INDIRECT TAX EVASION AND PRODUCTION EFFICIENCY

by

Arvind Virmani

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Development Research Department
Economics and Research Staff
World Bank

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Abstract

Though a number of papers have considered the issue of income tax evasion few have considered indirect tax evasion. One of the exceptions is a paper which considers the very special case of a risk-averse monopolist producer. The present paper addresses the issue of indirect tax evasion in competitive markets. Evasion in such a market is shown to be associated with production inefficiency. The results also suggest that Laffer-type curves could arise due to evasion.
1. **INTRODUCTION**

Since the papers by Allingham and Sandmo (1972) and Srinivasan (1972) a substantial literature has grown up on tax evasion. 1/ The analytic literature has been concerned primarily with direct tax evasion. One interesting exception to the neglect of indirect taxes is a paper by Marrelli (1984) dealing with output taxes. Marrelli deals with a very special case, that of a monopoly industry run by risk averse managers. The present paper investigates the effects of output tax evasion in a conventional competitive industry with risk neutral firms.

The focus of the literature on direct taxes is perhaps a reflection of their relative importance in total tax revenues in developed countries. In many developing countries the situation is the reverse, with indirect taxes being much more important as a source of revenue (Tanzi 1987). Nevertheless the value added tax is an important source of revenue in Europe. Evasion problems arising in indirect taxation have been reflected in discussions of the cross-check features of this tax (Aaron, 1983). The possibility of replacing income by consumption or VAT type taxes has also attracted attention in the USA, and an understanding of evasion aspects of indirect taxes may be helpful.

When first imposing a VAT or when extending VAT type taxes, one issue that often arises is whether to include retailers. The large number and small size of the establishments to be covered is seen as a potential source of collection and administration problems. More generally, developing countries

1. See Cowell (1985) for a survey and further references.
often exemp. firms below a certain size -- defined in terms of assets, employment or sales -- from output (excise) taxation. Analysis however provides limited guidance on what conditions should determine the cut-off size. The present paper makes a start in this direction by providing a definition of "small firms" in the context of evasion, and analyzing their evasion behavior.

The limited evidence available suggests that tax evasion varies across products (industries) and by size of firm. A US Internal Revenue showed that farm proprietors had the lowest proportion of reported income at minus 19%. Informal suppliers (including home and auto repair, food, child services and domestics) reported 20%, non-farm proprietors and partnerships and small business corporations reported 50%. Skolka (1985) has noted the concentration of evasion in retail trade, restaurants and hotels, and to a lesser extent in construction and transportation. One purpose of this paper is to investigate how industry characteristics such as the minimum efficient scale of production might influence evasion.

Recent papers by Malcomson (1986) and Fullerton (1982) and others have analyzed the Laffer curve in the context of general equilibrium models. An interesting issue, the role of evasion in evoking a negative effect of taxes on revenues, was not considered formally in these and other papers on the Laffer curve. The present paper shows, in a partial equilibrium framework, that there is a tax threshold below which firms will not evade

3. Reported a loss instead of the actual profits.
4. Scu, however, Feige and McGee (1982 a b, 1983). Also see Malcomson for other references on the Laffer curve.
taxes. Above this threshold either complete or partial evasion can result depending on the direct resource costs of evasion. In the former case revenues will fall sharply as the tax rate crosses the threshold, but may do so even in the latter.

It is also shown that in equilibrium evasion will be associated with less than efficient scale of production under the conventional U shaped curve of micro theory. Thus output tax evasion can have real resource costs to the economy even when there are no direct costs of evasion. This has implications for optimal tax theory which need to be explored in future work.

The basic model without any direct evasion costs is presented and analysed in the next section. Resource costs of evasion are introduced in section 3, which first considers linear costs. The final part of this section deals with non-linear resource costs. Section 4 summarizes the conclusions, and outlines some possibilities for future work. Formal justifications for the propositions in the text are given in the appendix.

2. THE BASIC MODEL

Earlier papers on income tax evasion have made detection probability a function of income evaded or income declared (Srinivasan (1973), Nayak

5. This contrasts with the case of import rationed firms investigated by Krueger (1974) in which firms have excessive capacity.
(1978), Sproule et al. (1980). The latter can be interpreted as an audit rule based only on information contained in the tax files. The former implies that besides declared income, government authorities have other information about firms. This is particularly relevant if the taxpayer does not declare anything. Though in general both tax filer and outside information play a role in determining detection probability, the present paper focuses on the latter aspect.

Firms (usually) have a fixed place of business, which is well known to local authorities and easily accessible to tax authorities. Government provided services, which include public utilities in many countries, are directly observable from accounts. Output related information such as capital stock and number of employees is also relatively easy to obtain. Based on this information, an output measure which is monotonically related to actual output seems feasible. Though such a measure is unlikely to meet the standard of legal proof required to convict individual tax evaders, it will influence detection probabilities.

It is assumed in the paper that detection probability is an exogenously fixed function of output. The results are essentially the same if

6. Marelli (1984) also assumes that probability is a positive function of declared firm revenues.

7. These papers take the rule as given, while Graets et al. (1984), Reinganum and Wilde (1984ab), and Corchon (1984) analyze the choice of such audit rules or probabilities by the tax collection agency.

8. The output index Q is such that $Q = S(K,L)$, where K and L are capital and labor input measures. Thus if the production function is $q = F(K,L)$, it is assumed that q and Q are positively (monotonically) related. I understand that this is done in Israel.
detection probability is a function of firm revenue. Casual empiricism lends some support to the link between firm size and detection probability. Thus the largest corporations in a country are likely to have an audit probability close to one, while small street vendors have close to zero probability. This is certainly true for instance in India. The "common wisdom" in developing countries, is that evasion by small firms is both easier and much more commonplace (see e.g. Virmani (1985)). Though detection probabilities of zero and one at the extremes of the output range do not ensure a smooth progression in between, a detection probability rising with output appears plausible.

Previous models of tax evasion have assumed that no direct expenses are incurred in evading taxes. In the sales tax context this is a very good assumption if evasion involves understatement of price at which sales were made. In discussion of the issue of ad valorem versus specific taxes the assertion is often made by policy makers that revenue (price) is easier to conceal than output. If evasion entails under statement of current output and sales volume, costs are likely to be incurred in concealing current output

9. The results are modified slightly if detection probability is a function of declared output, as shown at the end of section 2.

10. Though output is a good measure of firm size within an industry (the focus of the formal model presented), a revenue based measure has some attraction for cross-industry comparisons. It has its own problems, however, because factor inputs (capital and employment) are more directly and easily observable, than the price received and consequently the value of sales.

11. A common excuse made to tax inspectors is that actual prices received were lower than printed or standard prices because of discounts.

12. This is one justification for basing audit probabilities on output rather than on revenues.
and inputs from tax collectors. The model without resource costs of evasion is considered in the present section. These cost are introduced and analyzed in the next section.

Conventional assumptions are made about firms and about industry equilibrium. Firms are risk neutral and produce a single good and have access to a standard technology represented by an (identical) U shaped average cost function. Though the formal analysis does not preclude a short run interpretation, these can be thought of as long run cost curves, as the focus of the analysis is on long run equilibrium. Industry is competitive with free entry and exit, with the former (latter) occurring when profits are positive (negative). A stable equilibrium will therefore be characterized by zero profits. The industry is subject to a positive tax at rate t, on output or sales (no distinction is made between the two). Firms declare a fraction (a) of revenues to the tax collector, and pay tax on it. Evasion is detected with probability $P(q)$, where q is firm output. A penalty is levied on evaded taxes which raises the effective tax on evaded sales by a factor e (greater than one), to et. The notation used in the paper is summarized in table 1.

13. The common excuse for any discrepancy between declared output/sales and capacity is low capacity utilization due to poor market demand.

14. As noted earlier, the results are unchanged if it is a function of revenue, but the exposition becomes a little more complicated.
Table 1: List of Variables

- **p**
  Consumer price of output.

- **q**
  Output per firm. \( q' \) is the minimum marginal cost point.

- **C(q)**
  Cost function for firm, \( AC(q) = C(q)/q \).

- **AC_i**
  \( C(q_i)/q_i \) where \( i \) is either \( m, a \) or \( o \).

- **q_m**
  Minimum efficient scale of production/min. average cost output.

- **t**
  Ad valorem tax rate on output. Collected from firm.

- **a**
  Proportion of revenue declared to tax authorities.

- **P**
  Probability that evasion will be detected, \( P = P(q), P^0 > 0, P'' > 0 \).

- **e**
  Penalty proportion on detected tax evasion.

- **T**
  Expected tax per unit of revenue.

- **T_m**
  Marginal effective tax rate.

- **g**
  Average (per unit of output) Resource cost of evasion. The three cases considered are, \( g=0 \), \( b(1-a) \) and \( g(1-a), g^0 > 0 g'' > 0 \).

- **q_o**
  Firm output if potential equilibrium with complete evasion prevails.

- **P_o**
  Price under the above equilibrium.

- **q_a**
  Firm output if interior potential equilibrium with partial aversion prevails.

- **P_a**
  Price under the above equilibrium.

- **P_m**
  Price under a potential equilibrium with honest declaration.

- **q_c**
  Critical output level at which expected penalty factor is one.

- **t_m**
  Tax rate below which honesty is ensured (evasion ruled out).

- **t_o**
  Tax rate above which complete evasion is ensured (honesty ruled out).

- **t_m_o**
  Tax rate at which firms switch from honesty to complete evasion.

- **t_m_a \ (t_a_o)**
  Tax rate at which firms switch from complete honesty to (partial evasion) to partial (complete) evasion.
The representative risk neutral firm's objective is to maximize expectation of firm profits $\pi$. This reduces to,

$$\text{Max } \pi = (1-T)pq-C(q), \quad T = t(a+(1-a)eP(q)), \quad P' > 0, \quad P'' > 0, \quad P(0) = 0,$$

where the usual U shaped cost function $C(q)$ with elasticity $E(q)$, can be defined as in (2).

$$C(q)/q \Rightarrow C'(q) \text{ as } q \Rightarrow q_m, \quad C'(q) \Rightarrow 0 \text{ for } q \Rightarrow q', \quad 0 < q' < q_m, \quad E(q) = qC'(q)/C(q).$$

$p$ is price of output, $q_m$ is the minimum efficient scale or minimum average cost point, $q'$ the minimum marginal cost point, and a prime (') on a function represents the differential with respect to the argument. The effective tax rate $T$ will be positive and less than one if $eP$ is less than 1. This will ensure some production. The second differential of the probability function is assumed to be non-negative for expositional simplicity, though a weaker condition would be sufficient for our results. The cost elasticity $E$, for the standard smooth U shaped average cost curve defined here, is one at the minimum average cost point $q_m$. $q'=0(q'>0)$ represents the situation in which there are fixed costs (no fixed costs) of production. In the absence of fixed costs, elasticity would also be one at the zero output point.

The necessary conditions for firm profit maximization are obtained by differentiating (1) with respect to firm output and tax declaration. Ignoring
the case in which firm output is zero, the first condition is

\[ C'(q) = (1-T_m)p, \quad T_m = t(a+(1-a)e(P(q)+qP'(q))) = T+t(1-a)eqP' < 1 \quad (3) \]

where \( T_m \) is the marginal effective tax on output. Equation (3) therefore says that for any given evasion level, marginal cost of production must be expanded to the point at which it equals marginal net of tax revenue. The second condition is,

\[ eP(q) - 1 \triangleq 0 \quad \text{implies} \quad a = 1, a^* \text{ or } 0, \quad 0 < a^* < 1. \quad (4) \]

As \( e \) and \( P(.) \) are assumed to be exogenously fixed throughout this paper, we can define a critical output level \( q_c \), as follows:

\[ eP(q_c) = 1 \quad (5) \]

Equation (4) can then be restated as

\[ a = 0, a^* \text{ or } 1 \quad \text{as} \quad q \leq q_c \quad (4') \]

The critical output level is intimately related to the determination of the level of evasion. It represents an output level at which evasion is a

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15. The penalty factor has been taken as exogenously fixed in earlier papers. Detection probability has been viewed similarly in most papers, but some have assumed that it can be changed by incurring costs (Hansson (1985)).
fair gamble. Risk neutral firms are therefore indifferent to the level of evasion if output is at $q_c$. In this case the precise level of evasion is determined by (3) rather than by (4). If $q$ is different from $q_c$, firms are either honest ($a=1$) or declare nothing to the authorities. The solution of the simultaneous equations (3) and (4) for $q$ and $a$ is therefore much more complicated than the simple model of (1) and (2) suggests. The rest of this section unravels the implications of the solution in stages.

For subsequent reference, note that $q_c$ defined in (5) can also be seen in a different perspective. From a comparative (static) perspective $q_c$ can be viewed as an Index of Tax Enforcement, which will be different for different tax systems (or countries). A low $q_c$ represents a country or system with strong and a high $q_c$ one with weak enforcement.

At a stable equilibrium the expectation of profit equation (1) must be zero, so that equilibrium price $p$ is,

$$p = \frac{C(q)}{q(1-T)}, \quad T = t(a+(1-a)eP(q))$$  \hspace{1cm} (6)

Substitution of (6) in (3) yields the equation for determining equilibrium output, which can be written as (7) or (7')

$$E(q) = x(q,a), \quad x = \frac{(1-T_m)}{(1-T)} (= 1-y),$$  \hspace{0.5cm} (7)

$$C(q)/q - C'(q) = yC(q)/q, \quad y = t(1-a)eP'/\left(1-T\right).$$  \hspace{0.5cm} (7')

where $E$ was defined in (2).

Equations (4), (6) and (7) define the competitive equilibrium. Putting $a=1$ in (7) and comparing with (2), we have the usual no evasion equilibrium with production at the minimum efficient scale $q_m$. Let the quantity-price pairs, satisfying (6) and (7) conditional on full ($a=1$), partial ($1>a>0$) and no declaration ($a=0$), be $(q_m, p_m)$, $(q_c, p_c)$ and $(q_o, p_o)$.
respectively. For expositional brevity, these will be referred to as potential equilibria. Note that,

\[ p_0 = C(q_0)/(q_0(1-t-eP)), \quad E(q_0) = x(q_0,0), \]
\[ p_c = C(q_c)/(q_c(1-t)), \quad \text{and} \quad p_m = C(q_m)/(q_m(1-t)). \]  

(6')

By differentiation of (7') the slope of \( x \) with respect to \( q \), the shift in \( x \) with evasion proportion, and the change in firm output with tax rate (for fixed \( a \)) are found to be,

\[ \frac{dx}{dq} \approx 0, \quad \frac{dx}{da} \leq 0 \quad \text{and} \quad \frac{dq}{dt} \leq 0 \quad \text{as} \quad a \leq 1. \]  

(8)

Equilibrium can be depicted as in figure 1a and 1b (or figure 2) for two possible shapes of the cost function. The three potential equilibria are shown in figure 1a. 16/ Which of these is the actual equilibrium depends among other things, on (a) whether the minimum average cost is less than or greater than the critical output level, and (b) what the tax rate is. Figure 1b shows two cases with complete evasion, only one of which is a feasible equilibrium.

The rest of this section investigates the characteristics of the three potential equilibria and the conditions under which each will prevail. The notable results are highlighted by presenting them as propositions, while intermediate results are given as lemmas, the formal proofs are in the appendix. From figure 1 (i.e. either 1a or 1b) or equation (6'), it is apparent that, if a stable equilibrium exists, firm output \( q^* \) must be less than or equal to minimum efficient scale of production \( q_m \). That is,

16. Again, I am ignoring the zero output solution, by assuming that the cost elasticity does not lie above \( x(\cdot) \) at all output levels below \( q_m \).
Figure 1a: Fixed Costs

Figure 1b: No Fixed Costs
Proposition 1. Evasion Leads To Production Inefficiency

If evasion exists in equilibrium \((a<1)\) firm output will be less than minimum efficient scale \((q^*<q_m)\).

The existence of tax evasion will be associated with a less than efficient scale of production. As there is no inefficiency when detection probability is constant, this result arises directly from the assumption that detection probability rises with output. When a firm is evading taxes any increase in output (below \(q_m\)) not only lowers average cost but also increases the probability of detection. Thus the effective marginal cost (price) is higher (lower) than that indicated by production cost conditions alone, and the equilibrium must be to the left of the optimal scale (figure 2).

Equation (4) suggests the type of industry in which firms will evade taxes in equilibrium. From the evasion perspective we can define "small" firms as ones in which the minimum efficient scale of production is less than the critical output level \(q_c\). Proposition 2 shows that such small firms will evade taxes in the absence of resource costs of evasion.

Proposition 2. Small Scale Firms \((q_m < q_c)\) will Evade Taxes

An industry in which the minimum efficient scale is smaller than the enforcement index will be characterized by complete evasion and production inefficiency.

Corollary 2.1. The inefficiency of evading (or small firms) will increase with the tax rate and penalty factor.

In this case partial evasion with equilibrium at \(q_c\) is ruled out (by proposition 1) because this output is greater than the minimum efficient scale. The potential equilibrium with full disclosure cannot prevail because it results in a contradiction. From (6) equilibrium is at the efficient scale.
Figure 2a: Fixed Costs

Figure 2b: No Fixed Costs
if $a=1$, but (4) implies that $a=0$ at this production level. Basically complete evasion is the only feasible equilibrium under the assumed conditions.

A rise in the tax rate or the penalty factor increases the effective marginal cost. As the effective tax rate $T$ equals $teP$ under complete evasion, the cost of any increase in detection probability is a multiple of the tax rate and penalty factor. It therefore results in a decline in firm output and a rise in average production costs. This can be shown in figure 1, as a clockwise rotation of $x$ around the zero output point, which moves it down the cost elasticity curve. Note that in the case of figures 1b and 2b this cannot continue indefinitely, as there may be no equilibrium at an output $q$ such that $0<q<q'$.

Turning from small firms to the polar case of large firms one might conjecture that they will never evade taxes. From the definition of 'small', firms with minimum efficient scale larger than the enforcement index ($q_m>q_c$) can be considered medium and large firms. In equation (6') output $q_0$ was defined as the solution of (6) with disclosure parameter, $a$, set at zero. From the evasion perspective, firms may then be considered medium or large as $q_0$ is less than or greater than the enforcement index $q_c$. Though this distinction between medium and large firms is shown below to be an elusive one, consider the case in which $q_m>q_0>q_c$. Equilibrium at $q=q_0$ can be ruled out because $eP(q_0)>1$ implies $a=1$ (equation (4)), a contradiction. A partial evasion

17. Differentiation of $x$ with respect to tax rate (or penalty factor) shows that they are negatively related.
equilibrium at \( q = q_c \) is a feasible one. Thus it is not obvious a priori whether "large" firms will evade part of their taxes or not. \(^{18}\)

The difficulty in distinguishing between medium and large firms in the way suggested above (with \( q_m > q_c \)), is evident from lemma 1. This lemma is also the first step in determining the evasion behavior of such firms under different conditions.

**Lemma 1. The Critical Tax Rate (\( t_c \))**

If the tax enforcement index is greater than the minimum marginal cost point \( q' \), there exists a tax rate \( t_c < 1 \) at which the potential complete evasion equilibrium point \( q_0 \) equals the tax enforcement index \( q_c \).

If we solve (7) for \( t \) after putting \( a = 0 \) and \( q = q_c \), we obtain,

\[
    t_c = \frac{(1 - E(q_c))/(1 - E(q_c) + q_c P'(q_c))}{1} < 1 \quad \text{as} \quad E(q_c) < 1, \quad P' > 0.
\]

The conditions in the proposition ensures that the two curves in figure 1 actually intersect, so that a solution at \( q = q_0 \) is feasible. Thus the potential full evasion equilibrium \( q_0 \) depends on the tax rate imposed on the industry, and cannot be used to distinguish large from medium scale firms.

This contrasts with the definition of small scale firms which depends only on

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18. With constant probability all firms with \( eP(q_m) \) less (more) than one will evade all (no) taxes.
the enforcement parameters which are likely to be relatively stable across industries.\footnote{There may be a problem in using the \( P(q) \) function across industries, because of the problem of comparing \( q \) across industries. This problem may not be too serious if the tax collectors' actions are based on a quantity index derived from factor inputs as suggested in the introduction and footnote 8. Further as noted at the beginning of the section, the basic results apply even when detection probability is a function of revenue.}

The conjecture that large firms will not evade taxes, turns out to be partially true, but applies to both medium and large firms.

**Proposition 3. Honesty under Low Tax Rates**

If the minimum efficient scale of an industry \( (q_m) \) is greater than the enforcement index \( (q_c) \), and the tax rate is at or below the critical level \( (t_c) \) there will be no evasion in equilibrium.

As the effective tax rate is identical under the two potential equilibria \( (a=1, \text{ or } 1>a>0) \), the only difference is in production costs. The full declaration equilibrium, with the lowest unit costs, is therefore the only sustainable one. The formal proof of proposition 3 suggests that the equilibrium consumer (gross) price under the alternative evasion assumption plays a critical role in determining which behavior will prevail.

Consider therefore the case in which the tax rate is greater than the critical rate, and a complete evasion equilibrium is feasible.

**Lemma 2. Gross Price and Evasion Equilibrium**

Equilibrium pre-tax (gross) prices must be lower than the price which would prevail under any alternative feasible potential equilibrium.

The equilibrium pre-tax price is the ratio of average cost of production to the retention rate \( (1 \text{ minus the tax rate}) \). Both of these are higher...
(effective tax rate lower) under the potential equilibrium with complete evasion than under complete honesty. Thus the price under the former relative to that under the latter, \( \frac{p_0}{p_m} \) may be greater or less than 1 in general.

Consider the hypothetical situation in which new firms with access to the same technology enter with a different evasion behavior. This is possible if and only if prices can fall without making profits negative for the new entrant. As existing firms have zero profits by condition (6), this is only feasible if the zero profit price under the new (entrants) evasion behavior is less than under the existing one. Thus a "potential equilibrium" can be the actual equilibrium if, and only if, the corresponding price is the lowest of all "potential equilibrium prices."

When the tax rate is greater than the critical rate firms may either evade or honestly pay their taxes. To determine what actually happens we have to look at the price ratio \( p_r \) and its differential with respect to tax rates.

\[
 p_r = \frac{p_0}{p_m} = \frac{(1-t)/(1-te)}{(C(q_0)/q_0)/(C(q_m)/q_m)} \tag{9}
\]

Taking the log of the above equation, differentiating with respect to \( t \) and simplifying using equations (4) to (7), we have

\[
 \frac{dp_r}{dt}/p_r = \frac{eP/(1-te)}{(1/(1-t))} = -\frac{1-eP}{(1-teP)(1-t)} \leq 0 \quad \text{as} \quad eP(q_o) \leq 1. \tag{10}
\]

This price ratio can then be drawn as in figure 3.

Examination of figures 1 and 3 suggests that if \( E(q) \) continues to intersect \( x(q,0) \), then there must be a tax rate \( t_{mo} \) at which \( p_r \) becomes less than one. That is,

**Proposition 4. Threshold Tax Rate \( t_{mo} \)**

If the minimum efficient scale is larger than the enforcement index \( q_c \), and the minimum marginal cost point \( q' \) is zero, then there must exist a
tax threshold above which there is complete evasion.

Corollary 4.1 If the minimum marginal cost point \( q' \) is greater than zero there can be a case in which tax evasion will not occur at any tax rate.

The threshold rate can be obtained from (9) by putting \( p_r \) equal to one. That is,

\[
t_m^0 = \frac{(AC_0 - AC_m)}{(AC_0 - eP(q_0)AC_m)}, \quad AC_i = C(q_i)/q_i, \quad i = m \text{ or } 0,
\]

\[0 < t_m^0 < 1 \text{ as } eP(q_0) < 1 \text{ for } t_m^0 > t_c, \text{ and } AC_0 > AC_m.\]

At low values of the tax rate the benefit from evading taxes is low, relative to the difference in average costs of production. As the tax rate rises the benefit from evasion rises and exceed the average cost differential beyond the threshold rate.

When \( q' \) is greater than zero there can be an output range, \( 0 < q < q''( < q') \), in which the firm will not produce. In this case proposition 4 will be modified for tax rates above the threshold rate, along the lines in figure 4. The curve marked A shows the case of proposition 4 with \( q' = 0 \). The curves marked B and C show the two cases possible when \( q' > q'' > 0 \). Solution curve B(C) is obtained if \( q'' \) is less (more) than the full evasion output, \( q_0 \), at the threshold tax rate \( t_m^0 \). Thus in case C firms will be honest at all tax rates.

Proposition 4 implies that there will be a sharp fall in declared tax revenues when tax rates are raised above the threshold rate. 20/ Thus a Laffer-type curve could arise as a consequence of a rise in average indirect tax rates, as more and more industries switch to tax evasion. As some revenues are still collected through the detection penalty mechanism, this

20. Except for case C of Figure 4.
Figure 3. Evasion Tax Threshold

Figure 4. Medium-Large Firm Case
drop in revenues with average tax rates is not irreversible. Thus a global maxima could be preceded or succeeded by local maxima. The existence of production inefficiency in industries with tax evasion also suggests that optimal tax rate formulas must be adjusted. Tax rates will tend to be lower if appropriate account is taken of these resource costs.

A crude feel for the dividing line between complete and no tax evasion can be obtained as follows. Using a value of 3 (or 2) for the penalty factor, yields a separating probability \( P(q_c) \) of one third (or half) at an output equal to the critical level \( q_c \). The present results imply that firms with a minimum efficient scale less than that at which detection probability is less than one third (half) will declare nothing. If the monetary equivalent of \( r \) potential jail term is added to the monetary penalty however this probability could be reduced to a small fraction. A separating probability \( P(q_c) \) of the order of one tenth is quite plausible.

That firms either evade all taxes or declare everything is a weakness of the model considered so far. Though both these actions are observed in practice, so is partial evasion. One factor which may explain partial evasion is the resource cost of evasion (considered in the next section). Another potential factor is more surprising. A detection probability which is a
function of evaded, instead of total, output smooths the evasion progression. 21/

To show how this happens, I digress briefly to consider the case in which the probability function is, \( P = P((1-a)q) \). In this case the maximization of firm profits with respect to evasion proportion \( a \) yields, 
\[ tp(eS-1) < 0 \text{ implies } a < 1, \text{ as } S = P+(1-a)qP' = 0 \text{ if } a=1. \]

Full declaration or complete honesty is now completely ruled out, and partial evasion takes its place. Except for this substitution, the basic results are virtually unchanged. Thus complete evasion still occurs in the "small firm" case \( q_m < q_c \). In the case of other firms \( q_m > q_c \), there is now partial evasion below the same critical rate \( t_c \). The move from partial to complete evasion similarly occurs at a threshold tax rate. 22/

The reason for the elimination of the honest solution is that the marginal increase in detection probability with evasion is now zero at an honest equilibrium. As the loss in production efficiency is of second order (flat portion of the average cost curve) compared to the gain from evasion, it never pays to be completely honest.

3. RESOURCE COSTS OF EVASION

One well known cost of evasion by firms is the need to keep two books of account. Costs may also be incurred in concealing output and the raw

21. One scenario for such an outcome is that taxpayers are audited completely randomly, but an audit does not result in complete detection. Because of the legal requirement of proof the legally effective detection will be positively related to the evasion proportion (see Virmani (1987)).

22. This can now be determined by reference to relative production levels under the two potential equilibria.
material inputs which go into its production. In this section it is assumed
that evasion cost depends on total output evaded, with average costs a func-
tion of the proportion of output evaded. Firm profit equation (1) can there-
fore be rewritten as,
\[ \pi = (1-T)pq-C(q)-qg(1-a), \quad g(0)=0, \quad g'>0, \quad g'' \geq 0, \]
(1')
The rest of the assumptions are identical to those in the previous section.

Maximizing (1') with respect to a and q, and substituting equilibrium
price from the zero profit condition (6'), we obtain,
\[ (g'(1-a) - (1-eP(q))tp)q \leq 0 \implies a=0, \text{ or } 1, \]
\[ \text{or } g'(1-a) \leq (1-eP(q))tAC(q)/(1-T) \implies a=0, \text{ or } 1 \]
(12)
\[ (1-y)C(q)/q - C'(q) = yg(1-a), \quad y = t(1-a)eP'/(1-T), \]
(13)
\[ p = (C(q)/q + b(1-a))/(1-T), \quad T = t(a+(1-a)eP) \]
(6')

The potential equilibria with none and complete evasion can be
defined using equations (13) and (6'). The first \((q_m, p_m)\) is identical to
that obtained earlier. The second \((q_0, p_0)\) has essentially the same form, but
involves even greater inefficiency.

**Linear Costs**

Resource costs of evasion have two notable effects. One of these,
the effect on evasion by small firms is most clearly seen if we assume evasion
costs to be linear. \(^{23/}\) For the present assume therefore that \(g(\cdot)\) is given
by (14).

---

23. Linear resource costs of evasion have the same effect in the case in
which detection probability is a function of evaded output. Somewhat
more surprisingly this case becomes identical in form to the one
considered in the text, on the introduction of evasion costs.
\[ g(1-a) = (1-a)b, \quad b > 0 \]  \tag{14}

where \( b \) is a constant. Equation (12) simplifies to (12'), and
\[ a = 0, \ a^* \text{ or } 1 \text{ as } B = (1-t)b/t \leq (1-eP(q)) \frac{C(q)}{q} = f(q) \]  \tag{12'}

Recalling that the small firm case is one in which the minimum average cost point is smaller than the enforcement index, (12') can be shown graphically as in figure 5.

If \( B \) is less than or equal to \( f(q_m) \), as shown, an honest solution is ruled out (12'). When there are no resource costs \( B \) is identically zero and this condition always holds. Small firms evade all taxes as shown in the previous section. In the presence of resource costs a low enough tax rate will push \( B \) above \( f(q_m) \) so that a solution with complete honesty is feasible. 24/ This happens when the tax rate is less than \( t_0 \), where \( t_0 \) is obtained from (12') by solving with equality.

\[ t_0 = b/(b + (1-eP(q_m))AC_m), \quad AC_m = C(q_m)/q_m \]

As both honest and corrupt solutions are possible for rates below \( t_0 \), it is similar \( t_0 \), but the inverse of the critical tax rate \( t_c \) obtained in section 2 for the other firms. We therefore expect the following modification of the results obtained for small firms in the no resource cost case.

**Proposition 5. Threshold Tax Rate for Small Firms**

If the minimum efficient scale is larger than the enforcement index \( q_c \) and resource costs of evasion are positive, then there must exist a tax rate \( (t_m) \) below which firms do not evade taxes.

24. The honest solution is no longer ruled out for the case in which probability is a function of evaded output. The existence of resource costs results in a positive marginal cost of evasion at the honest potential equilibrium.
Figure 5: Linear Resource Costs of Evasion
The simplest way to see that this will happen is by noting that $B$ approaches infinity as $t$ approaches zero. We also know from earlier analysis that the full evasion output $q_0$ approaches the efficient point $q_m$ as taxes are lowered. Therefore there must be a tax rate at which $B$ is greater than $f(q_0)$, and complete evasion results. The critical rate $t_m$ in this case is (from (12')),

$$ t_m = \frac{b}{b + (1-eP_oAC_0)} < t_o \text{ as } AC_0 > AC_m' $$

where $q_0$ is obtained by solving (13) with $a=0$. The existence of resource costs shifts the advantage from complete evasion to complete honesty. This is manifested in the small firm case in a rise in the critical rate from zero to a positive value ($t_m$). 25/

The two critical values, $t_m$ and $t_o$, give the bounds outside which either complete evasion or honest declaration must prevail. If the tax rate is in between these two values either case is possible along with partial evasion. As shown in lemma 2, which one will actually prevail depends on the potential equilibrium price. The interior solution entails a higher price than the honest solution, and can again be ruled out as an equilibrium. It can also be shown that the potential equilibrium price with complete evasion is less than the potential equilibrium price under complete honesty for tax rates above the threshold (appendix). The critical tax rate $t_m$ therefore constitutes the threshold rate for small firms, with complete (no) evasion if rates are higher (lower).

25. The same results are obtained when detection probability is a function of evaded output. With complete honesty a feasible solution (footnote 24), partial evasion is ruled out because it results in higher prices.
The analysis for the other firms or equivalently for an industry in which the minimum efficient scale is less than the enforcement index is basically unchanged. The only difference is in the threshold rate $t_{mo}$. From equation (13) the price for the potential equilibrium with complete and no evasion are determined and equated. That is,

$$AC_m/(1-t) = (AC_o + b)/(1-teP(q_o)), AC_i = C(q_i)/q_i,$$

where $i$ is $m$ or $0$, or

$$t_{mo} = (b+AC_o - AC_m)/(b+AC_o - eP(q_o)AC_m) < 1 \text{ as } eP(q_o) < 1 , \quad (16)$$

which is identical to the threshold rate obtained earlier if $b=0$. Thus this industry will be characterized by complete (no) evasion as the tax rate is greater (lower) than this threshold rate $t_{mo}$.

**Convex Costs**

Return now to the case in which average evasion costs are strictly convex in the evasion proportion. That this case is fundamentally different from the audit model is apparent from equation (12). The sufficient condition for an interior solution for evasion proportion will now be satisfied more generally ($-g'' > 0$). As the price prevailing under a potential interior equilibrium can now be less than or equal to the honest equilibrium, there can be partial evasion over a range of tax rates.

Equation (12) defines the critical tax rates $t_m$ and $t_o$, with the honest solution ruled out above $t_o$, and the complete evasion solution ruled out below $t_m$.

$$t_o = g'(0)/(g'(0) + (1-eP)(g(0)+AC_m)), \quad (17)$$

$$t_m = g'(1)/(g'(1)eP(q_o) + (1-eP(q_o))(g(1)+AC_0)) \quad (18)$$

The solution can be analyzed by plotting the two curves given by equations (12) and (13) on the output evasion-proportion plane. Differentiation of $(6')$ shows that in equilibrium (i.e. given (12) and either (13)
with equality or a constant) the price changes only with the tax rate. Using the differential of \( p \) from (6'), the slopes of the output (12) and evasion (13) curves are both found to be positive (appendix). As taxes rise, the output curve rotates clockwise around the \((q=q_m, a=0)\) point, while the evasion curve rotates anti-clock wise around the enforcement index point \((q_c)\). This is illustrated for the medium-large firm case \((q_m>q_c)\) in figure 6.

The most surprising result of this section is that even non-linear resource costs do not result in partial evasion in the medium-large firm case. Thus resource costs have no effect on the form of the solution, which will change from honesty to complete evasion at some threshold tax rate. As equation (18) shows, resource costs can eliminate the switch to evasion by raising the critical rate \( t_m \) above one. This will happen if \( g'(l) > AC_o - g(l) \). At equilibrium the term to the right of the inequality is related to marginal costs of production. Therefore, loosely speaking medium-large firms are honest at all tax rates, when the marginal cost of evasion is greater than the marginal cost of output at the complete evasion point. Evasion is not profitable in this case (and firms may shut down completely).

Consider next the small firm case \((q_m<q_c)\). As before, evasion is ruled out for any output between these levels as \( eP(q_a) > eP(q_0) > 1 \) and it contradicts equation (12). Analysis along the lines given in figure 6 shows that there are three possibilities. The comparative statics (of an interior solution) show that fraction of output declared \((a)\) and firm output \( q \) are negatively related to the tax rate. The basic solution can be represented as in figure 7, with two threshold rates \( t_{ma} \) and \( t_{ao} \). Firms are honest if tax rates are below the first threshold, partially evade taxes between the two thresholds, and declare nothing if the tax rate is above the second threshold.
Figure 6: Output and Evasion by Large-Medium Firms ($q_m > q_c$)

Figure 7: Output and Evasion by Small Firms ($q_m < q_c$)
From equation (17) we can see that the critical rate $t_0$ is zero if the marginal cost of evasion is zero at the honest point in this case firms are never completely honest, and the first threshold in figure 7 is reduced to zero. When the marginal resource cost is constant (and positive) as analyzed in the first part of the paper, the two threshold rates collapse to one. Partial evasion never occurs in this case.

4. CONCLUSIONS

Two important conclusions emerge from this paper. One is that evasion is associated with production distortions. This conclusion is fairly robust to the form of the detection probability functions. The only case in which this link may be absent, is when auditing involves complete detection, and audit probability is a non-negative function of declared output (or revenue) alone. Formally, this would imply that firms can reduce audit probability to the lowest possible level by declaring nothing, even if they are large well known firms.

It is possible that tax authorities use an audit rule which allocates firms into different classes according to their size, with audit probability within a class constant or positively related to declared revenue. Complete detection on audit is, however, much less plausible because effective detection requires that evasion be proved in a court of law.\footnote{As noted by Greenberg (1984), this is the most important problem faced by the IRS.} Thus effective detection is likely to be a positive function of evasion proportion. When
this is coupled with a constant audit probability the results will be similar to those obtained above. 27/

One implication of production distortions is that optimal commodity tax rates can be quite different in the presence of tax evasion. The results of the paper suggest that the problem is likely to be more serious for industries in which the scale of production is relatively low. This suggests the hypothesis that optimal taxation will require lower tax rates on such industries if evasion exists. Alternatively the presence of evasion, may require exemption of small firms from taxation. The results of the paper are merely suggestive, and the issue needs to be pursued in future work.

The association between evasion and inefficiency provides a potential explanation for the observed dualism in developing countries. In these countries an organized and relatively efficient modern sector containing large firms, coexists with an unorganized sector containing small firms. Tax collections are confined primarily to the former (see Virmani (1986)). The model of section (2) shows that if tax collectors find it easier to detect evasion by large firms, this is exactly the outcome we would expect. If this explanation is valid, strategies for changing detection probabilities through modification of audit rules should be investigated in future work.

The second result is the variation in the pattern of output tax evasion with the tax rate. In general there are likely to be two critical tax rates, with no evasion below the first, complete evasion above the second, and partial evasion in between. The paper derived certain conditions under which

27. When coupled with a probability which is a positive function of declared income, an excessively high scale of production may also be possible.
the two thresholds collapse to a single one, with no partial evasion. Revenues were shown to drop sharply when tax rates cross this unified threshold, and are likely to do so even if the thresholds are separate. 28/

The paper showed that under certain conditions the first (honesty) threshold rate is zero so that there is evasion at all tax rates. As one of these conditions is small firm size, measured either by minimum efficient scale or revenue, one can hypothesize that this threshold is positively related to firm size. A more formal investigation of this issue could also be pursued in future work.

28. The latter will ensue if equilibrium output per firm is less than the minimum efficient scale. Effectively the production inefficiency is exactly equal to the revenue loss, as prices remain unchanged.
REFERENCES


APPENDIX

Proposition 1. Evasion Leads To Production Inefficiency

If evasion exists in equilibrium (a<1) firm output will be less than minimum efficient scale (q*<q_m).

Proof. Given the assumed shape of the cost curve, for q>q_m marginal cost is greater than the average cost and E(q)>1. From (7) and (7') and the modeling assumptions, y>0 and x<1. This violates equation (7). Therefore q>q_m cannot be an equilibrium. Contrary to proposition 1 assume that there is an equilibrium with q=q_m and a<1. In this case 1 = E(q) > x (<1) again contradicting (7). QED.

Proposition 2. Small Scale Firms will Evade Taxes

An industry in which the minimum efficient scale is smaller than the enforcement index will be characterized by complete evasion and production inefficiency.

Proof. Assume to the contrary that there is no evasion i.e. a=1. From equations (6) and (6') x and consequently E must be one, and equilibrium must be at q=q_m. With q_m<q_c, equation (4) shows that eP(q_m)<1 (as P'>0) and a=0, a contradiction. An equilibrium at q=q_c is ruled out by proposition 1. Therefore the only possible solution is at a=0 with complete evasion. QED.

Lemma 1. The Critical Tax Rate

If the tax enforcement index is greater than the minimum elasticity point q', there exists a tax rate t_c<1 at which the (potential) complete evasion equilibrium point q_o equals the tax enforcement index q_c.
Corollary \( q_0 \leq q_c \) for \( t \geq t_c \) as long as \( q_0 \geq q' \).

Proof. By assumption, \( E(q_c) \) must lie between zero and one for each output below the minimum efficient scale. From (7) and (7') \( x(q_c,0) = 1-y \),

\[
y(t) = t e q_c p'(q_c)/(1-t) \] using \( e p(q_c) = 1 \). Now \( y \) is zero at \( t=0 \), infinite at \( t=1 \) and has a positive slope for \( 0 < t < 1 \), such that \( E(q_c) = x(q_c,0) = 1-y(t_c) \) where

\[
y(t_c) = t_c e q_c p'(q_c)/(1-t_c) \]. Therefore

\[
t_c = (1-E(q_c))/(1-E(q_c)+e q_c p'(q_c)) \].

A sufficient condition for a positive \( q_0 \) to be a maxima conditional on all firms evading taxes, and thus satisfying (7) is that \( q_0 > q' \). This means that a potential equilibrium at \( q = q_0 \) is feasible when \( t = t_c \). It is defined by \( E(q_0) = x(q_0,0) \) and is consequently identical to \( q_c \).

If \( t \) is raised above \( t_c \), \( y \) must rise and \( x \) fall. With \( E'(q) \) positive above \( q' \), (7) can only be satisfied if \( q_0 \) falls. It will continue to be satisfied as long as \( q_0 > q' \). In the converse case of \( t < t_c \) the last condition is automatically satisfied. This proves the corollary. QED.

Proposition 3. Honesty under Low Tax Rates

If the minimum efficient scale of an industry is greater than the enforcement index, and the tax rate is at or below the critical level there will be no evasion in equilibrium.

Proof. From lemma 1 and its corollary \( t < t_c \) implies \( q_c < q_0 \). Equilibrium at \( q = q_0 \) can be ruled out because \( e p(q_0) > 1 \) implies \( a = 1 \) (equation (4)), a contradiction. The only feasible equilibria are \( q = q_c \) or \( q_m \). Assume,

---

30. For the no-fixed-cost case the minimum marginal cost point must be to the right of \( q' \), so that \( c''(q') > 0 \) and the sufficient condition is satisfied.
contrary to the proposition that equilibrium is at \( q = q_c, p = p_c (0 < a < 1) \). Using (4) in (6), the zero profit condition is,

\[
0 = (1-t)p_c q_c - C(q_c) = ((1-t)p_c - C(q_c)/q_c)q_c
\]

\[
< q_m((1-t)p - C(q_m)/q_m) = \pi(q = q_m).
\]

The inequality in the second line is obtained because the average cost is lower at \( q_m \) than at \( q_c < q_m \). Positive profits are therefore attainable by a new firm which enters the market and declares all its output. It cannot therefore be an equilibrium. Similarly it can be shown that a new entrant cannot make a profit if equilibrium is at \( (q_m, P_m) \).

**QED.**

**Corollary 3.1** \( P_m < P_c \) if \( q_c < q_m \).

**Proof.** \( P_m = (1-t)C(q_m)/q_m > (1-t)C(q_c)/q_c = p_c \) from the definition of the of \( q_m \).

**QED.**

**Lemma 2. Gross Price and Evasion Equilibrium**

Equilibrium pre-tax (gross) prices must be lower than the price which would prevail under any alternative feasible potential equilibrium.

**Proof.** From lemma 1, \( q_0 < q_c < q_m \). As in the proof of proposition 3, \( q_c \) can be eliminated by comparing its profits with those under \( q_m \). We have to show that equilibrium \( q^* = q_0 \) or \( q_m \), as \( p_0 < p_m \). Consider the case in which \( p_0 > p_m \).

Assume contrary to lemma 2, that equilibrium is at \( (q_0, p_0) \). Then from (1) and (6) the profits for a new entrant who produces \( q_m \) and declares all revenues is \( \pi \),

\[
\pi = q_m((1-t)p_0 - C(q_m)/q_m) = q_m(1-t)(p_0 - p_m) > 0
\]

where the last term is obtained by using the definition of \( p_m \). Firms will therefore enter and price will fall, contradicting the assumption that \( (q_0, p_0) \) was an equilibrium. Next consider the case in which \( p_0 < p_m \), and assume that equilibrium is at \( (q_m, P_m) \). Consider the profits of a new entrant who declares
nothing and produces \( q_0 \),
\[
\pi = q_0 ((1-teP)p_m - C(q_o) / q_o) = q_o (1-teP)(p_m - p_o) > 0
\]
where the last term is obtained by using the definition of \( p_o \).
This produces a contradiction. It is easy to show using the same logic that when equilibrium is at \( q_m \), \( p_m < p_o \) or at \( q_o \), \( p_o < p_m \), a new entrant cannot make positive profits. QED.

**Proposition 4. Threshold Tax Rate \((t_0)\)**

If the minimum efficient scale is larger than the enforcement index \( q_c \), and the minimum elasticity point \( q' \) is zero, then there must exist a tax threshold above which there is complete evasion.

Proof. From lemma 1, \( q_0 = q_c \) at \( t = t_c \). Equations (8) and (10) show that both \( q_0 \) and \( p_r \) must decline as \( t \) increases above \( t_c \). A potential complete evasion equilibrium continues to be feasible if marginal cost is non-decreasing. In this case \( p_r \) must fall to 1 at some tax rate which was defined as \( t_{mo} \) in equation (11). From the assumptions on the cost curve average cost for complete evasion \( AC_0 \) must increase with \( t \). Therefore \( t_{mo} \) must be less than one, and complete evasion will result if the tax exceeds this rate (lemma 2).

**Proposition 5. Threshold Tax Rate for "Small Firms".**

If the minimum efficient scale is larger than the enforcement index \( q_c \) and resource costs of evasion are positive, then there must exist a tax threshold below which firms do not evade taxes.

Proof. Consider the equilibrium when \( t < t_m \), and assume contrary to proposition 5 that it involves complete evasion. From the definition of \( t_m \), \( t < t_m \) implies, \( b(1-t)/t < b(1-t_m)/t_m = f(q_o) \). Now \( f(q_o) > b(1-t)/t \) from equation (12) given the assumption that the equilibrium is at \((a=0, q=q_o)\). This produces a contradiction, so equilibrium cannot be at \( a=0 \).
To prove that equilibrium will not involve partial evasion, compare the price that must prevail under it \( (p_a) \) with that under complete honesty.

\[
p_m = \frac{AC_m}{1-t} < \frac{AC_a}{1-t} \]

as \( q_a < q_m \) and \( \frac{dAC_i}{dq_i} < 0 \) for \( q_i < q_m \),

\[
= \frac{b}{t(1-eP(q_a))} = p_a,
\]

using the second and first lines of equation (12). Using the same reasoning as in the proof of lemma 2 it is easily shown that partial evasion cannot be an equilibrium (as \( p_a > p_m \)).

QED.
Solution and Analysis of Convex Cost Case

Using (1) of text

\[ d\pi = \pi_q dq + \pi_a da + \pi_p dp + \pi_t dt = \pi_a da + \pi_p dp + \pi_t dt = 0 \]

\[ \pi_p = (1-T)q ; \pi_t = -pq[a + (1-a)e_p] ; \pi_a \geq 0 \text{ as } \begin{cases} 0 \leq a < 1 \\ a = 0 \end{cases} \]

At interior: \( \pi_a = 0 \); At corner: \( da = 0 \)

\[ \frac{1}{p} \frac{\partial p}{\partial p} = \frac{1}{1-T} \frac{\partial T}{\partial t} \quad \text{dp} = \frac{p[a+(1-a)eP]}{1-T} \frac{\partial T}{\partial t} \]

Using (13) of text,

\[ \pi_{qq} = -[C''(q) + pte(1-a)S'] < 0 \quad , \quad \pi_{qt} = -p[a+(1-a)eS] < 0 \]

\[ \pi_{qp} = 1-T_m > 0 \quad , \quad \pi_{qa} = (1-teS) p + q' > 0 \text{ if } 1 > teS \]

\[ \pi_{qa} dq + \pi_{qa} da + (1-T_m) dp - \frac{\partial T}{\partial t} dt = 0 \quad -\pi_{qq} dq + \pi_{qa} da = \frac{PY}{t} dt \]

\[ \frac{\pi_{qa} dq + \pi_{qa} da = p [a+(1-a) eS - \frac{(1-T_m)}{1-p} [a+(1-a)eP]}{dt} dt - \frac{PY}{t} dt \]

Using eqn (12) of text

\[ \pi_{aq} = \pi_{aq} + tpe(1-a)P = \begin{cases} > 0 \text{ for } 1 \geq a > 0 \\ \geq 0 \text{ for } a = 0 \end{cases} \]
\[ \pi_{ap} = -t(1-eP)q < 0 \]

\[ \pi_{aa} = -[g'' + tepq P']q < 0 ; \quad \pi_{at} = -(1-ep)pq < 0 \]

\[ \pi_{aq} dq + \pi_{aa} da = q(1-eP)(tdp + pdt) = \frac{(1-ep)pq T}{1-T} \quad \text{dt} \geq 0 \quad \text{as} \quad q \leq q_c \]

**Interior Solution (da\neq0)**

\[
\begin{vmatrix}
 dq \\
 da \\
\end{vmatrix} = \frac{1}{J} \begin{vmatrix}
 \pi_{aa} - \pi_{qa} \\
 -\pi_{aq} & \pi_{aa} \\
\end{vmatrix} \quad \frac{py/t}{(1-eP)pqt} = \frac{pq}{J(1-T)} \begin{vmatrix}
 (1-a) ep & \pi_{aa} - (1-eP) T \pi_{aa} \\
 -(1-a) ep' & \pi_{aq} + (1-eP) T \pi_{qq} \\
\end{vmatrix} \quad \text{dt}
\]

\[ J = \pi_{aa} \pi_{qa} - \pi_{aq} \pi_{aq} > 0 + \frac{\partial q_a}{\partial t} < 0, \frac{\partial a}{\partial t} < 0 \]

**Corner Solution (a=0)**

\[ \frac{\partial q_0}{\partial t} = py(t \pi_{qq}) < 0 \quad , \quad \pi_{aa} < 0 \]
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