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A Novel Downside Risk Measure and Expected Returns

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Abstract

Several studies have found that the cross-section of stock returns reflects a risk premium for bearing downside risk; however, existing measures of downside risk have poor power for predicting returns. Therefore, this paper proposes a novel measure of downside risk, the ES-implied beta, to improve the prediction of the cross-section of asset returns. The ES-implied beta explains stock returns over the same period as well as the widely used downside beta, but also has strong predictive power over future returns. In the empirical analysis, although the widely used downside beta shows a weak relation with future expected returns, the ES-implied beta implies a statistically and economically significant risk premium of 0.5 percent per month. The predictive power of the ES-implied beta is not explained by the cross-sectional effects from the CAPM beta, size, book-to-market ratio, momentum, coskewness, cokurtosis or liquidity beta, nor does it depend on the design of the empirical analysis.

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1 Introduction

If an asset covaries strongly with the market in a declining market, this asset is undesirable and demands a risk premium, since it provides low payoffs precisely when investors are poor and protection is most needed. The covariation between asset returns and market returns when the market declines is estimated by the downside beta. Existing downside betas are found to relate to the cross-section of asset returns over the same period, but have limited power for predicting future cross-sectional returns. Thus this paper proposes a downside beta to improve the prediction of the cross-section of asset returns.

According to the capital asset pricing model (CAPM), introduced by Sharpe (1964), Treynor (1961), Lintner (1965a, 1965b), Mossin (1966) independently, the expected return of an asset in excess of the risk-free rate is determined by the expected excess return of the market and the covariation between asset returns and market returns. The CAPM beta is used to estimate the covariation between asset returns and market returns. Stocks bearing low beta covary little with the market and help smooth consumption (Lucas & Robert, 1978; Stiglitz, 1970); yet stocks having high beta covary strongly with the market and induce uncertainty, thus having to provide high expected returns to entice buyers.

The CAPM beta, however, is based on the mean-variance analysis in Markowitz (1952), and is criticized for treating downside losses and upside gains as equivalent, since investors care more about downside losses than upside gains in reality. Actually, in the same year as the mean-variance analysis is developed, Roy suggests minimizing only extreme losses for portfolio selection. Even Markowitz himself advocates using only low returns later. In Markowitz (1959), he wrote, "Analyses based on S (semi-variance) tend to produce better portfolios than those based on V (variance)" (p. 194). More recently, Gul (1991) develops a disappointment aversion model to allow agents to place greater weight on downside than on upside in their utility functions. This model is further extended by Routledge and Zin (2010) and applied by Faragó, Dahlquist, and Tédongap (2015) in asset allocation.

By focusing on low returns, downside betas offset the problem of the CAPM beta not distinguishing between downside losses and upside gains. Bawa and Lindenberg (1977) include the CAPM beta as a special case of the downside beta in a mean-lower partial moment framework so that the downside beta explains returns at least as well as the CAPM beta, which is known as performing poorly in predicting future stock returns [See Fama and French (1992) for example]. The predictive power of this downside beta, however, is also dubious. Jahankhani (1976) fails to find any improvement of downside beta over the CAPM beta when predicting returns of portfolios sorted by the CAPM beta. Although Estrada (2002) shows that downside beta explains the variability in the cross-section of asset returns in emerging markets, and Lettau, Maggiori, and Weber (2014) demonstrate the explanatory power of the downside beta for pricing the cross-section of currency returns, they use the whole sample period to compute downside betas and introduce look-ahead bias. Ang, Chen, and Xing (2006) identify a significantly positive relation between the downside beta and cross-sectional returns over the same period, but they find that the downside beta could not predict cross-sectional returns with high volatility. Therefore, a novel measure of downside risk that has predictive power for expected returns is of high demand.

This paper proposes an alternative measure of downside risk based on the expected shortfall (ES): the ES-implied beta. The ES measures the average of losses in extreme cases. This ES-implied beta exhibits high persistence in the empirical analysis, i.e., the ES-implied beta in the past predicts the ES-implied beta in the future. As a result, the ES-implied beta has predictive power for returns in the future as long as there is a significant relation between returns and the ES-implied beta over the same period.

To study the role that the ES-implied beta plays in pricing the cross-section of asset returns, this paper starts from analyzing the relation between average returns and the ES-implied beta over the same period. The performance of the ES-implied beta is compared to a downside beta conditional on market returns falling below the average of the market. This downside beta, referred to as the usual downside beta, is used most widely in the literature. If downside beta is a risk characteristic, there should be a pattern between returns of the portfolios formed by high downside beta and low downside beta. The empirical analysis shows that returns of portfolios sorted by either the ES-implied beta or the usual downside beta over the same period increase monotonically from low to high.

To have predictive power over returns in the future, downside betas also need to be persistent. Since the CAPM beta captures variation on both the downside and upside, it is correlated with downside betas. Therefore, this paper focuses on using relative downside betas by subtracting the CAPM beta from downside betas. While the relative usual downside beta in the past explains 3% of the variability in the relative usual downside beta, the relative ES-implied beta in the past explains 26% of the variability in the relative ES-implied beta. Other variables together explain 4% and 6% of the variability in the relative usual downside beta and the relative ES-implied beta, respectively. Therefore, the relative ES-implied beta is mainly explained by its lagged value and is highly persistent.

Considering that the relative ES-implied beta has a strong relationship with expected returns over the same period and is highly persistent, the relative ES-implied beta is likely to predict expected returns in the future. To study the performance of the relative ES-implied beta, I use two strategies, one by sorting portfolios according to downside betas in the past, and the other by running Fama-MacBeth regressions to estimate the risk premium directly. As the relative usual downside beta increases, returns of portfolios sorted by it decrease first and then increase. The spread between the highest and lowest portfolios, although significant at 5% using two-tailed tests, is only 0.2% per month and becomes substantially less significant after adjusting for other risk factors. In contrast, returns sorted by the relative ES-implied beta increase monotonically from low portfolio to high portfolio. The spread between the highest and lowest portfolios is 0.5% per month and remains highly significant after adjusting for other risk factors. Results of Fama-MacBeth regressions also indicate that the ES-implied beta has stronger predictive power over future returns than the relative usual downside beta does. The relative usual downside beta implies a risk premium of 0.2% when there is no other control and becomes insignificant after controlling for other risk factor loadings and characteristics. In comparison, the relative ES-implied beta implies an economically and statistically significant risk premium of 0.6% per month when used alone. After controlling for other predictors, although this risk premium decreases to 0.2%, it is still highly significant. Thus the relative ES-implied beta has stronger and more significant predictive power over future returns than the past relative usual downside beta does.

This paper further examines the relation between the predictive power of the relative ES-implied beta and other variables. I start from examining the relation between the relative usual downside beta and the relative ES-implied beta by double sorting portfolios. The relative ES-implied beta generates a significant spread within all quintiles sorted by the relative usual downside beta, whereas the relative usual downside beta only generates a significant spread within some quintiles sorted by the relative ES-implied beta, suggesting that the predictive power of the relative ES-implied beta is robust after controlling for the relative usual downside beta. Since the literature documents a high correlation between the usual downside beta and coskewness, I also check the predictive power of downside betas by controlling for coskewness. The relative usual downside beta only has predictive power for stocks in the first and second lowest coskewness-sort quintiles, while the relative ES-implied beta has predictive power for stock returns within all coskewness-sort quintiles. Therefore, the predictive power of the relative ES-implied beta is robust after controlling for coskewness, while the predictive power of the relative usual downside beta is limited to stocks with low coskewness. Moreover, I investigate the predictive power of the ES-implied beta after controlling for size, cokurtosis and volatility, exclusively. On average, the relative ES-implied beta implies a significant risk premium, but the risk premium is higher for stocks with small size, low cokurtosis and high volatility. Last but not least, I conduct several robustness checks to show that the predictive power of the relative ES-implied beta is not driven by the sample used in the main analysis nor the breakpoints used to construct portfolios.

This research relates to two strands of the literature. First, the ES-implied beta involves a dependence measure focusing on tails, the ES-implied correlation, into asset pricing. The ES-implied correlation is a development of the VaR-implied correlation and is proposed by Liu (2019), where the ES-implied correlation is found to be more trustworthy and stable than the VaR-implied correlation in simulations and empirical analysis. The VaR-implied correlation is pioneered by Campbell, Koedijk, and Kofman (2002). They use it to test the normality assumption. Cotter and Longin (2011) investigate the impact of portfolio weights, the type of position, data frequency, and the probability level on the VaR-implied correlation by using the US and the UK equity indexes. Mittnik (2014) extends the pairwise method used in these papers to joint estimation.

Second, this research contributes to measures of downside risk. As early as 1974, Hogan and Warren have pioneered a downside beta based on semi-deviation. Bawa and Lindenberg (1977) generalize Hogan and Warren's work and propose a downside beta conditional on the risk-less rate. Harlow and Rao (1989) extend the downside beta conditional on any arbitrary threshold in a generalized mean-lower partial moment framework. Estrada (2002), Ang et al. (2006) and Lettau et al. (2014) apply downside betas with different thresholds in predicting the cross-section of equity returns in emerging markets, stock returns in the United States, and international currency returns, respectively.

This paper proceeds as follows. Section 2 presents the methodology of the ES-implied beta and the model. Section 3 introduces the data. Section 4 studies the contemporaneous relation between asset returns and the relative ES-implied beta. Section 5 examines persistence and predictive power of the relative ES-implied beta. Section 6 concludes.

2 Methodology

2.1 Downside betas

This section begins from introducing the most widely used downside beta in the literature and then presents the ES-implied beta.

2.1.1 Existing downside betas

Various downside betas exist in the literature. The main difference is in the choice of thresholds, which determines the definition of downside. For example, Hogan and Warren (1974) and Bawa and Lindenberg (1977) investigate a downside beta conditional on risk-less rate. Estrada (2002) defines a downside beta conditional on asset returns and market returns falling below their own averages, respectively. Lettau et al. (2014) use a downside beta conditional on one standard deviation below the sample mean of market returns. Harlow and Rao (1989) demonstrate that the empirical data favour the downside beta conditional on the average market return rather than the one conditional on the risk-less rate. Ang et al. (2006) also use the downside beta conditional on the average of market returns. Galagedera (2007) proves that when the data are generated according to the CAPM model, the downside beta conditional on average market returns is numerically equal to the CAPM beta. As the beta conditional on the average market return is used most widely, this paper uses it as a benchmark and refers to it as the usual downside beta.

The formula of the usual downside beta is

$$\beta_{ed}^{-} = \frac{cov(r_i, r_m | r_m < \bar{r}_m)}{var(r_m | r_m < \bar{r}_m)}, \tag{1}$$

where r_i denotes returns of asset i, r_m denotes market returns, and $\bar{r}_m$ is the average of market returns. The usual downside beta can also be estimated by running the following regression,

$$r_i = a^- + \beta_{ed}^- r_m + \varepsilon_i^-, \quad \text{for} \quad r_m \leq \bar{r}_m.$$

2.1.2 The ES-implied beta

Notice that the CAPM beta is a product of the linear correlation and the relative volatility of the asset return to the market return, that is,

$$\beta_0 = \frac{cov(r_i, r_m)}{var(r_m)} = \rho_0 \frac{\sigma_i}{\sigma_m}, \tag{2}$$

where ρ_0 is the usual linear correlation, σ_i and σ_m correspond to the volatility of asset i and the market, respectively. Correlation reflects the dependence between market returns and asset returns. The higher the dependence, the riskier the asset. The ratio of asset volatility to market volatility assesses the risk of the asset relative to the market. However, neither the linear correlation nor the volatility ratio distinguishes among market situations, leading the CAPM beta not to reflect downside risk. Since many empirical studies have found that dependence between asset returns usually increases during market downturns and decreases during market upturns [see e.g., Ang and Bekaert (2002), "longinextreme2001" (n.d.) and Patton (2006)], it is natural to replace the linear correlation with a correlation measure that focuses on tails, such as the ES-implied correlation.

The ES-implied correlation is based on ES. The traditional definition of ES at probability level α is the average of returns falling below the α-quantile q_α, i.e.,

$$ES_\alpha = E(r | r \leq q_\alpha) \tag{3}$$

when the return distribution is continuous.[1]

Recall that the linear correlation between assets 1 and 2 could be calculated from

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2 w_1 w_2 \rho \sigma_1 \sigma_2, \tag{4}$$

[1] When the distribution is discontinuous, $ES(r)_\alpha = \frac{E(rI(r \leq q_\alpha)) + q_\alpha(\alpha - Pr(r \leq q_\alpha))}{\alpha}$, where $I()$ is an indicator function. See Acerbi and Tasche (2002) and McNeil, Frey, and Embrechts (2005).

where σ_p is the standard deviation of a portfolio composed of assets 1 and 2 with weights w_1 and w_2, $w_1 + w_2 = 1$.

Since standard deviations are constant across distributions, the linear correlation is also constant and fails to capture the increase in correlations when the market declines. Considering that ES can characterize risk in the tails, using ES to substitute standard deviations in equation (4) would yield a correlation measure varying across distributions.

ES is linked to standard deviations based on the assumption that returns of asset i belong to the location-scale family and are characterized by a location parameter μ_i, a scale parameter σ_i and a zero-location, unit-scale distribution F_{Z_i}, referred to as the standard distribution,

$$r_i = \mu_i + \sigma_i Z_i, \tag{5}$$

where Z_i follows the standard distribution F_{Z_i}.

Given a probability level α, the ES of asset i is

$$ES_{i,\alpha} = \mu_i + \sigma_i ES(Z_i)_\alpha, \tag{6}$$

where $ES(Z_i)_\alpha$ denotes the ES of the standard distribution of asset i at α-quantile.

Substituting equation (6) into (4) and dropping the ESs of standard distributions by following Campbell et al. (2002), Cotter and Longin (2011), and Mittnik (2014) results in the ES-implied correlation

$$\rho_{ES,\alpha} = \frac{(ES_{p,\alpha} - \mu_p)^2 - w_1^2(ES_{1,\alpha} - \mu_1)^2 - w_2^2(ES_{2,\alpha} - \mu_2)^2}{2w_1 w_2 (ES_{1,\alpha} - \mu_1)(ES_{2,\alpha} - \mu_2)}. \tag{7}$$

The ES-implied correlation is computed based on the fact that the difference between the ES of the portfolio and the weighted sum of ESs of the individual assets conveys the information of the correlation between the assets. Intuitively, the ES of a portfolio is determined by ESs of the individual assets and their correlation. If correlation does not change across probability levels in the return distribution, the ES of the portfolio minus the weighted sum of the individual ESs is constant everywhere. However, if correlation increases at a probability level, ES of the portfolio will exceed the weighted sum of individual ESs at that probability level. Reversing this process results in the ES-implied correlation.

When asset returns follow multivariate normal distribution, ESs of the standard distributions of assets and the portfolio are identical at every probability level. Equation (7) is equivalent to the linear correlation. When asset returns do not follow multivariate normal distribution, equation (7) provides a correlation measure that reflects standard distributions of assets and the portfolio.

The ES-implied correlation could be viewed as the linear correlation ρ_0 plus the dependence that is underestimated or overestimated by the linear correlation. When asset returns follow the multivariate normal distribution, the difference between the ES-implied correlation and the linear correlation, referred to as the relative correlation, is 0.

Based on the ES-implied correlation, the ES-implied beta is defined as follows:

$$\beta_{ES,\alpha} = \rho_{ES,\alpha} \frac{\sigma_i}{\sigma_m}. \tag{8}$$

where $\rho_{ES,\alpha}$ is the dependence between an individual asset and the market at α-quantile. When returns follow a multivariate normal distribution, $\rho_{ES,\alpha}$ is identical to the linear correlation, and the ES-implied beta therefore equals the CAPM beta.

To estimate the ES-implied beta, I first estimate ES using the empirical distribution of returns, then calculate the ES-implied correlation according to equation (7) and the ES-implied beta according to equation (8). The weights $w_1 = 0.5$ and $w_2 = 0.5$ are used for computing the ES-implied correlation. Liu (2019) develops several test statistics and demonstrates that the choice of weights does not have significant impact on the ES-implied correlation.

Similar to existing downside betas, the ES-implied beta needs a threshold to define downside. The question of which threshold is optimal is hard to answer, since how investors set the threshold is unknown. This paper uses 50%-quantile to include as many observations on the downside as possible.[2]

2.2 Model

This paper employs the Fama-MacBeth regression (Fama & MacBeth, 1973) to explore the relation between the ES-implied beta and cross-sectional returns. The Fama-MacBeth regression divides the whole sample period into several sub-periods and conducts cross-sectional ordinary least squares (OLS) for each sub-period. The risk premia then are the time-series averages of the cross-sectional estimators. Standard deviations of the risk premia are the dispersion of the time-series estimates. By deriving the standard deviation from the variation in the estimates, the Fama-MacBeth regression avoids the non-trivial procedure of correcting cross-sectional correlation in one single cross-sectional regression with the full sample. The Fama-MacBeth regression is very easy to implement and consists of two stages. The first stage involves calculation of risk factor loadings and characteristics for each sub-period. The second stage regresses cross-sectional returns on the factor loadings and risk characteristics calculated in the first stage for each sub-period, i.e.,

$$R_t = B_t \Lambda_t + \Xi_t, \tag{9}$$

[2]Robustness checks concerning alternative threshold values are not included but available from the author.

where $R_t = [\bar{r}_1, ..., \bar{r}_N]'_t$ is a vector of average excess returns within sub-period t, $B_t = [b_1, ..., b_k]_t$ is a $N \times k$ matrix containing k risk characteristics $b_1, ..., b_k$ computed in stage 1, $\Lambda_t = [\lambda_1, ...\lambda_k]'_t$ are the risk premia of the k factors at sub-period t and Ξ_t is a vector of residuals.

For a risk characteristic b_h with estimated risk premia $\hat{\lambda}_{h_1}, ..., \hat{\lambda}_{h_T}$ from sub-periods 1 to T, the estimate of its risk premium is $\hat{\lambda}_h = \frac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_{h_t}$ with a standard deviation $\hat{\sigma}(\hat{\lambda}_h) = \frac{1}{T}\sqrt{\sum_{t=1}^{T}(\hat{\lambda}_{h_t} - \hat{\lambda}_h)^2}$. The t-statistic for testing the significance of the risk premium then is $t(\hat{\lambda}_h) = \frac{\hat{\lambda}_h}{\hat{\sigma}(\hat{\lambda}_h)}$.

In this paper, I focus on examining the predictive power of the downside beta on the part of returns that the CAPM beta could not explain. Thus for each period, I control for the CAPM beta and regress expected returns on the CAPM beta, and the relative downside beta in the past,[3]

$$\bar{r}_t = \lambda_{0t} + \lambda_t \beta_{0,t-1} + \lambda_t^-(\beta^- - \beta_0)_{t-1}, \tag{10}$$

where $\bar{r}_t$ is the average return of assets in excess of the risk-free rate during sub-period t, $\beta_{0,t-1}$ is the CAPM beta in the past, $(\beta^- - \beta_0)_{t-1}$ is the relative downside beta in the past, $\lambda_{0t}, \lambda_t, \text{and} \lambda_t^-$ are the constant, and risk premia of the CAPM beta and the relative downside beta, respectively.

The model reduces to the CAPM when the relative downside risk does not price the cross-section of asset returns, i.e., the risk premium on the relative downside beta $\lambda_t^- = 0$, or the relative downside risk is 0.

In another regression, I control for size, book-to-market ratio, returns, coskewness, cokurtosis and liqudity beta in the past. Fama and French (1993) indicate that besides the CAPM beta, size and value also explain the cross-section of stock returns as the empirical analysis demonstrates that small-cap stocks and value stocks tend to have high expected returns. The effect of liquidity is examined in the paper of Pástor and Stambaugh (2003), where stocks with higher sensitivities to liquidity exhibit higher expected returns than stocks with lower sensitivities. Using the strategy of buying past winners and selling past losers, Jegadeesh and Titman (1993) show that stocks which provide high payoffs in the last period keep outperforming stocks that provide low payoffs. Friend and Westerfield (1980) show that coskewness affects asset returns, and Fang and Lai (1997) find the same for cokurtosis.

3 Data description

This section introduces the data used in the subsequent analysis and checks correlations between different variables.

[3] The model used in this paper differs from the dual-beta model [see, e.g., Howton and Peterson (1998), Faff (2001) and Price, Price, and Nantell (1982)] in that this paper estimates the risk premium on downside risk directly while the dual-beta model splits the full sample into two subsamples according to whether excess returns are positive or negative and then estimates the risk premium on each subsample.

This paper uses daily returns of stocks listed on the NYSE, AMEX and NASDAQ with share code 10 and 11 from July 1963 to December 2017 to compute downside betas. The data are obtained from the Center for Research in Security Prices (CRSP). Stocks with prices less than five dollars are excluded. In order to ensure enough observations, stocks must have at least 100 observations in the year that downside betas are constructed. All returns reported in this paper are excess monthly returns, where risk-free returns are approximated by the one-month Treasury Bill rate provided by Ibbotson Associates. The proxy for the market return is the value-weighted return of all stocks included in CRSP.

Firm characteristics are obtained from COMPUSTAT. Size is computed as the log of total assets. For computing liquidity beta, this paper obtains Pástor and Stambaugh liquidity risk factor from Professor Luboš Pástor's website. To compute risk-adjusted returns, this paper also acquires the Fama-French three factors, five factors and the momentum factor from Professor Ken French's website. coskewness and cokurtosis are defined as

$$\text{coskewness} = \frac{E[(r_i - \mu_i)(r_m - \mu_m)^2]}{\sqrt{var(r_i)}var(r_m)} \tag{11}$$

and

$$\text{cokurtosis} = \frac{E[(r_i - \mu_i)(r_m - \mu_m)^3]}{\sqrt{var(r_i)}var(r_m)^{3/2}}, \tag{12}$$

respectively.

Panel A in Table 1 reports summary statistics of various betas, i.e., CAPM beta, usual downside beta "β_{ed}^-", ES-implied beta "β_{es}^-", relative usual downside beta "Relative β_{ed}^-" and relative ES-implied downside beta "Relative β_{es}^-". Columns "Mean", "SD", "Q1", "Median", and "Q3" present their mean, standard deviation, first quartile, median, and third quartile, respectively. The CAPM beta on average is less than 1, suggesting that the sample in this paper overall is less correlated with the market and the majority may be small firms. The usual downside beta and the ES-implied downside beta are larger and more volatile than the CAPM beta. Subtracting the CAPM beta, the relative usual downside beta is centered around 0. In contrast, the relative ES-implied beta is still significantly positive. Remember that the ES-implied beta should equal the CAPM beta if returns follow normal distribution. Therefore, the summary statistics of the relative ES-implied beta imply that stock returns diverge from normal distribution.

Panel B in Table 1 reports correlations among betas and their correlations with stock returns "Return", Size, book-to-market ratio "BM", coskewness, cokurtosis, and liquidity beta β_L. Considering that a high downside beta always implies a high CAPM beta, it is not surprising to see a high correlation of 0.8 between the CAPM beta and the usual downside beta and a correlation of 0.7 between the CAPM beta and the ES-implied beta. Subtracting the CAPM beta, the relative usual downside beta and the relative ES-implied beta are little correlated with the CAPM beta. Therefore, it is better to use relative downside betas in the analysis to prevent the predictive power of the CAPM beta being affected by downside betas. Consistent with Ang et al. (2006), this paper finds a high correlation between the relative usual downside beta and coskewness with a coefficient of -0.3. In comparison, the relative ES-implied downside beta is not correlated with coskewness. Noticing that the relative ES-implied downside beta is mildly correlated with size and cokurtosis, Section 5.5 examines the predictive power of the relative ES-implied beta after controlling for size and cokurtosis exclusively.

4 Downside risk and realized returns

Since Ang et al. (2006) document that the usual downside beta has strong predictive power for returns over the same period, but not for returns in the future, this paper presents the explanatory power of downside betas for returns over the same period first to compare the in-sample performance with the out-of-sample performance later. To investigate the explanatory power of downside betas, I sort stocks into quintile portfolios. If a downside beta determines the price of assets, there should be a pattern in the portfolio returns. Portfolios sorted by higher relative downside beta should have higher average returns. On the contrary, if the downside beta is not priced or does not characterize risk correctly, there would be no significant difference in returns across the portfolios.

I use rolling windows when sorting portfolios. Each month, I sort portfolios based on betas computed over a horizon. There is a trade-off in choosing the horizon. On the one hand, a large sample size is needed in order to have enough observations on the downside. On the other hand, too long a horizon is impossible to capture time-varying betas. Following Ang et al. (2006) and Fama and French (2006), this paper uses 12 months as the horizon. Since betas are computed at one-year horizon and portfolios are formed every month, there exist some overlapping periods. Therefore, the reported t-statistics are adjusted using Newey and West (1987) heteroskedastic-robust standard errors with 12 lags.

The detailed procedure used for forming portfolios is as follows.

(1) At the beginning of each month, estimate the CAPM beta and downside betas using daily returns in the next 12 months.

(2) Sort stocks into quintile portfolios according to the beta computed above.

(3) Calculate returns of each quintile over the same period and report the average of portfolio returns.

Table 2 reports the time-series averages of excess returns of portfolios sorted by different betas, i.e., CAPM beta "β_0", usual downside beta "β_{ed}^-", ES-implied downside beta "β_{es}^-"", relative usual downside beta "Relative β_{ed}^{-}" (computed as $\beta_{ed}^{-} - \beta_0$), and relative ES-implied downside beta "Relative β_{es}^{-}" (computed as $\beta_{es}^{-} - \beta_0$). Columns "Low" to "High" present returns over the same period that are converted to monthly. Column "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio. Column "t-stat" reports associated t-statistics. Panel A and Panel B report returns of equal-weighted portfolios and value-weighted portfolios, respectively. Returns sorted by all betas increase monotonically from low portfolio to high portfolio. The CAPM beta, which is known to have poor predictive power, also shows a significant relation with returns over the same period. However, this does not guarantee that CAPM holds since CAPM implies that only the CAPM beta can explain expected returns. Downside betas generate a more significant spread between the lowest and highest portfolios than the CAPM beta does. Subtracting the CAPM beta, the relative usual downside beta and the relative ES-implied beta still generate a highly significant spread around 0.7% per month.

5 Predictive power of downside risk

5.1 Determinants of future downside risk

If a downside beta could be explained by other factors, then the downside beta is replicable and has little value. To the contrary, if the downside beta is mainly explained by its own value in the past, then the downside beta is persistent and is likely to have significant predictive power for future returns when there is a strong relationship between the downside beta and returns over the same period.

Table 3 reports results of predicting the relative usual downside beta and the relative ES-implied beta. Models (1) and (2) report the regression results of the relative usual downside beta and the relative ES-implied beta on their own one-lag values, respectively. Although both the relative usual downside beta and the relative ES-implied beta provide a significant prediction for their future values, the coefficient of the past relative ES-implied beta is more significant than the coefficient of the past relative usual downside beta economically and statistically. Accordingly, the past relative ES-implied beta explains 26% of the variability in the relative ES-implied beta, while the relative usual downside beta in the past explains only 3% of the variability in the relative usual downside beta. Models (3) and (4) control for size, book-to-market ratio "BM", return "Return", coskewness, cokurtosis, liquidity beta β_L, leverage, and return on assets "ROA" in the past 12 months. Together these variables explain 4% of the variability in the relative usual downside betas and 6% of the variability in the relative ES-implied beta. Compared to 26% of the variability explained by the past relative ES-implied beta, the predictive power of other variables for the relative ES-implied beta is minor. Results in models (3) and (4) also imply that stocks with smaller size and lower ROA in the past tend to have higher relative usual downside beta and relative ES-implied beta. Value stocks, illiquid stocks and stocks with lower returns, higher coskewness, and higher leverage in the past tend to have higher relative ES-implied beta.

In summary, the relative ES-implied beta is more persistent than the relative usual downside beta and cannot be explained by other variables.

5.2 Downside risk and future returns

Since there is a strong relationship between the ES-implied beta and returns over the same period and the ES-implied beta is highly persistent, the ES-implied beta is likely to have predictive power over returns in the future. This section examines the predictive power of downside betas by sorting portfolios and running Fama-MacBeth regressions.

5.3 Portfolios sorted by past relative downside betas

This section investigates the predictive power of betas over future returns by sorting portfolios. At the beginning of each month, stocks are sorted into quintile portfolios based on betas calculated using daily returns in the previous 12 months. The holding periods are one month and twelve months, respectively.

Table 4 reports average returns of holding the equal-weighted portfolios for one month across time. Row "Return" reports average returns in excess of the one-month Treasury-bill rate, row "CAPM alpha" reports risk-adjusted alphas from regressions of stock returns on CAPM beta, row "FF3 alpha" reports risk-adjusted alphas from regressions of stock returns on Fama and French (1996) 3 factors, row "FF3+Mom alpha" reports alphas from regressions of stock returns on Fama and French 3 factors and Momentum factor, and row "FF5 alpha" reports alphas from regressions of stock returns on the Fama and French (2015) 5 factors.

Panel A reports equal-weighted returns sorted by the CAPM beta. The flat returns across the portfolios indicate that the CAPM beta does not have significant predictive power for returns in the future. Panel B reports equal-weighted returns sorted by the relative usual downside beta $\beta_{ed}^{-} - \beta_0$. All returns decrease first and then increase. The t-statistics suggest that the relative usual downside beta generates a spread significant at 5% in unadjusted returns between the highest and lowest portfolios, but this predictive power becomes much less significant after controlling for other factors.

Panel C reports equal-weighted returns sorted by the relative ES-implied beta $\beta_{es}^{-} - \beta_0$. The unadjusted returns increase monotonically from low portfolio to high portfolio. A trading strategy that buys stocks with high relative ES-implied beta and sells stocks with low relative ES-implied beta in the former period generates a highly significant average return of 0.5% one month later.

The spread between the highest and lowest portfolios remains significant at 1% for risk-adjusted returns, suggesting that the predictive power of the relative ES-implied beta for future returns is robust after controlling for other risk factor loadings.

Table 5 reports the value-weighted returns sorted by the CAPM beta, the relative usual downside beta and the relative ES-implied downside beta for a holding period of one month, respectively. The value-weighted returns exhibit similar pattern as equal-weighted returns. There is still no significant difference between value-weighted returns sorted by the CAPM beta. Returns sorted by the relative usual downside beta decrease first and then increase, and the spread is significant at 5% only for unadjusted returns, while the relative ES-implied beta generates a spread significant for unadjusted and risk-adjusted returns.

I further investigate the predictive power of downside risk over returns in the next 12 months. Each month, I sort stocks into quintile portfolios based on a downside beta computed using daily returns in the previous 12 months and hold the portfolios for 12 months. Panel A and Panel B in Table 6 report equal-weighted and value-weighted monthly returns of portfolios sorted based on the relative usual downside beta "relative β_{ed}^{-}", respectively. Panel C and Panel D correspond to equal-weighted and value-weighted portfolios sorted by the relative ES-implied beta "relative β_{es}^{-}" in the previous 12 months. Similar to the results for one month holding period, the unadjusted returns of portfolios sorted based on past relative usual downside beta decrease first and then increase. The spread in the risk-adjusted returns, which is significant at 10% for one month holding period portfolios, now becomes trivial and insignificant. In contrast, unadjusted and risk-adjusted returns of portfolios sorted by the relative ES-implied beta increase monotonically from low portfolio to high portfolio. In addition, the spread between the highest and lowest portfolios is much higher in returns sorted by the relative ES-implied beta than in returns sorted by the relative usual downside beta. A trading strategy that buys the portfolio with the highest relative ES-implied beta and sells the portfolio with the lowest relative ES-implied beta on average generates an average return around 6% for a holding period of one year, while the same strategy using the relative usual downside beta leads to an annual return less than 2%.

5.4 Out-of-sample performance of Fama-MacBeth regressions

This section estimates the risk premium of downside betas using the Fama-MacBeth regression. At the beginning of each month, I regress returns in the month on the betas computed using daily returns in the past 12 months, and then compute mean and standard deviation of the estimates.

Table 7 reports the mean and t-statistics of the estimates. Model (1) reports the regression results on the CAPM beta. Consistent with the literature, the CAPM beta implies no significant risk premium to predict future returns. Models (2) and (3) report regression results on the relative usual downside beta and the relative ES-implied downside beta, respectively. While the relative usual downside beta implies a risk premium of 0.2% per month, the relative ES-implied downside beta implies a more economically and statistically significant risk premium of 0.6% per month. Models (4), (5) and (6) report regression results controlling for size, book-to-market ratio, returns, coskewness, cokurtosis, and liquidity beta in the previous 12 months. Small-cap stocks and value stocks tend to have significantly higher returns in the next period. Model (5) demonstrates that the relative usual downside beta fails to predict returns after controlling for other commonly used variables. In contrast, the relative ES-implied beta implies a significant risk premium around 0.2% per month. Model (7) runs a horse-race between relative usual downside beta and relative ES-implied downside beta. Due to the weak correlation between them, the results change little: the relative usual downside beta remains insignificant, while the relative ES-implied downside beta still implies a significant risk premium of 0.2%.

To sum up, the past ES-implied beta provides a significant risk premium and this risk premium is robust after controlling for other cross-sectional effects.

5.5 ES-implied beta and other risk characteristics

This section explores the relation between the predictive power of the relative ES-implied downside beta and other risk characteristics. I first study the relation between the relative ES-implied beta and the relative usual downside beta, then the relation between relative downside betas and coskewness, and finally the relation between relative ES-implied beta and other risk characteristics, including size, cokurtosis and volatility.

To study the relation between the relative ES-implied downside beta and other variables, I sort stocks based on the relative ES-implied downside beta after controlling for other cross-sectional effect. To be exact, each month stocks are first sorted into quintile portfolios based on the control variable in the past. Within each first-sort quintile, stocks are then sorted into additional quintiles based on relative ES-implied downside beta computed from daily returns in the previous 12 months. In this way, stocks are identical in terms of the control variable within each first-sort quintile. If the relative ES-implied downside beta's predictive power is different from the control variable's, it should be able to generate a significant spread within each first-sort quintile.

Table 8 presents the relation between the predictive power of the relative ES-implied beta and the relative usual downside beta. I first explore the predictive power of the relative usual downside beta after controlling for the relative ES-implied beta by sorting portfolios based on the relative ES-implied downside beta first and then the relative usual downside beta. Panel A reports the equal-weighted returns of the 25 portfolios. Column "Average" reports equal-weighted returns of stocks in each second-sort quintile. Row "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio and Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. The relative usual downside beta on average generates a significant spread of 0.18% and a spread significant at 5% within the first, second and fourth quintile sorted by the relative ES-implied beta. Panel B demonstrates that the relative ES-implied beta generates a significant spread at 1% within all quintiles sorted by the relative usual downside beta, suggesting that the predictive power of the relative ES-implied beta over returns prevails within all stocks regardless of their levels of relative usual downside beta. The results are similar for value-weighted returns and thus are not reported.

I then study the relation between the predictive power of relative downside betas and coskewness, since the literature documents a strong association between the predictive power of the relative usual downside beta and coskewness and Panel B in Table 1 shows a high correlation between the two. To find whether the predictive power of relative downside betas is affected by coskewness, I first sort stocks into quintile portfolios based on coskewness in the previous 12 months and then on the relative downside beta in the previous 12 months. Panel A and Panel B in Table 9 report equal-weighted returns of the 25 portfolios sorted by the relative usual downside beta and the relative ES-implied beta after controlling for coskewness, respectively. The relative usual downside beta only generates a significant spread within the first and second coskewness-sort quintiles, implying that the predictive power of the relative usual downside beta is limited to low coskewness stocks. In contrast, the relative ES-implied downside beta generates a highly significant spread around 0.5% within all coskewness-sort quintiles as Panel B in Table 9 shows. Therefore, while the relative usual downside beta only has significant predictive power over returns of stocks with low coskewness, the predictive power of the relative ES-implied downside beta over future returns is prevalent within stocks with any level of coskewness.

I further study the relation between the relative ES-implied downside beta and size, cokurtosis and volatility. Size and cokurtosis demonstrate a mild correlation with the relative ES-implied beta in Panel B of Table 1. As another measure of risk, volatility is also likely to explain the predictive power of the relative ES-implied beta over returns. Panel A in Table 10 reports equal-weighted returns of portfolios sorted by size first and then the relative ES-implied downside beta. On average, the relative ES-implied downside beta generates a significant spread of 0.2% between the highest and lowest portfolios after controlling for size. The overall decreasing spread from low size-sort quintile to high size-sort quintile suggests that the predicitve power of the relative ES-implied beta may be stronger for small-cap stocks than for large-cap stocks.

Panel B in Table 10 reports equal-weighted returns of portfolios sorted by cokurtosis first and then the relative ES-implied downside beta. On average, the relative ES-implied beta generates a significant spread of 0.45%. Within each cokurtosis-sort quintile, the relative ES-implied beta also generates a spread significant at 5% except in the highest cokurtosis quintile, where the spread is only significant at 10%. The spread also tends to be lower within high cokurtosis-quintile. The reason is that as a measure of the covariance between a stock's return and the cube of the market return, cokurtosis does not distinguish between upside and downside risk. A stock with low cokurtosis tends to have high returns when market returns are low or low returns when market returns are high. Low covariation during either market downside or market upside can lead to a stock bearing low cokurtosis, which means, a stock with low cokurtosis does not necessarily imply that the stock has low downside risk. In contrast, there is little asymmetry in a stock with high cokurtosis. A high-cokurtosis stock must have high covariation with market returns during both market downside and upside. Therefore, the stock must have high downside risk. The little variation in the relative ES-implied downside beta within the highest-cokurtosis quintile leads to low spread in portfolio returns within this quintile.

Panel C reports equal-weighted returns of portfolios sorted by volatility first and then the relative ES-implied downside beta. On average, the relative ES-implied downside beta generates a significant spread of 0.24%, but the spread is highest within the highest volatility quintile. The reason is that volatility treats upside and downside risk symmetrically. Since there is little asymmetry when volatility is low, low volatility implies low downside risk, hence a narrow spread in the relative ES-implied beta. On the other hand, high volatility could be caused by high variation during market downside or upside, leading to more variation in the relative ES-implied beta.

5.6 Robustness check

This section checks the robustness of the predictive power of the ES-implied beta over future returns to ensure the findings in the main analysis are driven by variation in the ES-implied beta rather than by testing methods. Table 11 reports portfolio returns sorted by the relative ES-implied downside beta in the previous 12 months using different settings. Instead of requiring stock price to be above $5, row "All stocks" includes all stocks. Row "NYSE stocks" sorts stocks listed on NYSE only to focus on large-cap stocks. Instead of assigning the same number of stocks into each portfolio, row "NYSE breakpoints" sorts portfolios using quintile breakpoints based on NYSE stocks so that small-cap stocks can fall into the same quintile. Panel A reports equal-weighted returns and Panel B reports value-weighted returns. In all cases, the relative ES-implied beta generates a significant spread. Without the restriction on price, the relative ES-implied beta generates an equal-weighted spread of 0.8% between the highest and lowest portfolios, even higher than the one in the main analysis. Recall that Panel A in Table 10 suggests that the predictive power of the relative ES-implied beta is stronger for small-cap stocks. Row "NYSE stocks" shows that when limiting to stocks listed on NYSE, the spread becomes smaller but is still significant. Using breakpoints based on NYSE stocks, the relative ES-implied beta still generates a highly significant spread of 0.36%. The value-weighted returns show the similar pattern. Therefore, the predicitive power of the relative ES-implied beta does not hinge on the empirical setting.

6 Conclusion

Downside risk plays a crucial role in asset pricing. As early as 1952, economists have realized that investors are more concerned about losses than gains. If the distribution of returns is symmetric, the analysis based on the whole distribution would be identical to the analysis based on low returns. However, it has been long recognized that return distribution is asymmetric. The recent literature, thereby, places the emphasis on low returns, and recommends the usage of downside beta, a beta conditional on returns falling below a threshold, in addition to the CAPM beta. Existing downside betas show a close relation to the cross-section of stock returns over the same period, but their predictive power over future returns is limited. Therefore, this paper proposes an alternative measure of downside beta using the expected shortfall, referred to as the ES-implied beta.

This paper compares the performance of the ES-implied beta with a downside beta that is widely used in the literature, calculated as covariation between asset returns and market returns conditional on market returns falling below market average and referred to as the usual downside beta. Consistent with the literature, the usual downside beta exhibits strong explanatory power for the cross-sectional returns over the same period. Returns of portfolios sorted by the relative usual downside beta over the same period increase monotonically from the low to high portfolios. The spread between the highest and lowest portfolios is 0.7% per month and highly significant. However, the relative usual downside beta demonstrates very limited power for predicting returns in the future. Returns of portfolios sorted by the relative usual downside beta in the past 12 months do not exhibit an increasing pattern. Furthermore, although the relative usual downside beta generates a spread of 0.2%, significant at 5%, between the highest and lowest portfolios in the unadjusted returns, it only generates a spread around 0.12% and significant at 10% in risk-adjusted returns. Results from Fama-MacBeth regressions also suggests that the predicitve power of the relative usual downside beta over futures is limited. The relative usual downside beta implies a significant risk premium of 0.2% when only controlling for the CAPM beta; however, this risk premium becomes insignificant after controlling for other variables.

In contrast, the performance of the relative ES-implied beta is relatively stable. Its in-sample performance is as good as the relative usual downside beta. The spread between the highest and lowest portfolios sorted by the relative ES-implied beta over the same period is 0.66% for equal-weighted portfolios and 0.61% for value-weighted portfolios. More importantly, the relative ES-implied beta is highly persistent and demonstrates strong predictive power over future returns. A trading strategy that buys stocks with high relative ES-implied beta and sells stocks with low relative ES-implied beta generates a highly significant return of 0.5% per month. Results from the Fama-MacBeth regressions show that the relative ES-implied beta implies a highly significant risk premium of 0.6% when controlling for the CAPM beta and remains significant after controlling for other variables.

A series of robustness checks is conducted to confirm that the predictive power of the relative ES-implied beta is not explained by other variables, nor does it depend on empirical settings. First, I examine the predictive power of the relative ES-implied beta and the relative usual downside beta after controlling for each other. While the relative usual downside beta only generates a significant spread within some quintiles sorted by the relative ES-implied beta, the relative ES-implied beta generates a significant spread within all quintiles sorted by the relative usual downside beta. I then examine the preditive power of the relative ES-implied beta and the relative usual downside beta after controlling for coskewness. Although the relative usual downside beta only generates a significant spread within stocks with low coskewness, the relative ES-implied beta generates a significant spread within all coskewness-sort quintiles. I also examine the predictive power of the relative ES-implied beta over future returns after controlling for size, cokurtosis and volatility, which are likely to capture the predictive power of the relative ES-implied beta. Results of double sorting portfolios indicate that risk premium on the relative ES-implied beta is overall prevalent among all stocks, although the premium appears to be stronger for small-cap stocks and stocks with low cokurtosis and high volatility. Last but not least, I change several empirical settings in the main analysis. The results suggest that the predictive power of the relative ES-implied beta is not driven by the sample nor the breakpoints used in the main analysis.

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Table 1: Data description

This table reports summary statistics of betas in Panel A and the correlation between betas and other variables in Panel B. Panel A presents mean, standard deviation, first quartile, median, and third quartile of CAPM beta, usual downside beta "β_{ed}^{-}", ES-implied beta "β_{es}^{-}", relative usual downside beta "Relative β_{ed}^{-}" and relative ES-implied downside beta "Relative β_{es}^{-}", respectively. The relative usual downside beta is computed as the difference between the usual downside beta and CAPM beta and the relative ES-implied beta is the difference between ES-implied beta and CAPM beta. Panel B presents correlations between betas and their correlation with stock returns "Return", Size, book-to-market ratio "BM", coskewness, cokurtosis, and liquidity beta β_L. CAPM beta, usual downside beta, ES-implied beta, coskewness, cokurtosis and liquidity beta are computed every month using daily returns of stocks listed on NYSE, AMEX and NASDAQ with prices above $5 in the past 12 months. Size is computed as the log of total assets and book-to-market ratio is book value of equity divided by market value of equity. The sample period for all variables except liquidity beta is from July 1963 to December 2017. For liquidity beta, the sample is from January 1968 to December 2017. The number of observations varies across time from 953 to 4662 stocks.

	Panel A: Summary statistics				
	Mean	SD	Q1	Median	Q3
β_0	0.85	0.59	0.41	0.80	1.21
β_{ed}^{-}	0.93	0.71	0.46	0.87	1.32
β_{es}^{-}	1.23	0.71	0.73	1.12	1.59
Relative β_{ed}^{-}	0.08	0.41	-0.14	0.05	0.28
Relative β_{es}^{-}	0.37	0.51	0.10	0.24	0.49

	Panel B: Correlations				
	β_0	β_{ed}^{-}	β_{es}^{-}	Relative β_{ed}^{-}	Relative β_{es}^{-}
β_0	1.00	0.81	0.71	-0.04	-0.02
β_{ed}^{-}	0.81	1.00	0.59	0.55	-0.11
β_{es}^{-}	0.71	0.59	1.00	-0.00	0.58
Relative β_{ed}^{-}	-0.04	0.55	-0.00	1.00	0.04
Relative β_{es}^{-}	-0.02	-0.11	0.58	0.04	1.00
Return	0.06	0.07	0.07	0.03	0.03
Size	0.13	0.01	-0.14	-0.17	-0.34
BM	-0.23	-0.19	-0.13	0.00	0.07
Coskewness	-0.02	-0.17	0.04	-0.27	0.08
Cokurtosis	0.24	0.20	0.03	0.00	-0.24
β_L	-0.01	-0.03	-0.04	-0.03	-0.04

Table 2: Stocks sorted by contemporaneous downside betas

This table reports time-series averages of returns sorted by CAPM beta "β_0", usual downside beta "β_{ed}^-", ES-implied beta downside "β_{es}^-"", relative usual downside beta "Relative β_{ed}^-" (computed as $\beta_{ed}^- - \beta_0$), and relative downside ES-implied beta "Relative β_{es}^-" (computed as $\beta_{es}^- - \beta_0$), respectively. Each month stocks are sorted into quintile portfolios based on one of the betas, which are computed using daily returns in the next 12 months. Panel A reports the equal-weighted returns in excess of the one-month Treasury-bill rate over the same period and Panel B reports the corresponding value-weighted returns. Column "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio. Column "t-stat" reports associated t-statistics. Portfolios are based on NYSE/AMEX/NASDAQ stocks with prices above $5 from July 1963 to December 2017. Returns are in percentage.

	Low	2	3	4	High	High-Low	t-stat
			Panel A: Equal-weighted				
β_0	0.82	0.89	0.97	1.12	1.65	0.84	3.88
β_{ed}^-	0.77	0.81	0.91	1.13	1.82	1.05	5.06
β_{es}^-	0.61	0.84	0.99	1.22	1.78	1.17	5.26
Relative β_{ed}^-	0.84	0.90	0.98	1.15	1.58	0.74	8.86
Relative β_{es}^-	0.84	0.91	1.04	1.17	1.49	0.66	7.69
			Panel B: Value-weighted				
β_0	0.79	0.83	0.88	1.00	1.46	0.68	3.29
β_{ed}^-	0.73	0.75	0.85	1.04	1.67	0.94	4.60
β_{es}^-	0.63	0.81	0.93	1.13	1.62	1.00	4.62
Relative β_{ed}^-	0.74	0.84	0.92	1.08	1.47	0.74	8.73
Relative β_{es}^-	0.78	0.87	0.98	1.09	1.39	0.61	6.88

Table 3: Modelling relative downside betas

This table reports results of Fama-MacBeth (1973) regressions of relative usual downside beta "Relative β_{ed}^{-}" and relative ES-implied downside beta "Relative β_{es}^{-}" on past firm characteristics and risk factors. In models (1) and (2), relative downside betas are regressed on CAPM beta "β_0" and their own values "Relative β^{-}" in the past 12 months, respectively. Models (3) and (4) control for size, book-to-market ratio "BM", return "Return", coskewness, cokurtosis, liquidity beta β_L, leverage, and return on assets "ROA" in the past 12 months. Size is the log of total assets, book-to-market ratio is book value of equity divided by market value of equity, leverage is computed as the sum of the market value of equity and the book value of liabilities divided by the market value of equity, and ROA is the ratio of earnings before extraordinary items to total assets. The betas, coskewness and cokurtosis are computed using daily returns in the past 12 months. The sample includes stocks listed on NYSE, AMEX and NASDAQ with prices above $5. The sample period for models (1) and (2) is from July 1963 to December 2017 and the sample period for models (3) and (4) is from January 1968 to December 2017. The t-statistics in square brackets are computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. The superscript *, ** and *** represent significance at 10%, 5%, and 1%, respectively.

Model	(1)	(2)	(3)	(4)
Variables	Relative β_{ed}^{-}	Relative β_{es}^{-}	Relative β_{ed}^{-}	Relative β_{es}^{-}
Intercept	0.084***	0.170***	0.173***	0.448***
	[13.475]	[28.479]	[9.723]	[25.267]
Past β_0	-0.038***	-0.016**	-0.007	0.007
	[5.563]	[2.474]	[0.637]	[0.668]
Past relative β^{-}	0.072***	0.497***	0.056***	0.385***
	[13.781]	[19.589]	[11.176]	[14.978]
Past size			-0.015***	-0.047***
			[5.786]	[17.263]
Past BM			0.000	0.024***
			[0.047]	[5.024]
Past return			0.021***	-0.089***
			[4.145]	[10.494]
Past coskewness			-0.029**	0.043***
			[2.369]	[4.584]
Past cokurtosis			-0.027***	0.004
			[5.093]	[0.522]
Past β_L			-0.001	-0.003**
			[0.450]	[2.450]
Past leverage			0.001	0.003***
			[0.943]	[9.121]
Past ROA			-0.071***	-0.054***
			[5.336]	[5.029]
Mean R^2	0.031	0.259	0.072	0.318
Mean RMSE	0.347	0.317	0.338	0.299

Table 4: Equal-weighted stocks sorted by past relative downside betas over one month horizon

This table reports time-series averages of equal-weighted returns sorted by CAPM beta β_0, relative usual downside beta "relative β^-_{ed}", and relative downside ES-implied beta "relative β^-_{es}" in Panels A, B and C, respectively. Each month stocks are sorted into quintile portfolios based on betas computed from daily returns in the previous 12 months. Columns "Low" to "High" report portfolio returns for a holding period of one month. Column "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio and Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. Row "Return" reports average returns in excess of the one-month Treasury-bill rate, row "CAPM alpha" reports risk-adjusted alphas from regressions of stock returns on CAPM beta, row "FF3 alpha" reports risk-adjusted alphas from regressions of stock returns on Fama and French (1996) 3 factors, row "FF3+Mom alpha" reports alphas from regressions of stock returns on Fama and French 3 factors and Momentum factor, and row "FF5 alpha" reports alphas from regressions of stock returns on the 5 factors presented in Fama and French (2015). Portfolios are based on NYSE/AMEX/NASDAQ stocks with prices above $5 from July 1963 to December 2017. Returns are in percentage.

	Low	2	3	4	High	High-Low	t-stat
			Panel A: past β_0-sorted portfolios				
Return	0.95	0.98	1.00	1.01	1.12	0.17	0.78
CAPM alpha	0.10	0.03	-0.01	-0.06	-0.12	-0.22	-1.36
FF3 alpha	0.08	0.02	-0.03	-0.08	-0.11	-0.19	-1.49
FF3+Mom alpha	0.08	0.03	-0.03	-0.08	-0.11	-0.19	-1.50
FF5 alpha	0.07	0.02	-0.04	-0.10	-0.11	-0.17	-1.35
			Panel B: past relative β^-_{ed}-sorted portfolios				
Return	0.98	0.89	0.94	1.06	1.18	0.20	2.33
CAPM alpha	-0.06	-0.09	-0.04	0.05	0.09	0.15	1.84
FF3 alpha	-0.06	-0.11	-0.05	0.04	0.05	0.12	1.84
FF3+Mom alpha	-0.06	-0.10	-0.05	0.05	0.06	0.12	1.92
FF5 alpha	-0.07	-0.10	-0.05	0.03	0.04	0.11	1.74
			Panel C: past relative β^-_{es}-sorted portfolios				
Return	0.85	0.86	0.95	1.04	1.36	0.51	5.49
CAPM alpha	-0.11	-0.10	-0.07	-0.01	0.23	0.34	3.81
FF3 alpha	-0.09	-0.10	-0.08	-0.04	0.17	0.25	4.43
FF3+Mom alpha	-0.08	-0.09	-0.07	-0.03	0.17	0.25	4.35
FF5 alpha	-0.09	-0.10	-0.08	-0.04	0.15	0.24	4.31

Table 5: Value-weighted stocks sorted by past relative downside betas over one month horizon

This table reports time-series averages of value-weighted returns sorted by CAPM beta β_0, relative usual downside beta "relative β^-_{ed}", and relative ES-implied beta "relative β^-_{es}" in Panels A, B and C, respectively. Each month stocks are sorted into quintile portfolios based on betas computed from daily returns in the previous 12 months. Columns "Low" to "High Column" report portfolio returns for a holding period of one month. Column "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio and Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. Row "Return" reports average returns in excess of the one-month Treasury-bill rate, row "CAPM alpha" reports risk-adjusted alphas from regressions of stock returns on CAPM beta, row "FF3 alpha" reports risk-adjusted alphas from regressions of stock returns on Fama and French (1996) 3 factors, row "FF3+Mom alpha" reports alphas from regressions of stock returns on Fama and French 3 factors and Momentum factor, and row "FF5 alpha" reports alphas from regressions of stock returns on the 5 factors presented in Fama and French (2015). Portfolios are based on NYSE/AMEX/NASDAQ stocks with prices above $5 from July 1963 to December 2017. Returns are in percentage.

	Low	2	3	4	High	High-Low	t-stat
			Panel A: past β_0-sorted portfolios				
Return	0.88	0.89	0.91	0.93	1.04	0.16	0.78
CAPM alpha	0.06	-0.02	-0.05	-0.10	-0.14	-0.20	-1.29
FF3 alpha	0.05	-0.02	-0.06	-0.11	-0.12	-0.17	-1.37
FF3+Mom alpha	0.06	-0.01	-0.06	-0.11	-0.12	-0.18	-1.40
FF5 alpha	0.04	-0.02	-0.07	-0.12	-0.11	-0.15	-1.22
			Panel B: past relative β^-_{ed}-sorted portfolios				
Return	0.90	0.82	0.88	0.99	1.11	0.21	2.29
CAPM alpha	-0.10	-0.13	-0.07	0.01	0.05	0.15	1.80
FF3 alpha	-0.09	-0.14	-0.07	0.01	0.02	0.12	1.82
FF3+Mom alpha	-0.09	-0.13	-0.07	0.02	0.03	0.12	1.93
FF5 alpha	-0.09	-0.13	-0.07	-0.00	0.01	0.10	1.62
			Panel C: past relative β^-_{es}-sorted portfolios				
Return	0.80	0.81	0.90	0.99	1.28	0.48	4.90
CAPM alpha	-0.12	-0.12	-0.09	-0.03	0.17	0.29	3.16
FF3 alpha	-0.10	-0.11	-0.09	-0.05	0.12	0.22	3.80
FF3+Mom alpha	-0.10	-0.11	-0.09	-0.04	0.12	0.22	3.69
FF5 alpha	-0.10	-0.12	-0.10	-0.05	0.11	0.20	3.67

Table 6: Stocks sorted by past relative downside betas over one year horizon

This table reports times series averages of returns sorted by relative usual downside beta "relative β_{ed}^{-}", and relative downside ES-implied beta "relative β_{es}^{-}" for a holding period of one year. Panel A and Panel B report equal-weighted and value-weighted returns sorted by the relative usual downside beta, respectively. Panel C and Panel D report equal-weighted and value-weighted returns sorted by the relative ES-implied beta. Column "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio and Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. Row "Return" reports average returns in excess of the one-month Treasury-bill rate, row "CAPM alpha" reports risk-adjusted alphas from regressions of stock returns on CAPM beta, row "FF3 alpha" reports risk-adjusted alphas from regressions of stock returns on Fama and French (1996) 3 factors, row "FF3+Mom alpha" reports alphas from regressions of stock returns on Fama and French 3 factors and Momentum factor, and row "FF5 alpha" reports alphas from regressions of stock returns on the 5 factors presented in Fama and French (2015). Portfolios are based on NYSE/AMEX/NASDAQ stocks with prices above $5 from July 1963 to December 2017. Returns are in percentage.

	Low	2	3	4	High	High-Low	t-stat
Panel A: equal-weighted portfolios sorted by past relative β_{ed}^{-}							
Return	0.94	0.89	0.92	0.97	1.07	0.13	3.13
CAPM alpha	-0.13	-0.13	-0.11	-0.09	-0.10	0.02	0.80
FF3 alpha	-0.11	-0.12	-0.10	-0.08	-0.09	0.02	0.61
FF3+Mom alpha	-0.10	-0.11	-0.09	-0.07	-0.08	0.02	0.75
FF5 alpha	-0.11	-0.12	-0.10	-0.09	-0.09	0.02	0.89
Panel B: value-weighted portfolios sorted by past relative β_{ed}^{-}							
Return	0.88	0.83	0.86	0.92	1.02	0.15	3.57
CAPM alpha	-0.15	-0.14	-0.12	-0.10	-0.11	0.03	1.07
FF3 alpha	-0.12	-0.13	-0.11	-0.09	-0.10	0.03	1.00
FF3+Mom alpha	-0.12	-0.12	-0.10	-0.08	-0.09	0.03	1.18
FF5 alpha	-0.13	-0.13	-0.11	-0.09	-0.10	0.03	1.12
Panel C: equal-weighted portfolios sorted by past relative β_{es}^{-}							
Return	0.78	0.81	0.92	1.02	1.27	0.49	6.85
CAPM alpha	-0.22	-0.20	-0.14	-0.08	0.08	0.29	4.99
FF3 alpha	-0.18	-0.17	-0.13	-0.08	0.05	0.22	5.86
FF3+Mom alpha	-0.17	-0.16	-0.12	-0.07	0.06	0.23	5.86
FF5 alpha	-0.17	-0.17	-0.13	-0.08	0.04	0.21	5.56
Panel D: value-weighted portfolios sorted by past relative β_{es}^{-}							
Return	0.75	0.77	0.89	0.99	1.23	0.48	6.57
CAPM alpha	-0.21	-0.19	-0.14	-0.08	0.07	0.28	4.60
FF3 alpha	-0.17	-0.16	-0.12	-0.07	0.05	0.22	5.86
FF3+Mom alpha	-0.16	-0.16	-0.12	-0.07	0.05	0.22	5.82
FF5 alpha	-0.17	-0.17	-0.13	-0.08	0.04	0.20	5.57

Table 7: Out-of-sample performance of Fama-MacBeth regressions

This table reports results of Fama-MacBeth (1973) regressions of monthly excess returns on relative usual downside beta "relative β_{ed}^{-}" and relative downside ES-implied beta "relative β_{es}^{-}". Model (1) reports regression results of excess returns on CAPM beta "β_0", model (2) on CAPM beta and relative usual downside beta, model (3) on CAPM beta and relative downside ES-implied beta, models (4) and (5) control for size, book-to-market ratio "BM", return "Return", coskewness, cokurtosis and liquidity beta β_L. Size is the log of total assets and book-to-market ratio is book value of equity divided by market value of equity. The betas, coskewness and cokurtosis are computed using daily returns in the past 12 months. The sample includes stocks listed on NYSE, AMEX and NASDAQ with prices above \$5. The sample period for models (1)-(3) is from July 1963 to December 2017 and the sample period for models (4)-(7) is from January 1968 to December 2017. The t-statistics in square brackets are computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. The superscript *, ** and *** represent significance at 10%, 5%, and 1%, respectively. Coefficients are in percentage.

Model Variables	(1) Return	(2) Return	(3) Return	(4) Return	(5) Return	(6) Return	(7) Return
Intercept	0.917***	0.897***	0.693***	1.398***	1.390***	1.269***	1.256***
	[5.985]	[5.889]	[4.497]	[6.115]	[6.094]	[5.601]	[5.557]
Past β_0	0.094	0.075	0.072	0.123	0.117	0.075	0.062
	[0.635]	[0.509]	[0.507]	[0.641]	[0.610]	[0.399]	[0.330]
Past relative β_{ed}^{-}		0.213**			-0.002		-0.033
		[2.412]			[0.020]		[0.322]
Past relative β_{es}^{-}			0.611***			0.165**	0.185**
			[4.768]			[2.001]	[2.411]
Past size				-0.150***	-0.148***	-0.141***	-0.140***
				[5.235]	[5.164]	[4.937]	[4.894]
Past BM				0.373***	0.369***	0.357***	0.355***
				[6.424]	[6.432]	[6.193]	[6.210]
Past return				-0.139	-0.133	-0.146	-0.138
				[0.822]	[0.788]	[0.865]	[0.827]
Past coskewness				-0.157	-0.185	-0.076	-0.172
				[0.821]	[0.880]	[0.399]	[0.818]
Past cokurtosis				0.049	0.037	0.079	0.076
				[0.690]	[0.510]	[1.139]	[1.061]
Past β_L				0.007	0.010	0.008	0.010
				[0.219]	[0.306]	[0.228]	[0.310]
Mean R^2	0.034	0.038	0.041	0.068	0.070	0.070	0.071
Mean RMSE	0.095	0.095	0.095	0.094	0.094	0.094	0.094

Table 8: Relation between the relative ES-implied beta and the usual downside beta

This table examines the relation between relative usual downside beta "relative β_{ed}^-" and relative ES-implied downside beta "relative β_{es}^-". Panel A reports equal-weighted returns of stocks sorted by relative ES-implied downside beta first and then relative usual downside beta. At the beginning of each month, stocks are sorted into quintile portfolios based on relative ES-implied downside beta computed from daily returns in the previous 12 months. Within each first-sort quintile, stocks are then sorted into additional quintiles based on relative usual downside beta computed from daily returns in the previous 12 months. Panel B reverses the order and sorts portfolios based on the relative usual downside beta first and then on the relative ES-implied beta. Column "Average" reports equal-weighted return of stocks in each second-sort quintile. Row "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio and Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. Portfolios are based on NYSE/AMEX/NASDAQ stocks with prices above \$5 from July 1963 to December 2017. Returns are in percentage.

	Panel A: relative β_{ed}^- sorts controlling for relative β_{es}^-					
	Low	2	3	4	High	Average
Low relative β_{ed}^-	0.77	0.75	0.97	0.99	1.33	0.96
2	0.73	0.77	0.84	0.95	1.28	0.91
3	0.78	0.85	0.97	1.01	1.31	0.98
4	0.91	0.94	1.01	1.05	1.40	1.06
High relative β_{ed}^-	1.06	1.00	0.97	1.20	1.48	1.14
High-Low	0.29	0.25	0.00	0.21	0.15	0.18
t-stat	2.39	2.62	0.00	2.20	1.39	2.19

	Panel B: relative β_{es}^- sorts controlling for relative β_{ed}^-					
	Low	2	3	4	High	Average
Low relative β_{es}^-	0.81	0.73	0.78	0.89	1.05	0.85
2	0.81	0.74	0.84	0.98	1.01	0.88
3	0.86	0.85	0.97	1.02	1.05	0.95
4	1.07	0.94	0.87	1.10	1.29	1.06
High relative β_{es}^-	1.35	1.24	1.23	1.33	1.51	1.33
High-Low	0.54	0.51	0.45	0.44	0.45	0.48
t-stat	4.27	4.95	4.40	4.31	3.96	5.66

Table 9: Relation between downside betas and coskewness

This table examines the relation between relative downside betas and coskewness. Panel A reports equal-weighted returns of stocks sorted by coskewness first and then relative usual downside beta. At the beginning of each month, stocks are sorted into quintile portfolios based on coskewness computed from daily returns in the previous 12 months. Within each first-sort quintile, stocks are then sorted into additional quintiles based on relative usual downside beta computed from daily returns in the previous 12 months. Panel B sorts on coskewness first and then on relative ES-implied downside beta. Column "Average" reports average return of stocks in each second-sort quintile. Row "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio and Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. Portfolios are based on NYSE/AMEX/NASDAQ stocks with prices above $5 from July 1963 to December 2017. Returns are in percentage.

	Panel A: relative β_{ed}^{-} sorts controlling for coskewness					
	Low coskewness	2	3	4	High coskewness	Average
Low relative β_{ed}^{-}	1.01	1.01	1.08	0.98	0.96	1.01
2	1.01	0.99	0.93	0.85	0.91	0.94
3	1.13	1.04	0.97	0.89	0.82	0.97
4	1.12	1.07	1.07	0.97	0.81	1.01
High relative β_{ed}^{-}	1.28	1.32	1.17	1.02	0.90	1.14
High-Low	0.27	0.31	0.09	0.05	-0.06	0.13
t-stat	2.08	2.92	0.90	0.48	-0.56	1.74

	Panel B: relative β_{es}^{-} sorts controlling for coskewness					
	Low coskewness	2	3	4	High coskewness	Average
Low relative β_{es}^{-}	0.97	0.94	0.87	0.79	0.71	0.86
2	0.92	0.96	0.95	0.79	0.71	0.87
3	1.01	0.97	0.98	0.91	0.81	0.93
4	1.13	1.15	1.03	1.02	0.93	1.05
High relative β_{es}^{-}	1.51	1.43	1.40	1.20	1.25	1.36
High-Low	0.54	0.49	0.53	0.41	0.54	0.50
t-stat	4.33	4.79	4.90	3.81	4.73	5.70

Table 10: Sorts of the relative ES-implied beta controlling for other factors

This table presents time-series averages of excess returns of portfolios sorted by the relative ES-implied beta after controlling for size, cokurtosis and volatility, exclusively. Stocks are sorted into quintile portfolios according to the control variable in the past 12 months at the beginning of each month. Within each first-sort quintile, stocks are then assigned into quintiles based on the relative ES-implied beta in the previous 12 months. Column "Average" reports equal-weighted returns of stocks in each second-sort quintile. Row "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio and Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. Portfolios are based on NYSE/AMEX/NASDAQ stocks with prices above $5 from July 1963 to December 2017. Returns are in percentage.

	Panel A: Sorts of the relative ES-implied beta controlling for size					
	Low size	2	3	4	High size	Average
Low beta	1.33	1.00	0.96	0.79	0.73	0.96
2	1.31	1.07	0.87	0.80	0.62	0.93
3	1.39	0.99	0.97	0.85	0.67	0.97
4	1.57	1.15	0.93	0.80	0.73	1.04
High beta	1.68	1.23	1.10	0.98	0.79	1.16
High-Low	0.35	0.24	0.15	0.19	0.06	0.20
t-stat	3.28	2.21	1.73	2.01	0.56	2.82

	Panel B: Sorts of the relative ES-implied beta controlling for cokurtosis					
	Low cokurtosis	2	3	4	High cokurtosis	Average
Low beta	0.85	0.84	0.90	0.84	0.86	0.86
2	0.90	0.94	0.99	0.84	0.85	0.90
3	1.03	1.04	0.95	0.91	0.78	0.94
4	1.21	1.17	1.11	1.04	0.74	1.05
High beta	1.38	1.50	1.39	1.19	1.07	1.31
High-Low	0.53	0.65	0.49	0.35	0.21	0.45
t-stat	4.62	5.27	3.53	2.56	1.64	4.43

	Panel C: Sorts of the relative ES-implied beta controlling for volatility					
	Low volatility	2	3	4	High volatility	Average
Low beta	0.67	0.78	0.85	1.05	1.20	0.91
2	0.72	0.77	0.94	1.04	1.24	0.94
3	0.74	0.83	0.92	1.16	1.40	1.01
4	0.81	0.86	0.98	1.18	1.44	1.05
High beta	0.83	0.95	1.11	1.23	1.64	1.15
High-Low	0.15	0.17	0.26	0.19	0.44	0.24
t-stat	1.76	1.74	2.75	1.74	3.28	3.07

Table 11: Robustness checks

This table reports time-series averages of returns sorted by relative ES-implied beta using other settings. Each month stocks are sorted into quintile portfolios based on the relative ES-implied downside beta computed from daily returns in the previous 12 months. Panel A reports equal-weighted returns and Panel B reports value-weighted returns. Row "All stocks" reports returns of portfolios by sorting all stocks listed on NYSE/NASDAQ/AMEX, row "NYSE stocks" sorts stocks listed on NYSE only, and row "NYSE breakpoints" sorts portfolios using quintile breakpoints based on NYSE stocks. Column "High-Low" reports the difference between returns of the highest portfolio and the lowest portfolio. Column "t-stat" reports associated t-statistics computed using Newey-West (1987) heteroskedastic-robust standard errors with 12 lags. The sample is from July 1963 to December 2017. Returns are in percentage.

	Panel A: Equal-weighted						
	Low	2	3	4	High	High-Low	t-stat
All stocks	0.67	0.71	0.70	0.79	1.47	0.80	3.84
NYSE stocks	0.77	0.76	0.80	0.85	1.01	0.24	2.55
NYSE breakpoints	0.86	0.82	0.90	0.94	1.22	0.36	4.45

	Panel B: Value-weighted						
	Low	2	3	4	High	High-Low	t-stat
All stocks	0.66	0.70	0.69	0.78	1.37	0.71	3.31
NYSE stocks	0.75	0.73	0.77	0.83	0.99	0.24	2.50
NYSE breakpoints	0.81	0.77	0.85	0.90	1.14	0.34	4.02