Measuring growth in total factor productivity

Gains in total factor productivity (TFP), reflecting more efficient use of inputs, have long been recognized as an important source of improvements in income and welfare. Cross-country differences in income levels and growth rates are mostly due to differences in productivity (Klenow and Rodriguez-Clare 1997; Easterly and Levine 2000).

Measuring TFP is therefore important in assessing countries’ past and potential economic performance. But it is also difficult, for two reasons. Fairly innocuous differences in assumptions can lead to very different estimates of TFP growth. And the interpretation of measured TFP growth can be problematic when such growth reflects factors other than purely technical change—such as increasing returns to scale, markups due to imperfect competition, or gains from sectoral reallocations.

This note discusses some of these difficulties using data for the Republic of Korea in 1960–97 for illustration. An Excel worksheet is available to readers who wish to reproduce the exercises described here for other countries; see PREMnotes or the Growth Thematic Group Web site under PREMnet.

The starting point for estimating TFP is a production function that represents how inputs are combined to produce output. For example, suppose that GDP ($Y$) is produced using two factors, physical capital ($K$) and human-capital-adjusted labor input ($H$), using a Cobb-Douglas production function:

\[ Y = A (K^\alpha H^{1-\alpha})^\gamma, \]

where $A$ is TFP, $\gamma$ measures the extent of returns to scale, and $\alpha$ measures the importance of physical capital in output. If $\gamma = 1$ ($\gamma > 1$) ($\gamma < 1$), there are constant (increasing) (decreasing) returns to scale. By expressing equation 1 in growth rates and rearranging the variables, TFP growth can be written as growth in output less a weighted average of growth in inputs:

\[ g_A = g_Y - \gamma [\alpha g_K + (1 - \alpha) g_H], \]

where $g_X$ is the growth rate of variable $X$.

Given data on the growth rates of $Y$, $K$, and $H$ and information on the parameters of the production function, we can obtain estimates of productivity growth as the difference between output growth and a weighted average of growth in inputs. This note considers these two ingredients and interprets the results.

The data matter

While real GDP growth rates are easy to obtain, measuring the growth rates of $K$ and $H$ is more difficult. In most cases physical capital is measured using the perpetual inventory method, which uses an estimate of the capital stock in a base year, assumptions on depreciation, and the flow of new investment. But measured growth rates of capital stock—and thus estimates of TFP growth—can be very sensitive to assumptions about initial stocks and...
Small differences in assumptions can lead to big differences in estimates of TFP growth. Small differences in assumptions can lead to big differences in estimates of TFP growth. Small differences in assumptions can lead to big differences in estimates of TFP growth. Small differences in assumptions can lead to big differences in estimates of TFP growth. Small differences in assumptions can lead to big differences in estimates of TFP growth. Small differences in assumptions can lead to big differences in estimates of TFP growth. Small differences in assumptions can lead to big differences in estimates of TFP growth.

The production function also depends on human-capital-adjusted labor input ($H$), which is a way of summarizing the contribution of “brains” (education) and “brawn” (the size of the labor force) to labor input. One way to do this is to adjust the number of workers for their years of schooling ($S$) by assuming that each additional year raises workers’ productivity by a given percentage. Various estimates suggest that, defined in this way, the returns to education are about 10 percent. The number of workers is the product of the working-age population ($L$) and the participation rate ($P$). Thus we can express:

\[
H = L \cdot P \cdot e^{0.15}
\]

How important is it to measure labor input in this way rather than simply as population? In many countries it can be very important. In Korea the combined effect of growth in the working-age population, in participation rates, and in average education levels increased the human-capital-adjusted labor force by about 5 percent a year between 1960 and 1997—almost three times as fast as the growth in population of 1.6 percent a year. Such differences can have substantial effects on estimated productivity.

The production function matters

With data on $Y$, $K$, and $H$, we now require only information on the parameters of the production function to estimate productivity growth. Because these parameters are not directly observable, it is common to make some assumptions.

For a constant returns to scale production function ($\gamma = 1$), it is common to assume values of $\alpha$ between 0.3 and 0.5. Depending on the observed growth rates of physical and human capital, the value chosen can matter a lot for estimates of TFP growth. The top panel of table 1 reports average annual growth rates of $Y$, $K$, and $H$ in Korea in 1960–97. The first row in the middle panel shows how TFP growth rates change as the value of $\alpha$ is increased from 0.3 (resulting in TFP growth of 1.6 percent a year) to 0.5 (resulting in TFP growth of just 0.3 percent a year). By increasing $\alpha$, we are increasing the weight on the fastest-growing factor of production in equation 2, physical capital, resulting in lower estimated TFP growth.

Estimates of TFP growth are also very sensitive to assumptions about the degree of scale economies. The remaining rows of the middle panel fall with increasing returns to scale, because some of the increase in output previously attributed to productivity improvements is instead attributed to scale economies. In some cases the distinction between the two may be relevant. If, for example, there are increasing returns to scale at low levels of development, and decreasing returns to scale at high levels of development, the distinction would be important in identifying how much of measured TFP growth is likely to be sustained over the long run.

So far we have assumed that the parameters of the production function do not change over time. But they might. If, for example, the production function is of the constant elasticity of substitution (CES) type with an elasticity different from 1, the weight on capital in equation 1 can change.

### Table 1: Estimates of TFP growth are sensitive to assumptions about the production function (percent)

<table>
<thead>
<tr>
<th>Input and output growth</th>
<th>Average annual growth in Korea, 1960–97</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y$</td>
<td>8.5</td>
</tr>
<tr>
<td>$K$</td>
<td>11.6</td>
</tr>
<tr>
<td>$H$</td>
<td>4.9</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Sensitivity of TFP growth estimates 1</th>
<th>Average annual TFP growth in Korea, 1960–97</th>
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</thead>
<tbody>
<tr>
<td>$\alpha = 0.3$</td>
<td>$\alpha = 0.4$</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td></td>
</tr>
<tr>
<td>$\gamma = 1.0$</td>
<td>1.6</td>
</tr>
<tr>
<td>$\gamma = 1.2$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\gamma = 0.8$</td>
<td>0.3</td>
</tr>
</tbody>
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<tr>
<th>Sensitivity of TFP growth estimates 2</th>
<th>Average annual TFP growth in Korea, 1960–97</th>
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</thead>
<tbody>
<tr>
<td>$\sigma = 0.8$</td>
<td>$\sigma = 1.0$</td>
</tr>
<tr>
<td>$\sigma = 1.2$</td>
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</tr>
<tr>
<td>$\alpha = 0.3$</td>
<td>3.3</td>
</tr>
<tr>
<td>$\alpha = 0.8$</td>
<td>1.6</td>
</tr>
<tr>
<td>$\alpha = 0.5$</td>
<td>0.7</td>
</tr>
</tbody>
</table>

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over time, depending on the size of the elasticity of substitution and the rate at which $K$ grows relative to $H$. This effect is shown in the bottom panel of table 1 for some plausible values of the elasticity of substitution between physical and human capital ($\sigma$).

Finally, it is worth noting that estimates of TFP growth constructed in this way can track output growth closely—and so may not be very informative—because they subtract growth of measured physical and human capital (which by construction are quite smooth) from output growth (which typically is not). Thus estimates of the variation in TFP growth over time can be quite sensitive to the period for which they are calculated, and the robustness of the results to the sample period should be checked carefully.

The interpretation matters

What do these estimates of TFP growth actually measure? At the firm level it is fairly straightforward to equate $g_A$ with technical progress. At the aggregate level it is less clear. If resources shift from less productive to more productive firms, perhaps as a result of economic reforms or if more productive firms simply grow faster, aggregate TFP growth may be observed even in the absence of technical progress at the firm level.

Although we may also be interested in these other effects, $g_A$ measures more than just the fundamental improvements in technology that underpin growth in the long run. Distinguishing between these different sources of TFP growth is important because there are likely to be natural limits to the gains from structural change once resources have been reallocated to the most efficient sectors.

What can be said of TFP growth relative to output growth? It is often asserted that TFP growth “accounts for” a certain fraction of growth. For example, given output growth of 8.2 percent in Korea in 1960–97, and estimated TFP growth of 1.4 percent, one might think that TFP growth accounted for just 18 percent of output growth (1.4 percent divided by 8.2 percent). But this conclusion is misguided, because it ignores the fact that decisions to invest in physical and human capital are themselves likely to depend on TFP growth. A simple ratio ignores the portion of growth in inputs induced by productivity growth, which should also be attributed to TFP growth.

A better way of assessing the importance of TFP growth is to ask the following question: if output growth is above average, how much of this additional growth can be attributed to TFP growth? One way to answer this question is to consider the covariance between productivity growth and output growth divided by the variance of output growth (see Klenow and Rodriguez-Clare 1997). Because this calculation reflects TFP-induced increases in inputs, rather than assuming inputs to be exogenous, it gives a more accurate picture of the role of TFP growth in the growth process. In Korea the share of growth attributable to productivity growth jumps from the naive estimate of 18 percent to more than 80 percent. While the second figure may be high, it illustrates the importance of taking into account the endogeneity of factor inputs when assessing the importance of TFP growth.

Where do we go from here?

Estimated TFP growth is very sensitive to assumptions about the underlying parameters in the production function. Thus a natural step is to try to estimate the production function directly. There are two ways to do so. One is to assume that the economy in question is characterized by constant returns to scale ($\gamma = 1$) and perfect competition. Then it is possible to view the parameter $\alpha$ as 1 minus the share of wages in value added (see Young 1995 and Hsieh 1999). But in many developing countries it seems implausible to assume constant returns to scale and perfect competition—and we have already seen how sensitive estimates of productivity growth are to assumptions on constant returns to scale. Moreover, reliable data on wages are often not available for the economy as a whole.

A second approach is to try to estimate the production function econometrically—The endogeneity of factor inputs should be considered when assessing the importance of TFP growth
for example, by regressing output growth on input growth (see Basu and Fernald 1995). While this approach does not require assumptions of constant returns and perfect competition, the problem of inputs responding endogenously to TFP growth returns to the fore. In the econometric approach the residual in the regression is interpreted as TFP growth. But we have already seen that the residual is correlated with growth in inputs, the right-hand variables in the regression. Thus ordinary least squares estimation will be invalid and it becomes necessary to find good instrumental variables—a difficult task.

But the econometric approach has its rewards. Given appropriate data and instrumental variables techniques, it is possible to retrieve estimates not only of TFP growth but also of the extent of scale economies and deviations from perfect competition. These can shed light on important policy issues that are as interesting as TFP growth. In the recent Korea Economic Report (World Bank 1999), for instance, the econometric approach is used to examine questions such as the extent to which key Korean manufacturing sectors had been exposed to competition prior to the East Asian crisis, and whether less competition was associated with lower productivity growth.

**Further reading**


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If you are interested in similar topics, consider joining the Growth Thematic Group. Contact Sandeep Mahajan (x80287) or click on Thematic Groups on PREMnet.