

Input Substitutability and Cross-Country Variation in Sectoral Linkages

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WORLD BANK GROUP

Development Economics
Development Research Group
March 2019

Abstract

Using panel data on input-output intensities and expenditure prices from 28 countries, this paper finds the elasticity of substitution across sectoral inputs to be less than one in each of the three broad sectors of the economy. Intermediates are most complementary in the production of services while it is easiest to substitute across intermediates in the production of agricultural goods. Differences

in relative prices alone account for a non-trivial fraction of the cross-country variation in sectoral linkages. Abstracting from the price channel that allows for substitution across inputs in response to changes in relative prices delivers biased aggregate implications of changes in productivity and distortions.

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Keywords: Sectoral Linkages, Input Substitutability, Distortions in Intermediate Markets

JEL Classification: O11, O47

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1 Introduction

Sectoral linkages – the extent to which goods from different sectors are utilized as intermediates in the production of sectoral goods, bear a systematic relationship with GDP per capita. The share of services in intermediate expenditure is higher in the developed countries. While this holds in the production of all sectoral goods, the gap in the services share across countries is starker in the production of services itself (Sposi, 2018). An interpretation that helps reconcile the cross-country variation is that distortions exist in the intermediate markets.

Nevertheless, such analyses typically employ a production technology with a Cobb-Douglas (CD) association across sectoral inputs. Hence, linkages do not respond to changes in relative prices. On the other hand, a well-documented fact in the development literature is that the relative price of services rises with GDP per capita.² If intermediates from different sectors enter production as complements, then the higher relative price of services in developed countries will lead to the sector commanding a larger share of the aggregate intermediate expenditure seen in the data.³ The objective of this note is to examine if the elasticity of substitution across sectoral inputs differs from unity, making the price channel a factor in accounting for the cross-country variation in linkages. Using panel data on input-output intensities and expenditure prices from 28 countries, I find the elasticity to be less than one in each of the three sectors. Intermediates are most complementary in the production of services, while it is easiest to substitute across intermediates in the production of agricultural goods.

2 Theoretical Framework and Data

The gross output Q_j of a sector j is produced using the intermediate bundle X_j and a composite Z_j aggregated from all other factors. The production technology is CD in X_j and Z_j . In addition to price-effects, I also allow for scale-effects in production technology, i.e., the intermediate expenditure shares react to changes in X_j . The scale-effect establishes an additional link between incomes and linkages as the size of the intermediate bundle rises with income. Borrowing from Comin et al. (2018), X_j is a scaled CES aggregate of intermediates from all sectors implicitly defined as

$$\sum_k \left(v_{jkc} X_{jct}^{\epsilon_{jk}} \right)^{1/\theta_j} X_{jkc}^{(\theta_j-1)/\theta_j} = 1 \quad (1)$$

where X_{jkc} denotes the quantity of sectoral good k used in the production of sectoral good j and c, t are subscripts corresponding to country and time respectively. The elasticity of input substitution θ_j can vary across the sectors so that the ease with which intermediates are

² Balassa (1964) is an early example that documents this relationship. More recently, Duarte & Restuccia (2010) provide the within-country evidence.

³ The idea of complimentary in sectoral linkages goes way back (Leontief (1936), Hirschman (1958)) and is more recently investigated in Jones (2011).

substituted for one another might be higher in the production of some sectoral good relative to others. The parameter ϵ_{jr} disciplines the scale elasticity of intermediate demand and the sector-weights v_{jkc} secure the relative importance of inputs from different sectors.

Given the technology in (1), the intermediate expenditure share of sector k relative to r in the production of sectoral good j is

$$\frac{\gamma_{jkct}}{\gamma_{jrct}} = \left(\frac{P_{kct}}{P_{rct}}\right)^{1-\theta_j} X_{jct}^{\epsilon_{jk}-\epsilon_{jr}} \left(\frac{v_{jkc}}{v_{jrc}}\right) \quad (2)$$

where P_{kct} is the price of good k in country c at time t . Equation (2) ties the rising intermediate share of services with income growth. If the inputs enter production as complements ($\theta_j < 0$), then a higher share of intermediate expenditure is allocated towards services as services become relatively more expensive compared to goods of other sectors with income as seen in the data. Choosing some base sector r , the scale elasticity and sector-weights can be normalized by setting $\epsilon_{jr} = v_{jrc} = 1 \forall j, c$. Replacing intermediate bundle X_{jct} in (2) as a function of total intermediate expenditure $E_{jct} (= \sum_k P_{kct} X_{jkct})$ and base sector price and intermediate share yields

$$\ln\left(\frac{\gamma_{jkct}}{\gamma_{jrct}}\right) = (1 - \theta_j) \ln\left(\frac{P_{kct}}{P_{rct}}\right) + (1 - \theta_j)(\epsilon_k - 1) \ln\left(\frac{E_{jct}}{P_{rct}}\right) + (\epsilon_k - 1) \ln(\gamma_{jrct}) + \ln(v_{jkc}) \quad (3)$$

which I use to estimate the parameters by employing the iterated non-linear seemingly unrelated regression method.⁴ I choose industry as the base sector and transform the constrained parameters such that the estimation becomes unconstrained.⁵

The quantitative analysis requires data on intermediate shares and sectoral prices. I use the OECD input-output database (OECD, 2017b) to collect the intermediate shares for many countries. To obtain the sectoral prices, I utilize the final consumption expenditure data from the OECD national accounts database (OECD, 2017a). The database contains expenditures by sectors in both current and constant prices (country-specific base year) in national currencies, which I use to construct price series indexed to unity for a base year for each country. Finally, I adjust the price series using the harmonized price data for 2005 from the GGDC Productivity Level Database (Inklaar & Timmer, 2014) to make prices comparable across countries and time. A limitation of the price series is that it corresponds to the consumption side of the economy. The consumer prices differ from the producer prices as the former include certain margins and distortions.

All sources contain disaggregated data into finer sector classification, which I aggregate up to three sectors – agriculture (a), industry (m) and services (s), using the International Standard Industrial Classification of All Economic Activities, Revision 3 (ISIC Rev. 3.1, 2002). The aggregate expenditure is normalized to per capita terms using population data from Feenstra et al. (2015). Merging price and linkage data from the multiple sources yields an unbalanced panel

⁴ Details in chapter 14.9 of Greene (2011).

⁵ The [appendix](#) contains details of the parameter restrictions and transformation scheme.

featuring 28 countries over the period 1995–2011. The panel includes countries with very high incomes but lacks the representation of low-income countries. Latvia, Lithuania and South Africa are the countries with the lowest incomes in the sample. Even so, there is a substantial variation in income. GDP per capita in the richest country is seven times larger than that of the poorest. The appendix contains more details on the data.

3 Results

[Figure 1](#) plots the point estimates of the substitution elasticity and the accompanying 95 percent confidence intervals (CIs). The squares represent the baseline estimates in which both price- and scale-effects operate together. The baseline specification overwhelmingly rejects the assumption of a unit elasticity. Moreover, this result is not sensitive to the introduction of scale-effects. The estimates in circles correspond to a standard CES specification so that only the price-effect endures. All the point estimates are lower than one, though the assumption of a CD relationship for the industrial sector is not rejected owing to a lower precision. The baseline estimates show that the elasticity is highest in agriculture followed closely by industry. The substitutability drops considerably in the production of services.

Next, I consider if there exists a systematic link between elasticity of input substitution and development. It is possible that the nature of production becomes more complex as countries grow richer making inputs less substitutable as they start serving special functions. For example, farm machinery, fertilizers and agronomical know-how are more easily substitutable when the organic content of agricultural produce is not a primary concern. Substitutability diminishes if the organic content becomes critical. To this end, I divide the sample into two equal sub-samples of low- (LI) and high-income (HI) countries based on mean GDP per capita.⁶ [Table 1](#) lists the estimates. The elasticities for the two groups are very similar for agriculture and industry, with the difference in estimates not being significant in the latter case. The point estimates suggest that the substitutability between inputs in services production is considerably lower for the HI group. Still, the estimate of elasticity for the HI group is noisy with the upper end of its CI lying close to the lower end of the estimate of the LI group.

How effective are the price- and scale-effects in explaining the cross-country variation in linkages? Panel A of [Table 2](#) presents the explained variation⁷ in intermediate shares. Column (1) corresponds to the baseline specification. Irrespective of the production sector, the two effects jointly explain more than 70 percent of the variation in the intermediate shares of agriculture. The explained variation is even higher when considering the shares of industry and services. Columns (2) and (3) shed light on the relative importance of the two effects. The former relates to the case when only the scale-effect is active while the latter holds the sole operation of the

⁶ The composition of sub-samples is stable over time – the 14 countries with the highest incomes in 2003 (the earliest when all 28 countries feature in the sample) also have the highest incomes in 2011.

⁷ $\text{var}(\ln(\widehat{y}_{jk})) / \text{var}(\ln(y_{jk}))$, where \widehat{y}_{jk} is the predicted share.

price-effect. On their own, the two effects come close to explaining what is explained considering their joint action. The scale-effect dominates the baseline specification in industrial production while the price-effect comes on top in accounting for the intermediate share of agriculture in agricultural and services production. The figures in columns (1)–(3) hide the importance of country-specific sector weights. Column (4) lists the explained variation when sector-weights are invariant across countries and only price differences drive the cross-country variation. The explained variation drops appreciably. The loss is most severe in agricultural production, with the substitution elasticity being nearer to unity. The relevance of sector-weights in explaining the variation suggests the influence of other country-specific factors including distortions. Nonetheless, the price-effect still accounts for a meaningful variation for industrial and services production. The channel is especially effective in explaining the shares of the sectors themselves where it accounts for 40–70 percent of the total variation.

Lastly, I compare the absolute fit of the specifications by reporting the root mean square errors (RMSE) in Panel B. Unsurprisingly, the RMSEs are highest when sector-weights are invariant. Comparing columns (6) and (7) reveals that the price channel does a better job at fitting the data for agriculture and industry. The scale channel dominates when it comes to services production, surpassing even the fit achieved in the presence of both channels. Introducing both effects taxes the absolute fit even though the substitution elasticity in services lies notably below one.

4 Conclusion

There exists a massive cross-country variation in how sectors interact with each other. The allocation of resources depends critically on these linkages and hence has implications for efficiency and growth. The analysis finds the elasticity of substitution across intermediates from the three broad sectors to be less than one.⁸ A non-trivial fraction of the variation in intermediate shares is accounted for by nothing but the variation in prices across countries.⁹ A ramification is that models of sectoral linkages must allow for an effective price channel to operate so that shares respond to changes in relative prices. The aggregate implications of changes in productivity and distortions derived in absence of the price channel will be biased.¹⁰

⁸ Peter and Ruane (2018) (PR) examine input substitutability using data on Indian firms. Their point estimate of elasticity for inputs aggregated at a broad level is less than one. However, they cannot rule out the CD association. The estimates in [figure 1](#) and [table 1](#) lie in the CI of the estimates in PR. Their analysis also highlights the relevance of aggregation as they conclusively find the elasticity to be greater than one when inputs are disaggregated to within-sector levels.

⁹ From a development accounting perspective, the price channel owing to complementarity in inputs implies that sectoral linkages constrict income differences as richer countries allocate more resources into the sector with a higher relative price. Fadinger et al. (2018) also report the same effect, though they take variations in linkages to be exogenous.

¹⁰ Jones (2011) discusses the relationship between aggregate output and input substitutability. Peter and Ruane (2018) quantitatively investigate the issue using data on Indian firms.

Furthermore, the estimates of distortions themselves are biased if they are inferred using a model with no price-effects.

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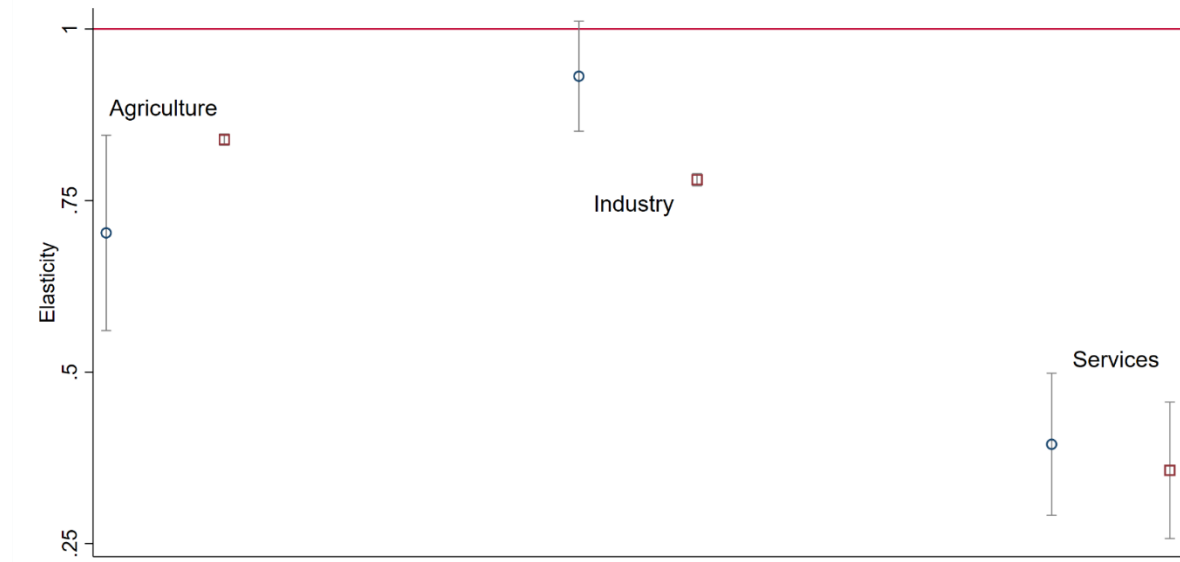


Figure 1: Estimates of Substitution Elasticity θ_j

¹Estimates of substitution elasticity θ_j and the corresponding 95 percent confidence intervals. The squares refer to estimates when both price- and scale-effects operate while only price-effects are active in the case of estimates shown as circles.

	θ_a (1)	θ_m (2)	θ_s (3)
All Countries	0.839 (0.004)	0.780 (0.005)	0.357 (0.051)
Low-Income Group	0.872 (0.002)	0.781 (0.005)	0.575 (0.011)
High-Income Group	0.861 (0.001)	0.777 (0.003)	0.375 (0.081)

Table 1: Estimates of Substitution Elasticity by Income Groups

¹Standard errors in parentheses. All estimates significant at 1 percent.

²Estimation corresponds to the specification in which both price- and scale-channels are active.

Channels	Panel A: Explained Variation				Panel B: Root Mean Square Error			
	Price & Scale	Scale Only	Price Only		Price & Scale	Scale Only	Price Only	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
γ_{aa}	0.718	0.711	0.861	0.059	0.044	0.064	0.047	0.096
γ_{am}	0.859	0.714	0.796	0.037	0.022	0.058	0.039	0.090
γ_{as}	0.805	0.741	0.734	0.268	0.032	0.031	0.029	0.053
γ_{ma}	0.740	0.865	0.714	0.207	0.012	0.020	0.014	0.020
γ_{mm}	0.844	0.911	0.825	0.392	0.019	0.039	0.022	0.047
γ_{ms}	0.860	0.857	0.854	0.490	0.019	0.027	0.020	0.039
γ_{sa}	0.705	0.823	0.726	0.227	0.005	0.005	0.005	0.006
γ_{sm}	0.941	0.897	0.917	0.578	0.022	0.015	0.021	0.045
γ_{ss}	0.898	0.836	0.856	0.689	0.024	0.014	0.023	0.045
Country Specific Sector-Weights	Yes	Yes	Yes	No	Yes	Yes	Yes	No

Table 2: Explained Variation and Root Mean Square Error

Appendix

A.1 Data

The national accounts database of the OECD (OECD, 2017a) reports the household final consumption expenditures across several categories. The categories correspond to the Classification of Individual Consumption According to Purpose (COICOP). The data are available both in current and constant prices, though the base years vary across the countries. I aggregate the expenditures across reported categories to make them consistent with the three broad sectors (see [table A.1](#) for the aggregation scheme). The ratio of sectoral expenditures in current prices to sectoral expenditures in constant prices yields the prices of the sectoral goods. These prices are indexed to one for a specific base year in each country. To make them comparable across countries, I use the internationally comparable data from the GGDC productivity level database (Inklaar & Timmer, 2014). The price data are available on a 10-sector basis for 2005 which I aggregate up to the three-sector level using gross-output weights from the input-output database (OECD, 2017b). Having internationally comparable sectoral prices for 2005, I adjust the price series for each country obtained earlier. [Table A.1](#) reports the aggregation scheme used to harmonize data from the three sources. The sample in the quantitative analysis contains data on 28 countries for which both intermediate share and price data are available. [Table A.2](#) lists the country coverage together with the sub-sample classification used in dividing them into low- and high-income groups.

A.2 Parameter Constraints and Normalization

The parameters of the scaled CES production technology represented by equation (1) have the following constraints attached to them: 1) $\theta_j > 0$ ($\theta_j \neq 1$), 2) $v_{jkc} > 0$ and 3) $\theta_j < 1 \Rightarrow \epsilon_{jk} > 0$; $\theta_j > 1 \Rightarrow \epsilon_{jk} < 0 \forall k$.¹¹ The scale elasticities ϵ_{jk} and sector-weights v_{jkc} can be scaled by a positive constant without any impact on the observable behavior (Comin et al. (2018)). Assuming $\theta_j < 1$ such that $\epsilon_{jm} > 0$, the normalization $\epsilon_{jm} = v_{jkc} = 1$ yields the system of equations described by equation (3). I transform the parameters as follows to make the underlying estimation unconstrained: $\epsilon_{jk} = \exp(e_{jk})$ and $\theta_j = 1/(1 + \exp(a_j))$. In contrast, assuming $\theta_j > 1$ such that $\epsilon_{jm} < 0$, the normalization $\epsilon_{jm} = -1$ yields the system below

$$\ln\left(\frac{\gamma_{jkct}}{\gamma_{jmct}}\right) = (1 - \theta_j) \ln\left(\frac{P_{kct}}{P_{mct}}\right) + (\theta_j - 1)(\epsilon_k + 1) \ln\left(\frac{E_{jct}}{P_{mct}}\right) - (\epsilon_k + 1) \ln(\gamma_{jmct}) + \ln(v_{jkc}) \quad (1)$$

The transformation under this normalization amounts to $\epsilon_{jk} = -\exp(e_{jk})$ and $\theta_j = 1 + \exp(a_j)$ and the estimation yields substitution elasticities that converge to 1 ($\theta_a, \theta_m, \theta_s \rightarrow 1$). I do not show the results separately for brevity which are almost indistinguishable from what is reported when only the scale-effects are in effect. The above transformations for ϵ_{jk} extend when estimating the scale-only specification together with setting either $\theta_j = 1 - \Delta$ or $\theta_j = 1 + \Delta$ with $\Delta \rightarrow 0$. In the price-only specification, the scale CES technology reduces to a standard CES stipulation and the estimation system is given by

¹¹ See Comin et al. (2018) for more details.

$$\ln\left(\frac{Y_{jkct}}{Y_{jrct}}\right) = (1 - \theta_j) \ln\left(\frac{P_{kct}}{P_{rct}}\right) + \ln\left(\frac{v_{jkc}}{v_{jmc}}\right) \quad (2)$$

The constraints amount to $\theta_j > 0$ and $v_{jkc} > 0$ ($\sum_k v_{jkc} = 1$) and are unconstrained transformation amounts to $\theta_j = \exp(a_j)$ and $v_{jac} = \frac{1}{1+\exp(n_{jmc})+\exp(n_{jsc})}$, $v_{jkc} = \frac{\exp(n_{jkc})}{1+\exp(n_{jmc})+\exp(n_{jsc})}$ for $k \in \{m, s\}$.

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Agriculture	Aggregated Sectors Industry	Services
	<u>OECD Input-Output Database</u>	
Agriculture and Fishing	Mining Manufacturing: Food, Beverage & Tobacco Manufacturing: Textile, Dress, Leather Manufacturing: Wood Manufacturing: Paper & Printing Manufacturing: Petrol Manufacturing: Chemicals Manufacturing: Rubber & Plastics Manufacturing: Non-metallic Minerals Manufacturing: Basic Metals Manufacturing: Fabricated Metals Manufacturing: Machinery Manufacturing: Advanced Machinery Manufacturing: Electrical Machinery Manufacturing: Motor Vehicles Manufacturing: Transport Equipment Manufacturing: Furniture & Recycling Utilities Construction	Wholesale & Retail Trade Hotels & Restaurants Transport & Storage Post & Communication Financial Intermediation Real Estate Renting of Machinery & Equipment Computer Related Activities R&D & Other Business Activities Public Administration & Defense Education Health & Social Work Community & Personal Service Private HHs with Employed Persons
	<u>OECD National Accounts Database (table 5)</u>	
Food & non-Alcoholic Beverages	Durable Goods	Services
Alcoholic Beverages, Tobacco & Narcotics	Semi-Durable Goods Non-Durable Goods (<i>less agriculture</i>)	
	<u>GGDC Productivity Level Database</u>	
Agriculture, Forestry & Fishing	Mining & Quarrying Manufacturing Utilities Construction	Wholesale & Retail Trade Hotels & Restaurants Transport & Communications Financial & Business Services Community, Social & Personal Services

Table A.1: Aggregation Scheme

Australia	Austria	Canada	Czech Republic*	Denmark
Estonia*	Finland	France	Germany	Greece*
Hungary*	Italy	Japan	Korea*	Latvia*
Lithuania*	Luxembourg	Mexico*	Netherlands	Poland*
Portugal*	Slovakia*	Slovenia*	South Africa*	Spain*
Sweden	United Kingdom	United States		

Table A.2: Country Coverage

*Classified in the low-income group.

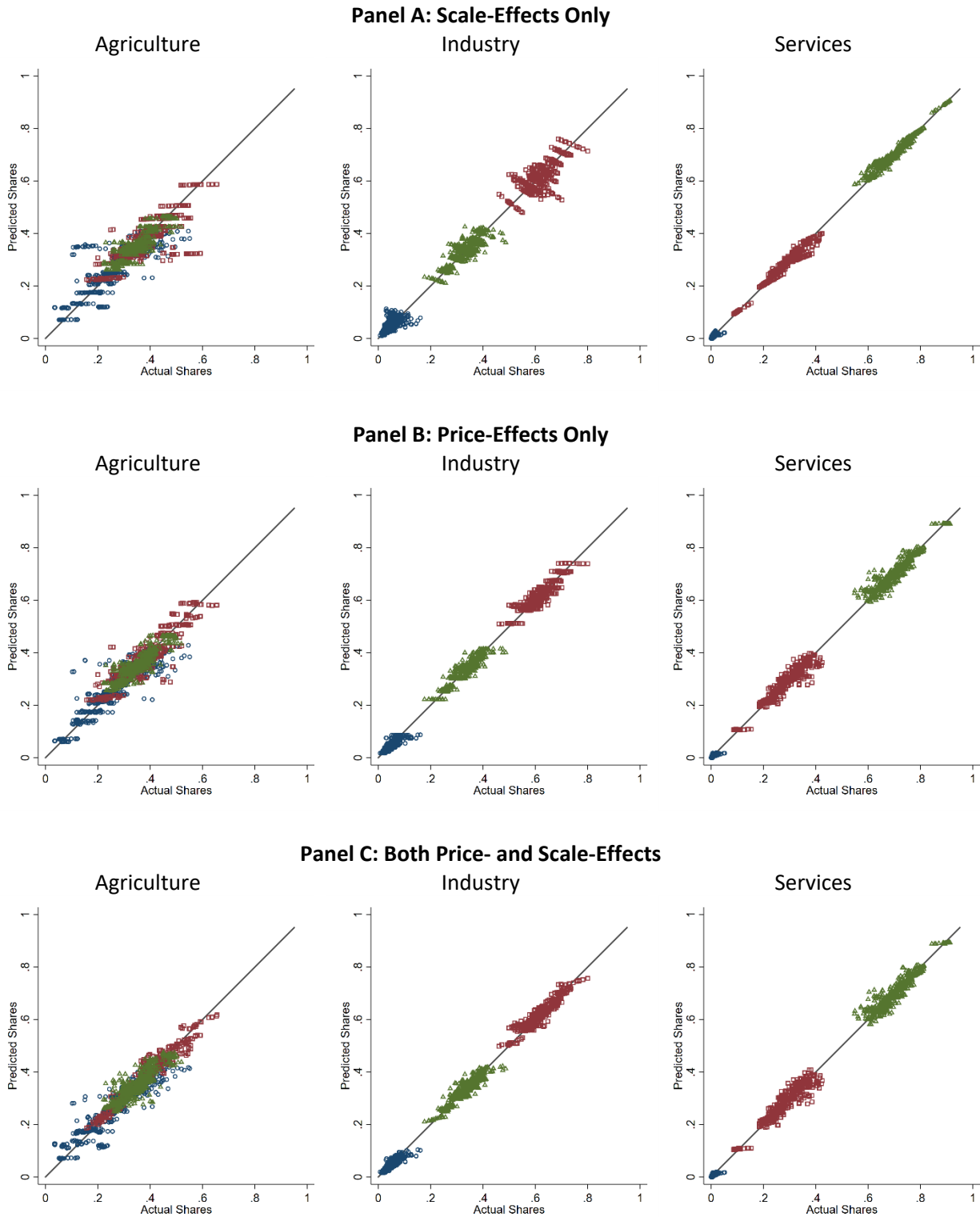


Figure A.1: Actual and Predicted Intermediate Shares

The circles (blue), squares (red), and triangles (green) correspond to the intermediate shares of agriculture, industry, and services respectively.