General Equilibrium Analysis of the Benefits of Large Transportation Improvements

GENERAL EQUILIBRIUM ANALYSIS OF THE BENEFITS OF LARGE TRANSPORTATION IMPROVEMENTS*

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This paper considers three benefit measures of a large transportation improvement in a general equilibrium framework, i.e., the Marshall-Dupuit consumer's surplus, the compensating variation, and the compensating surplus. First, we examine whether or not the measures can be reduced to the area to the left of a suitably defined transportation demand curve. Second, the measures are expressed as functions of various price and income elasticities which can be empirically estimated and we analyze factors affecting the magnitudes of the general equilibrium benefit measures. Third, the general equilibrium measures are compared with the partial equilibrium measures.

1. Introduction

In a partial equilibrium framework, the benefit of a transportation improvement is measured by a change in the area to the left of a transportation demand curve, where an uncompensated demand curve is used in the Marshall-Dupuit consumer's surplus and a compensated demand curve in the compensating variation and the equivalent variation. Quite often, however, a transportation investment affects many sectors in the economy and its general equilibrium repercussions cannot be ignored. In this paper, we examine several economic issues associated with the benefit evaluation of a large transportation project in a simple general-equilibrium framework.

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The first issue is whether or not the area to the left of a suitably defined transportation demand curve can serve as a benefit measure also in a general equilibrium model. The concept of the consumer's surplus has been extended to a general equilibrium framework by Harberger (1964, 1971), Mohring (1971), Silberberg (1972), and others. They showed that the line integral, 

\[-\int \sum x_i dp_i,\]

is a general equilibrium version of the consumer's surplus, where \(p_i\) and \(x_i\) are respectively the price of and demand for good \(i\). This measure becomes the Marshall-Dupuit or the compensating variation type depending on whether uncompensated or compensated demand functions are used in defining \(x_i's\) along the path of integration. Since the integration is carried out with respect to all the prices that change, the induced changes in prices of goods and services other than transportation services must be taken into account.

In a first-best economy with no price distortions, however, the effects of the induced changes in prices cancel out each other if the improvement is infinitesimally small, and the benefit is simply the initial level of traffic flow times the transportation cost reduction per unit flow.\(^1\) Our problem is then equivalent to one of examining whether or not this result carries over to the case of a large improvement.\(^2\) If the answer is affirmative, then the line integral can be reduced to the integral of the transportation demand only, and the task of benefit evaluation will be greatly simplified. It will also imply that the induced increase in the production of other sectors in the economy should not be included in the benefit calculation, contrary to the claim made by Adler (1971) and others.

It was shown by Lesourne (1975, pp. 71-72) that if the benefit is measured by the Marshall-Dupuit consumer's surplus (MD), the result for a marginal improvement can be extended to a large improvement.\(^3\) As shown, for

\(^1\)See, for example, Wheaton (1977), Harberger (1971, p. 791), Broadway (1975, p. 364), and Lesourne (1975, p. 35, Theorem 4). Sasaki (1983) also considered a marginal transportation improvement in a general equilibrium setting and obtained a formula which extends Wheaton's result to include a change in time costs. Solow (1973), Kanemoto (1977), and Arnott (1979) showed that the induced changes cannot be ignored in the second-best world with distortions such as unpriced congestion. See, also, Harberger (1971), Broadway (1975), Dasgupta and Stiglitz (1974), Bruce and Harris (1982), and Diewert (1983) for similar results in distorted economies with commodity taxes and/or tariffs.

\(^2\)Negishi (1972) and Harris (1978) also examined the choice of large projects but their focus is different from ours. Their benefit-cost criterion involves the value of the change in the production of all goods in the economy evaluated either at before-project prices or at after-project prices. In order to apply this criterion, one has to calculate the general-equilibrium effects of a project on all sectors in the economy. Our criterion is much easier to use because it requires the information on the change in transportation demand only. Of course, the change in transportation demand in our criterion includes all the general-equilibrium repercussions and the general equilibrium system must be solved to forecast the change accurately. People in practice, however, usually have much better 'feelings' on the change in transportation demand than on the induced changes in all other sectors.

\(^3\)This result is implicit in Harberger's formula of welfare change [1971, p. 789, eq. (6)]. Mohring (1976, ch. 8) provides an intuitive explanation.
example, by Mohring (1971), however, the MD measure has many undesirable properties: it is path dependent and yields neither a sufficient nor a necessary condition for a proposed project to satisfy a compensation test. In this paper, therefore, we consider two benefit measures in addition to MD: the Hicksian compensating variation (CV) and the compensating surplus (CS). MD and CV are well known and need no explanation. The last measure is similar to the coefficient of resource utilization introduced by Debreu (1951) and calculates the maximum cost that the society can pay for a transportation improvement when the utility levels of all households are to remain the same as the pre-improvement levels.  

We show that, unlike MD, CV cannot in general be represented by a change in the area to the left of a transportation demand curve. CS permits such a representation, however, if a numeraire good is suitably chosen.

Next, we illustrate, in a simple general equilibrium model, how an approximate measure of the increase in consumer's surplus can be obtained by using various price and income elasticities which can be empirically estimated. This approach provides a more convenient alternative to the usual one which simulates the effects of a transport project in a large-scale interregional model. Our approach is similar to that adopted by Diewert (1981, 1984) which used price derivatives of demand and supply functions in obtaining the second-order approximation of the deadweight loss associated with distortionary taxes. Combining his approach with Willig's (1976) which used the income elasticity of demand in deriving upper and lower bounds on the errors of approximating the compensating and equivalent variations with consumer's surplus, we obtain upper and lower bounds for the benefit of a large transportation improvement. Although our model is too simple to be applied directly to actual project evaluations, the method developed in this paper can be used to derive benefit measures in more realistic models.

In practice, many transport projects are evaluated on the basis of transport cost savings for the observed traffic flow and only in sophisticated cases the partial equilibrium measure of the area to the left of a transport demand curve is used. It is, therefore, of interest to examine the quantitative importance of the errors introduced by these naive estimation methods.

We carry out two types of comparisons: comparisons between the general-equilibrium (GE) benefit measures and the transportation cost reduction obtained for the initial traffic flow, and those between the GE measures and their partial equilibrium (PE) counterparts. Tinbergen (1957) pioneered the first type of comparisons. In a simple three-region model, he calculated

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4Although the same terminology is used, our compensating surplus is different from the compensating surplus concept in Hicks (1956, pp. 95-100) and Moss (1976).

5Although we do not consider the equivalent variation in this paper, it is easy to see that the induced changes cannot be ignored also in this measure.

6This case is equivalent to that suggested by Morisugi (1983).
numerical examples of the ratio between the two, called the 'multipliers'. In order to obtain a better insight on the magnitude of the Tinbergen multiplier, we express the multiplier in terms of price and income elasticities of demand for transported goods and examine factors affecting the magnitude of the multiplier.

Although our analyses in this paper are limited to the benefits of transportation improvements, they are applicable to the benefit evaluation problem of any large project. Especially, it is easy to see that our first result on the reducibility of the GE consumer's surplus measures to the area to the left of a transportation demand curve can be extended to any project evaluation problem.

The organization of this paper is as follows. A simple general equilibrium model of a two-region economy is constructed in section 2. In section 3, we define three benefit measures, MD, CV, and CS, and examine whether or not they can be reduced to a change in the area to the left of a transportation demand curve. Corresponding to the three benefit measures, three Tinbergen multipliers can be defined. In section 4, the multipliers are obtained as functions of the price elasticities of import demand, and we examine how the multipliers are affected by the elasticities and the size of the improvement. Finally, the GE measures are compared with the PE measures in section 5. The proofs of all propositions and a corollary are relegated to the appendix.

2. The model

Consider a two-region model corresponding to the complete specialization case in the international trade theory, where one region specializes in the production of one type of good. The good produced in the other region is used as an intermediate input and one of the consumption goods. Transportation costs are incurred when the good is imported and the main concern of this paper is how to measure the benefit of a large reduction in the transportation costs. For that purpose, we characterize competitive equilibria corresponding to different levels of transportation costs and examine how much the equilibrium level of social welfare rises when transportation costs are reduced.

Regions 1 and 2 produce goods 1 and 2 respectively. The production function of good $i$, $i=1,2$, is written $X_i = f_i(x_j^3)$, $j \neq i, j=1,2$, where $X_i$ is the amount of good $i$ produced and $x_j^3$ the amount of the good produced in the other region (good $j$, $j \neq i$) used in producing good $i$. The amounts of primary inputs such as labor and land are assumed to be fixed and suppressed in the production function.

Although one region produces only one good, each region consumes both two goods. The utility function of the representative consumer in region $j$ is $U^j(x^j)$, where $x^j = (x_1^j, x_2^j)$, $j=1,2$, denotes the consumption vector in region $j$ and $x_1^j$ is the consumption of good $i$ in region $j$. 

Exports of good $i$ from region $i$ to the other region $j (\neq i)$, which we denote by $z_i$, are the sum of consumption and production demands in region $j$, $z_i = x_i^j + x_i^3, j \neq i$. For simplicity, transportation costs are assumed to take the form of disappeared products. Assuming a linear transportation cost function, we can write transportation costs of exports from region $i$ to region $j$ as $x_i^4 = (t_i - 1)z_i$, where $t_i - 1$ is the transportation costs per unit quantity and $t_i$ is called the transportation factor. For region $j (\neq i)$ to obtain $z_i$ of good $i$, region $i$ must send $t_i z_i$ of the good, since $(t_i - 1)z_i$ disappears in the process of transportation.

Transportation improvements require inputs of goods 1 and 2. The needed amount of good $i$ is denoted by $x_i^5, i = 1, 2$, where $x_i^5 = 0$ and $x_i^5 \geq 0$ before and after the improvement respectively. The market clearing condition is then $X_i = \sum_{j=1}^5 x_i^j, i = 1, 2$.

If the price of good $i$ in region $j$ is denoted by $p_i^j$, the consumer and the
producer in region \( j \) face the price vector \( p^j = (p_1^j, p_2^j), j = 1, 2 \). For notational simplicity, define \( p_1 = p_1^1, p_2 = p_2^2, \) and \( p = p_2/p_1 \). Then, \( p_1 \) is the price of good 1 in region 1; \( p_2 \) the price of good 2 in region 2; and \( p \) the price ratio between good 2 and good 1 with each good evaluated at the price in the region where it is produced. Using the new notation, we can rewrite the price vector as \( p^1 = (p_1, t_2p_1p) \) and \( p^2 = (t_1p_1, p_1p) \).

The behavior of the consumer is represented by the expenditure function, \( e^i(p^i, u^i) = \min_{x^i} \{ p^i x^i : U^i(x^i) \geq u^i \} \). By Shephard's Lemma, the compensated demand function for good \( i \) is \( x^i_e = e^i(p^i, u^i) = \partial e^i(p^i, u^i) / \partial p^i_j \).

The profit maximization of the representative producer can be represented by the profit function, \( \pi^j(p^j) = \max_{x^j} \{ \int x^j \} - p^j x^j \}, i = 1, 2, j = 1, 2, j \neq i \). Note that, since primary inputs are suppressed in the production function, the profit here includes returns to labor and land inputs. Hotelling's Lemma yields output supply and input demand functions, \( X_j = Z_j(p^j) = \partial \pi^j(p^j) / \partial p^j_j \) and \( x^j_i = -\pi^i(p^i) = -\partial \pi^i(p^i) / \partial p^i_i \), \( i \neq j \).

It is assumed that the profit of the producer is given to the consumer in the region where the producer is located. The consumer pays the tax, \( T^j \), to finance the transportation improvement. Then, the budget constraint can be written \( \pi^j(p^j) = e^j(p^j, u^j) + T^j \), \( j = 1, 2 \).

Now, define the excess expenditure function, \( s^j(p^j, u^j) = e^j(p^j, u^j) - \pi^j(p^j) \), which is the expenditure function minus the profit function. Then, \( s^j(p^j, u^j) = \partial s^j(p^j, u^j) / \partial p^j_j = (\partial \pi^j(p^j) / \partial p^j_j - s^j) \), \( i \neq j \), is the compensated import demand function of region \( i \) and \( s^j(p^j, u^j) = \partial s^j(p^j, u^j) / \partial p^j_j = -s^j = -(X_j - x^j) \) is the negative of the compensated export supply function of region \( j \). Using the excess expenditure function, we can rewrite the market clearing conditions and income constraints as

\[
\begin{align*}
s^5(p^i, u^i) + T^i + x^5_i &= 0, \quad i = 1, 2, \quad j = 1, 2, \quad i \neq j, \\
s^5(p^i, u^i) + T^i &= 0, \quad j = 1, 2.
\end{align*}
\]

Given the choice of numeraire, i.e., \( p_1 \), and the values of \( x^5_i \) and \( T^j \), four equations in (1) and (2) contain three unknowns, \( p, u^1 \), and \( u^2 \). Since only three of the four equations are independent by Walras' Law, these equations determine the three unknowns. Note that, by the linear homogeneity of \( s^j(\ ) \) with respect to \( p^j \), eqs. (1) and (2) yield the obvious condition that the sum of the taxes to finance the transportation improvement equals the market value of the resources required for the improvement: \( T^1 + T^2 = p_1 x^5_1 + p_2 x^5_2 \).

Before proceeding to the discussion of benefit measures, we define the elasticities

\[
\sigma_{ij}^j = p_j s_{ij}^j / s_j^j, \quad i = 1, 2, \quad j = 1, 2.
\]

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where \( s_{ij} = \frac{\partial^2 s}{\partial p \partial p_j} \). For \( i \neq j \), \( \sigma_{ij} \) is the price elasticity of the compensated import demand function of region \( j \), and, for \( i = j \), it is the price elasticity of the negative of the compensated export supply function of region \( j \). Since the demand and supply functions are homogeneous of degree zero in prices, it can be easily seen that

\[
s_{ij} = s_{ji} = \frac{\partial^2 s}{\partial p_i \partial p_j} = -\sigma_{ij}[s_i / p_j], \quad i \neq j,
\]

and

\[
\sigma_{ji} = \sigma_{ij}(p_i / p_j s_j), \quad i \neq j.
\]

Note that, from (2) and the linear homogeneity of \( s'( ) \) in \( p_i \), \( \sigma_{ij} = -\sigma_{ji} \), if \( T^j = 0 \).

3. The benefit measures of a large transportation improvement

Corresponding to different definitions of the consumer's surpluses, there are many ways of measuring the benefit of a transportation improvement. In a general equilibrium situation, there is an added complexity that the benefit depends on how high the cost of the improvement is and how the cost is allocated among individuals. In this paper, we ignore the cost side and consider only the gross benefit. This is in accord with the usual practice where the gross benefit is first estimated independently of the cost of the project and in the next step of project evaluation it is compared with the cost. We also restrict our attention to only three measures: the Marshall-Dupuit consumer's surplus (MD), the compensating variation (CV), and the compensating surplus (CS).

In our model, goods are transported in two directions, good 1 from region 1 to region 2 and good 2 from region 2 to region 1, and a transportation improvement reduces the transportation costs of both two goods. For simplicity, we assume a proportionate decrease in the transportation factors from \( t^0 = (t^0_1, t^0_2) \) to \( t^1 = (1-h)t^0 = ((1-h)t^0_1, (1-h)t^0_2) \), where \( h \) is a scalar between 0 and 1.

First, in measuring the gross benefit, MD and CV assume that the cost of an improvement is zero and compare the pre-improvement and post-improvement equilibria. The welfare change caused by the improvement is measured in pecuniary terms by MD and CV. When the cost of the improvement is zero, eqs. (1) and (2) with \( x^5 = 0 \) and \( T^j = 0 \) determine the equilibrium relative price, \( p \), and the equilibrium utility levels, \( u^1 \) and \( u^2 \), corresponding to transportation factors before and after the improvement, \( t^0 \) and \( t^1 \).

Although the same terminology is used, our compensating surplus is different from the compensating surplus concept in Hicks (1956, pp. 95–100) and Moss (1976).
In order to define MD, the path between $t^0$ and $t^1$ must be specified, since, as is well known, MD is in general path dependent. We consider the simplest case of a line segment between $t^0$ and $t^1$ represented by $t^*_{a}(a) = (t^0_{a}(a), t^1_{a}(a)) = (1 - ha)t^0$, $0 \leq a \leq 1$. For any value of $a$ between 0 and 1, eqs. (1) and (2) with $x^0 = 0$ and $T^1 = 0$ define an equilibrium allocation. The equilibrium price ratio and utility levels are written $p^*_{a}(a)$, $u^1_{a}(a)$, and $u^2_{a}(a)$, respectively. For any arbitrary choice of numeraire, we obtain the price path of good 1 denoted by $p^1_{a}(a)$. For example, if good 1 is a numeraire, $p^1_{a}(a) = 1$, and if good 2 is a numeraire, $p^2_{a}(a) = 1/p^*_{a}(a)$. Given the path of $p^1_{a}(a)$, the price path of good 2 is $p^2_{a}(a) = p^1_{a}(a)p^*_{a}(a)$ and the equilibrium price vectors in regions 1 and 2 are $p^1_{a}(a) = (p^1_{a}(a), t^1_{a}(a)p^1_{a}(a)p^1_{a}(a))$ and $p^2_{a}(a) = (t^1_{a}(a)p^1_{a}(a), p^1_{a}(a)p^2_{a}(a))$.

For a marginal improvement, MD equals an induced rise in utility divided by the marginal utility of income. Following Silberberg (1972) and others, MD for a large change can be defined as an integral of the marginal measure

$$V = \int \left[ \lambda^1_{a}(a)u^1_{a}(a) + \lambda^2_{a}(a)u^2_{a}(a) \right] da,$$

where the weight $\lambda^j_{a}(a)$ is the reciprocal of the marginal utility of income in region $j$ at the equilibrium allocation with $t^*_{a}(a)$, i.e., $\lambda^j_{a}(a) = \partial e_{a}(p^j_{a}, u^j_{a})/\partial u^j_{a} = e^j_{a}(p^j_{a}, u^j_{a})$ at $p^j_{a} = p^*_{a}(a)$ and $u^j_{a} = u^j_{a}(a)$.

By the income constraints (2), we have $s_{a}(p^j_{a}(a), u^j_{a}(a)) = 0$, $j = 1, 2$, for any $a$. Hence, $u^1_{a}(a) = -(1/e^1_{a})[p^1_{a}t^1_{a}(a) + s^1_{a}p^1_{a}(a) + t^2_{a}s^1_{a}p^2_{a}(a)]$ and $u^2_{a}(a) = -(1/e^2_{a})[p^1_{a}t^1_{a}(a) + t^2_{a}s^2_{a}p^1_{a}(a) + s^2_{a}p^2_{a}(a)]$. MD can then be rewritten

$$V = -\int \left\{ \left[ p^1_{a}s^1_{a}t^1_{a}(a) + p^2_{a}s^2_{a}t^2_{a}(a) \right] \right.$$

$$+ \left[ (s^1_{a} + t^1_{a}s^2_{a})p^1_{a}(a) + (t^2_{a}s^1_{a} + s^2_{a})p^2_{a}(a) \right] \right\} da.$$

The first square bracket in the integral represents the direct benefit with fixed prices and the second bracket the indirect benefit through the induced changes in prices. By the market clearing conditions (1), however, the second square bracket vanishes and we finally obtain

$$V = -\int \left[ p^1_{a}(a)z^1_{a}(a)t^1_{a}(a) + p^2_{a}(a)z^2_{a}(a)t^2_{a}(a) \right] da,$$

where $z^1_{a}(a) = s_{a}(p^1_{a}(a), u^1_{a}(a))$, $i \neq j$, denotes the quantity of good $i$ exported from region $i$ at $t = t^*_{a}(a)$.

This result is equivalent to eq. (3.80) in Lesourne (1975, p. 72) and shows that our MD measure is a natural extension of the traditional consumer’s surplus, i.e., the area to the left of the demand curve. The only
difference from the traditional consumer’s surplus is that the transportation demand function here incorporates all the general equilibrium repercussions whereas the consumer’s surplus is usually defined with a partial equilibrium demand curve.

Notice that the market clearing conditions (1) play a crucial role in reducing MD to the area to the left of the transportation demand curve. Since the conditions are satisfied at any point along the equilibrium path, we can consider the benefit of a marginal improvement at each a. For such a small change, the welfare effects of the induced change in prices cancel out each other. Integrating the marginal benefit obtained in this way from a=0 to a=1 yields MD for a large improvement.

Hicks originally defined CV as the amount of compensation which individuals can pay at the post-improvement prices to remain at the pre-improvement utility levels, \( e^i(p^*(0), u^i(0)) - e^i(p^*(1), u^i(0)) \). In a general equilibrium model, this definition must be extended to account for a change in incomes due to a change in production. Following Diamond and McFadden (1974), Chipman and Moore (1980), King (1983), and Boadway and Bruce (1984), we define CV as

\[
C = \sum_j [e^j(p^*(1), u^j(1)) - e^j(p^*(1), u^j(0))] = \sum_j s^j(p^*(1), u^j(0)).
\]  

The second equality is obtained from (2) with \( T^j = 0 \), since it implies that \( \pi^j(p^*(a)) = e^j(p^*(a), u^j(a)) \), \( j = 1, 2 \), at \( a = 0, 1 \), and \( s^j(p^*(1), u^j(1)) = 0, j = 1, 2 \). This shows that CV equals the total amount of the surplus that the society as a whole can generate at the post-improvement equilibrium prices while attaining the pre-improvement utility levels.

CV is obviously path independent and has more desirable properties than MD as argued by Mohring (1971) and others. For example, Foster (1976) showed that a positive sum of consumers’ compensating variations is necessary for a proposed change to satisfy a weak compensation test in the first-best world where consumers’ prices are undistorted. Moreover, \( e^j(p^*(1), u^j) \) with constant reference prices, \( p^*(1) \), can be interpreted as a cardinal representation of individual utilities called ‘money metric utility’ by Samuelson (1974). CV therefore measures the utility differences between pre-improvement and post-improvement equilibria using money metric utility functions.\(^8\) As will be seen later, however, MD is much easier to calculate in

\(^8\)Chipman and Moore (1980) showed that CV cannot in general consistently rank many alternative projects. The reason is that the reference prices used in CV are after-project prices which are different between different projects. The equivalent variation (EV), however, uses the initial prices as reference prices and can consistently rank many alternative projects. This is one of the reasons why McKenzie and Pearce (1982) advocated the use of EV. In our analysis, however, we deal with only one project and this problem does not arise.
a general equilibrium setting and there may be circumstances where the measure is still useful as a first approximation.

Noting that \( s'(p^T(0), u^T(0)) = 0 \), we can rewrite (8) in the integral form,

\[
C = \int_0^1 \left[ s^1(p^{1*}(a), u^{1*}(0)) + s^2(p^{2*}(a), u^{2*}(0)) \right] da \\
= \int_0^1 \left\{ [p^{1*}(a)z_1(a)t^{1*}(a) + p^{2*}(a)z_2(a)t^{2*}(a)] \\
+ [(s^1 + t_1s_1^2)p^{1*}(a) + (t_2s_2^2 + s_2^2)p^{2*}(a)] \right\} da,
\]

where \( z_i(a) = s_i'(p^T(0), u^T(0)) \) is the compensated import demand for good \( i \) at \( t = t^*(a) \). Since \( s_i' \)'s here are compensated demand functions with utility levels fixed at the pre-improvement levels, they do not in general satisfy the market clearing conditions (1) except at \( a = 0 \). Unlike in the MD case, therefore, the second bracket in the last integral does not vanish. This leads to an important observation that, in a general equilibrium framework, CV does not in general equal the area to the left of the compensated transportation demand curve.

Finally, CS considers equilibria with constant utility levels and calculates the amount of surplus that the transportation improvement generates. That is, the compensating surplus is the maximum cost that the society can bear while remaining at the pre-improvement welfare position. In our model, the surplus measure is the sum of taxes, \( B = T^1 + T^2 \), that can be collected at an equilibrium with the post-improvement transportation factors and the pre-improvement utility levels. Note that the post-improvement equilibrium used in CS is obtained under the assumption that utility levels remain constant and the improvement cost is positive, whereas that in MD and CV assumes zero improvement cost and different utility levels. Thus, the post-improvement prices in CS are different from those in MD and CV.

CS is similar to the coefficient of resource utilization defined by Debreu (1951) in that both measures consider the surplus generated by a public project when the utility levels remain constant. However, the surplus measure cannot be defined unless we determine which combination of goods is left as the surplus. The coefficient of resource utilization assumes the special case where all the primal inputs are reduced proportionally. In this paper, we consider some other cases where produced goods are left as the surplus.

Regardless of the way in which taxes are collected, the surplus measure can be written

\[
B = -s^1(p^{1*}(1), u^{1*}(0)) - s^2(p^{2*}(1), u^{2*}(0)),
\]

(10)
where $p^\prime p'(1)$ is the equilibrium price vector with constant utility levels. Note that (10) is formally the same as CV in (8). The only difference lies in how the equilibrium prices are obtained. CS assumes that the utility levels are fixed and CV that the cost of the transportation improvement is zero.

The integral form of (10) is identical to (9) and as in the CV case CS does not in general equal the area to the left of the compensated demand curve. There is, however, an interesting special case where they are equal. If the taxes are paid only in the form of the numeraire, the effects of the induced changes in prices can be ignored, since the market clearing conditions for the goods whose prices change hold with $x_j^3=0$. For example, if good 1 is a numeraire, and if the taxes are paid only in good 1, then we obtain $p^\prime(p'(a)=0$ and $t_2s_2^1+s_2^3=-x_j^3=0$. The second square bracket in (9) is then zero and CS equals the change in the area to the left of the compensated transportation demand curve.

Although the above case is attractive in its simplicity, it has a drawback that the two regions are not treated symmetrically: the price of the export good of one region remains constant while that of the other region changes, and only the export good of the former region is used to pay the taxes. In the following sections we consider another case where the consumer in each region pays the tax in the export good, i.e., $T^j=p_jx_j^5$, $j=1,2$.

4. The Tinbergen multipliers

In a general equilibrium context, the benefit of a large transportation project can be estimated by explicitly computing the post-improvement equilibrium allocation (and the equilibrium path between pre- and post-improvement equilibria in the MD case). In practice, this amounts to building a large interregional model and simulating the effects of a transportation project. There are several drawbacks in this approach. Building a large-scale general-equilibrium model and computing the post-improvement equilibrium allocation in such a model is very costly: the reliability of the benefit estimates is difficult to check; and it is difficult to know what factors are most important in determining the size of the benefit. In this section, we propose an alternative approach which expresses the benefit of an improvement as a function of various price and income elasticities. This is done analytically and we can obtain a benefit estimate simply by substituting empirical estimates of these elasticities into the function. An advantage of our approach is that there is no need for computing the post-improvement equilibrium allocation which can be quite costly.

With a downward sloping transportation demand curve, the benefit of a large transportation improvement tends to exceed the total transportation cost reduction evaluated at the pre-improvement transportation demand. Instead of considering the benefit itself, we follow Tinbergen (1957) and
examine the magnitude of the ratio between the two which is called the Tinbergen multiplier. If the benefit is measured by MD, the multiplier is

\[ M_V = V/[(t_1^0 - t_1^1) p_1^*(0) \varepsilon_1^*(0) + (t_2^0 - t_2^1) p_2^*(0) \varepsilon_2^*(0)]. \quad (11) \]

The multipliers measured by CV and CS, \( M_C \) and \( M_B \), are defined by replacing \( V \) in (11) by \( C \) and \( B \) respectively. The magnitudes of the multiplier depend on the choice of numeraire. In the rest of the paper, we normalize the prices so that \( p_1 p_2 = 1 \) in order to preserve the symmetry between the two regions. Then, the prices must satisfy \( p_1 = p^{-1}_1 \) and \( p_2 = p^1 \).

In section 1, we defined the price elasticity of compensated import demand, \( \sigma_{i,j} \). The price elasticity of uncompensated demand can also be defined. In a general equilibrium model, however, the income of a consumer depends on prices and the usual uncompensated demand function is not very useful. In our model, the income of the consumer in region \( j \) is given by the profit in that region, \( \pi^i(p^i) \), which depends on the price vector there. We therefore use an extended version of the uncompensated demand function with endogenous income, \( \tilde{s}(p^i) \equiv s_i(p^i, \tilde{u}(p^i)) \), where \( \tilde{u}(p^i) \) is the utility level that is consistent with the budget constraint given the price vector \( p^i \), i.e., \( s_i(p^i, \tilde{u}(p^i)) = 0 \). The own price elasticity of this uncompensated import demand function is then

\[ \xi_i \equiv (p_i / s_i) \tilde{s}_i(p^i), \quad j \neq i, \quad i = 1, 2, \quad j = 1, 2, \quad (12) \]

where \( \mu_i \equiv p_i |x_i| e^i \) is the share of the imported good in the total expenditure of the consumer in region \( j \) and \( \eta_i \equiv (e^i / x_i)(e_i^i / e^i) \) can be easily seen to equal the income elasticity of demand for the imported good in region \( j \). Note that, since the price elasticity of the compensated import demand is always non-positive, \( \sigma_i \leq 0 \), the price elasticity of the uncompensated import demand is non-positive, \( \xi_i \leq 0 \), if the import good is a normal good, \( \eta_i \geq 0 \).

If the elasticities, \( \xi_i \)'s, were to remain constant along the equilibrium path from \( t^0 \) to \( t^1 \), then the exact estimate of the multiplier could be obtained. If \( \xi_i \)'s are not constant, then the exact estimate cannot be obtained, but, by using the lower and upper bounds for \( \xi_i \)'s, the bounds for the multiplier can be obtained in a manner similar to Willig's. First, we obtain the estimate of the multiplier, assuming that \( \xi_i \)'s are constant. Define

\[ y^*(\xi_1, \xi_2) \equiv \left[ \frac{1}{2} (\xi_1 + \xi_2) + 2 \xi_1 \xi_2 / (1 + \xi_1 + \xi_2) \right], \quad (13) \]

\[ m(y, h) \equiv \left[ 1 - (1 - h)^{1+y} \right] / [h(1+y)]. \quad (14) \]
Then, the following proposition yields the estimate of the multiplier in terms of the Marshall–Dupuit consumer’s surplus.

**Proposition 1.** If the price elasticities of import demand, $\xi_1$ and $\xi_2$, are constant, then the Tinbergen multiplier in terms of the Marshall–Dupuit consumer’s surplus is

$$M_V = m(y^*(\xi_1, \xi_2), h).$$

The multiplier is an increasing function of the size of the improvement, $h$, if $y^*(\xi_1, \xi_2) < 0$, and a decreasing function if $y^*(\xi_1, \xi_2) > 0$. If the Marshall–Lerner stability condition, $1 + \xi_1 + \xi_2 < 0$, holds, then the multiplier is a decreasing function of the price elasticity of import demand in either region, $\xi_i$.

In the normal case where the price elasticities of import demand are negative, $y^*(\xi_1, \xi_2)$ is negative and the multiplier increases as the size of the improvement becomes larger. Proposition 1 also shows that, given the size of the improvement, the multiplier is larger, the more price elastic is the price elasticity of import demand in either region. The multiplier can be extremely large if both the absolute value of the price elasticity and the size of the improvement are large. For example, if $h = 0.5$ and $\xi_1 = \xi_2 = -5.0$, then the multiplier is 7.5.

In a general case where $\xi_i$’s are not constant, the upper and lower bounds for the multiplier can be obtained from the following corollary.

**Corollary 1.** If lower and upper bounds for $\xi_i$ are $\xi_i$ and $\xi_i$, respectively, then the multiplier is between $m(y^*(\xi_1, \xi_2), h)$ and $m(y^*(\xi_1, \xi_2), h)$:

$$m(y^*(\xi_1, \xi_2), h) \leq M_V \leq m(y^*(\xi_1, \xi_2), h).$$

Next, we obtain the multipliers measured by CV and CS, $M_C$ and $M_B$.

**Proposition 2.** If the price elasticities, $\xi_i$ and $\sigma_{ij}$, $i=1,2, j=1,2$, are constant, then the multiplier in terms of the compensating variation is

$$M_C = \frac{1}{2} \{m(\sigma_{22}^1 + \delta(\sigma_{22}^1 + \frac{1}{2}), h) + m(\sigma_{11}^2 - \delta(\sigma_{11}^2 + \frac{1}{2}), h)$$

$$+ \delta[m(\sigma_{22}^1 + \delta(\sigma_{22}^1 + \frac{1}{2}), h) - m(\sigma_{11}^2 - \delta(\sigma_{11}^2 + \frac{1}{2}), h)$$

$$+ m(-\sigma_{11}^1 - \delta(\sigma_{11}^1 + \frac{1}{2}) - 1, h) - m(-\sigma_{22}^1 + \delta(\sigma_{22}^1 + \frac{1}{2}) - 1, h])\}.$$

Since our model is formally a simple generalization of the standard two-country model in international trade theory, the stability of equilibrium in our model requires the familiar Marshall–Lerner condition. See, for example, Takayama (1972, ch. 8).

This result can be interpreted as an example of the differential sensitivity analysis introduced by Diewert (1984).
and that in terms of the compensating surplus is

\[ M_B = m(y^*(\sigma^2_{11}, \sigma^1_{22}), h) + \frac{1}{2} \rho [m(-\sigma^1_{11} - \rho(\sigma^1_{11} + \frac{1}{2}) - 1, h) - m(-\sigma^2_{22} + \rho(\sigma^2_{22} + \frac{1}{2}) - 1, h)]. \]

where \( \delta \equiv (\xi_1 - \xi_2)/(1 + \xi_1 + \xi_2) \) and \( \rho \equiv (\sigma^2_{11} - \sigma^1_{22})/(1 + \sigma^2_{11} + \sigma^1_{22}). \)

This proposition shows that the compensating variation and the compensating surplus yield much more complicated multiplier formulae than the Marshall-Dupuit consumer's surplus. As a first step calculation, therefore, the Marshall-Dupuit measure might be more useful, since in a more realistic model with many regions and many commodities the formulae would become extremely complicated. The difference between the compensating variation and the compensating surplus multipliers is that \( \delta \) in \( M_C \) is replaced by \( \rho \) in \( M_B \). The difference arises from the fact that the equilibrium relative price in the compensating variation is obtained by using uncompensated demand functions and that in the compensating surplus by using compensated demand functions.

In the case where the elasticities are not constant, we can obtain bounds for \( M_C \) and \( M_B \) in the same way as in the Marshall-Dupuit surplus measure. In order to save space, however, we do not spell the details out.

Next, we examine a Cobb-Douglas example in the case where the two regions are symmetric. If \( u^i(x_{11}^i, x_{12}^i) = x_i \log(x_{11}^i) + x_j \log(x_{12}^i), x_1 + x_2 = 1, \) and \( f_i(x_{i1}^i) = A(x_{i1}^i)^{\beta}, i \neq j, i = 1, 2, j = 1, 2, \) then it can be seen that \( \sigma^2_{11} = \sigma^2_{22} = -\gamma_1(1 - x_1) - (1 - \gamma_1)/(1 - \beta) \) and \( \xi_1 = \xi_2 = -\gamma_1(1 - x_1) - (1 - \gamma_1)/(1 - \beta), \)

where \( \gamma_1 \) is the share of consumption demand in the total import demand. Since \( \gamma_1 \) must be between 0 and 1, both \( \gamma_1(1 - x_1) \) and \( x_1 + \gamma_1(1 - x_1) \) are less than one. However, \( (1 - \gamma_1)(1 - \beta) \) can be very large if the share of production demand for imports, \( 1 - \gamma_1, \) is large and \( \beta \) is close to one. Therefore, at least in the Cobb-Douglas case, consumption demand for transported goods does not yield a very large multiplier, but the multiplier may be very large in the case where transported goods are used as intermediate inputs.

5. Comparison between partial and general equilibrium benefit measures

The Tinbergen multiplier compares the general equilibrium benefit of a transportation improvement with the transportation cost reduction obtained for the initial traffic flow. Since the partial equilibrium (PE) measure is often used in practice, comparison between partial and general equilibrium (GE) measures is useful.\(^{11}\) The two measures are different in two respects. First,

\(^{11}\)This comparison was suggested by Thawat Watanatada at the World Bank.
the incomes of consumers are assumed constant in the PE measure, whereas
the GE measure incorporates changes in incomes caused by an induced
change in production. If transported goods are normal, this effect works in
the direction of making the GE measure larger than the PE measure. Second,
the induced changes in prices of goods and services other than transport-
ation services are ignored in the PE measure but are fully accounted for in
the GE measure. Since the first effect is rather straightforward, we consider
only the second effect in this section.

First, consider MD which yields the simplest result. Define

$$H(a, p) = p^{-\frac{1}{2}}s_1^{-a}((1-ha)t_1^0, p)ht_1^0 + p^{\frac{1}{2}}s_2^{-a}(1,(1-ha)t_2^0)ht_2^0.$$ 

Then, the MD measure (7) can be written \(\int_0^1 H(a, p^*(a)) \, da\), and the PE
version of the measure is \(\int_0^1 H(a, p^*(0)) \, da\). The following proposition com-
pares these two measures.

**Proposition 3.** Suppose a generalized version of the Marshall–Lerner stability
condition holds: \((\partial/\partial p)[t_1^0s_1^{-a}((1-ha)t_1^0, p)] \leq (\partial/\partial p)[t_2^0s_2^{-a}(1,(1-ha)t_2^0, p)]\) for any
\(p\) between \(p^*(a)\) and \(p^*(0)\). If \(\xi_1 < \xi_2\) along the entire equilibrium path from
\(a=0\) to \(a=1\) or if \(\xi_1 > \xi_2\) along the path, then the general equilibrium MD
measure, \(V\), is smaller than the partial equilibrium MD measure, \(V^p\).

Note that the generalized stability condition coincides with the Marshall–
Lerner condition at \(p=p^*(a)\). Proposition 3 shows that the GE measure is
smaller than the PE measure if import demand in one region is more price
elastic than that in the other region along the entire equilibrium path.
Whether or not it holds also in the case where the elasticities cross each
other is still an open question. Although the proposition does not treat the
case where \(\xi_1 = \xi_2\) along the entire equilibrium path, it is obvious that \(V = V^p\)
in this case, since the relative price remains constant.

Proposition 3 can be explained intuitively as follows. Suppose that import
demand for good 1 is more price elastic than that for good 2. Since the
transportation improvement considered in this paper reduces transportation
costs of goods 1 and 2 by the same proportion, the initial effect is a
proportionate fall in the prices of imported goods in the two regions. This
causes an increase in import demand but demand for good 1 which is more
price elastic rises more than that for good 2. Hence, for the trade balance
equation to be maintained, the price of good 1 should rise relative to the
price of good 2.

Now, the PE measure assumes that the prices of goods 1 and 2 are
constant, whereas in the GE measure the relative price of good 1 rises along
the equilibrium path. The GE import demand for good 1 then becomes
smaller than the PE demand, but that for good 2 becomes larger. Since
import demand for good 1 is more price elastic, the sum of demands for the
two goods is smaller in the GE case and the GE transportation demand
curve is steeper than the PE curve. Thus, the GE measure is smaller than the
PE measure.12

If the benefit is measured by CV or CS, the comparison between PE and
GE measures is extremely complicated. The main reason is that these two
measures do not in general equal the area to the left of the compensated
transportation demand curve as shown in section 3. Unlike in the MD case,
the general equilibrium measure can be larger than the partial equilibrium
measure if the compensating variation or the compensating surplus is used as
the welfare measure.13 This result is caused by the fact that the effect of the
induced change in the relative price cannot be ignored in these cases. In the
compensating variation case, there is another complication that the uncom-
pensated demand functions are used to derive the equilibrium prices
although the welfare change is evaluated by the compensated demand
functions.

6. Conclusions

We have examined three benefit measures of a large transportation
improvement, i.e., Marshall–Dupuit consumer's surplus (MD), compensating
variation (CV), and compensating surplus (CS), in a general equilibrium
framework. In a partial equilibrium framework, these consumer's surplus
measures can be expressed as a change in the area to the left of a suitably
defined transportation demand curve (either an uncompensated or a com-
pensated demand curve). We showed that neither CV nor CS in general has
such a representation in a general equilibrium situation, although MD does.
Even in the MD case, however, the appropriate demand curve is not a usual
demand curve but the general-equilibrium transportation demand locus.

Such a representation would be possible, if the effects of induced changes
in prices other than the transportation price cancelled out each other.
Consider, for example, an induced rise in the price of a certain good. The rise
in price lowers the welfare levels of its demanders but raises those of
suppliers. If demand equals supply, the gains on the supply side and the
losses on the demand side cancel out each other when measured in pecuniary
terms. The compensated demand curves used in CV and CS, however, do not
satisfy market clearing conditions except at the initial point, and they cannot
be reduced to a change in the area to the left of a transportation demand
curve.

Although in general the induced change in prices cannot be ignored in the

12Since the MD measure depends both on the path of integration and the choice of
numeraire, this result naturally depends on both of them.
13See Proposition 5 of Kanemoto and Mera (1984) for the analysis of these cases.
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CS measure, there is an interesting special case where it can be ignored. If only the numeraire good is used as the surplus, CS equals a change in the area to the left of a compensated transportation demand curve.

Although MD has a practical advantage that it equals the change in the area to the left of the equilibrium transportation demand locus, it suffers from well-known problems such as path-dependency. CV is path independent and has more desirable properties than MD, but it cannot be reduced to the change in the area to the left of a transportation demand curve. CS might offer a better alternative, since it has all the desirable properties of CV as a welfare measure and can be reduced to the change in the area to the left of the compensated transportation demand curve by a suitable choice of numeraire.

Next, we showed that the general-equilibrium benefit measures can be approximated by functions of various price and income elasticities which can be empirically estimated. This approach provides an attractive alternative to the commonly used one which simulates the effects of a transportation project in an interregional general-equilibrium model.

This approximation method was also used to examine the magnitude of the Tinbergen multiplier which is defined as the ratio between the general-equilibrium benefit and the transportation cost reduction obtained for the initial traffic flow. In the MD case, the approximation of the multiplier is especially simple and it was shown that the multiplier is larger, the more price elastic the import demand is. It was also shown that the multiplier can be extremely large when both the absolute value of the price elasticity and the size of the improvement are large.

Finally, we compared the general-equilibrium (GE) benefit measures with their partial equilibrium (PE) counterparts. The two measures are different in two respects. First, the PE measure assumes that the incomes of consumers are constant, while in the GE measure the incomes are affected by a transportation improvement through a change in the production of transported goods. If transported goods are normal goods, this makes the GE measure larger than the PE measure. Second, the prices of goods and services other than transportation services are taken as fixed in the PE measure whereas their changes are fully taken into account in the GE measure. It is shown that the second aspect works in the opposite direction in the MD case: the GE measure tends to be smaller than the PE measure. In other cases, the results are ambiguous.

It is important to note that our analysis has been carried out in a first-best model with no price distortions. Although there is extensive literature on the cost-benefit analysis in a second-best economy [e.g., Solow (1973), Kanemoto (1977), Arnott (1979), Harberger (1971), Boadway (1975), Dasgupta and Stiglitz (1974), Bruce and Harris (1982), and Diewert (1983)], these studies consider only an infinitesimally small improvement. The extension of our
analysis to a distorted economy would be a fruitful direction of future research.

Appendix

A.1. Proof of Proposition 1

Combining the market clearing conditions (1) and the budget constraint (2) and noting that $T^j=0$ and $x_i^j=0$ yields the trade balance equation, $t^j_i(a)z^j_i(a)=t^j_x(a)p^*(a)c^j_x(a)$. The trade balance equation simplifies the multiplier as

$$M_V = \left[ \int_0^1 p^*(a)^{-\frac{1}{3}}z^j_i(a) da \right]/\left[ p^*(0)^{-\frac{1}{3}}z^j_i(0) \right].$$  \hspace{1cm} (A.1)

Using the uncompensated demand functions, we can rewrite the trade balance equation as $t^j_i(a)z^j_i(a)p^*(a) = t^j_x(a)p^*(a)z^j_x(1, t^j_x(a)p^*(a))$. Differentiating this equation with respect to $a$ yields $p^*(a) = -\delta(h/(1-ha))p^*(a)$, where $\delta \equiv (\xi_1-\xi_2)/(1+\xi_1+\xi_2)$. Under our assumption that $\xi_i$'s are constant, the solution to this differential equation is

$$p^*(a) = (1-ha)^\delta p^*(0).$$  \hspace{1cm} (A.2)

Next, $z^j_i(a)$ satisfies

$$z^j_i(a) = \bar{s}^2_{11}(-ht^0_1) + \bar{s}^2_{12}p^*(a)
= -\bar{\xi}_1(1-\delta)(h/(1-ha))z^j_i(0).$$

Hence, we obtain

$$z^j_i(a) = (1-ha)^{\bar{\xi}_1(1-\delta)}z^j_i(0).$$  \hspace{1cm} (A.3)

Substituting (A.2) and (A.3) into (A.1) yields

$$M_V = m(y^*(\xi_1, \xi_2), h).$$

where $y^*(\xi_1, \xi_2)$ and $m(y, h)$ are defined respectively by (13) and (14).

Now, $m(y, h)$ satisfies $\partial m(y, h)/\partial h = g(y, h)/(1+y)h^2$, where $g(y, h) \equiv h(1+y)(1-h)^y-1 + (1-h)^{1+y}$. Since $g(y, 0) = 0$ and $\partial g/\partial h = -h(y(1+y)(1-h)^{y-1},$ Taylor's Theorem ensures that there exists some $\tilde{h}$ between 0 and $h$ such that $g(y, h) = g(y, 0) + h \partial g/\partial h \equiv y(1+y)(1-h)^{y-1}$. Hence,

$$\partial m/\partial h = -(\tilde{h}/h)(1-h)^{y-1} \geq 0 \quad \text{as} \quad y \leq 0,$$
and by L'Hospital's Rule,
\[ \lim_{h \to 0} m(y, h) = 1. \]

The partial derivative of \( m(y, h) \) with respect to \( y \) is
\[ \frac{\partial m(y, h)}{\partial y} = -r(y, h)/h(1+y)^2, \]
where \( r(y, h) \equiv 1-(1-h)^{1+y}+(1+y)(1-h)^{1+y} \log(1-h). \)
Since \( r(-1, h) = 0 \) and \( \partial r/\partial y = (1+y)(1-h)^{1+y} \log(1-h)^2 \geq 0 \) as \( y \geq -1 \), we have \( r(y, h) > 0 \) for any \( y \) except \( y = -1 \). Hence \( \partial m/\partial y < 0 \) if \( y \neq -1 \). By L'Hospital's Rule, we also have
\[ \lim_{y \to -1} \frac{\partial m}{\partial y} = -(1-h)^{1+y} \log(1-h)^2/2h < 0, \]
and hence \( \partial m/\partial y < 0 \) for any \( y \).

Next, \( y^*(\xi_1, \xi_2) \) satisfies
\[ \frac{\partial y^*}{\partial \xi_i} = 2(\xi_j + \frac{1}{2})(1 + \xi_1 + \xi_2)^2 > 0, \quad j \neq i, \quad i = 1, 2, \quad j = 1, 2, \]
when the derivative exists. Although \( y^*( ) \) is not differentiable at \( \xi_1 + \xi_2 + 1 = 0 \), the Marshall–Lerner condition excludes this case and we obtain \( \partial m(y^*(\xi_1, \xi_2), h)/\partial \xi_i < 0, \quad i = 1, 2. \)

A.2. Proof of Corollary 1

The corollary follows directly from \( \partial m/\partial \xi_i < 0, \quad i = 1, 2. \)

A.3. Proof of Proposition 2

First, we obtain the multiplier in terms of the compensating variation. The derivative of \( z_1(a) = s_1^2(t_1^*(a), p^*(a), u^*(a)) \) with respect to \( a \) is
\[ z_1'(a) = s_1^2(-ht_0^0) + s_1^2 p^*(a) \]
\[ = -\sigma_1^2(1-\delta)(h/(1-ha))z_1(a). \]

Hence, in the constant elasticity case we have \( z_1(a) = z_1(0)(1-b)^{a_1^2(1-\delta)}. \) In the same way, we obtain \( z_2(a) = z_2(0)(1-ha)^{a_2^2(1+\delta)}. \) Define \( s_1(a) = s_1^2(p^*(a), u^*(0)). \) Then it is easy to see that \( s_1^2(0) = s_1^2(0)(1-ha)^{-a_1^2(1-\delta)}, \)
\[ s_2^2(a) = s_2^2(0)(1-ha)^{-a_2^2(1+\delta)}. \]
Substituting these equations and (A.2) into (9) and noting \( t_2^0 \tilde{x}_1(0) = t_2^0 p^*(0) \tilde{x}_2(0) \), \( \tilde{s}_1(0) = -t_1^0 \tilde{x}_1(0) \), and \( \tilde{s}_2(0) = -t_2^0 \tilde{x}_2(0) \) yields \( M_C \) in the proposition.

Next, in the compensating surplus case, the equilibrium price ratio satisfies
\[
t^*_2(a)s_1^2(t^*_1(a), p^*(a), u^2(0)) = t_2^*(a)p^*(a)s_2^1(1, t_2^*(a)p^*(a), u^1(0)).
\]
Hence, \( p^*(a) = \rho(h/(1-ha)p^*(a) \) and \( p^*(a) = (1-ha)p^*(0) \). Since the compensating surplus would be the same as the compensating variation if the equilibrium price ratio were replaced, the multiplier \( M_C \) is obtained when \( \delta \) in \( M_C \) is replaced by \( \rho \). \( M_B \) in the proposition then follows from
\[
\sigma_2^1(1+\rho) + \frac{1}{2} \rho = \sigma_2^1(1+\rho) - \frac{1}{2} \rho = y^*(\sigma_2^1, \sigma_2^2).
\]

A.4. Proof of Proposition 3

The partial derivative of \( H(a, p) \) with respect to \( p \) is
\[
\frac{\partial H(a, p)}{\partial p} = -hp^{-3/2}[t_1^0 s_1^2(\xi_1 + \frac{1}{2}) - (t_2^0 p s_2^1)(\xi_2 + \frac{1}{2})].
\]

Since by the trade balance equation we have \( t_1^0 s_1^2 = t_2^0 p s_2^1 \) at \( p = p^*(a) \), the partial derivative becomes
\[
\frac{\partial H(a, p)}{\partial p} = -hp^{-3/2}(t_1^0 s_1^2) [\xi_1 - \xi_2] \geq 0 \quad \text{as} \quad \xi_1 \leq \xi_2,
\]
at \( p = p^*(a) \).

Now, suppose \( \xi_1 < \xi_2 \) for any \( a \). Then, \( p^*(a) < 0 \) for any \( a \) and hence \( p^*(a) < p^*(0) \) for any positive \( a \). Since \( t_1^0 s_1^2 = t_2^0 p s_2^1 \) at \( p = p^*(a) \), the generalized stability condition yields
\[
t_1^0 s_1^2((1-ha)t_1^0, p) \geq t_2^0 p s_2^1(1, (1-ha)t_2^0(p) \text{ at any } p \geq p^*(a).
\]
Hence, noting that \( \xi_1 < -\frac{1}{2} \) from \( \xi_1 < \xi_2 \) and \( \xi_1 + \xi_2 + 1 < 0 \), we obtain
\[
\frac{\partial H}{\partial p} \geq -hp^{-3/2}(t_2^0 p s_2^1) [\xi_1 - \xi_2] > 0
\]
for any \( p \geq p^*(a) \). Thus, \( H(a, p^*(0)) > H(a, p^*(a)) \) for \( a > 0 \), which implies \( V^* < V^p \).

The case where \( \xi_1 > \xi_2 \) for any \( a \) can be proven in the same way.

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