposure to any one country. This can be assumed to emanate from the central bank or the depository insurance agency or to be exogenously set by the bank itself. This level is denoted by \( D_{\text{max}} \). Statutory and internal bankruptcy limits are set on the basis of the value of export earning assets and are not allowed to reach a level greater than some constant, \( b \), times the asset value: \( D_{\text{max}} = bX \). The minimum asset-debt ratio is thus set as: \( x_{\text{min}} = X/D_{\text{max}} = X/bX = 1/b \). When \( D \) reaches the full value, \( bX \), regulatory bankruptcy occurs, and the bank appropriates a fraction, \( \epsilon \), of the total asset value. Thus the market value of the debt when \( D \) reaches its maximum is the seizable fraction of total assets:

\[
C(D) = \epsilon X, \text{ when } D \geq bX
\]

Now consider that the model thus far specified includes two state variables: \( X \) and \( D \). Such models are inherently very difficult to solve numerically, and nonlinearity can be a serious problem. From an economically appealing point of view, the value of debt actually depends on only one state variable, \( x \), the ratio of asset value to debt. If we double the value of the asset and the face value of the outstanding debt, then the market value of the debt should also be doubled. If we denote \( c = C/D \), the ratio of the market value to the face value of the debt, the value of \( c \) can be specified as:

\[
(2) \quad c = c(x,t); c_x > 0
\]

where

\[
(3) \quad c = \epsilon x \text{ for } x \leq 1/b
\]

and

\[
(4) \quad c = \alpha \text{ for } x \geq \tilde{x}
\]

Equation 2 states that as the asset-debt ratio, \( x \), rises, \( c \) also rises. At very low levels of \( x \) (below the default threshold, \( 1/b \)), the value of \( c \) is exogenously given by the amount of the asset that can be seized by lenders, \( \epsilon x \) (equation 3). This is shown as the line \( OA \) in figure 1. Conversely, for very high levels of \( x \) (above \( \tilde{x} \), which is our finite proxy for infinity), the value of \( c \) equals its asymptotic limit, \( \alpha \)
Figure 1. **Exogenous and Endogenous Boundary Conditions**
The value of $c$ for interim levels of $x$, $1/b < x < \bar{x}$, is driven by the probability of receiving debt-service payments or of $x$ reaching $\bar{x}$ or $1/b$ over time.

To simplify the numerical analysis, we do not actually derive the cost of debt service as valued by the borrower, or the penalty, $P$. Given this, to ensure that repudiation is never chosen by the country, we require that the market value of debt be no greater than the penalty to the country, $C_t \leq P$, or equivalently, $c \leq \lambda x$. We show below, however, that the constraint can always be satisfied by an appropriate choice of $b$, the limit on indebtedness set by the regulatory authorities. In figure 1, the line $c = \lambda x$ provides an upper bound to the value of $c$, because for $c > \lambda x$, the country would choose to repudiate and lenders get only $ex$ per dollar of debt outstanding.

*Endogenously determined rescheduling boundaries.* So far, we have determined the value of debt, $c$, for two limit values of the ratio $x$: when it is very large and when it is very small. In all intermediate cases we assume that the bank has three alternative courses of action:

- Ask for a full repayment of the interest, $I_t$, plus the principal due, $A_t^d$. In this case the debt is reduced to:
  $$D_t = D_{t-1} - A_{t-1}^d$$

- Require the country to make only the interest payment on the debt, that is, $R$. The debt level remains unchanged. If the bank charges a fee, $F_1$, for this rescheduling, it is added to the face value of the debt:
  $$D_t = D_{t-1} + F_1$$

- Receive no payment at all. In this case the level of debt increases by the forgone interest plus a second type of rescheduling fee, $F_2$:
  $$D_t = D_{t-1} + I_{t-1} + F_2$$

If, in addition, the creditor decides to lend some additional funds, $L$, the debt becomes:
  $$D_t = D_{t-1} + I_{t-1} + F_2 + L_t$$

In each case, we assume that principal ($A$), interest ($I$), fees ($F$), and additional loans ($L$), and thus the repayment due at any point in time, can be calculated as some percentage of the remaining face value of the debt. This implies that even if the country makes the full repayment due each year, the debt is never completely repaid but the remaining debt progressively tends toward zero.

The optimal policy for the bank is reduced to the choice of the two rescheduling boundaries, $x_1$ and $x_2$, in figure 1, which differentiate the request for payment from the country: $x \geq x_1$ when the bank requests full payment; $x_1 > x \geq x_2$ when the bank requires only the interest payment; and $1/b < x < x_2$ when the bank asks for no payment at all and may even make new funds available to the country. An important point is that the rescheduling boundaries are func-
tions of $x$ alone rather than of $X$ and $D$ independently. This follows from the fact that the two exogenous limit boundaries can be written as functions of $x$, and that we have constant returns to scale.

We have restricted the bank's choice to three alternatives to simplify the argument, but the model could accommodate any number of alternative repayments. At every repayment date, the creditor decides on which repayment to require from the borrower. If the creditor could appropriate the full value of the productive assets in case of repudiation or default, it would be in its best interest to require full payment in all cases. As the debtor is a sovereign country, however, only the fraction, $\varepsilon$, of the value of the asset is appropriable in the event of repudiation or default. Hence the creditor will select a rescheduling policy which reduces the likelihood of repudiation or default in the future. The trade-off is thus between increased expected costs associated with default and a higher repayment at the present time. As the asset value increases, the incentive for the creditor to reschedule is lessened.

**The Valuation Process**

The value of debt at any point in time is determined by the value of the asset, the face value of debt, the rescheduling policy at all times in the future, and a set of parameters (the interest rates faced by the country and by the bank, the fraction of debt repaid as principal, and so forth). Once the function $c = c(x)$ is determined, the "market" price of debt, $c$, can be simply located on the vertical axis in figure 1, given any value for $x$.

One difficulty that is faced is the potential infinite maturity of the debt claim given rescheduling possibilities. This is overcome by observing that the valuation problem can be made stationary, or, to be more accurate, periodic. The model assumes the distribution of the asset value to be stationary, and the repayment schedule as well as the exogenous boundary conditions to be independent of time. Consequently, for a given level of $x$, $c$ is independent of time. The problem does not exhibit perfect stationarity because the relative value of debt between repayment dates depends on the time left to the next repayment; but it is periodic.

The periodicity property is essential to our valuation method in that it closes the system of equations of the problem. The classical differential equation of contingent claims pricing specifies the value of a claim as a function of its payoffs at maturity. This is done by valuing the claim just before maturity and then recurrently deriving its price at earlier points in time. As the maturity of the debt is infinite, this method does not apply. We can still derive the relationship between the value of the claim at the present repayment date and its value at the next repayment date, however, contingent on a given rescheduling policy. The value of the claim one year from now is only known for the exogenous boundaries, but for a given level of $X/D$ it is the same as the current value. This gives us a well-specified differential equation, which becomes a system of equations in the numerical approximation. Once the value of the claim is determined for any given rescheduling policy, summarized by the choice of the boundary values $x_1$.
and $x_2$, the optimal rescheduling strategy is found by maximizing the claim value over all possible policies. This last step yields both the optimal rescheduling boundaries and the value of debt to a fully rational lender.

III. Simulation Results: The Market Value of Debt

Based on the debt valuation model described above, simulations were run which allow us to compare the numerical values and the bank's optimal rescheduling policies obtained when we vary any one of the major parameters. The changes in the upper and lower boundary values indicate the resulting alteration in the debt values at which full payment or full rescheduling are optimal. From these simulations, it appears that the bank benefits substantially from voluntary rescheduling in realistic ranges of the parameters, in contrast to the old proverb "a bird in the hand is worth two in the bush." The expected costs of default are high enough for the bank to prefer to forgo part of an immediate payment even if the sovereign country is able to repay.

We review here the determinants of debt value and examine the comparative statics of the model for the ten parameters. Debt values are derived for 100 different levels of the reduced variable $X/D$, which range from 0.10 to 50 and thus include most realistic cases.

The determinants of debt value are listed in table 1, along with the levels we used for our base case. Figures 2 through 7 below present the simulation results for different values of the parameters. Each simulation differs from the base case only in the value of one parameter. Table 4 in the appendix summarizes our numerical results.

The Base Case

The market-to-face value of debt for the base parameter values given in table 1 is shown in figure 2 for a range of $x$ values, the asset-debt ratios. This ratio measures the indebtedness of the country and, consequently, the likelihood that the promised payments will be made. The value of debt is a linear function of $x$ at default and a concave function of $x > 1/b$ (see figure 1). In case of default, the market to face value of the debt, $c$, is small (1.6 percent of face value at the default boundary in the base case). It jumps upward, however, if default is not called. On the brink of default, $c$ equals 10 percent. This demonstrates the value to the lender of keeping alive the game by rescheduling rather than calling default.

The maximum value of $c$ is reached asymptotically: as the asset value increases, the likelihood of full repayment increases until debt becomes virtually riskless. The debt is worth its face value, and thus $c = 1$ when the asset value is

12. These values can be compared with those estimated for twenty developing countries in the appendix, which were calculated using a short-cut asset valuation technique.

13. At the limit of very large values of $x$, debt becomes a perpetuity. It is easy to show that its value is $(A + R)/(A + R^e)$ times face value, that is, a debt value–face value ratio of 1.1 in the base case.
Table 1. Debt Valuation Parameters: Base Case Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Representative value (percent)</th>
<th>Figure showing simulation results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Endogenous</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_1$ Upper rescheduling boundary for $X/D$ (full repayment required for $x \geq x_1$)</td>
<td>108</td>
<td>3</td>
</tr>
<tr>
<td>$x_2$ Lower rescheduling boundary for $X/D$ (interest payment required for $x_1 &gt; x \geq x_2$)</td>
<td>49</td>
<td>3</td>
</tr>
<tr>
<td><strong>Exogenous</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$ Interest rate charged to the country</td>
<td>13 11, 15</td>
<td>4</td>
</tr>
<tr>
<td>$R^*$ Riskless rate paid by bank</td>
<td>10 12</td>
<td>6</td>
</tr>
<tr>
<td>$A^d$ Fraction of principal repaid per year</td>
<td>20 25, 50, 100</td>
<td>5</td>
</tr>
<tr>
<td>$\sigma^2$ Yearly variance of asset value</td>
<td>16 50</td>
<td>—</td>
</tr>
<tr>
<td>$1/b$ Boundary value of $X/D$ for default</td>
<td>35 55, 100</td>
<td>7</td>
</tr>
<tr>
<td>$e$ Appropriate fraction of assets</td>
<td>5 10</td>
<td>—</td>
</tr>
<tr>
<td>$F$ Rescheduling fee (percent of face value)</td>
<td>0 1</td>
<td>—</td>
</tr>
<tr>
<td>$L$ Funding by bank (percent of face value)</td>
<td>0 5</td>
<td>—</td>
</tr>
</tbody>
</table>

— Not applicable.

approximately double the face value and $x = 2$. This is the credit rationing boundary. At this level of debt, a new lender that voluntarily extended additional funds, driving down $x$, would obtain a claim that would be worth less than the amount lent despite the risk premium built into the interest rate charged. The difference between full value (the value of debt if it were riskless) and face value measures the riskiness of debt (10 percent at the credit rationing boundary).

**Creditor Policies**

The value of debt is sensitive to changes in the rescheduling policy chosen by the lender. The rescheduling policy specifies the type of repayment requested by the lender as a function of the tradable wealth of the borrowing country. It is summarized by the parameters $x_1$ and $x_2$ in figure 1. If the bank is more “impatient,” that is, if it starts requesting full or partial payment for a lower level of $x$, it increases the risk of financial distress and the expected probability of default.

Our numerical analysis yields the optimal rescheduling policy and the value of debt for that optimal policy and for alternate (suboptimal) policies. The optimal rescheduling strategy in the base case is for the lender to request full repayment until the asset-debt ratio, $x$, falls below 61 percent. Below this level, continued regular debt service would be likely to quickly trigger bankruptcy, and future debt service payments would then be jeopardized. In this case, the lenders would only request payment of interest due. When $x$ falls below 48 percent, the lender is willing to reschedule both principal and interest. These optimal rescheduling boundaries are clearly sensitive to the level of $x$ at which default is called. For
Figure 2. Debt Valuation: The Base Case

Market value of debt / face value of debt (C/D = c)

Asset value / face value of debt (X/D = x)

Key: □ base case
example, when defaults occur at $x = 100$ percent, optimal rescheduling occurs at $x = 127$ percent. In this case, both interest and principal are always rescheduled together.

Figure 3 also demonstrates alternative rescheduling policies. Bond financing would not permit any recontracting. It is clear that this sharply reduces the value of debt. Just prior to default, debt in the form of bonds would only be worth 60 percent of debt in the form of a credit that could be rescheduled. As the country gets wealthier, this premium shrinks toward zero when debt is riskless. Similarly, the credit rationing boundary is restricted under bond financing. The value of $x$ for which $c = 1.0$ is about 10 percent higher than in the base case for credits; that is, bond lenders would cut off credit at lower levels of debt. Again, the numerical values are sensitive to the assumption about the size of $x$ when default is called. The higher this is, the greater is the advantage of credit financing over bond financing. Figure 3 also illustrates two ad hoc rescheduling policies: creditors may be too impatient, ask for full payment, and thereby increase the probability of default; or they may be too lax and reschedule debt when asset levels would allow full payment. In both cases, the value of debt declines relative to an optimal rescheduling policy.

The interest rate charged to the country ($R$), together with the principal ($A$), determines the size of the repayments requested by the bank at each year end. The difference between $R$ and the riskless rate, $R^*$, at which the bank can borrow in financial markets, compensates the bank for the riskiness of the loan. The "fair" interest rate $R$ is that for which $c = 1$; that is, the market value equals the face value of debt. This is clearly a function of $x$. Figure 4 shows that in a competitive capital market, the spread charged to the country will be an increasing function of the long-run debt-asset ratio. Our simulations indicate that a 1 percentage point increase in the spread would support an 8 percent increase in debt.

Although higher levels of debt can be sustained with a higher spread, figure 4 shows that the spread is not an effective instrument with which lenders can compensate for ex post excessive debt; regardless of the spread, default is called at the same level of $x$. A high spread has two offsetting features. On the one hand, it raises profits to the lender when debt is risk-free. On the other hand, it raises the probability of bankruptcy or default. With the higher spread, the rescheduling boundary is raised relative to the base case. When debt is sufficiently risky that a rescheduling of interest is optimal ($x = 0.67$ for a 5 percentage point spread), these two forces balance each other. The value of debt to creditors is actually raised by lowering the spread. This explains why actual reschedulings have rarely involved an increase in spreads. The additional risk of default outweighs the additional current income.

There are similar offsetting factors governing the maturity of debt. In figure 5, we show how $c$ varies as the proportion of debt to be repaid each year is increased. When debt is riskless, the maturity does not really matter, and $c$ is the same for both levels of repayment. Longer-term debt is preferred by creditors
Figure 3. Rescheduling Policy: Optimal versus Suboptimal Strategies

Key: □ optimal  ○ higher boundary conditions, x₁ and x₂  ● no rescheduling
because they lock in higher total interest payments. As $x$ falls and debt becomes more risky, lenders are keen to shorten maturities and recover their principal quickly. The shorter-maturity debt ($A = 25\text{ percent}$) has a greater value than the longer-maturity debt over the asset range from $x = 0.48$ to $x = 2.6$. Below this level, however, attempts by creditors to demand full repayment would precipitate bankruptcy more quickly. There is an incentive to lengthen maturities. In our simulations, we find that this switching point coincides approximately with the full rescheduling boundary, $x_2$ in figure 1. Thus, the model explains why short-term debt dominates as debt starts to become risky but is then rescheduled into longer-term maturities when the situation further deteriorates.

Front-end fees charged at rescheduling do not have the same intertemporal effects as raising spreads or shortening maturities. Our simulations reveal that the unit value of debt is unchanged by adding a 1 percent fee. The total portfolio of the creditor is increased, therefore, because the fee is assumed to be capitalized into the stock of debt. Fees are extremely flexible instruments which allow
Figure 5. Principal Repayment: Fraction of Face Value Repaid Each Year

Key: □ $A = 20\%$ of $D$  ▪ $A = 25\%$ of $D$
the creditor to extract the maximum possible from the debtor but do not force the debtor to repudiate and do not unduly increase the risk of bankruptcy.

Finally, the lender may also provide new money to a country in financial distress to help stave off the risk of default. The promise by a lender to provide an additional 5 percent of debt in new money as well as rescheduling all payments due is shown by our simulations to be able to increase the value of the lender's portfolio. This is not, however, a time-consistent policy. When a country actually has severe financial problems, new money can help raise the value of old debt, but it does not do so by enough to compensate for the greater volume of debt that is then at risk.

**Global Conditions**

Our final set of sensitivity runs deals with the impact of changing global conditions; the riskless interest rate, the variance of the asset, the level of the default boundary, and the proportion of assets seized by banks upon default.

For any given level of \( x \), a higher riskless interest rate lowers the value of debt because it raises the probability of default without increasing the value of debt if full repayment occurs. Figure 6 shows a rise of 2 percentage points in the riskless interest rate and in the rate charged to borrowers. The credit rationing boundary at which the face value equals the market value of debt shrinks by nearly 10 percent for a given level of \( x \). In fact, the impact on nominal debt will be substantially higher, because the value of the asset will be significantly affected by a change in the interest rate. If a country's discounted earnings were worth 200 at \( R^* = 10 \) percent, it could take on a debt of 100 (\( x = 2 \)). As \( R^* \) rose to 12 percent, however, the discounted value of earnings would fall (to, say, 160), while the rationing ceiling would also fall (\( x = 2.2 \)). Thus, the total nominal debt ceiling that would be imposed by lenders would fall from 100 to 73. The simulation confirms the extreme importance of the level of the interest rate in determining country creditworthiness.

As would be expected, an increase in the asset variance from the base case level of 0.16 to 0.5 also causes the value of debt to fall.\(^{14}\) In a highly uncertain world, country creditworthiness will decrease.

If the regulatory authorities tighten limits on lending relative to asset value by calling bank loans value-impaired at an earlier stage (a higher \( x \)), the impact on the value of debt is dramatic, even for cases in which the probability of default is low. Figure 7 shows the impact of raising the ceiling from \( x_{\text{min}} = 0.35 \) (\( b = 2.8 \)) to \( x_{\text{min}} = 0.5 \) (\( b = 2 \)) and \( x_{\text{min}} = 1.0 \) (\( b = 1 \)). The most obvious advantage of the lower ceiling is evident at low levels of \( x \), at which it is in the interests of the creditor and the borrower to continue to reschedule for as long as possible. What is perhaps more surprising is the strength and persistence of the effect at lower debt levels. If default is called at \( b = 1.0 \), then the credit rationing

\(^{14}\) A value of 0.5 is typical for a U.S. corporation in the petroleum, chemicals, or metals and mining industries.
Figure 6. Interbank Interest Rates

Key: □ $R^* = 10\%$  • $R^* = 12\%$
Figure 7. Default Boundary

Key: □ $x_{\text{min}}^{\text{min}} = 35$ percent  ● $x_{\text{min}}^{\text{min}} = 50$ percent  ○ $x_{\text{min}}^{\text{min}} = 100$ percent
boundary (when \( c = 1 \)) is shifted out from \( x = 2 \) to \( x = 3 \). That is, the maximum level of debt that would be voluntarily lent to a country, holding the spread constant, is lowered by one-third. It is clear why regulatory authorities have been reluctant to enforce stringent controls on international lending. It is also clear that a tougher regulatory environment would seriously limit the volume of debt that commercial lenders would be prepared to extend.

The final sensitivity analysis consists of raising the share of assets appropriated by lenders at default from 5 to 10 percent. This has almost no impact on debt value. The intuition is simple. At default, the asset value is very low, as is the amount that lenders recover. In order to raise the value of debt, it is more important to change the probability of reaching the default boundary than to change the payoff at that boundary.

IV. Conclusions and Inferences

Creditors have little legal recourse if a sovereign state repudiates its external debt obligations. There are, however, deadweight losses to the system as a whole that would arise in the case of a default. These may be thought of as the loss of mutually profitable opportunities for future loans, or as the imposition of efficiency reducing sanctions on international trade. It is in the interest of both parties to recontract to avoid such losses. The potential for rescheduling payments due on syndicated credits permits lenders to structure the incentives faced by borrowers in such a way that repudiation is never a rational action. In our framework, we show that rescheduling of principal, and sometimes of interest due, is in the best interests of both creditors and debtors as they strive to avoid damaging the country’s asset base, on which future debt servicing capability depends. That is to say, reschedulings can free resources to add to investment, making future debt service payments more likely.

We have developed a numerical method for valuing developing-country debt. Debt is viewed as a contingent claim on underlying assets that have a stochastic value, which implies that the net payoff to repudiation is stochastic. This is the major source of risk in international lending, and it cannot be completely controlled by credit rationing. As the value of the asset-debt ratio falls, the “market” value of debt falls below the face value at an increasing rate.

Our numerical results are broadly consistent with certain stylized characteristics of international credit markets. The point at which the market value of debt falls below the face value is critical, for example, as it indicates when voluntary lending will be completely shut off. Under the assumptions of our simulation analysis, this occurs at an asset-debt ratio of 2 for an average developing country with a traded sector incremental capital-output ratio of 4. The cutoff point for new lending is shown to be sensitive to international conditions. This helps explain why new lending was suddenly curtailed in a period of high real interest rates and low growth prospects for developing countries—factors likely to
sharply reduce the value of the assets underlying the debt. Conversely, the methodology could be used to estimate by how much interest rates would have to fall to restore creditworthiness to any particular country.

A rough-and-ready application of the methodology to estimate the market value of debt relative to the face value (the price) of debt for twenty developing countries at the end of 1983 is given in the appendix. In all cases, the estimated price is less than unity. This suggests that these countries had reached their commercial credit limits. Additional lending would be forthcoming only in amounts that would leave the asset-debt ratio unchanged. That is, the growth of asset value would determine the feasible growth of debt. Eight of the countries fall below the rescheduling boundary. These countries have "solvency" problems, in the sense that they may be unable to repay debt service if further adverse shocks occur. According to our analysis, creditors would willingly defer the receipt of payments from these countries in order to avoid damaging the asset base on which future debt payments depend. The other countries seemed to be faced with classical liquidity problems: no new lending will be made available because the debt price had fallen below unity, and yet they face the need to pay interest and principal on a large outstanding debt stock.

In the appendix, countries are also ranked according to their debt price; this gives a creditworthiness ordering that corresponds closely with rankings from other sources. A more detailed application of our methodology offers the potential for improving creditworthiness orderings, and for assessing the degree of adjustment required to regain full resumption of spontaneous lending.

There are several actions that lenders can take to raise the value of debt they hold. Foremost among these is the potential to reschedule debt service obligations. This gives a clear advantage to syndicated credits over other instruments such as bonds, for which reschedulings are more difficult because of the anonymous and dispersed nature of ownership. When the asset-to-debt value of a country is low, we find that credits have a greater value than bonds, particularly during periods when adverse shocks are common and debt becomes more risky. This higher valuation also permits a greater volume of credits to be extended for any given asset value. Second, it seems that shortening maturities is always in the interest of lenders when debt is slightly risky, but it adds to the risk of default as the degree of financial distress gets larger. Similarly, we see that raising the interest spread has a mixed effect on debt values. If debt is relatively safe, higher spreads add to profits. But during a rescheduling, when debt is risky, higher spreads increase the probability of a default. The net benefits to lenders are small or negative over a broad region. In fact, lenders may even gain by reducing spreads somewhat. There are, however, no such adverse effects with respect to front-end fees. These are almost pure profit for the banks. It is not surprising, therefore, that actual reschedulings have been characterized by a lengthening of maturities, relatively minor adjustments to spreads (and sometimes even a narrowing), and heavy fees. Finally, we show that the precommitment to provide additional funds during a rescheduling can raise the value of existing debt. These
should not be construed as "concessions" on the part of commercial lenders. Rather, they are safeguards to protect the value of existing debt.

Our results can also shed light on the trade-off involved in some current proposals for international actions to alleviate the debt crisis. The development of a secondary market in developing-country debt could provide lenders with extra liquidity, but at the cost of a reduction in debt value if the ability to reschedule was lessened as a result of such a market. Limiting the number of debt holders seems essential to offset the threat of repudiation. Thus, it is unclear whether lenders would gain from further development of secondary markets. Developing countries, however, would probably lose in the long run from such markets. They would then be faced with a financial market similar to the bond market, to which they have had relatively little access in the postwar period because of the particular risk characteristics of lending to them. Calls for tougher supervision by regulatory authorities could also be damaging to international lending. The longer lenders can stave off a repudiation by capitalizing interest into debt in dire times, the greater the value of the debt. Finally, tougher policies regarding the seizure of assets in the event of a repudiation, even if feasible, would not be in the interest of lenders. When defaults are called, the value of such assets is typically very small and hardly protects the lender from risk.

APPENDIX

Numerical Solution Method

Given the periodicity of the problem, we need only derive the relationship between the values of debt at two successive repayment dates, $T$ and $T + 1$. Suppose that the relative value of debt at the next repayment date $c_{T+1}$, is known for each possible asset-to-debt value, $x_{T+1}$. To evaluate $c_T$, we will derive the value of debt at the instant just prior to $T + 1$ and recurrently move backward to solve it at all prior times, $T < t < (T + 1)$. This yields the value of debt $c_T(x_T)$ at the beginning of the period as a linear function of all possible values at the end of the period, $c_{T+1}(x_{T+1})$. The periodicity argument implies that the two functions $c_T$ and $c_{T+1}$ are equal for any given value of the ratio, $x$. This reasoning gives us a well-specified equation which we solve numerically.

A variety of numerical methods can be used. We adopted the binomial method of Cox, Ross, and Rubinstein (1979), which provides a clear, economic understanding of the process. The continuous-time, lognormal distribution of $x_t$ can be approximated by a series of independent multiplicative binomial jumps over discrete intervals. Divide the one-year period between two repayments into $N$ equal intervals with length $\tau = 1/N$. Over each interval, the value of $x_t$ can jump to two different values: a higher one, $x_{t+\tau} = u \cdot x_t (u > 1)$, or a lower one, $x_{t-\tau} = d \cdot x_t (d < 1)$. This probability distribution is discrete in the sense that the set of possible values taken by the random variable, $x_t$, is a finite set. Furthermore, it is defined only at discrete points in time. This distribution converges to the
lognormal distribution (with annual variance $\sigma^2$) when the number $N$ of discrete intervals tends toward infinity and $u$ and $d$ are given by:

$$u = \exp(\sigma/\sqrt{\tau}) \quad d = \exp(-\sigma/\sqrt{\tau})$$

Using this approximation, we need to determine the value of the claim at all “nodes” of the “grid” defined by the possible values of $x$ at times $T + n \cdot \tau$ ($n = 0,1, \ldots, N$). In the simulations, we adopted the value $N = 100$, which yields a good approximation of the lognormal process.

In this discrete framework, we can use the arbitrage argument (due to Merton 1973) to derive the value of the claim at time $t$ as a function of its value at time $t + \tau$. Consider a portfolio composed of the contingent claim and a share, $h$, of the asset, $x$. The value of such a portfolio at time $t$ is:

$$c_{t+\tau}(u \cdot x_t) + h(u \cdot x_t) \quad \text{or} \quad c_{t+\tau}(d \cdot x_t) + h(d \cdot x_t)$$

If we choose the value $h = [c_{t+\tau}(d \cdot x_t) - c_{t+\tau}(u \cdot x_t)]/[u - d]x_t$, inspection of that equation shows that the value of the portfolio is the same in both cases. The return on investment in this portfolio is risk-free. It should thus be equal to the riskless rate, $r$, available to the bank in order to preclude arbitrage opportunities. We thus have the relationship:

$$c_t(x_t) = [pc_{t+\tau}(u \cdot x_t) + (1 - p)c_{t+\tau}(d \cdot x_t)]/r$$

where

$$p = (r - d)/(u - d), \text{ and } 0 < p < 1$$

We interpret $p$ as the “quasi-probability” of an upward move. The value of debt at time $t$ then becomes the expected value (for the quasi-probability $p$) of debt at time $t + \tau$ discounted at the riskless rate of interest. The expression for $c_t(x_t)$ depends in a linear fashion on the two possible values of $c_{t+\tau}$. This derivation can be iterated backward to determine the (linear) expression of $c_t$ as a function of all possible values of $c$ at time $T + 1$. The variable $c_T$ can thus be interpreted as the present value of $c_{T+1}$ associated with a series of multiplicative independent binomial processes defined by $u, d,$ and the quasi-probability, $p$.

As noted before, the recursive relationship is independent of the true probability of an upward move. This is because movements in the value of debt can be perfectly hedged by a position in assets, the size of which depends only on the amplitude of changes in the asset price.

Recall that the underlying asset (the foreign exchange value) of the country is not traded and its value must be derived. Our valuation procedure does not require a perfect duplication of changes in the asset price and changes in the value of traded assets which are effectively priced by the market. In a capital-asset pricing model framework, for example, only the nondiversifiable risk is priced. Thus the valuation method is very general. Furthermore, it allows the lender to offset the risk of its claim by selling traded assets, which are highly correlated with the country value.
Repayment Equations

So far, we determined the value of the claim at time T as a function of all possible values of the claim at the instant prior to the next repayment date, \( T + 1 \). When repayment occurs, the variable \( x \) decreases or increases as specified by the following equations.

Normal repayment (\( x \geq x_1 \)):\[
x = \frac{(x - A - R)}{(1 - A)}
\]

Interest-only payment (\( x_2 \leq x < x_1 \)):\[
x = \frac{(x - R)}{(1 + F_1)}
\]

No repayment (\( 1/b < x < x_1 \)):\[
x = \frac{x}{(1 + R + F_2)}
\]

These equations are linear; consequently, the value of the claim at date \( T \) (after repayment) is a linear function of the value of the claim at time \( T + 1 \) (after repayment). The equations can be formalized in matrix notation to an equality between a vector \( c_T \) and the product of a matrix \( \Omega \) by a vector \( c_{T+1} \), where the \( i \)th component of \( c_T \) is the value of debt for the \( i \)th discrete value of the state variable \( x \). Since the two vectors \( c_T \) and \( c_{T+1} \) are equal, the numerical problem reduces to:

\[
c_T = \Omega \cdot c_T
\]

The elements of the matrix \( \Omega \) are known functions of the fixed parameters and of the rescheduling policy characterized by \( x_1 \) and \( x_2 \). The optimal rescheduling policy and the value of debt associated with it are then obtained by maximizing the solution to the matrix equation over all possible rescheduling policies.

An Empirical Application of the Valuation Model

We have calculated the implicit value of debt for twenty developing countries at the end of 1983. The process requires evaluation of the country's underlying assets. The procedure used is as follows. We first took the total replacement value of the capital stock of each country by summing investment over the preceding fifteen years and assuming a 10 percent depreciation rate. The initial capital stock in 1968 was taken to be four times gross domestic product (GDP) of that year (four represents a "normal" capital-output ratio for a developing country). This is given in column 1 of table 2. We then make two adjustments to this figure to give a measure of the actual value of capital in the traded sector. The first converts the replacement value into the actual value. Because countries with low incremental capital-output ratios (ICORS) have many profitable investment opportunities remaining, the actual value of their firm exceeds the replacement value. Accordingly, we multiply the replacement value by \( 4/\text{ICOR} \). For the average developing country with an ICOR of 4, the actual value coincides with the replacement value. The second adjustment to obtain the share of capital in the
Table 2. Valuation of Tradable Assets, 1983
(millions of U.S. dollars)

<table>
<thead>
<tr>
<th>Country</th>
<th>Replacement capital(b) (1)</th>
<th>ICORs(b) (2)</th>
<th>Tradable share(c) (3)</th>
<th>Value of traded capital ((4) = (1) \times \frac{(4)}{(2)} \times (3)) (5)</th>
<th>Total reserves minus gold(d) (6) = (4) + (5)</th>
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</thead>
<tbody>
<tr>
<td>1. Egypt, Arab</td>
<td>72,629</td>
<td>2.7</td>
<td>0.249</td>
<td>26,792.0</td>
<td>771.0</td>
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<td>27,563.0</td>
</tr>
<tr>
<td>2. Venezuela</td>
<td>205,076</td>
<td>5.9</td>
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<td>57,153.2</td>
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<td></td>
<td>113,163.0</td>
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<td>4.2</td>
<td>0.271</td>
<td>109,759.0</td>
<td>4,355.0</td>
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<td>12. Ecuador</td>
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<td>7,796.0</td>
<td>644.5</td>
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<td>11.3</td>
<td>0.286</td>
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<td>17,731.1</td>
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<tr>
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<td>7.8</td>
<td>0.224</td>
<td>11,757.4</td>
<td>1,270.0</td>
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<td>2,115.4</td>
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<td>184.3</td>
</tr>
</tbody>
</table>

Note: Countries are listed in order of the estimated price of a dollar of commercial debt as shown in table 3, column 6.


c. (Mining and manufacturing)/GDP, average for 1970–81, at current prices.


traded sector is calculated by multiplying the total capital stock by the average
share of mining plus manufacturing in GDP. These sectors are chosen as they are
likely to be worst hit by an international embargo and thus may be considered by
creditors to be effective “collateral” on the country’s debt. Production of agricul-
tural exports could be quickly converted into production of domestic consump-
tion goods. The result of these adjustments gives the value of traded-sector
capital, shown in column 4 of table 2. As the final outcome of this process, the
total traded assets of the country (column 6) are obtained by adding total re-
serves minus gold to the value of traded capital.

We present two versions of the asset-debt ratio in table 3. In practice, coun-
tries have official debt, which is serviced prior to commercial obligations. For
example, World Bank and IMF loans have never been rescheduled. Paris Club
reschedulings on other official loans are conditional on similar treatment being
accorded commercial lenders, and thus we account for these as two debt catego-
ries. Values based on the first definition, given in column 4 of table 3, shows
total assets over total debt. In the second variant, column 5, we first subtract out
official debt from total assets, to give a measure of the net assets on which
commercial lenders have a claim, and divide this by commercial debt.

Given the appropriate asset-debt ratio for a selected country, the estimated
price of a dollar of commercial debt can be obtained from table 4. The first
column gives the range of possible asset-debt ratios. The relevant price of debt
under the base case assumptions is then read from the corresponding row of the
second column. The additional columns give the debt price for the various
simulations in which, for each column, the value of one of the nine determinants
is altered.

The estimated prices can be used in two ways. First, for some developing
countries there exists information on the value of Eurocredits implicit in swap
arrangements undertaken by commercial banks. For example, the New York
Times (June 1985) gives the following quotes: Bolivia, 0.20; Peru, 0.50; Argent-
tina, 0.70; Venezuela, 0.90. Other sources suggest Brazilian and Mexican debt
trades for about 0.80–0.85 cents on the U.S. dollar. A comparison of these
prices with our crude estimates shows a close correspondence. We take this as a
priori evidence that a model such as the one described above could be made
operational.

The second test of the relevance of the estimated price is to use it to rank the
twenty countries by creditworthiness, as shown in tables 2 and 3. Thus, we
would have placed Egypt as the most creditworthy country of our sample in
December 1983, Venezuela second, Colombia eighth, and so forth. We com-
pared this rank ordering with the 1984 Euromoney rankings (published Febru-
ary 1985). The Spearman correlation coefficient is 0.65. While this is a
reasonably positive association, inspection of the orderings shows that a major
difference of opinion exists with respect to Turkey. Excluding this, the correla-
tion coefficient for the nineteen remaining countries rises to 0.79. We conclude
### Table 3. Asset-Debt Ratios for Selected Developing Countries, 1983
(millions of U.S. dollars)

<table>
<thead>
<tr>
<th>Country</th>
<th>Official debt&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Commercial debt&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Total assets&lt;sup&gt;c&lt;/sup&gt;</th>
<th>Assets / total debt&lt;sup&gt;d&lt;/sup&gt;</th>
<th>Net assets / commercial debt&lt;sup&gt;e&lt;/sup&gt;</th>
<th>Estimated price: commercial debt&lt;sup&gt;f&lt;/sup&gt;</th>
<th>Euromoney rank, 1984&lt;sup&gt;g&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Egypt, Arab Rep.</td>
<td>12,304.6</td>
<td>7,534.2</td>
<td>27,563.0</td>
<td>1.39</td>
<td>2.02</td>
<td>0.99</td>
<td>6</td>
</tr>
<tr>
<td>2. Venezuela</td>
<td>237.0</td>
<td>32,711.4</td>
<td>61,588.4</td>
<td>1.87</td>
<td>1.88</td>
<td>0.98</td>
<td>9</td>
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<tr>
<td>3. Thailand</td>
<td>4,312.1</td>
<td>8,909.0</td>
<td>20,996.6</td>
<td>1.59</td>
<td>1.87</td>
<td>0.97</td>
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<tr>
<td>4. Korea, Rep.</td>
<td>9,313.2</td>
<td>27,224.5</td>
<td>57,153.2</td>
<td>1.56</td>
<td>1.76</td>
<td>0.96</td>
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<td>5. Malaysia</td>
<td>2,318.4</td>
<td>13,891.8</td>
<td>26,521.4</td>
<td>1.64</td>
<td>1.74</td>
<td>0.96</td>
<td>1</td>
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<td>6. Yugoslavia</td>
<td>4,252.7</td>
<td>14,935.4</td>
<td>29,691.0</td>
<td>1.55</td>
<td>1.70</td>
<td>0.95</td>
<td>11</td>
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<td>7. Cyprus</td>
<td>258.7</td>
<td>591.0</td>
<td>1,241.6</td>
<td>1.46</td>
<td>1.66</td>
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<td>8. Colombia</td>
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<td>14,508.4</td>
<td>25,210.0</td>
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<td>1.50</td>
<td>0.91</td>
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<td>9. Mexico</td>
<td>6,716.8</td>
<td>84,934.8</td>
<td>113,163.0</td>
<td>1.23</td>
<td>1.25</td>
<td>0.83</td>
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<td>10. Brazil</td>
<td>10,058.2</td>
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<td>114,144.0</td>
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<td>3,724.6</td>
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<td>1.22</td>
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<td>12. Ecuador</td>
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<td>1.09</td>
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<td>13. Chile</td>
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<td>14,740.3</td>
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<td>0.83</td>
<td>0.59</td>
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<td>17. Argentina</td>
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<td>0.34</td>
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<td>13,027.4</td>
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<td>0.29</td>
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<td>0.05</td>
<td>-1.54</td>
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<td>20</td>
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</table>

**Note:** Countries are listed in order of estimated price of their commercial debt.

b. Publicly guaranteed debt plus private nonguaranteed and short-term minus official debt.
c. For the definition, see table 2.
d. Estimated minimum price prior to regulatory default, for countries numbered 17–20.
e. Based on debt swap arrangements; reproduced for comparison with our estimate for country debt price.

**Sources:** World Bank (1985); Euromoney (February 1985).
Table 4. Sensitivity Simulations: Calculated Market Value of Debt / Face Value of Debt

<table>
<thead>
<tr>
<th>Asset value / debt face value</th>
<th>Interest rate spread (percent)</th>
<th>Principal due (percentage of debt)</th>
<th>Front-end Fee (1 percent of debt)</th>
<th>Loans (1 percent of debt)</th>
<th>Higher interest (up to 2 percent)</th>
<th>Higher variance (25 percent)</th>
<th>Default boundaries(^b)</th>
<th>Seizure rate: (10 percent of debt)</th>
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<td>Base case</td>
<td>Rescheduling(^a)</td>
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<td>1.00</td>
<td>1.00</td>
<td>0.96</td>
<td>1.05</td>
<td>1.01</td>
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<td>0.99</td>
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Note: The values shown are representative examples of the 100 points at which \( c \), the ratio of the market to the face value of the debt, is evaluated.

a. "Excess" corresponds to a higher than optimal value of \( x_3 \); "limited" to an \( x_3 \) value that is too low (see figure 1).

b. The variable \( b \) = the regulatory maximum of the debt-asset ratio.
that the ranking produced by our methodology corresponds closely to the ex post Euromoney ranking.

Individual country observations also seem reasonable: the relative positions of Korea, Malaysia, the Philippines, and Thailand clearly correspond to their actual economic and financial circumstances. Colombia and Venezuela emerge as the most creditworthy of Latin American countries. Brazil and Mexico have very similar positions, while Costa Rica and Nicaragua are in critical financial straits. These results suggest that this methodology could prove valuable in analyzing country creditworthiness.

References


Sadeq, S. 1985. "LDC Debt: Growth Opportunities, the Option to Repudiate and Debt Rescheduling." Master's degree dissertation, Sloan School of Management, Massachusetts Institute of Technology.