

Is There Enough Redistribution?

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Abstract

This paper asks whether there are welfare gains from additional redistribution. First, it derives a sufficient condition for the existence of welfare gains from a small increase in lump-sum transfers financed by a uniform increase in labor income taxes. A calibration suggests that, even under very conservative assumptions, most

countries would benefit from such a scheme. Second, it asks whether, given existing tax revenues, there are gains from diverting public funds from government investment projects toward redistributive programs. The analysis suggests that the answer is highly sensitive to parameter values and the rate of return on government investment.

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1 Introduction

The tradeoffs associated with redistributive policy are well-understood. On the one hand, if the marginal utility of consumption is higher at low income levels, then instituting transfers from rich to poor improves social welfare (from a utilitarian perspective). On the other hand, providing appropriate incentives for people to work puts limits on the efficient amounts of redistribution.

The characterization of the optimal redistributive policy has been shown to depend crucially on many aspects of the economic environment which may be difficult to measure, such as higher moments of the distribution of idiosyncratic productivity shocks (see, for example, Golosov et al. [2016]). Rather than characterizing the optimal policy, this paper studies the welfare implications of a simple perturbation to existing tax schemes. A calibration suggests that, for most countries, a small increase in transfers financed by a uniform increase in labor income taxes would improve social welfare.

Methodologically, the approach is similar to the pioneering work of Golosov et al. [2014], who develop a perturbation method which can be used to study a wide class of tax reforms. Here, I apply their method to the question of redistribution in a simple small open economy setting. The gains from redistribution can be calibrated based on the joint distribution of labor income and consumption, the elasticity of labor supply, the administrative cost of transfers, the labor income share, the share of taxes in GDP and the coefficient of relative risk aversion. While there is some controversy regarding the appropriate calibration of some of these parameters, I show that, in most countries, even conservative assumptions deliver positive welfare gains from additional redistribution.

The conclusion regarding the optimality of increasing tax revenues to finance additional government transfers is fairly robust. A related question is whether, given existing tax revenues, social welfare can be improved by shifting government spending from government investment programs to govern-

ment transfer programs. To address this question, the model is extended to include government capital that augments labor productivity. Here, the conclusions are more mixed: under a conservative parameterization, the welfare gains from additional government investment exceed the gains from additional redistribution. However, there are some realistic parameterizations for which the welfare gains from additional cash transfers are roughly equivalent to those of a government investment project with an annual rate of return of 20 percentage points. These estimates are broadly in the range of estimated rates of return to government investment, though these are notoriously imprecise (see Gramlich [1994] for a review). From a policy perspective, the findings suggest that while there is a strong case for increasing government spending through additional taxation, the optimal allocation of spending between transfers and government investment projects is sensitive to parameter values and the rate of return to government capital.

2 Model

There is a continuum of price-taking households indexed $i \in [0, 1]$. The household supplies labor, $l_{i,t}$, and accumulates savings, $a_{i,t}$.

In most models, inequality is generated by idiosyncratic shocks. It is therefore useful to assume that households face some uncertainty, captured by stochastic variables denoted $\{s_{i,t}\}_{i,t}$. The stochastic variables affect the disutility from labor. In this formulation, $s_{i,t}$ can be thought of as an individual productivity shock: a negative shock makes it more costly to supply the same amount of effective units of labor. It will be useful to denote the history of shocks by $s_i^t = (s_{i,0}, \dots, s_{i,t})$.

Household i faces a proportional tax of $\tau_t(l_{i,t}) \geq 0$ on its labor income and receives a government transfer of $b_t(l_{i,t}) \geq 0$. Note that taxes and transfers may depend only on the household's current labor income and not on

past labor income. This assumption is made mainly for simplification.¹ In addition, it will be useful to assume that at any time t , the set of possible state variables $\{a_{i,t}, s_i^t\}_{i \in [0,1]}$ is finite.

While there is idiosyncratic uncertainty that generates inequality, I assume that there is no aggregate uncertainty. Thus, wages and aggregate quantities do not depend on the shocks $\{s_{i,t}\}_{i,t}$.

In principle, household decisions and state variables depend on the history of shocks, s_i^t . However, to simplify notation, I will omit the dependence on s_i^t and write $l_{i,t} = l_{i,t}(s_i^t)$, $a_{i,t} = a_{i,t}(s_i^{t-1})$, etc.

Utility from consumption and leisure is separable. Households maximize expected utility subject to a budget constraint:

$$V_i(a_{i,0}, \{w_t, r_t, b_t(\cdot), \tau_t(\cdot)\}_{t=0}^\infty) = \max_{a_{i,t+1}, l_{i,t}, c_{i,t}} E\left[\sum_{t=0}^{\infty} \beta^t (u(c_{i,t}) + g(l_{i,t}, s_{i,t}))\right]$$

s.t. $a_{i,0}$ and:

$$c_{i,t} + a_{i,t+1} = (1 - \tau(l_{i,t}))w_t l_{i,t} + (1 + r_t)a_{i,t} + b_t(l_{i,t})$$

where $\beta \in (0, 1)$ is the time discount factor. It is useful to denote the set of optimal labor supply decisions by $\mathcal{L}_i(a_{i,0}, \{w_t, r_t, b_t(\cdot), \tau_t(\cdot)\}_{t=0}^\infty)$.

Production. There are two production inputs: capital (K_t) and labor ($L_t = \int_0^1 l_{i,t} di$). The production function, $F_t(K_t, L_t)$, exhibits constant returns to scale and satisfies the standard Inada conditions. Factors are paid their marginal products. The wage rate, w_t , and the capital rental rate, R_t , are given by:

$$w_t = \frac{\partial F_t(K_t, L_t)}{\partial L} \text{ and } R_t = \frac{\partial F_t(K_t, L_t)}{\partial K} \quad (1)$$

¹See Golosov et al. [2007] for a more general treatment.

Capital markets. I assume a small open economy in which the the global interest rate, r_t , pins down aggregate capital supply:

$$(1 - \tau_t^k)R_t + 1 - \delta = 1 + r_t \quad (2)$$

where τ_t^k is the tax on capital income and $\delta \in [0, 1]$ is the capital depreciation rate.

Note that, given the constant returns to scale assumption, the factor payments R_t and w_t depend only on the capital-labor ratio, which is pinned down by equation 2. Consequently, changes in labor supply will be met by proportional changes in capital, and will thus have no effect on equilibrium wages.

Social welfare. Given a tax and transfer schedule, $\{\tau_t(\cdot), \tau_t^k, b_t(\cdot)\}$, a sequence of interest rates, $\{r_t\}_{t=0}^\infty$, and a sequence of wages, $\{w_t\}_{t=0}^\infty$, social welfare is given by:

$$W(\{\tau_t(\cdot), \tau_t^k, b_t(\cdot), r_t, w_t\}_{t=0}^\infty) = \int_0^1 V_i(a_{i,0}, \{w_t, r_t, b_t(\cdot), \tau_t(\cdot)\}_{t=0}^\infty) di \quad (3)$$

The government's problem is to choose a tax and transfer schedule and a sequence of government debts, $\{x_t\}_{t=1}^\infty$, to maximize social welfare, subject to x_0 , equations 1-2, the government budget constraint:

$$\frac{1}{\mu} \int_0^1 b_t(l_{i,t}) di + x_{t+1} = \int_0^1 \tau_t(l_{i,t}) w_t l_{i,t} + \tau_t^k R_t K_t + (1 + r_t) x_t \quad (4)$$

the incentive compatibility constraints:

$$\{l_{i,t}\}_{t=0}^\infty \in \mathcal{L}_i(a_{i,0}, \{w_t, r_t, b_t(\cdot), \tau_t(\cdot)\}_{t=0}^\infty) \text{ for all } i \quad (5)$$

and the targeting constraint:

$$b_t(l) = b_t(\underline{l}_t) \text{ for all } l \leq \underline{l}_t \quad (6)$$

The parameter μ denotes the transfer multiplier: it costs the government $1/\mu$ dollars to finance a transfer of 1 dollar. For example, if there are administrative costs, $\mu < 1$.

The targeting constraint (equation 6) states that it is impossible to perfectly target transfer recipients. The government must give the same transfer to all workers with $l_{i,t} \leq \underline{l}_t$, where \underline{l}_t is exogenously given. It is useful to denote by I_t the set of workers for which $l_{i,t} \leq \underline{l}_t$.

The following assumption will be crucial for the analysis that follows:

Assumption 1 *Given existing taxes and transfers, the solution to the household's optimization problem is unique. This unique solution will be denoted by $a_{i,t+1}^*$, $l_{i,t}^*$ and $c_{i,t}^*$ (with the understanding that the optimal plan is a function of the state variables).*

This assumption is in the spirit of Assumption 2 in Golosov et al. [2014]. However, it is not innocuous. It is possible to construct examples in which it will not hold at the optimal policy. In particular, depending on parameters, the optimal policy may implement an equilibrium in which some agents are indifferent with respect to two different choices of labor supply. Appendix A presents an example to illustrate the conditions under which this Assumption 1 is plausible.

In general, an optimal tax code may or may not satisfy the uniqueness assumption. However, this assumption will be crucial for the analysis that follows as it guarantees that labor supply decisions are continuous with respect to small changes in tax policy.

An additional assumption will be made:

Assumption 2 *Let $T_t = \int_0^1 \tau_t(l_{i,t}^*) w_t l_{i,t}^* di + \tau_t^k R_t K_t$ denote the government's tax revenue. Then, given prevailing tax rates, a marginal change in the trans-*

fer size b_t has no effect on current or future equilibrium tax revenues: $\frac{\partial T_t}{\partial b_{t'}} = 0$ for all $t \geq t'$.

This assumption is reasonable, at least as an approximation, if transfer recipients generate a small share of aggregate labor income, especially if their labor income is taxed at a low (or zero) rate. Note that even if transfer recipients do not pay any taxes, their labor supply decisions may affect equilibrium tax revenues through capital taxation: lower labor supply would imply a lower capital stock which, if $\tau_t^k > 0$, would generate lower tax revenues. However, this general equilibrium effect will be small if transfer recipients supply a small share of aggregate labor inputs (note that labor inputs are specified in effective units - thus, the assumption is that labor supplied by transfer recipients is relatively unproductive). Alternatively, if a large proportion of households are transfer recipients, then the assumption is a reasonable approximation if, for more productive households, transfers constitute a small share of income and thus an increase in transfers does not lead to a large change in the incentives to work.

Note that while Assumption 2 rules out scenarios in which transfers are distortionary, it does not preclude the possibility that there are distortions associated with the taxation used to finance those transfers. The distortionary costs of taxation operate through the labor supply elasticity: higher taxes reduce the incentives to work, resulting in lower equilibrium labor supply and, consequently, lower equilibrium capital supply.

It will be useful to introduce the following notation to describe different aggregations of the labor supply elasticity. Let $\epsilon_{L,t} = \frac{\partial \ln(L_t)}{\partial \ln(w_t)}$ denote the aggregate elasticity of labor supply, and let $\epsilon_{i,t} = \frac{\partial \ln(l_{i,t}^*)}{\partial \ln(w_t)}$ denote the individual elasticity of labor supply. Note that both elasticities are equilibrium objects that may depend on the tax code and that, given Assumption 1, the elasticities are well-defined. The results will focus on $\tilde{\epsilon}_{L,t}$, which is defined as the maximum of the aggregate labor supply elasticity and a tax-weighted

average of individual labor supply elasticities:

$$\tilde{\epsilon}_{L,t} = \max\left\{\frac{\int_0^1 \tau_i l_i \epsilon_{i,t} di}{\int_0^1 \tau_i l_i di}, \epsilon_{L,t}\right\} \quad (7)$$

Note that the assumption of a finite set of state variables guarantees that the set of elasticities $\{\epsilon_{i,t}\}_{i \in [0,1]}$ is finite, so the integral exists.

It will be convenient to introduce the notation $\tau_{i,t} = \tau_t(l_{i,t}^*)$ to denote equilibrium average tax rates. Given a sequence of capital tax rate $\{\tau_t^k\}_{t=0}^\infty$ and a sequence of deficits $\{x_t\}_{t=0}^\infty$, it is said that the taxes $\{\tau_{i,t}\}_{i,t}$ are feasible if there exist a sequence of tax schedules $\{\tau_t(\cdot)\}_{t=0}^\infty$ such that $\tau_{i,t} = \tau_t(l_{i,t}^*(\tau_t(\cdot)))$.

The analysis that follows focuses on the welfare gains from implementing a tax schedule that induces a uniform shift in average tax rates, $\{\tau_{i,t} + \eta\}$ for some $\eta > 0$. Note that a uniform increase in the existing tax schedule of the form $\tilde{\tau}_t(\cdot) = \tau_t(\cdot) + \eta$ does not necessarily deliver the equilibrium tax rates $\{\tau_{i,t} + \eta\}$, because a uniform shift may have differential effects on individual labor supply decisions at different tax brackets. Achieving an equilibrium in which the average tax rates are $\{\tau_{i,t} + \eta\}$ might necessitate a non-uniform adjustment of the tax code.

Before proceeding, it is worth motivating the focus on this particular perturbation of the tax code. In particular, one might find it more natural to discuss issues of redistribution by perturbing the tax schedule in a way that is more targeted at the very rich. One example is greater taxation of capital income, which is highly concentrated among wealthy households. In the small open economy setup considered here, capital taxation would be equivalent to labor taxation (in a closed economy setup, this equivalence would hold at the steady state). The reason for this is that the rate of return on capital is pinned down by global market conditions. Thus an increase in capital taxation would lead to a decline in the capital stock and, subsequently, a decline in wages. This would result in a decline in household income that is proportional to its labor income. Because the general equilibrium effects are

similar, I chose the more direct approach of analyzing an increase in labor taxes.

A second example of a perturbation that is more targeted at wealthy households is an increase in top tax rates (see Saez [2001]). While such a perturbation may be feasible and desirable, a uniform increase in tax rates is more likely to satisfy the incentive compatibility constraints. Intuitively, an increase in the top tax rate will have a smaller effect on the labor supply of highly productive households when lower incomes are taxed at a higher rate as well. In the context of this exercise, the focus on a uniform tax increase is more conservative. Countries that are shown to benefit from such a scheme would also benefit from a more progressive tax reform, assuming that such a reform is feasible.

The following proposition provides a sufficient condition for the desirability of greater redistribution of labor income:

Proposition 1 *Let $s_T = (\int_0^1 \tau(l_i) w l_i di + \tau^k RK)/Y$ denote the share of tax revenue in output, and let $(1 - \alpha)$ denote the equilibrium labor income share. If the following condition holds:*

$$\frac{1 - \alpha}{s_T} \left(1 - \frac{\int_0^1 u'(c_{i,t}) \frac{l_{i,t}}{L_t} di}{\frac{\mu}{\int_{I_t} 1 di} \int_{I_t} u'(c_{i,t}) di} \right) > \tilde{\epsilon}_L \quad (8)$$

then it is possible to improve welfare by an increase in transfers financed by a uniform increase in labor taxes: $\tau_{i,t} \mapsto \tau_{i,t} + \eta$ for some $\eta > 0$.

The proof is provided in the appendix. Intuitively, the optimality of increasing labor taxes depends on the elasticity of labor supply. The elasticity of labor supply governs the extent to which an increase in tax rates reduces labor inputs. A more elastic labor supply implies that an increase in taxes generates a smaller increase in tax revenues and, consequently, smaller increases in transfers and equilibrium welfare.

Additionally, the welfare gains from increasing transfers depend on the

Table 1: Benchmark calibration

Parameter	Value or source
$u'(c)$	$c^{-\gamma}$
γ	0.5
$\tilde{\epsilon}_L$	2
μ	0.6
Consumption distribution	Lakner and Milanovic [2016]
Labor income distribution	Consumption distribution for the bottom 90%
s_T	Tax revenues to GDP, WDI
$(1 - \alpha)$	Labor income share, PWT
I	All households

level of risk aversion, which is captured by the marginal utilities $u'(c_{i,t})$. To illustrate, note that if $\mu = 1$ and there is no consumption inequality, then the condition above is violated, and there are no gains from increasing redistribution. However, when there is consumption inequality, then $\int_0^1 u'(c_{i,t}) \frac{l_{i,t}}{L_t} di \leq \frac{1}{\int_{I_t} 1 di} \int_{I_t} u'(c_{i,t}) di = E[u'(c_{i,t}) | i \in I_t]$. Thus, for $\mu > 0$ sufficiently large, the left hand side will be positive, which is a necessary but not sufficient condition for the inequality to hold.

Note that the condition in Proposition 1 depends on a weighted average of marginal utilities, where weights are given by the individual shares of labor income (l_i/L). An increase in transfers financed by a uniform increase in labor taxes is primarily financed by those whose labor income is higher. The amount of additional tax revenue collected from any given household is proportional to its labor income. Thus, the welfare cost of redistribution depends on the joint distribution of consumption and labor income.

2.1 Calibration

This section assesses the condition in Proposition 1 for a sample of countries. Table 1 summarizes the calibration approach.

To calibrate the joint distribution of consumption and labor income, I use data from Lakner and Milanovic [2016]. The data set contains survey-based measures of *either* household consumption or disposable income by decile. The calibration requires the joint distribution of labor income and consumption for computing the term $\int_0^1 u'(c_{i,t})l_{i,t}/L_t di$. I will assume that, for the bottom 9 deciles, consumption is given by disposable income and that income consists only of labor income ($c_{i,t} = (1 - \tau_t(l_{i,t}))w_t l_{i,t}$). The top 10% is assumed to have no labor income and consumes only capital income. Formally, using \hat{q}_i to denote the estimated consumption or income of decile i , I approximate:

$$\frac{\int_0^1 u'(c_{i,t})\frac{l_{i,t}}{L_t}}{E[u'(c_{i,t})|i \in I_t]} \approx \frac{\sum_{i=1}^9 u'(\hat{q}_i)\frac{\hat{q}_i}{\sum_{i=1}^9 \hat{q}_i}}{\frac{1}{I} \sum_{i=1}^I u'(\hat{q}_i)} \quad (9)$$

Note that ignoring the labor income of the top 10% vastly *understates* the benefits from additional redistribution. The highest decile likely receives a relatively large share of aggregate labor income, and has the lowest marginal utility of consumption. Thus, instituting transfers from this group to the rest of the population is highly desirable from a welfare perspective. By ignoring the top 10%, the burden of financing the transfers falls only on the bottom 90%, which is less favorable. The assumption that the top decile receives no labor income is thus highly conservative in this context.²

In addition, if taxes are progressive, then the approximation $l_i/L \approx \hat{q}_i/\sum_j \hat{q}_j$ understates the benefits from additional redistribution. Under the assumption that taxes are progressive, disposable income is more equally distributed than gross labor income. Note that the marginal benefits from

²At the same time, including the top 10% under the assumption that $c_{i,t} = (1 - \tau_{i,t})l_{i,t}$ may overstate the benefits from redistribution. The reason is that the top decile is likely to receive some capital income, which tends to be highly concentrated among the rich. Consequently, both the consumption level and the level of disposable income likely exceed labor income. The assumption $c_{i,t} = (1 - \tau_{i,t})l_{i,t}$ would therefore overstate the extent to which an increase in labor taxes would tax the rich. The most conservative approach is to ignore this group.

additional redistribution depend on gross labor income shares rather than on disposable labor income shares.

I assume that the marginal utility of consumption takes the form:

$$u'(c) = c^{-\gamma} \tag{10}$$

Harsanyi [1955] establishes that, for the purpose of welfare analysis, it is appropriate to interpret γ as the coefficient of relative risk aversion (rather than, for example, the intertemporal elasticity of substitution or a parameter that governs the degree of altruism). I choose $\gamma = 0.5$, a highly conservative estimate of risk aversion. The macro literature typically uses $\gamma = 2$, and much larger values are often used in the finance literature, which documents that large levels of risk aversion are necessary in order to account for the equity premium puzzle.

Estimated tax shares, s_T , are based on data from the World Development Indicators (WDI). Estimated labor shares, $(1 - \alpha)$, are taken from the Penn World Tables (PWT). I assume that $\mu = 0.6$, which is highly conservative given estimates of the administrative costs of cash transfer programs.

As a conservative benchmark, I assume that the targeting constraint does not allow any targeting of poor households, and that the government must transfer the same amount to every household. This is conservative because the welfare gains from cash transfer programs are higher when they target those who have higher marginal utilities of consumption.

Finally, there is substantial controversy regarding the labor supply elasticity. The micro literature typically finds an elasticity in the vicinity of 0.3, while the macro literature finds that a much larger elasticity of between 1 and 2 (see Keane [2011]). I take a conservative approach and assume that $\tilde{\epsilon}_L = 2$. Note that a high elasticity of labor supply implies that taxes are more distortionary.

Results. Despite its highly conservative assumptions, the calibration suggests that 103 out of the 111 countries in the sample satisfy the condition of Proposition 1, suggesting that there is too little redistribution in these countries. The exceptions are Botswana, Gabon, Mongolia, Norway, New Zealand, Senegal, Trinidad and Tobago and Turkey. Note that Proposition 1 provides a sufficient rather than a necessary condition; thus, additional redistribution may be desirable even in countries for which this condition is not satisfied, especially given the highly conservative calibration.

3 Extension: Government investment

This section extends the model to allow for productivity-augmenting government investment. The purpose of this extension is to address the policy question of how government spending should be allocated between investment and cash transfer programs. Especially in developing countries, the ability to raise taxes may be limited but there may be more flexibility in how to allocate government spending and foreign aid.

To extend the model, assume that, in addition to private capital and labor inputs (K and L), output depends on the level of government capital, G . The production function takes the form:

$$F_t(K_t, L_t, G_t) = A_t f(G_t) h(K_t, L_t) \quad (11)$$

where h is a constant returns to scale production function. This specification guarantees that there are no rents and that, when capital and labor are paid their marginal products, the sum of factor payments is equal to output (provided that the services of government capital are free of charge).

The government budget constraint is modified as follows to include gov-

ernment investment:

$$\frac{1}{\mu} \int_0^1 b_t(l_{i,t}) di + G_{t+1} + x_{t+1} = \quad (12)$$

$$\int_0^1 \tau_t(l_{i,t}) w_t l_{i,t} + \tau_t^k R_t K_t + (1 - \delta) G_t + (1 + r_t) x_t$$

The government's optimization problem is to maximize social welfare subject to the government budget constraint, the incentive compatibility constraints and the targeting constraint, taking taxes as given:

$$\max_{b_t(\cdot), l_{i,t}, G_{t+1}, x_{t+1}} \int_0^1 V_i(a_{i,0}, \{w_t, r_t, b_t(\cdot), \tau_t(\cdot)\}_{t=0}^\infty) \quad (13)$$

s.t. G_0, x_0 , equations 1-2, 5-6 and 12.

Proposition 2 *Assume that (a) for all t , it is strictly optimal to set $b_t(l_{i,t}) > 0$ for $i \in I_t$; (b) the production function is Cobb-Douglas in K and L ($h(K, L) = K^\alpha L^{1-\alpha}$), and (c) There is common elasticity of labor supply ($\epsilon_{i,t} = \epsilon_L$). Then, at the optimum, the rate of return to government investment is:*

$$\frac{\partial F_{t+1}}{\partial G} - \delta = \frac{\mu(r_{t+1} + \delta)}{\int_0^1 \frac{u'(c_{i,t+1})}{E[u'(c_{i,t+1}) | i \in I_{t+1}]} \frac{(1 - \tau_{i,t+1})^{l_{i,t+1}}}{L_{t+1}} di + \mu S_T \frac{1 + \epsilon_L}{1 - \alpha}} - \delta \quad (14)$$

The proof is in the appendix. Note that, similar to the condition of Proposition 1, the rate of return on government investment implied by the welfare maximization problem depends on a weighted average of marginal consumption utilities. However, in contrast to the condition in Proposition 1, weights are proportional to disposable labor income and do not necessarily sum to 1. Intuitively, an increase in government capital increases labor productivity, which increases disposable labor income. The welfare gains from government investment are thus proportional to disposable rather than to gross labor income. Similar to equation 9, for the purpose of the calibration,

I assume that:

$$\int_0^1 \frac{u'(c_{i,t+1})}{E[u'(c_{i,t+1})|i \in I_{t+1}]} \frac{(1 - \tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di \approx \frac{\sum_{i=1}^9 u'(\hat{q}_i) \frac{\hat{q}_i}{\sum_{i=1}^9 \hat{q}_i}}{\frac{1}{I} \sum_{i=1}^I u'(\hat{q}_i)} \quad (15)$$

To calibrate the optimal rates of return to government investment implied by proposition 2, it is necessary to take a stance on the values of r_{t+1} . I assume that r_{t+1} is given by the Euler condition, which relates the interest rate to the rate of consumption growth.³

The calibration based on the conservative parameterization described by Table 1 suggests that the optimal rate of return on government investment should be negative in most countries, and always below 2%. Given that most estimates suggest high rates of return to government investment, the conclusion is that, given current tax revenues, governments should divert resources from cash transfer programs to government investment projects.

However, this conclusion is a result of the highly conservative parameterization, and may not hold up in the presence of less conservative (and perhaps more realistic) parameters. Table 2 illustrates how the optimal rate of return to government capital varies with alternative calibration parameters.

Depending on the parameterization, the average optimal rate of return on government investment can be as high as 21%. This estimate is in the range of plausible estimates of rates of return to government investment projects, although those are notoriously imprecise. The calibrated optimal rates of return vary somewhat by country, and are reported in Appendix C.

³The Euler condition is $\beta(1 + r_{t+1}) = u'(c_t)/u'(c_{t+1})$. I assume that the rate of consumption growth is given by the rate of GDP per capita growth, which is calibrated based on data from the World Development Indicators.

Table 2: Average returns (percent): alternative calibrations

μ	$\tilde{\epsilon}_L$	I	γ	Average optimal return
0.6	2	All	0.5	0
1	2	All	0.5	1
0.6	0.3	All	0.5	1
1	0.3	All	0.5	3
0.6	2	Bottom decile	0.5	1
1	2	Bottom decile	0.5	2
0.6	0.3	Bottom decile	0.5	3
1	0.3	Bottom decile	0.5	5
0.6	2	All	2	3
1	2	All	2	5
0.6	0.3	All	2	7
1	0.3	All	2	11
0.6	2	Bottom decile	2	8
1	2	Bottom decile	2	9
0.6	0.3	Bottom decile	2	18
1	0.3	Bottom decile	2	21

4 Conclusion

This paper establishes that, even under highly conservative assumptions, there is a strong case for further redistribution. In theory, the gains from redistribution depend on a variety of structural parameters, such as the social aversion to inequality; the ability to target transfers at the poor; and the administrative costs of transfer programs. The calibration establishes that even at very low levels of inequality aversion, an un-targeted transfer program in which transfer recipients receive only 60 cents on the dollar would improve social welfare. The implication is that under any realistic parameterization, there are gains from further redistributive policy.

The analysis relies crucially on two assumptions that guarantee the feasibility of increasing transfers and tax revenues. The first assumption states that, given prevailing tax policy, optimal individual labor supplies are uniquely pinned down. While this assumption seems plausible, it may be violated under an optimal policy. The second assumption states that an increase in transfers (holding taxes fixed) has a negligible effect on tax revenues. This framework therefore refocuses the debate on the optimal extent of redistribution. Given that further redistribution is desirable even under the most conservative parameterizations, opponents of further redistribution must either argue that, although desirable, increasing tax revenues is not feasible, or that increasing transfers would result in a large drop in tax revenues.

When total government spending is fixed, whether or not it is optimal to increase transfers to the poor depends on the welfare gains from other forms of government spending. I extend the model to include productivity-enhancing government investment, and establish that the optimality of diverting public funds from government investment projects to cash transfer programs is highly sensitive to parameter values.

To conclude, this paper refocuses the debate on the optimal scope of redistribution. First, it establishes that this debate is viable only in cases in which increasing tax revenues is not feasible. Otherwise, further redistribu-

tion is desirable under any realistic parameterization. Second, it establishes that, when increasing tax revenues is not feasible, the optimality of diverting funds from government investment projects to redistributive programs depends on the rate of return to government investment, the administrative cost of transfers, the elasticity of labor supply and the social aversion to inequality.

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A An example to illustrate the conditions under which Assumption 1 holds

Example 1. Consider the following simple example in which $a_{i,0} = 0$ for all i and there are only two permanent states, $s_{i,t} \in \{0, 1\}$ (where $s_{i,t} = s_{i,0}$ for all t). The utility function takes the form:

$$u(c_{i,t}) = (1 - \phi) \ln(c_{i,t}), g(l_{i,t}, s_{i,t}) = \begin{cases} \phi \ln(1 - l_{i,t}) & \text{if } s_{i,t} = 1, l_{i,t} \in [0, 1) \\ 0 & \text{if } s_{i,t} = l_{i,t} = 0 \\ -\infty & \text{otherwise} \end{cases}$$

In this example, only agents with an $s_{i,t} = 1$ shock are able to supply labor. Agents with $s_{i,t} = 0$ are unable to supply any labor, because any positive amount of labor supply results in utility of $-\infty$. In addition, given that shocks are permanent and initial wealth is set to 0, they cannot borrow or consume out of savings. Thus, an optimal policy would allow them to choose $l_{i,t} = 0$ and provide them with some government transfers to consume.

Assume that $\beta(1 + r_t) = 1$ for all t . Under this assumption, a time invariant tax and transfer policy is optimal and the equilibrium can be solved in a static framework. Households with $s_{i,0} = 1$ choose their permanent labor supply, $l_{i,t} = l$, subject to a tax rate τ , and households with $s_{i,t} = 0$ consume the government transfer. Assuming a production function with no capital inputs ($F(K, L) = L$), the transfer is given by $b = \mu\tau l$.

The $s_{i,t} = 1$ household solves:

$$\max_{l,c} (1 - \phi) \ln(c) + \phi \ln(1 - l) \text{ s.t. } c = \begin{cases} (1 - \tau)wl & \text{if } l > 0 \\ b & \text{otherwise} \end{cases} \quad (16)$$

The optimal labor supply decision in the region $l > 0$ is:

$$l^o = 1 - \phi \quad (17)$$

The optimal labor choice, l^* , is then given by:

$$l^* = \begin{cases} l^o & \text{if } (1 - \phi) \ln((1 - \tau)wl^o) + \phi \ln(1 - l^o) \geq (1 - \phi) \ln(b) \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

Of course, the government will not be able to finance any transfers unless $l^* = l^o > 0$. The size of the transfer is limited by the government budget constraint $b = \mu\tau l^*$. Under the constraint that the government must induce productive agents to choose $l^* = l^o$, the government's optimization problem can be written as:

$$\max_{\tau,b} (1 - \phi) (\ln((1 - \tau)wl^o) + \ln(b)) + \phi \ln(1 - l^o) \quad (19)$$

s.t. $b = \mu\tau l^o$ and the incentive compatibility constraint:

$$(1 - \phi) \ln((1 - \tau)wl^o) + \phi \ln(1 - l^o) \geq (1 - \phi) \ln(b) \quad (20)$$

Ignoring the incentive compatibility constraints yields an optimal tax rate of $\tau = 0.5$ (note that given log utility, it is optimal for the government to spend an equal share of aggregate labor income on each household). The tax rate $\tau = 0.5$ is consistent with the incentive compatibility constraint only for μ sufficiently small. To see this, note that $b = \mu\tau l^o$ and hence the incentive

compatibility constraint is satisfied only when:

$$(1 - \phi) \ln(0.5wl^o) + \phi \ln(1 - l^o) \geq (1 - \phi) \ln(\mu 0.5wl^o) \quad (21)$$

Or:

$$\phi \ln(1 - l^o) \geq (1 - \phi) \ln(\mu) \quad (22)$$

Note that both sides of the inequality are negative. The inequality is satisfied for $\phi > 0$ sufficiently small and $\mu < 1$ sufficiently small. If we assume that the disutility from labor is low and that the administrative cost of transfers is large, then the incentive compatibility constraint will not be binding and the $s_{i,t} = 1$ household will strictly prefer the choice $l = l^o$ over the choice $l = 0$.

However, it is easy to see that this will not be the case when $\mu = 1$. In this case, $\tau = 0.5$ is not a feasible policy choice because productive households will prefer not to work and receive the government transfer. The optimal tax rate, τ , is then given by the incentive compatibility constraint:

$$(1 - \phi) \ln((1 - \tau)wl^o) + \phi \ln(1 - l^o) = (1 - \phi) \ln(\mu\tau wl^o) \quad (23)$$

Thus, under the optimal policy, the solution to the productive household's optimization problem is not unique: it is indifferent between $l = l^o$ and $l = 0$. To see this, note that if the household strictly prefers $l = l^o$, then the government can implement a small increase in the size of the transfer and in the tax rate. Given that $\tau < 0.5$, this alteration will improve social welfare.

To summarize, Assumption 1 holds in this example only when transfers are relatively costly (μ is small) and when the disutility of labor is not large (ϕ is small).

B Proofs

Proof of Proposition 1. Let λ_t denote the Lagrange multiplier on the government's budget constraint at time t . Applying the envelope theorem and ignoring the targeting constraint, the government's first order condition with respect to $b_t(l_i^t)$ yields:

$$u'(c_{i,t}) \leq \lambda_t \left(\frac{1}{\mu} - \frac{\partial T_t}{\partial b_{i,t}} \right) \quad (24)$$

Thus, given Assumption 2 the targeting constraint implies that:

$$E[u'(\underline{c}_{i,t}) | i \in I_t] \leq \lambda_t \left(\frac{1}{\mu} - \frac{\partial T_t}{\partial b_{i,t}} \right) = \frac{\lambda_t}{\mu} \quad (25)$$

Or:

$$\mu E[u'(\underline{c}_{i,t}) | i \in I_t] \leq \lambda_t \quad (26)$$

Consider the government's optimization problem with respect to labor taxes, $\tau_t(\cdot)$. It will be convenient to introduce the notation $\tau_{i,t} = \tau_t(l_{i,t}^*)$. This formulation assumes that the government can directly choose the tax rate faced by agent i , rather than the tax schedule. Note that $\tau_{i,t}$ must satisfy some incentive compatibility constraints, that induce agent i to "reveal his type" - in other words, there must exist some tax schedule $\tau_t(\cdot)$ such that, given that tax schedule, $\tau_{i,t} = \tau_t(l_{i,t}^*)$.

The assumption that the solution to the household's optimization problem is unique and that the set of state variables at any time t is finite implies that, for a sufficiently small $\bar{\eta} > 0$, it is possible to increase all labor tax rates by any $\eta \in (-\bar{\eta}, \bar{\eta})$ and distribute the tax receipts among agents in I_t .

Let $W(\eta)$ denote the welfare obtained from a uniform tax increase of η :

$$W(\eta) = \int_0^1 V_i(a_{i,0}, \{w_t, r_t, b_{i,t}, \tau_{i,t} + \eta, \tau_{i,t}^k\}_{t=0}^\infty) \quad (27)$$

s.t.

$$\frac{1}{\mu} \int_{I_t} b_t^\eta di + (1 + r_t)x_t^* = x_{t+1}^* + T_t \quad (28)$$

where T_t is the equilibrium tax revenue given the tax schedule $\{\tau_{i,t} + \eta\}$, and b_t^η is such that the increase in taxes and transfers is budget neutral.

The government can improve welfare by choosing $\eta > 0$ if $W'(\eta) > 0$. Note that given the small open economy assumption, the capital labor ratio is pinned down by global market conditions, and hence both R_t and w_t do not respond to changes in labor taxes. Thus, using the envelope theorem, this condition can be written as:

$$\int_0^1 u'(c_i)w_t l_{i,t} < \lambda_t \frac{\partial T_t}{\partial \eta} \quad (29)$$

Using equation 26, this condition holds if:

$$\int_0^1 u'(c_i)w_t l_{i,t} < \mu E[u'(c_{i,t})|i \in I_t] \frac{\partial T_t}{\partial \eta} \quad (30)$$

Note that taxes are given by $T = \int_0^1 ((\tau_i + \eta)w_i l_i + \tau_k R_t K_t) di$, and thus, using the observation that w_t and R_t are pinned down by global market conditions and do not respond to changes in labor inputs or taxes:

$$\frac{\partial T_t}{\partial \eta} = \int_0^1 w_i l_i di + \int_0^1 (\tau_i w_t \frac{\partial l_i}{\partial \eta} + \tau_k R_t \frac{\partial K_t}{\partial L_t} \frac{\partial L_t}{\partial \eta}) di \quad (31)$$

Rewriting:

$$\frac{\partial T_t}{\partial \eta} = \int_0^1 w_i l_i di + \int_0^1 (\tau_i w_t l_i \frac{\frac{\partial l_i}{\partial \eta}}{l_i} + \tau_k R_t \frac{\partial K_t}{\partial L_t} L_t \frac{\frac{\partial L_t}{\partial \eta}}{L_t}) di \quad (32)$$

Note that, given that the capital labor ratio is pinned down at some ζ ,

$$\frac{\partial K_t}{\partial L_t} L_t = K_t:$$

$$\frac{K}{L} = \zeta \Rightarrow K = \zeta L \Rightarrow \frac{\partial K}{\partial L} = \zeta \Rightarrow \frac{\partial K_t}{\partial L_t} L_t = \zeta L_t = K_t \quad (33)$$

Hence:

$$\frac{\partial T_t}{\partial \eta} = \int_0^1 w_i l_i di + \int_0^1 \left(\tau_i w_i l_i \frac{\partial l_i}{\partial \eta} + \tau_k R_t K_t \frac{\partial L_t}{\partial \eta} \right) di \quad (34)$$

Note that it follows that $\frac{\partial l_i}{\partial \eta} = -\epsilon_{i,t}$ and $\frac{\partial L_t}{\partial \eta} = -\epsilon_{L,t}$. Hence,

$$\begin{aligned} \frac{\partial T_t}{\partial \eta} &= \int_0^1 w_i l_i di - \int_0^1 \tau_i w_i l_i \epsilon_{i,t} di - \tau_k R_t K_t \epsilon_{L,t} \geq \quad (35) \\ \int_0^1 w_i l_i di - \tilde{\epsilon}_{L,t} \left(\int_0^1 \tau_i w_i l_i di - \tau_t^k R_t K_t \right) &= \int_0^1 w_i l_i di - \tilde{\epsilon}_{L,t} T \end{aligned}$$

Substituting back into equation 30,

$$\int_0^1 u'(c_i) w_t l_{i,t} di < \mu E[u'(c_{i,t}) | i \in I_t] \left(\int_0^1 w_i l_i di - \tilde{\epsilon}_{L,t} T \right) \quad (36)$$

Or:

$$\int_0^1 u'(c_i) w_t l_{i,t} di - \mu E[u'(c_{i,t}) | i \in I_t] \int_0^1 w_i l_i di < -\mu E[u'(c_{i,t}) | i \in I_t] \tilde{\epsilon}_{L,t} T \quad (37)$$

$$\frac{\int_0^1 u'(c_i) w_t l_{i,t} di}{\mu E[u'(c_{i,t}) | i \in I_t] T} - \frac{\int_0^1 w_t l_i di}{T} < -\tilde{\epsilon}_L \quad (38)$$

Or:

$$\frac{\int_0^1 u'(c_i) \frac{l_{i,t}}{L_t} di}{\mu E[u'(c_{i,t}) | i \in I_t] \frac{T}{w_t L_t}} - \frac{w_t L_t}{T} < -\tilde{\epsilon}_L \quad (39)$$

Note that $T = s_T Y$ and $w_t L_t = (1 - \alpha) Y$ where $1 - \alpha$ is the labor income share. Hence, the above can be rewritten as:

$$\frac{\int_0^1 u'(c_i) \frac{l_{i,t}}{L_t} di}{\mu E[u'(c_{i,t}) | i \in I_t] \frac{s_T}{1 - \alpha}} - \frac{1 - \alpha}{s_T} < -\tilde{\epsilon}_L \quad (40)$$

Or:

$$\frac{1 - \alpha}{s_T} \left(1 - \frac{\int_0^1 u'(c_i) \frac{l_{i,t}}{L_t} di}{\mu E[u'(c_{i,t}) | i \in I_t]} \right) > \tilde{\epsilon}_L \quad (41)$$

Proof of Proposition 2. Using λ_t to denote the Lagrange multiplier on the government's time t budget constraint, an application of the envelope theorem implies that the first order condition with respect to G_{t+1} is:

$$-\lambda_t + \beta \lambda_{t+1} \left(1 - \delta + \frac{\partial T_{t+1}}{\partial G_{t+1}} \right) + \quad (42)$$

$$\beta E_t \left[\int_0^1 u'(c_{i,t+1}) (1 - \tau_{i,t+1}) w'_{t+1}(G_{t+1}) l_{i,t+1}^* \right] = 0$$

where $w'_{t+1}(G_{t+1})$ is the derivative of equilibrium wages with respect to government capital. Note that this first order condition relies on the result that, for any $t' > 0$, $w_{t+t'}$ does not depend on G_t .

To simplify notation, it will be convenient to write $u'(\underline{c}_t) = E[u'(c_i, t) | i \in I_t]$. Assuming that positive transfers are strictly optimal, the inequality in equation 26 holds with equality. Combining the two first order conditions yields:

$$\mu u'(\underline{c}_t) = \beta \mu u'(\underline{c}_{t+1}) \left(1 - \delta + \frac{\partial T_{t+1}}{\partial G_{t+1}} \right) + \quad (43)$$

$$\beta E_t \left[\int_0^1 u'(c_{i,t+1}) (1 - \tau_{i,t+1}) w'_{t+1}(G_{t+1}) l_{i,t+1}^* \right]$$

To solve for $w'(G)$, note that, using the constant returns assumption and the assumption that factors are paid their marginal product:

$$wL + RK = F(K, L, G) \quad (44)$$

Using $L'(G)$ and $K'(G)$ to denote the derivatives of equilibrium labor and capital with respect to G , taking a derivative of both sides of the above

equation with respect to G yields:

$$w'(G)L + wL'(G) + RK'(G) = \frac{\partial F}{\partial G} + \frac{\partial F}{\partial L}L'(G) + \frac{\partial F}{\partial K}K'(G) \quad (45)$$

Note that R is pinned down by the global equilibrium condition. As factors are paid their marginal products, $w = \frac{\partial F}{\partial L}$ and $R = \frac{\partial F}{\partial K}$. Hence:

$$w'(G)L = \frac{\partial F}{\partial G} \quad (46)$$

Substituting into equation 43 yields:

$$\mu u'(\underline{c}_t) = \beta \mu u'(\underline{c}_{t+1}) \left(1 - \delta + \frac{\partial T_{t+1}}{\partial G_{t+1}}\right) + \quad (47)$$

$$\beta E_t \left[\int_0^1 u'(c_{i,t+1}) (1 - \tau_{i,t+1}) \frac{\partial F_{t+1}}{\partial G} \frac{l_{i,t+1}^*}{L_{t+1}} \right]$$

Rearranging yields:

$$\frac{\partial F}{\partial G} = \frac{\mu \left(\frac{1}{\beta} u'(\underline{c}_t) - u'(\underline{c}_{t+1}) \left(1 - \delta + \frac{\partial T_{t+1}}{\partial G_{t+1}}\right) \right)}{\int_0^1 u'(c_{i,t+1}) \frac{(1 - \tau_{i,t+1}) l_{i,t+1}^*}{L_{t+1}} di} \quad (48)$$

To calibrate the term $\frac{\partial T}{\partial G}$, it is useful to note that:

$$\frac{\partial T}{\partial G} = \frac{\partial T}{\partial \tilde{A}} \frac{\partial \tilde{A}}{\partial G} \quad (49)$$

where $\tilde{A} = Af(G)$. Note that household decisions depend only on factor payments, which can be written as a function of \tilde{A} rather than as a function of A and G . Thus, tax revenues depend only on \tilde{A} .

Expanding the above yields:

$$\frac{\partial T}{\partial G} = \frac{\partial T}{\partial \tilde{A}} \frac{\partial \tilde{A}}{\partial G} = \left(\frac{\frac{\partial T}{\partial \tilde{A}} \tilde{A}}{T} \right) \frac{TAf'(G)}{\tilde{A}} = \frac{\partial \ln(T)}{\partial \ln(\tilde{A})} \left(\frac{TAf'(G)}{Af(G)} \right) = \quad (50)$$

$$\frac{\partial \ln(T)}{\partial \ln(\tilde{A})} \left(\frac{TAf'(G)h(K, L)}{Af(G)h(K, L)} \right) = \frac{\partial \ln(T)}{\partial \ln(\tilde{A})} \left(\frac{T}{F(K, L, G)} \right) \frac{\partial F}{\partial G} = \frac{\partial \ln(T)}{\partial \ln(\tilde{A})} \frac{\partial F}{\partial G} s_T$$

Substituting into equation 48 yields:

$$\frac{\partial F}{\partial G} = \frac{\mu(\frac{1}{\beta}u'(\underline{c}_t) - u'(\underline{c}_{t+1}))(1 - \delta + s_T \frac{\partial \ln(T)}{\partial \ln(\tilde{A})} \frac{\partial F}{\partial G})}{\int_0^1 u'(c_{i,t+1}) \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di} \quad (51)$$

Rearranging:

$$\frac{\partial F}{\partial G} \left(1 + \frac{\mu u'(\underline{c}_{t+1}) s_T \frac{\partial \ln(T)}{\partial \ln(\tilde{A})}}{\int_0^1 u'(c_{i,t+1}) \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di} \right) = \frac{\mu(\frac{1}{\beta}u'(\underline{c}_t) - u'(\underline{c}_{t+1}))(1 - \delta)}{\int_0^1 u'(c_{i,t+1}) \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di} \quad (52)$$

$$\begin{aligned} \frac{\partial F}{\partial G} \left(\frac{\int_0^1 u'(c_{i,t+1}) \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di + \mu u'(\underline{c}_{t+1}) s_T \frac{\partial \ln(T)}{\partial \ln(\tilde{A})}}{\int_0^1 u'(c_{i,t+1}) \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di} \right) = & \quad (53) \\ \frac{\mu(\frac{1}{\beta}u'(\underline{c}_t) - u'(\underline{c}_{t+1}))(1 - \delta)}{\int_0^1 u'(c_{i,t+1}) \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di} & \end{aligned}$$

Yielding:

$$\frac{\partial F}{\partial G} = \frac{\mu(\frac{1}{\beta}u'(\underline{c}_t) - u'(\underline{c}_{t+1}))(1 - \delta)}{\int_0^1 u'(c_{i,t+1}) \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di + \mu u'(\underline{c}_{t+1}) s_T \frac{\partial \ln(T)}{\partial \ln(\tilde{A})}} \quad (54)$$

Or:

$$\frac{\partial F}{\partial G} = \frac{\mu(\frac{1}{\beta} \frac{u'(\underline{c}_t)}{u'(\underline{c}_{t+1})} - 1 + \delta)}{\int_0^1 \frac{u'(c_{i,t+1})}{u'(\underline{c}_{t+1})} \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di + \mu s_T \frac{\partial \ln(T)}{\partial \ln(\tilde{A})}} \quad (55)$$

The government's optimization problem with respect to x_{t+1} yields the standard Euler condition:

$$\frac{1}{\beta} \frac{u'(\underline{c}_t)}{u'(\underline{c}_{t+1})} = 1 + r_{t+1} \quad (56)$$

Thus, equation 55 can be rewritten as:

$$\frac{\partial F}{\partial G} = \frac{\mu(r_{t+1} + \delta)}{\int_0^1 \frac{u'(c_{i,t+1})}{u'(c_{t+1})} \frac{(1-\tau_{i,t+1})l_{i,t+1}}{L_{t+1}} di + \mu_{ST} \frac{\partial \ln(T)}{\partial \ln(A)}} \quad (57)$$

To conclude the proof, it is left to show that, under the Cobb-Douglas assumption:

$$\frac{\partial \ln(T)}{\partial \ln(\tilde{A})} = \frac{\partial \ln(T)}{\partial \ln(A)} = \frac{1 + \epsilon_L}{1 - \alpha} \quad (58)$$

To see this, note that the equalization of the marginal return to capital with the market rate implies that:

$$(1 - \tau_k)\alpha Af(G)\left(\frac{K}{L}\right)^{\alpha-1} - \delta = r \quad (59)$$

Rearranging yields the following expression for the capital-labor ratio as a function of A :

$$\frac{K}{L} = \left(\frac{(1 - \tau_k)\alpha Af(G)}{r + \delta}\right)^{\frac{1}{1-\alpha}} \quad (60)$$

Note that the derivative of the capital-labor ratio with respect to A is:

$$\frac{\partial \frac{K}{L}}{\partial A} = \frac{1}{(1 - \alpha)A} \left(\frac{(1 - \tau_k)\alpha Af(G)}{r + \delta}\right)^{\frac{1}{1-\alpha}} = \frac{1}{(1 - \alpha)A} \frac{K}{L} \quad (61)$$

and hence:

$$\frac{\partial \frac{K}{L}}{\partial \ln(A)} = \frac{\frac{K}{L}}{1 - \alpha} \quad (62)$$

Wages are equal to the marginal product of labor:

$$w = (1 - \alpha)Af(G)\left(\frac{K}{L}\right)^\alpha \quad (63)$$

Taking logs:

$$\ln(w) = \ln((1 - \alpha)f(G)) + \ln(A) + \alpha \ln\left(\frac{K}{L}\right) \quad (64)$$

The elasticity of the wage with respect to A is therefore:

$$\frac{\partial \ln(w)}{\partial \ln(A)} = 1 + \alpha \frac{\partial \ln(\frac{K}{L})}{\partial \ln(A)} = 1 + \frac{\alpha}{1 - \alpha} = \frac{1}{1 - \alpha} \quad (65)$$

As taxes are given by $T = \int \tau_i w l_i di + \tau_k R K$,

$$\frac{\partial \ln(T)}{\partial \ln(A)} = \frac{\partial T}{\partial A} \frac{A}{T} = \frac{1}{T} \left(\int \tau_i \left(\frac{\partial w}{\partial A} A l_i + w \frac{\partial l_i}{\partial w} \frac{\partial w}{\partial A} A \right) di + \tau_k R \frac{\partial K}{\partial A} A \right) \quad (66)$$

Note that the equilibrium rental rate, R , does not depend on A because the capital stock fully adjusts to meet the global rate of return. Rewriting yields:

$$\frac{\partial \ln(T)}{\partial \ln(A)} = \frac{1}{T} \left(\frac{\partial w}{\partial A} A \int \tau_i (l_i + w \frac{\partial l_i}{\partial w}) di + \tau_k R \frac{\partial K}{\partial A} A \right) = \quad (67)$$

$$\frac{1}{T} \left(\frac{\partial w}{\partial A} \frac{A}{w} \int \tau_i (w l_i + w \frac{\partial l_i}{\partial w} \frac{w}{l_i} l_i) di + \tau_k R \frac{\partial K}{\partial A} A \right) = \quad (68)$$

$$\frac{1}{T} \left(\frac{\partial \ln(w)}{\partial \ln(A)} \int \tau_i (w l_i + w \frac{\partial \ln(l_i)}{\partial \ln(w)} l_i) di + \tau_k R \frac{\partial K}{\partial A} A \right) = \quad (69)$$

$$\frac{1}{T} \left(\frac{1}{1 - \alpha} \int \tau_i w l_i (1 + \epsilon_L) di + \tau_k R \frac{\partial K}{\partial A} A \right) = \quad (70)$$

$$\frac{1}{T} \left(\frac{(1 + \epsilon_L)}{1 - \alpha} \int \tau_i w l_i di + \tau_k R \frac{\partial (\frac{K}{L} L)}{\partial A} A \right) = \quad (71)$$

$$\frac{1}{T} \left(\frac{(1 + \epsilon_L)}{1 - \alpha} \int \tau_i w l_i di + \tau_k R A \left(\frac{\partial (\frac{K}{L})}{\partial A} L + \frac{K}{L} \frac{\partial L}{\partial A} \right) \right) = \quad (72)$$

$$\frac{1}{T} \left(\frac{(1 + \epsilon_L)}{1 - \alpha} \int \tau_i w l_i di + \tau_k R \left(\frac{\frac{K}{L}}{1 - \alpha} L + \frac{K}{L} \frac{\partial L}{\partial w} \frac{\partial w}{\partial A} A \right) \right) = \quad (73)$$

$$\frac{1}{T} \left(\frac{(1 + \epsilon_L)}{1 - \alpha} \int \tau_i w l_i di + \tau_k R K \left(\frac{1}{1 - \alpha} + \frac{\partial L}{\partial w} \frac{w}{L} \frac{\partial w}{\partial A} \frac{A}{w} \right) \right) = \quad (74)$$

$$\frac{1}{T} \left(\frac{(1 + \epsilon_L)}{1 - \alpha} \int \tau_i w l_i di + \tau_k R K \left(\frac{1}{1 - \alpha} + \frac{\partial \ln(L)}{\partial \ln(w)} \frac{\partial \ln(w)}{\partial \ln(A)} \right) \right) = \quad (75)$$

$$\frac{1}{T} \left(\frac{1 + \epsilon_L}{1 - \alpha} \int \tau_i w l_i di + \tau_k RK \left(\frac{1}{1 - \alpha} + \epsilon_L \frac{1}{1 - \alpha} \right) \right) = \quad (76)$$

$$\frac{1 + \epsilon_L}{1 - \alpha} \frac{\int \tau_i w l_i di + \tau_k RK}{T} = \frac{1 + \epsilon_L}{1 - \alpha} \quad (77)$$

C Optimal rates of return on government capital, by country

The conservative calibration of the optimal rate of return to government investment uses the parameter values reported in table 1. The column labeled "Not Conservative" reflects an alternative calibration in which $\mu = 1$; $\gamma = 2$; $\epsilon = 0.3$; and transfers are targeted at the bottom decile of the distribution.

Table 3: Optimal rate of return on government investment (%), by country

Country	Conservative	Not conservative
Angola	0.5	23
Albania	0.2	21.1
Argentina	-0.4	14.8
Armenia	0.7	26.8
Australia	-1	11.2
Austria	-1.3	7.8
Azerbaijan	0.3	34.3
Burundi	-0.2	7.9
Belgium	-1.3	7
Benin	-0.5	17.4
Burkina Faso	0	22.2
Bangladesh	0.6	36.6
Bulgaria	-0.8	16.4
Bolivia	1	37.2
Brazil	0.1	21.2
Barbados	-0.6	8.2
Bhutan	1.6	69.4
Botswana	-3.3	5
Central African Republic	-1.7	-4.7
Canada	-0.7	18.2
Switzerland	-0.4	30.3
Chile	-0.2	16.9
China	1	52.6
Cote d'Ivoire	-0.7	19.5
Cameroon	-0.8	20.5
Congo, Rep.	-0.4	37.2
Colombia	0.3	32.8
Comoros	-0.2	7.5
Cape Verde	-0.3	19.1
Costa Rica	0	34.8
Czech Republic	-0.6	15.2
Germany	-0.4	20.4
Djibouti	0.5	25.6
Dominican Republic	0.4	28.4
Ecuador	-1.6	14.5
Egypt, Arab Rep.	-1.4	12.9
Estonia	0.5	59.8
Ethiopia	1.5	48.9
Fiji	-1.7	9.4

Table 4: Optimal rate of return on government investment (%), by country (continued)

Country	Conservative	Not conservative
France	-1	7.1
Gabon	-3	6.2
United Kingdom	-1.5	7.8
Ghana	0	38
Guinea	-2	7.5
Guinea-Bissau	-0.1	9.4
Greece	-1.5	-0.2
Hong Kong SAR, China	0.2	26.7
Honduras	0.7	26.2
Croatia	-0.5	9.8
India	-1.2	43.9
Iran, Islamic Rep.	-0.4	20.7
Israel	-0.9	13.5
Italy	-1.7	2.6
Jordan	-0.4	7.7
Japan	-1.4	16.5
Kazakhstan	-0.1	26.4
Kenya	-0.6	23.6
Cambodia	0.5	36
Korea, Rep.	-0.7	27.6
St. Lucia	-1.4	5.8
Sri Lanka	0.7	41.8
Luxembourg	-1.3	7.4
Morocco	-1.1	15.7
Madagascar	-0.8	14.8
Mexico	-0.3	15.7
Mali	-0.3	43.2
Mongolia	-0.4	22.8
Mozambique	0	19.2
Mauritania	0.1	18.9
Malawi	-0.7	20.7
Malaysia	-0.6	29.1
Namibia	-0.3	16.9
Niger	-0.4	12.6
Nigeria	0.8	65.3
Netherlands	32 -1	8.6
Norway	-1.3	4.3
Nepal	0.5	22.6
New Zealand	-1.6	6.1
Pakistan	-0.9	15.3

Table 5: Optimal rate of return on government investment (%), by country (continued)

Country	Conservative	Not conservative
Panama	0.3	48.6
Peru	-0.3	21.7
Philippines	-0.7	22.5
Poland	0.2	26.3
Portugal	-1.2	9.2
Paraguay	0.8	39.7
Russian Federation	-0.1	20
Rwanda	1.1	47.9
Sudan	0	24.7
Senegal	-1.2	6.9
Singapore	-0.6	23.4
Sierra Leone	0.1	22.8
El Salvador	-0.7	10.6
Slovak Republic	-0.2	28.6
Swaziland	-1.4	18.6
Syrian Arab Republic	-1	15.1
Chad	-0.1	14.7
Togo	0.1	18
Thailand	-1.7	16.5
Trinidad and Tobago	-2.1	6.8
Tunisia	-1.7	12.2
Turkey	-1.5	12
Taiwan	-1.5	13.8
Tanzania	-0.8	26.6
Uganda	0.3	31.2
Uruguay	-0.1	23.9
United States	-0.3	24.8
Uzbekistan	0.2	29.6
Venezuela, RB	-1	8.7
Yemen, Rep.	-1	0.9
South Africa	-1	9.8
Zambia	0.7	31.6