This paper investigates the optimal boundary between the public and private production sectors. The government in effect determines which activities to maintain within the public sector and which ones to privatize. In choosing the sectoral boundary, the government trades off the relative inefficiency of marginal government production against the capital income tax distortions affecting private production. Optimally the government sector is shown to be "too large" in the sense that the government carries out some activities where it has an efficiency disadvantage vis-à-vis the private sector. Furthermore, it invests more in these activities than the private sector would do. These results are obtained in open economy and closed economy versions of the model.

Keywords: privatization; capital income taxes

JEL Codes: D24; H21
1. Introduction

A large share of production is carried out by state-owned enterprises. Traditionally governments provide a large share of education, health services, road construction and maintenance. In addition, governments in many countries run essentially commercial enterprises that primarily sell goods and services. This type of (non-financial) government activity accounted for a weighted average of 4.9 percent of GDP in 8 industrialized countries during the 1978-1991 period, while for 40 developing countries the average was 10.7 percent.\(^1\) Reflecting considerable variation among industrialized countries, the government production share was 18.2 percent for Portugal, and 1.2 percent for the United States. Many government activities in principle can be carried out by the private sector, and in recent years there has been a strong movement towards privatizing government enterprises. In the United Kingdom, for instance, the share of government production fell from 6.6 percent in 1982 to 1.9 percent in 1991. After the fall of communism, the countries of Eastern Europe, of course, have been on a path of wholesale privatization. These trends reflect the feeling that privatization yields efficiency and welfare gains. Several studies, as surveyed by Boardman and Vining (1989), provide evidence that state-owned enterprises perform substantially worse than similar private firms.\(^2\)

This paper considers a model where government and private production coexist. Specifically, there is a range of production activities that in principle can be carried out by either the government or the private sector. The activities differ in the relative efficiency with which the government and the private sector can carry them out. Some activities, specifically, yield more output for given inputs when carried out by the government, and vice versa. The comparative advantage the government and the private sector have in carrying out certain activities depends, among other things, on production externalities if any, the public goods nature of the output, the feasibility of contracts in the private sector and, finally, the market structure that prevails in the private sector scenario.\(^3\) The government can alter the range of government activities by privatizations or instead by take-overs. Private production also differs from public production in that it is subject to a distorting investment tax. In addition, the government has access to saving taxation and some taxation of private profits. In this setting, the government jointly sets the range of state production activities, physical investment in these activities, and tax policy. The optimal range of government activities is shown to depend on the relative efficiency (or wastefulness) of government production as well as on the distortions created by the investment tax. Further, the required rate of return on investment in public activities is shown to generally be less than that in private firms, but greater than the net return to saving.

The benchmark model is of a small open economy that takes the external cost of capital as given. This analysis yields several interesting insights. First, the range of government activities is chosen efficiently (in the sense that the state and private sectors each carry out the activities they have an absolute advantage in), if the government can raise all required revenues through the nondistorting profit tax. Generally, however, the government has to levy positive investment and saving taxes. In this instance, the scale of the state sector is positively related to the optimal private investment tax rate. The reason is that a higher taxation
of private activity reduces the value of having borderline activities carried out by the private sector. As a result, the government takes on some activities that it has an absolute disadvantage in (ignoring taxes). The positive link between the investment tax and the scope for government production implies that everything that causes a higher investment tax also causes a larger state sector. Stronger preferences for public goods, for instance, result in higher tax levels and a larger state sector.

The paper also considers the case of a closed economy where the cost of capital reflects the internal saving-investment balance. Again, the size of the government sector is efficient, if the government can raise all required revenues through the nondistorting profit tax. With strong preferences for public goods, however, the government is required to impose a capital income tax that causes a wedge between the gross return to private investment and the net return to private saving. With such a distorting tax in place, the government faces a shadow price of capital in between the gross return to capital in the private sector and the net-of-tax return received by savers. The fact that the private and public perceived costs of capital differ again provides the government with the incentive to expand the government sector. As a result, also in the closed economy the government takes on some activities where it has an absolute production disadvantage, if in fact distorting capital income taxation is used.

An extensive literature (see, for instance, Diamond and Mirrlees (1971a,b), Dasgupta and Stiglitz (1972), Sandmo and Drèze (1971), Hagen (1988) and Stiglitz (1982)) has considered optimal taxation given joint public and private production in two separate sectors or firms. Diamond and Mirrlees (1971a) specifically show that the optimal tax scheme implies overall production efficiency in the presence of a full set of consumption taxes. In the present paper, similarly there is overall production efficiency if private sector profit tax revenues suffice. A focus of the earlier literature, and in particular of Sandmo and Drèze (1971), has been the appropriate social discount rate or shadow price of capital for public investment. The present paper similarly is concerned with the appropriate level of public investment given the range of public production activities. As in the earlier work, the implications of public investment for private investment and saving - and for tax revenues - are of considerable interest in the case of a closed economy. The main focus of the paper, however, is on the joint determination of the range of public activities and tax policy. This analysis yields interesting insights also for the case of a small open economy, where the shadow price of public capital is trivially equal to the international interest rate.

As a related matter, Bolton and Roland (1992) have previously considered how privatizations should be carried out so as to minimize the subsequent need for the government to raise revenue through distorting taxation. Specifically, they argue that the government may wish to consider non-cash bids at privatization auctions (in the form of debt or equity stakes in the newly privatized firms). This way, the government can realize some of the efficiency advantages of privatization without a need to impose undesirably high taxes on the newly created private firms. Laban and Wolf (1993), Roland and Verdiér (1994) and others further
address the coordination and other problems associated with large-scale privatizations, as they have taken place in Eastern Europe.

The remainder of this paper is organized as follows. Section 2 outlines the model of the optimal range of state production for the case of a small open economy. Section 3 extends the open economy model to include the foreign ownership of domestic private firms. Section 4 considers the range of government activities in a closed economy. Section 5 concludes.

2. The small open economy

To start, we consider a two-period framework for a small open economy that is financially well integrated with the rest of the world. The domestic interest rate equals the exogenously given world interest rate, \( r \). The economy can produce a single good by way of a range of production activities or projects with a total volume of unity. All production activities can in principle be carried out by either the public or the private sector. The continuum of production activities are indexed by a waste parameter \( \omega \) with support \( [-\omega, \omega] \), where we assume that \(-\omega < 0 < \omega\). The waste parameter \( \omega \) has a density function \( h(\omega) \), and a corresponding distribution function \( H(\omega) \). A project's output, if carried out by the private sector, is characterized by a strictly concave production function, \( F(K) \), where \( K \) is first-period project-specific investment and \( F(K) \) is the project's second-period output. The project's output, if carried out by the public sector, is given by \( (1 - \omega) F(K) \), where \( \omega \) is the project-specific waste parameter. Clearly for activities with \( \omega > 0 \), the private sector has an absolute production advantage, and vice versa. The waste parameter is short-hand for the fact that economic sectors differ in the extent of technological externalities, the degree to which the output is a public good, and the market structure (if privately organized). Below, the government optimally specializes in the range of activities with the lowest waste parameters. Of interest is the marginal public project, with waste parameter \( \omega \), that demarcates the public and private sectors. Production efficiency requires that \( \omega = 0 \) so that each sector carries out the production activities where it has an absolute efficiency advantage. With \( \omega > 0 \) (\( \omega < 0 \)) instead, the government sector is "too large" ("too small") in that the public (private) sector carries out some activities where it has an absolute efficiency disadvantage. The range of public activities, and thus \( \omega \), is affected by the privatization of public activities or by the public take-over of previously private projects. To account for such privatizations or take-overs, let \( \omega_0 \) be the waste parameter of the original marginal public production activity. We will assume that \( \omega < \omega_0 \) so that indeed there are privatizations.

In the first period, the representative agent receives an endowment of the single good denoted \( Y \). These resources are divided between first-period consumption, \( C_1 \), and saving, \( S \). In the second period, there is private consumption, \( C_2 \), and consumption of a public good, \( G \). This public good is a one-to-one transformation of the single produced good; it does not affect production possibilities, nor the marginal rate of substitution between first and second period consumption. To finance this public good, the government can impose a private investment tax at a rate \( v \), and a saving tax at a rate \( u \), both payable in the second period. In
addition, second-period profits are taxed at a rate $z$. We assume the investment tax is deductible from taxable profits. Profits can be thought to reflect some project-specific fixed factor such as land or labor. The feasible profit tax, $z$, is assumed to be limited to the range $0 < z < 1$. The upper limit on the profit tax may have to do with institutional constraints precluding complete taxation of profits.

The ordering of events is as follows. First, the government decides on the extent of privatization, on tax rates, and on investment in public activities. Next, private agents make their saving and investment decisions. Finally in the second period, the government receives the payment for privatized projects, and production and consumption of private and public goods take place. As indicated, with $< 0$ the government privatizes some public activities. In return, the government receives a second-period payment, $P$, from the representative household. This payment equals the after-tax profits the private sector can obtain from carrying out the newly privatized activities (given the investment and profit taxes) as follows,

\[
P = (\sigma_p - \sigma_p^0) (1 - z) [F(K_p) - (1 + r + v)K_p] - P
\]

where \( \sigma_p = 1 - H(\omega) \) and \( \sigma_p^0 = 1 - H(\omega^0) \) are the shares of projects in private hands after and before privatization (or loosely speaking the post- and pre-privatization sizes of the private sector), and where \( K_p \) is the private capital investment at any private-sector project.

The two period budget constraint of the representative household can now be stated as follows,

\[
C_2 = (1 + r - u) (Y_{1} - C_1) + \sigma_p (1 - z) [F(K_p) - (1 + r + v)K_p] - P
\]

Second-period consumption, $C_2$, thus reflects first-period saving, second-period after-tax profits and the payment to the government for any privatized activities.

The government's second-period budget constraint is as follows,

\[
G = u S + \sigma_p [F(K_p) - (1 + r + v)K_p] - v K_p + P
\]
where \( \{ \text{K sub } g \} \) is the investment at a particular government-sector project. On the income side in (3), the government receives saving, profit and investment tax revenues and the profits from government-run activities plus the sales receipts from privatized activities.

Using the expression for \( P \) in (1), the government's net gain from privatization can be written as follows,

\[
\{\sigma_p - \sigma_p^0\} \cdot \left[ F(K_p) - (1+r)K_p\right]
\]

\[
\left\{\int_{\omega^0}^{\omega^\prime} \left[ (1-\omega)F(K_g(\omega)) - (1+r)K_g(\omega)\right] h(\omega) d\omega \right\}
\]

Following privatization, the government receives the sum of the sales revenues, \( P \), in (3) and the investment and profit taxes levied on the privatized entities. This sum simply equals the entire pre-tax profits from the privatized activities as reflected in (4). In return, the government has to forego the profits it would obtain from running the privatized projects itself.

The lifetime utility function has the additive form given by \( U(C_1, C_2) + V(G) \). In the first period, the representative household decides on its saving and on the private sector investment level given rise to the following familiar optimality conditions:

\[
U_1 = (1 + r - u) U_2
\]

\[
F'(K_p) - (1 + r + v) = 0
\]

Eq. (6) immediately implies that the investment \( K_p \) is negatively related to \( v \) for any private-sector project.

Aggregate private and public investments, \( p \) and \( g \), are given by,

\[
\{\text{overline K} \} = \{ p \} - \{ \text{K sub } p \}
\]

\[
\left\{\int_{\underline{\omega}}^{\omega^\prime} \{ g \} \cdot h(\omega) d\omega \right\}
\]
Let us define \( \overline{F}(\bar{K}_g, \omega^\text{hat}) \) to be the maximum public output for a given aggregate government capital \( g \) and projects with waste parameters \( \omega^\text{hat} \) in the state sector. Note that maximizing government output \( \{ \overline{F}(\bar{K}_g, \omega^\text{hat}) \} \) (for a given capital stock \( g \)) implies that the waste-inclusive marginal product of capital, \( (\omega^\text{hat}) \overline{F}'(\bar{K}_g, \omega^\text{hat}) \), is constant for all government projects. Specifically, we have

\[
(\omega^\text{hat}) \overline{F}'(\bar{K}_g, \omega^\text{hat}) = (1 - \omega^\text{hat}) \overline{F}'(\bar{K}_g, \omega^\text{hat}) = \frac{\overline{F}'(\bar{K}_g, \omega^\text{hat})}{\bar{K}_g} \quad \text{for all} \quad \omega \leq \omega^\text{hat}.
\]

This implies that the level of public investment, \( K_g(\omega) \), for any government activity can be written as

\[
K_g(\omega) = \left( (1 - \omega^\text{hat}) \overline{F}'(\bar{K}_g, \omega^\text{hat}) / (1 - \omega) \right)
\]

given an investment, \( K_g(\omega) \), in the marginal public project with waste parameter \( \omega \). This formulation immediately implies that the public investment, \( K_g(\omega) \), is negatively related to the waste parameter \( \omega \). This is illustrated in Figure 1. In the figure, the level of investment, \( K_p(\omega) \), at any private project is constant for any \( \omega > \omega^\text{hat} \).

Against this background and given eq. (5) and (6), the government wishes to maximize the utility of the representative household subject to its budget constraint in (3). The government's choice variables are taken to be the tax rates \( u \) and \( v \), the volume of public goods, \( G \), the marginal activity in the public sector, \( \kappa_g \), the investment level in the marginal public activity \( K_g(\omega) \), and the profit tax \( \tau \). The government's problem corresponds to the following Lagrangean,

\[
L = U(C_{s1}, Y_{s1} - C_{s1}; 1 + r - u) + \sigma_p(1 + \tau) \left[ F(K_{p}) - (1 + r + v)K_{p} \right] + \lambda \left[ K_{p} - G \right]
\]

where \( \lambda \) is the Lagrange multiplier associated with the government budget constraint. 5

The first order conditions with respect to \( u, v, K_g(\omega) \), and \( G \) can be stated as follows,

\[
\text{(10)} \quad u \{ -U \} + \lambda \{ -1 + \omega e \} = 0
\]

\[
\text{(11)} \quad v \{ -U \} + \lambda \{ -\omega e \} = 0
\]

\[
\text{(12)} \quad (1 - \omega) \overline{F}'(\bar{K}_g, \omega^\text{hat}) = (1 + r)K_g(\omega) - \left[ F(K_p) - (1 + r)K_p \right]
\]

Let us define \( \{ \overline{F}(\bar{K}_g, \omega^\text{hat}) \} \) to be the maximum public output for a given aggregate government capital \( g \) and projects with waste parameters \( \omega^\text{hat} \) in the state sector. Note that maximizing government output \( \{ \overline{F}(\bar{K}_g, \omega^\text{hat}) \} \) (for a given capital stock \( g \)) implies that the waste-inclusive marginal product of capital, \( (\omega^\text{hat}) \overline{F}'(\bar{K}_g, \omega^\text{hat}) \), is constant for all government projects. Specifically, we have

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given an investment, \( K_g(\omega) \), in the marginal public project with waste parameter \( \omega \). This formulation immediately implies that the public investment, \( K_g(\omega) \), is negatively related to the waste parameter \( \omega \). This is illustrated in Figure 1. In the figure, the level of investment, \( K_p(\omega) \), at any private project is constant for any \( \omega > \omega^\text{hat} \).

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\[
L = U(C_{s1}, Y_{s1} - C_{s1}; 1 + r - u) + \sigma_p(1 + \tau) \left[ F(K_{p}) - (1 + r + v)K_{p} \right] + \lambda \left[ K_{p} - G \right]
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\]

\[
\text{(11)} \quad v \{ -U \} + \lambda \{ -\omega e \} = 0
\]

\[
\text{(12)} \quad (1 - \omega) \overline{F}'(\bar{K}_g, \omega^\text{hat}) = (1 + r)K_g(\omega) - \left[ F(K_p) - (1 + r)K_p \right]
\]
\[ \omega \hat{} \] is the uncompensated semi-elasticity of saving with respect to the tax rate \( u \) (which is assumed positive at a maximum in (10)), \( e_v = -\frac{dK}{d\nu}/K > 0 \) is the semi-elasticity of private investment with respect to the tax \( \nu \), and \( p \) is the marginal propensity to consume in the first period out of second-period income. Further, the parameter \( \eta = \lambda / U_2 \) is the post-privatization size of the private sector relative to the pre-privatization size of the private sector. Accordingly, a value of exceeding unity points at ongoing privatization, and vice versa.

We can solve for the optimal saving tax, \( u \), and the optimal investment tax, \( v \), from (10) and (11) as follows,

\[ u = \frac{1}{e_u} (1 - \eta) \] (15)

\[ v = \frac{1 - z}{\rho e_v} (1 + p/e_u) (1 - \eta) \] (16)

where \( \eta = \lambda / U_2 \) is the marginal cost of funds. At the same time, the optimal saving and investment taxes are related to each other as follows,

\[ v = (1 - z) e_u \] (17)

where \( e_u = -\frac{dS}{du}/S \) is the compensated semi-elasticity of saving with respect to the tax \( u \). Note from (16) and (17) that the greater is the extent of ongoing privatization, the smaller is the incentive for the government to utilize the investment tax. To see this, note from (12) that a higher investment tax imposed on newly privatized activities in fact reduces the privatization selling price by more than the additional investment tax revenues, as a higher investment tax further distorts the post-privatization investment choice.

Let us now consider the optimal range of production activities in the public sector. First, let us consider that the feasible profit tax revenues are sufficient to finance the optimal volume of government spending. In this instance, the marginal cost of funds \( \eta = \lambda / U_2 \) is unity. Eq. (15) and (16) then immediately imply that the saving tax, \( u \), and investment tax, \( v \), are optimally set to zero, while eq. (12) then yields that the optimal marginal waste parameter, \( \omega \hat{} \) also is equal to zero. In this case, the public and private sector each carry out the activities where they have an absolute efficiency
advantage, and overall production efficiency prevails. Next, let us consider that the available profit tax revenues are insufficient to finance the optimal provision of public goods. In this case, the profit tax, $z$, equals the maximum profit rate, $\overline{z}$, and the marginal cost of public funds, $\nu$, exceeds unity, and both the saving and investment taxes, $u$ and $v$, are positive in (15) and (16). For this case, we can show:

Proposition 1. In the open economy, the optimal privatization choice implies

(i) $\{\omega \hat{=} -0\}$ if $\{\eta \hat{=} -1\}$,
(ii) $\{K_g(\omega \hat{=}) > K_p\}$ if $\{\eta \hat{=} -1\}$,
(ii) $\nu$ and $\omega$ are positively related.

For a proof, see the Appendix. Part (i) of the proposition indicates that the government sector is "too large" with $\eta > 1$, as the public sector has taken over some activities from the private sector where the public sector has an absolute efficiency disadvantage. The government sector is expanded to the point where $\omega \hat{=} -0$ to balance the distortion of private investment with a positive investment tax, $\nu$.

Following eq. 12, the surplus $(1 - z)F(K_g) - (1 + r)K_p$ for the marginal public activity equals the surplus $F(K_p) - (1 + r)K_p$ for any private activity, as indicated by the two shaded areas in Figure 2. Note that parts $1 - z$ and $z$ of the private-sector surplus, i.e. $(1 - z)F(K_p) - (1 + r + v)K_p$ and $z(F(K_p) - (1 + r + v)K_p)$, accrue to the private and government sectors as after-tax profit income and profit tax revenues, respectively.

Part (ii) of the proposition states that with $\eta > 1$ the government invests more in the marginal public project than the private sector, i.e. $K_g(\omega)$ > $K_p$, as illustrated in Figure 3. In the figure, $K_g(\omega)$ maximizes the surplus $(1 - z)F(K) - (1 + r)K$, equal to the tax-ridden surplus, $F(K_p) - (1 + r)K_p$, attained at any private activity. Private investors instead choose, $K_p$, to maximize the expression $F(K_p) - (1 + r + v)K_p$, and thus $K_p < K_g(\omega)$. The result implies that the investment tax, $\nu$, distorts private investment more than the waste parameter, $\omega$, distorts the public investment in the marginal public project. Part (iii), finally, indicates that the waste parameter of the marginal public project, $\omega$, and thus the size of the state production sector, are positively related to the optimal investment tax, $\nu$. The reason is that a higher and thus more distorting investment tax increases the relative overall efficiency of public vs. private production.

Part (iii) of proposition 1 has the important implication that factors that lead to a higher optimal investment tax also lead to a larger optimal state production sector. This enables us to show the following:

Proposition 2.

(i) For given values of $e_u$, $(1 - z)/e_v$, and $p$, a higher value of $\nu$ leads to larger $u$, $\nu$ and $\omega$.
(ii) For given values of $e_u$, $p$, , a higher value of $(1 - z)/e_v$ increases $\nu$ and while $u$ remains the same.
(iii) For given values of $(1 - z)/e_v$, $p$, and , a higher value of $e_u$ reduces $u$ and $\nu$ and .
Part (i) indicates that stronger preferences for public goods, as proxied by a higher value of \( \theta \), lead to a larger state production sector. To see this, note that the saving and investment taxes, \( u \) and \( v \), increase with \( \theta \) in (15) and (16). The result then follows by applying part (iii) of proposition 1. Part (ii) states that a higher value of \( (1 - z) / e_u \), for instance through a lower value of \( z \), leads to a larger state sector. With a lower profit tax rate \( z \), the investment tax, \( v \), is higher in (16), which reflects that the investment tax acts as a second-best tax on private-sector profits. The result then again follows by applying part (iii) of proposition 1. Finally, part (iii) states that a higher compensated saving semi-elasticity, \( e_u^c \), leads to a smaller state sector. To see this, note equation (16) and part (iii) of proposition 1.

To conclude this section, consider the possibility that the state sector becomes more productive or - equivalently - less wasteful. In particular, consider an increase in the efficiency of government production, as indicated by smaller waste parameters for public activities with \( \omega < \omega^\hat{} \). From (3), we see that this public sector efficiency gain is equivalent to a lump sum expansion of government resources, as it increases the profits the government derives from operating its production activities. Such a revenue expansion causes a reduction in the marginal cost of public funds, \( \mu \). The government can then lower the saving and investment tax rates, \( u \) and \( v \), and the marginal waste parameter \( \mu \). An increase in the efficiency of government production thus, somewhat paradoxically, may lead to a reduction in the size of the government sector.

3. **International ownership of private firms**

The government's privatization decision generally balances the welfare losses stemming from the taxation of private investment against the relative inefficiency of (marginal) government production. These welfare losses, of course, are substantially mitigated if domestic projects or firms are partly foreign-owned, in which case the incidence of investment taxation is partly on the foreign owners of domestic firms. To examine this, this section introduces foreign ownership of domestic projects or firms, following Huizinga and Nielsen (forthcoming). In particular, we assume that a share of all private sector projects is owned by foreigners. In case of privatization, foreign residents also purchase a share of the equity of any newly privatized firms. The second-period budget constraint of the domestic representative agent then is modified as follows,

\[
C_2 \{ <\bot (1 + r - u) \} (Y_{1-C} \{1-\{1-\alpha\}(1-r-z)\} [F(K_{p'}) - (1 + r + v)K_{p'}] \}
\]

\[
\{ <\bot (1-\alpha)P + R \} \quad (18)
\]
where $R$ is taken to be the symmetric, after-tax receipts domestic agents receive from their ownership of any foreign private firms.

Equivalently to (17), we now have,

$$
\text{ital \{v\} = \{1 - \z\} over \rho \text{e sub \{v\} } \left[ (1 - \alpha)u \text{e sub \{u\} sup \{c\}} + \alpha \right] \} (19)
$$

From (19), we see that a higher foreign ownership share, $\z$, causes the government to shift the stress from saving to investment taxation. In particular, a higher value of $\z$ leads to a higher value of $v$, for a given value of $u$ if we assume $1 - e\text{e sub \{u\} u > 0}$. For a given value of $\z$, a larger foreign ownership share, $\z$, then also causes a larger state production sector as proxied by $\overline{\z}$. A larger foreign ownership share, however, reduces the cost of public funds, precisely because part of the incidence of the investment tax now is on foreign owners. This tends to lower $v$. The net effect on the investment tax rate is, however, ambiguous.

4. **The closed economy**

In the closed economy, the cost of capital no longer can be taken as given, but rather reflects the private supply of saving and the aggregate private and public demand for investment. In this setting, the government has to take into account how its demand for investment funds affects the general equilibrium in the economy including its saving, investment and profit tax revenues. This section investigates the joint government tax, public investment and privatization decisions for this case. The government is shown to face a shadow cost of capital between the gross return to investment in the private sector and the net-of-tax return to savings. The perceived private and public costs of capital thus are different. From the public perspective, this perceived difference in costs of capital is a distortion which can be corrected by expanding the public sector. Optimally, therefore, the government sector expands, and the government takes on activities where it has an absolute production disadvantage. As in the open economy, the government sector will be "too large".

In the closed economy, the tax authority has a single tax instrument, $x$, to introduce a wedge between the gross return to investment and the net return to saving. In particular, a single tax $x$ can be thought to be levied on the return to saving. As a result, the net-of-tax return to saving is $r - x$, while $r$ is the return to investment. Again, there is a profit tax $z$ with $\text{ital \{0 \leq z \leq \bar{z}, \text{bar = -1}\}}$and $K_p$ and $K_g(\omega)$ are the private and public project-specific investments. With aggregate private and public investments $p$ and $g$, the saving-investment balance implies that $\text{ital \{\overline{K} sub {p} + \overline{K} sub {g} = S\}}$. Again, we denote by $\text{ital \{(\omega, g)\}}$ maximum public output from the aggregate government capital $\text{ital \{K sub {g(\omega hat)}\}}$ given.

The government's optimization problem is given by the following Lagrangean:
\[ L = U(C_i, (Y - C_i)(1 + r - x) + \rho(I - z) [F(K_p) - (1 + r)K_p] - P) + V(G) + \]

\[ (xS + \rho z [F(K_p) - (1 + r)K_p] + (\rho) - (1 + r)G + P - G) \]

(20)

The government's choice variables are \( x, K_g() \), and \( G \) (for a given value of \( z \)). The corresponding first order conditions are:

\[ \lambda(1 - x^e_{s}) \frac{d}{d\lambda} \left[ \kappa_{g}^{0} \frac{d}{d\kappa_{g}} \right] = 0 \]

(21)

\[ U_{2} \left[ \kappa_{g}^{0} \frac{d}{d\kappa_{g}} \left( \frac{\overline{K}_{g}(\omega\hat{\omega}) \overline{F}(\overline{K}_{g}(\omega\hat{\omega}))}{1 + r} \right) \right] = 0 \]

(22)

\[ \lambda \left[ \frac{d}{d\lambda} \left[ \kappa_{g}^{0} \frac{d}{d\lambda} \left( \frac{\kappa_{g}^{0}}{K_{g}(\omega\hat{\omega})} \right) \right] \right] = 0 \]

(23)

\[ V'(G) - 0 = 0 \]

(24)

where \( e_u^{0} = e_u + p(1-z) \rho K_p/S \) and \( \kappa_{g}^{0} = \left( 1 - \frac{kappa_{g}^{0}}{K_{g}(\omega\hat{\omega})} \right) \frac{d}{d\kappa_{g}} \), with \( \kappa_{g}^{0} = \left( 1 - \frac{kappa_{g}^{0}}{K_{g}(\omega\hat{\omega})} \right) \frac{d}{d\kappa_{g}} \) being the effective, after-profit-tax share of capital owned by the private sector with no privatization.

In these expressions, \( e_s \) can be expressed as follows,

\[ e_{s} = \frac{1}{1 - \frac{d}{d\lambda}} e_{u} - \rho \left( 1 - z \right) \frac{sigma_{p}^{0} K_{p}}{S} \frac{d}{d\lambda} \]

(25)
Next, the saving-investment balance implies that $dr/dx$ can be found as,

$$\{d'r\over dx\}=-\{e_{s}\over \{v\}^{c}\} \over \{K_{p}\} \over \{S\}$$ \hspace{1cm} (26)

so that $e_{s}$ can be written as,

$$e_{s}=\{K_{s}\} \over \{e_{v}\} \over \{u\} \over \{0\}$$ \hspace{1cm} (27)

which is positive with $e_{u} > 0$. Using the saving-investment balance, we can also find that,

$$\{d'\over K_{g}\} \over \{\omega\} \over \{\rho\} \over \{\eta\} \over \{r\} \over \{f\} \over \{K_{g}\} \over \{\omega\} \over \{p\} \over \{K_{g}\} \over \{\omega\} \over \{p\}$$ \hspace{1cm} (28)

$$\{d'\over K_{g}\} \over \{\omega\} \over \{\rho\} \over \{\eta\} \over \{r\} \over \{f\} \over \{K_{g}\} \over \{\omega\} \over \{p\} \over \{K_{g}\} \over \{\omega\} \over \{p\}$$ \hspace{1cm} (29)

Expression (28) immediately shows that $x \geq 0$ as $\eta \geq 1$. Other things equal, ongoing privatization, with $\rho$ exceeding unity, reduces the optimal tax wedge, $x$, as a higher saving-investment tax wedge discourages any post-privatization investment. In (29), the left hand side is the marginal product of capital in the public sector. This is set equal to the shadow cost of public capital on the right hand side of (29). Note that this shadow cost of public capital is bounded by the cost of capital in the private
sector, i.e. \(I + r\), and the net-of-tax return to savings, i.e. \(I + r - x\), if \(\eta > 1\). As a matter of fact, the shadow cost of public capital, \(\frac{\partial \{\overline{F}\}}{\partial \overline{K}_g}\), can be written in the form of a weighted average of the private cost of capital, \(I + r\), and the net return to saving \(I + r - x\), with weights equal to \(\frac{e_v}{e_v + (1 - z)e_u^c}\) and \((1 - z)e_u^c/\left(e_v + (1 - z)e_u^c\right)\), respectively. This weighted average formula is a generalization of the main result in Sandmo and Dreze (1971) (as seen in their eqn. (20)) so as to include both profit taxation and some ongoing privatization. From (29), we see that the higher the rate of profit taxation, \(z\), and the larger the relative post-privatization private sector, \(\omega\), the closer will the shadow cost of public capital, \(\frac{\partial \{\overline{F}\}}{\partial \overline{K}_g}\), will be to the private cost of capital, \(I + r\). Interestingly, the shadow price of public capital in the open economy, equal to \(I + r\), can be written as a weighted average of the marginal productivity of capital in private activities, \(I + r + v\), and the net return to saving, \(I + r - u\), with equal weights as in (29).

Next, let us turn to the optimal size of the state production sector. As before, we can distinguish the cases where the profit tax, \(z\), is not strictly bound by its upper limit \(\bar{z}\). In the first case with \(z = 1\), equation (28) yields that \(x = 0\), while equations (29) and (30) jointly yield \(K_{g(\omega)} = K_{p(\omega)}\) and \(\omega = 0\) so that there is overall production efficiency in the economy. Next, with \(z > 1\) we find,

**Proposition 3.** In the closed economy with \(z > 1\), we have (i) \(\omega > 0\) and (ii) \(K_g() > K_p\).

For a proof, see the Appendix. The proposition implies that with distorting capital income taxation the government sector will be "too large" as the government expands into activities where it has an absolute production disadvantage. By expanding the government sector, the government counters the distortion created by the fact that the private cost of capital is higher than the shadow cost of public capital on the right hand side of (29). As a result, the private sector invests too little from the public perspective, and the marginal product of capital in the private sector exceeds the marginal product of capital in the public sector. The optimal size of the government sector balances the benefits for the government of being able to set investment volume against the production disadvantage vis-à-vis the private sector for any marginal public project. Note that proposition 3 exactly corresponds to parts (i) and (ii) of proposition 1. The government sector thus will optimally be "too large" regardless of whether the interest rate is given exogenously or determined endogenously through savings.

5. **Conclusion**

This paper has shown that the optimal range of public production activities and tax policy are closely related. In any privatization decision, the government in effect trades off the relative efficiency of public production against the private investment distortion created by tax policy. In an open economy, the private
investment decision is distorted by a source-based investment tax. In a closed economy, the private investment decision is distorted by either a private investment tax or a saving tax. Either tax produces a wedge between the gross return of investment and the net-of-tax return received by savers. On account of this tax wedge, the private cost of capital exceeds the shadow cost of public capital. A difference between the private and public perceived costs of capital implies that tax policy distorts the private investment decision. To correct this distortion of private investment, the government faces an incentive to expand the range of its production activities. In particular, the government will take on some activities where it has an absolute production disadvantage. In this sense, the government sector will be "too large". More generally, the size of the government sector is related positively to the investment tax wedge. It would be interesting to test this implication of the model in future work.

The level of investment taxes - and thus the size of the state production sector - may be affected by tax competition in the international economy. With increased international capital mobility, there appears to be more scope for international (investment) tax competition. As a result of tax competition, perhaps, corporate income tax rates have been on a downward trend in European countries. In Europe, the general lowering of corporate income tax rates has coincided with a trend towards privatizing government activities.

The analysis of this paper has focused on the relationship between capital income taxes and the size of the government production sector. Analogously, one could consider the relationship between labor income taxes and the size of the state sector. In this instance, the model predicts that a formerly state-owned enterprise - after privatization - reduces its payroll. Privatizations indeed seems to lead to reduced employment levels.
References


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World Bank, 1995, Bureaucrats in business, the economics and politics of government ownership, Oxford University Press, Oxford.
Appendix

Proof of Proposition 1:

(i) Eq. (12) implies that

\[-(1+r)K_{sub g} \equiv \{F(K_{sub p}) - (1+r)K_{sub p}\} \equiv 0.\]

Note that with \(\omega hat \equiv 0\), we have \(\Delta > 0\), as the private investment, \(K_{p}\), is distorted by the investment tax, \(v\). To see the result, note that \(d\Delta/d\omega hat < 0\).

(ii) From (12) and \(\Delta > 0\), we see that,

\[-(1+r)K_{sub g}(\omega hat) > F(K_{sub p}) - (1+r)K_{sub p}\]

The result follows by noting that \(F(K_{sub p}) - (1+r) > 0\) from (6).

(iii) Totally differentiating (12), we find that,

\[\frac{d\omega hat}{dv} \equiv \frac{\{F(K_{sub p}) - (1+r)\}}{F(K_{sub g}(\omega hat))} > 0\]

Proof of Proposition 3:

Defining \(\alpha = (1 - \eta)F(K_{g}(\omega hat)) - (1 + r - \eta)K_{g}(\omega hat) - \{F(K_{p}) - (1 + r - \eta)K_{p}\} \equiv 0\) with

\[\alpha = (I - \{1\} over \{eta\})\{``\{``(1 - \eta)\}``e sub u sup c over \{rhd`e sub v`e sub u``}\}\]

the proof is parallel to that for (i) and (ii) of Proposition 1.
Endnotes

1 These figures are from Worldbank (1995), Table A.1, pp. 268-271. For a description of covered government activities, see p. 26.

2 Galal, Jones, Tandon and Vogelsang (1994) present thorough case studies of the generally positive welfare consequences of divestitures in the United Kingdom, Chile, Malaysia and Mexico.

3 Hart, Shleifer and Vishny (1996) present a model where contract incompleteness affects a government's choice whether or not to privatize an activity, with an application to prisons.

4 The assumption that the government has an absolute production advantage in some activities does not affect the analysis.

5 In (9) the constraints on the profit tax rate, \( z \), are ignored.

6 Note that it is possible that \( x < 1 \) with optimally \( z = 0 \). This occurs if the profits that the government obtains from running state firms with \( u = 0, v = 0 \), and \( \omega \hat{=} \omega \) exceed the corresponding optimal provision of public goods, \( G \). In this instance, optimal policy implies \( x < 0, v < 0 \) and \( \omega \hat{=} \omega \), which implies that the state sector is "too small". This somewhat unrealistic case is ignored.

7 Consistent with the proposition, Bhaskar and Khan (1995) provide evidence that privatization of the jute industry in Bangladesh reduces employment with little change in output.

8 Note that (6) and (13) also imply that the marginal product of capital in the public sector, i.e. \( (1-\omega)F'(K_{g}(\omega)) \) for \( \omega \leq \omega \), is less than the marginal product of capital in the private sector, i.e. \( F'(K_{p}) \). This simply reflects that the investment tax, \( v \), distorts the private investment decision.

9 The issue of tariff reform in a small open economy with public production has been considered by Abe (1992).

10 Note that with complete profit taxation, i.e. \( z = 1 \), or no private projects to start with, i.e. infinitely large, the required returns on public and private investments are identical.