Emerging Market Fluctuations

What Makes the Difference?

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April 2009
Abstract

Aggregate fluctuations in emerging countries are quantitatively larger and qualitatively different in key respects from those in developed countries. Using data from Mexico and Canada, this paper decomposes these differences in terms of shocks to aggregate efficiency and shocks that distort the decisions of households about how much to invest, consume, and work in a standard model of a small open economy. The decomposition exercise suggests that most of these differences are explained by fluctuations in aggregate efficiency, distortions in labor decisions over the business cycle, and, most importantly, fluctuations in country risk. Other distortions are quantitatively less important.

This paper—a product of the Growth and the Macroeconomics Team, Development Research Group—is part of a larger effort in the department to understand macroeconomic volatility in developing countries. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at chevia@worldbank.org.
Emerging Market Fluctuations: What Makes the Difference?∗

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∗JEL Codes: E32, F41. Keywords: Business Cycles, Small Open Economy, Country Risk Premium
†I thank Aart Kray, Juan Pablo Nicolini, Demian Pouzo, Claudio Radlitz, and Luis Serven for helpful comments and suggestions. The findings, interpretations, and conclusions expressed in this paper are entirely those of the author. They do not necessarily represent the view of the World Bank, its Executive Directors, or the countries they represent.
1 Introduction

Aggregate fluctuations in emerging countries are different from those in developed countries. Table 1 illustrates this point. The first and second rows of the table display summary statistics of the business cycle in two representative emerging and developed countries, Mexico and Canada respectively. The third and fourth rows of the table report the same statistics averaged across 13 emerging and 13 developed countries. Overall, the table shows that, while economic aggregates are substantially more volatile in emerging countries (except, perhaps, hours worked), there is a deeper difference between these countries: consumption is more volatile than output in emerging countries but less volatile in developed countries; and the share of net exports on output is highly countercyclical in emerging countries but less so in developed countries. Moreover, these regularities hold across a large number of emerging and developed countries (Neumeyer and Perri, 2005; Aguiar and Gopinath, 2007).

Table 1: Business Cycles in Emerging and Developed Small Open Economies

<table>
<thead>
<tr>
<th>Country / Moment</th>
<th>( \sigma(y) )</th>
<th>( \sigma(l) )</th>
<th>( \sigma(m/y) )</th>
<th>( \sigma(c)/\sigma(y) )</th>
<th>( \sigma(x)/\sigma(y) )</th>
<th>( \rho(m/y, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>2.12 (.45)</td>
<td>1.4 (.2)</td>
<td>1.63 (.43)</td>
<td>1.28 (.05)</td>
<td>4.51 (.43)</td>
<td>-0.80 (.11)</td>
</tr>
<tr>
<td>Canada</td>
<td>1.44 (.21)</td>
<td>1.69 (.24)</td>
<td>0.91 (.1)</td>
<td>0.85 (.08)</td>
<td>4.6 (.23)</td>
<td>-0.05 (.24)</td>
</tr>
<tr>
<td>Emerging Countries</td>
<td>2.74</td>
<td>-</td>
<td>3.22</td>
<td>1.45</td>
<td>3.91</td>
<td>-0.51</td>
</tr>
<tr>
<td>Developed Countries</td>
<td>1.34</td>
<td>-</td>
<td>1.02</td>
<td>0.94</td>
<td>3.41</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

This table reports second moments of HP-filtered quarterly aggregate data. The statistics \( \sigma(x) \) and \( \rho(x, y) \) measure, respectively, the standard deviation of the series \( x \) and the correlation coefficient between the series \( x \) and \( y \). The variable \( y \) denotes output; \( l \), hours worked; \( m \), net exports; \( c \), private consumption; and \( x \), investment. GMM-based standard errors are reported in parentheses.

Sources: Rows 1 and 2, author’s calculations. Rows 3 and 4 taken from Aguiar and Gopinath (2007).

While researchers agree on these differences, they disagree on their causes: by introducing different frictions and shocks into their models, researchers provide alternative stories for why aggregate fluctuations in emerging countries are different. (Christiano, Gust, and Roldós, 2004; Neumeyer and Perri, 2005; Mendoza, 2008; Uribe and Yue, 2006; Aguiar and Gopinath,
These frictions and shocks, however, are usually chosen based on intuition or anecdotal evidence.

The purpose of this paper is to identify the type of reduced form distortions that explain the observed business cycle differences between emerging and developed countries. The logic of the experiment is as follows. At a conceptual level, shocks and frictions in most structural models drive a *wedge* between certain marginal rates of substitution and marginal rates of transformation relative to a prototype frictionless economy. Then, instead of testing whether a particular structural model generates the data, I estimate these wedges based on the prototype economy and measure their contribution to aggregate fluctuations. The estimated wedges, however, cannot be used to identify primitive shocks and frictions: models with different primitive shocks and frictions could induce movements in the same reduced form wedge in the prototype economy. Nevertheless, these wedges offer useful information: a successful model should induce reduced form wedges similar to those estimated based on the prototype economy; moreover, these wedges should account, quantitatively, for fluctuations in the economic aggregates.

Several authors have used variations of this approach applied to the U.S., including Parkin (1988), Hall (1997), Rotemberg and Woodford (1999), Mulligan (2002), Chari, Kehoe, and McGrattan (2007), and Shimer (2009). In this paper I follow more closely the methodology proposed by Chari, Kehoe, and McGrattan, labeled ‘Business Cycle Accounting’. In this methodology, the researcher interprets the data through the lens of a small open economy model. This prototype economy is subject to reduced form shocks that are interpreted literally as total factor productivity (the *efficiency wedge*), as labor and investment taxes (the *labor wedge* and *investment wedge*), as fluctuations in real interest rates (the *risk premium wedge*), and as government consumption (the *government consumption wedge*).

I estimate a stochastic process for these wedges using data from a representative emerging country, Mexico, and a representative developed open country, Canada. Next I estimate the realized wedges and study their contribution to aggregate fluctuations in both countries.
Besides considering aggregate fluctuations at the business cycle frequency, I also study two recession episodes: the 1995 Mexican crisis and the 1983 Canadian recession.

The main findings of this paper are the following: first, all wedges, except for the government consumption wedge, are substantially more volatile in Mexico; second, fluctuations in the risk premium wedge explain the ‘deep’ differences between Mexico and Canada: the excess volatility of consumption over output and the highly countercyclical share of net exports on output in Mexico; third, the risk premium wedge plays, if anything, a secondary role in Canada’s business cycle; fourth, the government consumption wedge has negligible effects in both countries; and finally, the investment wedge does not play any role in Mexico’s fluctuations but contributes somewhat to the behavior of aggregate investment and net exports in Canada.

Extending these results to the set of all emerging countries—the empirical regularities discussed in Neumeyer and Perri (2005) and Aguiar and Gopinath (2007) imply that, in principle, we could—these findings suggest that, to build models of emerging market fluctuations consistent with the data, researchers need to understand, first and foremost, what type of primitive shocks and transmission mechanisms drive fluctuations in the risk premium wedge, in the efficiency wedge, and in the labor wedge.

On the methodological side, this paper departs from Chari, Kehoe, and McGrattan’s procedure in using a different prototype economy to decompose fluctuations and in proposing an alternative strategy to estimate the wedges. Chari, Kehoe, and McGrattan base their analysis on a prototype closed economy. This paper uses a model of an open economy. Although the open economy model is observationally equivalent to a prototype closed economy, an open economy is the natural framework to study fluctuations in emerging countries. Indeed, a number of authors have explicitly introduced models with fluctuations in the price of foreign debt, together with additional frictions, to understand emerging market fluctuations (Neumeyer and Perri, 2005; Aguiar and Gopinath, 2006; Uribe and Yue, 2006; Arellano, 2008).

The paper is organized as follows. Section 2 discusses the prototype small open economy
and an observational equivalence. Section 3 describes the decomposition methodology. Section 4 applies the methodology using data from Mexico and Canada. Section 5 discusses the implications of the results for understanding emerging market fluctuations and concludes.

2 A Prototype Small Open Economy

This section describes a standard small open economy model with incomplete asset markets augmented with five stationary reduced form shocks, referred to as wedges. In the model, these reduced form shocks are interpreted literally as productivity shocks (the efficiency wedge), as labor income and investment taxes (the labor wedge and investment wedge), as fluctuations in interest rates (the risk premium wedge), and as government consumption (the government consumption wedge).

Time is denoted by \( t = 0, 1, 2, \ldots \) and the state of the economy at period \( t \) by \( s_t \). Let \( s^t = \{s_0, s_1, \ldots, s_t\} \) denote the history of the economy until time \( t \) and \( \pi(s^t) \) the probability of observing the history \( s^t \) as of time zero. Wedges in period \( t \) depend on the history \( s^t \). The efficiency wedge is denoted by \( A(s^t) \); the labor wedge, by \( (1 - \tau_l(s^t)) \); the investment wedge, by \( 1/(1 + \tau_i(s^t)) \); the risk premium wedge, by \( 1/Z(s^t) \), and the government consumption wedge, by \( g(s^t) \). Note that a drop in any of these wedges is interpreted as an increase in the corresponding distortion.\(^1\)

A representative household has preferences over contingent sequences of consumption, \( c(s^t) \), and hours worked, \( l(s^t) \), represented by the expected utility function

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t (1 + \eta)^t U(c(s^t), l(s^t)) \pi(s^t),
\]

where \( 0 < \beta < 1 \) is a subjective discount factor and \( \eta \) is the growth rate of the population.

Here and throughout this section, all variables are expressed in per capita terms.

\(^1\)While a drop in government expenditures may not mean an increase in any distortion, it does induce a drop in output.
Households own the stock of capital and are able to issue one period uncontingent discount bonds traded in international financial markets. Each bond is a contract to deliver one unit of the consumption good in the following period in exchange of \( q(s^t) \) units today. I decompose the discount price as

\[
q(s^t) = \frac{q^* (s^t)}{Z(s^t)},
\]

where \( q^* (s^t) \) is interpreted as the price of a risk-free bond and \( 1/Z(s^t) \) as a risk premium factor. Fluctuations in \( Z(s^t) \) introduce a wedge between the intertemporal marginal rate of substitution between consumption today and tomorrow, and the marginal rate of transformation in an economy that faces the relative price \( q^* (s^t) \).

The household has initial capital \( k_0 = k(s^0) \) and outstanding foreign debt \( b_0 = b(s^0) \), and face the budget constraint

\[
c(s^t) + (1 + \tau_x(s^t)) x(s^t) + b(s^{t-1}) = (1 - \tau_l(s^t)) w(s^t) l(s^t) + v(s^t) k(s^{t-1}) + T(s^t) \\
+ (1 + \eta) b(s^t) q(s^t).
\]

Here \( k(s^{t-1}) \) is the stock of capital chosen in period \( t - 1 \) and available for production in period \( t \), \( b(s^{t-1}) \) is the stock of foreign debt maturing at period \( t \), \( x(s^t) \) is investment, \( w(s^t) \) is the wage rate, \( v(s^t) \) is the rental rate of capital, \( \tau_x(s^t) \) is a tax on investment expenditures, \( \tau_l(s^t) \) is a labor income tax, and \( T(s^t) \) is a lump-sum transfer. Because the government has access to lump-sum transfers, I assume from now on, and without loss of generality, that all foreign debt is held by the household.

Competitive firms rent capital and labor from the household to produce consumption goods with the constant returns to scale technology

\[
y(s^t) = A(s^t) F \left( k(s^{t-1}), (1 + \gamma)^t l(s^t) \right),
\]

where \( \gamma \) is the rate of labor augmenting technical progress. These firms choose capital and
labor to maximize profits, given by

\[ A(s^t)F\left(k(s^{t-1}), (1 + \gamma)^t l(s^t)\right) - w(s^t)l(s^t) - v(s^t)k(s^{t-1}). \]

Households own a technology to produce capital goods. After being used in the production of consumption goods, the stock of capital can be mixed with investment expenditures to produce capital goods in the following period according to the constant returns to scale production function\(^2\)

\[(1 + \eta)k(s^t) = G(k(s^{t-1}), x(s^t)).\]

Feasibility in the final good sector requires

\[ c(s^t) + x(s^t) + (1 + \gamma)^t g(s^t) + m(s^t) = y(s^t). \]

Here \(g(s^t)\) is government consumption and \(m(s^t)\) represents net exports, given by

\[ m(s^t) = b(s^{t-1}) - (1 + \eta)b(s^t)q(s^t). \]

Standard small open economy models with exogenous world interest rates induce a unit root in the equilibrium quantities. Because the unit root complicates the numerical approximation of the equilibrium, the model is rendered stationary by imposing the following debt elastic discount price

\[ \frac{1}{q^*(s^t)} = 1 + r^* + \psi \left[ \exp \left( \frac{b(s^t)}{(1 + \gamma)y(s^t) - \varphi} \right) - 1 \right], \]

where \(r^*\) is a world interest rate, \(\varphi\) is the steady state debt to output ratio, and \(\psi > 0\) measures the sensitivity of the discount price to deviations in the debt to output ratio from \(\varphi\).\(^3\)

\(^2\)This specification allows for technologies with capital adjustment costs as well as the standard capital accumulation equation \((1 + \eta)k(s^t) = (1 - \delta)k(s^{t-1}) + x(s^t)\), where \(\delta\) is a capital depreciation rate.

\(^3\)Although there are several methods to induce stationarity, most of them imply similar business cycle
In equation (7), \( b(s^t) \) and \( y(s^t) \) refer to aggregate variables; therefore, the elasticity of the discount price to \( b(s^t)/y(s^t) \) is not internalized by the household. Nevertheless, when calibrating the model I set \( \psi = 0.001 \), which implies that movements in the debt to output ratio have a small effect on \( q^*(s^t) \)—yet it is sufficient to induce a unique steady state.

An equilibrium allocation of the prototype economy with initial conditions \( k(s^0) \) and \( b(s^0) \) is a path for output \( y(s^t) \), consumption \( c(s^t) \), labor \( l(s^t) \), capital \( k(s^t) \), foreign bonds \( b(s^t) \), and investment \( x(s^t) \) that satisfies the technological constraint (3), the capital accumulation equation (4); the feasibility condition (5); the net exports equation (6), where \( q(s^t) \) satisfies (2) and (7); and the optimality conditions

\[
- \frac{U_l(s^t)}{U_c(s^t)} = A(s^t)F_l(s^t)(1 + \gamma)^t (1 - \pi_l(s^t)) , \tag{8}
\]

\[
q(s^t)U_c(s^t) = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)U_c(s^{t+1}), \tag{9}
\]

\[
\frac{1 + \tau_x(s^t)}{G_x(s^t)} U_c(s^t) = \beta \sum_{s^{t+1}|s^t} \pi(s^{t+1}|s^t)U_c(s^{t+1}) \left\{ A(s^{t+1})F_k(s^{t+1}) + \frac{1 + \tau_x(s^{t+1})}{G_x(s^{t+1})}G_k(s^{t+1}) \right\} , \tag{10}
\]

The term \( \pi(s^{t+1}|s^t) \) is the probability of \( s^{t+1} \) conditional on \( s^t \); \( U_c(s^t) \) and \( U_l(s^t) \) are the marginal utility of consumption and labor; \( F_k(s^t) \) and \( F_l(s^t) \) are the marginal product of capital and labor in the final goods technology; and \( G_k(s^t) \) and \( G_x(s^t) \) are the marginal product of capital and investment in the capital goods technology, all in history \( s^t \).

Equation (8) summarizes the intratemporal labor-consumption choice and the demand for labor, (9) is the intertemporal first order condition with respect to foreign debt, and (10) summarizes the intertemporal Euler equation with respect to capital and the demand for capital.
Observational Equivalence

The five shocks of the previous model have a dubious structural interpretation. Indeed, few economists would agree that random labor or investment taxes are the main driving forces behind business cycle fluctuations. In a recent paper, however, Chari, Kehoe, and McGrattan (2007) prove that a large class of structural models with interpretable primitive shocks and frictions induce reduced form wedges similar to those in the prototype economy. Specifically, they show that we can find a stochastic process for the wedges such that the equilibrium allocation of the structural model coincides with that of the prototype economy. Frictions and shocks in a particular structural model manifest themselves as variations in one or more wedges in the prototype economy.

The mapping from the class of models to the prototype economy, however, is not one to one. More than one model with frictions could induce variations in the same wedge in the prototype economy. Thus, we cannot identify individual models (and therefore, primitive shocks and frictions) from the data. We can, on the other hand, limit the set of models consistent with them: a model is not consistent with the data if its primitive shocks and frictions induce wedges in the prototype economy that do not contribute to observed fluctuations. Equivalently, the identification of the wedges that contribute the most to observed fluctuations can be used as a guide for building detailed models consistent with the data.

In a series of papers, Chari, Kehoe, and McGrattan (2002; 2005; 2007) prove the observational equivalence of several models and a prototype closed economy. A model with sticky wages (Bordo, Erceg, and Evans, 2000) and a model with cartelization and unionization (Cole and Ohanian, 2004) are equivalent to a prototype economy with labor wedges; the models with financial frictions of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) induce investment wedges in the prototype economy; and models with input financing frictions could induce labor wedges (Neumeyer and Perri, 2005; Mendoza, 2008), or efficiency wedges (Christiano,
Gust, and Roldós, 2004; Mendoza, 2008). Because the prototype small open economy is observationally equivalent to Chari, Kehoe, and McGrattan’s closed economy, all these equivalence results extend to the prototype small open economy.

Moreover, some of the above models also induce a risk premium wedge if we interpret them in terms of the prototype open economy: the model with a simple borrowing constraint discussed in Chari, Kehoe, and McGrattan (2005) induces risk premium and government consumption wedges in the prototype open economy; financial frictions as in Kiyotaki and Moore (1997) and Mendoza (2008) induce investment and risk premium wedges in the prototype open economy; and disturbances in foreign interest rates as in Neumeyer and Perri (2005) and Uribe and Yue (2006) induce a risk premium wedge, and indirectly through input financing frictions, a labor wedge in the prototype open economy.

3 The Decomposition of Business Cycles

Having discussed the prototype economy and the equivalence result, this section describes the decomposition methodology. The methodology starts by parameterizing and calibrating the model; next, I discuss how to estimate a stochastic process for the wedges and their realized values; and finally, I measure the contribution of the wedges to aggregate fluctuations in Mexico and Canada by performing counterfactual experiments.

Counterfactual Experiments

The Business Cycle Accounting methodology decomposes the movements in some economic aggregates in terms of movements in one or more wedges. To measure the contribution of the wedges to aggregate fluctuations, I simulate counterfactual economies in which one or more wedges are active (they take their measured values), and the other wedges are inactive (they are set to constants).

The following experiment, for example, measures the contribution of the labor wedge to
aggregate fluctuations. Suppose that we know the stochastic process followed by the state $s_t$, we observe the history $s^t$, and we know the mappings $A(s^t)$, $\tau_l(s^t)$, $\tau_x(s^t)$, $Z(s^t)$, and $g(s^t)$. I construct a counterfactual economy as follows: given the true stochastic process for $s_t$ and history $s^t$, let the labor wedge be as in the actual economy, $1 - \hat{\tau}_l(s^t) = 1 - \tau_l(s^t)$, and map the other wedges to a constant, for instance, their values at time zero: $\hat{A}(s^t) = A(s^0)$, $\hat{\tau}_x(s^t) = \tau_x(s^0)$, $\hat{Z}(s^t) = Z(s^0)$, and $\hat{g}(s^t) = g(s^0)$ for all $t$. The contribution of the labor wedge to aggregate fluctuations is measured by comparing the time series generated by the counterfactual economy with the data. The contribution of the other wedges, in isolation or in combination, is measured in a similar way. Note that, in doing these counterfactual experiments, I change the mappings from $s^t$ to the wedges but keep both, the process followed by $s_t$ and its realized values, the same across experiments. We therefore need to identify $s_t$ and $\pi(s^t)$.

To identify $s_t$ and $\pi(s^t)$, I follow Chari, Kehoe, and McGrattan (2007) and assume that the state follows a five dimensional stationary autoregressive process

$$s_{t+1} - \bar{s} = P(s_t - \bar{s}) + \epsilon_{t+1},$$

where $\bar{s}$ is the mean of $s_t$, $P$ is the matrix on lagged values, and $\epsilon_{t+1}$ is an i.i.d. Gaussian process with mean zero and covariance matrix $V$; thus, the history of shocks $s^t$ is summarized by the current state $s_t$. To identify the state, I assume that there is a one to one mapping from the wedges in the prototype economy to $s_t$. Since the observation of the wedges uniquely identifies the state, let $s_t \equiv (\log A_t, \tau_{lt}, \tau_{xt}, \log Z_t, \log g_t)$ without loss of generality. Therefore, the problem of identifying $s_t$ is reduced to the problem of identifying the wedges.

**Parametrization and Calibration**

Now I state the functional forms representing preferences and the production functions for consumption and capital goods. Next I discuss the calibration of the prototype economy.
Each period in the model represents one quarter. Preferences are given by

\[ U(c, l) = \frac{c^\rho (1 - l)^{1 - \rho}}{1 - \sigma}, \]

where \( \sigma > 0 \), \( 0 < \rho < 1 \), and the time endowment is normalized to 1. The production function for consumption goods is given by \( AF(k, l) = Ak^\alpha l^{1 - \alpha} \) and the one for capital goods by

\[ G(k, x) = x + (1 - \delta)k - 0.5\phi \left( \frac{x}{k - \bar{x}/\bar{k}} \right)^2 k, \quad (12) \]

where \( \bar{x}/\bar{k} \) is the investment to capital ratio in a balanced growth path and \( \phi > 0 \). Note that \( G(k, x) \) is a standard capital accumulation equation with quadratic adjustment costs.

To solve for the equilibrium of the model, all variables dated \( t \) are normalized by level of technology \((1 + \gamma)^t\). Normalized consumption, for example, is given by \( c(s^t)/(1 + \gamma)^t \). Here and throughout the paper, a ‘bar’ above a variable refers to its normalized steady state value.

The population and productivity growth rates \( \eta \) and \( \gamma \), and the parameters \( \bar{s}, P, \) and \( V \) are country specific. The parameter \( \eta \) is set at the average growth rate of the working-age population; the parameter \( \gamma \), at the average growth rate of real output per working-age person; and the parameters \( \bar{s}, P, \) and \( V \) are estimated separately for Mexico and Canada. The element \( \bar{s}_d = \log Z \), however, is fixed by a steady state condition.

A subset of the remaining parameters take standard values: the coefficient of relative risk aversion is set at \( \sigma = 2 \); the capital share in output, at \( \alpha = 0.32 \); the debt elasticity term, at \( \psi = 0.001 \) (Schmitt-Grohé and Uribe, 2003; Aguiar and Gopinath, 2007); and the steady state

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4 The following are the exceptions: hours worked, \( l(s^t) \), and the rental rate of capital, \( v(s^t) \), are already stationary and need not be normalized; capital is normalized relative to the period in which it becomes available for production, \( k(s^{t-1})/(1 + \gamma)^t \); and debt is normalized relative to the period of maturity, \( b(s^{t-1})/(1 + \gamma)^t \).

5 Evaluating equation (7) at the steady state gives

\[ \bar{Z} (1 + r^* + \psi \left[ \exp \left( \bar{b}/\bar{y} - \varphi \right) \right]) \beta(1 + \gamma)^{(1 - \rho) - 1} = 1. \]

If we insist that \( \varphi = \bar{b}/\bar{y} \), this equation determines \( \bar{Z} \) as a function of the parameters. Alternatively, we could allow for \( \bar{b}/\bar{y} \neq \varphi \); then, as we estimate \( Z \), we use the previous equation to find \( \bar{b}/\bar{y} \). Unfortunately, the latter approach delivers highly counterfactual debt to output ratios (of about 300 percent). For this reason I follow the former approach, as does most of the literature, e.g. Aguiar and Gopinath (2007).
external debt to output ratio, $\varphi$, is set at 50 percent in annual terms—Reinhart, Rogoff, and Savastano (2003) report an average ratio of external debt to output of 44 percent for a group of emerging countries with some history of external default, 27 percent for a group of emerging countries with no history of default, and 54 percent for a group of industrial countries.

The remaining parameters are calibrated to match the average moments of a representative small open economy. In this economy, the population and productivity growth rates are 2 percent and 1 percent in annual terms; the world interest rate, $r^*$, and average risk premium, $\log \bar{Z}$, are 4 percent and 3 percent per year; the average efficiency wedge is $\bar{A} = 1$; the labor and investment wedges are $\bar{\tau}_l = \bar{\tau}_x = 0$; the government wedge, $\bar{g}$, is chosen to match a ratio of government consumption to output of 15 percent—the average government consumption to output ratio is 10 percent in Mexico and 20 percent in Canada—; the parameter $\rho$ is set to induce a steady state labor supply of $1/3$; the parameter $\delta$ is chosen to induce a steady state investment to output ratio of 20 percent—the average investment to output ratio is 20.5 percent in Mexico and 21 percent in Canada—; and the capital adjustment cost parameter $\phi$ is set to induce an elasticity of the price of capital with respect to the investment to capital ratio of 25 percent (Chari, Kehoe, and McGrattan, 2007). Table 2 summarizes these numbers.

**Estimation and Measurement of the Wedges**

The parameters $\bar{s}$, $P$, and $V$ of the process (11) are estimated using a maximum likelihood approach and data on output, investment, net exports, hours worked, and government consumption.

The data are quarterly observations on gross domestic product, aggregate investment, hours worked, net exports of goods and services, and government consumption expenditures. Data from Mexico covers the period 1987:1–2007:4; data from Canada, the period 1976:1–

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6Most papers in the literature calibrate the parameter $\phi$ to match the volatility of aggregate investment generated by the model with the data. In the Business Cycle Accounting methodology, however, the model matches the data exactly for any value the parameters. Because it is not possible to follow the traditional approach, I set this number to induce an elasticity of the price of capital with respect to the investment to capital ratio similar to what is estimated for the U.S., 25 percent.
Table 2: *Calibration of the Prototype Economy*

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Aversion (preferences)</td>
<td>$\sigma$</td>
<td>2</td>
</tr>
<tr>
<td>Consumption exponent (preferences)</td>
<td>$\rho$</td>
<td>0.32</td>
</tr>
<tr>
<td>Discount factor (preferences)</td>
<td>$\beta$</td>
<td>0.987</td>
</tr>
<tr>
<td>Capital depreciation rate (annual)</td>
<td>$\delta$</td>
<td>4.7%</td>
</tr>
<tr>
<td>Capital exponent (technology)</td>
<td>$\alpha$</td>
<td>0.32</td>
</tr>
<tr>
<td>Capital adjustment cost</td>
<td>$\phi$</td>
<td>13.1</td>
</tr>
<tr>
<td>World interest rate (annual)</td>
<td>$r^*$</td>
<td>4%</td>
</tr>
<tr>
<td>Debt elasticity term</td>
<td>$\psi$</td>
<td>0.001</td>
</tr>
<tr>
<td>Steady state debt/output ratio (annual)</td>
<td>$\varphi$</td>
<td>50%</td>
</tr>
</tbody>
</table>

The prototype economy is calibrated to match average moments of a representative small open economy.

2007:4. Hours worked is defined as average hours worked per worker in the manufacturing sector multiplied by total employment and divided by total hours available for work. The raw data were first transformed into per working-age person; next, output, investment, net exports, and government consumption were exponentially detrended using the average growth rate of output per working-age person in each country. The sources and additional details on the construction of the data are described in Appendix A.

To estimate the parameters, the model is log-linearized around the steady state and the likelihood function is evaluated using the Kalman filter. (Appendix B describes the estimation procedure in detail.) There are 44 parameters to estimate: 4 in $\bar{s}$, 25 in $P$, and 15 in $V$—because of the symmetry of the covariance matrix, only the lower triangular part of $V$ is estimated. The estimated coefficients for Mexico and Canada are reported in Table 3.

The wedges are estimated after the maximum likelihood step. The government consumption wedge is observed directly. The labor wedge is measured using the condition that equates the marginal rate of substitution between consumption and labor with the marginal product of labor distorted by the labor tax rate. Introducing the proposed functional forms into (8)
and solving for $1 - \tau_l(s^t)$ gives:

$$1 - \tau_l(s^t) = \frac{1 - \rho}{\rho(1 - \alpha)} \left( \frac{l(s^t)}{1 - l(s^t)} \right) c(s^t) / y(s^t).$$

Thus, the realized labor wedge is measured using the series of the consumption-output ratio and hours worked.

The efficiency wedge is measured as a Solow residual: given a guess for the initial capital stock, $k(s^0)$, I construct a series for the stock of capital $k(s^t)$ using investment data and the capital accumulation equation (4). Next, using the series of capital and data on output and hours worked, I obtain the efficiency wedge $A(s^t)$ as the residual in the output equation (3).

To measure the investment and risk premium wedges, I use the policy functions of the estimated model. Specifically, these wedges are estimated using a fixed-interval smoothing algorithm on the log-linearized model. A fixed-interval smoother computes the expectation of an unobservable state in a model written in state space form, conditional on all the information contained in the sample (Hamilton, 1994; Anderson and Moore, 2005). As a by-product, the smoother computes the best estimates, in the mean squared error sense, of capital and debt at the initial period. (Mechanically, all wedges were estimated using the smoother.)

Because there are five wedges and five observable variables, variations in the wedges explain all the movements in the data. Thus, the methodology decomposes fluctuations in the five observable variables in terms of five wedges.

4 Results

This section applies the decomposition methodology to measure the contribution of the wedges (the efficiency wedge, $A_t$, the labor wedge, $1 - \tau_l$, the investment wedge, $1/(1 + \tau_xt)$, the risk premium wedge, $1/Z_t$, and the government consumption wedge, $g_t$) to aggregate fluctuations in Mexico and Canada. I start by considering general fluctuations at the business cycle
Table 3: Estimated stochastic process for the wedges: Mexico and Canada$^a$

<table>
<thead>
<tr>
<th>Means $\bar{s}$</th>
<th>Matrix $P$ on lagged values</th>
<th>Matrix $Q$, where $V = QQ^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23</td>
<td>0.92 -0.05 0.08 1.40 0.02</td>
<td>1.17 0 0 0 0</td>
</tr>
<tr>
<td>0.29</td>
<td>-0.01 0.91 0.04 0.35 0.02</td>
<td>-0.08 1.20 0 0 0</td>
</tr>
<tr>
<td>0.19</td>
<td>-0.06 0.12 0.79 -5.65 0.07</td>
<td>0.83 -1.06 1.90 0 0</td>
</tr>
<tr>
<td>0.007</td>
<td>0.001 0.01 -0.004 0.87 -0.002</td>
<td>-0.09 0.02 0.00 0.05 0</td>
</tr>
<tr>
<td>-2.29</td>
<td>0.56 0.00 0.03 3.04 0.62</td>
<td>0.31 0.03 0.51 1.45 2.07</td>
</tr>
</tbody>
</table>

| 0.15            | 0.88 -0.01 -0.03 1.14 -0.02 | 0.76 0 0 0 0 |
| 0.51            | -0.11 0.95 -0.07 0.46 0.00 | 0.50 0.57 0 0 0 |
| -0.25           | -0.04 -0.05 0.95 -0.39 0.02 | 0.06 -0.39 0.91 0 0 |
| 0.09            | 0.01 0.01 0.01 0.83 0.00 | -0.07 -0.01 -0.01 0.06 0 |
| -1.46           | 0.05 -0.13 -0.13 -0.10 0.97 | 0.08 -0.18 0.50 -0.28 1.11 |

$^a$ This table shows the estimated parameters of the stochastic process (11) for the wedges. The model is log-linearized around the steady state and the likelihood function is evaluated using the Kalman filter. The observable variables are output, investment, net exports, hours, and government consumption.

$^b$ The entries of the matrix $Q$ are multiplied by 100 for easier reading.

frequency, and next I study two recession episodes: the 1995 Mexican crisis and the 1983 Canadian recession.

In summary, the results from this section can be described as follows. First, all wedges, except for the government consumption wedge, are substantially more volatile in Mexico. Second, fluctuations in the risk premium wedge explain the ‘deep’ differences between Mexico and Canada: the excess volatility of consumption over output and the highly countercyclical share of net exports on output in Mexico. Third, the risk premium wedge has a negligible effect in Canada’s business cycles, except to explain the volatility of net exports. Fourth, the government consumption wedge does not play any role in either country. And finally, the investment wedge has negligible effects in Mexico’s fluctuations, and only contributes to the behavior of investment and net exports in Canada.
Decomposition of Business Cycles

Consider, first, the properties of the estimated wedges. Tables 4 and 5 report some summary statistics of the estimated wedges in Mexico and Canada respectively. Each table is divided into two panels. The top panel reports the standard deviation of the estimated wedges and the correlation of the wedges with each of the observable variables: output, consumption, investment, hours, and net exports. The lower panel reports the standard deviation of the wedges relative that of output and the correlation matrix of the estimated wedges.

Comparing the second column of the top panel of Table 4 with that of Table 5, we observe that, except for the government consumption wedge, all wedges in Mexico are substantially more volatile than in Canada: the efficiency wedge by over 60 percent, the labor wedge by over 30 percent, the investment wedge by over 80 percent, and the risk premium wedge by 50 percent. Note that these magnitudes are in line with the volatility of output, which is almost 50 percent higher in Mexico (Table 1). These numbers, on the other hand, are somewhat different when measured relative to the volatility of output (second column of the lower panel of Tables 4 and 5). While the relative volatility of the efficiency and investment wedges is higher in Mexico, the relative volatility of the labor wedge is higher in Canada, and the relative volatility of the risk premium wedge is similar in both countries.

Consider now the correlation of the wedges with the data. The upper panel of Table 4 shows that, in Mexico, output, consumption, investment, and hours worked are all positively correlated with the efficiency wedge, with the labor wedge, with the risk premium wedge, and with the government consumption wedge. Output and consumption, however, are both negatively correlated with the investment wedge. In other words, these correlations suggest that downturns in Mexico are periods in which productivity is low, periods in which labor decisions and intertemporal consumption choices are relatively more distorted, but are periods

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7In computing these numbers, the estimated wedges were first HP-filtered with a smoothing parameter of 1600. GMM-based standard errors are reported in parentheses in these and all the tables that follow. The optimal weighting matrix is computed using the Newey-West estimator with a lag length equal to one fourth of the sample size.
in which investment decisions are relatively less distorted.

In Canada, on the other hand, the co-movements are somewhat different. First, the investment wedge is positively correlated with output and consumption, meaning that investment decisions are indeed relatively more distorted in downturns (upper panel of Table 5). Second, the risk premium wedge is virtually uncorrelated with output, consumption, and investment, but is negatively correlated with hours and net exports. Finally, and in contrast with Mexico, the government consumption wedge is positively correlated with output. This finding is consistent with the observation that fiscal policy is countercyclical in developed countries but procyclical in developing countries (Kaminsky, Reinhart, and Vegh, 2005).8

The remainder of this section describes the contribution of the different wedges, both in isolation and in combination with other wedges, to aggregate fluctuations in Mexico and Canada. Tables 6 and 7 display some summary statistics of the counterfactual economies with different active and inactive wedges. In reporting these results, I first added an exponential trend equal to the sum of the estimated productivity and population growth rates to the counterfactual time series, and next I HP-filtered the resulting series. This transformation replicates the transformation applied to the data.

The counterfactual experiments are divided into models with one active wedge (panel A), models with two active wedges (panel B), and so on. The second and third columns provide a measure of how well the different models match output: the second column reports the relative standard deviation of counterfactual output to actual output and the third column measures the correlation between the two output series. If both of these numbers are close to one, the model fits output reasonably well. The last four columns report some summary statistic of the counterfactual series: the volatility of the share of net exports on output, the volatility of consumption and investment relative to that of output, and the correlation between output and the share of net exports on output. The first row of these tables rewrites the summary

8The higher volatility of the risk premium wedge and its positive and significant correlation with most economic aggregates in Mexico but not in Canada is consistent with the findings in Neumeyer and Perri (2005) and Uribe and Yue (2006).
Table 4: Properties of estimated wedges in Mexico

<table>
<thead>
<tr>
<th>Wedge</th>
<th>$\sigma_w$</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Net Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>1.58 (.26)</td>
<td>0.92 (.03)</td>
<td>0.86 (.05)</td>
<td>0.73 (.14)</td>
<td>0.37 (.17)</td>
<td>-0.71 (.13)</td>
</tr>
<tr>
<td>Labor</td>
<td>2.35 (.41)</td>
<td>0.76 (.08)</td>
<td>0.83 (.05)</td>
<td>0.69 (.08)</td>
<td>0.86 (.02)</td>
<td>-0.79 (.08)</td>
</tr>
<tr>
<td>Investment</td>
<td>2.58 (.33)</td>
<td>-0.25 (.15)</td>
<td>-0.51 (.13)</td>
<td>0.21 (.09)</td>
<td>0.18 (.11)</td>
<td>0.19 (.11)</td>
</tr>
<tr>
<td>Risk Prem.</td>
<td>0.15 (.03)</td>
<td>0.75 (.07)</td>
<td>0.86 (.04)</td>
<td>0.63 (.11)</td>
<td>0.16 (.17)</td>
<td>-0.82 (.06)</td>
</tr>
<tr>
<td>Govt. Cons.</td>
<td>0.26 (.03)</td>
<td>0.41 (.05)</td>
<td>0.39 (.04)</td>
<td>0.19 (.09)</td>
<td>0.20 (.07)</td>
<td>-0.33 (.07)</td>
</tr>
</tbody>
</table>

Correlation matrix of wedges

<table>
<thead>
<tr>
<th>Wedge</th>
<th>$\sigma_w/\sigma_y$</th>
<th>Efficiency</th>
<th>Labor</th>
<th>Investment</th>
<th>Risk Prem.</th>
<th>Govt. Cons.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>0.74 (.04)</td>
<td>1</td>
<td>0.51 (.16)</td>
<td>-0.37 (.16)</td>
<td>0.86 (.04)</td>
<td>0.41 (.05)</td>
</tr>
<tr>
<td>Labor</td>
<td>1.11 (.08)</td>
<td>1</td>
<td></td>
<td>-0.24 (.10)</td>
<td>0.50 (.14)</td>
<td>0.27 (.07)</td>
</tr>
<tr>
<td>Investment</td>
<td>1.22 (.19)</td>
<td>1</td>
<td></td>
<td>-0.51 (.10)</td>
<td></td>
<td>-0.35 (.07)</td>
</tr>
<tr>
<td>Risk Prem.</td>
<td>0.07 (.01)</td>
<td>1</td>
<td></td>
<td></td>
<td>0.26 (.07)</td>
<td></td>
</tr>
<tr>
<td>Govt. Cons.</td>
<td>0.12 (.02)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

This table shows the properties of the estimated wedges in Mexico for the period 1987:1–2007:4. The estimated wedges are detrended using the Hodrick Prescott filter with smoothing parameter of 1600. The column $\sigma_w$ reports the standard deviation of the wedges, while $\sigma_w/\sigma_y$ reports the standard deviation of the wedges relative to the standard deviation of output. GMM-based standard errors are reported in parentheses.

In measuring the contribution of, say, the labor wedge to aggregate fluctuations, two counterfactual economies are especially illustrative. In one economy, only the labor wedge moves as in the data and the other wedges are fixed at their average values in the sample. In the other economy, the labor wedge is fixed at its average value and the rest of the wedges move as in the data. The first experiment measures the direct contribution of the labor wedge: the closer are the counterfactual time series to the data, the more important is the direct contribution of the labor wedge to aggregate fluctuations. The second experiment measures the contribution of the labor wedge when combined with other wedges: the farther away are the predicted time series from the actual data, the more important is the contribution of the
Table 5: Properties of estimated wedges in Canada

<table>
<thead>
<tr>
<th>Wedge</th>
<th>$\sigma_w$</th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Hours</th>
<th>Net Exports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency</td>
<td>0.96 (.10)</td>
<td>0.61 (.07)</td>
<td>0.20 (.14)</td>
<td>0.35 (.10)</td>
<td>−0.08 (.22)</td>
<td>0.35 (.08)</td>
</tr>
<tr>
<td>Labor</td>
<td>1.76 (.17)</td>
<td>0.36 (.25)</td>
<td>0.75 (.08)</td>
<td>0.37 (.21)</td>
<td>0.81 (.06)</td>
<td>−0.46 (.06)</td>
</tr>
<tr>
<td>Investment</td>
<td>1.41 (.18)</td>
<td>0.67 (.08)</td>
<td>0.16 (.16)</td>
<td>0.91 (.02)</td>
<td>0.64 (.11)</td>
<td>−0.16 (.15)</td>
</tr>
<tr>
<td>Risk Prem.</td>
<td>0.10 (.01)</td>
<td>0.01 (.10)</td>
<td>0.12 (.10)</td>
<td>0.07 (.12)</td>
<td>−0.49 (.09)</td>
<td>−0.30 (.07)</td>
</tr>
<tr>
<td>Govt. Cons.</td>
<td>0.34 (.03)</td>
<td>−0.33 (.12)</td>
<td>−0.04 (.15)</td>
<td>−0.51 (.06)</td>
<td>−0.37 (.13)</td>
<td>−0.12 (.09)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Correlation matrix of wedges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wedge</td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Efficiency</td>
</tr>
<tr>
<td>Labor</td>
</tr>
<tr>
<td>Investment</td>
</tr>
<tr>
<td>Risk Prem.</td>
</tr>
<tr>
<td>Govt. Cons.</td>
</tr>
</tbody>
</table>

This table shows the properties of the estimated wedges in Canada for the period 1976:1–2007:4. The estimated wedges are detrended using the Hodrick Prescott filter with smoothing parameter of 1600. The column $\sigma_w$ reports the standard deviation of the wedges, while $\sigma_w/\sigma_y$ reports the standard deviation of the wedges relative to the standard deviation of output. GMM-based standard errors are reported in parentheses.

Consider, first, counterfactual economies with only one active wedge. These experiments, displayed in panel A of Tables 6 and 7, suggest that only the efficiency and labor wedges could account, by themselves, for an important fraction of output fluctuations in both countries. The models with just the efficiency wedge and just the labor wedge, however, completely miss the volatility of consumption and investment relative to that of output, and the negative correlation between output and the share of net exports on output in Mexico. Likewise, in Canada, the models with only efficiency wedges and only labor wedges predict a large positive correlation between output and the share of net exports in output, and understate

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\footnote{Because the government consumption wedge plays a negligible role in both countries, panels B and C of Tables 6 and 7 do not include experiments with the government consumption wedge.}
the volatility of investment relative to that of output. These models, on the other hand, are consistent with the smaller volatility of consumption relative to that of output in Canada.

The investment, risk premium, and government consumption wedges, by themselves, cannot explain aggregate fluctuations in either country. In Mexico, the economy with just the investment wedge accounts for only 10 percent of output volatility, grossly overstates the volatility of consumption and investment relative to that of output, and misses the negative correlation between output and the share of net exports on output. In the economy with just the risk premium wedge, predicted and actual output are negatively correlated, and the correlation of output with the share of net exports on output is extremely large. Finally, the economy with just the government consumption wedge accounts for only 4 percent of the output volatility and misses the volatility of the share of net exports on output and the correlation between output and the share of net exports on output. Likewise, in Canada, the model with just the investment wedge explains only 16 percent of the volatility of output and also completely overstates the volatility of consumption and investment relative to that of output. In the model with just the risk premium wedge, predicted output is negatively correlated with actual output, consumption is more volatile than output, and the correlation of output and the share of net exports on output is almost one. Finally, the model with just the government consumption wedge accounts for only 7 percent of the volatility of output.

Consider now counterfactual economies with all wedges but one. Panel D of Table 6 suggests that three wedges are essential to understand business cycles in Mexico: the efficiency wedge, the labor wedge, and the risk premium wedge. Eliminating any of these wedges causes the model to miss the data in some dimension. In the model with no efficiency wedges, predicted output is 54 percent less volatile than actual output and their correlation is just 0.53, investment is over eight times more volatile than output, and the correlation between output and the share of net exports on output is only -0.17. In the model with no labor wedges, predicted output is 43 percent less volatile than actual output and their correlation is only 0.41, the volatility of consumption and investment relative to that of output is overstated, and
Table 6: Contribution of the wedges to aggregate fluctuations in Mexico

<table>
<thead>
<tr>
<th>Wedges/Moments</th>
<th>$\sigma(y^o)/\sigma(y)$</th>
<th>$\rho(y^o, y)$</th>
<th>$\sigma(m^o/y^o)$</th>
<th>$\sigma(c^o)/\sigma(y^o)$</th>
<th>$\sigma(x^o)/\sigma(y^o)$</th>
<th>$\rho(m^o/y^o, y^o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>–</td>
<td>–</td>
<td>1.63 (.43)</td>
<td>1.28 (.05)</td>
<td>4.51 (.43)</td>
<td>−0.80 (.11)</td>
</tr>
<tr>
<td><strong>A. Economies with just one wedge</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency</td>
<td>0.85 (.05)</td>
<td>0.89 (.04)</td>
<td>1.12 (.18)</td>
<td>0.71 (.06)</td>
<td>1.78 (.26)</td>
<td>0.48 (.14)</td>
</tr>
<tr>
<td>Labor</td>
<td>0.93 (.05)</td>
<td>0.83 (.06)</td>
<td>1.70 (.31)</td>
<td>0.34 (.06)</td>
<td>0.73 (.09)</td>
<td>0.95 (.02)</td>
</tr>
<tr>
<td>Investment</td>
<td>0.10 (.01)</td>
<td>0.46 (.16)</td>
<td>1.32 (.19)</td>
<td>3.01 (.23)</td>
<td>32.9 (4.7)</td>
<td>0.24 (0.09)</td>
</tr>
<tr>
<td>Risk Prem.</td>
<td>0.66 (.02)</td>
<td>−0.91 (.03)</td>
<td>3.39 (.71)</td>
<td>1.01 (.02)</td>
<td>4.33 (.13)</td>
<td>0.99 (.03)</td>
</tr>
<tr>
<td>Government</td>
<td>0.04 (.01)</td>
<td>0.65 (.08)</td>
<td>0.23 (.03)</td>
<td>1.05 (.05)</td>
<td>2.67 (.16)</td>
<td>0.04 (.20)</td>
</tr>
<tr>
<td><strong>B. Economies with two active wedges</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effic., Labor</td>
<td>1.55 (.03)</td>
<td>0.99 (.02)</td>
<td>1.89 (.52)</td>
<td>0.40 (.03)</td>
<td>0.99 (.07)</td>
<td>0.95 (.02)</td>
</tr>
<tr>
<td>Effic., Invest.</td>
<td>0.88 (.03)</td>
<td>0.91 (.04)</td>
<td>0.68 (.07)</td>
<td>0.81 (.12)</td>
<td>3.33 (.26)</td>
<td>0.72 (.03)</td>
</tr>
<tr>
<td>Effic., Risk Pr.</td>
<td>0.57 (.09)</td>
<td>0.27 (.07)</td>
<td>3.65 (.60)</td>
<td>2.38 (.35)</td>
<td>7.41 (.90)</td>
<td>0.23 (.08)</td>
</tr>
<tr>
<td>Labor, Invest.</td>
<td>0.99 (.06)</td>
<td>0.83 (.05)</td>
<td>2.65 (.35)</td>
<td>0.46 (.07)</td>
<td>3.82 (.61)</td>
<td>0.77 (.06)</td>
</tr>
<tr>
<td>Labor, Risk Pr.</td>
<td>0.41 (.08)</td>
<td>0.41 (.07)</td>
<td>1.99 (.48)</td>
<td>2.00 (.40)</td>
<td>7.50 (1.5)</td>
<td>−0.26 (.11)</td>
</tr>
<tr>
<td>Invest., Risk Pr.</td>
<td>0.61 (.03)</td>
<td>−0.91 (.03)</td>
<td>3.12 (.85)</td>
<td>0.92 (.02)</td>
<td>5.91 (.49)</td>
<td>0.94 (.03)</td>
</tr>
<tr>
<td><strong>C. Economies with three active wedges</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eff., Lab., Inv.</td>
<td>1.60 (.03)</td>
<td>0.98 (.03)</td>
<td>2.20 (.36)</td>
<td>0.46 (.06)</td>
<td>2.05 (.22)</td>
<td>0.92 (.03)</td>
</tr>
<tr>
<td>Eff., Lab., R. Pr.</td>
<td>0.93 (.02)</td>
<td>0.99 (.05)</td>
<td>2.15 (.38)</td>
<td>1.53 (.07)</td>
<td>4.66 (.19)</td>
<td>−0.66 (.05)</td>
</tr>
<tr>
<td>Eff., Inv., R. Pr.</td>
<td>0.55 (.08)</td>
<td>0.36 (.07)</td>
<td>2.99 (.07)</td>
<td>2.16 (.27)</td>
<td>7.45 (1.4)</td>
<td>0.02 (.13)</td>
</tr>
<tr>
<td>Lab., Inv., R. Pr.</td>
<td>0.45 (.07)</td>
<td>0.487 (.10)</td>
<td>2.16 (.56)</td>
<td>1.63 (.21)</td>
<td>8.88 (1.7)</td>
<td>−0.12 (.14)</td>
</tr>
<tr>
<td><strong>D. Economies with four active wedges</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All but Efficiency</td>
<td>0.46 (.07)</td>
<td>0.53 (.09)</td>
<td>2.15 (.60)</td>
<td>1.58 (.21)</td>
<td>8.74 (1.7)</td>
<td>−0.17 (.14)</td>
</tr>
<tr>
<td>All but Labor</td>
<td>0.57 (.09)</td>
<td>0.41 (.07)</td>
<td>3.02 (.70)</td>
<td>2.15 (.27)</td>
<td>7.37 (1.4)</td>
<td>−0.02 (.13)</td>
</tr>
<tr>
<td>All but Investment</td>
<td>0.95 (.02)</td>
<td>1.00 (.00)</td>
<td>2.18 (.39)</td>
<td>1.51 (.07)</td>
<td>4.63 (.18)</td>
<td>−0.67 (.04)</td>
</tr>
<tr>
<td>All but Risk Prem.</td>
<td>1.62 (.03)</td>
<td>0.98 (.03)</td>
<td>2.19 (.35)</td>
<td>0.47 (.06)</td>
<td>2.00 (.20)</td>
<td>0.92 (.02)</td>
</tr>
<tr>
<td>All but Government</td>
<td>0.97 (.00)</td>
<td>1.00 (.00)</td>
<td>1.61 (.43)</td>
<td>1.30 (.05)</td>
<td>4.59 (.42)</td>
<td>−0.79 (.12)</td>
</tr>
</tbody>
</table>

This table reports the contribution of the wedges to aggregate fluctuations in Mexico. For consistency when comparing the models with the data, the counterfactual series were exponentially detrended using the average population and output growth rates, and then detrended using the Hodrick Prescott filter with a smoothing parameter of 1600. The statistic $\sigma(x)$ measures the standard deviation of the series $x$, while $\rho(x, y)$ measures the correlation coefficient between the series $x$ and $y$. Counterfactual time series are denoted with a superscript $\circ$. The variable $y$ denotes output; $m$, net exports; and $c$, private consumption. GMM-based standard errors are reported in parentheses.
Table 7: Contribution of the wedges to aggregate fluctuations in Canada

<table>
<thead>
<tr>
<th>Wedges/Moments</th>
<th>$\sigma(y^o)/\sigma(y)$</th>
<th>$\rho(y^o,y)$</th>
<th>$\sigma(m^o/y^o)$</th>
<th>$\sigma(c^o)/\sigma(y^o)$</th>
<th>$\sigma(x^o)/\sigma(y^o)$</th>
<th>$\rho(m^o/y^o,y^o)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>–</td>
<td>–</td>
<td>0.91 (.10)</td>
<td>0.85 (.08)</td>
<td>4.60 (.23)</td>
<td>–0.05 (.24)</td>
</tr>
<tr>
<td><strong>A. Economies with just one wedge</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency</td>
<td>1.09 (.23)</td>
<td>0.61 (.06)</td>
<td>1.23 (.11)</td>
<td>0.64 (.07)</td>
<td>0.86 (.07)</td>
<td>0.93 (.02)</td>
</tr>
<tr>
<td>Labor</td>
<td>0.99 (.16)</td>
<td>0.32 (.25)</td>
<td>0.87 (.10)</td>
<td>0.78 (.07)</td>
<td>1.12 (.05)</td>
<td>0.77 (.02)</td>
</tr>
<tr>
<td>Investment</td>
<td>0.16 (.01)</td>
<td>0.11 (.13)</td>
<td>1.29 (.17)</td>
<td>4.53 (.50)</td>
<td>22.8 (1.6)</td>
<td>–0.17 (.07)</td>
</tr>
<tr>
<td>Risk Prem.</td>
<td>0.33 (.07)</td>
<td>–0.10 (.22)</td>
<td>1.01 (.11)</td>
<td>1.35 (.20)</td>
<td>3.70 (.22)</td>
<td>0.97 (.00)</td>
</tr>
<tr>
<td>Government</td>
<td>0.07 (.01)</td>
<td>0.59 (.04)</td>
<td>0.33 (.02)</td>
<td>1.08 (.14)</td>
<td>3.76 (.50)</td>
<td>0.14 (.11)</td>
</tr>
<tr>
<td><strong>B. Economies with two active wedges</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eff., Labor</td>
<td>1.07 (.07)</td>
<td>0.91 (.04)</td>
<td>0.88 (.14)</td>
<td>0.68 (.06)</td>
<td>1.17 (.07)</td>
<td>0.87 (.04)</td>
</tr>
<tr>
<td>Eff., Invest.</td>
<td>1.11 (.24)</td>
<td>0.61 (.06)</td>
<td>1.86 (.14)</td>
<td>1.00 (.15)</td>
<td>3.71 (.69)</td>
<td>0.54 (.13)</td>
</tr>
<tr>
<td>Eff., Risk Pr.</td>
<td>1.09 (.27)</td>
<td>0.58 (.09)</td>
<td>1.14 (.12)</td>
<td>0.54 (.06)</td>
<td>1.10 (.06)</td>
<td>0.92 (.01)</td>
</tr>
<tr>
<td>Labor, Invest.</td>
<td>0.96 (.15)</td>
<td>0.35 (.24)</td>
<td>0.90 (.05)</td>
<td>0.89 (.10)</td>
<td>4.01 (.60)</td>
<td>0.12 (.29)</td>
</tr>
<tr>
<td>Labor, Risk Pr.</td>
<td>0.93 (.16)</td>
<td>0.31 (.21)</td>
<td>1.29 (.09)</td>
<td>0.90 (.07)</td>
<td>1.93 (.17)</td>
<td>0.62 (.04)</td>
</tr>
<tr>
<td>Invest., Risk Pr.</td>
<td>0.42 (.07)</td>
<td>–0.04 (.21)</td>
<td>1.81 (.21)</td>
<td>1.80 (.15)</td>
<td>10.0 (1.3)</td>
<td>0.52 (.08)</td>
</tr>
<tr>
<td><strong>C. Economies with three active wedges</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eff., Lab., Inv.</td>
<td>1.06 (.05)</td>
<td>0.94 (.03)</td>
<td>0.71 (.04)</td>
<td>0.89 (.07)</td>
<td>3.92 (.18)</td>
<td>–0.01 (.15)</td>
</tr>
<tr>
<td>Eff., Lab., R. Pr.</td>
<td>0.96 (.02)</td>
<td>0.98 (.00)</td>
<td>0.86 (.06)</td>
<td>0.79 (.04)</td>
<td>1.69 (.18)</td>
<td>0.66 (.07)</td>
</tr>
<tr>
<td>Eff., Inv., R. Pr.</td>
<td>1.12 (.27)</td>
<td>0.57 (.09)</td>
<td>1.89 (.20)</td>
<td>0.82 (.13)</td>
<td>3.93 (.85)</td>
<td>0.51 (.14)</td>
</tr>
<tr>
<td>Lab., Inv., R. Pr.</td>
<td>0.92 (.16)</td>
<td>0.33 (.19)</td>
<td>1.50 (.13)</td>
<td>1.03 (.10)</td>
<td>4.71 (.69)</td>
<td>0.20 (.22)</td>
</tr>
<tr>
<td><strong>D. Economies with four active wedges</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All but Efficiency</td>
<td>0.93 (.15)</td>
<td>0.36 (.19)</td>
<td>1.54 (.13)</td>
<td>1.03 (.05)</td>
<td>4.74 (.66)</td>
<td>0.28 (.20)</td>
</tr>
<tr>
<td>All but Labor</td>
<td>1.16 (.28)</td>
<td>0.59 (.09)</td>
<td>1.80 (.19)</td>
<td>0.56 (.08)</td>
<td>3.94 (.81)</td>
<td>0.57 (.13)</td>
</tr>
<tr>
<td>All but Investment</td>
<td>1.00 (.02)</td>
<td>0.99 (.00)</td>
<td>1.10 (.09)</td>
<td>0.65 (.06)</td>
<td>1.56 (.15)</td>
<td>0.70 (.07)</td>
</tr>
<tr>
<td>All but Risk Prem.</td>
<td>1.08 (.05)</td>
<td>0.95 (.02)</td>
<td>0.56 (.04)</td>
<td>0.66 (.03)</td>
<td>3.94 (.20)</td>
<td>0.22 (.09)</td>
</tr>
<tr>
<td>All but Government</td>
<td>0.96 (.01)</td>
<td>1.00 (.00)</td>
<td>0.93 (.10)</td>
<td>0.98 (.06)</td>
<td>4.67 (.25)</td>
<td>–0.26 (.24)</td>
</tr>
</tbody>
</table>

This table reports the contribution of the wedges to aggregate fluctuations in Canada. For consistency when comparing the models with the data, the counterfactual series were exponentially retrended using the average population and output growth rates, and then detrended using the Hodrick Prescott filter with a smoothing parameter of 1600. The statistic $\sigma(x)$ measures the standard deviation of the series $x$, while $\rho(x,y)$ measures the correlation coefficient between the series $x$ and $y$. Counterfactual time series are denoted with a superscript $o$. The variable $y$ denotes output; $m$, net exports; and $c$, private consumption. GMM-based standard errors are reported in parentheses.
the correlation of output with the share of net exports on output is zero. Finally, in the model with no risk premium wedges, predicted output is 62 percent more volatile than actual output, consumption is substantially less volatile than output, the volatility of investment relative to that of output is understated, and the share of net exports on output is highly procyclical. The investment and government consumption wedges, on the other hand, can be eliminated from the model without severely affecting the ability of the model to match the data. Interestingly, the investment wedge does not even contribute to the volatility of aggregate investment: the model with no investment wedges matches the volatility of investment relative to that of output remarkably well.

Panel D of table 7, on the other hand, suggest that two wedges are essential to understand business cycles in Canada: the efficiency wedge and the labor wedge. There is, however, some role for the investment wedge and a small role for the risk premium wedge. The model with no investment wedges predicts a substantially lower volatility of investment compared to that in the data and a highly procyclical share of net exports on output; and the model with no risk premium wedges understate the volatility of the share of net exports on output.

Overall, these findings suggest that business cycles in Mexico can be explained by the combined effect of the efficiency wedge, the labor wedge, and the risk premium wedge. Moreover, the risk premium wedge is the key to understand the excess volatility of consumption over output and the highly countercyclical share of net exports on output. Business cycles in Canada, on the other hand, can be explained reasonably well by the combined effect of the efficiency wedge and the labor wedge. The investment wedge contributes somewhat to the behavior of investment and net exports, and the risk premium wedge has, if anything, a secondary role.

Two Recession Episodes

Now I apply the decomposition methodology to study two recession episodes: the 1995 Mexican ‘Tequila’ crisis and the 1983 Canadian recession. In summary, the results of this section
are consistent with the findings in the previous section with two exceptions: the investment wedge contributes to the recovery of output and labor in Canada, and the worsening in the risk premium wedge prevented an even larger drop in output and labor in Canada.

The 1995 Mexican Crisis

The upper panel of Figure 1 shows output and the estimated wedges in the 1995 Mexican crisis. The series have been normalized to equal 100 at the beginning of the episode and the units of the risk premium wedge are shown on the right vertical axis. Output falls 12 percent in two quarters and remains below trend until 2000. The efficiency, labor, and risk premium wedges deteriorate throughout the recession, although by 1998, the labor wedge fully recovers. The investment wedge, on the other hand, improves substantially throughout the episode, suggesting that investment decisions were, in fact, relatively less distorted during the Mexican crisis. The solid blue lines in Figure 2 display Mexican data. Labor (hours) is normalized to equal 100 at the beginning of the recession; investment and net exports are normalized by the initial level of output, and are shown in percentage points. Labor initially drops 3 percent but soon recovers, and by 1998 it is over 4 percent above its initial value. Investment drops abruptly and co-moves closely with output. Finally, net exports move sharply from a deficit of 4 percent to a surplus of 4 percent and remain in surplus until 1998.

The contribution of the wedges to the Mexican crisis is shown in Figures 2 and 3. These figures report counterfactual experiments in which the active wedges are set to their estimated values and the inactive wedges are set at their values at the beginning of the crisis. Consider, first, the direct contribution of the efficiency wedge. The model with just the efficiency wedge, shown in the upper panel of Figure 2, matches the evolution of output and labor remarkably well until 1997. Thereafter, the model predicts a somewhat slower recovery compared with that in the data. The model also misses the evolution of investment and net exports throughout the episode: predicted investment declines by a small amount and predicted net exports actually drops instead of increasing.
Figure 1: Output and four wedges in two recessions. (Data normalized to equal 100 before the recession; risk premium wedge is on left axis.)
Consider next the labor wedge. The model with just the labor wedge, also shown in the upper panel of Figure 2, predicts a fall in output of about one third of the fall in the data and a fall in labor larger than that in the data. This model captures the recovery and most ups and downs in output and labor, but also fails to match investment and net exports.

The models with just the investment wedge and with just the risk premium wedge, shown in the lower panel of Figure 2, cannot explain the crisis. The model with just the investment wedge completely misses the data: it predicts a steady increase in output and labor, and a small initial decline in investment followed by a strong increase since mid-1995. The model with just the risk premium wedge predicts an increase in output and labor, a small decline in investment, and a large increase in net exports. Note, however, that the risk premium wedge is the only wedge that, by itself, drives an increase in net exports.

Consider now economies with all wedges but one. In the model with no efficiency wedge, displayed in the upper panel of Figure 3, predicted output actually increases and predicted labor fluctuates until mid-1996 but matches the data closely thereafter. The model predicts an initial fall in investment of about two thirds of the fall in the data and matches its recovery almost perfectly thereafter. In addition, the model overstates the initial increase in net exports but matches that series closely after 1996. In other words, the efficiency wedge contributes substantially to the behavior of output, to the initial decline in labor, and somewhat to the drop in investment. Note, however, that because this model predicts a recovery similar to that in the data (saving the level of output), it has to be that other wedges are mainly responsible for the recovery after 1996.

Consider now the model with no labor wedge, also shown in the upper panel of Figure 3. In this model, predicted output drops about half of the drop in the data and recovers faster than actual output does; predicted labor increases throughout the episode; predicted investment matches the data almost perfectly; and predicted net exports increase substantially more than actual net exports do. That is, the labor wedge contributes substantially to the behavior of output, labor, and net exports; and to the recovery since 1996.
Figure 2: Data and counterfactual models with one wedge.
Figure 3: Data and counterfactual models with four wedges.
Now consider the model with no risk premium wedge, shown in the lower panel of Figure 3. This model predicts a drop in output and labor substantially larger than those in the data; a drop in investment about half of the drop in the data, and a large drop in net exports. In addition, this model does not match the behavior of consumption either (not shown in these figures). In other words, the risk premium wedge contributes substantially to the Mexican crisis. While its contribution to the behavior of output and labor is important, the most important role of the risk premium wedge is in the behavior of net exports: without that wedge, the model is unable to explain the large increase in net exports and the behavior of consumption.\footnote{The large drop in output and labor in the model with no risk premium wedges is likely to disappear if I change the period utility function and adopt the preferences proposed by Greenwood, Hercowitz, and Huffman (1988). With these preferences, changes in interest rates do not have a wealth effect on labor supply; therefore, the worsening in the risk premium wedge need not imply an increase in hours and, therefore, in output. These preferences, however, are inconsistent with a balanced growth path unless one assumes that the disutility of work increases at the growth rate of technology, an undesirable feature in preferences.}

Consider, finally, the model with no investment wedges. The plots in the lower panel of Figure 3 show that shutting down the investment wedge does not affect the ability of the model to match output, labor, and net exports. Because the model with no investment wedges cannot explain the full drop in investment and its recovery, I conclude that, if anything, the investment wedge only contributes to the behavior of investment in the Mexican crisis.

Summarizing, these findings are consistent with those of the previous section and suggest that the efficiency wedge, the labor wedge, and the risk premium wedge account for most of the aggregate behavior in the Mexican 1995 crisis. Moreover, among all wedges, only the risk premium wedge is able to account for the behavior of net exports in the Mexican crisis.

The 1983 Canadian Recession

Now I apply the decomposition methodology to the 1983 Canadian recession. The lower panel of Figure 1 displays output and the estimated wedges in that episode. Output drops by 9 percent at the trough of the recession. There is a small decline in the efficiency wedge, which
returns to trend by 1984. The labor wedge worsens substantially and tracks output closely. The investment wedge also follows output closely until 1984; afterwards, output recovers but the investment wedge does not. The risk premium wedge, on the other hand, remains flat until mid-1983; then it worsens slightly but recovers by the second quarter of 1984. The solid blue lines in Figure 4 displays Canadian data. Investment and labor co-move closely with output, although investment recovers more slowly than output does. Net exports, on the other hand, increase from 0 to over 4 percent and remain in surplus for several years.

Consider first counterfactual economies with only one active wedge. The upper panel of Figure 4 shows the prediction of the models with just the efficiency wedge and just the labor wedge. The model with the efficiency wedge alone misses the drop in output, labor, and investment, and predicts a small decline in net exports. The model with just the labor wedge matches hours remarkably well and accounts for about two thirds of the drop in output. This model, however, misses investment and net exports. The model with just the investment wedge, shown in the lower panel of Figure 4, completely misses output and labor, but matches investment and net exports reasonably well. Finally, the model with just the risk premium wedge predicts an increase in output, an increase in labor, misses the behavior of investment, but matches net exports well.

Consider now models with all wedges but one. The top panel of Figure 5 displays the predictions of the models with no efficiency wedge and with no labor wedge. The lower panel of that figure shows the models with no investment wedge and no risk premium wedge. The model with no efficiency wedge matches labor, investment, and net exports closely, but predicts a somewhat smaller decline and slower recovery in output compared with those in the data. The model with no labor wedge completely misses output and labor, matches investment closely, and overstates the increase in net exports. The model with no investment wedge matches output and labor closely, although it predicts a somewhat faster recovery than that in the data. This model, on the other hand, misses the behavior of investment and net exports. Consider, finally, the model with no risk premium wedge. This model misses net exports.
Figure 4: Data and counterfactual models with one wedge.
Figure 5: Data and counterfactual models with four wedges.
exports and predicts a larger drop in output and labor, and a somewhat smaller decline in investment compared with those in the data.

Overall, these findings suggest that most of the drop in output and labor is due to the labor wedge; the efficiency wedge has a somewhat smaller role; the investment wedge accounts for the behavior of investment and somewhat for the recovery; and the slight worsening in the risk premium wedge in 1983-1984 acted to avoid a larger drop in output and labor—see, however, the discussion in footnote 10.

5 Discussion and Conclusion

In this paper I have decomposed fluctuations in Mexico and in Canada in terms of reduced form shocks that drive a wedge between certain marginal rates of substitution and marginal rates of transformation relative to a prototype frictionless economy. I have found that the business cycle and the 1995 crisis in Mexico can be explained, to a large extent, by the combined effect of the efficiency wedge, the labor wedge, and the risk premium wedge. And what is most important, the risk premium wedge accounts for the large increase in net exports in the 1995 crisis, the excess volatility of consumption over output, and the highly countercyclical share of net exports on output in Mexico. Business cycles in Canada, on the other hand, are mostly driven by fluctuations in the efficiency wedge and in the labor wedge.

Investment wedges do not contribute at all to fluctuations in Mexico, not even to fluctuations in aggregate investment. Moreover, the investment wedge is countercyclical and improved substantially during the 1995 crisis. In other words, investment decisions in Mexico are less distorted in downturns and more distorted in booms relative to a frictionless economy. In Canada, on the other hand, the investment wedge is procyclical and contributes somewhat to the behavior of aggregate investment and net exports.

If Mexico is, indeed, a representative emerging country and Canada a representative developed small open country, these findings have implications for model development. Successful
models of emerging market fluctuations should generate the type of wedges that we observe in
the data, and these wedges should account, quantitatively, for aggregate fluctuations. Thus,
we need to understand what type of primitive shocks and transmission mechanisms drive fluc-
tuations in the risk premium wedge, in the efficiency wedge, and in the labor wedge. Moreover,
because a successful model should also induce countercyclical investment wedges, the trans-
mision mechanisms and the spillovers from the primitive shocks to the wedges should be
strong enough to mitigate the countercyclical effect of the investment wedge.

There are models consistent with some of these requirements. The working capital require-
ment on labor demand stressed by Neumeyer and Perri (2005), Uribe and Yue (2006), and
Mendoza (2008); and on imported inputs, stressed by Mendoza (2008), translates shocks to the
interest rate into labor and efficiency wedges respectively. The problem is that in those models
fluctuations in interest rates are exogenous and, therefore, the issue of why the risk premium
wedge in Mexico is different from that in Canada is left unexplained. The paper by Mendoza,
however, also includes a collateral constraint that, when binding, induces an endogenous risk
premium wedge that amplifies the labor and efficiency wedges induced by the working capital
constraints. Moreover, as Chari, Kehoe, and McGrattan (2005) show, a tightening in the
collateral constraint could manifest itself as an improvement in the investment wedge, which
is, in fact, observed in the Mexican crisis. Still, in periods in which the collateral constraint
is slack, Mendoza’s model reduces to a model with exogenous fluctuations in interest rates.
These fluctuations need to be understood. A step in that direction are the endogenous default
models of Aguiar and Gopinath (2006) and Arellano (2008). These models, however, are still
at an early stage of development and are either endowment or labor only economies; it is not
clear if full-blown versions of these models will be consistent with all the business cycles facts.

This paper can be extended in a number of directions. The most obvious is to apply
the methodology to other emerging countries and check whether the results are robust. The
problem here is with the length of the time series and the large number of parameters that
need to be estimated. Because most emerging countries do not have sufficiently long time
series, the estimates of the 44 parameters will likely be inaccurate. Note, however, that the similarity of the business cycle within the group of emerging countries suggests that the most important results will remain intact.

A second possibility is to extend the framework along the lines of Aguiar and Gopinath (2007). In that paper, they argue that the most important difference between emerging and developed countries lies in the persistence of the total productivity shocks they face. Emerging countries, they claim, are mostly subject to trend shocks, while developed countries are mostly subject to temporary shocks. It is possible to extend the prototype economy along these lines by posing two types of efficiency wedges, one temporary and one permanent. With this extension we could study whether the differences between emerging and developed countries are due to the different type of productivity shocks they face, to different risk premium wedges, or to both. This extension, however, could be difficult to implement because the number of parameters to estimate increases by 50 percent, making the maximization of the likelihood function—already challenging in the present formulation—substantially more difficult.\footnote{The number of parameters to estimate increases to 63: 6 means, 36 coefficients in the matrix on lagged values, and 21 coefficients in the covariance matrix.}
Appendix A  Sources and Construction of the Data

Mexico

All series, except population data, are from Instituto Nacional de Estadísticas y Geografía—http://dgenesyp.inegi.gob.mx/cgi-bin/bdieintsi.exe. Data are quarterly series on output, investment, labor, net exports, and government consumption for the period 1987:1–2007:4. Output is “gross domestic product”, investment is “gross fixed capital formation” plus “change in inventories”, net exports are “exports of goods and services” minus “imports of goods and services”, and government consumption is “government consumption expenditures”, all at 1993 prices. The data were seasonally adjusted using the Census Bureau’s X-12 ARIMA program.

Labor is \((Average\ hours\ worked) \times (Employment) / (Available\ Hours)\). Data on \(Average\ hours\ worked\) are from the Encuesta Industrial Mensual. For the period 1987:M01–1995:M12, I use the survey with 129 activity classes; for the period 1994:M01–2007:M12, the survey with 205 activity classes. Overlapping periods are averaged, and quarterly figures are averages of monthly data. \(Employment\) is \((1-unemployment\ rate) \times (Rate\ of\ activity\ of\ population\ over\ 14\ years\ of\ age) \times (Total\ population\ over\ 14\ years\ of\ age)\). For the period 1987:M01–2000:M04, unemployment data are from the Encuesta Nacional de Empleo Urbano (ENEU). For the period 2000:M04–2007:M12, unemployment data are from the Encuesta Nacional de Ocupación y Empleo (ENOE). The data on the \(Rate\ of\ activity\ of\ population\ over\ 14\ years\ of\ age\) are in quarterly frequency. For the period 1987:01–2004:04, data is from ENU: “Economically active population over 12 years of age”. For the period 2005:01–2007:04, data is from ENOE: “Economically active population over 14 years of age”. To match the levels of the series, I added a 1% to the series before 2005:01. This procedure roughly transforms the first series to economically active population over 14 years of age. Data on \(Total\ population\ over\ 14\ years\ of\ age\) are from the World Development Indicators. Quarterly figures are interpolated from annual data. Lastly, \(Available\ hours\) is \((Total\ population\ over\ 14\ years\ of\ age) \times 100\), assuming 100 hours per week for work or leisure.

Canada

National accounts data are from OECD Quarterly National Accounts Statistics. Seasonally adjusted series at current prices (CAN.CARSA.S1 series) are deflated using the GDP deflator series (CAN.DOBSA2000). Labor data is from LABORSTA Labor Statistics Database. Labor is constructed as in Mexico, except that \(Employment\) is directly observed. \(Average\ hours\ worked\) is from series “B6 Hours of work per week in manufacturing”. I use the series “hours paid for wage earners, total men and women”; missing observations are filled using “hours paid for employees, total men and women”. Employment is from series “B1 Employment, general level”. The employment series has a structural break in January 1995 due to a methodological change. The break in the trend was adjusted using an extended Hodrick-Prescott filter that allows for structural breaks (Schlicht, forthcoming). Labor data were seasonally adjusted using the Census Bureau’s X-12 ARIMA program, and quarterly figures are averages over monthly data. Population data is from the World Development Indicators.
Appendix B  Maximum Likelihood Estimation

The model is stationary in terms of the variables $\bar{x}_t = x_t/(1 + \gamma)^t$ for any $x_t$ except for labor and the rental rate of capital.\footnote{12} Let $\bar{x}_t$ be the steady state value of any variable $x_t$. To approximate the equilibrium, the model is first log-linearized around the steady state and then written in the state space form

$$X_{t+1} = M(\theta)X_t + \nu_{t+1}$$  \hspace{1cm} (B.1)
$$Y_t = N(\theta)X_t,$$  \hspace{1cm} (B.2)

Here (B.1) is the state transition equation; (B.2) is the observation equation; $X_t$ and $Y_t$ are the state vector and observation vector, both in terms of deviations from the steady state and given by

$$X_t = \begin{bmatrix} \log \left( \tilde{k}_t/\bar{k} \right), \log \left( \tilde{b}_t/b \right), \log (A_t/A), \tau_t - \bar{\tau}, \tau_{xt} - \bar{\tau}_x, \log \left( Z_t/Z \right), \log \left( g_t/\bar{g} \right) \end{bmatrix} \quad \text{and} \quad Y_t = \begin{bmatrix} \log (\tilde{y}_t/\bar{y}), \log (\bar{x}_t/\bar{x}), \log (l_t/\bar{l}), \bar{m} - m, \log (g_t/\bar{g}) \end{bmatrix};$$

the noise process in (B.1) is given by $\nu_{t+1} = [0, 0, \epsilon_{t+1}]^\prime$, where $\epsilon_{t+1}$ is the innovation of the stochastic process (11); and the matrices $M(\theta)$ and $N(\theta)$ are nonlinear functions of the parameters $\theta = \{s, P, V\}$. The remaining parameters of the model ($\sigma, \rho$, and so on) are held fixed throughout the estimation.

I first construct the empirical analogs of $\tilde{y}_t$, $\bar{x}_t$, $l_t$, $\bar{m}_t$, and $g_t$ using data on output, investment, labor, net exports, and government consumption. This is done by exponentially detrending the data using the average growth rate of the population and output. (Hours worked are only detrended using the population growth rate.)

Given a guess $\theta$, I solve for the log-linearized policy functions $M(\theta)$ and $N(\theta)$, and construct a time series $Y_t$ using the transformed data and the steady state values. The likelihood function is then evaluated using the Kalman filter on the system (B.1–B.2). To maximize the log-likelihood function, I follow Chari, Kehoe, and McGrattan (2007), and apply the following transformations: to induce positive definiteness of $V$, I estimate the lower triangular Cholesky decomposition of $V$; to induce stationarity, I add a penalty term of $5 \times 10^5 \max (\bar{\lambda} - 0.995, 0)^2$ to the likelihood function, where $\bar{\lambda}$ is the eigenvalue of $P$ with the largest absolute value.

The maximization step is difficult: the log-likelihood is a nonlinear function of 44 parameters. I first use a simulated annealing algorithm (Kirkpatrick, Gelatt, and Vecchi, 1983) to obtain an approximation to the global optimum. Next I use the estimate from the annealing step to initialize a local search algorithm: a Quasi-Newton algorithm, a Nelder-Mead simplex algorithm, or both. Next I perturb the estimate from the previous step and use it to restart the local search algorithm. After repeating this cycle several times, I use the best estimate obtained so far to restart the simulated annealing algorithm, and repeat the process until no further improvements in the log-likelihood can be obtained.

\footnote{12}In the case of capital and debt, let $\tilde{k}_t = k(\bar{s}^{t-1})/(1 + \gamma)^t$ and $\tilde{b}_t = k(\bar{s}^{t-1})/(1 + \gamma)^t$.}

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References


