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A LINEAR PROGRAMMING MODEL OF THE OUTPUT AND EMPLOYMENT POSSIBILITIES OF THE ECONOMY OF JAMAICA 1965-1975; PRELIMINARY RESULTS

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Introduction

This paper, together with paper No. 7, was prepared as part of a project on Jamaica within the work program of the Division, which is concerned with the use of quantitative techniques as aids for country economic analysis. The choice of Jamaica, made in the consultation with the Area Department, was based on the need to include within the work program an example of a small country. It was also felt that Jamaica was a country in which the Bank was likely to be operationally involved. The project consists of three interrelated parts; (1) a structural model (circulated in draft form in November, 1966) and now under revision. (2) special studies of problems relating to employment, wage levels, productivity and savings behavior of which this paper and paper No. 7 represent the first draft conclusions. (The work on savings behavior will be incorporated in the revised version of the structural model). The problems of employment and wage structure were raised by the Western Hemisphere Department as likely to be crucial areas for consideration in their continuing economic work on Jamaica.

The Jamaican economy has been characterized for some years by a rapid rate of population growth and a high level of unemployment. In any development perspective, therefore, the question of employment opportunities is very important. This paper uses the linear programming technique to explore the structure of the Jamaican economy as it is at present, with particular reference to its employment possibilities, and as it may appear over the next decade. The object is to derive a function that will represent the costs of various choices, open to the economy over the next decade, involving more or less employment possibilities. This function can be said to illustrate the possible "trade-off" between the maximum potential growth in income in Jamaica and the maximum potential increase in employment. It is stressed that the results presented here are from a preliminary formulation of the model based on imperfect statistics available here in the Bank. They should not be interpreted as quantitative predictions, but are intended to indicate possible overall patterns of the development of the structure of the economy. The model will be applied to more up-to-date data as it becomes available through country economic work.

The calculations were performed using the Control Data "Allegro" Linear Programming system. The author wishes to thank Alfred Conrad, Edward Hawkins, Atle Elsaas, and Marshall Hall for their helpful comments and criticisms; he maintains the traditional responsibility for any remaining errors. The author also notes with appreciation the statistical assistance of W.L. van der Valk and Dawn Elvis.

The setting of the problem

During the 1950's, the Jamaican economy, while growing rapidly, experienced substantial emigration (mainly to the U.K.) and, at the same time, continued to suffer from chronic unemployment. With the advent of limitations on emmigration to the U.K. in the early 60's the employment problem became even more serious. Thus it seems that probably the most important question facing the Jamaican government is whether the economy can continue to grow as it has in the past while at the same time absorbing a substantially larger number of people into the labor force. This paper undertakes to explore the potential possibilities in terms of output and employment for the economy over the coming years up to 1975. As 1965 is the most recent year for which we have good data, we look at the entire decade 1965-75. The tentative conclusions seem to indicate that the range of possibilities for increasing employment beyond the pattern that pertains if we simply maximize output, is very limited, and that although the employment problem will be serious the economy will face a more fundamental problem arising from a lack of foreign exchange.

In many instances no conflict need be expected between the objectives of increased output and increased employment. If the output of an economy, or even a sector, arises we expect that employment will rise also and probably in proportion. Thus over a large range of possibilities maximization of one objective will result in the maximization of the other. If, however, we introduce the concept of limited supplies of complementary factors such as foreign exchange, capital, and skilled labor, the goal of increasing output may conflict with that of reducing unemployment.

For example, investment of a limited amount of capital in the mining industry will bring about a high increase in output with only a nominal increase in employment. On the other hand investment in agriculture may employ many more people, but the increases in output will be much smaller than if the same amount had been put into mining. Thus the choice that appears before the planners, given a general scarcity of capital, foreign exchange, and skilled labor, is whether to encourage industries with a high employment to resource ratio, or those with a high output to resource ratio. Certainly the former is a more popular strategy and it carries with it a connotation of a more equitable income distribution, but this may take place only at the cost of substantial amounts of potential output. What we wish to explore in this paper is the nature of the so-called "trade-off" between output and employment; specifically how much choice do the planners really have and what are the costs in terms of foregone potential output of a policy of maximization of employment?

Method of Approach

The first problem is how to analyze the situation. Looking at the past and trying to derive some relevant overall ratios is rather difficult. In the first place the structural changes brought about first by the outmigration of labor and then by the cessation of this activity make it difficult to make any quantitative statements about the relationships of unemployment and investment over the period for which we have good data. Furthermore, looking simply at aggregates gives us no notion of the problem of choice between sectors. In addition there is a problem of identifying the connection between changes in output and employment over the past and particular patterns of resource availability. Moreover there

is no way of telling if a particular change was made in response to any constraint at all.

What we propose to do is to treat the problem in an optimizing fashion, maximizing the two objectives (output and employment) subject to the structure of the economy and the scarcity of factors such as foreign exchange and capital. For this purpose all that is needed from the past are the average relationships that pertain in the economy. Then, given the projections of resources for the future, we can look at the differing sectoral output patterns that result when we shift from one objective to the other, always maintaining the resource constraints. It should be noted at this point that we do not consider labor to be a scarce factor in this particular context. We assume that skilled labor is necessary, but that it is not as scarce as the other factors, and thus never provides a binding constraint to the expansion of the economy. Unskilled labor we treat as a free good in terms of opportunity costs, so that it never imposes a constraint on growth.

The approach can be illustrated by Figure I which shows the choices open to the economy between the output of goods and services and the amount of employment necessary to produce goods and services, given the resources available to the economy. These choices can be visualised in the form of a "production possibility" curve, or frontier; the possibility of any choice at all between the two objectives is represented by drawing the curve ACB sloping downwards from left to right, implying that, with all other factors given, it is only possible to obtain additional employment at the expense of a smaller Gross Domestic Product (and vice versa). It is obvious, however, that employment and output are

highly complementary. To expand output always necessitates some additional labor and an expansion of employment will not take place unless there is some additional output, however small, as a result.

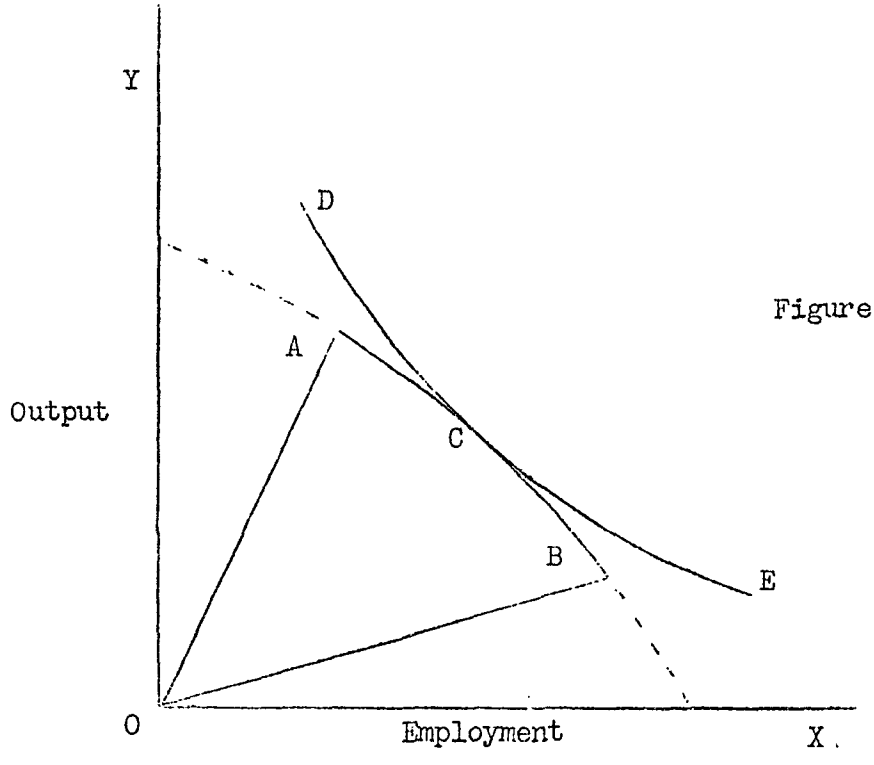

Figure I

In terms of the Figure I this means that the curve ACB will never be continuous over the whole range from the Y to the X axes (the sections shown as broken lines need never be considered). It will never be possible to achieve high outputs with very low levels of employment, or high levels of employment with low levels of output.[1/] There must exist

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1/ The reductio ad absurdum would be the cases where the production possibility curve cut the axes; a point on the Y axis, for example, would imply maximum output, with zero employment.

limits, represented in Fig. I by the lines OA and OB, within which the possible choices for the economy are contained. The line OA, for example, shows the possible expansion of output for the economy when the maximization of output is the sole objective; a movement outwards from O towards A involves relatively large increases in output associated with relatively small increases in employment. Similarly the line OB shows the possible combinations when the maximization of employment is the sole objective. If there is no choice open the two lines OA and OB will coincide and there need be no conflict between the two objectives. The argument of this paper is that there does exist for Jamaica a range of choices notionally represented by a line ACB and an attempt is made to trace out this frontier of possibilities.

It is customary in the pedagogic treatment of this case to go beyond the above argument to try and establish how the choice might be made once the curve ACB is established. This is done by postulating a preference function for the economy showing the preferred combinations of output and employment at all possible combinations of the two; this function can then be represented by a set of community indifference curves (of which DCE in Fig. I is one such curve). The best position for the economy will then be a point such as C, where the production possibility curve is tangential to the highest possible community indifference curve; at C the rate at which the economy can balance the twin objectives, given all its resources, coincides with the relative preferences of the community. It is generally agreed that it may not be possible (even in principle) to estimate a community preference function and this paper does not enter

into that question. It has the more limited aim of mapping out the shape of ACB, thus providing all the potential choices for the consideration of the authorities charged by the community with the task of making the necessary policy decisions.

The problem, therefore, is how to solve for the various points along AB. If the economy was of one or a few sectors the solution could easily be obtained either by a classical maximization, subject to constraints, or by an exhaustive enumeration of the various possibilities. However, if we wish to look at the problem on a multisectoral basis (which we must in order to get the effects of differences in sectoral factor and output intensities) we must use some form of programming model. In this case we choose to make linear assumptions about the economy and thus to take advantage of the technique of linear programming. It will be readily admitted that such a process may introduce distortions into the model as a representation of the economy, but the power of the technique over alternate methods is so great as to override most of the objections so long as we are always fully aware of the limitations of the assumptions made. In addition at a later stage of refinement, we shall relax the assumption to introduce some non-linearities.

Data

In order to make such a model we need certain pieces of data, foremost of which is an input-output table. Such a table exists for Jamaica for the year 1958, and although in general a rather unsatisfactory piece of work[1/] we have projected it forward to 1965 and have adapted it to our

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1/ See Note on the input-output table in Annex 1.

model. Most of the rest of the information used comes from the industrial surveys of 1954, 1960 and 1963[1/] and from the national income accounts for the period 1959-65. Finally some employment data was taken from the employment surveys and from the census of population.[2/] In general there are a number of inadequacies in this data and further research on the model will initially entail the use of more up-to-date information.

The Model

Having described the background to the model and the problem we are seeking to solve, we proceed to describe the equations that make up the system. The first consideration is the items that are maximized, specifically output and employment. If we let Z stand for some measure of national welfare[3/] then at maximization of the relative increase in output,[4/]

$$(1a) \quad Z=(Q-Q_0)/Q_0$$

where Q is output in time t and Q_0 is output at time zero. When we are at maximization of the relative increase in employment then

$$(1b) \quad Z=(L-L_0)/L_0$$

1/ Report on a Survey of Establishments, 1954, 1960, 1963. Department of Statistics, Kingston, Jamaica.

2/ Annual Abstract of Statistics 1966, No. 25. pp. 126-131. Department of Statistics, Kingston, Jamaica. O.C. Francis, The People of Modern Jamaica, Department of Statistics, Kingston, Jamaica 1963.

3/ It should be stressed that although we are dealing in actual units, Z is an ordinal variable. We are interested in the ability to say one state is preferred to another, and do not intend any judgment as to the absolute magnitudes involved.

4/ In actual running of the model we have used consumption instead of output. The difference is conceptual and computational and it does not affect the results.

where L stands for employment, and the subscript as before refers to the base period. At some combination of the two objectives,

$$(1c) \quad Z = a(Q-Q_0)/Q_0 + b(L-L_0)/L_0$$

where a + b=1, as we can normalize Z without any loss of ordinality. Furthermore, we can use the fact that we only need an ordinal measure to state,[1/]

$$(1) \quad Z = aQ + b\,(Q_0/L_0)L$$

which will be the objective function (the function which is maximized) in our model. As we vary "a" from 1 to 0 subject to the normalization constraint we will in effect be moving the point C in Figure I along the possibilities curve from A to B.

The next step is the structural equations of the economy. We have divided the economy up into twelve sectors following partly the input-output table and partly the availability of other data. The sectors are: 1) Sugar--this includes both the growing of cane and the manufacture of sugar, as well as the manufacture of rum. 2) Agriculture--covers the rest of agriculture including both subsistence and market production whether or not for export. 3) Mining--this is mainly the bauxite mines, but also includes gypsum and quarrying. 4) Construction--this includes all capital formation and maintenance. 5-8) Manufacturing sectors covering food,

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1/ $Z = aQ/Q_0 - a + bL/L_0 - b$. Since $a+b=1$, $Z = aQ/Q_0 + bL/L_0 - 1$, multiplying by Q_0 and rearranging,

$$ZQ_0 + Q_0 = aQ + b(Q_0/L_0)L$$

and since we are only looking for an ordinal measure and Q_0 is a constant, we can ignore it on the left hand side.

clothing and textiles, heavy industry, and miscellaneous. 9) utilities 10) distribution, 11) transport 12) services--including ownership of dwellings, but not including administration of government. In each sector we state,

(2a) Supply $_i$ = Demand $_i$

and this relationship is stated mathematically as;

$$(2) \quad X_i = C_i + I_i + N_i + E_i + G_i + \sum_j a_{ij}X_j, \; i = 1 \ldots 12$$

Where X stands for production, (gross output or deliveries from sector i), C for consumption, I for investment goods, N for inventories, E for exports, G for government, and the final term for the interindustry uses of the output from industry i. The coefficients a_{ij} are from the input table and simply state the amount of input required from the ith industry for a unit of output in the jth industry.[1/]

Now we need to relate X (gross output) to Q (net output). If we let M_i stand for the ratio of imports for intermediate use in the ith sector to the gross output in that sector, then

$$(3) \quad Q_i = X_i \left(1 - \sum_j a_{ji} - m_i\right)$$

and summing up,

1/ It should be made clear that the demands for goods in this model are highly structured and thus output in a particular sector is limited by this structure rather than by factor availabilities, although overall output is factor determined. There are no activities, other than linear programming slacks, that allow unlimited disposals of sector output.

$$(4) \quad Q = \sum_i Q_i + V_g$$

where V_g is value added in government.

Both the government and the export sectors are exogenous to the system we are considering. As a good forecast of the probable activity in these sectors we have used estimates contained, or implied, in the most recent Bank economic report (WH-154a-December 1965). Although most of the analysis contained therein is with regard to 1970, we have projected the trends through to 1975.

We project government activity to grow at about 5.2% per annum and thus total government expenditure G is estimated to be at a level of £59.0 million in 1975. Of this £35.78 million will be value added (V_g) and about £15 million will be demand for goods and services to be purchased from the domestic economy. This demand (G_d) is broken down as follows;

$$(5) \quad G_i = g_i Gd$$

where the coefficients g_i reflect the distribution of demand in the base period; thus we assume that there will be no change in the nature of the government demand over the ten year period.

In the case of exports, we have made detailed projections of most of the sectoral exports. Sugar, as a result of the Commonwealth sugar agreement, is not expected to earn more than £22 million in 1975 having earned £16 million in 1965. However, this is a demand limitation, it may very well happen that the economy will not want to export this much (or may not be able to) but we feel that it will export at least £19 million worth. Thus (in £ million)

(6) $19 \leq E_1 \leq 22$

Similarly for exports of food agricultural products,

(7) $14 \leq E_2 \leq 16.5$

where the level in the base period was 11.5. Mining also faces limitations as the amount that can be exported depends on the capacity of the foreign owned facilities for bauxite. The level of such exports was £36 million in 1965; we expect in 1975,

(8) $45 \leq E_3 \leq 50$

Sectors 4 (construction) and 9 (utilities) cannot export their output, and the distributive sectors (10-12) export indirectly. Sectors 5-8, the manufacturing sectors exported about £7 million in 1965. We expect these exports to grow at about 7% per annum[1/] so that in 1975,

(9) $18 \leq E_{5-8} \leq 22.5$,

the distribution of this amount over the four sectors being assumed to be the same as in the base period. The largest export prospect will, of course, be tourism. From a level of £28 million in 1966, we expect a growth of 10% per annum to about a level of £66 million. Thus where

(10) $44.5 \leq E_T \leq 65.9$

1/ This may be slightly on the high side. For example, Jamaica has just concluded a textile agreement with the U.S. (the major market for Jamaican textile exports) which specifies a growth of 5% per annum.

the lower level is, as above intended to allow for the possibility that this type of export is not as favorable as others, or a limitation of capacity. This export activity is divided among the three distributive sectors (transport, services, and distribution) in the same ratio as the base period. Finally there is a miscellaneous export category (consisting mainly of distributive margins) that amounted to 11.5% of the value of physical exports in the base year. We have assumed this proportion to remain during the decade and also that the distribution of this export item over sectors 10-12 will not change.

The export picture, when viewed in the aggregate, is not very promising. The limitations on the exports of sectors 1-3 coupled with the relatively small amount of those from sectors 5-8 means that, taken at the maximum level exports will only grow at about 4% per annum whereas in the past (1960-65) they have grown at close to 7% per annum.

Consumption is determined from a savings equation which states that marginal consumption (public and private combined) must be at least a given percentage of marginal value added, 1/ thus

$$(11) \quad C-C_0 + G-G_0 \geq r(Q-Q_0)$$

1/ What we are saying here is that consumption is determined at the margin in a behavioral fashion. Consumption behavior then determines savings, at the margin, as a residual. (We feel there is strong reason to believe that consumption behavior is quite stable over a number of years). Thus equation (11) creates a capital scarcity by not allowing investment to increase at the expense of consumption, particularly when this implies a deviation from past trends. We have, of course, built into "r" an allowance for institutional increases in savings such as taxes and savings campaigns.

where r is the national marginal propensity to consume. From this aggregate level of consumption we can go to private consumption (since C_o, G_o, and G are all exogenous to, the system) once we have the level of Q. This total private consumption we allocate among the twelve sectors in the same proportions that pertained in the base period.[1/] [2/]

$$(12) \quad C_i = c_i C$$

Inventories are handled in a simple fashion, it being assumed that there is a constant inventory to production ratio for each industry, the ratios being taken from the original input-output table. Thus

$$(13) \quad N_i = n_i X_i$$

where n_i is the inventory to production ratio in the ith industry.

Finally we must explain the treatment of investment. As a result of the particular way in which the input-output table was constructed, there is an assumption that only the construction industry supplies capital goods, thus total fixed investment If is;

$$(14) \quad I_f = I_4 \text{ and } I_i = 0, \; i \neq 4$$

1/ As the result of the particular nature of the input-output table, imports for consumption are channelled through the distribution sector instead of being charged directly to consumption. In contrast, government imports are directly identified.

2/ This, of course, restricts the consumer to the constant proportions of various goods in aggregate consumption over the whole time period and raises a number of questions both with respect to income (Engel's curves) and also prices. Constant base year proportions are, however, a rough approximation to low income consumer behavior and to refine the treatment would require approximations of non-linear functions to a certain extent and this must remain until a later stage of the analysis.

This, of course, must in turn be equal to the sum of the demand by the economy for investment goods. If we call the demand in a particular sector V_i then

$$(15)\ \Sigma V_i = I_f$$

and the next problem is to find how much investment each sector needs. If we assume that each sector has a constant incremental capital-to-output ratio, then the total amount of sectoral capital needed over the ten year period will be a direct multiple of the change in sectoral output between 1965 and 1975. However, the solution to the model looks only at the flows in the final year, 1975, so that we need some method of relating the total investment over the period to the investment in the last year. If we assume that investment increases in a monotonic fashion it can be shown that an appropriate stock to flow conversion factor might be 0.2 1/. Thus

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1/ Let us assume that investment increases in a steady fashion, then

$$I_t = I_o e^{rt}$$

where r is the rate of growth of investment. The total investment over the ten year period is thus,

$$I_t = \frac{I_o(e^{10r}-1)}{r}$$

and the ratio of the final year investment to the total is;

$$s = \frac{r}{1-e^{10r}}$$

If we let r=16% per annum then w=0.20. This is just an approximate factor; it may be that 16% is too high and that thus w ought to be less than 0.2. However, all this would do would be to put a smaller percentage of the total investment in the terminal period, and thus allow the model to get more total capital over the entire period. This would not affect our results materially as we have seen that the foreign exchange constraint is the most important factor limitation.

(16) $V_i = 0.2\ k_i(Q_i - Q_{io})$

where k_i is the incremental capital-to-output ratio in the ith sector.[1/]

The above set of equations (3-16) describes the variables that form a part of equation 2. The next step is to describe the other equations that represent the economy.

Imports are related to sectoral production and to various external factors,

(17) $M = \Sigma\ m_i X_i + FIP + T + M_g$,

where M is total imports and m_i is the ratio of imports for intermediate use to total production in sector i. FIP and T are factor income payments and transfers and since these have not been included in the export equations we state them in equation (17) as net figures; they are in any case exogenous to the model. M_g represents imports by the government.

In the model we have treated imports as being entirely noncompetitive. That is to say we do not allow the economy to produce goods that it is currently importing; all it can do is to increase the output in the activities in which it is currently engaged. To treat the competitive

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1/ The way in which this equation is set up allows the model to use a negative value for V_i and thus there is a possibility that the model will "run backwards" and convert already fixed capital in one sector into capital in another sector. In order to prevent this we have added an additional set of constraints to the effect that;

$$X_i \geq 0,\ i=1...12$$

in actual point of fact this set of constraints is never needed in our model.

import problem we would have to go to an import substitution model complete with hypothetical production function vectors. Such a model can become quite complicated and thus we defer it to the future.

Having now the import and export equations we can express the constraint of foreign exchange with an equation,

$$(18) \quad E_i + E_o + E_T + F \geq M$$

where F is the expected inflow on capital account (i.e. the allowable imbalance on current account). We have tentatively set F at £9 million. The inequality of (18) is in order that foreign exchange be a constraint that binds only as long as imports try to exceed exports. If it happens that the earnings from exports plus the capital inflow are more than desired imports then foreign exchange is not a constraint on the system.

We summarize the foreign exchange situation as follows. The crucial equation is (18) and it is in terms of the value of an increase in F (exchange without a current cost) that we will be able to determine the scarcity of foreign exchange. Imports are in most cases fixed proportions of sectoral output and thus can only be "saved" by shifting output patterns. Exports are all limited by factors assumed to be related to external demand conditions, however, we have included lower levels to prevent the complete removal of the export activity (as the result of the nature of the linear programming system) should such activity become too costly in terms of output and employment gained per unit of factor expenditure. This case of export activities going to lower bounds should only occur when factors other than foreign exchange serve as the binding constraints on the system.

Finally the amount of employment must be related to the levels of production,

$$(19) \quad L = \Sigma\, l_i\, X_i$$

where l_i is the employment to production ratio for the ith industry. Here again we assume the ratio to be constant over the ten year period.

Equations (1-19) complete the basic system. (A tableau of the system is shown in Figure II; this is mainly for the purpose of arranging the information for coding on the computer, but is also useful for an overall visualization). This system has a total of 27 "activities" and a total of 49 possible constraints. In the programming framework the solution will be such that there is one constraint for each activity and such that the value of Z in equation (1) is maximized. The basic problem is to determine levels of each of the activities at this maximum point and to find how these levels change when the coefficients of equation (1) (a and b) are varied.

Results

The table shows a set of preliminary results from subjecting the system described above to a linear programming process. Four separate results are shown (cases 1-4), based on different assumptions. The first three are with a marginal propensity to consume of 0.85, while the fourth one uses a figure of 0.80. The first and fourth results place the emphasis entirely on increasing output, while the third emphasises only employment. The second set of results is a case where equal emphasis is given to both objectives, (a=b=0.5). For purposes of comparison the initial

LINEAR PROGRAMMING TABLEAU

	X_1 ---------- X_{12}	I_f	I_n	L	Q	C	E_1	E_2	E_3	E_5	E_6	E_7	E_8	E_T	E_0	M		R.H.S.
				$b\frac{Q_0}{L_0}$	a													
Max X_i	[I-A-N] Matrix	1.0				C_1 ... C_{12}	1.0	1.0	1.0	1.0	1.0	1.0	1.0	E_{T10} ... E_{T12}	E_{o10} ... E_{o12}		$\leq$	$-C_i$
Minimum X_i	1.0 ... 1.0 ... 1.0																$\geq$	X_{10} ... X_{120}
Minimum E_i							1.0 ... 1.0 ... 1.0										$\geq$	E_{1L} ... E_{TL}
Maximum E_i							1.0 ... 1.0 ... 1.0										$\leq$	E_{1H} ... E_{TH}
Labor	ℓ_i			-1.0													=	Lg
Import	m_i															-1.0	=	-Mg-FIP-T
Investment	$0.2K_i$	-1.0															=	$(0.2\Sigma K_i X_{io})$
Inventory	n_i		-1.0														=	0
Output	$(1-\Sigma a_{ij}-m_i)$				-1.0												=	-Vg
Savings					r	-1.0											$\leq$	$(r\mathcal{C}_b+G-G_o-C_o)$
E_o							e_{10} ---------- e_{80}								-1.0		=	0
Foreign Exchange							1.0 ---------- 1.0									-1.0	$\geq$	F

IBRD-3585

FIGURE II

JAMAICA: Results of Linear Programming (all values in £000)

	1	2	3	4	Initial Level (1965)	Growth rate 1965-75 (% p.a.) 1/
Sugar	33,710	33,682	33,668	33,978	23,568	3.7
Agriculture	57,026	59,292	59,410	60,644	47,175	2.5
Mining	54,637	51,396	49,568	54,824	35,116	4.6
Construction	97,510	97,048	96,896	107,849	76,523	3.5
Mfg. 1	65,138	64,971	64,912	66,439	46,978	3.5
Mfg. 2	15,160	15,169	16,391	16,702	9,502	5.8
Mfg. 3	27,376	26,514	27,295	28,882	19,197	4.2
Mfg. 4	13,701	13,684	13,680	13,958	9,393	4.0
Utilities	7,112	7,104	7,116	7,288	5,257	3.3
Distribution	265,508	265,126	264,850	273,432	203,009	3.0
Transport	55,887	55,670	55,510	57,185	40,951	3.4
Services	121,934	122,655	122,497	124,715	82,972	4.2
I. Fixed	72,334	72,096	72,041	82,088	58,620	3.4
Emp. 2/	1,409,895	1,426,397	1,428,119	1,468,820	1,027,396	3.6
Output	411,897	409,983	409,219	425,278	311,779	3.2

Cases:
1. Output alone maximized - marginal savings rate= 0.15.
2. Output and employment both maximized (equal weights)-marginal savings rate= 0.15.
3. Employment alone maximized - marginal savings rate= 0.15.
4. Output alone maximized - marginal savings rate= 0.20.

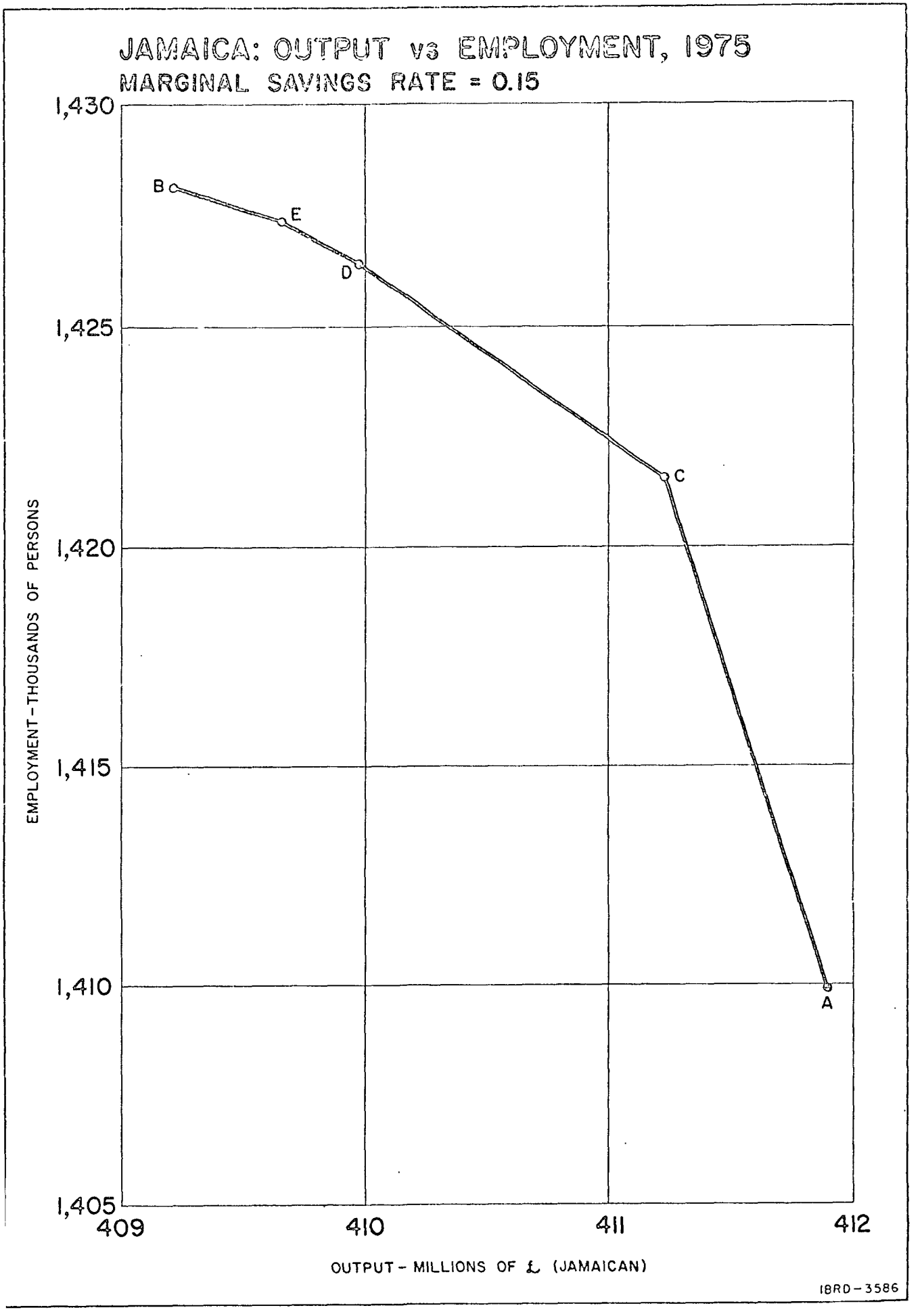
1/ Growth from base period to 1975 case 4 (Savings rate = 0.20).
2/ Number of people employed.

levels (1965) of the variables are presented, as are the implied annual compound growth rates from the 1965 level to the 1975 case 4, where, with a marginal propensity to consume of 0.8, the marginal national savings rate is 0.20. Case 4 was chosen for the comparison as it represents the maximum amount of growth that can be had from the economy; the scarcity of foreign exchange binds the economy to a single solution - a shift of emphasis to employment does not change the results.

The rows of the table show the output (production) levels of each sector, as well as the level of fixed investment, employment, and output. It should be noted in this respect that the employment includes agricultural employment. (However, if the model is restated so as to look only at non-agricultural employment, the results, as far as sectoral output patterns are concerned, are the same.) We should make clear what distinguishes cases 1-3 from case 4. In the first three the crucial constraint is capital, (resulting from setting the marginal savings rate at 0.15) and the segments of the curve are the result of alternate ways in which the foreign exchange earned can be used to produce greater or lesser amounts of output and employment. Case 4 is a relaxation of the savings constraint to 0.20, and we then find that foreign exchange is the crucial constraint. In these circumstances there is only one possible optimal point.

Figure III shows the transformation curve (ACB in figure I) that we have been looking for. It shows five different points representing the three presented in the table for a savings rate of 0.15, and two others not presented which are in between (a=0.55, a=0.35). As can be seen the curve is the shape that we expected, but what is surprising is how small a difference it makes whether one is at maximum employment or at

FIGURE III


JAMAICA: OUTPUT vs EMPLOYMENT, 1975
MARGINAL SAVINGS RATE = 0.15
1,430
1,425
1,420
1,415
1,410
1,405
EMPLOYMENT-THOUSANDS OF PERSONS
B
E
D
C
A
409
410
411
412
OUTPUT - MILLIONS OF £ (JAMAICAN)
IBRD-3586

maximum output. The total trade-off between case 1 and case 3 (and thus between employment and output or between points A and B in figure I) is £147 of output lost per additional person employed, which is equivalent to an elasticity of about 2.0 (i.e. for a 1% loss in potential output we can get a 2% increase in employment). However, the total difference in employment possible is only slightly over 1%. Furthermore, if we look just at the first increment (between points A and C) the "cost" of a job is only £57 and the elasticity is over 5, but the difference is only 0.5%.

The reason the range of possibilities is so small is that the constraints that bind the system to a maximum level are not too far removed from the initial level. Thus none of the many possible growth paths have much latitude to change the patterns of the base period. This means that the range of possibilities, when viewed from the terminal period, show very little variation.

It is interesting to look at the mechanism which causes the segments. At all points capital and foreign exchange are scarce factors. However, as we move from A to B different sectoral exports as a source of foreign exchange move from their upper bounds to their lower bounds thus indicating their relative profitabilities as earners of foreign exchange. An activity at a lower bound is an indication of a greater cost (in terms of capital) to produce the good for export than to use the same capital to make consumption and employment. Similarly at the upper bound, the model gains more from the export of the good and the use of the resulting foreign exchange to make consumption and employment than from using the capital to make these directly. At point A where the emphasis is on

output and not at all on employment, exports of agriculture, tourism, and food and clothing are inferior activities (they are all labor intensive) and are all at their lower levels; all other export activities are at upper levels meaning they are favored modes of earning exchange. This situation lasts until we move to point C where Agriculture becomes worthwhile and exports of heavy manufactures drops out. Then at point D, mining (being not at all, labor intensive) drops out and tourism (which uses a lot of labor) becomes a worthwhile export. Point E brings heavy manufacturing back in, (as the stress is placed on anything that can employ people) and finally to get to point B the exports of food and clothing become a good way to employ people and earn foreign exchange. If we look at the overall change from A to B we find the three exports that were not favored at A(agriculture, food and clothing, and tourism) as they are labor using, are all favored at B while the export of mining which uses very little labor has dropped out.

Case 4 shows the results when we relax the savings constraint by a large amount. This was done by dropping the marginal consumption rate down to 0.80, and in fact the model is only able to get to a marginal rate of 0.83, (equivalent to a marginal savings rate of 0.17) at which point it is constrained from saving (and thus investing and thus growing) any more by the limits of foreign exchange. If we look at the last column of the table it can be seen that this is equivalent to a rate of growth of only 3.2% per annum, which is low compared to the rate of 5.2% (in constant prices) experienced over the period 1960-65. However as mentioned above, this growth rate can be traced directly to the growth of exports which in the 60-65 period was 7% and in the 65-75 period will be 4%.

In point of fact, the use of the optimizing process has pointed the way for a slightly more efficient use of foreign exchange; in the past it has taken about a 1.3% growth in exports for a 1% growth in output, in the model this ratio has been dropped to a 1.2% in exports for every 1% in output.

It is interesting to calculate the "shadow price" of foreign exchange in case 4, where the only maximand is consumption. The shadow price in this instance is defined as the amount of additional consumption that will be generated by the system per additional unit of foreign exchange. It turns out that £1 of foreign exchange is worth £1.5 in consumption in the model.

As for the problem of employment, it is very difficult to get a precise idea of the past trends in this field because the coverage in employment surveys has increased from year to year. It does seem, however, that the annual increase has been on the order of 2-3% which is somewhat lower than the 3.6% attainable by the model over the projection period. On the basis of this rough estimate we conclude that the employment problem, although important, may not be quite as serious as the foreign exchange question.

Significance of the preliminary results

The work described above is only the first stage of this study. Its purpose has been to demonstrate that the question of the relationship and possible conflict between employment and output can be handled in a linear programming, optimization framework.

It has been shown that there is indeed a trade-off between a maximum rate of growth of employment and a maximum rate of growth of output. However, the range of allowable variation is fairly small. We feel that this is perhaps an indication of an impending problem in the area of foreign exchange, however, at this stage in the analysis it cannot be taken as a prediction. The present analysis shows the supply of foreign exchange to be a binding constraint on the potential growth of the economy; this may be due to the structure of the economy, but on the other hand it may well be the result of the manner in which the model represents the economy. In the first place by assumption, we have allowed the economy no room for explicit import substitution, which may provide some relaxation of the foreign exchange constraint. Another limitation of the model is it uses average rather than marginal import coefficients, which could result in an overstatement of the import requirements of the system. Thus an obvious next step is to refine the import functions through the use of appropriate econometric statistical analysis and incorporate the results into the model.

As was pointed out earlier, the model as now stated, does not allow competitive import activities. To the extent that the economy uses less foreign exchange per unit of growth, the model has described a certain amount of import substitution. This does not occur in a particular sector as the result of the changing of an import coefficient, but rather in the economy as a whole as the result of a changed sector mix. We are implying here that were it not for the foreign exchange limit, the substitution process could be carried even further. To incorporate

explicit import substitution behavior into the model would require a formulation of some complexity. This, however, is one of the more obvious directions in which future versions of the model might move.

A further refinement would be to look at the various types of labor and attempt to define skilled labor as a possible binding constraint; such an analysis might well show that the employment problem will be relatively more important than it appears to be in this version of the model. With defined skill limits the problem might be approached by expanding the economy (as represented by the model) to these limits and then recasting the system as a problem in the minimization of the use of foreign exchange.

Another improvement would be to define savings in terms of its functional components, so as to get a better representation of consumption and capital formation. Finally, the consumption function can be adjusted in a quasi-non-linear fashion so as to reflect more nearly actual consumer behavior, and possibly some of the production functions can be modified to account for non-linearities.

Conclusions

The object of the present work was to examine in a quantitative manner some of the issues that appear to be of major significance in any continuing economic work on Jamaica. It has been customary in that work to pay special attention to the employment problem, given the rate of growth of the population, the limitations on emmigration and the already existing burden of unemployment. On the other hand, Jamaica has not, so far, been considered as an economy likely to suffer from a severe foreign exchange constraint. The present analysis gives some ground for a

reconsideration of both these presuppositions, by providing a clearer picture of the relationship of employment and output and of the dependence of both upon factor availabilities.

It should be stressed that these results have been obtained with the use of data that is deficient in many respects. Better information is known to exist and it should be possible, within the framework of normal country economic work, to improve the model by incorporating it in an improved version. The main object has been to demonstrate the possible value of a technique as an aid to country economic analysis. The results obtained throw some doubt on widely held views as to the role of key factors in the economic development of Jamaica. The next step should be to test out how far such results depend either on the assumptions of the present model, or on the limited and imperfect data available with which to test it. The technique demonstrated here provides a way of handling these interrelated aspects of economic development that is not easily done through more conventional methods of country analysis.

It should enable us to get a clearer idea of the manner in which the economy can develop so as to make best use of its resources and so as to deal with the probable pattern of external events over the next decade.

ANNEX I

A note on the input-output table used in this paper

The input-output table has been adapted from that published in: "Long-term Growth of the Economy of Jamaica" by C.O'Loughlin, Social and Economic Studies, Volume 12, No. 3, September 1963 pp. 246-282. The article contains matrices for 1958, 1965, 1970, and 1975. The last three are projections and are highly conjectural. We have used the 1958 matrix and made suitable adjustments to 1965 in the light of actual data. The biggest problem with this matrix is that it has been put together in such a fashion as to reflect the paucity of explicit data available. This means that there is a large amount of information, normally inferred in the process of building such a table, that has been left out of the analysis. Thus for example, instead of making some kind of general assumption about distributive margins thereby allowing goods to be identified by the producing sector with margins from the distributive sectors, in most of the cases goods are channelled through the distribution sector so all source information is lost. This puts a much heavier burden on the distributive sectors than one would like and subjects us to restrictive assumptions about the composition of the sources of the flows. The same type of thing occurs with investment where everything is channelled through the construction sector. Also imports are almost all treated as going through the distributive sector, in the case of capital equipment the goods enter distribution and then go through construction before appearing in investment--in an ideal table these goods would appear directly as imports

for capital formation. These objections to the input-output table are substantial, but are not enough to make us undertake the very burdensome job of making one ourselves. It is hoped that we can obtain an up-to-date matrix that is reported to exist in the Ministry of Planning.