Indexation and Inflationary Inertia: Brazil 1964–1985

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Some governments facing high rates of inflation have adopted wage and exchange rate indexation in an attempt to offset the costs to workers and exporters. But indexation based on past values of inflation is considered to be a powerful source of current and future inflation that undermines fiscal and monetary stringency designed to reduce inflation. Brazil indexed wages and the exchange rate for more than twenty years, and we examine Brazilian evidence on these issues. We derive a transfer function and estimate time-varying parameters to allow for the changes in indexing policy over the period. We find that the increased frequency of wage and exchange rate adjustment amplified the effect of past inflation and made it less sensitive to monetary and fiscal policies and more vulnerable to domestic agricultural supply shocks. Indexing did not eliminate the effectiveness of monetary policy, however, which retained a significant effect throughout the indexation period.

This article analyzes the effect of wage and exchange rate indexing on inflationary adjustment and examines how the changes in indexation affect the inflationary process and the responsiveness of inflation to monetary correction. The critical issue is the extent of feedback from indexation to inflation. Brazil is the largest country in the world undergoing sustained periods of high inflation, and it has had more than twenty years of wage indexation, which changed in important ways over time. We thus use the Brazilian experience to examine these relationships.

Economists instrumental in the design of Brazil's 1986 stabilization plan, the Plan Cruzado, claimed that the feedback process was so strong in Brazil that it dominated the effects of excess demand factors on inflation and created inertial inflation (Lopes 1986, Arida and Lara-Resende 1985). This inertial inflation, it is claimed, makes monetary and fiscal policies less effective, requiring greater...
monetary-fiscal cuts to reduce inflation by a given amount, and it increases the
sensitivity of inflation to foreign price and supply shocks. Arida and Lara-
Resende thus advocated abolition of the indexing system as a "necessary con-
dition" for successful stabilization in succeeding stages with monetary-fiscal
correction (1985, p. 43).

"Heterodox shocks" designed to account for inertial inflation typically in-
clude price freezes, as opposed to the more orthodox stabilization programs
which disdain their use (see Kiguel and Liviatan 1988 for an analysis of "ortho-
dox" stabilization policies in Latin America). Dornbusch and Simonsen (1986)
have argued that the heterodox approach may provide a valuable "breathing
spell," in which stable prices can usher in political support for making the more
difficult fiscal corrections.

The temptation to mistake the breathing spell for success is enormous, how-
ever, as Dornbusch and Simonsen acknowledge, and the fundamental fiscal
adjustments may never occur. This is what happened in Brazil. By the end of
1987 the annual inflation rate in Brazil was more than 300 percent, and by
mid-1989 it was more than 1000 percent. The resurgence of inflation suggests
that policymakers in Brazil underestimated the importance of monetary-fiscal
correction during the disindexation phase of the stabilization program.

In this article we analyze these issues using a model of industrial and agricul-
tural price dynamics, which incorporates indexing rules for wages and the
exchange rate. A transfer function for inflation, similar to a reduced form, is
derived from this structural model. The transfer function, however, relates an
endogenous variable (inflation in this case) to its own lagged variables and to
the current and lagged exogenous variables of the model. This permits us to
analyze the feedback effects of past inflation upon itself and to evaluate the
effects of current and past policy variables. Other lagged endogenous variables
do not appear in the equation. By comparison, a reduced form equation relates
an endogenous variable to all the predetermined variables of the model, includ-
ing the lagged endogenous and exogenous variables.

In order to trace the relation of indexing to inflation accurately, it is neces-
sary to account for the changes in indexing rules that occur over the period of
observation. We do not evaluate the transition from a free to indexed system—
both the inflationary process and the system of government and private sector
indexing evolved over time. But the government's official nationwide wage
indexing process was adopted in 1965 and revised from an annual to semestral
or biannual basis in 1979, whereas the system of exchange rate indexation was
temporarily suspended from December 1979 to the second semester of 1980.
Thus estimation under the conventional assumption of constant coefficients is
not appropriate. Kalman filtering is used here to obtain estimates of these time-
varying parameters of the transfer function. Previous studies of Brazilian infla-
tion which supported the inertial inflation hypothesis assumed constant coeffi-
cients in their estimating equations (see, for example, Lara-Resende and Lopes
The following section contains a description of the model and a derivation of the transfer function for inflation. Section II presents the results of ordinary least squares estimation, describes the Kalman filtering method, and evaluates the time-varying parameter estimates in terms of their significance for indexing policy. The policy implications of our findings are discussed in the conclusion.

I. THE MODEL: ACCOUNTING FOR FEEDBACK AND PARAMETER CHANGES

Many factors contribute to the inflationary process; our model accounts for industrial and agricultural price dynamics, indexing rules for wages and the exchange rate, the domestic interest rate, and money demand and growth of output (see Barbosa 1987 for the original development of the model). The transfer function for overall inflation is derived from the relationships of this model. Overall inflation, \( \dot{p} \), is a linear combination of industrial and agricultural inflation rates:

\[
\dot{p} = (1 - \alpha) \dot{p}_i + \alpha \dot{p}_a
\]

where \( P, P_i, \) and \( P_a \) are the indexes for the overall price level, the industrial price level, and the agricultural price level. Upper-case letters denote the actual values of each variable, while lower-case letters denote logarithmic values. The symbol "\(^\prime\)" over a lower-case letter denotes the rate of change of the variable. The weights of the two price indexes in the overall price index are denoted by \( \alpha \), a constant parameter.

Agricultural and industrial inflation rates differ when growth in agricultural output, \( \hat{y}_A \), varies from its trend, \( \hat{y}_A \):

\[
\dot{p}_a - \dot{p}_i = \Theta(\hat{y}_A - \hat{y}_A) = -\Theta A
\]

where \( A \) is the deviation of the actual agricultural growth rate from its trend (that is, agricultural supply shocks). The constant reflects the effect of this agricultural output gap on the relative prices of agricultural and industrial output. This equation is neither a structural demand nor supply equation but an equilibrium relationship between two endogenous variables in the agricultural sector.

Industrial price changes respond to changes in nominal wages, \( \hat{w} \), net of productivity growth, \( q \), and the external costs of imported raw materials, \( P_i^* \) (in local currency):

\[
\dot{p}_i = \tau(\hat{w} - q) + (1 - \tau)\dot{p}_i^*
\]

where \( \tau \) is a constant parameter indicating the relative weights of labor and imported raw material costs in the pricing of industrial output, and \( \dot{p}_i^* \) is the sum of the rate of change of the exchange rate and world price of raw materials:

\[
\dot{p}_i^* = \dot{e}_i + \dot{p}_{mt}
\]
The rate of change of the nominal exchange rate, \( \dot{e}_t \), is given in terms of domestic over foreign currency, and \( \dot{p}_M \) is the rate of change of the foreign price of raw materials.

The rate of change of the exchange rate is determined by the indexing policy of the government and is linked to the difference between domestic and foreign inflation:

\[
(5) \quad \dot{e}_t = \Phi_c (\dot{p}_t - \dot{p}_M) + \varepsilon_t
\]

If the government follows a purchasing-power-parity rule, then \( \Phi_c = 1 \). If instead it follows a policy of permitting the exchange rate to appreciate in real terms, then \( 0 < \Phi_c < 1 \). The symbol \( \dot{p}_f \) represents the foreign inflation rate, and \( \varepsilon \) represents exogenous shifts in the rate of depreciation of the domestic currency. This term reflects the occurrence of maxidevaluations—changes in the exchange rate not related in a continuous way to differences between domestic and foreign inflation.

The rate of change of wages is a function of government indexing policy and excess capacity. Wages adjust in the following way:

\[
(6) \quad \dot{w}_t = \sum_{i=1}^{n} \Phi_{ui} \dot{p}_{t-i} - \beta h_t + \bar{w}, \quad n = 4 \text{ (annual adjustment indexing)}
\]
\[
= 2 \text{ (semestral adjustment indexing)}
\]

The parameters \( \Phi_{ui} \) are the policy-determined indexing parameters for wages, with \( 0 \leq \Phi_{ui} \leq 1 \). From 1965 to 1979, wage indexation was implemented on an annual basis. Thus using quarterly data, we would expect a fourth-order lag (going back one year) to be appropriate, and we set \( n = 4 \) under this regime. After the 1979 change to semiannual indexing lags beyond two quarters may become less important, and we use \( n = 2 \). The symbol \( h \) represents the level of excess capacity in the economy, and \( \bar{w} \) is an exogenous component affecting wage growth.

The level of excess capacity, \( h_t \), changes with deviations of actual output growth, \( \hat{y}_t \), from its constant trend \( \hat{y} \):

\[
(7) \quad h_t - h_{t-1} = - (\hat{y}_t - \hat{y})
\]

Combining equations 1 through 6, we obtain the following supply or cost-determined inflation equation:

\[
(8) \quad \dot{p}_t = \tau (\bar{w} - q) + \tau \sum_{i=1}^{n} \Phi_{ui} (\dot{p}_{t-i}) + (1-\tau) (\dot{p}_M + \Phi_c \dot{p}_f + \varepsilon_t)
\]
\[
- \sigma OA_t - \beta \bar{w} h_t [1-(1-\tau) \Phi_c]^{-1}
\]

This equation models supply-side inflation as a positive function of wage growth net of productivity increases, prior inflation, and international price factors (imported input prices, foreign inflation, and the exchange rate), and as a
negative function of changes in agricultural sector output and excess capacity. The effect of these determinants is modified by the relative weights of wages and imported inputs in industrial prices and by the government's wage and exchange rate indexing system.

The demand side of the economy picks up the effects of money demand and the adjustment of the nominal interest rate, $I$. Money demand depends on real output and the interest rate, and equals supply. Although this assumption seems strong, estimates of demand for money based on quarterly data in Brazil show fast adjustment. When we modified this assumption to allow a stock adjustment mechanism, estimates of the lag money stock variables turned out to be insignificant and invariant over time. Thus changes in the adjustment speed of money balances appear to have minor effects on changes in the inflationary feedback process.

Hence we write the demand for real money balances as:

$$m_t - p_t = \alpha_0 + \alpha_1 y_t - \alpha_2 I_t$$

The nominal interest rate can be characterized as a function of the expected rate of inflation and of the ratio of the fiscal deficit to gross national product. Financial sector indexation ensured that the ex-ante real interest rate was nonnegative, and we reflect this factor with a constant, $r \geq 0$. Thus the interest rate represents the opportunity cost of holding money. Accurate data for the deficit do not exist in Brazil, however, so that we omit it from our specification. Thus the interest rate, $I$, is written:

$$I_t = r + \pi$$

where $\pi$ is the expected rate of inflation. We assume, for simplicity, that the expected rate of inflation is an "optimal forecast" of future inflation based on past inflation:

$$\pi_t = \delta_0 + \sum_{i=1}^{m} \delta_i \hat{\pi}_{t-i}$$

where $m > 1$

The parameters of this equation are time-varying, assuming that agents form expectations of future inflation through weighting or discounting past data to obtain one-period forecasts of inflation. The process by which this optimal forecasting coefficient is obtained is described in section II.

Equations 9, 10, and 11 may be combined to obtain a demand-side expression for current prices:

$$p_t = -[\alpha_0 - \alpha_2 (r + \delta_0)] - \alpha_1 y_t + m_t + \alpha_2 \sum_{i=1}^{m} \delta_i \hat{\pi}_{t-i}$$

1. Using only the most recent realization of inflation (with $m = 1$) is optimal only when the variable follows a random walk. Muth (1960) has shown that an optimal forecast is a geometrically weighted average of past realizations if inflation follows an integrated moving average process. We have set $m = 4$ in our estimation. Longer lags were insignificant.
Equation 12, the demand-determined price equation, may be combined with equation 7, which describes excess capacity, and equation 8, the supply-determined inflation equation, to yield the following transfer function for the rate of inflation, after first-differencing of both equations:

\[
\hat{p}_t = a_0 \hat{y} + a_1 \hat{p}_{t-1} + a_2 \hat{p}_{t-2} + a_3 \hat{p}_{t-3} + a_4 \hat{p}_{t-4} + b_1 \hat{m}_t + c_1 (\bar{w} - q) + c_2 (\hat{p}_M - \Phi \hat{p}_h + \varepsilon) + c_3 A_t
\]

The parameters of this equation are defined in Table 1. (See Zellner and Palm 1974 on the use of transfer functions for testing structural econometric models with time series models.)

Regardless of the degree of indexation of wages and the exchange rate, long-run homogeneity requires that the coefficients of the four past inflation rates and the monetary variable add up to one:

\[
a_1 + a_2 + a_3 + a_4 + b_1 = 1
\]

If the monetary policy variable is totally ineffective in controlling inflation \((b_1 = 0)\), then the inflation process would be unstable, since the coefficients of the lag values would sum to unity. The model thus suggests that there may be

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{y})</td>
<td>Trend growth in output</td>
<td>(a_0 = -\alpha_1 \tau \theta / \Omega)</td>
</tr>
<tr>
<td>(\hat{p}_{t-1})</td>
<td>Inflation—one-quarter lag</td>
<td>(a_1 = {\alpha_1 [1 - (1 - \tau) \Phi] } / \Omega)</td>
</tr>
<tr>
<td>(\hat{p}_{t-2})</td>
<td>Inflation—two-quarter lag</td>
<td>(a_2 = {\alpha_1 \tau (\Phi_{a_2} - \Phi_{a_1}) } / \Omega)</td>
</tr>
<tr>
<td>(\hat{p}_{t-3})</td>
<td>Inflation—three-quarter lag</td>
<td>(a_3 = {\alpha_1 \tau (\Phi_{a_3} - \Phi_{a_2}) } / \Omega)</td>
</tr>
<tr>
<td>(\hat{p}_{t-4})</td>
<td>Inflation—four-quarter lag</td>
<td>(a_4 = {\alpha_1 \tau (\Phi_{a_4} - \Phi_{a_3}) } / \Omega)</td>
</tr>
<tr>
<td>(\hat{m}_t)</td>
<td>Rate of growth of money supply</td>
<td>(b_1 = \tau \beta / \Omega)</td>
</tr>
<tr>
<td>((\bar{w} - q))</td>
<td>Exogenous wage growth less</td>
<td>(c_1 = \alpha_1 \tau (1 - L) / \Omega)</td>
</tr>
<tr>
<td></td>
<td>productivity</td>
<td></td>
</tr>
<tr>
<td>((\hat{p}_M - \Phi \hat{p}_h + \varepsilon))</td>
<td>Determinants of exchange rate</td>
<td>(c_2 = \alpha_2 (1 - \tau) (1 - L) / \Omega)</td>
</tr>
<tr>
<td></td>
<td>devaluation</td>
<td></td>
</tr>
<tr>
<td>(A_t)</td>
<td>Agricultural supply deviations</td>
<td>(c_3 = -\alpha_1 \sigma \Theta (1 - L) / \Omega)</td>
</tr>
<tr>
<td></td>
<td>from trend</td>
<td></td>
</tr>
</tbody>
</table>

Note: \(\Omega = \{[1 - (1 - \tau) \Phi] \alpha_1 + \beta \theta\}\). The symbol \(L\) represents the lag operator. The coterms of these coefficients are thus first-differenced.
Table 2. The Effects of Changes in Indexing Policies and Expectation Formation on Coefficients of the Inflation Transfer Function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lag structure (in quarters)</th>
<th>Coefficient of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$\phi_{wi}$ - wages</td>
<td>+</td>
<td>n.a.</td>
</tr>
<tr>
<td>$\phi_{wi}$ - wages</td>
<td>n.a.</td>
<td>+</td>
</tr>
<tr>
<td>$\phi_c$ - exchange rate policy</td>
<td>+</td>
<td>?</td>
</tr>
<tr>
<td>$\delta_t$ - expectations</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: $\phi_{wi}$ = wage indexing parameter for past quarter $i$; $\phi_c$ = exchange rate devaluation parameter; $\delta_t$ = inflation expectations parameter for first-quarter lag of inflation. + = positive effect; - = negative effect; ? = ambiguous effect; n.a. = not applicable. Parameters and coefficients are fully defined in Table 1.

II. EMPIRICAL RESULTS

We analyze the experience of Brazil, where inflationary expectations are a way of life and the indexing policy has changed in different ways over the twenty years for which we have data. Because our purpose is to study the time...
profile of the coefficients in the inflation transfer function across regime changes in indexing policy, we did not split the sample into different subperiods for different indexing regimes. For the sake of comparison with the time-varying parameter estimates, we first discuss the results of the model estimated with ordinary least squares (OLS) under the assumption of time-invariant parameters. We then provide a description of the Kalman filtering method and discuss the results estimated using this method under the assumption of time-varying parameters.

The Data

The price index used in this study is the quarterly cost of living index compiled by the Getulio Vargas Foundation in Brazil for the first quarter of 1963 through first quarter 1985. The monetary growth rate and exchange rate data come from the Central Bank of Brazil. The monetary variable makes use of the base component of the money stock. The price of foreign goods and agricultural output data used to compute the external (foreign) and internal (agricultural) shock variables come from the National Institute of Statistics in Rio de Janeiro. The price of foreign goods is a trade-weighted price index.

**OLS Results: Constant Coefficient Assumption**

Table 3 presents the OLS estimates of the transfer function for inflation given by equation 13, adapted for estimation as follows. The variables for change in trend growth of output (\(\dot{y}\)), and for the change in productivity and the average wage (\(\bar{w} - q\)) do not appear in the estimated equation, as appropriate data for these variables are not available for Brazil. To the extent that fiscal deficits have been financed by new money creation, note that the monetary “demand” variable, \(m_t\), indirectly registers the effects of fiscal deficit changes as well as monetary policy. Thus, the variable \(m\) represents a policy-determined excess demand variable for the economy as a whole and not simply a monetary policy instrument. We define our money demand variable as the difference between monetary base growth \(m'\), and last period inflation, \(\pi_{t-1}\) because we are interested in the effect of “surprise” or unexpected changes in domestic credit, over and above inflation.

All variables except the second and third lags on inflation and the constant term are significant. The lack of significance and the negative signs of the second and third lags may be due to multicollinearity of the inflation arguments over long periods of sustained inflation. As expected, the coefficient of the fourth quarter lagged inflation is significant due to the long period (1965–79) of annual indexation. The adding-up restrictions of equation 14 hold at the 5 percent level (but not at the 1 percent level). The OLS estimates thus support the hypothesis that Brazilian inflation is to a large extent an autoregressive process, which is an expected outcome given the widespread backward-looking indexing policies followed. Yet inflation is not purely inertial, because the
Table 3. OLS Estimates of Determinants of Inflation With Time-Invariant
Parameters, Brazil, 1963–85

<table>
<thead>
<tr>
<th>Variable</th>
<th>Parameter</th>
<th>Parameter estimate</th>
<th>Standard error</th>
<th>T Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation, past periods</td>
<td>$a_1$</td>
<td>0.774</td>
<td>0.091</td>
<td>8.529</td>
</tr>
<tr>
<td></td>
<td>$a_2$</td>
<td>-0.058</td>
<td>0.120</td>
<td>0.480</td>
</tr>
<tr>
<td></td>
<td>$a_3$</td>
<td>-0.180</td>
<td>0.119</td>
<td>1.505</td>
</tr>
<tr>
<td></td>
<td>$a_4$</td>
<td>0.393</td>
<td>0.097</td>
<td>4.051</td>
</tr>
<tr>
<td>Money growth, $\hat{m}<em>t - \hat{p}</em>{t-1}$</td>
<td>$b_1$</td>
<td>0.165</td>
<td>0.062</td>
<td>2.675</td>
</tr>
<tr>
<td>Exchange rate determinants,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$(\hat{p}<em>{2t-4}, \hat{p}</em>{2t}, \hat{m}_t + \hat{e}_t)$</td>
<td>$c_1$</td>
<td>0.102</td>
<td>0.042</td>
<td>2.403</td>
</tr>
<tr>
<td>Agricultural supply deviations,</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>$a_0$</td>
<td>-0.008</td>
<td>0.006</td>
<td>1.310</td>
</tr>
</tbody>
</table>

Note: $R^2 = 0.9123$; standard deviation = 0.0285; Durbin-Watson statistic = 1.7694; Box-Pierce Q(5) = 9.759; $F(1/81) = 120.4$; autoregressive conditional heteroskedasticity test (4) = 2.072; number of observations = 89; numbers in parentheses are degrees of freedom.

Test of exogeneity with dependent variable: Argument Granger causality, $F(4/80)$

$\hat{m} - \hat{p}_{t-1}$

$\hat{m} = \Phi \hat{p}_{t-1} + \hat{e}_{t-1}$

$\hat{p}_{t-1}$

$A$

Test of restriction: $a_1 + a_2 + a_3 + b_1 = 1$: $F(1/81) = 4.895$

$b_1 = c_2 = c_3 = 0$: $F(3/81) = 10.15$

$a_2 = a_3 = a_4 = 0$: $F(3/81) = 5.81$

results show that it is also responsive to monetary policy. Inflation also responds to internal agricultural price and external price shocks. These results differ from Lara-Resende and Lopes (1981) inasmuch as our demand term has significant and positive effects on inflation throughout the indexation period, whereas their proxy variable for demand is not significant.

The Durbin-Watson statistic for autocorrelation fell in the indeterminate region (at the 5 percent level)—we were not able to tell if the hypothesis of serial independence of the disturbance terms could or could not be rejected. We thus calculated the Box-Pierce statistic, Q, which is more specifically tailored to time-series data. The Q statistic found does not allow us to reject the hypothesis of white noise (that is, we may assume that the residuals are not autocorrelated). The autoregressive conditional heteroskedasticity test, distributed as a chi-square variate, also is consistent with a white noise error term. Finally, we also cannot reject the hypothesis that the three terms representing the deviations of monetary growth, foreign prices, and agricultural outputs are econometrically exogenous, as shown by the $F$-statistics for the Granger test of causality. The conditions satisfied by these statistics permit the use of Kalman filtering.

Expectations, of course, play a role in the autoregressive process. Even if there were no indexation, adjustment to inflation may take place in a manner
consistent with our empirical estimates, through a backward-looking expectations mechanism. As can be seen from the coefficient definitions in table 1, our estimates do not allow us to identify separately the expectations and the indexation coefficients of our model. In a rational expectations or "model consistent" framework, expectations depend on the policy regime. Thus their effect would be consistent with and reinforce the effects of policy-determined backward-looking indexing. Whether the policy changes affect the parameters directly or through a more complicated expectations mechanism is an issue that is beyond the scope of this paper.

The Kalman Filter Method

In the context of time-varying parameters, we evaluate the effect of indexing policy switches on the parameters of the inflation equation. We wish to find out if the inflation coefficients are synchronized with indexing policy changes in ways suggested by our model and current macroeconomic theory.

The statistical technique we employ to cope with time-varying parameters is the Kalman filter. The filter allows us to track changes in the coefficients of the inflation determinants as the variables, and most specifically the indexing policies, have changed. The filter is developed in a two-stage process in which first, for period \( t = 1 \), an optimal predictor of inflation for the next period, \( P_{t-1} \), is formed on the basis of all the information available at \( t - 1 \). This prediction equation includes the forecast value of the time-varying coefficients. In the second stage, the predictor is updated to incorporate new observations on the variables as they become available, from which it obtains updated parameter values. Thus new coefficient values are generated for each period. (Expanded treatments of this approach are available in Hannan 1970, Anderson and Moore 1979, Pagan 1980, Harvey 1981, McNelis and Neftci 1982, Bomhoff 1983, and Chow 1984.)

In application, estimation proceeds in three steps: (1) specification of starting values for the time-varying regression coefficients, the coefficient variance-covariance matrix, and the variance of the disturbance term of the regression model at time \( t = 0 \); (2) specification of stochastic processes for the evolution of the time-varying coefficients, as well as specification or estimation of the autoregressive parameters and the variance of the disturbance terms of these stochastic processes; and (3) an iterative solution to find the best one-period predictor of the dependent variable at each period.

Intuitively, the Kalman filter may be described as an "optimal discounting" of past data to find the best one-period forward predictors. Rolling regressions, conversely, give equal weight to past and present data in giving one-period forecasts.

Because the Kalman filter is a single-equation linear projection operator, its use requires that the right-hand side variables in the estimating equation be econometrically exogenous, and that the disturbance term be independently and identically distributed. Tests satisfying these conditions, discussed above,
appear in table 3. Tests on the residuals of the equation with time-varying parameter estimates also cannot reject serial independence.

Although the time-paths of the coefficients may show some variation, one question arises: is the variation significant? The exact distribution of the coefficients estimated with Kalman filtering is not known, and likelihood tests are known to be downwardly biased (see Chow 1984). In figures 1–3 below, we graph movements in the coefficients to informally evaluate their “significance” and present in each graph the upper and lower bounds for a 90 percent confidence interval around the ordinary least squares estimate of each coefficient.

Although the graphs allow us to compare the Kalman filter estimates with the time-invariant OLS estimates, they do not tell us if the movements of the Kalman estimates are significantly different from each other from one period to the next. Because we cannot say if the parameters show significant change from one period to the next, we can only see if the changes in the coefficients are synchronized and correlated in the “right” direction with policy changes,

Figure 1. *Time Variation of the One-Period Feedback Parameter, a*,

![Time variation of the one-period feedback parameter, a.](image)

**Key:**
- preindexation;  
- annual indexation;  
- biannual indexation.

**Note:** The dashed lines indicate the upper and lower bounds of a 90 percent confidence interval around the OLS estimate of table 3.
Figure 2. *Time-Variation of One-Period and Four-Period Feedback Effects, \( a_1 \) and \( a_4 \)

<table>
<thead>
<tr>
<th>Year</th>
<th>0.75</th>
<th>0.70</th>
<th>0.65</th>
<th>0.60</th>
<th>0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>0.80</td>
<td>0.75</td>
<td>0.70</td>
<td>0.65</td>
<td>0.60</td>
</tr>
<tr>
<td>1966</td>
<td>0.75</td>
<td>0.70</td>
<td>0.65</td>
<td>0.60</td>
<td>0.55</td>
</tr>
<tr>
<td>1968</td>
<td>0.70</td>
<td>0.65</td>
<td>0.60</td>
<td>0.55</td>
<td>0.50</td>
</tr>
<tr>
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<td>0.45</td>
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<tr>
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<td>0.45</td>
<td>0.40</td>
</tr>
<tr>
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<td>0.55</td>
<td>0.50</td>
<td>0.45</td>
<td>0.40</td>
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</tr>
<tr>
<td>1976</td>
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<td>0.45</td>
<td>0.40</td>
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</tr>
<tr>
<td>1978</td>
<td>0.45</td>
<td>0.40</td>
<td>0.35</td>
<td>0.30</td>
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</table>

Key: ——— one-period feedback parameter; ——— four-period feedback parameter; 
|          | preindexation; | annual indexation; | biannual indexation. |

particularly changes in the indexation system. For this reason, we have shaded the figures on which the Kalman estimates appear to indicate three different regimes: the preindexing period before 1965, the system of annual indexation in place from 1965 through 1979, and the system of semestral or biannual indexation from 1980 until the end of the sample period.

*The Time-Varying Estimates*

The Kalman filter estimates for two feedback coefficients of the transfer function for inflation, equation 13, appear in figures 1 and 2.

Figure 1 shows considerable variation in the one-period feedback coefficient of the transfer function, \( a_1 \). The first increase occurs in 1965, when the indexing laws were introduced. The other major increase begins in 1979, when the wage indexing adjustment interval was lowered from one year to six months, to compensate for erosion due to inflation. Figure 1 also shows that the coefficient movement remains below the \( OLS \) estimates although it lies within the 90 percent confidence interval for most of the estimation period.
Figure 2 compares the variation of the one- and four-period feedback effects. It shows a rise in the four-period effects at the time the annual indexing laws were imposed in 1964, and a decline at the time annual indexing was changed to semestral indexing. However, the variation of the coefficients lies within the 90 percent confidence interval given by the OLS estimates (not shown).

The path of the monetary coefficient appears in figure 3. We see that there is a gradual decline in the effectiveness of this variable on inflation as indexing spread and became increasingly expected and accounted for in Brazil. By the end of the period its value was slightly above 0.15, however. Because its time-varying standard deviation at the end of the period was approximately 0.06, this suggests that money growth still had a “significant” effect on inflation.

Figure 4 depicts the variation of the coefficient of external shocks measured by changes in imported input prices and foreign inflation and by exogenous shifts in the exchange rate. The coefficient is fairly constant after 1967, when administered prices were imposed on imports. We see a slight rise in 1973, at the time of the oil shock and the removal of administered price controls, a
decline in 1980 and 1981 after exchange rate indexation was interrupted, and a sharp rise in 1982–83. The exchange rate disindexation from December 1979 to the second semester of 1980 temporarily reduced the pace of devaluation to rates below domestic and foreign inflation differentials in order to reduce expectations. (For a discussion of exchange rate policies and the use of administered prices for reducing inflation, see Pereira and Nakano 1987, p. 128, 158).

The adjustment of the agricultural supply coefficient appears in figure 5. The pattern of increases in the coefficient over the three periods suggests that inflation became more responsive to supply shocks as indexing became more widespread and increasingly a "way of life." The changes in the indexing adjustment period after 1979, in particular, made inflation more sensitive to these internal supply shocks.

Figures 1 through 5 show how Kalman filtering techniques can provide valuable information on the entropy of Brazilian inflation. The figures suggest that inflation moves up when wage indexation intervals are shortened (as in 1979, from twelve to six months), even if this change reduces the memory of
inflation. The figures also suggest that stringent monetary policies can still fight inflation, but with a lower response speed, and that inflation becomes more vulnerable to supply shocks as the indexing system becomes more widespread.

IV. CONCLUSION

We have presented results which link the spread and increasing degree of wage indexing with the Brazilian inflationary process during the past twenty years. Exchange rate indexing policies through passive crawling peg regimes appear to have relatively minor or insignificant effects on the degree of inertia in the inflation process. Changes in wage or exchange-rate indexing appeared to weaken the response of inflation to monetary correction and to increase inflationary sensitivity to agricultural supply shocks. However, the indexing system did not eliminate or make the effects of monetary correction insignificant.

The model used to derive the transfer function for inflation neglects the
effects of exchange rate expectations, changes in administered prices, and tax indexing on the inflationary process. The model also cannot separate out the inertial effects of wage indexing from the direct effects of inflationary expectations. Given the extended duration and formal nature of the indexing system in Brazil, however, we can reasonably assume that economic agents took the system into account in estimating current and future inflation. Thus one would presume that the effects of expectations would be consistent with the effects of the indexing policy. A more complex model explicitly treating these phenomena would provide more detail and perhaps give more accurate measures of the movements of the inflation transfer function coefficients.

Despite these limitations, this study provides evidence linking the wage indexing system with increasing inertia in the Brazilian inflationary process. However, it does not link changes in wage or exchange rate indexation with a trivial or insignificant response of inflation to monetary stringency.

Our results thus challenge the assumption that a suspension of indexation is a necessary precondition for making inflation responsive to monetary correction. We do not deny that a suspension of indexation may reduce the output costs of the stabilization program. A similar empirical analysis of a transfer function for the output gap or unemployment may show that stabilization may be much more painful without some form of disindexation. However, our results suggest the importance of fundamental monetary correction, even during the disindexation phase of the stabilization program.

**Appendix: The Kalman Filter Technique**

According to Harvey (1981), consider the following measurement equation:

\[ y_t = Z_t \alpha_t + \epsilon_t, \quad t = 1, \ldots, N \]

where \( y_t \) is an observed variable, \( Z_t \) a \( k \)-element vector of observed variables, and \( \epsilon_t \) is a serially independent, normally distributed error term with zero mean and \( \sigma^2 \) variance. The only difference between equation 15 and the standard general linear model is the time dependence of the \( \alpha \) vector of behavioral coefficients. The term \( \alpha_{u_t} \) is a state variable, and \( \alpha_{t} \) is a state vector.

The process governing the evolution of the coefficients \( \alpha_t \) through time is given by the transition equation:

\[ \alpha_t = T \alpha_{t-1} + R v_t \]

The values of the variables in the \( T \) and \( R \) matrices are fixed, and \( v_t \) is a vector of disturbances with mean zero and covariance matrix \( Q_t \).

Let \( a_t \) denote the minimum mean square linear estimator (MMSLE) of \( \alpha_t \) based on all information up to and including the current observation \( y_t \). Let \( a_{t-1} \) denote the predicted values, or MMSLE of \( \alpha_t \) at \((t - 1)\). All the information available at time \((t - 1)\) is incorporated in \( a_{t-1} \), which has a covariance matrix
\[ a_{t/t-1} = T \alpha_{t-1} \]

Given that the expected values of \( T\alpha_{t-1} \) is \( \alpha_t \), if we subtract \( \alpha_t \) from both sides of 17 and take expectations, we find that \( E(a_{t/t-1}) = \alpha_t \). Thus, \( a_{t/t-1} \) is unconditionally unbiased (see Harvey 1981, p. 104).

Given that \( a_{t/t-1} \) is the MMSE of \( \alpha_t \) at \( t-1 \), the MMSE of \( y_t \) at time \( t-1 \) is:

\[ y_{t/t-1} = Z_t a_{t/t-1} \]

for any of the possible values of \( Z_t \). The associated error in prediction of \( y_t \) is:

\[ \nu_t = y_t - y_{t/t-1} = Z_t(\alpha_t - a_{t/t-1}) + \epsilon_t. \]

Because the expected value of both \( \alpha_t - a_{t/t-1} \) and \( \epsilon_t \) are zero, \( E(\nu_t) = 0 \). The variance of the prediction error is then:

\[ E(\nu_t^2) = E(y_t - y_{t/t-1})^2 = \sigma_t^2 f_t \]

where \( f_t = Z_t P_{t/t-1} Z_t' + I_t \) and where \( I_t \) is the \( t \)-dimensional identity matrix for a sample size of \( N \) observations and \( I_t \) is the \( t \)th diagonal.

The main elements of this approach are the prediction equations for the state vector \( \alpha_t \) (equation 17), the prediction error for \( y_t \) (equation 19), and its covariance matrix (equation 20).

Updating equations are now used to incorporate the new information on the observed variables for the current period \( (y_t, \text{ and } Z_t) \) with information already available in the optimal predictor \( a_{t/t-1} \) to obtain an estimate of \( \alpha_t \). This is essentially a Bayesian decision process, in which the prior information on \( a_{t/t-1} \) is combined with the sample information consisting of the period \( t \) observations on \( y \) and \( Z \).

Equipped with this information the state updating equation is given by:

\[ a_{t+1/t} = T a_{t/t-1} + P_{t/t-1} Z_t \nu_t / f_t \]

The prediction error for \( y_t \), \( \nu_t \), and the Kalman gain, defined as \( (P_{t/t-1} Z_t / f_t) \), are the two main inputs used to update our estimate of \( \alpha_t \) given \( a_{t/t-1} \). \( P_{t/t-1} \) is independent of \( y_t \) and can thus be calculated in advance by the filtering process. Intuitively, equation 21 tells us that the coefficient will be revised in period \( (t - 1) \) according to the specified transition process and in proportion to the current prediction errors. This proportion is determined by the size of the current arguments \( Z_t \) and the coefficient variance relative to the variance of the total prediction error.

Before proceeding to our application, we need to determine the starting values for the state variables \( a_n \) and to specify the transition matrix \( T \) and matrices \( Q, R, \) and \( \sigma_t^2 I \). Our starting estimates come from generalized least
squares estimation, based on the approach suggested by Sarris (1973), with $k \times k$ matrix $Q = 1.0\sigma^2(Z'Z)^{-1}$. Because the OLS estimates are minimum variance estimates, this selection initially may bias the results toward less variation, but we also specify the transition matrix, $T$, as an identity matrix and hence assume a random walk process for the time-varying coefficients. This may bias the results toward greater variation. We have also set $Q$ at values of $0.1\sigma^2(Z'Z)^{-1}$ and $10\sigma^2(Z'Z)^{-1}$ to see how sensitive the results of the estimation process were to the specification of this matrix. In our presentation, we discuss the results for $Q = \sigma^2(Z'Z)^{-1}$ because the results did not turn out to be markedly different.

2

References


2. Copies of the results under the alternative assumptions, as well as a printout of the program for *Regression Analysis of Time Series (RATS, 3.0)* developed by Andres Gomez-Lobo are available from the authors upon request.

