Dipak Mazumdar

The Rural-Urban Wage Gap, Migration, and the Shadow Wage

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(continued on inside back cover)
THE RURAL-URBAN WAGE GAP, MIGRATION, AND THE SHADOW WAGE

By DIPAK MAZUMDAR

Introduction

This paper investigates some of the issues discussed in the recent literature on the shadow wage of labour in the presence of an institutionally determined rural-urban wage gap. The analysis is presented within the framework of the Little–Mirrlees formulation [5] which distinguishes two elements in the opportunity cost of employing an additional worker in the urban sector: (i) the value of the output forgone due to the use of a unit of labour; and (ii) the cost to the economy of the additional consumption due to the transfer of a worker from the traditional to the new activity. The latter is important because the urban wage gap is higher than the level of consumption a worker would enjoy in his alternative rural employment. The rural–urban wage gap is also connected with the generation of urban unemployment as migrants to the urban labour market are induced to ‘invest’ in a spell of unemployment in the expectation of obtaining employment at a higher wage. Cf. Todaro [12]. A formulation of the shadow wage in the presence of the wage gap must then take into account the effect of creating an additional job in the urban sector on the part of the urban unemployed as well as the two factors specified by Little–Mirrlees.

The present discussion concentrates on two points which have been inadequately dealt with in the papers on the subject by Harberger [2], Harris and Todaro [3], Lal [4], and Stiglitz [10, 11]. The first is concerned with the problem of defining a migration function which will determine the volume of migration in response to the rural–urban wage gap, and hence the size of the pool of the unemployed in the urban market. Both Harberger and Stiglitz reach the result that the shadow wage will equal the market wage if the equilibrium rate of unemployment remains unchanged in the urban area after the creation of an additional job in the area. But neither of them recognizes the point that the actual magnitude of the equilibrium rate of unemployment is crucial to the determination of the value of the shadow wage—and that the magnitude of the unemployment rate depends very much on the migration function implicit in the specific formulation of the equilibrium condition of the labour market. We will show how

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1 I am grateful to Deepak Lal, Johannes Linn, Amartya Sen, Lyn Squire, for carefully reading earlier drafts of this paper and making helpful suggestions. The last version was issued as IBRD Working Paper No. 197. I acknowledge with thanks valuable comments on this version from two anonymous referees which enabled me to correct some errors and develop some points further.
**Table I**

*Summary of notations*

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>SWR</td>
<td>shadow wage rate</td>
</tr>
<tr>
<td>S</td>
<td>premium on savings relative to consumption</td>
</tr>
<tr>
<td>W</td>
<td>wage in the 'formal' urban sector</td>
</tr>
<tr>
<td>AP</td>
<td>average product of a worker in agriculture (rural sector)</td>
</tr>
<tr>
<td>MP</td>
<td>marginal product of a worker in agriculture (rural sector)</td>
</tr>
<tr>
<td>C</td>
<td>opportunity cost of a migrant from the rural to the urban area</td>
</tr>
<tr>
<td>p</td>
<td>probability of obtaining a formal sector job</td>
</tr>
<tr>
<td>L</td>
<td>total urban labour force</td>
</tr>
<tr>
<td>N</td>
<td>total employed in the urban formal sector</td>
</tr>
<tr>
<td>U</td>
<td>total number of unemployed or those in the informal sector who are seeking jobs in the urban formal sector</td>
</tr>
<tr>
<td>j</td>
<td>fraction of W which equals the income of the migrant in town while he is searching for a job in the formal sector</td>
</tr>
<tr>
<td>m</td>
<td>fraction of W which equals C</td>
</tr>
<tr>
<td>a</td>
<td>fraction of W which equals AP</td>
</tr>
<tr>
<td>Y (= yN)</td>
<td>the income produced in the formal (O) sector</td>
</tr>
<tr>
<td>P</td>
<td>the income produced in the informal (U) sector</td>
</tr>
<tr>
<td>γ</td>
<td>the percentage rate of growth of employment in the O-sector</td>
</tr>
<tr>
<td>α</td>
<td>the marginal propensity to consume U-goods on the part of the U-sector</td>
</tr>
<tr>
<td>β</td>
<td>the marginal propensity to consume U-goods for the O-sector</td>
</tr>
<tr>
<td>W_A</td>
<td>income per period in the rural area given up by the migrant or his family</td>
</tr>
<tr>
<td>F(A)</td>
<td>present value of the incomes stream of W_A per period</td>
</tr>
<tr>
<td>E(H)</td>
<td>present value of the expected income stream in town during the migrant’s job search</td>
</tr>
<tr>
<td>E(V)</td>
<td>present value of the income stream of the migrant from his formal sector wage</td>
</tr>
<tr>
<td>h</td>
<td>fraction of W which the migrant gets per period in town during his job search</td>
</tr>
<tr>
<td>i</td>
<td>effective rate of discount used by the migrant</td>
</tr>
<tr>
<td>D</td>
<td>expected duration of the formal sector job</td>
</tr>
<tr>
<td>q</td>
<td>average rate of turnover in the formal sector</td>
</tr>
<tr>
<td>tu</td>
<td>expected duration of unemployment in town before the migrant gets a formal sector job</td>
</tr>
</tbody>
</table>

Different values of the shadow wage are reached with different specifications of the equilibrium condition.

The second objective of the paper is to incorporate explicitly in the analysis the problem of financing the urban unemployed during their period of search—a subject which has been curiously neglected in the literature of the shadow wage. Two specific cases of financing during the migrants’ job search are considered: (i) the case in which the migrant is maintained by income transfers from his rural family (‘rural financing’); and (ii) the case in which the migrant maintains himself by working part-time in the urban traditional informal sector (‘self-financing’).

In Section I we present a formulation of the shadow wage which combines a simple (and extreme) migration function with the assumption of rural
financing of the unemployed—leading to a Harberger-type result that the 
shadow wage would equal the market wage. In Section II an expression 
for the shadow wage is derived with a more reasonable (and realistic) 
migration function, but still retaining the assumption of rural financing. 
Section III considers the problem on the alternative assumption of ‘self-
financing’. The models of Section II and III (unlike that of Section I, as 
we shall see) require a multi-period setting. A specific example of a multi-
period model is considered in this paper, and alternative values for the 
shadow wage are derived in terms of this example. In the Appendix we 
turn briefly to the formulation of Stiglitz who also considers a multi-period 
model, but it is shown that his neglect of the key question of financing the 
unemployed leads to a gross exaggeration of the equilibrium unemployment 
rate—and hence to the false Harberger-type result that the shadow wage 
will equal the market wage if the unemployment rate remains unchanged.

Section I

It will be helpful to the subsequent discussion if we briefly recall the 
Little-Mirrlees formulation of the shadow wage rate. When an additional 
worker is employed in the ‘urban’ sector, there is a loss in output equal to 
the marginal product of the worker in the rural sector (= MP). There is 
also an increase in consumption equal to the difference between the wage 
at which the worker is employed (W) and the marginal product in the rural 
sector (MP). The expression for the shadow wage is then:

$$SWR = MP + \left(1 - \frac{1}{S}\right)(W - MP)$$

where $S (> 1)$ is the value of a unit of savings in terms of consumption.

The economics behind the second term in the expression is that the entire 
increase in consumption is not a cost to the economy: a portion $1/S$ of the 
increase in consumption has to be subtracted from the total to give the true 
social cost in terms of the numeraire ‘savings’. The above expression 
reduces to

$$SWR = W - \frac{1}{S}(W - MP).$$

The term within brackets being positive, the shadow wage is below the 
market wage.

We can use this framework to interpret a model of the shadow wage which 
gives a Harberger-type result.\footnote{Harberger’s formalization is not available in published form. Apart from the verbal 
statement given in [2], a more formal model presented by him at a seminar in Oxford is 
reported by Sen [9] and Lal [4]. It is a formulation given by the latter which is used here. 
See [9], p. 498, and [10], p. 118.} The basic equilibrium in the labour market
is achieved by the condition that the expected wage of a job seeker migrating to the urban area is equated to his output foregone ($MP$):\footnote{The assumption here is that of income maximization. Sen has pointed out that the assumption of utility maximization on the part of the migrant will give a different result if there is diminishing return to income.}

\begin{equation}
MP = pW.
\end{equation}

'\(p'\) the probability of obtaining an urban job is given by the ratio of the employed (\(N\)) to the total labour force (\(L\)) in the urban area:

\begin{equation}
p = \frac{N}{L}.
\end{equation}

From (4)

\begin{equation}
\frac{dL}{dN} = \frac{1}{p}.
\end{equation}

Thus creating an additional job in the urban area adds \(1/p\) workers to the labour force. The increase in the number of unemployed is \((1/p) - 1\). The marginal rate of unemployment is \(\frac{(1 - 1/p)}{p}\) or \((1 - p)\), which is the same as the average rate. Thus the migration function implied in (4) ensures that the rate of urban unemployment remains the same, even with additional job creation in the urban area.

Given the foregoing adjustment in the urban labour market through induced migration, we can calculate the shadow wage rate by exploring (a) the loss in output, and (b) the increase in consumption in the economy à la Little–Mirrlees when an additional job is created in the urban area.

(a) Since \(1/p\) migrants are attracted from the rural sector due to the job creation,

\begin{equation}
\text{loss in output} = \frac{1}{p} MP = W \quad \text{(using (3))}.
\end{equation}

(b) What about the increase in consumption due to the migration of \(1/p\) workers? In Harberger's view, a unit of savings at the margin is equally valuable as a unit of consumption, i.e. \(S\) in (1 a) is equal to unity. If this is so, then the 'consumption effect' need not be considered. But it will be shown now that even if the value of \(S\) is greater than unity so that the consumption effect becomes important the Harberger result can be derived if we make a specific assumption about the method of financing the job-seeker during his period of search. It is that the migrant searching for a job in the urban area is maintained by his rural family. That is to say, even though the migrant is separated from the rural family, all the workers still form part of the same economic unit in sharing the total income accruing to the unit.

Given this assumption about financing the job-seeker during his period of search, the increase in consumption due to the employment of an
additional worker in the urban area is easily calculated. The employed worker increases consumption to the extent of the wage earned. \((W)\). \(1/p\) migrants are attracted to the urban area. Thus the income of (and consumption of) the rural families (including the migrants) falls by \((1/p)MP\).

Thus the net increase of consumption in the economy equals:

\[
W - \frac{1}{p} MP = 0 \quad \text{(using (3))}. \quad (7)
\]

(Note that it does not matter how the decrease in consumption is distributed between the migrants and the members of his family remaining in the rural area.)

We can then conclude that the shadow wage rate is given solely by the loss in output due to the induced migration and from (6) is equal to the market wage.

This 'catchy' result is thus seen to rest on three assumptions:

(i) the financing of the job-seeker by the rural family just discussed;
(ii) the migration function implicit in equation (4);
(iii) the equilibrium condition given in equation (3).

Before turning to an examination of the migration function, it might be relevant to discuss the specific nature of the assumption (iii). It has been noted in the literature that when the migrant is a personal income maximizer, his supply price will be given by the average product of the family farm rather than the marginal product (which will be the case when he is motivated by the ethic of maximizing family income).\(^1\) From this it might be concluded that all we have to do is to replace \(MP\) by \(AP\) in equation (3). The analysis, however, needs to be a bit more complicated than this for the following reason. If there is rural financing of the migrant during his job search, then if he is unable to find a job in town he receives an income equal to \(gW\) from his rural family. The equilibrium condition for such a migrant, equating his loss in the rural area to the expected gain in town will be given by:\(^2\)

\[
AP = pW + (1-p)gW. \quad (3')
\]

Now if \(gW\) is equal to \(AP\), i.e. the migrant gets from his family in town exactly what he got in the rural area then equation (3') breaks down. Migration for the individual is costless, even though the family left in the rural area makes a loss. Clearly, for migration with personal income maximization we must have \(gW < AP\). The rural family can enter into a whole range of arrangements with the potential migrant to the town about the level of financing required—and the agreement may also include a provision

\(^1\) Mazumdar [6], Cline [1], and Sen [9].

\(^2\) In the case of family maximization considered earlier, unemployment for the migrant with a probability of \((1-p)\) is associated with zero income for the family and hence does not show up in equation (3). I am indebted to J. Linn for suggesting this point.
for transfer of income back to the family if the migrant gets a job. Many solutions exist to the problem we are considering. It may, however, be worth while to confine ourselves to what might be called the case of 'pure' personal income maximization. This case occurs when migration of an individual has a cost from the point of view of the migrant, but is costless from the point of view of the rest of the family. It is given by the condition:

$$gW = AP - MP.$$ \hfill (3'')

The family, in other words, compensates the migrant in town to the full extent of the saving due to his departure (the difference between his claim on consumption and his contribution to production in the farm household). From (3') and (3'') we get the equilibrium condition:

$$MP = \frac{1}{p}[W - (AP - MP)].$$ \hfill (3 a)

Comparing (3 a) with (3) it is clear that the value of $p$ is higher in this case, and the equilibrium unemployment rate in town $(1 - p)$ is lower.

As before, the loss in output is $MP/p$ and the increase in consumption in the economy $\{W - (MP/p)\}$, when $1/p$ persons migrate in response to an additional worker being hired at the wage $W$. In terms of the L-M formula, and using (3 a), in this case:

$$SWR = W - \frac{1}{S}(AP - MP).$$ \hfill (8)

Since $AP > MP$, this case of 'pure' personal income maximization yields the conclusion that the shadow wage will be less than the market wage.\footnote{We are implicitly working with the assumption in this argument that the income produced in the family farm is shared out equally among the productive members of the family. This is not too much of an error so long as the unproductive members are only small children who do not consume very much.}

The question arises: how likely is it that the average product of a rural family would be greater than the marginal product of a family member. It should be remembered that even if the marginal product of a unit of labour time (after taking into account the hours of work contributed in the aggregate by the family) is very low, the marginal product of a particular worker could be high because of work-sharing arrangements within the family. With equal work-sharing, the marginal product will be the same for every worker and near to the average product (ignoring non-productive members).\footnote{Note that throughout this discussion average product refers to the 'net income' per person in the family farm—i.e. net of any outflow for payments of rent to fixed factors.}

Another point, however, has to be made in this connection. As individuals
differ in terms of their efficiency, so they would differ in their leisure preferences. If there are larger differences involved in the latter, then equal or nearly equal work-sharing would be difficult to achieve in the family. The head of the family might allocate work so as to equalize the values of disutility of effort for the productive workers at the margin. Looked at this way, potential migrants from particular families may well have very low values of their marginal product, relative to the average product of the family. This is particularly likely for personal income maximizers whose ties with the other family members have been, by definition, loosened.

Section II

Let us now turn to an examination of the migration function implied in (4). The probability of obtaining an urban job in any period will be given by the ratio of new jobs becoming available in the period to the total number of job-seekers competing for the jobs. Thus when we have an expression for the probability given by the ratio of the employed to the total labour force as in (4) it is implicitly assumed that there is a complete turnover of labour in every period—i.e. no job lasts for more than one period, and the entire labour force (including the employed and the unemployed) compete afresh for all jobs in the next period. Only on this assumption will the creation of one extra job induce in-migration of $1/p$ new job-seekers. Clearly, such an extreme assumption about the turnover of labour grossly exaggerates both the rate of urban unemployment for a given wage-gap and the size of induced migration. If, for example, the urban wage is three times the marginal product of labour in agriculture, the rate of unemployment $(1-p)$ will be 66 per cent—from (3), and the creation of one extra job will induce three workers to join the urban labour force.

Evidently the unrealistic nature of the expression for the probability of obtaining an urban job of the implied migration function is a consequence of the use of a simplified one-period model. If we assume that a potential migrant thinks in terms of a job in the urban organized sector lasting one period only as the model in Section I does, the logical corollary is the assumption of a total turnover of labour in each period. In order to be able to consider a more realistic model of the urban labour market in which jobs do not just last for one period, we should really work with a multi-period model.

It should, however, appear on reflection, and should be clearer from the subsequent analysis, that a multi-period framework is relevant to only one of the elements involved in the formula of the shadow wage. This is the equilibrium condition which equates the expected gain from migration to its expected cost, and hence determines the volume of job-seekers in the urban market in any period. This volume, given the rate of job creation...
in the market, determines the probability of obtaining an urban job \((p)\) in each period. Other parts of the shadow wage formula—the loss in output or the extra consumption generated—are values per period which are not affected by the specification of the time-horizon of the model. In what follows we shall first derive an expression for the shadow wage on the basis of an undetermined \(p\). The value of \(p\) will be subsequently determined from the equilibrium condition of migration in a multi-period setting.

Let urban employment grow at a steady constant rate (depending on the rate of net expansion and of labour turnover). Then the number of jobs created at any period of time is \(\gamma N\), where \(N\) is the stock of urban jobs in this period. These jobs will be competed for by \((U + \gamma N)\) number of job-seekers, of which \(\gamma N\) are absorbed into employment and \(U\) remain unemployed. (\(U\) is left to be determined at a later stage from the equilibrium condition of migration.)

The probability of getting an urban job at any time period can then be redefined as:

\[
p = \frac{\gamma N}{U + \gamma N}.
\] (4a)

With this definition, unemployment \((U)\) as a percentage of the labour force \(L\) \(= U + N\) is

\[
\frac{U}{L} = \frac{\gamma(1-p)}{\gamma(1-p)+p}.
\] (4a)

We have from (4a):

\[
L = U + N = \left[\frac{\gamma(1-p)}{p} + 1\right]N.
\] (9)

Hence,

\[
\frac{dL}{dN} = \frac{\gamma(1-p)}{p} + 1,
\] (10)

assuming that a small increase in the number of urban jobs does not increase the rate of growth of urban employment perceptibly.

It might be objected that in some cases the public investment projects for which we need the shadow wage would have more than a marginal impact on the rate of growth of employment in the particular labour market. It is, however, important to realize that \(\gamma\)—like the accounting rate of interest—is a parameter which is given for a particular bundle of projects, and depends partly on other factors in the labour market concerned. The shadow wage derived on the basis of the given \(\gamma\) helps us to choose between the specific projects within the bundle. If the size of the bundle being considered is significantly altered a different \(\gamma\) will have to be adopted for the next round of calculations.

Thus, in terms of (10), when one extra job is created, using the values of the parameters of our example, \(\gamma(1-p)/p\) number of workers migrate in search of urban jobs in addition to the one required to fill the new job.
The rate of unemployment in the urban area remains unchanged. It is worth emphasizing that this model gives the same result as that of Section I in suggesting that induced migration will be such as to maintain the rate of unemployment: the difference is in the magnitude of the unemployment and migration rates predicted.

We now derive an expression for the shadow wage, taking into account (a) the loss in output and (b) the increase in consumption, given the number of new migrants as suggested by (10).

$$\text{Loss in output} = \frac{dL}{dN} \cdot MP. \quad (11)$$

$$\text{Increase in consumption} = W - \frac{dL}{dN} \cdot MP. \quad (12)$$

Hence

$$SWR = \frac{dL}{dN} \cdot MP + \left(1 - \frac{1}{S}\right) \left(W - \frac{dL}{dN} \cdot MP\right)$$

$$= \left(1 - \frac{1}{S}\right)W + \frac{1}{S} \frac{dL}{dN} \cdot MP$$

$$= W \left[1 - \frac{1}{S}\left(1 - m - m\gamma\left(\frac{1-p}{p}\right)\right)\right] \quad (13)$$

substituting for the value of $dL/dN$ from (10).

The relationship of (13) to the Herberger-type formulation of Section I can be seen if we treat the latter as a special case of (13) in which $\gamma = 1$, and $p = m$. The $SWR$ in (13) then reduces to the market wage $W$. When we allow for the possibility of an urban job which lasts more than one period both of these restrictive conditions will have to be dropped.

The equilibrium rate of migration and the determination of $p$

We can now replace the equilibrium condition of migration given in equations (3) and (3 a) of Section I by a better specified model with a multi-period horizon which determine $U$ and hence $p$ in equation (4 a).

The simplifying assumptions made are the following:

(i) Migrants coming to the urban labour market make a once-for-all decision on migration on the basis of expectations of obtaining a job in the formal sector of the urban market. That is to say, once the decision to migrate is taken there is no 'looking back', and migrants stay in the urban market until they get an urban job.

(ii) The fixed costs of migration are small in relation to the cost of output forgone and of maintenance in the urban area during the period of search.

---

1 In algebraic terms $dU = (\gamma(1-p))/p$, and $dL = (\gamma(1-p)+p)/p$ so that the marginal rate of unemployment is $(\gamma(1-p))/(\gamma(1-p)+p)$, the same as the average rate. Lal [4], p. 118, is incorrect in suggesting that induced migration will be $1/p$ as long as the equilibrium unemployment rate is unchanged.
(iii) Once a migrant obtains an urban job it lasts 'for ever', that is to say, the service in the urban job once obtained, is expected to be long compared to the period spent in searching for such a job.

(iv) The alternative to migration is the certainty of a rural job at the going level of productivity (or earnings).

Consider a typical migrant at the point of time when he is deciding if it pays him to migrate to look for a job in town. He has the certainty of an income stream in the rural sector whose present value would be given by (assuming constant agricultural earnings \(W_d\) for simplicity):

\[
P(A) = \sum_{t=1}^{\infty} W_d(1+i)^{-t} = \frac{W_d}{i}. \quad (14)
\]

If he migrates to town he must consider the expected present value of his urban income \(E(V)\), and the expected present value of the income he will get while he is finding a permanent urban job, \(E(H)\). (Note that \(E(H)\) could be negative.)

We must have, for equilibrium:

\[
P(A) = E(V) + E(H). \quad (15)
\]

Let \(p\) = the probability of getting a job in any period,

\[V_t = \text{the present value of his income stream from urban employment if he finds employment in period } t.\]

We have:

\[V_1 = \sum_{t=1}^{\infty} W(1+i)^{-t} = \frac{W}{i} \]

and

\[V_t = \frac{V_1}{(1+i)^{t-1}},\]

since he is getting the same income \((t-1)\) years later (assuming his total lifetime employment is long compared to the period of search).

We can then calculate the expected present value of the urban income stream as:

\[
E(V) = p[V_1 + (1-p)V_2 + (1-p)^2V_3 + \ldots] = \frac{p}{i} \left[ V_1 + \frac{(1-p)}{(1+i)} V_1 + \frac{(1-p)^2}{(1+i)^2} V_1 + \ldots \right] = \frac{p}{i} \left[ \frac{(1+i)}{(p+i)} V_1 \right] = \frac{p}{i} \left[ \frac{(1+i)}{(p+i)} \right] W. \quad (18)
\]

Let \(hW\) = the fraction of \(W\) per period which the migrant gets during his job search—a sort of unemployment pay from his rural family.

Then

\[
E(H) = \frac{1-p}{1+i} hW + \frac{(1-p)^2}{(1+i)^2} hW + \frac{(1-p)^3}{(1+i)^3} hW + \ldots = \frac{1-p}{(p+i)} hW. \quad (19)
\]
Substituting (14), (18), and (19) into the basic equation (15), we have that when the volume of migration to town is in equilibrium:

\[ W_A = \left\{ \frac{(1-p)}{(p+i)} \right\} h + \frac{p(1+i)}{i(p+i)} W; \]

whence we have (remembering that \( W_A = jW \)):

\[ p = \frac{i(j-h)}{1-j+i(1-h)}. \]

With family income maximization \( j = m \), and \( h = 0 \), on the assumption that the migrant can be financed during his period of search by income transfers within the family. If in fact the family has to borrow money which has an interest cost, \( h \) will be negative.

If in the case of pure personal maximization of Section I, the migrant is able to subsist during his period of search on the amount the family is willing to transfer to him, we will have \( j = a \) and \( h = a - m \).

**The determination of \( U \) and the SWR**

The equilibrium rate of unemployment in the urban market \( U/L (= u) \), it will be recalled, is given by the expression \( y(1-p)/y(1-p)+p \). Let us now see how the predicted unemployment rate from our model turns out for alternative values of the relevant variables. The rate of job creation in the formal urban sector is quite modest in most LDCs (including that due to labour turnover); say \( y = 0.1 \). The following table gives us the values of \( u \) (with \( p \) being determined by the alternative values of \( m \) and \( i \) from equation (21)), examples A referring to family income maximization and example B to personal income maximization.

<table>
<thead>
<tr>
<th>Values of the equilibrium unemployment rate for ( \gamma = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
</tr>
<tr>
<td>A.</td>
</tr>
<tr>
<td>B.</td>
</tr>
<tr>
<td>B.</td>
</tr>
</tbody>
</table>

It is seen that the predicted unemployment rate is reduced significantly from the results of the Harberger-type single-period model. It would also appear that \( u \) is more sensitive to variations in the value of \( m \) than those of \( i \) for family income maximization (examples A). The predicted values of \( u \) are also significantly less for personal income maximization as can be seen from the line B of the table, but only for relatively high interest rates.

\(^1\) In the 'normal' developed country case \( a = m = 1 \); hence \( p \) reduces to unity whether we have family or personal income maximization, i.e. the equilibrium unemployment rate is zero since there is no rural–urban wage gap.
Which are the implications for the value of the $SWR$ of the equilibrium rate of unemployment in terms of the model of this section? The expression for the $SWR$ given in equation (13) can be rewritten as:

$$SWR = W\left[1 - \frac{1}{S}\left(1 - m - \frac{mu}{1-u}\right)\right]. \tag{13a}$$

Compare (13a) with the L-M version of the $SWR$ (1a) which can be rewritten as:

$$SWR = W\left[1 - \frac{1}{S}(1-m)\right]. \tag{1b}$$

The expression (13a) is different from L-M in only having an extra positive term $m/S \cdot u/(1-u)$ which increases the value of the $SWR$ as a fraction of the market wage. But the order of magnitude of this extra term is small. For example with $m = 0.3$, $S = 2$, and $u$ having a relatively high value of 0.2, the L-M version gives the value of the ratio $SWR/W$ as 0.65, and expression (13a) as 0.69.

It is clear, however, that (13a) could theoretically give a high value of the $SWR$. Indeed, if both $m$ and $u$ are sufficiently large fractions (e.g. a bit more than 0.5), it is possible for the $SWR$ to be greater than the market wage. But empirically $u$ can hardly be more than 0.2, so that $SWR$ would approach $W$ only for values of $m$ of 0.8 or more.\(^1\)

Section III

It has been assumed in the last section that migrants coming to the urban market are financed during their period of search by the families from which they came. It has become increasingly apparent to observers recently that one of the major characteristics of the urban labour market in LDCs is the existence of a large ‘informal’ sector.\(^2\) The characteristic of the informal sector is that entry is easy, so that a migrant seeking to obtain employment in the high-wage sector can finance himself by participating in the former. At the same time unlike employment in the ‘formal’ sector—which tends to be contractual and full-time—much of the work in the informal sector is either self-employment or is in the market for casual labour which allows for variable hours or days of work. Thus, it is feasible for the migrant to search for a job in the formal sector even when he is earning a living in the informal sector.

Harberger [2] puts forward the proposition that the wage rate in the informal sector should be accepted as the shadow wage, since in the absence

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\(^1\) The reader may note from (13a) that the condition for the $SWR$ to equal $W$ can be expressed alternatively as $m = 1 - u$, or as $\gamma = \gamma/(1 + \gamma)$. Thus if $\gamma$ is 0.1, $\gamma$ will need to be 0.11.

\(^2\) The christening of the sector as ‘informal’ was done by the ILO Employment Mission to Kenya which has also served to give the stamp of official approval on the distinction between the two sectors. It has been called the ‘unprotected’ sector by Harberger [2] and the ‘unorganized’ sector by Mazumdar [7].
of any institutional maintenance of his wage rate in this sector the market will be cleared, and the established wage should represent the supply price of labour. There are, however, several important arguments against this proposal.

(i) If the 'informal' sector provides an opportunity for migrants to earn a living wage while searching for a more permanent job in the formal sector, then the market clearing wage in the former will be less than the supply price of labour to the urban market. Migrants, in other words, will be willing to take a 'cut' in their supply-price during the period of job search in the expectation that they will be enjoying a higher stream of earnings if they do break into the formal sector. The extent of the 'cut' will depend on the differential in wages between the formal sector and the rural area, and on the probability of obtaining a job in the formal sector.¹

(ii) A distinction has to be made between the daily wage rate and the average earnings of labour in the 'informal' sector. It is by no means certain that even if entry is easy for labour in this sector, the daily wage rate will be equated to the disutility price of a day's work. The wage rate in a labour market is strongly influenced by the community's idea of a generally acceptable 'social minimum' and also by the demonstration effect of the wage rate in the 'formal' sector. If this minimum is significantly above the disutility price of a day's labour there will be competition for the available number of jobs on any particular day. The labour market in the 'informal' sector is usually organized along the lines of a casual labour market. Thus, the over-supply of labour does not lead to total unemployment of a portion of the labour force, but to a sharing out of the available jobs among the job-seekers, so that over the period of a month everybody gets a certain number of days of work.² Equilibrium in the market is established so that the average earnings of a job-seeker for the month is equated to his supply price (ignoring the point made in the last paragraph). Thus, the opportunity cost of labour in the urban sector should be nearer the average earnings of a worker in the informal sector than the wage rate.

(iii) Apart from this problem of measuring the 'true' supply price of labour by looking at the wage/earnings data in the 'informal' urban market, we must take into account the consumption effect of creating an additional job in the 'formal' sector. It seems likely that when migrants are financing themselves during their job search by working in the informal sector, the consumption effect will be more significant than when they were being financed by their rural families.

¹ A fuller discussion of this point within the framework of a model of migration with a two-tier urban labour market will be found in Mazumdar [7].

² Cf. the evidence cited in Mazumdar [8].
Let us then follow the same procedure as in the models in Sections I and II for estimating the shadow wage. We assume that migrants to the urban market expect to obtain employment in the urban formal sector, but if they fail to do so are not totally unemployed, but find some employment in the urban informal sector. The seekers of formal sector jobs—defined as \( U \) in the models of Sections I and II—are in this case the total number employed (though not full-time) in the informal sector. The average earnings of a worker in the informal sector is not exogenously given, but is determined by the conditions of equilibrium in the urban market.

**A model of the urban labour market**

A simple model of the urban labour market could be formulated by making a reasonably realistic assumption about the linkages between the two subsectors in the process of income generation in the market. It is assumed that the informal or unorganized sector (\( U \)-sector) sells its output entirely to the population in the urban market—who derive their income from the \( U \)-sector as well as from the formal or 'organized' sector (\( O \)-sector). The \( O \)-sector, however, sells much of its output to the population outside this particular urban market. Consequently, \( U \)-sector output is dependent on the income of the \( O \)-sector but \( O \)-sector output is autonomously determined.

Let \( y \) = the productivity of a worker in the \( O \)-sector at a point of time

\[
Y = yN = \text{the income produced in the } O-\text{sector}
\]

\[
P = \text{the income produced in the } U-\text{sector}
\]

\[
\alpha = \text{the marginal propensity to consume } U-\text{goods on the part of the } U-\text{sector}
\]

\[
\beta = \text{the marginal propensity to consume } U-\text{goods for the } O-\text{sector}.
\]

Then, we have,

\[
P = \frac{\beta}{1-\alpha} \cdot Y. \tag{22}
\]

The total demand for goods and services from the informal sector gets randomly distributed among the workers found in the sector. Since each worker is as good (or as lucky) as another, the income generated in this sector is thus shared equally by all the workers participating in the sector, so that we get average earnings in the sector determined by the following:

\[
gW = \frac{P}{U}. \tag{23}
\]

\(^1\) It has been found, in several studies, that a large proportion of the output of the \( U \)-sector consists of services which are sold to the residents of the city. Cf. the review by Mazumdar [7].
The probability of getting an O-sector job for those now in the U-sector is as given in Section II:

\[ p = \frac{\gamma N}{U + \gamma N} \quad \text{as in (4a).} \]

The equilibrium condition for migration will now have to be redefined to take into account the possibility of the migrant finding some work in the U-sector if he does not succeed in breaking into the O-sector. This means that, in terms of the analysis of the determination of the equilibrium volume of migration presented in the last section, the migrants receive an 'unemployment pay' equal to \( gW \) during their period of search. Thus the equilibrium condition given by equation (21) remains unchanged, but we note, in this case, with family income maximization \( h = g, \) and \( j = m. \) But with pure personal income maximization \( h = a - m + g, \) and \( j = a. \)

The value of \( g \) is determined endogenously in the system. \( P \) is determined by (22). The three equations (21), (23), and (4a) are sufficient to determine simultaneously the three unknowns \( p, g, \) and \( U. \)

What happens when \( N \) changes? Generally by altering the probability of getting an O-sector job it will have an effect on \( U, \) so that \( g \) changes: and if the average earnings in the U-sector changes we can expect to see further feedbacks on \( P \) (and hence \( g \)) through changes in \( \alpha \) and \( \beta. \) It is, however, possible to conceive of a state of dynamic equilibrium in the labour market, in which \( g \) remains unchanged over time for a given value of \( \gamma, \) and it is this value of \( g \) which should enter into the calculation of the shadow wage for this type of model.

The requirements for the dynamic equilibrium in which employment and earnings in both the urban subsectors grow at the same rate are twofold: (i) \( \alpha \) and \( \beta \) remain constant so that the multiplier mechanism of equation (22) ensures that the growth rates of income in the two urban subsectors are equalized over time; and (ii) labour productivity and wage in the urban formal sector grow at the same rate as the supply price of labour in the rural sector, and the productivity of a day's work in the urban informal sector also grows at this rate. This second condition implies that the growth rates of employment in the two urban subsectors are the same, and that average earnings in the U-sector increase at the same rate as the O-sector wage and the supply price of rural migrants.

Given these conditions a particular value of \( \gamma \) will determine the ratios of employment and earnings in the two urban subsectors, as it determined the equilibrium rate of unemployment in the model of Section II. Thus the values of these ratios as well as that of \( p \) can be determined for any date on the growth path from the system of equations already given for the static model.

1 Cf. the discussion in Mazumdar [8].
We now turn to the determination of the shadow wage in the steady state characterized by a particular value of \( \gamma \). It should be remembered that the shadow wage thus calculated has a constant value relative to the \( O \)-sector market wage \( W \)—and hence it will have a time trend same as \( W \). This factor will have to be taken into account in using the \( SWR \) for the evaluation of projects having a life longer than one time period.

**Derivation of the shadow wage**

The migration function in this model is the same as in the model of Section II and is implicit in the specification of \( p \) in equation (4a). When one more worker is employed in the formal urban sector, \( \{\gamma(1-p)+1\}/p \) migrants are attracted to the urban area. These migrants participate in the urban informal sector and have earnings equal to \( gW \).

The loss in output due to this migration is (i) the fall in agricultural output as in the previous models minus (ii) the increase in the output of the urban informal sector. Thus the loss in output

\[
\frac{dL}{dN} MP - (\frac{dL}{dN} - 1) gW
\]

\[
= \frac{dL}{dN} (MP - gW) + gW. \tag{24}
\]

The increase in consumption in the economy will include three elements:

(i) The worker employed in the formal sector increases his consumption by \( (W - gW) \).

(ii) The migrants coming into the urban informal sector suffer a loss in consumption. Assuming they were sharing equally in the family pot in the rural area, the loss in consumption is \( (AP - gW) \).

(iii) The families from which the migrants come gain in income (and consumption) to the extent of \( (AP - MP) \) for each migrant.

Thus, the total increase in consumption:

\[
= W - gW + \frac{dL}{dN} (AP - MP - AP + gW)
\]

\[
= W - gW - \frac{dL}{dN} (MP - gW). \tag{25}
\]

Using the familiar \( L-M \) formula, the value of the shadow wage in this model then reduces to:

\[
SWR = W - \frac{1}{\bar{S}} \left[ W - gW - \frac{dL}{dN} (MP - gW) \right]
\]

\[
= W \left[ 1 - \frac{1}{\bar{S}} \left( 1 - \frac{\gamma(1-p)}{p} (m-g) \right) \right]. \tag{26}
\]

We can substitute in (26) for the value of \( p \) from the equilibrium condition:
given in equation (21). Take as an example the case of family income maximization. With our present model, we then have:

\[ p = \frac{i(m-g)}{1-m+i(1-g)} \]  

(21a)

From (21a) and (26):

\[ SWR = W \left[ 1 - \frac{1}{S} \left( 1 - m - \frac{\gamma}{i} (1-m)(1+i) \right) \right]. \]  

(26a)

Compare this expression with the formulation for the SWR given in the case of family income maximization with rural financing (i.e. without the informal sector of Section II). Substituting the appropriate value of \( p \) in equation (13), p. 414, the interesting result is reached that the expression for the SWR is exactly the same as in equation (26a).

A more intuitive understanding of this result is possible if we remember that in both cases—with or without the informal sector—the decision on migration is being taken by the rural family as a unit when we have family income maximization. In the model with the informal sector the loss in output per migrant is reduced by \( gW \) compared with the situation in which the migrant remained unemployed during the period of search. But the cost to the family unit of migration is reduced by exactly this amount, so that we get more migration, given \( \gamma \) and \( i \), in the present case. The two effects cancel each other out, giving an identical value for the shadow wage, provided that the discounting factor applied by migrants \( (i) \) is the same in the case of rural and self-financing.

There is, however, no reason to presume that the value of \( i \) will be the same in the two cases. With expected capital market imperfections in LDCs, migrants will have a substantially higher \( i \) if they have to be financed by their rural families than if they financed their urban living by participating in the informal sector. Thus the ratio of \( SWR/W \) could be significantly higher in the present case in spite of the identical expressions for the shadow wage. In particular, the conclusion reached at the end of Section II—that for all practical purposes the SWR in a model with induced migration would not be very much greater than the L–M result—need not hold for the current model.\(^1\) The SWR could indeed be higher than the market wage. The condition for this is, however, stringent and is in the following form (26a)

\[ \gamma > \frac{i}{1+i}. \]  

(27)

Since \( \gamma \) is small, \( i \) has to be very low indeed for this condition to be met.

Finally, with personal income maximization the decisions by migrants are taken as individuals separated from the rural family. The fall in output

\(^1\) Note that while the unemployment rate could scarcely be expected to be more than 20 per cent, the ratio of informal sector employment to total urban employment could be very much higher.
per migrant is \((m-g)W\) rather than \(mW\) as in the model without the informal sector. The finance available to the migrant during his search period is also different: \((a-m)W\) in the model without the informal sector, and \(gW\) in the informal sector case. Using these values in the equations (21) for \(p\), and substituting for \(p\) in equations (13) and (26) we get the following expressions for the \(SWR\) in the two cases:

For personal income maximization with rural financing:

\[
SWR = W \left[ 1 - \frac{1}{S} \left( 1 - m - \frac{\gamma}{i} (1+i)(1-a) \right) \right].
\]  
(26b)

With informal sector financing:

\[
SWR = W \left[ 1 - \frac{1}{S} \left( 1 - m - \frac{\gamma(1+i)(1-a)(m-g)}{i(a-g)} \right) \right].
\]  
(26c)

In either case the \(SWR\) is smaller than in the corresponding family maximization case. Thus the conditions for the \(SWR/W\) to be equal to or greater than unity are more stringent.

Conclusions

The paper explored the various expressions for the shadow wage which emerge when we try to combine the effects of the loss in output in the alternative occupation, the extra consumption involved, and of induced migration in the presence of an institutionally determined (high) wage in the urban formal sector. Two types of migration function implicit in the definition of the probability of getting an urban job were discussed, as were two different methods of financing the migrant during his period of search for an urban formal sector job.

1. The proposition that the shadow wage should equal the market wage when we take into account induced migration (à la Harberger–Stiglitz) does not depend on the condition that the equilibrium rate of unemployment in the urban market is stable. It is crucially dependent, on the other hand, on the assumption of a migration function which predicts an exaggerated rate of urban unemployment and hence an exaggerated volume of induced migration (Stiglitz’s multi-period model which gives this result is dealt with in the Appendix).

2. With a more reasonable definition of the migration function it was shown in Section II that the effect of induced migration on the shadow wage will be small when there is ‘rural financing’ of the job-seeker in the urban market, and the \(L-M\) result is only slightly modified.

3. In Section III the value of the shadow wage was considered when migrants financed themselves by participating in the informal sector. An expression for the \(SWR\) could be derived for a labour market with a dynamic equilibrium growth in which a specific (constant) value of \(\gamma\) determines not
only the proportion of employment in the informal sector (in the same way as the rate of unemployment in the model of Section II), but also the ratio of earnings in the sector to the formal sector wage. It was seen that primarily because migrants could be expected to apply a lower discount rate to their expected income stream, the induced migration effect could be significant and the shadow wage may exceed the market wage depending on the actual magnitudes of the variables involved.

We do not as yet have detailed knowledge about the urban labour markets of LDCs to assess which type of financing of the rural–urban migrant is more dominant. The large (and probably growing) size of the informal sector is apparent in most urban markets of the LDCs. But it does not follow that the hypothesis of Section III—that migrants are able to search for jobs in the ‘formal’ sector simultaneously as they earn a living wage in the ‘informal’ sector—is necessarily valid. It should be remembered that the job-seekers have to compete for the available volume of work in the ‘informal’ sector with another type of migrant who is interested in doing a spell of work in the ‘informal’ sector as an end in itself and under certain conditions the latter might be the dominant group in this sector.\footnote{For a more extended analysis of this point, see Mazumdar [8].}

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APPENDIX

Stiglitz [11] uses a migration function related to that of Section II of the text implicit in his equation:

\[ p = \frac{\gamma N}{U} \]  \hspace{1cm} (4 b)

and not the equation of Section I based on an idea of the instantaneous turnover of labour. Yet he obtains an expression for the equilibrium unemployment rate of

\[ \left(1 - \frac{MP}{W}\right), \]

equally like the 'Harberger-type' model of Section I. How does he do it?

It is seemingly based on a multi-period model, but is not really so because of its neglect of the problem of financing and the rate of discount. Stiglitz's argument is as follows:

Let \( D \) be the expected duration on the job \((= 1/q)\)

\( tu \) be the expected duration of unemployment.

Now \( tu \) will be equal to \( 1/p \), \( p \) being the probability of being hired at any period.

The fraction of total time spent in the urban market actually on the job when obtained will be equal to:

\[ \frac{D}{D+tu} \]

Thus the expected wage is equal to:

\[ W_u\left(\frac{D}{D+tu}\right) = \frac{W}{1+tuq} = \frac{W}{1+(q/p)}. \]  \hspace{1cm} (29)

Assuming family income maximization:

\[ MP = \frac{W}{1+(q/p)}, \]  \hspace{1cm} (30)

whence

\[ p = \frac{MP}{W-MP} q. \]  \hspace{1cm} (30 a)

If we now assume there is no net increase in employment in the urban market \( \gamma = q \), and remembering that from (4 b) the rate of unemployment is equal to:

\[ \frac{\gamma}{\gamma+p}, \]

we have

\[ \frac{U}{L} = 1 - \frac{MP}{W}. \]  \hspace{1cm} (31) from (29)

Apart from the special assumptions of family income maximization and zero net growth in employment, the model allows for only one aspect of the problem of search: i.e. there is net loss of the alternative output \((= MP)\) per period during the period of waiting. The cost of maintenance of the job-seekers during this period is ignored. Even if we assume with rural financing and family income maximization that the family maintains the job-seekers in town at the level of consumption they would have earned in the village anyway (with no compensation for cost-of-living differences) there would be a net cost due to the urban income stream starting at a later date than it would have if the migrants had stayed at home. This cost should have been taken care of by obtaining present values at the appropriate rate of discount, and for the appropriate periods of time involved.

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