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Strategic Interactions and Portfolio Choice in Money Management: Theory and Evidence

Alvaro Pedraza Morales*

Abstract

I study portfolio choice of strategic fund managers in the presence of a peer-based underperformance penalty. While the penalty generates herding behavior, correlated trading among managers is exacerbated when a strategic setting is considered. The equilibrium portfolios are driven by the least restricted manager, who may vary according to the realization of returns. I compare model predictions to evidence from the Colombian pension fund management industry, where six asset managers are in charge of portfolio allocation for the mandatory contributions of the working population. These managers are subject to a peer based underperformance penalty, known as the Minimum Return Guarantee (MRG). I study trading behavior by managers before and after a change in the strictness of the MRG in June 2007. The evidence suggests that a tighter MRG results in more trading in the direction of peers, a behavior that is more pronounced for underperforming managers. I show that these findings are consistent with the qualitative and quantitative predictions of the theoretical model.

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In financial markets, institutional investors manage a significant portion of the total assets and comprise an even greater portion of the trading volume. Given the size of the portfolio management industry, models addressing the agency issues of delegated portfolio management and their effects on asset pricing have become popular over the last few years.

In this paper, I study portfolio choice of strategic fund managers in the presence of a peer-based underperformance penalty. While the penalty might generate crowd effects among managers even under competitive behavior, correlated trading is potentially exacerbated when strategic behavior is considered.

Relative performance concerns among managers may be present for several reasons. The most common explicit compensation schemes in the asset management industry depend linearly on the volume of assets under management and non-linearly on excess performance relative to a benchmark (as exemplified by success fees or performance bonuses). Another implicit source for relative performance concerns is the potential increase in funds flowing towards the best performing managers. Such empirical regularities have been documented by Chevalier and Ellison (1997) for mutual funds and Agarwal, Daniel and Naik (2004) for the hedge fund industry. The evidence suggests that a manager will get additional money flows and thus a higher future compensation if her relative return is above a threshold.

A less studied source of relative performance concerns comes from regulation. In particular, in countries that have moved from pay-as-you-go (PAYGO) pension systems to Defined Contributions (DC) systems based on individual accounts, regulation typically includes a Minimum Return Guarantee or an underperformance penalty levied on portfolio managers. The rationale for such regulations is to discourage excessive risk taking by the managers of these accounts. In most cases, the formula to determine the MRG is calculated based on peer performance. Hence, managers have an explicit reason to care about the returns of their peers.
Relative performance concerns generated by excess performance fees or the performance-flow relationship typically imply a convex payoff based on relative performance, which gives rise to more risk taking among managers. In contrast, an underperformance penalty represents the opposite kind of performance incentive—a serious penalty for being the loser, as opposed to a big price for being the winner—and therefore one would expect to find the opposite sort of behavior, namely herding. More specifically, an underperformance penalty based on peer returns introduces an explicit reason for managers to track each others’ portfolios, possibly generating crowd effects as managers minimize the risk from behaving differently from others. Of course managers might also herd into (or out of) the same securities over some period of time for other reasons not related to underperformance penalties. First, managers may receive correlated private information, perhaps from analyzing the same indicator (Hirshleifer, Subrahmanyam and Titman, 1994). Second, a manager might infer private information from the prior trades of better-informed managers and trade in the same direction (Bikhchandani, Hirshleifer and Welch, 1992; Sias, 2004). Third, managers might disregard private information and trade with the crowd due to the reputational risk of acting differently from other managers (Scharfstein and Stein, 1990). Finally, managers might simply have correlated specific preferences over certain types of securities. An underperformance penalty, such as the MRG, resembles a reputational risk, in that the manager might be penalized for having lower returns than her peers. With the MRG, the risk is explicit as the manager will be penalized financially if returns are below the maximum allowed shortfall relative to the peer benchmark.

When the number of competing money managers is small, relative performance concerns might lead to strategic behavior. In this environment, strategic interactions imply that a manager’s optimal portfolio choice needs to take into account the impact of his trades on other managers’ decisions. In such setting, the effects from a peer performance penalty on portfolio strategies and trading dynamics might be more pronounced.

The DC pension industries in several Latin American and Eastern European countries are
natural candidates to display strategic interaction, because they consist of a small number of competing Pension Fund Administrators (PFA), who act as asset managers and make portfolio choices on behalf of the working population.

I use detailed data on the security allocations chosen by Colombian PFAs between 2004 and 2010 to study the strategic interaction between managers under relative performance concerns. The Colombian institutional set up satisfies several key conditions necessary for testing for strategic behavior of managers. PFAs manage the savings of a captive market, namely individual retirement accounts, so their set of competitors is restricted to other PFAs, and excludes other asset managers. Each PFA must comply with a MRG that is calculated based on peer performance, creating an explicit incentive to care about the portfolio choice and performance of other managers. Since the number of PFAs is small (six), strategic behavior might be more pronounced than with a large number of competitors. Previous empirical work on strategic behavior has examined data on managers’ broad asset allocation or overall portfolio returns. By using monthly detailed portfolio holdings, I am able to test richer implications of models with strategic behavior. Finally, the Colombian government changed the MRG formula in June 2007, increasing the maximum allowed shortfall and thereby loosening the MRG. This policy experiment allows me to measure the change in behavior associated with the change in the underperformance penalty, arguably holding constant other possible explanations for correlated trading.

The evidence suggests that a tighter MRG results in more trading in the direction of peers and a smaller cross-section dispersion of returns between pension funds. Moreover, the ranking among managers in terms of performance seems to play a role in portfolio balancing decisions. With a tighter MRG, underperforming managers are more likely than their competitors to trade in the direction of their peers. This is done by buying stocks in which the manager has smaller weights in her portfolio relative to her peers, as opposed to selling stocks with larger weights relative to her peers.

I next present a partial equilibrium model of portfolio choice in which a small number
of managers behave strategically in the presence of relative performance concerns. Relative performance concerns arise from a peer-based underperformance penalty similar to the MRG. Fund managers are endowed with the wealth of a group of fund investors and charge a management fee for carrying out the investment strategy. Managers take prices/returns as given and are strategic in the sense that they internalize that their choices affect the strategy of their peers and vice-versa due to the underperformance penalty. I calculate the equilibrium policies and trading strategies. I calibrate the model to the data before the change in regulation and simulate the quantitative effects of a change in regulation comparable to the one carried out by the Colombian government. The model captures the observed change in behavior after the change in policy. In particular it can account both qualitatively and quantitatively for the observed changes in the extent of correlated trading and the observed increase in dispersion of returns across managers.

The model shows that the presence of relative performance concerns through a peer based underperformance penalty affects the asset allocation in two distinct ways. First, the MRG regulation gives rise to time varying investment policies, with the managers making procyclical trades, buying more of the asset that performed well in the previous period. Second, given the strategic nature of the managers, the model suggests that the equilibrium portfolios are driven by the choices of the least restricted manager (i.e. the manager that has superior accumulated returns at any period in time), while the more restricted manager is likely to end up selecting a portfolio that is similar to her competitor. The strategic nature of the managers exacerbates the extent of procyclical trading, as both the more and less restricted manager recognize their relative position and the action of their competitor. For example, the overperforming manager moves heavily towards her normal portfolio (the optimal portfolio without relative performance concerns) as she recognizes that the underperforming manager, who is more exposed to the penalty, will rebalance her portfolio in the same direction. The overall extent of this correlated trading is more pronounced than in a setting with no strategic interactions among managers.
The rest of the document is organized as follows: In section 1.1 I review the leading literature on strategic behavior by fund managers. The empirical evidence is presented in section 2, where I conduct two empirical exercises that describe the overall trading behavior of Colombian PFAs. In section 3, I introduce a model of strategic fund managers in the presence of an underperformance penalty. I calculate the optimal portfolio choice and trading strategies. Finally, in section 4, I present the conclusions and discuss future work.

1.1 Related Literature

This paper is related to several strands of the literature. The empirical literature on strategic behavior of money managers has focused on the trading strategies of asset managers competing for leadership to gain status, higher compensation or increased future flows of funds. The game is similar to a typical tournament, where winners get a large prize and the losers end up with much less. In such tournaments, managers optimally increase their risk taking to maximize the probability of reaching a top position at some target date (usually at year end). Using U.S. data, Chevalier and Ellison (1997) document strong gambling incentives among top-performing mutual funds. Examining strategic behavior in the context of fund families, Kempf and Ruenzi (2008) document that mutual fund managers belonging to families with a small number of funds behave differently from managers belonging to large families. They argue that this result is driven by strategic interactions that might be more pronounced in small fund families. For UK funds, Jans and Otten (2008) present evidence of strategic behavior, finding that fund managers recognize the impact of their own decisions on the actions of their peers, rather than treating competing managers as exogenous benchmarks. A possible explanation for the findings of Kempf and Ruenzi (2008) and Jans and Otten (2008) is that, with strategic managers, the interim leader expects the laggard to increase risk, and therefore the leader also increases risk to maintain his lead (Taylor, 2003).

I complement this literature by presenting empirical evidence on the trading behavior of Pension Fund Administrators in Colombia, where a small number of managers compete and
set their strategies to avoid a peer-based underperformance penalty. With only six PFAs, it is highly likely that managers are strategic, as they recognize that the other managers will react to their own portfolio choice as it will affect each manager’s future compensation. In contrast to the previous literature, where risk taking behavior arises as managers try to outperform their peers, I study the effects of the opposite kind of performance incentive, a serious penalty for being the loser. In this setting one would expect to find the opposite outcome, meaning herding among managers.

Despite strong theoretical foundations and a common perception that professional investors herd, earlier studies found little evidence of herding behavior, and in most cases herding was mostly associated with only particular types of assets, like small stocks (Wermers (1999) for US mutual funds and Lakonishok, Shleifer and Vishny (1992) for US pension funds). In a more recent study, however, Sias (2004) shows that changes in security positions of institutional asset managers over a quarter are strongly correlated with the the trades of other institutions over the previous quarter. The author also finds that changes in positions on particular stocks are weakly but positively related to returns over the following year. The results favor the hypothesis that herding is a result of institutions inferring information from each other’s trades. Raddatz and Schmukler (2013) find that Chilean pension funds, also subject to a peer-based MRG, tend to herd, buying and selling the same assets at the same time. The authors also find differences in the extent of herding across assets. By comparing the trading behavior of PFAs before and after the MRG change, I am able to identify the effects of the underperformance penalty holding other factors constant.

On the theoretical side, papers on portfolio choice with underperformance constraints include Deelstra, Grasselli and Koehl (2003) and Tepla (2001). In a general equilibrium framework Cuoco and Kaniel (2011) study asset price effects of different performance-based fees for money managers, including both excess performance bonuses and underperformance penalties. Other papers studying relative performance concerns, examine the effects of delegation on limits to arbitrage, trading volume and price discovery. These papers include
Shleifer and Vishny (1997), Cuoco and Kaniel (2011), Guerrieri and Kondor (2012) and Kaniel and Kondor (2013). However, these authors abstract from the potential role of strategic interactions in trading behavior. In a partial equilibrium framework, Basak and Makarov (2014) model strategic interactions between two managers competing for additional flows. My model is related to theirs in that I assume a discrete and small (two) number of strategic managers, but I focus on the effects of an underperformance penalty based on peer returns, as opposed to the effects of payoffs that are convex in portfolio returns due to a flow-performance relationship. In my setting herding is the optimal strategy, in contrast to gambling which can be present when managers face a convex payoff structure as in Basak and Makarov (2014). In my model, the overperforming manager moves heavily towards her preferred portfolio as she recognizes that the underperforming manager has to rebalance her portfolio in the same direction. The extent of this correlated trading is more pronounced given the strategic nature of the managers and the aggregate portfolio follows the preferences of the least restricted manager.

2. EMPIRICAL EVIDENCE

2.1 The Colombian Private Pension Industry

In 1993 the Colombian Congress approved Law 100, which among other reforms introduced major changes in the pension system. The country adopted a dual pension scheme, in which a defined contribution (DC) system of individual accounts was created in addition to the already existing defined benefit system. Under the new system, pensions were financed by compulsory contributions made by both the employer and the employee. The law also provided guiding principles for the establishment, operation and supervision of Pension Fund Administrators (PFAs). Under the new scheme, all workers who chose the DC system were required to select a PFA to manage their retirement accounts. The worker’s investment decision was restricted to the choice of the PFA, while the government regulated PFAs’
portfolio strategies by imposing limits on specific asset classes and individual securities, and through other provisions such as banning short selling. Workers were allowed to switch PFAs every six months.

The law also determined the compensation structure of the PFAs and the Minimum Return Guarantee (MRG). The PFAs were allowed to charge fees for collecting contributions, managing the fund and giving benefits. In particular, PFAs charge a front end load fee of 5.5% on new contributions. On average, the fee on new contributions represents close to 90% of the annual compensation of the PFAs. If a worker makes consistent contributions he does not face any additional charges.

The MRG is a lower threshold of returns that each individual PFA needs to guarantee for its investors. If a PFA fails to provide at least this return, the PFA must transfer part of its own net worth to the fund to make up the shortfall. The MRG is assessed monthly by comparing the fund’s average annual return over the previous three years to the average of the six PFAs. Between January 2004 and June 2007, the minimum return guarantee was calculated as the average across PFAs of the average annual return over the previous three years ($\Pi_t$), minus 30%, so that $MRG_t = 70\% \Pi_t$. After June 2007 the government changed the formula to $MRG_t = \min\{70\% \Pi_t, \Pi_t - 2.6\%\}$. For average industry returns below 8.66%, the new formula implies a MRG equal to $\Pi_t - 2.6\%$, as $70\% \Pi_t > \Pi_t - 2.6\%$. Effectively, for this set of returns, the new formula yielded a lower MRG (equivalently, a larger allowed shortfall) than what would have been calculated before June 2007.

Within this institutional setting, the MRG creates an explicit reason for each PFA to track peer portfolios and performance. The penalty for falling too far behind the industry average returns may lead the PFA to bankruptcy. Given the size of each PFA, and the total value of assets under management, a typical Colombian PFA falling 50bps below the MRG threshold would use up its entire net worth compensating its investors. With such a severe penalty, one should expect that the MRG is of first order importance when PFAs set their strategies.
Data on Colombian pension funds was provided by ASOFONDOS (Colombian Association of Pension Fund Administrators). The database includes the detailed security allocations for the funds managed by each of the six PFAs, on a monthly basis for the period 2004:1 to 2010:12. Summary statistics for this data set are presented in Table 1 at two-year intervals. As of June 2010, total assets under management were US$44.1 billion (equal to 17% of Colombian GDP). At that time, 32% of these funds were invested in Colombian stocks, which amounted to 7.1% of the total domestic market capitalization. Throughout the sample period, net flows to these funds were positive, which reflects the fact that most of the workers contributing to these funds were still young (more than 70% were younger than 40 years old).

In addition to the pension funds, PFAs manage voluntary retirement funds in separate accounts. These voluntary accounts supplement the compulsory retirement savings in the pension funds. Contrary to pension funds, these accounts are subject to very few regulations. In particular, they are not subject to the MRG and do not have limits on individual securities or asset classes. Moreover, workers are typically directly involved in the asset allocation of their voluntary portfolios. Panels D and E in Table 1 present summary statistics of the voluntary funds.

In the following sections I present two empirical exercises suggesting that relative performance concerns are important for the portfolio dynamics of PFAs. For this, I introduce two measures that describe trading activity by PFAs. The first is an aggregate fund measure that describes how each manager rebalances her portfolio relative to the peer portfolio. The second focuses on fund trades of individual stocks. For both empirical exercises, I focus on the trading behavior across domestic stocks. While these represent only a fraction of the total portfolio, correlated behavior among managers is likely to be more pronounced for these securities, which display higher dispersion of returns than other assets in PFAs portfolios.
2.2 Trading Strategies and Relative Performance

In this section I introduce a measure of the direction of a PFA’s trades relative to its peers. The objective is to summarize the trading behavior and strategies of Colombian pension fund managers in a parsimonious way.

At the end of each month, each fund’s location is defined by its portfolio weights. The vector of portfolio weights for a fund $i$ in month $t$ is denoted $w_i^t \in \mathbb{R}^{S+1}$, where each element $s = \{1, 2, \ldots, S\}$ represents a domestic stock in the fund’s portfolio, $w_{st}^i = \frac{\text{shares}_{st}^i \times p_{st}}{VF_i^t}$. Here $\text{shares}$ is the number of shares of stock $s$ held by the fund, $p$ is the stock price and $VF_i^t$ is the total value of the fund. The element $w_{S+1}^t$ in the vector of portfolio weights represents the fund’s participation in assets other than domestic stocks (i.e. domestic corporate debt and government debt). For each PFA $i = 1, 2, \ldots, 6$, the average peer fund portfolio has weights denoted by the vector $\pi_i^t = \frac{1}{5} \sum_{-i} w_i^t$, where $\sum_{-i}$ is the sum of all funds excluding fund $i$.

To measure a fund’s trading strategy, or its change in portfolio weights, I first adjust for passive portfolio evolution due to changes in prices. Including changes in weights due to price changes may overstate the degree of coordination among funds. If the gross return of stock $s$ between period $t$ and $t+1$ is defined as $\text{ret}_{st}$, the adjusted vector of weight changes for fund $i$ from $t$ to $t+1$ can be denoted $\Delta w_i^t$, where each element $s$ is defined by $\Delta w_{st}^i = w_{st+1}^i - w_{st}^i \times \text{ret}_{st}$. The last term accounts for the change in the weights due to differences in returns among stocks in the portfolio. To measure the position of fund i’s portfolio relative to its peers at period $t$, I calculate a vector of differences between the fund and its competitors, $d_i^t = \pi_i^t - w_i^t$. To capture the direction of portfolio weight changes, I measure the angle between the change of a PFA’s weights and the distance from its peers’ portfolio, as follows:

$$\text{direction}_i^t = \cos(\theta) = \frac{\Delta w_i^t \cdot d_i^t}{||\Delta w_i^t|| \ ||d_i^t||}$$ (1)

In this specification, $\text{direction}$ measures the correlation across securities between portfolio weight changes for fund $i$ and the initial distance between $i$ and its peers. If fund $i$ is moving
exactly towards its peers, the angle is zero and direction is equal to 1. If the manager is rebalancing the portfolio in exactly the opposite direction of its peers, the angle is 180 degrees and the direction measure equals -1.

Figure 1 displays two examples of the angle between the vector of weight changes for fund manager i and the vector of initial distance between i and the other managers. This figure assumes that there are three securities; given that the portfolio weights add up to one, the third dimension is redundant. Initially, the peer portfolio $\pi_i^t$ has a larger share of stock A than manager i. In panel (a) the manager increases her participation in stock A, moving towards peers. In panel (b) the manager increases her participation in stock B, moving away from the peer portfolio.

If there is a constraint on short selling, the space becomes a Simplex of portfolio weights and the measure would be naturally biased towards higher values of $direction$. For example, if fund $i$ is currently invested only in stock B, it is located along the vertical axis, and the only way to continue to move away from its competitors would be to move along the axis, in which case the angle would be smaller than 180 degrees and $direction$ would be greater than -1. In this example, moving away from one’s peers would mean buying more of what you already own, as opposed to short selling securities in which your peers have larger weights.

Figure 2 depicts the time series behavior of the measure of direction for both pension funds and voluntary funds. For each month in the sample, I calculate the direction of weight changes for each PFA over the next quarter, and take the average across PFAs. A high value indicates that PFAs on average are moving towards their peers. Evidently, for the pension funds, PFAs on average traded more in the direction of their peers prior to the MRG formula change in June 2007 than after this date. For the voluntary funds, the behavior of direction seems to be same before and after the policy change.

Table 2 presents summary statistics on $direction$. The statistics are split for the period before and after the change in the MRG. For the pension funds, mean direction fell from 0.32 in the early period to 0.14 after the change in the MRG, suggesting that the policy change
may have affected managers’ behavior.

Table 2 also reports statistics on the relative performance between pension funds before and after the MRG change. Relative performance with respect to the peer portfolio is defined as $rel^i_t = R^i_t - R^{-i}_t$, where $R_t$ are 36 month returns prior to $t$ (consistent with the measurement period of the MRG). The relative performance variable $rel^i_t$ measures whether fund $i$ is over-performing ($rel^i_t > 0$) or under-performing ($rel^i_t < 0$) at time $t$ relative to the other managers. After June 2007, there seems to be some increase in the cross-section dispersion of PFA returns. If portfolios are less alike, returns are likely to vary more cross-sectionally.

A separate question is whether managers’ strategies depend on relative performance. Panel C in Table 2 presents the correlation between $direction_{it}$ and $rel^i_t$. The negative correlation between relative performance and $direction$ indicates that before June 2007, PFAs with poor relative performance tended to move more strongly towards peers. After June 2007 there is no evidence that relative performance is correlated with the direction of trades.

To summarize, the loosening in the MRG in June 2007 is associated with three important changes in the data for pension funds: (F1) Less trading in the direction of peers; (F2) Increase in cross-section dispersion of returns between funds. (F3) A disappearance of the negative correlation between relative performance and trading in the direction of peers.

### 2.3 Individual Stocks and Trading Strategies

In this section I further investigate herding behavior using data on individual stock trades. For each stock, the fund’s distance to the peer benchmark is measured as $d^i_{st} = \pi^i_{st} - w^i_{st}$, where the fund can be overexposed ($d^i_{st} < 0$), underexposed ($d^i_{st} > 0$) or have the same weight ($d^i_{st} = 0$) as its peers. I estimate the following model of a fund’s changes in individual stock weights:
\[
\Delta w^i_{st} = \beta_0 + \sum_{m=1}^{M} \beta_m x^i_{st} + \gamma_0 MRG_t + \sum_{m=1}^{M} \gamma_m MRG_t \cdot x^i_{st} + \varepsilon^i_{st}
\]  

(2)

where \(\Delta w^i_{st}\) is adjusted for stock returns as in the previous section, \(x^i_{st}\) are fund and stock specific characteristics and \(MRG_t\) is a time dummy equal to one for dates before July 2007 and zero thereafter, representing the policy change. The objective here is twofold, first to determine what fund based characteristics determine PFA trading on individual stocks, and second to measure whether there was any change in the impact of these characteristics after the MRG formula was modified.

More specifically, I set \(x^i_{st} = (d^i_{st}, rel^i_t, d^i_{st} \times rel^i_t, size_t, Controls_{st}, Market^i_{st})\). Here \(size_t\) is the share of assets under management of fund i relative to the industry. The vector of \(Controls_{st}\) contains stock specific variables. I introduce lagged returns at one, three, six and twelve months to account for momentum trading, defined as purchasing (selling) assets with positive (negative) past returns.\(^9\) This popular investment strategy has been widely documented for institutional investors.\(^10\) Chan, Jegadeesh and Lakonishok (1996) suggests that momentum trading may be caused by a delayed reaction of investors to the information in past returns and past earnings. I also control for firm size and liquidity, as institutional investors may share an aversion to stocks with certain characteristics, as documented by Wermers (1999), who found evidence that US mutual funds tend to herd in small stocks. Given the potential persistence in trading strategies, I allow the error term \((\varepsilon^i_{st})\) to be correlated within stocks and correct the standard errors as in Petersen (2009).

Finally, to verify that the results are driven by managers trading relative to their peers and not by trading relative to a broad market benchmark, I calculate \(Market\ Distance\) as the difference between the IGBC index weight on stock s and fund i’s weight in stock s for each period, \(Market^i_{st} = \Pi^i_{st} - w^i_{st}\). The IGBC is a widely used value and liquidity based index for the Colombian stock market. I also interact this measure with relative performance.

This previous specification is motivated by Basak, Pavlova and Shapiro (2007), who find different behavior in U.S. equity mutual funds depending on whether managers are ahead
or behind the S&P 500 index. In their specification, the authors define *risk shifting* as an increase in the absolute difference between a fund’s returns and the S&P 500 returns. They regress this variable on an interaction between current relative returns and the market returns. Their question is whether underperforming funds move towards or away from the market index, thereby increasing or decreasing the size of deviations from market returns. My specification is analogous to theirs, in that one of my objectives is to measure whether underperforming funds move towards or away from a reference portfolio, the peers’ portfolio ($F_3$), by increasing or decreasing their holdings of stocks in which they are underexposed or overexposed. Note that while Basak, Pavlova and Shapiro (2007) only observe return outcomes, I observe portfolio weights and thus the actual strategy of the manager. In a setting with a small number of managers it might be hard to distinguish if changes in the cross-section dispersion of returns are due to managers’ strategies or to the realization of stock returns.

Table 3 documents the results of the linear regression for adjusted weight changes $\Delta w_{st}^i$. The results suggest that regardless of relative performance, managers were more likely to increase their holdings of stocks in which they were already overexposed after the MRG was loosened in June 2007 than before this date. That is, there was less trading towards peers once the MRG was loosened. This change in behavior associated with the change in regulation is consistent with the average behavior of trading direction presented in Figure 2 and Table 2.

Figure 3 presents differences in marginal effects of *distance* on adjustments in portfolio weights before and after the policy change $\left( \frac{\partial \Delta w(MRG=1)}{\partial d} - \frac{\partial \Delta w(MRG=0)}{\partial d} \right)$ along with corresponding confidence intervals. Underperforming managers ($rel < 0$) were more likely to increase their holdings of stocks in which they were underexposed ($d > 0$) prior to June 2007 than after this date. This result for individual stocks is consistent with the decrease after June 2007 in the correlation between direction and relative performance documented in Table 2-Panel C. To give a sense of the quantitative importance of these estimates, the
results indicate that a fund lagging in returns by 200bps relative to its peers, and with 5% underexposure in an individual stock \( s \), would increase the weight on \( s \) by 0.42% more prior to June 2007 than after the MRG was loosened.

For high performing managers (with relative returns above 320bps their peers), the estimated marginal effects on distance are negative but not statistically significant (not shown in Figure 3). That is, there is no evidence that the change in the strictness of the penalty affected the way top-performing managers traded stocks with overexposure, perhaps because the MRG impacts more strongly the average and worst-performing managers. Top-performing managers, unconstrained by the MRG, might deviate from the peer portfolio to possibly attract more funds. However, as the results indicate, the MRG policy change doesn’t seem appropriate to identify the effects of such incentives for managers in the high end of the return spectrum.

Specification (2) at Table 3 adds variables including distance from the market portfolio to the benchmark specification. These variables are insignificant both before and after the MRG policy change. This suggests that managers’ trading strategies are sensitive to the position relative to their PFA peers in particular, rather than to their position relative to the market portfolio.

Columns (3) and (4) at Table 3 display the standard errors corrected for within fund correlation. The overall results are robust to this form of clustering. I also performed additional robustness test that are omitted for brevity but available upon request. In these tests, I show that the main findings hold for different measures of changes in the portfolio weights.\(^{12}\)

### 2.3.1 Buy and Sell Strategies

In a final empirical exercise, I complement the above results by distinguishing buys and sells of individual stocks. This is a discrete version of the previous specification. Here, fund \( i \)’s trading strategy for a particular stock \( s \) is measured by whether the fund buys or sells the
stock prior to the following period:

$$(buy^i_{st}, sell^i_{st}) = \begin{cases} 
(1,0) & \text{if } shares^i_{st+1} > shares^i_{st} \\
(0,1) & \text{if } shares^i_{st+1} < shares^i_{st} \\
(0,0) & \text{if } shares^i_{st+1} = shares^i_{st} 
\end{cases}$$

corrected for stock splits at period $t$. In this setting, the analog to the direction measure introduced before is as follows: When a fund buys shares in stocks in which it’s already overexposed (underexposed), it moves away from (towards) the peer benchmark. When a fund sells shares in stocks in which it’s already overexposed (underexposed) it moves towards (away from) the peer benchmark.

Panel B in Table 1 shows some trading statistics for different months within the data set. For example, in June 2008, PFAs collectively held 44 different stocks and each fund on average had 26.3 stocks in its portfolio. That month, each PFA traded on average 8.33 stocks, with 6.66 of those trades as buys. In this setting a trading strategy is measured by the probability at time $t$ that fund $i$ buys or sells stock $s$ within the set of stocks owned by all PFAs (i.e. the probability of fund $i$ buying stock $s$ from among the 44 total stocks is the likelihood that $s$ was among the 6.6 stocks that fund $i$ bought that period).

**Note on Short Selling:** As was the case for the direction measure discussed in the previous section, the short selling ban for these funds introduces a bias against using sales to move away from peers. Consider the previous example. The average PFA sold 1.67 stocks during June 2008. Given the short selling constraint, those sells must come from the set of stocks owned in the previous month (25.8 as of May 31, 2007) not from the total set of stocks held by the PFA industry (44 as of May 31, 2007). Hence, a measure of the probability of selling a stock that considers the entire set of securities is naturally biased towards smaller values, as opposed to a measure of the probability of buying a stock since a fund can buy any stock in the peer portfolio whether owned at the beginning of the period or not. For this
reason I examine buying and selling strategies separately in what follows. Moreover, when estimating the probability of selling a stock, I condition on stock ownership at the beginning of the previous period.

I estimate a Probit specification of the probability of buying ($y = buy$) or selling ($y = sell$) a stock as $Pr(y_{ist} = 1) = \Phi (\beta_0 + \sum_m \beta_m x_{ist} + \sum_m \gamma_m MRG_t \cdot x_{ist})$, where $\Phi$ is the cumulative distribution function of the standard normal, and the vector of independent variables $x$ is the same as in equation (2).

Columns one and two of Table 4 document the results of the Probit regression for the probability of buying a stock ($buy_{ist}$). Consistent with the results in the continuous regression, managers were more likely to buy stocks in which they were already overexposed after the MRG was loosened in June 2007 than before. Meanwhile, an underperforming manager ($rel < 0$) was more likely to buy stocks in which she was underexposed ($d > 0$) prior to June 2007 than after this date. Columns three and four of Table 4 present the results from the Probit regression for the probability of selling a stock ($sell_{ist}$) conditional on stock ownership. The coefficients on the interactions between MRG, Peer Distance and Relative Performance are all indistinguishable from zero, suggesting that the policy change in June 2007 had no impact on how PFAs sold stocks in which they were overexposed, regardless of relative performance. Given that between 2004 and 2010 the yearly net flows to these funds were about 8.5% of the value of the fund, underperforming managers had the option of reducing their relative participation in any given stock by holding their number of shares constant, as opposed to selling shares.

To summarize the main empirical findings, the evidence suggests that a more strict MRG prior to June 2007 is associated with more trading in the direction of peers, and in particular more buying of stocks in which managers were underexposed. Meanwhile, underperforming managers traded more heavily towards the peer portfolio prior to June 2007, by buying stocks in which they were underexposed, as opposed to selling stocks in which they were overexposed. This asymmetric behavior between buys and sells could be explained by the
fact that these funds were growing within the sample period. As I will show in section 3, these results are largely in line with the predictions of a model in which money managers behave strategically due to relative performance concerns.

2.4 Alternative Explanations

The specification strategy above assumes that the policy change is exogenous to the domestic stocks’ return process. In the estimation I control for stock-specific attributes such as past returns and trading volume. However, one cannot control for all stock characteristics that might have changed after July 2007 and that might have induced the funds to adjust their trading behavior. For example, PFAs might have received more good signals about the fundamentals of stocks in which they were underexposed prior to July 2007 than after, inducing them to buy more of those stocks before the policy change than after. The shortcoming of this argument is that if a PFA is underexposed in a particular stock relative to the peer portfolio, by construction there must at least one PFA overexposed in the same stock. As favorable new information arrives about a stock, both underexposed and overexposed PFAs should increase their holdings. Hence, one would need some sort of argument for why PFAs with underexposure were the only ones receiving good signals.

Another possible explanation for the results is that PFAs altered their trading strategies due to managerial changes around the time of the policy change. For example, trading strategies might result from changes in management within the firms or shifts in preferences among the top investment officials. A closer look at PFAs’ CEO replacement indicates that, while there were some changes in management over the sample period, there is no evidence of an industry wide event before or after the MRG adjustment. In terms of preference shocks, interactions between PFAs dummies and the MRG dummy should account for individual PFA changes before and after the policy experiment. However, an industry-wide taste shock occurring in mid 2007 would be indistinguishable from the policy experiment. While such event is unlikely, it cannot be ruled out under the current empirical specification.
3. THE GENERAL MODEL

Motivated by the above empirical findings, I consider a model in which the presence of relative performance concerns among a small number of money managers leads to strategic behavior. In particular, I focus on portfolio choice when there is an underperformance penalty, such as the MRG described in the previous section. I show that the behavior of institutional asset managers in the presence of the MRG is consistent qualitatively and quantitatively with the observed data.

3.1 Model Assumptions

I consider a finite horizon economy \( t = 0, 1, \ldots, T \), modeled as follows:

**Securities:** The investment opportunities are represented by a riskless bond and a risky stock. The bond is a claim to a riskless payoff \( B > 0 \). Without loss of generality the net interest rate is normalized to zero (B price is normalized to \( B = 1 \)).

The gross stock return follows a random process with states \( r^s = \{r^H, r^L\} \) and probabilities \( \{p, \ 1-p\} \). This is a partial equilibrium model as the return process is exogenous. The return between \( t \) and \( t+1 \) on a portfolio with fraction \( \phi_t \) of wealth invested in the stock and \( 1 - \phi_t \) in the risk free bond is denoted by

\[
R^s_{t+1}(\phi_t) = \phi_t (r^s_{t+1} - B) + B
\]

**Fund Managers:** I consider two fund managers \( i \) and \( j \). Each manager chooses an investment policy \( \{\phi^j_t\}_{t=0}^{T-1} \). For this, they are compensated at time \( T \) with a management fee \( F_{iT} \), which is a function of the terminal value of their portfolio \( W_{iT} \) and that of their competitor \( W_{jT} \). Specifically I assume that

\[
F_{iT} = F(W_{iT}, W_{jT}) = \beta W_{iT} + \gamma W_{i0} \min \left\{ 0, \frac{W_{iT}}{W_{i0}} - \frac{W_{jT}}{W_{j0}} + x \right\}
\]
In this specification, the fund managers’ compensation at time $T$ consists of two components: a proportional fee, which depends on the final value of the portfolio $\beta W_{iT}$, and the underperformance penalty $\gamma W_{i0} \min \left\{ 0, \frac{W_{iT}}{W_{i0}} - \frac{W_{jT}}{W_{j0}} + x \right\}$ which depends on the manager’s performance relative to the other manager $j$. Here $x \geq 0$ is the allowed shortfall, i.e., $x = \infty$ implies that the manager is unrestricted, while $x = 2\%$ means that the maximum return shortfall allowed is 2\% relative to the peer returns. Any cumulative returns below this threshold result in a penalty that reduces the manager’s net fee, possibly to compensate the investors for the lack of returns. The size of the penalty is modeled by $\gamma$. In the Colombian setting, the PFAs face an underperformance penalty with $\gamma = 1$, since under the Colombian MRG the manager pays a penalty that guarantees that the investors’ net returns are exactly the peer benchmark $\frac{W_{jT}}{W_{j0}} - x$.

Throughout this document I refer to a manager that pays the penalty in any given state as the loser in that state. In this setting strategic interactions arise as each manager needs to consider each other’s policy reaction so as to avoid the underperformance penalty.

Fund managers are assumed not to have any private wealth. They therefore act so as to maximize the expected utility $E_0 [u_i(F(W_{iT}, W_{jT}))]$ given initial wealth $W_{i0}$, subject to the period by period budget constraint $W_{i,t+1} = R_{s}^{*} (\phi_{it}) W_{it}$.

**The Normal Policy:** In the rest of the document I refer to the normal policy $\phi_{it}^{NP}$ of fund manager $i$ as the optimal portfolio allocation when no relative performance concerns are present. In this case $\gamma = 0$ and each fund manager solves a standard portfolio choice problem. The optimal share in stocks is given by the first order condition

$$p(r^H - B) u_i' \left( \frac{R_{i,t+1}^H (\phi_{it}) W_{it}}{W_{i0}} \right) + (1 - p) (r^L - B) u_i' \left( \frac{R_{i,t+1}^L (\phi_{it}) W_{it}}{W_{i0}} \right) = 0 \quad (5)$$

for $t = 0, 1, \ldots, T - 1$.

Fund managers $(i, j)$ are assumed to have CRRA preferences defined over their final wealth, $u_m(W) = \frac{1}{1-\sigma_m} W^{1-\sigma_m}$ with $m = i, j$. To generate differences in the normal policy, I assume that manager $i$ is less risk averse than manager $j$, $\sigma_i < \sigma_j$. Using data from
U.S. mutual funds, Koijen (2008) documents substantial heterogeneity in estimates of fund managers’ risk aversion. Portfolios between managers may also differ because of ability, information, or pay for performance incentives. Here, my objective is to study how the introduction of an underperformance penalty such as the MRG causes fund managers to deviate from their normal strategies.

In this paper, I appeal to the Nash equilibrium notion to characterize managers’ strategic interactions in the presence of relative performance concerns. Below, I define the structure of the game between managers at time 0, where each manager draws up a plan of how she is going to invest throughout the whole time period \( \{0, 1, \ldots, T\} \). This definition of the game eases exposition, and is without loss of generality, since neither manager would want to deviate from a policy chosen initially at any subsequent date \( t \), so that the equilibrium policies are time-consistent.

**Information sets:** I consider a complete information game at time 0, in which each manager knows all the primitives and parameters of the model described above, namely the stock return process, own initial wealth and risk aversion and those of the other manager. Finally, each manager knows the functional form of the underperformance penalty (\( \gamma \) and \( x \)).

**Strategy sets:** A strategy of manager \( i \) is a function \( \phi_i(t, W_{it}, W_{jt}) \) defined over the space \( \{0, 1, \ldots, T\} \times (0, +\infty) \times (0, +\infty) \), where \( \phi_i(t, W_{it}, W_{jt}) \) is manager \( i \)'s investment policy at time \( t \) for given values of wealth under management, \( W_{it} \), and that of her opponent, \( W_{jt} \). For convenience, I will use \( \phi_{it} \) as a shorthand notation for manager \( i \)'s time \( t \) investment strategy and drop its arguments.

**Manager’s payoffs:** The manager’s payoffs for policy vectors \( \{\phi_{it}, \phi_{jt}\}_{t=0}^{T-1} \) are given as follows. First, period T wealth is obtained by substituting \( \phi_{it} \) and \( \phi_{jt} \) into the dynamic wealth process of each manager. Given terminal wealth \( W_{iT} \) and \( W_{jT} \), fees are computed according to (4), yielding the final payoff.
3.2 Two Period Model ($T = 1$): Portfolio Choice

I start by solving the model in a two period version of the above economy. The objective here is to show how best response and equilibrium policies are calculated and to address in the simplest environment the effects of the underperformance penalty on managers’ equilibrium portfolios.

In a two period and two state economy ($s = \{H, L\}$), for a given initial level of wealth under management $W_{i0}$, a portfolio allocation $\phi_{i0}$ by manager $i$ at period 0 determines two possible values of final wealth $W_{i1}^H$ and $W_{i1}^L$ according to equation (3). For each value of final wealth there is an associated management fee, calculated using (4). With no underperformance penalty $\gamma = 0$, the manager’s income is a proportional fee on the final wealth under management ($\beta W_{i1}^s$). With the underperformance penalty, the net fee depends on whether manager $i$’s returns are above or below the peer benchmark $W_{j1} - x$ for each of the two states. Hence a portfolio choice $\phi_{i0}$, for a given choice $\phi_{j0}$, is effectively a choice of a possible pair of management fees in both high and low states of the economy.

Figure 4 depicts the optimization problem in the space of returns. In the left panel, the normal policy is such that in both the high and low states, manager $i$’s returns are above her peer or below her peer by less than $x$. In this case the underperformance penalty is not binding and the manager’s optimal portfolio is her normal policy. In the right panel, if the normal policy was played, the manager’s returns would be below the peer benchmark in the low state (loser in low). The manager optimally chooses a portfolio with less exposure to the risky asset ($\hat{\phi}_{i0} < \phi_i^{NP}$), generating smaller returns in the high state but greater returns in the low state than in her normal policy. Basically the manager is giving up a higher income in the high state to increase the income in the low state in order to reduce the penalty. In this example, manager $i$ still pays the penalty in the low state, but less than what she would have paid had she played her normal policy.

In the rest of the document I will use $\hat{\phi}$ to denote best response functions and $\phi^*$ to
denote equilibrium policies.

**PROPOSITION 1.** For a given manager $j$ portfolio choice $\phi_{j0}$, manager $i$’s best response $\hat{\phi}_{i0}$ is given by

$$
\hat{\phi}_{i0} = \begin{cases}
    a_i + b_i \phi_j - c_i x & \phi_i^{NP} \leq \phi_{j0} - \frac{x}{r_B - B} \quad \text{(loser in high)} \\
    \phi_i^{NP} & \phi_{j0} - \frac{x}{r_B - B} < \phi_i^{NP} < \phi_{j0} + \frac{x}{B - r_L} \\
    \bar{a}_i + \bar{b}_i \phi_j + \bar{c}_i x & \phi_i^{NP} \geq \phi_{j0} + \frac{x}{B - r_L} \quad \text{(loser in low)}
\end{cases}
$$

(6) \begin{align*}
\phi_i^{NP} - \frac{x}{r_B - B} < \phi_i^{NP} < \phi_{j0} + \frac{x}{B - r_L} \\
\phi_i^{NP} \geq \phi_{j0} + \frac{x}{B - r_L}
\end{align*}

(7) \begin{align*}
\phi_i^{NP} \leq \phi_{j0} - \frac{x}{r_B - B} \quad \text{(loser in high)}
\end{align*}

(8)

with $b_i, \bar{b}_i, c_i, \bar{c}_i \geq 0$. Switching subscripts $i$ and $j$ above yields manager $j$’s best response. The proof of the proposition and definitions for $a_i, \bar{a}_i, b_i, \bar{b}_i, c_i$ and $\bar{c}_i$ in terms of the underlying parameters of the model are presented in Appendix C.

**DEFINITION 1.** The Shifting Region is the region in the parameter space such that a manager’s best response policy is to play a strategy different than her normal policy $\hat{\phi} \neq \phi^{NP}$.

In this two period economy, this region is defined by the conditions in (6) and (8). In other words, we are in the shifting region if $\phi_i^{NP} \in (-\infty, \phi_{j0} - \frac{x}{r_B - B}] \cup [\phi_{j0} + \frac{x}{B - r_L}, \infty)$ which is the region to the left of $\frac{W_i}{W_j} - x$ and below $\frac{W_i}{W_j} - x$ in Figure 4.

In the Shifting Region, $\bar{b}_i, \bar{b}_i \geq 0$ indicates that the manager optimally chooses a portfolio that is shifted towards her peer, buying more or less of the risky asset depending on the other manager’s portfolio. If $x$ increases (e.g. the allowed shortfall is larger, implying a looser MRG) the shifting region becomes smaller, which means that the manager will play her normal policy for a larger set of parameters.

Moreover, in the shifting region, a larger (smaller) $x$ implies a smaller (larger) shift. As an example, consider the best response when manager $i$ plays to lose in the low state (8). Here the manager selects an allocation with a greater share in the risky asset than her competitor ($\hat{\phi}_{i0} > \phi_{j0}$). More specifically, the allocation is such that the manager pays the underperformance penalty in the low state (loser in Low). A larger $x$ implies a greater share in the risky asset, or less shift from her normal policy. On the contrary, a smaller $x$ implies
a smaller share in the risky asset or a larger absolute shift. A similar analysis can be made for the best response in (6). Note that when the manager is playing to pay the performance penalty in the high state (loser in high) she has a lower share of the risky asset than her competitor. As $x$ decreases, her participation in the risky asset increases, moving farther away from her normal policy and closer to her peer.

To summarize, the maximum allowed shortfall $x$ determines both the size of the shifting region and the size of the shift in the best response functions.

**COROLLARY 2.** In the shifting region, if the manager is playing to lose in the low state, the best response satisfies $\hat{\phi}_{i0} \in [\phi_{j0} + \frac{x}{B-r_L}, \phi_{j0}^{NP}]$. If the manager is playing to lose in the high state, the best response satisfies $\hat{\phi}_{i0} \in [\phi_{i0}^{NP}, \phi_{j0} - \frac{x}{r_H-B}]$.

Corollary 2 states that within the shifting region, the maximum shift is up to the point where the maximum allowed shortfall is met.

**COROLL{ARY 3.** If $\gamma >> \beta$, manager $i$’s best response in the shifting region is $\hat{\phi}_{i0} = \phi_{j0} + \frac{x}{B-r_L}$ when playing to lose in the low state, and $\hat{\phi}_{i0} = \phi_{j0} - \frac{x}{r_H-B}$ when playing to lose in the high state.

Corollary 3 refers to the case when the size of the underperformance penalty $\gamma$ is significantly larger than the proportional fee $\beta$. When facing a large enough penalty the manager shifts her strategy to the point where she is never below the underperformance benchmark, and the optimal portfolio implies that the manager hits the maximum allowed shortfall exactly in either the high or the low state, such that $\frac{W_{it}}{W_{i0}} = \frac{W_{jt}}{W_{j0}} - x$, in which case the manager does not pay the penalty and receives the proportional fee $\beta W_{it}$. In figure 4, this would look like a horizontal line in the left shifting region as $\frac{\beta}{\beta+\gamma} \to 0$. The manager is giving up a higher income in the high state, to guarantee that in the low state she doesn’t have to pay the underperformance penalty.

Given the definition of the MRG in the Colombian case, where the PFA is required to pay in full ($\gamma = 1$) any shortfall in returns below the benchmark, the penalty is significantly
greater than the average proportional fee $\beta = 0.008$ on the assets under management (see Appendix B for more details). In this case, corollary 3 suggests that the managers will chose a strategy to avoid the penalty in every state. As it turns out, as of December 2013, no PFA has ever fell below the MRG threshold. Even in the turmoil of October 2008, the PFA with the lowest returns managed to have returns 118bps above the MRG (this is the closest any PFA was to the MRG in the sample period).

Up to this point, I have characterized the best response functions. In this setting with the competition restricted to a small number of managers, one should expect that the managers anticipate each others’ reactions to their strategies. In order to describe the behavior of these strategic managers I appeal to Nash Equilibrium, in which strategies are mutual best responses.

**DEFINITION 2. Nash Equilibrium** A pure-strategy Nash equilibrium is a pair of portfolio choices $(\phi^i, \phi^j)$ that solve the fixed point equations $\phi^i_t = \hat{\phi}_i(\hat{\phi}_j^*)$ and $\phi^j_t = \hat{\phi}_j(\hat{\phi}_i^*)$.

Proposition 4 describes these equilibrium policies for each manager and the conditions that determine when each one of these equilibria is played.

**PROPOSITION 4.** The Nash equilibrium policies are given by:

\[
(\phi^*_{i0}, \phi^*_{j0}) = \begin{cases} 
(\phi_{iN}, \phi_{jN}) & \Phi_{i0} \geq \phi_{iN} \text{ and } \Phi_{j0} \leq \phi_{jN} \\
(\phi_{iN}, a_j + b_j \phi_{iN} - c_j x) & \Phi_{i0} \geq \phi_{iN} \text{ and } \Phi_{j0} > \phi_{jN} \\
(\overline{\alpha}_i + \overline{\tau}_i \phi_{jN} + \overline{\tau}_j x, \phi_{jN}) & \Phi_{i0} < \phi_{iN} \text{ and } \Phi_{j0} \leq \phi_{jN} \\
(\Phi_{i0}, \Phi_{j0}) & \Phi_{i0} < \phi_{iN} \text{ and } \Phi_{j0} > \phi_{jN} 
\end{cases} \tag{9}
\]

where $\Phi_{i0} = \frac{(\tau_i + \overline{\alpha}_i + (\tau_i - \overline{\tau}_i) x)}{1 - \overline{\beta}_i \overline{\beta}_j}$, $\Phi_{j0} = \frac{(a_j + b_j \phi_{iN} - c_j x)}{1 - \overline{\beta}_i \overline{\beta}_j}$. Moreover, when $x > (r^H - B) (\phi_{iN} - \phi_{jN})$ and $x > (B - r^L) (\phi_{iN} - \phi_{jN})$ the conditions in (9) are always satisfied and managers play their normal policy. $r^H - B > B - r^L$ is a necessary condition for (10) to be an equilibrium and $r^H - B < B - r^L$ is a necessary condition for (11) to be an equilibrium.
The equilibrium portfolios take a simple form: (i) Both managers play their normal policy if (9) holds. (ii) One manager plays her normal policy and the other shifts her portfolio if (10) or (11) hold. (iii) Both shift their portfolio towards their competitor if (12) holds.

As expected, for a large enough \( x \), both managers play their normal policies, as they are guaranteed under their normal policies of not paying the penalty in either state. Within the shifting region, when condition (12) is satisfied, both managers shift their portfolios towards each other, but they do so taking into account that the other manager will also shift, resulting in a shift that is less than in a non-strategic case. In this equilibrium it is as if both managers agree to move from their normal portfolio towards the other manager. As a result, neither individual needs to shift as much. This case is illustrated in panel (a) of Figure 5. The equilibrium portfolios in (10) and (11) are particularly interesting as they clearly illustrate the impact of strategic behavior on equilibria. In each case the equilibrium portfolios are highly shifted toward one of the two managers. In (10), the less risk averse manager (i throughout this document) plays her normal policy and the more risk averse manager (j) shifts her portfolio, buying a higher share in the risky asset than her normal policy. This case is illustrated in panel (b) of Figure 5. Suppose that manager \( i \) initially conjectures that manager \( j \) will play her normal policy. In the shifting region, \( i \) responds by playing a portfolio that is shifted towards \( j \) (\( \hat{\phi}_i^A \) in Figure 5). For \( \hat{\phi}_i^A \), manager \( j \) is in the shifting region and responds by increasing her share in the risky asset, thus moving towards \( i \). In response, manager \( i \) plays a portfolio with less shift, as in \( \hat{\phi}_i^B \) in Figure 5, for which \( j \) responds with still more shifting towards \( i \). In equilibrium, \( i \) plays her normal policy and \( j \) does all the shifting even though relative to their normal policies both managers are in the shifting region. Note that \( r^H - B > B - r^L \) is a necessary condition for this equilibrium to exist. With returns skewed to the right, the more risk averse manager \( j \) is more exposed to the underperformance penalty, as manager \( i \) can select a portfolio to avoid the penalty in the low state, at the same time that manager \( j \) has to pay the penalty in the high state.

To summarize the previous discussion, the shifting region and size of the shift depend
among other things on the tightness of the MRG constraint $x$. A prominent characteristic of the equilibria with strategic managers and an underperformance penalty based on peer performance is that managers might end up playing portfolios that are heavily shifted towards the normal policy of one of the two managers, more so than in a non-strategic environment.

### 3.3 Three Period Model ($T=2$): Trading Strategies

To study how underperformance penalties affect manager’s trading strategies, and how this relationship depends on the manager’s current level of underperformance, I solve a three period version of the model with periods $t=0,1$ and 2 (details are presented in Appendix A).

The structure of the equilibrium policies in the intermediate period $t = 1$ is the same as in the initial period of the two period case. However the conditions under which each manager shifts from the normal policy and the size of the shift varies according to the accumulated returns. More specifically, Proposition 5 states that in the shifting region, the outperforming manager is likely to play her normal policy, while the underperforming manager does all the shifting. Here the outperforming manager realizes not only that she is far away from the penalty threshold, but that her competitor is lagging in returns and is facing the risk of paying a higher penalty.\(^{14}\)

The main results of the three period model are summarized in Proposition 6. In period 0, either both managers select their normal policies, one shifts and the other plays her normal policy or both shift. After the first period, if high returns are realized in period 1, the manager with the greater share in the risky asset (manager $i$ throughout this document) will either increase the share of the risky asset in her portfolio or continue to play her normal policy if that was her equilibrium strategy in the first period. Manager $j$, who has a smaller share in the risky asset is now more vulnerable to the underperformance penalty, will then buy more shares of the risky asset, shifting more from her normal policy. Here both managers end up (weakly) increasing their shares in the risky asset, thus trading in the same direction. If instead low returns are realized in period 1, both managers (weakly) decrease
their participation in the risky asset. Using the terminology from the previous section, the model states that the overperforming manager will move toward her normal policy while the underperforming manager moves toward her peer. The game is to follow the leader, where the interim winner moves away from the peer portfolio and the interim loser tries to catch up to minimize the risk of paying the underperformance penalty in the last period.

The size of these portfolio changes, depends among other things on the strictness of the MRG (x). This correlated trading may look as if both managers are chasing returns, but in fact they are simply chasing each other. When a manager is overperforming she is less exposed to the penalty, so she can now move toward her normal policy, which happens to be more of the asset that performed well. The underperforming manager is more constrained by the penalty and realizes that the leading manager will move toward her normal policy, so she will have to shift her portfolio more in this direction.

The model shows that the introduction of relative performance concerns through a peer based underperformance penalty affects the asset allocation in important ways. First, the MRG regulation gives rise to time varying investment policies, with the managers making procyclical trades, buying more of the asset that performed well in the previous period. Second, given the strategic interactions of the managers, the model suggests that the equilibrium portfolios and individual shifts are driven by the least restricted manager, and depending on the parameters, the more restricted manager may end up doing all the shifting. Hence the combined portfolio might be highly tilted towards the preferences of the best-performing manager.

As an additional exercise, I calibrate the three period model to the estimated data prior to June 2007 and calculate the change in the trading behavior predicted by the model in response to a change in regulation equivalent to the change in the Colombian MRG formula introduced in June 2007 (details are presented in Appendix B). The model does a reasonably good job explaining the observed magnitudes of the decline in correlated trading, the observed increase in dispersion of returns between managers and the observed reduction
in the correlation between trading direction and relative performance.

4. CONCLUSIONS

In this paper I study portfolio choices of strategic fund managers in the presence of a peer-based underperformance penalty. The penalty generates herding behavior, and the extent of correlated trading is exacerbated when a strategic setting is considered.

I document empirical evidence suggesting strategic behavior of asset managers when facing a peer-based underperformance penalty. The evidence is taken from the Colombian pension industry, where six Pension Fund Administrators compete to manage the saving accounts of the working population, and are subject to a Minimum Return Guarantee based on peer performance. The evidence suggests that a tighter MRG is associated with more trading in the direction of peers and a smaller cross-section dispersion of returns between pension funds. Moreover, the ranking among managers in terms of performance seems to affect how PFAs rebalance their portfolio. When the MRG is tight, underperforming managers are more likely than their competitors to trade in the direction of their peers; this is not true when the MRG is slack. Underperforming managers rebalance their portfolio by buying more heavily stocks in which the manager is underexposed relative to her peers, as opposed to selling stocks in which she is overexposed. Since these pension funds were in an accumulation stage during the sample period, with new flows accounting for an average of 8.5% the value of the fund each year, managers were presumably able to reduce their participation in stocks to which they were overexposed simply by maintaining a fixed number of shares.

There is an interesting time dimension that is not studied in this paper: managers’ behavior close to the MRG evaluation date. Unfortunately, since the data provided was for monthly portfolio holdings, same as the MRG evaluation period, I cannot study whether PFAs altered their trading strategies by the end of the month. This is an important analysis but it requires access to portfolio data at higher frequencies.
Finally, I present a model of portfolio choice and strategic interactions among managers facing a peer-based underperformance penalty similar to the MRG. In the model, two fund managers select the trading strategy on behalf of their investors and are compensated based on their assets under management and their relative performance. The model shows that the introduction of a peer based underperformance penalty induces managers to make procyclical trades, buying more of the asset that performed well in the previous period. Such behavior is exacerbated when strategic managers are considered and the aggregate portfolio of the managers is highly tilted towards the preferences of the least restricted manager.

In the model analyzed in this paper asset prices and returns are exogenous. Given the size of the assets under management by the PFAs, their procyclical trading might have price effects; for example, procyclical trading might increase both volatility and correlation between stock prices. It would be interesting to study asset prices in a general equilibrium version of the above model where fund managers interact with other market participants. The challenge for such an extension is that since there is only a small number of managers, they might be strategic about the effect of their trading on asset prices, recognizing their market power. Previous work starting with Lindenberg (1979) suggests that these considerations might reduce the size of the trades but the direction of trading would still be the same. Finally, since I have data on individual security allocations for PFAs, any model prediction could be further tested with individual stocks. These extensions are left for future work.
APPENDIX A: THREE PERIOD MODEL SOLUTION

The timeline of events is as follows: In period \( t = 0 \), each manager chooses some \( \phi_{i0} \) and \( \phi_{j0} \). Returns for the risky asset \( r_s^1 \) are realized and managers enter period \( t = 1 \) with a new level of wealth \( R_i^1 W_0 \) (to simplify notation \( R_i^1 \equiv R_i^1 (\phi_0) \) where \( R_i^1 (\phi_0) \) is defined according (3)). In this period each manager chooses a portfolio allocation \( \phi_{i1} \) and \( \phi_{j1} \). The last period returns (\( t=2 \)) for the risky asset \( r_s^2 \) are realized, managers observe their relative wealth, and fees are calculated depending on their relative performance. Here I define a trading strategy as the change in portfolio allocation between period 0 and 1 (\( \phi_{i1} - \phi_{i0} \)). I solve this problem by backward induction.

**Period \( t=1 \) equilibrium policies:** Starting at period 1, each manager observes the realized returns \( R_i^s \) and \( R_j^s \) given portfolio allocations at time 0, \( \phi_{i0} \) and \( \phi_{j0} \), and the realization of the uncertainty \( s \) at the high or low state. At this point, the best response functions and equilibrium policies are calculated as in the two period economy in the previous section. The only difference is that the underperformance penalty for manager \( i \) now satisfies

\[
\frac{w_i^S}{w_{i0}} - \frac{w_j^S}{w_{j0}} + x = R_i^s R_j^s (\phi_{i1}) - R_j^s R_i^s (\phi_{j1}) + x \text{ using the dynamic wealth process.}
\]

The shifting region, which was given by conditions in (6) and (8) in Proposition 1 for the two period case, is now

\[
\phi_{i1}^{NP} \in \left( -\infty, \frac{R_{j1}}{R_{i1}} \phi_{j1} - \frac{x - B (R_{j1} - R_{i1})}{R_{i1} (r_H - B)} \right] \cup \left[ \frac{R_{j1}}{R_{i1}} \phi_{j1} + \frac{x - B (R_{j1} - R_{i1})}{R_{i1} (B - r_L)}, \infty \right) \tag{A1}
\]

The best response functions in period 1 for this case are similar to Proposition 1 and are omitted here. Instead in Proposition 5 I present the equilibrium policies for both managers at period \( t = 1 \) for a given pair of accumulated returns \( R_i^s \) and \( R_j^s \).

**PROPOSITION 5.** The Nash equilibrium policies for managers \( i \) and \( j \) at period \( t = 1 \) for a given pair of accumulated returns \( R_i^s \) and \( R_j^s \) by manager \( i \) and \( j \) respectively are given
by:

\[
(\phi^*_i, \phi^*_j) = \begin{cases} 
(\phi^N_i, \phi^N_j) & \Phi_{i1} \geq \phi^N_i \text{ and } \Phi_{j1} \leq \phi^N_j \\
(\phi^N_i, a_i + b_i \phi^N_j - c_i x) & \Phi_{i1} \geq \phi^N_i \text{ and } \Phi_{j1} > \phi^N_j \\
(a_i + b_i \phi^N_j + c_i x, \phi^N_j) & \Phi_{i1} < \phi^N_i \text{ and } \Phi_{j1} \leq \phi^N_j \\
(\Phi_{i1}, \Phi_{j1}) & \Phi_{i1} < \phi^N_i \text{ and } \Phi_{j1} > \phi^N_j
\end{cases} \tag{A2}
\]

where \( \Phi_{i0} = \frac{(a_i + b_i a_i) + (c_i - b_i c_i)}{1 - b_i b_j} \), \( \Phi_{j0} = \frac{(a_j + b_j a_j) - (c_j - b_j c_j)}{1 - b_i b_j} \) and the coefficients are defined as:

\[a_i = a_i - \gamma \left(1 - \frac{R^i_{t1}}{R^j_{t1}}\right)/A_i, \quad b_i = \frac{R^i_{t1}}{R^j_{t1}}b_i, \quad c_i = \frac{c_i}{R^j_{t1}}, \quad a_j = \bar{a}_j - \gamma \left(1 - \frac{R^i_{t1}}{R^j_{t1}}\right)/\bar{A}_j, \quad b_j = \frac{R^i_{t1}}{R^j_{t1}}\bar{b}_j, \]

\[c_j = \frac{c_j}{R^j_{t1}} \quad \text{and} \quad \bar{A}_j, A_i, a_i, \bar{a}_j, b_i, \bar{b}_j, c_i, \bar{c}_j \] were defined in the proof of Proposition 1 in Appendix C. Moreover,

i. If \( x > (r^H - B) (R^s_{i1} \phi^N_i - R^s_{j1} \phi^N_j) + (R^s_{i1} - R^s_{j1}) B \) and \( x > (B - r^L) (R^s_{i1} \phi^N_i - R^s_{j1} \phi^N_j) - (R^s_{i1} - R^s_{j1}) B \), the conditions in (A2) are always satisfied and managers play their normal policy at \( t = 1 \).

ii. \( R^s_{i1} (r^H - B) > R^s_{j1} (B - r^L) \) is a necessary condition for (A3) to be an equilibrium

iii. \( R^s_{i1} (r^H - B) < R^s_{j1} (B - r^L) \) is a necessary condition for (A4) to be an equilibrium

Intuitively, a manager that enters period 1 with smaller returns than her competitor \( \left(\frac{R^s_{i1}}{R^j_{j1}} < 1\right) \) is more likely to pay the underperformance penalty at \( t = 2 \). From (A1), the shifting region for \( i \) becomes larger as her accumulated relative returns with respect to her peer are smaller.

Nash equilibrium strategies at period 1 are functions of the portfolio choices by both managers at period 0 and the state of returns \( s \) at period 1. Formally \( \phi^*_i = \phi^*_i (\phi_{i0}, \phi_{j0}, s) \).

**Period \( t = 0 \) equilibrium policies:** With this set up one can write the maximization problem for manager \( i \) as a portfolio choice \( \phi_{i0} \) at period 0 that maximizes expected terminal utility for a given policy by her peer \( \phi_j \), with the equilibrium portfolios in period 1 for realized returns \( r^s_1 \) given by Proposition 5.
Formally, the wealth process can be written as $W_{iT} = R_1^s(\phi_{i0}) R_2^s(\phi_{i1}(\phi_{i0}, \phi_{j0}, s))$, the manager’s compensation is calculated according to (4) and the expectation is calculated over the four possible states $\{sS\} = \{HH, HL, LH, LL\}$.

**PROPOSITION 6.** The unique Nash equilibrium policies are functions of $x$ and have the following properties:

i. At time 0, $\phi_{i0}^* \leq \phi_{i0}^{NP}$ and $\phi_{j0}^* \geq \phi_{j0}^{NP}$

ii. If the high returns are realized in period 1 ($r_1 = r^H$) then $\phi_{i1}^{NP} \geq \phi_{i1}^* \geq \phi_{i0}^*$ and $\phi_{j1}^* \geq \phi_{j0}^*$

iii. If the low returns are realized in period 1 ($r_1 = r^L$) then $\phi_{i1}^* \leq \phi_{i0}^*$ and $\phi_{j0}^* \geq \phi_{j1}^* \geq \phi_{j}^{NP}$
APPENDIX B: NUMERICAL ANALYSIS

In this section I describe the calibration of the three period model and evaluate its quantitative implications for portfolio choice and trading strategies. I first calibrate the trading behavior of fund managers to the observed behavior in the Colombian pension industry before June 2007, and show that the change in behavior implied by the model following an exogenous change in the formula of the MRG is quantitatively similar to the changes in PFA behavior observed in the data after June 2007.

Before proceeding with the calibration it is important to keep in mind that I have constructed a simple model to highlight the potential impact of an MRG on portfolio choice with strategic managers. To obtain a parsimonious and tractable set up I have made two important assumptions. First, managers differ only in their risk aversion. Second, there are only two securities, a risk-free and a risky asset. With two securities $\Delta w$ and $d$ in section 2 are one dimensional objects, so direction as defined in equation (1) can only take three possible values $\{-1,0,1\}$, as the measure is normalized by the magnitude of both $\Delta w$ and $d$. To give a sense of the magnitude of the adjustment in portfolio weights I redefine direction in period 1 as follows:

$$direction_{i1} = \frac{(\phi^*_{i1} - \phi^*_0) (\phi^*_{j0} - \phi^*_0)}{(\phi^*_{NP} - \phi^*_0)^2}$$ (B1)

where the two terms in the numerator are the model analogues for $\Delta w$ and $d$ respectively. The denominator in (B1) normalizes the measure by the maximum distance between the two portfolios when the managers are choosing their normal strategies. Finally, measuring direction in period 1 alone might underestimate important adjustments to the portfolio allocations carried out in period 0. For this period, I introduce an alternative measure of direction based on deviations from the normal policy, as follows:
\[ \text{direction}_{i0} = \frac{(\phi_{i0}^* - \phi_{i}^{NP})(\phi_{j}^{NP} - \phi_{i}^{NP})}{(\phi_{i}^{NP} - \phi_{j}^{NP})^2} \] (B2)

Direction for manager j is obtained by switching subscripts i for j in equations (B1) and (B2). Here \( \text{direction} \in [0, 1] \) is zero when both managers choose their normal policies in each period and takes positive values when the manager moves her portfolio towards the other manager.

A period in the model represents a quarter. The timing of events is as follows: Each manager chooses their equilibrium portfolio at \( t = 0 \). After the first quarter, returns for the risky asset are realized and managers choose their optimal policy at \( t = 1 \), changing their portfolio allocation depending on their relative performance. The risky asset returns for the second quarter are realized, and the MRG is enforced if a manager’s cumulative returns over the two quarters are below the benchmark.

Using historic returns for the Colombian stock market calculated with the IGBC index (see Table 5), I estimate the mean, standard deviation and skewness of these overall returns, and then solve for \( p \), \( r^H \) and \( r^L \) to match these three moments. In a two manager setting, the MRG between January 2004 and June 2007 can be written as \( \text{MRG} = 70\% \left( \frac{R_i + R_j}{2} \right) \) where \( R \) are individual fund 3-year annual returns. The equivalent MRG measure for fund \( i \) in terms of the compensation formula (4) defined in the model is calculated as \( 70\% \left( \frac{R_i + R_j}{2} \right) = R_j - x \). In the data, the yearly average PFA return over a three year window is 11%. This would imply a yearly \( x = 1.65\% \). In the model economy, the MRG is applied to the cumulative returns of two quarters, so the actual measure of MRG for a semester is adjusted by a factor of two, so that \( x^0 = 3.3\% \).

Finally I calibrate the risk aversion parameters \( \sigma_i \) and \( \sigma_j \) to match the average \( \text{direction} \) of trading and the average cross-section dispersion of portfolio returns across managers prior to June 2007. Average \( \text{direction} \) in the model is calculated as the average of \( \text{direction}_{i0} \) and \( \text{direction}_{i1} \) for both managers and across all states. Cross-section dispersion of returns is calculated for the observed returns of both managers in both periods 1 and 2. The
model moments implied by this calibration are presented in Table 6. Compared to the data, the model generates too strong of a negative correlation between relative performance and trading towards ones peers prior to June 2007. Table 6 also presents the implied model moments generated by the change in the MRG formula to $x = 2.6\%$ starting in July 2007 (in terms of the model analogue this represents a change in the MRG formula to $x = 5.2\%$).

After the change in regulation, model managers trade less towards their peers and exhibit a less negative correlation between relative performance and the direction of trading. In addition, the cross section dispersion of returns increases. All of these moment changes are consistent with the data.

Figures 6a and 6b present the equilibrium strategies for each manager for the calibrated model. At period 0, if the MRG is strict enough, manager $j$ optimally shifts her portfolio towards $i$, while $i$ plays her normal policy. If the stock yields high returns in period 1, manager $j$ finds herself behind the other manager and then moves her portfolio closer to $i$ ($\phi^*_j1 \geq \phi^*_j0$), buying more shares in the stock, while manager $i$ plays her normal strategy again and doesn’t rebalance her portfolio. If the low returns are realized, manager $j$ is overperforming and can play her normal policy given that she is now not constrained by the MRG, potentially rebalancing her portfolio by increasing her participation in the risk-free asset ($\phi^*_j1 \leq \phi^*_j0$), moving away from the other manager. Manager $i$, underperforming in this state, finds it optimal to shift her strategy towards $j$ to avoid the underperformance penalty, and does so by increasing her participation in the risk-free asset as well ($\phi^*_i1 \leq \phi^*_i0$). Aggregate trading behavior is procyclical in this partial equilibrium model, increasing the risky asset share after good returns and decreasing it after bad returns. However, managers are not chasing returns, but are instead chasing each other, setting their portfolio to avoid being below the MRG.

Figures 6c and 6d display the average of direction across funds for both periods 0 and 1 and the correlation between direction and relative performance for different values of $x$. As expected, a tighter MRG constraint (small $x$) results in more shifting towards one’s peers.
The U-shape of the correlation measure can be explained as follows. With a tight MRG, both managers set their portfolio close to each other. Hence, the cross-section dispersion of returns is small, and subsequent portfolio adjustments are small, even for underperforming managers. As the MRG is loosened, cross-section dispersion of returns increases, but the MRG might still bind, and portfolio adjustments also increase after returns are realized to avoid the penalty. In the limit, with a loose MRG (large $x$), managers can simply play their normal policies, the cross-section dispersion of returns is larger, but no portfolio adjustment is required.
APPENDIX C: PROOFS

Proof of Proposition 1: In the absence of an underperformance penalty, by definition the normal policy yields a higher expected utility than any other strategy. With the underperformance penalty in place, for a given portfolio choice by the other manager $\phi_j$, if the normal policy can be implemented without triggering the penalty in any state, the manager optimally chooses her normal policy as if there were no relative performance concerns. Note that if this is the case, the compensation for the manager in each state is $\beta W_i T$, which is the same as without the penalty. The maximum allowed shortfall for each state, $\frac{W^H}{W_{i0}} \geq \frac{W^H}{W_{j0}} - x$ and $\frac{W^L}{W_{i0}} \geq \frac{W^L}{W_{j0}} - x$, can be written in terms of the share in the risky asset $\phi$ using (3) as $\phi_j + \frac{x}{B - r_L} \geq \phi_i \geq \phi_j - \frac{x}{B - r_H}$. Hence if $\phi_i^{NP}$ is in this region the manager’s optimal policy is the normal policy.

If the manager cannot implement her normal policy without avoiding the underperformance penalty, the compensation (4) needs to be calculated accordingly with a penalty in that state. Also, noting that a manager cannot be a loser in both states, to solve the optimization problem I split the problem into two regions, one where the manager is a loser in the low state and another where the manager is a loser in the high state.

For the manager playing to lose in the low state the problem is transformed into a constrained maximization as follows (loser in the low state):

$$\max_{\phi_i} EU \left[ F^S_i (\phi_i, \phi_j) \right] + \mu \left[ \phi_i - \phi_j - \frac{x}{B - r_L} \right] \quad (C1)$$

The Kuhn-Tucker conditions are

$$(1 - p)(\beta + \gamma)B - r^L U' \left[ F^L_i (\phi_i, \phi_j) \right] + \beta (r^H - B) p U' \left[ F^H_i (\phi_i, \phi_j) \right] + \mu = 0 \quad (C2)$$
\[ \mu \left( \phi_i - \phi_j - \frac{x}{B - r_L} \right) = 0, \mu \geq 0 \quad (C3) \]

which imply that the best response in this region is:

\[ \hat{\phi}_{i0} = \begin{cases} 
\varphi_{i0} & \varphi_{i0} > \phi_{j0} + \frac{x}{B - r_L} \quad \text{(interior solution)} \\
\phi_{j0} + \frac{x}{B - r_L} & \text{otherwise (corner solution)}
\end{cases} \quad (C4) \]

where \( \varphi_{i0} \) solves (C2) with \( \mu = 0 \). Solving for \( \varphi_{i0} \), the best response in this region can be written as a linear function of \( \phi_{j0} \) and \( x \) as \( \hat{\phi}_{i0} = a_i + b_i \phi_{j0} + c_i x \), where the coefficients take the following values:

**Interior solution**

\[ a_i = (\beta + \delta_i \gamma) A_i B; \quad b_i = \delta_i \gamma (r_H - B) A_i; \quad c_i = \delta_i \gamma A_i \]

where

\[ A_i = \left[ \delta_i (\beta + \gamma)(r_H - B) + \beta (B - r_L) \right]^{-1} \]

and

\[ \delta_i = \left[ \frac{(\beta + \gamma)p(r_H - B)}{\beta (1-p) (B - r_L)} \right]^{-1/\sigma_i} \]

**Corner solution**

\[ a_i = 0; \quad b_i = 1; \quad c_i = \frac{1}{r_H - B} \]

To solve the optimal portfolio in the region where manager \( i \) pays the underperformance penalty in the high state (loser in the high state), I proceed in similar fashion. The best response function of manager \( i \) in this region is again linear in \( x \) and \( \phi_{j0} \) as \( \hat{\phi}_{i0} = \overline{a}_i + \overline{b}_i \phi_{j0} - \overline{c}_i x \), with the coefficients defined as follows:

**Interior solution**

\[ \overline{a}_i = (1 - \overline{\sigma}_i) \beta A_i B; \quad \overline{b}_i = \gamma (B - r_L) A_i; \quad \overline{c}_i = \gamma A_i \]
where

$$\bar{A}_i = \left[ \epsilon_i \beta (r^H - B) + (\beta + \gamma) (B - r^L) \right]^{-1}$$

and

$$\bar{\epsilon}_i = \left[ \frac{\beta p (r^H - B)}{(\beta + \gamma) (1 - p) (B - r^L)} \right]^{-1/\sigma_i}$$

**Corner solution**

$$\bar{a}_i = 0; \bar{b}_i = 1; \bar{c}_i = \frac{1}{B - r^L}$$

**Proof of Corollary 2:** The assertion follows directly from the corner solutions in Proposition 1 that effectively determine the boundaries of the shift.

**Q.E.D**

**Proof of Corollary 3:** The optimality condition for an interior solution for a manager playing to lose in the low state reads

$$\frac{(1 - p) (B - r^L)}{p (r^H - B)} \frac{u'(F^L)}{u'(F^H)} = \frac{\beta}{\beta + \gamma} \to 0 \quad (C5)$$

In order for the marginal rate of substitution between the compensation in the low and high state to go to zero, the manager must end up with all the wealth in the low state, which implies $\varphi \to -\infty$. Hence the boundary condition $\varphi_{i0} \leq \phi_{j0} + \frac{x}{B - r^L}$ in (C4) is always satisfied and the manager’s best response is $\hat{\phi}_{i0} = \phi_{j0} + \frac{x}{B - r^L}$.

The optimality condition for an interior solution for a manager playing to lose in the high state reads

$$\frac{(1 - p) (B - r^L)}{p (r^H - B)} \frac{u'(F^L)}{u'(F^H)} = \frac{\beta + \gamma}{\beta} \to \infty \quad (C6)$$

42
In order for the marginal rate of substitution between the compensation in the low and high state to go to infinity, the manager must end up with all the wealth in the high state, which implies \( \varphi \to \infty \). Hence the boundary condition \( \varphi_{i0} \geq \phi_{j0} - \frac{x}{r - B} \) is always satisfied and the manager’s best response is \( \hat{\phi}_{i0} = \phi_{j0} - \frac{x}{r - B} \).

\[Q.E.D\]
References


Notes

1 See for instance Turner and Rajnes (2001) for a review on these systems. Castaneda and Rudolph (2010) present a theoretical analysis of portfolio choice under peer-based and index-based MRG.

2 In June 2008 some of the limits were: (i) Maximum 50% in domestic government debt. (ii) Maximum 40% in equity securities. (iii) Maximum 40% in foreign securities.

3 Other smaller fees apply in special cases (e.g. there is a fee when the worker changes PFA, as well as a proportional fee on the value of the account when the worker has not made a contribution for six consecutive months.)

4 A similar provision is in place in other countries, including Chile, Peru, the Dominican Republic and Uruguay.

5 As an example of how this new formula loosened the MRG constraint (increased the maximum allowed shortfall) consider the date December 31, 2009. Between December 31, 2006 and December 31, 2009 the industry annual average returns were 6.01%. With the new formula in place, the MRG was 3.41%, instead of the 4.20% that would have occurred under the older formula.

6 In the 15 year history of the private pension system (between 1996 and 2010), no PFA ever yielded returns below the MRG. Even in the turmoil of October 2008, the PFA with the lowest returns managed to have returns 118bps above the MRG (this is the closest any PFA was to the MRG in the sample period)

7 The only major restriction that these funds share is the short selling ban.

8 A similar measure of direction was first introduced by Koch (2012). Here I define the angle between the active change in weights and the initial distance to the peer benchmark, as opposed to the angle between the active change in weights and the peer benchmark active change in weights as in Koch (2012).

9 Selling past losers can also be explained by window dressing. For US pension funds see Lakonishok et al. (1991)


11 To reduce the effect of outliers $\Delta w_{it}$ is winsorized at 1% each tail.

12 Alternatively to the adjusted weight change, a PFA might opt for a “pasive” rebalancing of its portfolio by accounting for changes in security prices. For this reason, I estimate equation (2) using the unadjusted change in weights, i.e. $\Delta w_{it} = w_{it+1} - w_{it}$.

13 According to ASOFONDOS, four of the six PFAs had only one CEO replacement each during the sample period, occurring on the following dates: October 2006, February 2008, October 2008 and May 2010. The other two PFAs changed their CEO four times each between January 2004 and December 2010.
One should note that if both managers have the same returns to start the period ($R_{i1} = R_{j1}$) the equilibrium strategies are exactly the same as in the two period model.

These not need to be the ones calculated for the two period economy. I call them $\phi_0$ just to be consistent with the notation.
Table 1

Summary statistics for Colombian pension funds and voluntary fund holdings

Key statistics are provided below (at two-year intervals) for the Colombian pension funds and voluntary funds. For each column, statistics are shown for the portfolios reported by June 30 of each year, except as noted. The database, made available by the Association of Pension Fund Administrators (ASOFONDOS), includes monthly portfolio holdings of each security in every pension fund and voluntary fund from January 31, 2004 to December 31, 2010. Panel A documents the total number of funds, the total assets under management and the share invested in stocks traded publicly in the domestic capital market. Panel B shows the average number of stocks held per fund at each date, the number of different stocks held by all six pension funds as a group and the number of stocks in the IGBC index, which is a major stock index for the Colombian stock market. Panel B also provides trading data, inferred from the difference in portfolio holding between May 31 and June 30 of each year. Panel C shows key statistics on relative performance between funds and portfolio differences between each fund and the peer portfolio and between each fund and the market portfolio. Relative performance is measured as the difference between the peer returns and individual fund returns, for annual returns measured over a three year rolling window. Distance measures a fund’s exposure to each stock relative to the benchmark: \( d_{st} = \pi_{st} - w_{st} \). Panels D and E present key statistics for voluntary funds.

<table>
<thead>
<tr>
<th>Year</th>
<th>2004</th>
<th>2006</th>
<th>2008</th>
<th>2010</th>
</tr>
</thead>
</table>

Panel A. Pension fund count, assets and asset allocation

<table>
<thead>
<tr>
<th></th>
<th>2004</th>
<th>2006</th>
<th>2008</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of funds</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Total assets ($billions)</td>
<td>8.2</td>
<td>13.8</td>
<td>27.8</td>
<td>44.1</td>
</tr>
<tr>
<td>Net flows (contributions minus withdraws $billions)</td>
<td>0.8</td>
<td>1.5</td>
<td>2.4</td>
<td>1.7</td>
</tr>
<tr>
<td>Percent invested in domestic stocks</td>
<td>5.0</td>
<td>12.6</td>
<td>22.4</td>
<td>32.1</td>
</tr>
<tr>
<td>Largest fund share (percentage over the pension industry)</td>
<td>27.1</td>
<td>26.6</td>
<td>27.2</td>
<td>27.2</td>
</tr>
<tr>
<td>Smallest fund share (percentage over the pension industry)</td>
<td>2.9</td>
<td>3.8</td>
<td>4.5</td>
<td>4.8</td>
</tr>
</tbody>
</table>

Panel B. Pension funds domestic stock count and trading statistics

<table>
<thead>
<tr>
<th></th>
<th>2004</th>
<th>2006</th>
<th>2008</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of stocks held per fund</td>
<td>16.2</td>
<td>21.2</td>
<td>26.3</td>
<td>30.0</td>
</tr>
<tr>
<td>Number of distinct stocks held by all pension funds</td>
<td>41</td>
<td>50</td>
<td>44</td>
<td>47</td>
</tr>
<tr>
<td>Number of stocks in the market index</td>
<td>26</td>
<td>33</td>
<td>27</td>
<td>32</td>
</tr>
<tr>
<td>Average stocks traded per fund</td>
<td>7.2</td>
<td>5.2</td>
<td>8.3</td>
<td>7.0</td>
</tr>
<tr>
<td>Proportion of trades that are buy (percent)</td>
<td>65.1</td>
<td>61.3</td>
<td>80.0</td>
<td>54.8</td>
</tr>
<tr>
<td>Total buys ($millions)</td>
<td>14.1</td>
<td>20.3</td>
<td>82.7</td>
<td>50.7</td>
</tr>
<tr>
<td>Total sells ($millions)</td>
<td>4.5</td>
<td>16.3</td>
<td>23.9</td>
<td>80.0</td>
</tr>
<tr>
<td>Average yearly sells (percentage of sell volume over total trades)</td>
<td>27.4</td>
<td>25.4</td>
<td>29.2</td>
<td>65.4</td>
</tr>
</tbody>
</table>

Panel C. Pension funds performance and portfolio differences (standard deviation in parenthesis)

<table>
<thead>
<tr>
<th></th>
<th>2004</th>
<th>2006</th>
<th>2008</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average relative returns (percent)</td>
<td>-0.10</td>
<td>0.13</td>
<td>0.27</td>
<td>0.29</td>
</tr>
<tr>
<td>(0.49) (0.79) (1.30) (1.30)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average peer distance (percent)</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(3.83) (2.01) (1.23) (2.09)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average market distance (percent)</td>
<td>0.06</td>
<td>0.00</td>
<td>-0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>(5.39) (3.10) (2.94) (3.08)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel D. Voluntary funds count, assets and asset allocation

<table>
<thead>
<tr>
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<th>2004</th>
<th>2006</th>
<th>2008</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total assets ($billions)</td>
<td>1.2</td>
<td>2.1</td>
<td>4.8</td>
<td>4.3</td>
</tr>
<tr>
<td>Percent invested in domestic stocks</td>
<td>1.8</td>
<td>4.5</td>
<td>9.2</td>
<td>14.5</td>
</tr>
<tr>
<td>Largest fund share (percentage over the pension industry)</td>
<td>38.1</td>
<td>28.0</td>
<td>47.0</td>
<td>36.1</td>
</tr>
<tr>
<td>Smallest fund share (percentage over the pension industry)</td>
<td>1.9</td>
<td>3.5</td>
<td>2.1</td>
<td>4.7</td>
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</table>

Panel E. Voluntary funds domestic stock count and trading statistics

<table>
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<tr>
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<th>2004</th>
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<th>2008</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of stocks held per fund</td>
<td>5.2</td>
<td>13.8</td>
<td>17.8</td>
<td>20.3</td>
</tr>
<tr>
<td>Number of distinct stocks held by all pension funds</td>
<td>23</td>
<td>30</td>
<td>36</td>
<td>34</td>
</tr>
<tr>
<td>Average stocks traded per fund</td>
<td>0.8</td>
<td>6.5</td>
<td>10.2</td>
<td>12.8</td>
</tr>
<tr>
<td>Proportion of trades that are buy (percent)</td>
<td>79.5</td>
<td>38.5</td>
<td>39.4</td>
<td>59.2</td>
</tr>
<tr>
<td>Total buys ($millions)</td>
<td>1.9</td>
<td>10.9</td>
<td>12.8</td>
<td>34.3</td>
</tr>
<tr>
<td>Total sells ($millions)</td>
<td>0.5</td>
<td>7.75</td>
<td>32.9</td>
<td>77.8</td>
</tr>
<tr>
<td>Average yearly sells (percentage of sell volume over total trades)</td>
<td>19.8</td>
<td>41.5</td>
<td>71.9</td>
<td>69.4</td>
</tr>
</tbody>
</table>
Table 2
Direction of portfolio weight changes

The direction measure, $direction_i^t$, for a given fund $i$ at some month $t$ equals $\frac{\Delta w_i^t d_i^t}{||\Delta w_i^t|| ||d_i^t||}$, where $\Delta w_i^t$ is the active change in portfolio weights between $t$ and $t + 1$ adjusted by stock individual returns. $d_i^t$ is the distance in month $t$ between fund $i$’s portfolio and the peer portfolio. Statistics are calculated for measures of direction across funds ($direction_i^t$). Direction captures whether each fund is moving towards or away from the peer benchmark.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
<th>std dev</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Statistics for direction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pension Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>0.32</td>
<td>0.32</td>
<td>-0.21</td>
<td>0.71</td>
<td>0.19</td>
</tr>
<tr>
<td>After June 2007</td>
<td>0.14</td>
<td>0.16</td>
<td>-0.68</td>
<td>0.50</td>
<td>0.18</td>
</tr>
<tr>
<td>Voluntary Funds</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>0.13</td>
<td>0.10</td>
<td>-0.73</td>
<td>0.99</td>
<td>0.38</td>
</tr>
<tr>
<td>After June 2007</td>
<td>0.13</td>
<td>0.16</td>
<td>-0.93</td>
<td>0.96</td>
<td>0.42</td>
</tr>
<tr>
<td><strong>Panel B. Statistics for pension funds relative performance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Relative Returns (in bps)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>0.07</td>
<td>0.13</td>
<td>-3.63</td>
<td>3.71</td>
<td>1.29</td>
</tr>
<tr>
<td>After June 2007</td>
<td>-0.07</td>
<td>-0.35</td>
<td>-4.98</td>
<td>5.45</td>
<td>1.86</td>
</tr>
<tr>
<td><strong>Panel C. Correlation between direction and relative performance for pension funds</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Before June 2007</td>
<td>$-0.31^{***}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>After June 2007</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3

Linear regression for adjusted weight changes

The dependent variable is the change in weight $\Delta w_{st+1}^i$ for stock $s$ between period $t$ and $t+1$ for fund $i$ adjusted by the stock returns as follows: $\Delta w_{st+1}^i = w_{st+1}^i - \frac{w_{st+1}^i \times \text{ret}_{st}}{\sum_s w_{st}^i \times \text{ret}_{st}}$, where $\text{ret}_{st}$ are the gross returns for stock $s$ between $t$ and $t+1$. The unit of observation is a month. “MRG” is a dummy variable, equal to one for dates prior June 2007 and zero thereafter. “Peer (Market) Distance” is the difference between the weight of stock $s$ in the peer (market) portfolio and the weight of $s$ in fund $i$’s portfolio. The market portfolio is the IGBC, a major index in the Colombian stock market. “Relative Performance” is the difference in returns between manager $i$ and the overall pension industry, measured over the previous 36 months for each date. “Size” is the share of assets under management of pension fund $i$ as a percentage of the entire pension industry. Standard errors in parenthesis are adjusted for within-stock clustering in (1) and (2) and adjusted for within-funds clustering in (3) and (4). Note: ***/**/* indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Within stocks clustering</th>
<th>Within funds clustering</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>MRG x Peer Distance</td>
<td>0.0515**</td>
<td>0.0403*</td>
</tr>
<tr>
<td></td>
<td>(0.0255)</td>
<td>(0.0220)</td>
</tr>
<tr>
<td>MRG x Relative Performance</td>
<td>0.0024</td>
<td>0.0024</td>
</tr>
<tr>
<td></td>
<td>(0.0027)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>MRG x Peer Distance x Relative Performance</td>
<td>-1.5505*</td>
<td>-1.9248*</td>
</tr>
<tr>
<td></td>
<td>(0.9447)</td>
<td>(1.0123)</td>
</tr>
<tr>
<td>MRG x Size</td>
<td>0.0001</td>
<td>-0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.0006)</td>
<td>(0.0006)</td>
</tr>
<tr>
<td>MRG x Size x Peer Distance</td>
<td>-0.2201*</td>
<td>-0.2464*</td>
</tr>
<tr>
<td></td>
<td>(0.1218)</td>
<td>(0.1336)</td>
</tr>
<tr>
<td>MRG</td>
<td>0.001</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
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<tr>
<td>Peer Distance</td>
<td>0.1068***</td>
<td>0.1040***</td>
</tr>
<tr>
<td></td>
<td>(0.0166)</td>
<td>(0.0177)</td>
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<tr>
<td>Relative Performance</td>
<td>-0.0035*</td>
<td>-0.0034*</td>
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<tr>
<td></td>
<td>(0.0019)</td>
<td>(0.0019)</td>
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<td>Peer Distance x Relative Performance</td>
<td>0.8529</td>
<td>0.7604</td>
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<tr>
<td></td>
<td>(0.8067)</td>
<td>(0.8053)</td>
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<tr>
<td>Size</td>
<td>-0.0017***</td>
<td>-0.0017***</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>Size x Peer Distance</td>
<td>-0.3464***</td>
<td>-0.3475***</td>
</tr>
<tr>
<td></td>
<td>(0.0875)</td>
<td>(0.0872)</td>
</tr>
<tr>
<td>MRG x Market Distance</td>
<td>0.0198</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0153)</td>
<td>(0.0121)</td>
</tr>
<tr>
<td>MRG x Market Distance x Relative Performance</td>
<td>0.0519</td>
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</tr>
<tr>
<td></td>
<td>(0.4751)</td>
<td>(0.6943)</td>
</tr>
<tr>
<td>Market Distance</td>
<td>0.0032</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0056)</td>
<td>(0.0023)</td>
</tr>
<tr>
<td>Market Distance x Relative Performance</td>
<td>0.0742</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.2499)</td>
<td>(0.2698)</td>
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<tr>
<td>Controls</td>
<td>YES</td>
<td>YES</td>
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<tr>
<td>Pension fund fixed effects</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Number of observations</td>
<td>18960</td>
<td>18960</td>
</tr>
</tbody>
</table>
Table 4
Probit regression of buying or selling a stock

The dependent variable is the dummy variable \( \text{buy}_{it+1} \) or \( \text{sell}_{it+1} \), which indicates whether a given fund \( i \) in period \( t + 1 \) increases or decreases the number of shares in stock \( s \). The unit of observation is a month. “MRG” is a dummy variable equal to one for dates prior June 2007 and zero thereafter. “Peer (Market) Distance” is the difference between the weight of stock \( s \) in the peer (market) portfolio and the weight of \( s \) in fund \( i \)’s portfolio. The market portfolio is the IGBC, a major index in the Colombian stock market. “Relative Performance” is the difference in returns between manager \( i \) and the overall pension industry, measured over the previous 36 months for each date. “Size” is the share of assets under management of pension fund \( i \) as a percentage of the entire pension industry. Standard errors in parenthesis are adjusted for within-stock clustering. Note: ***/**/* indicate that the coefficient estimates are significantly different from zero at the 1%/5%/10% level.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Probability of buying conditional on ownership</th>
<th>Probability of selling conditional on ownership</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>MRG x Peer Distance</td>
<td>86.51***</td>
<td>79.64***</td>
</tr>
<tr>
<td></td>
<td>(26.14)</td>
<td>(25.64)</td>
</tr>
<tr>
<td>MRG x Relative Performance</td>
<td>6.97</td>
<td>7.25</td>
</tr>
<tr>
<td></td>
<td>(4.61)</td>
<td>(4.94)</td>
</tr>
<tr>
<td>MRG x Peer Distance x Relative Performance</td>
<td>-2105.06*</td>
<td>-3980.69**</td>
</tr>
<tr>
<td></td>
<td>(1259.69)</td>
<td>(1629.30)</td>
</tr>
<tr>
<td>MRG x Size</td>
<td>2.27***</td>
<td>2.28***</td>
</tr>
<tr>
<td></td>
<td>(0.74)</td>
<td>(0.86)</td>
</tr>
<tr>
<td>MRG x Size x Peer Distance</td>
<td>-427.09***</td>
<td>-445.82**</td>
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<tr>
<td></td>
<td>(191.90)</td>
<td>(195.08)</td>
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<tr>
<td>MRG</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Peer Distance</td>
<td>-54.59*</td>
<td>-45.18</td>
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<tr>
<td></td>
<td>(28.37)</td>
<td>(31.86)</td>
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<tr>
<td>Relative Performance</td>
<td>-8.68***</td>
<td>-8.16**</td>
</tr>
<tr>
<td></td>
<td>(3.21)</td>
<td>(3.49)</td>
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<tr>
<td>Peer Distance x Relative Performance</td>
<td>1491.73</td>
<td>809.40</td>
</tr>
<tr>
<td></td>
<td>(1043.67)</td>
<td>(1130.83)</td>
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<tr>
<td>Size</td>
<td>-4.41***</td>
<td>-4.48***</td>
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<tr>
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<td>(0.51)</td>
<td>(0.53)</td>
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<tr>
<td>Size x Peer Distance</td>
<td>216.03</td>
<td>210.56</td>
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<td>(145.99)</td>
<td>(144.45)</td>
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<tr>
<td>MRG x Market Distance</td>
<td>-0.39</td>
<td>-15.24</td>
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<tr>
<td></td>
<td>(36.22)</td>
<td>(22.34)</td>
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<tr>
<td>MRG x Market Distance x Relative Performance</td>
<td>1587.83</td>
<td>1565.04</td>
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<td></td>
<td>(1870.28)</td>
<td>(1709.24)</td>
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<tr>
<td>Market Distance</td>
<td>-8.45</td>
<td>3.26</td>
</tr>
<tr>
<td></td>
<td>(12.80)</td>
<td>(3.47)</td>
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<tr>
<td>Market Distance x Relative Performance</td>
<td>663.93</td>
<td>-131.84</td>
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<tr>
<td></td>
<td>(415.18)</td>
<td>(262.43)</td>
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<tr>
<td>Constant</td>
<td>35.15*</td>
<td>35.45*</td>
</tr>
</tbody>
</table>

Pension fund fixed effects | YES | YES | YES | YES
Number of observations | 18960 | 18960 | 11299 | 11299
Table 5
Calibration
The return process of the risky asset is calibrated according to the Colombian stock market historical returns (Data available after 1987). The risk aversion parameters $\sigma_i$ and $\sigma_j$ are calibrated to match the average direction of trading and the average cross-section dispersion of returns prior to June 2007.

<table>
<thead>
<tr>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk Premium</td>
<td>12.5%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>19%</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.33</td>
</tr>
</tbody>
</table>

Panel B. Model parameters

<table>
<thead>
<tr>
<th>Probability of high state</th>
<th>p=0.55</th>
<th>Mean, variance, skewness historical returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>High returns $r^H$</td>
<td>25%</td>
<td>Mean, variance, skewness historical returns</td>
</tr>
<tr>
<td>Low returns $r^L$</td>
<td>-10%</td>
<td>Mean, variance, skewness historical returns</td>
</tr>
<tr>
<td>Minimum Return Guarantee $x^0$</td>
<td>3.3%</td>
<td>Before June 2007</td>
</tr>
<tr>
<td>$x^1$</td>
<td>5.2%</td>
<td>After June 2007</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1</td>
<td>Defined by Colombian Government</td>
</tr>
<tr>
<td>PFA management fee $\beta$</td>
<td>0.8%</td>
<td>Management fees</td>
</tr>
</tbody>
</table>

Panel C. Calibration

| Risk Aversion | $\sigma_i = 2.75$, $\sigma_j = 2.21$ | Direction and cross-section variation of returns before June 2007 |

Table 6
Effects from the change in the Minimum Return Guarantee formula
Empirical and model-implied moments before and after the change in the MRG formula in June 2007. The parameters are set according to the calibration in Table 5.

<table>
<thead>
<tr>
<th></th>
<th>Before June 07</th>
<th>After June 07</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
</tr>
<tr>
<td>Mean Direction</td>
<td>0.32</td>
<td>0.32</td>
</tr>
<tr>
<td>Std. dev. of relative returns</td>
<td>1.19%</td>
<td>1.08%</td>
</tr>
<tr>
<td>$Corr(direction, rel)$</td>
<td>-0.31</td>
<td>-0.90</td>
</tr>
</tbody>
</table>
Figure 1. Economy with three assets. This figure presents two examples of changes in the portfolio composition in the space of weights for stocks A and B. The portfolio of fund i moves from \( w_i^t \) to \( w_i^t + \Delta w_i^t \). The distance vector is \( d_i^t = \Pi_i^t - w_i^t \), which represents the initial difference between fund i and the peer portfolio at the beginning of the period \( t \). \( \theta \) is the angle formed between the change in the portfolio of manager i and the distance vector. Direction is defined as \( \cos \theta \). When manager i moves towards the peer portfolio, \( \theta \) is smaller and direction is closer to 1, as in panel (a). In panel (b) the manager moves away from the peer benchmark and direction takes smaller values as the angle increases.

(a)  
(b)

Figure 2. Average direction of portfolio change. The direction measure, \( \text{direction}_i^t \), for a given fund i at some month \( t \) equals \( \frac{\Delta w_i^t d_i^t}{||\Delta w_i^t|| ||d_i^t||} \), where \( \Delta w_i^t \) is the active change in portfolio weights between \( t \) and \( t + 1 \) adjusted by stock individual returns. \( d_i^t \) is the distance between fund i’s portfolio and the peer portfolio as of month \( t \). The figure reports the monthly value of direction averaged across the six PFAs, for pension funds (solid line) and voluntary funds (dotted line).
Figure 3. Marginal effects. Difference in marginal effects of distance on adjustments in portfolios weights before and after the policy change $\frac{\partial \Delta w(MRG=1)}{\partial d} - \frac{\partial \Delta w(MRG=0)}{\partial d}$ with 90% confidence intervals.

Figure 4. Utility maximization for manager $i$ for a given portfolio choice by $j$ $\phi_j$. When the final wealth of manager $i$ $\frac{W_i}{W_0}$ is below the peer benchmark $\frac{W_j}{W_0}$ $- x$ for either the high or low state she pays the underperformance penalty, thus reducing the net compensation. In panel (a) the manager’s best response is to play her normal policy $\hat{\phi}_i = \phi_i^{NP}$. Here, she doesn’t pay the underperformance penalty in any state. In panel (b), the manager’s best response is a portfolio with a lower share in the risky asset than in her normal policy $\phi_i < \phi_i^{NP}$, to reduce the underperformance penalty that is paid if the low state of returns is realized.
Figure 5. Nash Equilibrium Portfolios \((\phi^*_i, \phi^*_j)\). Best responses by managers i (solid line) and j (dash-dotted line). In panel (a) both managers optimally shift their portfolio towards their peer. In panel (b) manager j does all the shift, while manager i plays her normal policy.
Figure 6. Nash Equilibria and trading behavior in a calibrated three period economy for different values of \( x \). Panels (a) and (b) present the Nash equilibrium policies in period 0 and period 1 for managers \( i \) and \( j \) respectively. Direction is plotted in panel (c) using the formula \( \frac{(\phi_{i0}^* - \phi_{NP}^*) (\phi_{j1}^* - \phi_{NP}^*)}{(\phi_{NP}^* - \phi_{NP}^*)^2} \) for period 0 and \( \frac{(\phi_{i1}^* - \phi_{i0}^*) (\phi_{j0}^* - \phi_{j1}^*)}{(\phi_{NP}^* - \phi_{NP}^*)^2} \) for period 1. Direction is calculated for each manager and averaged across \( i \) and \( j \) for periods 0 and 1. The correlation between relative performance and direction presented in panel (d) is calculated for period 1 for each pair of direction and relative returns for both managers. Parameters are set according to the calibration in Table 5.