DISCUSSION PAPER

THE QUANTITATIVE ANALYSIS OF OPTIMAL PUBLIC POLICY

by

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March 1985

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Abstract

This paper presents an overview of the authors' work on developing a quantitative framework for the analysis of optimal public policy. That work has proceeded both by extending the theory and by using numerical analysis to elucidate its implications. The paper focuses on the results that such an approach makes possible in the areas of taxation and pricing, public investment and trade policy. The authors' ongoing work is concerned with the impact of public policy under different assumptions about the functioning of factor markets.
1. Introduction

The purpose of this paper is to present an overview of the authors' work on optimal public policy. This work has been concerned with developing a quantitative framework for the analysis of policy both by extending the theory and using numerical analysis to elucidate the theory's implications. We have studied policy issues in the following areas:

1. Taxation and Pricing
2. Public Investment
3. Trade Intervention

The study of policy in these areas is interesting because of the perceived inadequacies of a completely laissez faire government. There are of course many such inadequacies stemming from a variety of sources, such as non-competitive markets, externalities, public goods and income inequality. Our work has concentrated on a concern for income inequality as a motive for active government policy and, in order to achieve this concentration, has assumed that no other market imperfections are present. However our methodology can be readily extended to include other market failures.

This concentration on distributional motives does not imply a lack of interest in the revenue-raising role of taxation. Rather, we regard the revenue-raising role of distortionary taxation as being inescapably linked to its distributional role. The reason for this is that a government which has no concern for inequality could raise any required revenue by non-distortionary poll taxes. Thus an argument for the use of distortionary taxes to raise revenue is essentially an argument against the distributional effect of poll taxes, combined with the observation that it is
impossible to achieve redistribution by the use of lump-sum transfers.

This explains our interest in distortionary taxation (including the pricing of nationalised industry outputs above or below marginal cost) but does not necessarily justify government intervention in either investment of international trade. Indeed, Diamond and Mirrlees (1971) have shown that if the government can set optimal taxes on all commodities and if there are no after-tax private sector pure profits then the economy should be productively efficient. This result implies that the government should use private profitability as the criterion for selecting investment projects and that a "small" country (one that cannot influence its terms of trade) should allow free trade. Thus distortionary tax policy should be accompanied by laissez faire investment and trade policy.

Despite the significance of this result, it must be emphasised that it rests on strong assumptions. We will leave aside the issue of positive after-tax pure profits as there are no reliable data which can be used to assess their significance. However, much of our work has been concerned with the importance of the other assumption: optimal taxation of all commodities (including factors of production). In order for this assumption to be satisfied the government must, for example, be able to tax capital income at a different rate.
from labour income, be able to tax the income of different types of labour at different rates and be able to tax a farm household's consumption of its own products. Also, it must be able to give up the usual practice of taxing consumer goods at only two of three different rates and be prepared to tax each good at a separate rate. We do not believe that all governments have these tax powers and some of our work has explored the policy implications of restricted taxation.

The significance of tax restrictions is that they rob the government of an ability to control consumer prices independently of producer prices. It is this independence that is crucial to the argument for production efficiency because it implies that the organisation of production need have no effect on the distribution of income.

When taxes are restricted consumer prices depend, to some extent, on producer prices and so some degree of production inefficiency can be desirable if its effect on producer prices alters consumer prices in a way that raises extra revenue or improves the distribution of income. Thus a commercially unprofitable investment project might be socially desirable if it raised the wages for a group of poor workers. Similarly, trade intervention might be justified through its effect on producer, and thus consumer, prices. In this case neither consumer prices nor producer prices are adequate for evaluating government projects. We will therefore have to consider a third set of prices: shadow prices, which can be used in the analysis of public sector projects.

These considerations imply that we should study the following policy instruments. First, taxes (and subsidies) on trade between house-
holds and the rest of the economy. These taxes imply that the households face consumer prices which are different from producer prices. The taxes can be levied on the sale of factors as well as on the purchase of goods and services. Thus we include linear income taxation as a possible policy and this can be extended to a negative income tax system if the government can make uniform lump-sum payments to all households. Second, shadow prices for evaluating public sector projects. These shadow prices will equal private sector producer prices if taxes are unrestricted. Third, import and export taxes (and subsidies) which apply to the private sectors transactions with the rest of the world. These will be zero for a small country with unrestricted taxes.

Our work has proceeded by constructing analytical general equilibrium models that capture the essential features of the policies under consideration. These models are used to derive as many general results as possible about the characteristics of optimal government policy. Such results provide valuable insights but they usually are unable to give much idea of the magnitude of optimal taxes, shadow prices or trade taxes. Nor are they able to fully demonstrate the sensitivity of these magnitudes to changes in government objectives, the degree of government control for the general economic environment.

It is in order to overcome this problem that numerical analysis is used. This numerical analysis consists of replacing the general social welfare function, utility functions, production functions and resource endowments of the analytical model with specific functional
forms and parameter values for a specific, and usually small, number of households and industries. The optimal policies can then be calculated using numerical optimisation routines. This procedure can be repeated for different parameter values and/or different functional forms in order to provide an insight into the sensitivity of the results to the assumptions made.

The parameter values are typically chosen so that the model, before optimisation, approximately replicates observed values of outputs, prices and income distribution for a particular country in a particular year. However, it must be stressed that the exploratory models used to date are not meant to provide a strictly accurate description of any economy. Instead, they are designed to deepen our understanding of the theory, and to analyse the sensitivity of policy to different assumptions. The use of actual data in constructing the models provides 'reasonable' parameters values, which can form the central case for a series of sensitivity experiments.

We have not used detailed models which accurately describe particular economics for two reasons. First, we believe that a thorough understanding of the theory is essential before practical application is attempted, and such understanding is best obtained by detailed analysis of the properties of simplified models. The understanding that we have gained has allowed us to progress to models that include greater institutional realism and are thus near to practical application. Second, optimal policies often result in sets of relative prices that are significantly different from those that have been observed in practice. Thus any estimates of demand and supply behaviour in the neighbourhood of these prices must be based on extrapolation of relationships well beyond the range of observation. This extrapolation is typically achieved
by assuming the constancy of certain key parameters, such as elasticities of substitution or marginal budget shares. While the maintenance of such assumptions does not impair the theoretical value of the results, it would clearly be unwise to base large policy changes on them. A promising way out in empirical work is to apply the theory developed by us to guide tax, price and tariff reform. This proceeds by comparing the marginal costs and benefits of changes in instruments of government policy and uses econometrically estimable information about basic behavioural relationships. A reform is possible when the benefits of a policy change outweigh its costs: at an optimum, the two are, by definition, offsetting.

While local reform is thus easier to analyse, many situations call for more thoroughgoing change. Although we are not able, for data reasons, to say exactly which policies are best for any particular country, sensitivity analysis gives us an idea of the range within which optimal policies lie. Moreover, the sensitivity analysis tells us which parameters are particularly significant in the determination of optimal policy and enables econometric work to concentrate in those areas with the highest return for policy making.

The rest of this paper is organised as follows. Section 2 explains the relationship of our work to other literature in the field of public policy. Section 3 describes a model that captures many of the issues that we have tackled. Section 4 gives an overview of the analytical results we have obtained so far. Section 5 describes the numerical optimisation methods that we have used, while Section 6 gives examples of the numerical results we have obtained. Finally, Section 7 concludes by considering the shortcomings of the models and the ways in which future research can overcome them.
2. Relationship to the Literature

Our first paper (Heady and Mitra: 1980) was directly inspired by the path-breaking theoretical work of Diamond and Mirrlees (1971) on optimal taxation and public production, and represented an attempt to elucidate the quantitative implications of their tax formulae. The work of Diamond and Mirrlees stimulated further theoretical work, recently surveyed by Mirrlees (1982). Some of this work dealt with the possible consequences of restrictions on the government's tax powers, for example Stiglitz and Dasgupta (1971), Diamond and Mirrlees (1976) and Munk (1980). Our second paper (Heady and Mitra: 1982) contributed to these theoretical developments and provided numerical results to illustrate their significance. The numerical methods used in that work are explained in Heady and Mitra (1984c). More recently, we have extended the application of the theory of restricted taxation to trade policy (Heady and Mitra: 1984a) and to government policy in a dual economy (Heady and Mitra: 1984b and 1985). Our work is related to several separate strands of the public finance literature and this section describes these relationships.

The attempt to quantify the implications of theoretical developments in public finance parallels the work on optimal income taxation by Mirrlees (1971) and Stern (1976). There have also been other examples of the quantification of optimal unrestricted commodity taxation. Atkinson and Stiglitz (1972) and Deaton (1977) calculated optimal taxes in models where the structure of the economy had been simplified to enable the use of relatively simple numerical techniques. Our work constitutes an advance over their analysis in that it does not require such strong simplifying assumptions and this permits a much wider range of sensitivity experiments.

This means that we do not have to assume a particular distribution of
income or a particular class of demand system in order to obtain results. More significant, in our view, is the fact that we do not have to assume constant producer prices and are therefore able to analyse the interaction between commodity taxes and the production side of the economy\(^2\). This ability to incorporate flexible producer prices is of great importance in our work on restricted taxes, as is explained in Section 3 below.

This work on optimal taxation and public production analyses a situation where the government is simultaneously choosing optimal taxes, optimal public investment and optimal trade policies. In contrast, the literature on project evaluation in underdeveloped countries has concentrated on just one aspect of government policy: public investment. Thus, both Dasgupta, Marglin and Sen (1972) and Little and Mirrlees (1974) are concerned with the position of a project evaluator who has no control over tax policy and only limited control over trade policy. In such a situation it is not appropriate for the project evaluator to assume that tax or trade policy is optimal. Diamond and Mirrlees (1976) argue that this is equivalent to the case of restricted taxes when the restrictions imposed by administrative problems are supplemented by the sub-optimal behaviour of the tax and trade authorities. Thus our analysis of public investment policy in the presence of tax restrictions helps provide an understanding of how shadow prices for project evaluation
depend both on the general economic environment and on the degree of suboptimality in the government's tax policies.

A different quantitative analysis of government policy is provided by Ahmad and Stern in their work on tax reform in India (1981, 1983a, 1983b, 1983c and 1984). This involves looking for small tax changes that can improve social welfare without reducing government revenue. This is clearly related to work on optimal policy in that no such tax changes exist at an optimum. However, it has the practical advantage that, as only small changes are considered, it does not involve extrapolation of estimated demand systems outside the range of observation. Also, it is often only politically possible to carry out tax changes gradually. We regard this area of work as complementary to our own in that we are seeking to characterise the final state which would be approached by a continual process of reform.

Another complementary area of work is that of tax and trade policy simulation, recently surveyed by Shoven and Whalley (1984). This work involves no government optimisation but simulates the consequences of alternative policies. A welfare analysis of these consequences can produce useful rankings of specific policies but, as the range of possible policies is so large, it would be impractical to use these techniques to search for optimal policy.

Simulation, of course, uses considerably less computer time than optimisation and so the models used in this area typically have much greater detail, with more households and industries. This makes them more useful in practical policy analysis for specific countries.
Also, by analysing specific policy proposals the practitioners of this technique usually restrict themselves to fairly small changes in key variables so that as with the tax reform literature, they avoid the problems of extrapolation.

3. A Typical Model

The purpose of this section is to outline a model that captures the main issues we have studied. This model was used in our first paper on restricted taxes (Heady and Mitra: 1982) and in our paper on trade policy (Heady and Mitra: 1984a). We will also discuss the relationship of this model to the standard model of unrestricted commodity taxation that we analysed in our first paper (Heady and Mitra: 1980) and to the model we have developed for dual economies (Heady and Mitra: 1984a and 1985). In this model there are two sets of private agents: household and firms.

Households

Each household, indexed by h, has an endowment of goods, such as capital or a particular type of labour. It is the differences in these endowments that generate income inequality. The household sells some or all of its endowment and uses the proceeds to buy consumer goods and services. These transactions can be represented by a vector, $x^h$, whose positive elements correspond to quantities purchased and whose negative elements correspond to quantities sold. If the total number of commodities (factors, consumer goods and services) is n then $x^h$ is an n-dimensional vector.

All households face an n-dimensional vector of consumer prices, $q$. This means that their budget constraint can be written as:

$$q^T x^h = 0$$

where $T$ denotes the transpose operation.
It should be noted that equation (1) does not allow households to receive pure profits, from non-competitive firms or competitive firms with decreasing returns, although it does allow for a return to capital owned by the household. This is a specification that is shared with much of the theoretical literature and all of the numerical analysis of government policy referred to in Section 2. This omission does affect the theoretical results but a practical reason for its maintenance is that there are no data that enable us to distinguish pure profits from the return to capital.

However, equation (1) does allow the possibility of households receiving lump-sum payments from the government. This is simply achieved by including in each household's endowment a good with a positive price that cannot be consumed or used in production: it is simply sold to the government. As lump-sum payments cannot normally be tailored to each individual household this does not remove the distributional motive for distortionary taxation. Our analysis has always assumed that any lump-sum payments are uniform, which together with uniform taxes on factor sales would constitute a negative income tax scheme.

In these circumstances, the transactions and utility of household \( h \) will depend on its given endowment and the prices that it faces. It is therefore possible to define a demand function, \( x^h(q) \), and an indirect utility function, \( v^h(q) \), for each household. These functions will be homogeneous of degree zero in \( q \). It should be noted that the cardinalisation of the indirect utility function is significant as the government's objective is assumed to be the maximisation of a social
welfare function, based on household utility levels:

\[ V(q) = W(v_1(q), \ldots, v_H(q)) \]  \hspace{1cm} (2)

It is this social welfare function that incorporates the government's distributional goals and in the numerical analysis we have used the specific form:

\[ W = \begin{cases} \frac{1}{p} \sum_{h=1}^{H} (v_h)^\rho, & \text{when } p \neq 0 \smallskip \cr \sum_{h=1}^{H} \log (v_h), & \text{when } p = 0 \end{cases} \]  \hspace{1cm} (3)

Different degrees of aversion to inequality can be obtained by varying the value of \( \rho \), from utilitarianism when \( \rho = 1 \) to Rawlsian 'maximin' when \( \rho \) tends to minus infinity. However, the value of \( \rho \) represents the degree of aversion to inequality of utility and so the degree of aversion to inequality of income will depend both on the value of \( \rho \) and on the extent to which the indirect utility function displays diminishing marginal utility of income. In most of our numerical analysis we have chosen a cardinalisation of the indirect utility function which is linear in income, this would not be possible for some types of preference patterns.

**Firms**

Private production is assumed to be organised in a series of industries, indexed by \( f \). They are assumed to operate under constant returns to scale and to behave competitively. Thus, as stated above, they earn no pure profits. In our numerical analysis we have always assumed that each industry produces only one good because of the way that data are presented in input-output tables. However, joint production would not cause any difficulties.
The firms in any one industry are assumed to have access to the same technology and thus behave identically (although each firm's scale of operation is indeterminate). The firms choose production techniques in order to maximise profits at producer prices, \( p \). Because of constant returns, we can represent this choice by a vector, \( A^f \), which describes the inputs (negative elements) and outputs (positive elements) associated with operating the technique at unit level. This vector will depend on producer prices and we therefore have a unit supply function, \( A^f(p) \). As we will be interested in the response of private sector activity to tax changes, this supply function is assumed to be continuously differentiable.

The firms in each industry will operate their chosen technique at a total level of \( y_f \). Thus total private sector supply will be given as \( \sum_A^f(p)y_f \). In order for \( y_f \) to be a finite number in equilibrium the industry cannot make a profit. Thus the following condition must be satisfied:

\[
p^T A^f(p) \leq 0 \quad \text{for all } f \tag{4}
\]

Also, in order for firms to choose a positive \( y_f \) the activity must not make a loss. Therefore:

\[
\sum_f p^T A^f(p)y_f = 0 \tag{5}
\]

**Government**

The model also allows for government production of some or all of the goods, either as a monopoly or in competition with private producers. In an open economy international trade can also be regarded as a government activity because it is subject to possible government control, either directly or through the use of trade taxes and subsidies.

We have always assumed that government production is subject to
constant returns to scale, although the analysis can easily be altered
to accommodate either increasing or decreasing returns to scale. In
terms of international trade, the constant returns assumption implies
that the country is 'small' and thus faces given international
prices for its imports and exports.

The public production is described by a matrix G whose columns represent
the activities available to the public sector. The levels of each
activity are denoted by the vector z so that public sector
supply is represented by Gz. The use of a finite number of activities
is in contrast to the assumption of smooth substitution made about the
private sector and is a reflection of the fact that, as the public
sector is directly controlled, we are not concerned with its response
to private sector price changes. However, the distinction between a finite
number of activities and smooth substitution is more apparent than
real because the number of activities can be made very large. Indeed,
in most of our numerical analysis the columns of G are chosen to give
a very good approximation to a continuous spectrum of techniques.

The model also allows for the government to have a net revenue raising
requirement to cover administration and public good provision. As
prices are endogenous, it makes sense to represent this as a vector, R,
of physical requirements such as labour and different types of goods.
In principle, the components of R could be government choice variables
to be determined along with taxes and public production. However,
this is an unnecessary complication for our purposes and we have not
pursued it. Indeed, we have often assumed that there was no such
requirement as our main concern has been with redistribution.
In an equilibrium supply must be at least as great as demand for each good and this can be represented as:

$$\sum_{i} A_i^f(p)y_i^f + Gz - X(q) - R \geq 0$$  \hspace{1cm} (6)

Where $X(q) = \sum_{h} X^h(q)$, the vector of aggregate net trades of households.

In addition to public production the government controls taxes that produce divergences between private producer prices and consumer prices. However, it has been essential to much of our work that the government does not have unrestricted tax powers and it is important to consider how restrictions should be specified. At first sight it might appear that there are two different types of restriction:

(1) that certain goods must be taxed at the same rate, as in the case of different types of labour;

(2) that certain goods should have a zero rate of tax, as in the case of goods produced and consumed within an agricultural household.

However, it should be noted that the demand and supply functions are homogeneous of degree zero in consumer and producer prices, respectively. Thus, consumer prices (for example) can be scaled up and down without affecting consumer demand or producer prices. This scaling could be manipulated so that the difference between the consumer prices and the producer price (i.e. the tax) on any one good is zero. Thus specifying that any one good should be untaxed would have no effect on the real possibilities open to the government.

To give an example suppose that the government wished to lower the consumer price of capital services below its producer price, and thus raise revenue, but that capital income could not be taxed. The government could instead use taxes to raise the consumer prices of all
other goods, have the same effect on relative prices and thus on consumer demands and government revenue.

Note however that this scaling of consumer prices could only make several goods untaxed if their tax rates were the same. Thus a restriction that a group of goods be untaxed is really just like the restriction that the goods should all be taxed at the same rate. This means that both types of restriction are of type (1), to the extent that they are restrictions at all, and can be represented by partitioning the set of all commodities into K pre-selected groups, where all elements of group \( k \) must be taxed at an ad valorem rate of \( (t_k - 1) \) common to that group. Thus, for any good i in group k:

\[
q_i = t_k p_i
\]  

(7)

It is worth noting that the revenue raised by such a tax on good i in group k will be \((1 - t_k) X_i (q)\) (where \( X_i \) is ith element of X).

Thus a tax which raises the consumer price of a factor (for which \( X_i \) is negative) will result in a loss of revenue, while a tax which lowers it will gain revenue.

Equation (7) enables us to write the vector of aggregate net trades, \( X \), and the social welfare function as functions of \( p \) and \( t \). Thus, the government problem of maximising (2) subject to (4), (5) and (6) can be written as:

\[
\text{Max } V(t, p) \\
\text{subject to: } \sum_f A^f(p) y_f + Gz - X(t, p) - R \geq 0 \quad (6^*) \\
p^T A^f(p) \leq 0 \text{ for all } f \quad (4)
\]
\[ \frac{\partial}{\partial x} p A_f(p) y_f = 0 \]  

(5)

In practice the government controls only \( t \) and \( z \) while the market determines \( p \) and the \( y_f \). However, in this problem we can regard the government as also choosing \( p \) and the \( y_f \) because any choice that satisfies the constraints will be a market equilibrium, so long as the government disposes of any goods that are in excess supply. In other words, the government choice of \( t \) and \( z \) implies (via the constraints) a choice of \( p \) and the \( y_f \).

Thus the solution to this problem must satisfy first-order conditions with respect to \( t \), \( p \), \( z \) and the \( y_f \):

\[ V_t - \lambda X^T_t s \leq 0 \quad (t > 0), \]  

(8)

\[ V_p - \lambda X^T_p s + \lambda (\sum_{f} A^f_y(p)) Y_f^T s - \sum_{f} t^T A^f(p) \]  

\[ + u \sum_{f} A^f(p) y_f \leq 0 \quad (p \geq 0) \]  

(9)

\[ \lambda s T^T A^f(p) + u p A^f(p) \leq 0 \quad (y_f \geq 0) \]  

(10)

\[ s^T G \leq 0 \quad (z \geq 0) \]  

(11)

Where subscripts denote derivatives and each inequality bears the relationship of complementary slackness with the corresponding variable appearing in parentheses on the right. \( s \) is a vector of shadow prices (corresponding to (6')), \( \lambda \) is a scalar which enables \( s \) to be normalized, the \( \mu^T \) are scalar multipliers corresponding to (4) and \( u \) is a scalar multiplier corresponding to (5).
The results that can be obtained from the analysis of conditions (8) - (11) will be discussed in Section 4 below. The rest of this Section will be concerned with the relationship of this model to both the standard model of unrestricted taxation and our more recent work on dual economics.

Looking first at the model of unrestricted taxation, the obvious difference is that each good can be taxed at a separate rate. This implies that the government can choose producer and consumer prices independently and therefore can manipulate private production (via producer prices) without any concern for its effect on consumer prices and welfare. Thus the government has as much control over the private sector as the public sector and in the absence of private sector profits, there is no need to distinguish between them. They should therefore face the same prices, \( p = s \). This means that condition (9) is no longer necessary for determining \( p \) and condition (10) simplifies to:

\[
s^T A^e(s) \leq 0 \quad (y^e_f \geq 0)
\]

(10')

Note that (10'), combined with \( p = s \), means that conditions (4) and (5) are automatically satisfied by an optimal solution and do not have to be imposed. The assumption of unrestricted taxation therefore simplifies the analysis of optimal policy considerably. This analytical simplification is accompanied by a simplification of the computational technique, see Section 5 below.

In contrast to the relaxation of tax restrictions, the analysis of dual economies introduces additional complications. The basic idea of dual economy models is that agriculture in underdeveloped countries is organized in a fundamentally different way from manufacturing. From the point of view of public finance a major difference is that it is typically more difficult to tax agriculture than manufacturing. This is because many farms consume much of their own produce and
mainly use their own land and labour in the production process. This means that the returns to labour and land do not correspond to identifiable financial transactions, that can be taxed. It is therefore often only practicable to tax agriculture through the transactions it has with the rest of the economy. Thus agricultural land and labour, together with the agricultural sector's consumption of its own output, are non-taxable and this constitutes a significant tax restriction. Moreover, the fact that agriculture can be taxed through its transactions with the rest of the economy implies that agricultural prices could well be different from both urban producer and urban consumer prices. This means that the government has to choose two sets of taxes: (1) the wedge between urban producer and consumer prices; (2) the wedge between urban producer and agricultural prices. However, the government may not be able to choose these taxes completely independently because of the possibility of arbitrage between the urban and rural sectors. For example, if taxes were set so that the relative prices of food and clothing were different in the rural areas from the urban areas people would be able to make profits by trading between the two sectors. In order to avoid this problem the government must satisfy commodity specific tax restrictions, although it should be noted that they need not apply to all goods. Electricity, for example, can be sold at different prices in different areas without any problems.

The other new feature of the model is the introduction of migration from the rural areas to urban areas. We follow Lewis (1954) in supposing that rural residents only migrate to urban areas if there is a job available and that the wage differential between the two
sectors is just enough to offset the differences in the cost of living and in non-pecuniary benefits. Thus there is no unemployment and the utility of a migrant is equal to the utility of an equally endowed non-migrant. One issue that arises in connection with specifying the migration mechanism is whether migrants lose their rights to land. At one extreme one might suppose that migrants can rent or sell their land for its full value, while at the other extreme it could be assumed that migrants give up all rights to their land, which is then distributed to non-migrants. Looking at different countries, one can find examples of both extremes, as well as cases where migrants retain some rights. In order to investigate the sensitivity of optimal policy to this issue we have analysed both extreme cases. The results of this analysis are given in Heady and Mitra (1984b).

In summary, the analysis of a dual economy involves the introduction of a second set of taxes, a non-taxable sector and an additional equilibrium condition (that the utility of migrants should equal that of equally endowed non-migrants).

4. Analytic Results

The purpose of this Section is to outline the analytic results that we have obtained (Heady and Mitra: 1982, 1984a, 1984b, 1985). We deal first with the results from the model of restricted taxation as set out in Section 3, and then with the extension of these results to the dual economy model. The analytic results of the model of unrestricted taxation are derived from conditions
(8) - (11). Condition (11) is the standard condition that constant returns government sector projects should not make profits at shadow prices. Equation (10) implies that any private activity that is in operation, and thus breaks even at producer prices, must also break even at shadow prices. This result was originally demonstrated by Diamond and Mirrless (1976).

Our work has mainly been concerned with conditions (8) and (9), as they provide conditions for the choice of taxes and the divergence between shadow prices and producer prices. The analysis of these conditions is simplified if we regard the tax on a good (the difference between its consumer price and producer price) as being the result of two divergences: the difference between the consumer price and the shadow price, \( \tau_i \), and the difference between the producer price and the shadow prices, \( \omega_i \). Thus:

\[
\tau = q - s; \quad \omega = p - s
\]

We will call the vector \( \tau \) "shadow consumer taxes" and the vector \( \omega \) "shadow producer taxes". However, it must always be remembered that the actual taxes paid, \((t_k - 1)p_i\), is the difference between the two taxes \((\tau_i - \omega_i)\) and it is this that is subject to the restrictions.

This enables us to rewrite equation (8) and (9) as (disregarding the inequalities):

\[
-\sum_{i \in k} \sum_{h} \frac{p_i}{p_i^{ch}} x_{ij} \tau = 1 - \sum_{i \in k} \sum_{h} b_i p_i^{h} x_{i}^{h} \sum_{i \in k} p_i^{X_i}
\]
\[- \sum_{f} \sum_{j} a_{ij}^{f} \frac{\partial y_{f}}{\partial X_{i}} = - \frac{\mu}{\lambda} + \frac{\sum_{f} a_{ij}^{f}}{\lambda \sum_{f} \frac{X_{i}}{A_{i}^{f} y_{f}}} \]

\[
\frac{-c_{i} X_{i}}{\sum_{f} \frac{X_{i}}{A_{i}^{f} y_{f}}} \left[ \sum_{h} \sum_{j} x_{ij}^{ch} \frac{\partial c_{h}}{\partial X_{i}} + 1 - \frac{b_{h} x_{ij}^{h}}{\lambda X_{i}} \right]
\]

where \( k \) it will be recalled is the tax group index and \( x_{ij}^{ch} \) is the derivative of household \( h \)'s compensated net demand for good \( i \) with respect to the prices of good \( j \);

\( A_{i}^{f} \) is the output of the \( i \)th good from the unit activity chosen by industry \( f \);

\( a_{ij}^{f} \) is the derivative of \( A_{i}^{f} \) with respect to the price of good \( j \);

\( b_{h} \) is the net social marginal utility of income for household \( h \)

\[
( = \frac{\partial w_{h}}{\partial \nu_{h}} + \lambda \sum_{j} \frac{\partial x_{ij}^{h}}{\partial \nu_{h}} )
\]

where \( I \) is the (possibly zero) lump-sum income of household \( h \); the first term represents the social value placed on the extra utility of the household, while the second term represents the social value of the extra taxes that the household will pay).

Equation (13) is similar to the unrestricted tax rule in Atkinson and Stiglitz (1976), except that both sides have been averaged over the goods within each tax group. The reason for this averaging is that the government is restricted to set a uniform tax for all goods in the group. Thus the left-hand side represents the proportionate reduction in value, at producer prices, of compensated demand for the goods in tax group \( k \) that would result from a small equi-proportional
intensification of shadow consumer taxes if shadow prices were constant. The right-hand side is smaller for groups of goods that are mainly consumed by households with high net social marginal utility of income (typically the poor). Thus equation (13) tells us that the proportionate reduction is smaller for such groups.
The left-hand side of equation (14) is similar to the left-hand side of equation (13) but is measuring the distortionary effect of shadow prices, rather than consumer taxes. Also, the effect is not averaged over tax groups because the government can use trade policy or public production to vary producer prices independently of taxes. The main reason for the introduction of production distortions can be seen in the final term in equation (14), as the first two terms are simply ensuring that producer price changes satisfy the zero profit conditions (4) and (5). The term in square brackets represents the extent to which the shadow consumer tax on good i is not optimal, and equation (13) assures us that this term would be zero if good i were the only good in its tax group. Thus production distortions are warranted by the fact that tax restrictions have prevented the government from setting the optimal consumer prices on some goods and that variation of producer prices might get those prices nearer the optimum. However, as Munk (1980) points out, this produces distortions on the production side of the economy. Thus equation (14) represents a balance between improving consumer prices and distorting production.

This result, that tax restrictions make it desirable to introduce a difference between private producer prices and public shadow prices, has far-reaching implications for public investment and trade policy. Dealing first with public investment, the difference between the two sets of prices implies that it is possible for socially desirable public investment in the production of private goods to make losses at market prices. This might occur if the project had a desirable
effect on private producer prices and thus, via the tax restrictions, on consumer prices. The implications for trade policy follow from the fact that, for a small country, public shadow prices equal world prices (a result that follows from condition (11)). Thus, a divergence between shadow prices and private producer prices implies that there should be trade taxes and subsidies. For example, a tariff might be justified if the government wished to increase the consumer prices of the good, or of factors which produced the good, but could not do so through the tax system. The detailed implications for trade are examined in Heady and Mitra (1984a).

A further implication for public investment and trade policy follows from the fact that producer prices affect consumer prices when taxes are restricted. Looking back at the government's maximisation problem, we see that producer prices are constrained by equation (5), which requires private activities in use to break even. This raises the possibility that government production or trade could replace some private activities and thus relax the constraint imposed by equation (5) on some prices. This would give the government more latitude in setting producer prices, and thus consumer prices, and could result in improvements in Social Welfare. Our paper on trade policy (Heady and Mitra: 1984a) gives some numerical examples of this possibility.

In extending these results to the dual economy (Heady and Mitra: 1984b and 1985) we have avoided issues of income distribution by assuming that all households are equally endowed. This assumption was made in order to concentrate on the special features of the model: the non-taxability of trades within agriculture and the migration from rural to urban areas. The non-taxability issue can be dealt with by regarding each farm as a consumer that purchases goods from the rest of the economy and finances these purchases.
by sales of agricultural products. The fact that the agricultural products are produced by labour and land is irrelevant to the government as the supplies of the factors cannot be taxed. Thus it is only the urban sector that needs its production to be modelled explicitly.

The introduction of migration means that the government must take account of the effect of its policies on intersectoral labour mobility. However, this does not cause any difficulties as we specify the migration equilibrium condition in terms of utility. Thus, any policy that increases the utility of urban residents will promote rural-urban migration, while any policy that increases the utility of rural residents will reduce such migration. This means that if, for example, the government wishes to promote rural-urban migration the tax policies would be the same as if it put a higher weight on urban utility and a lower weight on rural utility. Thus we can still represent the government as maximising a function of individual utilities and equation (13) must still describe optimal taxes, although the values of $b^k$ will be altered by the migration consideration.

However, it should be noted that in the absence of commodity specific tax restrictions there will be two sets of taxes, each of which satisfy (13). Thus taxes on urban consumers and rural farms will separately satisfy the tax rule described above, and even if all households are equally endowed the proportionate reductions will generally be different in the two sectors. If commodity specific tax restrictions are in operation, the proportionate reductions would have to alter for those goods subject to the restrictions.

One aspect of tax policy that arises in this model is the question of
how the tax burden is divided between agriculture and non-agriculture. The answer to this question depends on whether migrants retain the rights to their land income. If they do then the tax burden per capita should be the same in each sector. The reason for this is that marginal migrants do not gain from migrating and, if they keep their land rights, no other households gain either. However, if the per capita tax burdens were unequal the government could gain revenue by altering the number of urban jobs. Such possibilities of gain mean that government policy is not optimal. Therefore optimal policy must be characterised by equal per capita tax burdens.

This result has to be modified if migrants lose some of their land rights because non-migrants will benefit from the migration of others. In this case the per capita tax burden in agriculture should be higher than in non-agriculture so that, at the optimum, the gain to non-migrants of further migration is balanced by the loss to government revenue.

Another way of looking at this result is that, when migrants lose land rights, the act of migration has a beneficial externality which is rewarded by lower taxation of urban residents. However, when migrants keep all their land rights no such externalities arise and there is no reason for differential taxation.

Turning to production and trade policy, we have already argued that agricultural prices should be different from shadow prices. As these prices cover inter alia agricultural inputs such as fertilizer and agricultural output, this implies that farmers are not facing shadow prices when making their production decisions. Thus it will generally
be optimal to tax international trade in agricultural inputs and outputs. Also, government projects in rural areas should use shadow prices that differ from local market prices.

In contrast, the inability to tax trades within agriculture does not constitute a reason for introducing production inefficiency into urban production. This is true even if urban taxes on some goods are constrained to be the same as rural taxes (the commodity specific tax restrictions) because, as long as consumer prices can be set independently of urban producer prices, there is no advantage to be gained from manipulating private urban production. Thus the issue of whether shadow prices should equal urban producer prices, with all that that implies for production and trade policy, depends only on whether there are tax restrictions within the urban sector itself.

Finally, this summary of theoretical results confirms what was said in the Introduction. The theory provides us with useful insights into optimal government but the results are not in a form which makes it easy to draw direct conclusions about the likely values of optimal taxes, tariffs or shadow prices. It is for this reason that we have supplemented this theoretical analysis with the numerical analysis that is described in the next two sections.

5. Computational Techniques

This section gives a brief description of the computational techniques we have used in our work. As stated above, our first work in this area (Ready and Mitra: 1980) was concerned with obtaining numerical solutions to the optimal tax formula derived by Diamond and Mirrless (1971). This work involved adapting Scarf's algorithm (Scarf, 1973...
and 1982) to the special requirements of the problem. We have continued
to use Scarf's algorithm and have extended it to deal with restricted
taxes (Heady and Mitra, 1984c). However, in our recent work on dual
economics we have used a dynamic non-linear programming algorithm
called CONOPT (Drud, 1984). The main reason for this was that we also
wanted to analyse the dynamic properties of the dual economy model,
work that is still in progress.

As the operation of CONOPT is outside our area of expertise, the rest
of this Section will be concerned with the use of Scarf's algorithm
in our work. Also, as the precise details and necessary proofs are
published, we will confine ourselves to an informal sketch of our
procedures, concentrating on the simpler case of computing unrestricted
taxes.

In setting up the model of unrestricted taxation we could follow
Section 3 in specifying a choice of taxes and producer prices. However,
that approach was only really required because of the fact that, with
restricted taxes, producer prices have a real effect on consumer
prices. When taxes are unrestricted we can regard the government
as directly controlling consumer prices so that the problem becomes:

Maximise \[ V(q) \]

Subject to: \[ Gz - X(q) \geq 0 \]

where we have disregarded any government net revenue raising
requirement and have made use of the fact that no distinction between
private and public production need be made.

The first-order conditions for maximisation are:

\[ q \lambda [X_q] \leq 0 \quad (q \geq 0) \]
This problem is similar to the type of non-linear programme which has been solved by Hansen (1969), using the Scarf algorithm. However, Hansen's procedure dealt with concave programmes and used a standard constraint qualification to prove convergence to the optimum. In contrast, this problem is not concave and so Hansen's argument cannot be used directly.

A second difficulty with Hansen's algorithm is that it searches over a simplex of all the control variables, which in our case would be the elements of q and z. Now z can be a vector of very high dimension, especially if there are wide substitution possibilities. Thus the algorithm might have to search over a simplex with a large number of dimensions and this would make it very slow.

Our procedure solves the second problem by using a parametric linear programme to find a z that corresponds to each q. Thus, the algorithm need only search over the simplex of consumer prices, q.

We then showed that our modified version of Hansen's algorithm would converge to a solution of the first-order conditions of our problem provided that a new constraint qualification was satisfied. This constraint qualification required that when the government budget is in deficit there is always a small tax change that will reduce the deficit. This is equivalent to ruling out Edgeworth's tax paradox, but only when the budget is in deficit. Unfortunately, the lack of concavity means that a solution to the first-order conditions might not be a
maximum. However, the algorithm can be started at different points in an attempt to find all the solutions to the first-order conditions, and the value of social welfare at the different solutions can be compared to find the maximum.

Thus our algorithm searches over a grid of vectors $q^1, \ldots, q^k$ on a unit simplex and it gives a vector label to each vector in the grid. For vectors on the sides of the simplex, the standard Scarf-Hansen labels are used. However for each vector, $q$, in the interior of the simplex, the following linear programme is solved:

choose $z$ and $r$ to minimise $r$

subject to $Gz + re_n \geq X(q)$ \hspace{1cm} (19)

Where $e_n$ is an n-dimensional vector with unit entries.

This programme is choosing production levels, $z$, which minimise the largest difference between demand and supply. Note, that when $r = 0$, the feasibility condition (16) is satisfied and so this programme provides a test for the feasibility of $X(q)$. Also, corresponding to (19) there will be a set of shadow prices, $s$, which by duality will satisfy condition (18):

$\begin{align*}
    s^T G &< 0 \\
    (z &> 0)
\end{align*}$ \hspace{1cm} (18)

If the minimal value of $r$ is negative or zero, $X(q)$ is feasible and we give $q$ the label:

$[e_n + V_q]$

If the minimal value of $r$ is positive, $X(q)$ is infeasible and we give $q$ the label:

$[e_n - [X_q]^T s]$

where $s$ was derived from the linear programme.
Scarf's algorithm can then be applied to the labelled simplex and it will terminate at a primitive set which we show provides an approximation to a vector $\hat{q}$ that satisfies feasibility (16) and the first-order condition (17), provided that the regularity condition is satisfied. Furthermore, the labelling was constructed so that (18) is satisfied and so we have an approximate solution to the first-order conditions. This approximation is typically more accurate if the grid is finer and we use Merrill's (1972) grid refinement technique to obtain solutions that are as accurate as we require.

The extension of this technique to restricted taxes requires us to search over a grid of tax rates and private producer prices, while shadow prices are generated by a linear programme which incorporates private sector activities. Apart from that, the only difference is that special devices have to be used to ensure that private activities satisfy the non-positive profit and zero profit constraints (4) and (5). This is technically difficult because of the complementary slackness involved in these constraints and interested readers are referred to our paper on computing restricted taxes (Heady and Mitra: 1984c).

The exact time taken by these algorithms depends on the precise nature of the problem and on the computer used. However, in the models we have investigated, which usually have two factors and three consumer goods, it has usually taken well under a minute to obtain a solution with a final grid size of 10,000.

6. Numerical Examples

The purpose of this Section is to give some ideas of the sort of results that numerical analysis of optimal public policy can provide.
### Table 1
Input Requirements for Unit Activity Levels

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Good 1</th>
<th>Activities Good 2</th>
<th>Producing Good 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good 1</td>
<td>0</td>
<td>0.01</td>
<td>0.10</td>
</tr>
<tr>
<td>Good 2</td>
<td>0.04</td>
<td>0</td>
<td>0.12</td>
</tr>
<tr>
<td>Good 3</td>
<td>0.10</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
<td>Unskilled Labor</td>
<td>0.77</td>
<td>0.38</td>
<td>0.43</td>
</tr>
<tr>
<td>Skilled Labor</td>
<td>0.09</td>
<td>0.66</td>
<td>0.35</td>
</tr>
</tbody>
</table>

### Table 2
Consumption Parameters

<table>
<thead>
<tr>
<th></th>
<th>Good 1</th>
<th>Good 2</th>
<th>Good 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0.35</td>
<td>0.25</td>
<td>0.40</td>
</tr>
<tr>
<td>c</td>
<td>11.0</td>
<td>2.5</td>
<td>0.0</td>
</tr>
</tbody>
</table>

### Table 3
Labor Endowments in Efficiency Units

<table>
<thead>
<tr>
<th></th>
<th>Class I</th>
<th>Class II</th>
<th>Class III</th>
<th>Class IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion of Population</td>
<td>0.54</td>
<td>0.38</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>Unskilled Labor</td>
<td>20.48</td>
<td>35.95</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Skilled Labor</td>
<td>0.0</td>
<td>0.0</td>
<td>108.5</td>
<td>289.0</td>
</tr>
</tbody>
</table>
We will give two rather brief examples here and the reader is referred to our other papers for more detailed and extensive analysis. The first example comes from our paper on trade policy (1984a) and deals with the distributional case for tariffs, while the second example looks at optimal taxes for revenue raising in the dual economy model (1984b). As stated in the introduction, one of the main reasons for interest in numerical analysis of optimal public policy is the ability to conduct sensitivity analysis, and we will concentrate on that aspect here.

(a) Redistributive Tariffs

In this model there are two types of labour, skilled and unskilled, which the government must tax at the same rate. This tax restriction raises the possibility of using tariffs because the imposition of a tariff would raise the producer price, and thus the consumer price, of the type of labour that is used intensively in producing the protected good. The effect of a tariff on the relative wages of the two types of labour will depend on the production technology. In particular one would expect the effect to be greater if there are greater differences in the skill mix between industries and if the elasticity of substitution is low. Our examples will concentrate on the significance of the elasticity of substitution, although an earlier paper (1982) presents results on skill mix differences.

For these examples there is assumed to be no substitution between intermediate inputs or between any intermediate input and either type of labour. However, there is substitution between the two types of labour. Table 1 presents the input requirements for activities producing one unit of each output. The labour inputs reported represent the least
cost combinations when the factor prices are equal. The elasticity of substitution is assumed to be the same for each industry but is varied parametrically for the sensitivity analysis.

It is assumed that households supply their labour inelastically but that their demands for consumer goods can be represented by the linear expenditure system. The utility function which generates these demands is:

$$\log (u_h) = \sum_{i=1}^{3} b_i \log (x_i^h - c_i), \quad \sum_{i=1}^{3} b_i = 1$$

The parameter values used in our examples are reported in Table 2.

Inequality in the model is generated by the fact that households differ in both the type (skilled or unskilled) and efficiency of the labour they supply. The distribution between four classes is given in Table 3.

The government has no net revenue raising requirement and there is no public production. Its objective is to reduce the degree of inequality, but it is also concerned about the average utility level. These concerns are represented by a social welfare function of the type discussed in Section 3, above.

$$W = \frac{1}{\rho} \sum_{h} u_h^{\rho}, \quad \text{when } \rho \neq 0$$

$$W = \sum_{h} \log(u_h), \quad \text{when } \rho = 0$$

Clearly, the value of $\rho$ will affect the choice of optimal tariff and sensitivity to this parameter is analysed below.

In the calculations reported in Table 4 it is assumed that the only tax restriction is the one on the two types of labour. This means
that taxes on consumer goods can be manipulated to offset some of the
effects of tariffs on consumer prices. It is also assumed that the
country is small and thus faces given terms of trade, whose value
(0.87) is such that the country imports good 1 and exports good 3.
Good 2 is assumed to be non-traded. For simplicity, the producer prices
are scaled so that the world price of good 3 equals its domestic
producer price. Thus the choice of tariff for good 1 is the only
element of trade policy. The optimal choice for this tariff is given in
Table 4, for different values of $\rho$ and $\sigma$ (the elasticity of substitution
between the two types of labour.

Table 4

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>0.0</th>
<th>0.0</th>
<th>0.0</th>
<th>0.25</th>
<th>-1.0</th>
<th>-5.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>2.5</td>
<td>3.2</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>optimal tariff</td>
<td>18.6%</td>
<td>11.7%</td>
<td>14.2%</td>
<td>10.9%</td>
<td>18.0%</td>
<td>26.9%</td>
</tr>
</tbody>
</table>

The first three columns of Table 4 show that the tariff becomes
considerably larger when the elasticity of substitution is reduced by a
small amount. Indeed, the effect of the elasticity of substitution is
so strong that the optimal tariff becomes prohibitive for elasticities
slightly less than 2.5 and domestic production of good 1 is completely
abandoned for elasticities slightly greater than 3.2. The fact that
smaller elasticities of substitution increase the tariff is strongly
suggested by the theory: small elasticities imply a greater effect on
relative prices and smaller distortionary cost. However, the theory
could not indicate just how sensitive the optimal tariff is to this
parameter.
The last four columns show how sensitive the optimal tariff is to variations in $\rho$, the degree of concern for inequality. As one would expect, the tariff becomes larger when the government is more egalitarian. The effect is significant, but this example shows that this subjective parameter is no more important than the (objective) elasticity of substitution, a characteristic that is common to many of our numerical examples.

(b) Revenue-raising Taxes

In this example, we look at the effect of changing the government's revenue-raising requirement on taxes in the dual-economy model. In this particular version of the model all households are equally endowed, so no issues of equity arise. Also, the government owns all of the capital stock in the manufacturing sector and so receives significant capital income. This means that when the government's revenue requirement is small it has money to distribute, but we assume that this must be distributed through subsidies (negative taxes) rather than direct payments to households.

The government can set taxes on all urban goods and factors, as well as trades between the urban and rural sectors. However, the government cannot tax trades within the rural sector and is subject to the commodity-specific tax restriction that food and clothing must have the same relative prices in the two sectors. However, the other consumer good, services, can have different prices in the two sectors.

In addition to the distortions caused by taxes, we assume that rural-urban migrants lose their rights to land and thus generate a beneficial externality.
The agricultural sector produces food using inputs of land, labour and imported fertilizer. We assume a nested CES technology with land and fertilizer forming a subaggregate (with $\sigma = 1.5$) which is then combined with labour to produce food (with $\sigma = 1.1$).

The urban sector produces clothing and services, again with a nested CES technology. However they use capital and imported energy to produce the subaggregate (with $\sigma = 0.5$) for both industries which is then combined with labour to produce output (with $\sigma = 1.1$ for clothing, $\sigma = 0.9$ for services).

Households in both sectors have identical linear expenditure system utility functions, as in example (a). However, labour is not inelastically supplied and must therefore be included in the utility function (thus extending it to four goods). The consumption parameters are given in Table 5.

Table 5

<table>
<thead>
<tr>
<th></th>
<th>Food</th>
<th>Clothing</th>
<th>Services</th>
<th>Labour*</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>0.06</td>
<td>0.15</td>
<td>0.3</td>
<td>0.49</td>
</tr>
<tr>
<td>c</td>
<td>1.7</td>
<td>1.0</td>
<td>0.4</td>
<td>-9.53</td>
</tr>
</tbody>
</table>

* Note that labour supply is a negative number so that a reduction in labour supplied will ceteris paribus increase utility.

As all households have the same utility level, the government's objective is simply to maximise that level subject to raising the necessary revenue.
The optimal taxes for this purpose are given in Table 6 for a range of revenue requirements, expressed as a percentage of national income. These results are presented graphically in Figure 1.

The revenue requirement of 36.6% corresponds to a situation where the government's capital income just covers its revenue requirement. Thus the taxes raised on clothing, services and fertilizers are simply financing the payments to urban labour (represented as a positive tax because it produces a consumer price that is greater than the corresponding producer price). These payments are made to urban workers in order to encourage rural-urban migration, which generates a beneficial externality. The pattern of taxation on clothing, services and fertilizer is determined by the equal proportionate reductions rule derived in Section 4, modified so that clothing is taxed equally in the two sectors. The most striking feature of the tax pattern is the low tax on fertilizers. Thus, although, as predicted in Section 4, production inefficiency is caused by what is in effect a tariff on fertilizer imports, the extent of the distortion is not very great.

When we compare the pattern of taxes (except that on urban labour) when the revenue requirement is increased, we see that all of the taxes increase but that their relative positions remain generally unchanged. Also, when the revenue requirement is zero the goods which were previously highly taxed are now highly subsidised. This pattern is unsurprising as one would expect that the relative distortionary costs of taxes on different goods would be approximately the same at different rates of taxation, including negative rates. However, a different functional form for the demand system might yield a less consistent pattern.
### Table 6

Optimal Taxes with Varying Revenue Requirement

<table>
<thead>
<tr>
<th>Revenue Requirements</th>
<th>0%</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>36.6%</th>
<th>50%</th>
<th>60%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Urban</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothing</td>
<td>-22.2%</td>
<td>-16.8%</td>
<td>-10.7%</td>
<td>4.6%</td>
<td>25.4%</td>
<td>42.9%</td>
<td>103.7%</td>
<td>247.1%</td>
</tr>
<tr>
<td>Services</td>
<td>-7.7%</td>
<td>-3.1%</td>
<td>1.7%</td>
<td>12.5%</td>
<td>24.8%</td>
<td>33.5%</td>
<td>53.3%</td>
<td>76.8%</td>
</tr>
<tr>
<td>Labour</td>
<td>77.9%</td>
<td>76.1%</td>
<td>73.8%</td>
<td>68.3%</td>
<td>61.3%</td>
<td>56.8%</td>
<td>45.1%</td>
<td>46.5%</td>
</tr>
<tr>
<td><strong>Rural</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clothing</td>
<td>-22.2%</td>
<td>-16.8%</td>
<td>-10.7%</td>
<td>4.6%</td>
<td>25.4%</td>
<td>42.9%</td>
<td>103.7%</td>
<td>247.1%</td>
</tr>
<tr>
<td>Services</td>
<td>-31.2%</td>
<td>-25.3%</td>
<td>-18.5%</td>
<td>-0.3%</td>
<td>26.1%</td>
<td>49.7%</td>
<td>128.3%</td>
<td>188.5%</td>
</tr>
<tr>
<td>Fertilizer</td>
<td>-6.0%</td>
<td>-4.7%</td>
<td>-3.4%</td>
<td>-0.8%</td>
<td>1.6%</td>
<td>3.0%</td>
<td>4.9%</td>
<td>4.9%</td>
</tr>
</tbody>
</table>
FIGURE 1

SENSITIVITY OF TAX STRUCTURE TO REVENUE REQUIREMENT

Source: Table 6
However, the fact that the sign of the taxes on different goods change at different levels of revenue requirement shows that the determination of optimal taxes is not quite as simple as the remarks above might suggest.

7. Conclusions

This paper has outlined our work on public policy and has shown the sort of contribution that our methodology can make. We believe that our work to date makes a substantial contribution to the understanding of what determines optimal public policy. We have extended the theory to include such institutional features as tax restrictions and the dualistic nature of many underdeveloped economies. Our numerical work has illustrated the significance of these institutional features as well as demonstrating the sensitivity of optimal policy to changes in key parameters.

However, we are also aware of several shortcomings in our models. The most important shortcomings, and the ones we will discuss here, are concerned with the way in which we model the markets for factors. This is most obvious in the case of capital because the models reviewed here are static while the supply of savings (which augment the capital stock) must depend on expectations about the future. Thus those of our models which include capital (supplied either elastically or inelastically with respect to the current rate of return) are omitting important influences on its supply.
We have made some progress in making the dual economy model dynamic, using CONOPT to solve the numerical examples, but that work is still incomplete. However, even in the dynamic model it is only the government that is undertaking savings. What is really required is a model of private sector savings, which will require the modelling of household expectations.

Turning to the labour market, all of our models assume that the labour market clears and that the total number of workers of each type is fixed. It would clearly be desirable to include the possibility of unemployment, especially for underdeveloped countries. Also, it would be interesting to introduce training explicitly so that the number of each type of worker could vary in response to policy changes.

We plan to extend our dual economy model to include urban unemployment by introducing a rigid urban wage and induced migration, as in Harris and Todaro (1970), but have not yet addressed the problems of modelling training.
FOOTNOTES

1. There would still, of course, be an argument for trade intervention if the country was not "small".

2. Harris and Mackinnon (1979) also developed a technique for computing optimal commodity taxes which permitted flexible producer prices and arbitrary income distributions and demand systems. Unfortunately, they have not extended their techniques to the analysis of restricted taxes.

3. Two of the exceptions are Stiglitz and Dasgupta (1971) and Munk (1978).

4. We have generally used the linear expenditure system and cardinalised the indirect utility function so that for any set of consumer prices utility is proportional to the value of endowments minus the value of the minimum consumption levels.

5. If there are multiple equilibria we must assume that the government can get the economy to its preferred equilibrium.

6. This does not mean that there will be no shadow producer taxes on goods whose taxes are unrestricted. The need for producer prices to satisfy (4) and (5) will mean that a change in one producer price will necessarily involve the change of at least one other.

7. We cannot guarantee that we can find them all.

8. Consumer prices have been scaled so that food is untaxed.

9. A different form of production function or utility function of agricultural households might imply a higher tariff.

10. The exception is that the tax on rural services falls below that on clothing for very high revenue requirements.
References Cited


