The Tree-Crop Problem

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This paper is an abbreviated version of our earlier paper (Bellman and Hartley [1983/84]).


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Abstract

The paper considers the problem of formulating and calibrating a general neoclassical prototype model for the supply of tree-crops. Previous approaches to the problem are reviewed and various agronomic considerations are discussed. The decisions of producers are of three types: (1) land allocation, (2) cultivation, and (3) harvesting. Land-allocation in a mixed crop situation is treated as a Markovian process, and the perennial technology is dynamic. The producer is assumed to make all decisions jointly within the year to maximize the discounted present value of profits over a finite planning horizon. An anticipations-formation process is formulated and methods to calibrate the parameters of the technology and anticipations process are formulated.
1. Introduction ................................................................. 1
2. Previous Approaches to the Problem................................. 4
3. Agronomic Aspects of Perennial Supply ............................. 8
4. The Structure of a Prototype Model for Perennial Crops ........... 14
   4.1 Land Allocation and Rotation ..................................... 17
   4.2 The Technology of a Perennial Crop ............................ 26
       Planting Decision Variables .................................... 32
       Cultivation Decision Variables ............................... 34
       The Harvesting Decision Variable ........................... 35
       Density ........................................................... 36
       Potential Yield Per Tree ...................................... 39
       Actual Yield Per Tree ......................................... 41
   4.3 Alternative Crops .................................................. 43
   4.4 Labor and Materials Requirements ............................... 43
5. A Neoclassical Model for the Supply of a Perennial ................. 44
   5.1 Optimal Decisions and Dynamic Programming ..................... 46
   5.2 Direct Estimation/Calibration of the Technology Parameters .. 57
   5.3 The Anticipations-Formation Process .......................... 59
   5.4 Model Calibration ................................................ 61
6. Conclusions ................................................................. 66

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1/ We wish to thank Miriam Bailey for her excellent word-processing.
1. Introduction:

The "tree-crop problem," i.e., the development of well-specified structural econometric models for the supply of perennial crops, has long defied a satisfactory solution. In view of the fact that perennial crops account for a major proportion of gross domestic product, employment, foreign exchange earnings and government revenues of many LDCs within the tropics, it is also a problem of considerable importance. Indeed, establishment, replanting and rehabilitation of various perennial crops have historically accounted for a significant proportion of the World Bank's agricultural investment portfolio.

For most perennials, it is alleged that both demand and supply functions are inelastic with respect to current price. Accordingly, world market prices -- and, pari passu, the export earnings of producers -- exhibit great instability. In addition, the attempt to establish producer cartels, which may stabilize world prices (through judicious manipulation of output quotas, prices and buffer stocks), has not generally proved successful (de Vries [1983]). Further, balancing the needs of governments to extract sufficient surplus from the export of such cash crops (through imposition of export duties) against the need of farmers for sufficient profits (to warrant continued cultivation and replanting with new varieties) has proved to be an extremely difficult problem. Despite the plethora of subsidies and other incentives, such policies have rarely succeeded in stimulating the smallholder. As a result, in many LDCs the existing stock is not only overaged, but also contains an excessive proportion of lower-yielding planting material.
Serious research on developing appropriately realistic models of tree crop supply has been hampered by a number of difficulties. First, data constraints have often precluded formulation of a general structural model. One of the major obstacles to a thorough examination of the tree-crop problem has been the lack of systematic data on the age-distribution and clonal composition of existing stands in each year. Unfortunately, in many LDCs the available data on perennials consist only of macro time-series on output, producer prices and area under cultivation, along with the array of government taxes, subsidies and other policy instruments which were operative in each year. In some cases, data on labor and materials inputs (notably fertilizer) and associated prices, on new plantings and replantings, and on climatic conditions are also available.

As a result of differences or gaps in the available data base, researchers have felt obliged to specify overly simple stochastic reduced-form models, which fail to capture many of the essential features of the problem. The inability to incorporate the age distribution endogenously within the model (in the form of a complete dynamic vintage capital model of perennial supply) has severely hampered a quantitative assessment of the effects of alternative tax and incentive schemes on input use, replantings, new plantings and, hence, on the trajectory of future output. Further, it has made the design of optimal policy, relative to whatever the government's objective, nearly impossible -- given the inherent complexity of the problem.

Second, the econometric tools have not been appropriate to the problem at hand. We seek only a model which numerically defines a perennial supply function -- i.e., we seek to determine the optimal land-use and input decision levels relative to a farmer's objective function given: (1) the technology of crop cultivation, and (2) the farmer's anticipated trajectory of
current and future state variables (prices, weather conditions, etc.) over a
finite planning horizon. The optimal values of such decision variables are
then inserted into the perennial technology to determine current output
levels. This will be recognized as a dynamic programming problem (Bellman
[1957] and Bellman and Dreyfus [1962]). The problem remaining is that even if
the technology of the problem is known, or can be estimated directly, the
parameters in the process governing the formation of producers' anticipations
of the various state variables cannot be taken as known to the investigator,
and can only be "sorted out" by either: (a) anticipations surveys, or (b) by
their reflection in the decisions taken by cultivators and the associated non-
decision endogenous variables. In either case there are dynamic constraints
on the domain of the decision variables of a complex and interdependent
nature. We consider only case (b), as reliable anticipations surveys over an
extended planning horizon are rarely available.

Given a data set on all endogenous (decision and non-decision)
variables and all state variables (prices, weather, holding and producer
characteristics, etc.), the neoclassical econometric problem is the inverse of
predicting the output trajectory of given decision sequences. In particular,
under any maintained hypothesis regarding the technology and anticipations-
formations process, our interest is either in calibrating the parameters (in a
deterministic model) or estimating their "true values" (in a stochastic
analogue). In most applications this problem will be complicated by two
further issues: (i) the data will be provided at various levels of aggregation
and/or within various taxonomic breakdowns, and (ii) the configuration of
available data on all of these may be quite arbitrary, depending upon the
empirical application at hand. The combination of these and other
complications -- see Hartley [1982] -- suggests the use of neoclassical
econometric calibration methods on a deterministic vintage production model (see Hartley [1981a, 1981b, 1982, 1983, 1984b]), as opposed to the virtual impossibility of using anything but Iterated Monte-Carlo methods on any realistic stochastic analogue (Hartley [1984a]).

It should be evident from our brief description that the "tree-crop problem" is an extremely challenging, important, but inherently difficulty problem to solve. In addition, the interface between the available data base, the characteristics of the perennial per se, and the frequently changing institutions and government policies in any particular country application combine in such a way as to produce extremely complex econometric calibration/estimation problems. Therefore, even with a fairly complete "theory of perennial supply" in hand,

"...the economist embarking (upon) research in this area must move in essentially uncharted waters, and remain ultrasensitive to the peculiarities of his chosen concern, even though he begins at essentially the same starting point as his colleague interested in annual crops--the basic price expectations, area adjustment supply model--and has available to him the results of other studies of perennial crops." [1]

2. Previous Approaches to the Problem: [2]

There is an extensive literature on attempts to estimate supply functions for perennial crops. Most model specifications, however, represent fairly simple modifications of the now standard Nerlove (1958) model of the dynamics of supply response for annual crops in order to accommodate certain of

1/ Askari and Cummings [1977], p. 223, parentheses added.
2/ All matters considered in this and the next section have been discussed in greater detail elsewhere (Hartley [1982]). We include here a brief overview to provide continuity to the paper.
the characteristic features of a perennial. 1/

The basic Nerlovian model consists of three equations:

(a) a first-order adaptive expectations model of future prices (Cagan [1956]),

(b) "desired" area under cultivation, determined as a function of expected "normal" or long run future prices, and

(c) a first-order partial adjustment model, here relating actual area planted to "desired" area. 2/

This model, whether couched in terms of area or output, leads to the familiar geometric distributed lag model (Koyck [1954]) in current and past prices, and to a reduced form "supply function" in which current area-planted/output is a linear function of current and one-period-lagged prices and lagged area-planted/output.

Modifications of this model for perennials attempt to account for:

(1) the existence of a gestation period of (say) G years between planting and first bearing, and

(2) the fact that yields vary, ceteris paribus, in a systematic fashion over the economic life of an existing stand -- the so-called age-yield profile.

The consequence of (1) and (2) is that the observed total output in any year is the sum, across all mature vintages, of the area under cultivation of stock of a given age multiplied by the yield per unit area associated with each age. A perennial "supply function" is then obtained by utilizing agronomic

---

1/ See Askari and Cummings [1977], chapter 7, for an excellent survey of this literature, which highlights the Nerlovian approach. See also Nerlove [1979].

2/ See, e.g., Knox [1952] for an early reference from investment theory.
knowledge as to the length of the gestation period and a simple parameterization of the relationship between yields of the mature vintages and age to define the potential output of the existing age-composition. Departures of actual from potential output then arise via a harvesting equation -- capturing the effect of inclement weather or disease at harvest-time or the result of low current producer prices relative to the marginal cost of harvesting. Finally, the area under cultivation of a given age (say k) in a given year (say t) is modeled as a consequence of a planting decision taken in year t-k+1 -- an investment decision, usually motivated (see Bateman [1965] or Wickens and Greenfield [1973]) by comparing the (anticipated) net-present value in that year of a newly planted perennial with that of certain alternative land uses over a planning horizon of, say, N years. Thus, the planting decision depends, inter alia, upon a future trajectory of price anticipations, formed on the basis of information available in year t-k+1.

The structure of this prototype model implies a "supply function", with short-run price elasticities arising from the harvesting decision, and a series of long-run price elasticities associated with the planting decisions made in year t-k+1 for each of the currently mature vintages (ages k > G).

The standard approach is to analytically derive (or otherwise postulate) an explicit reduced form equation from a structural model (such as sketched above) under some simplifying assumptions. These represent a judicious compromise between realism (i.e., attempting to capture whatever agronomic information can be ascertained regarding the length of the gestation period, the shape of the age-yield profile, the formation of price anticipations governing the planting decision, etc.) and the need for simplicity and parsimony in parametrization (in order to obtain an estimable analytic closed-form expression for the resulting reduced-form "supply" equation).
This compromise usually involves the following types of simplifying assumptions (see, e.g., Bateman [1965] or Behrman [1968]):

(i) age-yield profiles are zero up to age $G$ and constant (at some average level) thereafter,

(ii) price expectations are formed (say) adaptively, and determine a single long-run "normal" price, assumed constant over the entire planning horizon of $N$ years,

(iii) "desired" area to be planted in any year is a function of a constant long-run expected output price, relative to that of an alternative crop, and

(iv) actual area planted partially adjusts to "desired" area.

Even if such a model were plausible, the resulting reduced form equation does not permit unique identification of any of the structural parameters, and leads to an equation in which current output depends on a distributed lag of current and past relative prices and lagged area. Indeed, in the most rigorous attempt to date (Wickens and Greenfield [1973] on Brazilian coffee), where the age-yield profile is essentially permitted to be a set of free parameters, the net empirical result is a model identical to Nerlove's original model for annual crops.1/

Three fundamental problems with this approach (Hartley [1982]) are:

1) the original structure and behavioral assumptions are largely ad hoc and, since they lead to a reduced form equation in which the structural parameters are logically unidentifiable, it is not possible to separately test which of the maintained structural

1/ More recently, Hartley, Nerlove and Peters [1984] have expanded upon this model by separating the new planting from the replanting decision--both of which are explicitly modeled.
(2) the resulting supply equation, since it is not formally derived from a solution to an underlying constrained optimizing model of producer behavior -- say, maximizing the discounted present value of the future stream of profits, is not really a supply function at all, but rather an "empirical association" between output and a distributed lag on prices, etc.

(3) the approach takes no account cultivation and harvesting input decisions and fails to properly capture the vintage features of land-use decisions.

The implication of (1), (2) and (3) is that, even if the resulting "supply function" predicts well, it is generally not possible to place an interpretation on the meaning of the reduced form parameters or to properly utilize the estimated model to analyze most policy interventions. Worse still, if the model fails to predict well, it is not possible to pinpoint which of the several jointly maintained simplifying assumptions are the source of the difficulty. At best, we obtain "elasticity" estimates of a linearized so-called "pseudo supply function" (Hartley [1982]).

3. Agronomic Aspects of Perennial Supply:

Unlike the case of annual crops, in which a planting decision each year determines the total area under cultivation, the area adjustment and land allocation decisions required to model the change in the age-composition of the area under a perennial crop from year to year are quite complex. Producers may make gross additions to the existing area under cultivation by new planting of land formerly having alternative uses. Gross reductions in the total area arise through removal decisions (Arak [1968, 1969] and French
and Mathews (1971]) — either through the abandonment of existing stands to fallow land or through uprooting for diversification to alternative uses.

Finally, producers may decide to change the age-composition of their holdings without changing the total area under cultivation. This is accomplished by a decision to uproot and replant existing vintages with new perennial stock. All remaining vintages, by default, will then move up one age-class — resulting in a different age-composition. Thus, the change in area under cultivation and its age-composition from one year to the next represents the net effect of all of these area adjustment decisions. Clearly, to specify how each of these decisions affects the aggregate age-composition requires modeling the area under stock of each age-class that is removed or replanted in any year. In short, we require a complete land-rotation system — as given in section 4.1 below.

For most perennials the age-yield profile is not a biologically fixed relationship. Rather, the potential yield which may be obtained from a given variety of a given age depends significantly upon the current and preceding levels of fertilizer applications and maintenance activities that have been employed.1/ Further, for most perennials the application of fertilizer in a given year produces a yield response which is physically distributed over both

1/ For example, while the total area under tea in Sri Lanka has remained largely constant since the early 1930's, yields per acre have more than doubled. This is alleged to have occurred (Forrest [1967]) as a consequence of substantial increases in the use of fertilizer and, since 1953, the gradual replacement of the traditional varieties of tea bushes by vegetatively-propagated clonal varieties.
current and subsequent years. In short, production functions for perennials are inherently dynamic — both current and past levels of labor and materials inputs at any age influence current and future yields.

The majority of perennials yields a crop which is harvested at a particular time of the year. Such discrete harvests occur when the fruits, beans, pods, nuts, etc. are ripe. Here, rainfall, disease and other weather conditions at harvest-time may reduce the potential crop that may be collected. In addition, the producer may make a deliberate economic decision of whether to take in the entire potential crop or only a portion — generally based on comparing the marginal cost of harvesting with the prevailing crop price.

Other perennials, such as rubber or tea, may be harvested continuously over the year. Here, the yield which may be obtained varies with the intensity of harvesting. With rubber, this may be controlled by adjusting the length of the cut made in the bark of the tree, the number of separate cuts or the number of days between tappings. Similarly, with tea, the intensity varies with the length of the interval between plucking rounds. In the case of rubber, a decision to increase the current relative tapping intensity,

1/ Agronomic evidence for rubber (de Gaus [1973]) indicates the total response to a given recommended dose of fertilizer is distributed over five years, with a modal response occurring three years after application.

2/ This may be of particular importance if current crop prices are very low relative to levels which prevailed at the time of previous land-use and input decisions.

3/ An ex ante index of "relative tapping intensity" (RTI) is a common summary measure (Peries [1970]), with an ex post adjustment to "actual" (ATI) to account for unanticipated rainfall. Chemical stimulants (e.g., ethyryl) may also be used to increase current yields. This is usually initiated during the latter years of the bearing cycle, but such practices vary with time and place.
ceteris paribus, will generally reduce the potential yield that could have been obtained in the following year(s). Here, the harvesting-intensity decision involves a physical trade-off between current and future yields.

The density of the existing stand (i.e., the number of trees-per-unit-area) is an important determinant of yields. With rubber (Barlow [1978]), yields-per-acre increase with density up to a point, whilst yields-per-tree decline with density (due to the increased competition among trees for limited light and nutrients). This is likely to be the case in general.

The ability of the producer to control the course of a stand's density over its life-cycle depends upon the perennial at issue. The selection of the initial stand density at planting is common to all perennials. In most cases, however, the formation of a leaf canopy after the second or third year following planting precludes resupply of vacancies created in the initial stand due to inclement weather, disease, pests, etc. In other cases (such as tea) vacancies may be "infilled" after each pruning cycle. In the absence of resupplying, densities will decline monotonically with age, and, ceteris paribus, with a concommitant decline in yield-per-acre. Preventative measures with respect to tree losses from disease and pests may be taken through the use of insecticides, pesticides, etc. Extreme weather conditions are almost impossible to control for -- an issue of major importance in cultivation of oranges, coffee or tea, where a severe frost or drought may, on occasion and unexpectedly, reduce densities to levels such that continued cultivation is no longer profitable. In such cases, replanting or diversification to an alternative crop are the only viable options -- with obvious consequences for output and, if a significant portion of the total
crop, for prices over the ensuing gestation period.1/

The effect of weather on the output of a perennial has three separate courses. First, as noted above, severe weather conditions may reduce stand densities. Second, weather conditions affect the growth of trees (and hence future potential output). Third, they affect the size of and ability to harvest the currently available crop. In most cases, rainfall has a positive long-run effect on tree growth and a negative immediate effect on the harvested crop.2/ Thus, the types of weather indices required to model each of the separate effects of weather on the stock of trees, the growth of trees and on harvesting of the crop requires knowledge of the agronomy of the perennial and the country at issue if we are to properly control for the effect of weather variation on output.3/

Finally, we note the importance of the availability and adoption of new higher yielding varieties of a given perennial on secular trends in yields. This point is of extreme importance, as it characterizes the

---

1/ The reaction of "futures market" commodity prices to such events is ample testament to their significance.

2/ For example, with rubber, rainfall interferes with tapping — reducing current yields. In other cases, where harvests are discrete (such as with cocoa), untimely rainfall at harvest-time interferes with the collection, drying and ability to transport beans to market.

3/ This point, though well-known to agronomists, has not been properly appreciated by perennial supply modelers.
The principal way by which technological change occurs.\textsuperscript{1} The purpose of the foregoing discussion has been to exhibit certain features of the production technology associated with a perennial crop. A particular producer (or their aggregate) may be viewed in any year as having a capital stock (area under cultivation) consisting of various vintages (area under a given variety of a given age).\textsuperscript{2} Changes in the age-composition of the stock from one year to the next arise through the natural process of ageing -- as each vintage moves up one age-class, except for the decisions of the producer to uproot for replanting; to diversify to alternative crops; to abandon existing stock; or to plant new areas with the perennial. The yield per unit of capital (area or per tree) varies with each vintage according to an age-yield profile. The profile, however, may be shifted both by the current and preceding sequence of cultivation inputs (fertilizer, pesticides, weeding labor, etc.) and harvesting intensities, and by exogenous natural events (rainfall, disease, drought, frost, etc.) beyond the control of the producer. The effect of current levels of both controlled decisions and uncontrolled exogenous events is dynamic, in that both current and future output levels are affected.

The input decisions which a particular producer must make in any year consist of the set of possible area adjustments (whether to continue, abandon or uproot each existing vintage; whether to replant or diversify any uprooted

\textsuperscript{1} Introduction and adoption of fertilizer, stimulants and pesticides are other examples. The hybrid varieties of rice developed at the International Rice Research Institute (IRRI) are excellent examples of the sharp rise in yields, whereas its collection of "gene pools" of present-standard failures, along with documented field performance, permits the possibility of recovering from any, otherwise irreversible, mistakes.

\textsuperscript{2} Christopher Gilbert has suggested use of the term "cohort", as opposed to "vintage", may be more appropriate.
area; and whether or not to newly plant areas previously in alternative uses; etc.). For all plantings, the decisions on initial densities and varietal types; and, for all age-classes, the levels of cultivation and harvesting inputs (fertilizer application, maintenance labor, pesticide sprays, harvesting intensity, etc.) — where all such decisions are made jointly over each vintage currently under cultivation and in anticipation of future decisions and prices.1/

4. The Structure of a Prototype Model for Perennial Crops:

In this section we consider the specification of a prototype model of a perennial, incorporating the basic types of decisions which cultivators must make and the agronomic and technological constraints to which they are subject.

We shall develop a model for a given producer, i, and time period, t. Each producer is assumed to operate a holding with a constant area, \( a_i \), which must be allocated each year between perennial cultivation, alternative crops and other land uses. The presence of a perennial, however, introduces various restrictions on the admissible land allocations. These may be conveniently displayed through the use of a land rotation system.2/

---

1/ E.g., if a producer of rubber anticipates that prices will fall in the coming years and decides to uproot a given vintage in (say) two years, he will not apply fertilizer currently (unless fertilizer prices are extremely low), since he will fail to realize the full marginal product, distributed over the next five years, and he will raise his tapping intensity without regard for the long-run yield consequences. Such strategies are not easy to incorporate within the traditional, Nerlovian-based approaches to modeling perennial supply.

2/ The land rotation system is akin to the transition matrix of a first-order Markov process. The main difference is that we shall be concerned with modeling the transition probabilities.
Conditional upon the land-allocation decision, the producer must then decide upon the appropriate levels inputs for each of the crops. In the case of the perennial we distinguish between the decisions associated with each age-class, $k = 1, 2, \ldots, K_1$, where $K_1$ is the maximum economic life-span of the perennial. Also, for each age-class we consider the various planting, cultivation and harvesting decisions separately. For each of these, agronomic considerations indicate that the effect of a current decision is physically distributed over current and future periods, so that the technology is fundamentally dynamic in nature. In contrast, for the $K_2$ possible alternative crops—taken here to be annuals, the full output response obtains within the year.

The dynamic aspects of perennial cultivation imply that the producer must have a multi-period objective function. We shall consider the standard case in which the producer makes his decisions on land-allocation and crop-inputs so as to maximize the discounted present value of the current and future stream of profits over an $N$ period future planning horizon. Except in highly simplified models, it is not possible to obtain analytic expressions for the optimal decisions. Accordingly, to calibrate the parameters of a model of perennial supply and obtain the values of the optimal decisions requires an interface between dynamic programming (Bellman [1957], Bellman and Dreyfus [1962]) and neoclassical econometrics (Hartley [1981a, 1981b, 1983, 1985]).

---

1/ We do not mean to imply that these decision-making processes are necessarily sequential within a given year—see below. However, across years, they are dynamic multi-stage decisions, since time is irreversible.

2/ We distinguish between the natural life span and the economic life of a perennial. The latter is less than the former and occurs at the point when the farmer does not anticipate continued cultivation to be profitable relative to available alternatives.
We shall consider these issues in the next section and restrict present discussion to the structure of the prototype model. 1

Let \( \pi_{it} \) denote the profit of producer \( i \) in year \( t \), where

\[
\pi_{it} = p_{it}^1 \cdot q_{it}^1 + p_{it}^2 \cdot q_{it}^2 - \frac{1}{2} p_{it} \cdot \xi_{it} - \frac{1}{2} p_{it} \cdot \xi_{it}^* - m_{it} \cdot \xi_{it}^*
\]  

The complete price-vector associated with producer \( i \), is defined by

\[
P_{it} = (p_{it}^1, p_{it}^2, \xi_{it}, m_{it})',
\]

where \( p_{it}^1 \) is the price of the perennial crop; \( p_{it}^2 \) is a \( K_2 \)-vector of alternative crop prices; \( \xi_{it} \) is an \( L \)-vector (\( L \geq 1 \)) of wage rates for each type of labor required, \( \xi_{it}^* \); and \( m_{it} \) is a \( M \)-vector (\( M \geq 1 \)) of unit prices or rental rates associated with the materials/equipment requirements contained in the vector, \( m_{it} \). 2 The total output of the perennial crop is \( q_{it}^1 \); and \( q_{it}^2 \) is a \( K_2 \)-vector of alternative crop outputs.

Our prototype model will thus involve three classes of decision-variables, contained within the vector, \( \pi_{it} \):

---

1/ Our objective is to develop a general prototype model which, in any particular application, can be collapsed (under suitable simplifying assumptions) to meet the exigencies of available data.

2/ This taxonomy distinguishes between input decisions and the labor and materials requirements associated with each such decision. This requires an additional specification of the labor and materials "process technology" associated with each planting, cultivation and harvesting input. In addition, the notation permits the producer price to vary across producers. E.g., smallholders who sell to dealers may receive a lower price than large estates.
\[
\begin{bmatrix}
0 \\
x_{1i}^0 \\
x_{2i}^0
\end{bmatrix}
\]  
\begin{align*}
\text{(land allocation decisions)} \\
\text{(perennial crop decisions)} \\
\text{(alternative crop decisions)}.
\end{align*}

To complete the model, it is necessary to discuss the land-rotation system and associated decision-vector, \( x_{1i}^0 \); as well as the technology associated with the decision-vectors, \( x_{1i}^1 \) and \( x_{1i}^2 \), which determine the output levels, \( q_{1i} \) and \( q_{2i} \), respectively. We then close the model by defining the labor and materials requirements, \( L_{1i} \) and \( M_{1i} \), associated with the decision-vector, \( x_{1i} \).

### 4.1 Land Allocation and Rotation:

Let \( a_{1i} \) denote a \( K \)-element land-use vector, defined by

\[
a_{1i} = (a_{1i}^1(k)) = [a_{1i}^1 \ a_{1i}^2 \ a_{1i}^3]^T,
\]

where

\[
a_{1i}^j = [a_{1i}^j(1) \ a_{1i}^j(2) \ \ldots \ a_{1i}^j(K_i)]^T, \quad j = 1, 2, 3,
\]

and

\[
a_{1i}^j(k) = a_{1i}^j(K_i),
\]

in which \( a_{1i}^1(k) \) is the area under perennial stock of age \( k \) in year \( i \); \( a_{1i}^2(k) \) is the area under alternative crop \( k \); \( a_{1i}^3(k) \) denotes the area in the \( k \)th

\[
(4.4)
\]
other land-use and $a_i$ denotes the constant total area of the $i$th holding.¹
Thus, treating each age-class as a separate "land-use", we have a total of
\[ K = K_1 + K_2 + K_3 \] alternatives.

For convenience, we shall assume that all alternative crops are
annuals -- though the model can easily be extended to accommodate other
perennials among the alternatives by allocating a separate land-use category
to each age-class, as in $a_{it}^1$. Also, different varietal types of a given
perennial can be treated as separate perennials. We shall also assume that
$a_{it}^3$ contains three other land uses:²

\[
\begin{align*}
    a_{it}^3(1) & = \text{area uprooted and cleared during year $t$, but suitable for}\cr & \text{cultivation in year $t+1$.} \\
    a_{it}^3(2) & = \text{area fallow, but suitable for cultivation upon uprooting}\cr & \text{and clearing.} \\
    a_{it}^3(3) & = \text{area unsuitable for cultivation or alienated (after}\cr & \text{clearing and uprooting) for non-agricultural uses.}
\end{align*}
\]

Thus, via use of the elements of $a_{it}^3$ we may determine decisions to uproot
and clear $(a_{it-1}^3(1) - a_{it}^3(2) - a_{it}^3(1))$, abandon $(a_{it-1}^1 - a_{it}^3(2))$,
replant $(a_{it-1}^3(1) - a_{it}^1(1))$, new plant $(a_{it-1}^2 - a_{it}^1(1))$ and diversify
$(a_{it-1}^3(1) - a_{it}^2)$ land under a perennial. Finally, we shall treat $a_{it}^3(3)$ as a
"sink", in the sense that, once land enters this state, it may never again

¹/ This is a simplification, since the size of a given holding may change
over time. E.g., upon nationalization of the larger rubber estates in Sri
Lanka in 1975, portions of a given holding were redistributed to other
management units.

²/ We are indebted to Graham Pyatt for suggesting a simplification leading to
the present taxonomy.
return to cultivation. 1/

Suppose in year $t-1$ the producer has allocated all available land on his holding according to the land-allocation vector, $a_{it-1}$. It is a characteristic of perennials that, given $a_{it-1}$, the range of possible allocations in year $t$ is thereby constrained. To illustrate, consider the area under the perennial of age $k-1$ in year $t-1$, i.e., $a^1_{it-1}(k-1)$. In the next year, $t$, there are only three possible land-uses for such an area: either to continue the land under perennial cultivation (as stock then of age-class $k$); or uproot for subsequent replanting or diversification to alternative uses; or abandon the area as fallow land.

In general, whatever the taxonomy of land uses adopted, we can represent the complete land-allocation vector in year $t$, given $a_{it-1}$, by means of the land-rotation system,

$$a_{it} = x^0_{it} \cdot a_{it-1}$$

where

$$x^0_{it} = \left( x^0_{it}(k,k') \right)$$

is a $K \times K$ matrix, $K = K_1 + K_2 + K_3$, with elements, $x^0_{it}(k,k')$, denoting the proportion of the area, $a_{it-1}(k')$, devoted to land-use $k'$ in the preceding

---

1/ This permits the model to account for secular declines in area potentially available for cultivation, as land is alienated for roads, buildings, urban development and other such purposes (French and Mathews [1971]).
year, t-1, which is then allocated to land-use k in year t. 1/ 

A land-rotation system therefore represents the simple fact, whatever the allocation of land in the previous year, all land in the current year must be allocated within the same taxonomy. 2/ Any restrictions on transitions — say, from use k' to use k — in successive years can be incorporated by setting the associated cells, \( x_{it}^0(k,k') \), identically to zero. All remaining cells are subject to the producer’s choice, and thus are decision-variables.

Let \( D^0 \) denote the appropriate land-allocation restriction matrix,

\[
D^0 = (\delta^0(k,k')) \tag{4.8}
\]

a K\times K matrix with known elements,

\[
\delta^0(k,k') = \begin{cases} 
1, & \text{if land may pass from use } k' \text{ to use } k \text{ in successive years} \\
0, & \text{if not.} 
\end{cases} \tag{4.9}
\]

Then, clearly, the \( \{x_{it}^0(k,k')\} \) must satisfy the following conditions:

\[
x_{it}^0(k,k') = 0, \text{ if } \delta^0(k,k') = 0 \text{ or } a_{it-1}(k') = 0 \text{ (fixed cells)} \tag{4.10a}
\]

\[
x_{it}^0(k,k') \geq 0, \text{ if } \delta^0(k,k') = 1 \text{ and } a_{it-1}(k') > 0 \text{ (decision cells)}, \tag{4.10b}
\]

\[1/ \text{ The reason for representing land allocation decisions as proportions is, that this transforms the feasible decision-space into a subset of the multi-dimensional unit "cube." This is convenient for purposes of dynamic programming, where we are not indifferent to units of measurement.}\]

\[2/ \text{ We assume the area on the holding remains intact over time. The purchase and sale of land, however, can easily be incorporated within this framework, if important.}\]
for $k,k'=1,\ldots,K$, and the set of column-restrictions,

$$\sum_{k=1}^{K} s^O(k,k') \cdot x^O_{it(k,k')} = 1, \text{ (proportionality restrictions)},$$

for $k'=1,\ldots,K$. Thus, (4.10a) captures all origin-destination restrictions, while (4.10b) and (4.10c) reflect the proportional allocation of $a_{it-1}(k')$ to all admissible land-uses in the next year.

It is worth remarking that the crop rotation system, (4.6) and (4.10a)-(4.10c), resembles a first-order Markov process in which the $\{x^O_{it(k,k')}\}$ are akin to transition probabilities. In the present case, however, rather than being constants, the transition probabilities represent a deliberate allocation decision on the part of each producer (see Bellman [1977]).

For notational convenience we define the land-allocation decision-vector, $x^O_{it}$, as the set of all elements in the matrix, $X^O_{it}$, which are not identically zero, i.e.,

$$x^O_{it} \equiv \text{vec}(x^O_{it(k,k')} ; s^O(k,k') = 1).$$

We illustrate the land-rotation system under the present taxonomy by displaying a possible choice of the land-allocation restriction matrix, $D^O$, in Figure 1. Each admissible transition cell, $k' - k$, is shaded, while inadmissible states are left blank.

Columns 1 to $K_1 - 1$ indicate that all perennial areas of these ages in year $t-1$ must either shift-up one age-class from $k'= k - 1$ to $k$; be uprooted and cleared in year $t$ (row $K_1 + K_2 + 1$); or be abandoned as fallow land (row $K_1 + K_2 + 2$). Column $K_1$, referring to the maximum economic age-class, implies...
<table>
<thead>
<tr>
<th>Land Use in Period $t$</th>
<th>Land Use in Period $t-1$</th>
<th>Perennial</th>
<th>Alternative Crops</th>
<th>Other Uses</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k'$</td>
<td></td>
<td>$a_1^{(1)}$</td>
<td>$a_1^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>$k$</td>
<td></td>
<td>$a_{k-1}^{(1)}$</td>
<td>$a_{k-1}^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>$k_{t-1}$</td>
<td></td>
<td>$a_{k_{t-1}}^{(1)}$</td>
<td>$a_{k_{t-1}}^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>$k_1$</td>
<td></td>
<td>$a_{k_1}^{(1)}$</td>
<td>$a_{k_1}^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>$k_{t-2}$</td>
<td></td>
<td>$a_{k_{t-2}}^{(1)}$</td>
<td>$a_{k_{t-2}}^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>$k_2$</td>
<td></td>
<td>$a_{k_2}^{(1)}$</td>
<td>$a_{k_2}^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>$k_{t-3}$</td>
<td></td>
<td>$a_{k_{t-3}}^{(1)}$</td>
<td>$a_{k_{t-3}}^{(2)}$</td>
<td></td>
</tr>
<tr>
<td>$k_{t-4}$</td>
<td></td>
<td>$a_{k_{t-4}}^{(1)}$</td>
<td>$a_{k_{t-4}}^{(2)}$</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 1:** Admissible Crop Rotations -- The Pattern of Matrix $D^0$. 
the area must either be uprooted and cleared or abandoned, and continuation to the next age class is ruled out.

Before proceeding further, we should remark that in the case of a pure perennial holding with no alternative crops, fallow land or land in non-agricultural uses, then the land-allocation restriction matrix would be of the simple form,

\[
d^0 = \begin{bmatrix}
0^0 & 0 & 0 \\
d_{21}^0 & 0 & 0 \\
\vdots & \vdots & \vdots \\
d_{31}^0 & d_{32}^0 & d_{33}^0 \\
\end{bmatrix}
\]

where \(d_{21}^0\) is a \((K_1-1)\times(K_1-1)\) identity matrix, \(d_{31}^0\) is the unit vector and \(d_{13}^0, d_{32}^0, d_{33}^0\) are all one. A pure rotation system, with a one-year time-cost for uprooting and clearing, emerges in (4.12) in the special case where no uprootings occur until the maximum age \(K_1\) has been reached (i.e.,

\[
d_{21}^0 = r_{K_1-1}, \quad d_{13}^0 = 1, \quad d_{31}^0 = 0, \quad d_{32}^0 = 1 \text{ and } d_{33}^0 = 0 \).
\]

Such a process is known in the statistical time-series literature as a circulant (Anderson [1971]), with a period of \(K_1-1\) years, implying that each parcel of land will rotate through its complete lifecycle, plus one year for uprooting and clearing.
before replanting, in $K_1+1$ years. 1

Returning to the example of Figure 1, note that we are treating the
case in which all alternative crops are annuals and such land can be
"completely rotated" between all such crops in any successive pair of
years. 2 Hence, with unrestricted rotation among the alternative crops, we
obtain the $K_2 \times K_2$ "unit matrix" in the diagonal block for such transitions.
Also, we permit all land formerly under alternative crops (columns
$K_1+1, \ldots, K_1+K_2$) either to be newly planted with the perennial (row 1) or
removed from cultivation (row $K_1+K_2+3$). Both (here) are assumed to occur
without a lag.

1 A common assumption in the economic literature on perennial supply
(Bateman [1965], Behrman [1968], Wickens and Greenfield [1973], etc.) is
that the age-yield profile, $[y_{it}(k) : k=1, \ldots, K_1]$, is constant for each age,
$k$, regardless of the producer $i$ and year $t$ -- see section 4.2 below. If
so, and if the area under the perennial were constant, then the output
sequence, $[q_{it}^k : t=1,2, \ldots]$, would exhibit a regular periodic cycle about a
constant mean with a period of $K_1+1$ years. If, in addition, area were
uniformly distributed across the $K_1$ age-classes and the uprooting state,
then such a land allocation would eliminate any oscillations in the output
stream $[q_{it}]$. The realism of this assumption -- a constant age-yield
profile -- rests upon whether or not cultivators decisions or exogenous
shocks can influence the trajectory of yields and, if so, whether or not
the cultivation and harvesting decision do deviate from "recommended
practice" in response to changes in prices or other extraneous random
events. In short, the existing literature fails to account for the effect
on both yield and density of the input decisions and exogenous variables,
and limits discussion largely to land-allocation decisions through new
planting and replanting.

2 If a second perennial crop is among the alternatives, then we would
introduce a second "bordered diagonal block", as in (4.12), into the land-
allocation restriction matrix, $D^0$, of (4.18).
We now turn to other land uses ($j=3$). Note first that column $K_1+K_2+1$ indicates that all land uprooted and cleared from the preceding (perennial) use may either be replanted (row 1); diversified into any of the alternative crops (rows $K_1+1, ..., K_1+K_2$); or alienated for non-agricultural uses (row $K$). Column $K_1+K_2+2$ implies that fallow land either remains fallow (row $K_1+K_2+2$) or must be uprooted and cleared (row $K_1+K_2+1$) -- which takes a year -- before it is available for cultivation or other non-agricultural uses. Finally, element $(K,K)$ exhibits the "sink" -- once land is alienated for non-agricultural purposes, it never returns to cultivation.

The particular example in Figure 1 is meant to be illustrative of the flexibility of a land-rotation system in (4.6). Many useful extensions are possible. The choice of elements, $\delta^0(k,k')$ in $D^0$ can be made to represent land-allocation decisions in a mixed agricultural system (containing perennials of different life-spans and annual crops), as well as capture land on annual crops which also proceeds through a crop rotation sequence, involving, where appropriate, fallow periods.1/ In addition, one can also read single-and double-cropping sequences within a given year--of particular importance with respect to annual crops such as rice.

Figure 1 also provides several examples of costs-of-adjustment, measured in units of time required to effect a transition from one land-use to another.2/ Other direct costs-of-adjustment arise as a result of the labor and materials required to effect all selected transitions -- see section

---

1/ A fallow state for annual crops simply requires treating one of the "alternative crops," say $K_2$, as land which is non-producing -- presumably for soil conservation, etc.

2/ Here, time has an implicit (opportunity) cost given by the profit which would have been earned on alternative land-uses during that period.
4.4. The various time, labor, and materials costs-of-adjustment associated with land preparation thus act as penalty functions on certain types of land-use transitions. In general, the natural ageing process of a perennial is (virtually) costless, whereas uprooting and clearing of an existing perennial stand or previously fallow land acts as a barrier to replanting and/or new planting. An exception (see below) occurs when minimum maintenance standards are required.

4.2 The Technology of a Perennial Crop:

Having determined the age-composition and area under the perennial, \( a_{it}^1 \), as a consequence of the decision-vector, \( x_{it}^0 \), we consider next the technology of perennial crops and the associated planting, cultivation and harvesting decisions contained in the vector, \( x_{it}^1 \), of (4.3).

We assume that the perennial has a minimum gestation period of \( G \) years, during which no crop obtains, and a maximum economic life-span of \( K_l \) years. It follows that the total output, \( q_{it}^1 \), may be represented as the sum, across all vintages, of the product of the area, \( a_{it}^1(k) \), and the yield-per-acre, \( y_{it}(k) \), of age \( k \), i.e.,

\[
q_{it}^1 = a_{it}^1 \cdot y_{it} = \sum_{k=1}^{K_l} a_{it}^1(k) \cdot y_{it}(k). \tag{4.13}
\]

The standard approach is to assume that the relationship between yield-per-acre and age -- the so-called age-yield profile -- is a set of unknown, biologically-determined constants,

\[
y_{it}(k) = \begin{cases} 
0 & \text{if } 1 \leq k \leq G \\
\delta_k & \text{if } G < k \leq K_l.
\end{cases} \tag{4.14}
\]
Various formulations of this model have been given. Bateman [1965] assumes the \( \beta_k \) for cocoa follow a two-level step-function, with one level between first-bearing, \( G+1 \), and full-bearing age, \( G_{\text{max}} \), and a second constant level thereafter. Etherington [1973] assumes yields for tea steadily rise between ages \( G+1 \) and \( G_{\text{max}} \), and are constant thereafter -- apart from oscillations over the pruning cycle. \(^{1/}\) Smit [1976] assumes (for rubber) that the \( \beta_k \) follow a low-degree polynomial. Wickens and Greenfield [1973] essentially permit the \( \beta_k \) to be free parameters. In all cases, however, agronomic considerations suggest that the \( y_{it}(k) \) also vary with current and preceding levels of input decisions and exogenous factors. We now consider how such a model may be formulated.

Let the decision-vector for the perennial, \( x_{it}^1 \), be written as

\[
x_{it}^1 = (x_{it}^1(1), x_{it}^1(2), \ldots, x_{it}^1(K_t))',
\]

where \( x_{it}^1(k) \) refers to all decisions taken with respect to age-class \( k \) in year \( t \). Further, we shall decompose \( x_{it}^1(k) \) as follows:

\[
x_{it}^1(k) = \begin{bmatrix} x_{it}^{1,1}(k) \\ x_{it}^{1,2}(k) \\ x_{it}^{1,3}(k) \end{bmatrix}
\]

(planting decisions)

(4.15)

We now require some notational preliminaries.

\(^{1/}\) See Eden [1965] for a classic description of tea cultivation.
For any vector, say \( v_{it}(k) \), referring to age-class \( k \) of a perennial in year \( t \), we denote the current and preceding sequence of \( n \) lagged vectors as

\[
V_{it}(k,n) = [v_{it}(k) v_{it-1}(k-1) \ldots v_{it-n}(k-n)].
\]

(4.17)

Thus, e.g., \( \chi_{it}^{1}(k,k-1) \) would represent the complete history of decisions taken with respect to the perennial stock from the year of planting, \( t-k+1 \), to its current age, \( k \), in year \( t \).\(^1\) We also introduce the \( K_1 \times (n+1) \) matrix, \( V_{it}(n) \), with vector "elements:"

\[
V_{it}(n) = \begin{bmatrix}
v_{it}(1) & 0 & \ldots & 0 \\
v_{it}(2) & v_{it-1}(1) & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
v_{it}(n+1) & v_{it-1}(n) & \ldots & v_{it-n}(1) \\
\vdots & \vdots & \ddots & \vdots \\
v_{it}(K_1) & v_{it-1}(K_1-1) & \ldots & v_{it-n}(K_1-n)
\end{bmatrix} \equiv \begin{bmatrix}
v_{it}(1,n) \\
v_{it}(2,n) \\
\vdots \\
v_{it}(n+1,n) \\
v_{it}(K_1,n)
\end{bmatrix}
\]

(4.18)

so that, e.g., \( \chi_{it}^{1}(K_1) \) would denote the set of all possible decision-vectors associated with every age class in year \( t \) on a perennial holding containing \( K_1 \) age-classes.

Thus, between years \( t-1 \) and \( t \), we have the following recursion relation.

\(^1\) If \( n > k \), we add a suitable number of zero vectors as in (4.18).
This notation defines a rotation system for any lagged sequence of vectors associated with the separate age-classes of a perennial crop for any choice of \( n = 1, \ldots, K_1 \), and is applicable to decision-vectors as well as subsequent definitions of other non-decision endogenous variables. Note that \( \mathbf{v}_{it} \) in (4.18) is the "innovation vector" in year \( t \), whilst \( \mathbf{v}_{it-1}(K_1, n) \) is the "vector" (defined by (4.17)) associated with the maximum age-class, \( K_1 \), in the preceding year — applicable in year \( t-1 \), and thus contained in \( \mathbf{v}_{it-1}(n) \), but no longer relevant in year \( t \). Indeed, by writing \( \mathbf{v}_{it-1}(n) \) in the form as given in the second expression of (4.18), we can immediately recognize the portion of \( \mathbf{v}_{it-1}(n) \) contained again in \( \mathbf{v}_{it}(n) \), which we denote by

\[
\mathbf{v}^g_{it-1}(n) = [\mathbf{v}_{it-1}(1, n)' \mathbf{v}_{it-1}(2, n)' \ldots \mathbf{v}_{it-1}(K_1 - 1, n)'],
\]

i.e., the set of vectors associated with all but the maximum age class \( K_1 \) in year \( t-1 \). Thus, the set of lagged decision vectors, which serve as state-variables in year \( t \) are contained in the matrix, \( \mathbf{X}^g_{it}(n_1 - 1) \), for suitable choice of the lag-length, \( n_1 \). This notation will subsequently be employed to define the relevant state-variables in our discussion of the dynamic programming formulation of a perennial supply model.
We shall be interested in a model determining the following endogenous variables in year $t$:

$$
\mathbf{d}_{it} \equiv [d_{it}(1) \ d_{it}(2) \ldots \ d_{it}(K_t)]' \quad \text{(density)}
$$

$$
\mathbf{Y}_{it}^{\text{Pot}} \equiv [y_{it}^{\text{Pot}}(1) \ y_{it}^{\text{Pot}}(2) \ldots \ y_{it}^{\text{Pot}}(K_t)]' \quad \text{(potential yield per tree)}
$$

$$
\mathbf{Y}_{it}^{T} \equiv [y_{it}^T(1) \ y_{it}^T(2) \ldots \ y_{it}^T(K_t)]' \quad \text{(actual yield per tree)}
$$

$$
\mathbf{X}_{it} \equiv [y_{it}(1) \ y_{it}(2) \ldots \ y_{it}(K_t)]' \quad \text{(actual yield per unit area)}
$$

of which $\mathbf{Y}_{it}^{\text{Pot}}$ is generally not reported as data, and the remaining variables are connected by the identity,$^1$\footnote{1/ This implicitly defines the dot product ($\cdot$).}

$$
\mathbf{X}_{it} = \mathbf{Y}_{it}^{T} \cdot \mathbf{d}_{it} \equiv (y_{it}^T(k) \cdot d_{it}(k))
$$

-- i.e., actual yields per unit area are the dot product of actual yields per tree and the number of surviving trees per unit area. It remains, therefore, to develop a model for the trajectories of yields per tree, $\{y_{it}^T(\cdot)\}$, and density, $\{d_{it}(\cdot)\}$, utilizing the intervening trajectory of potential yields per tree, $\{y_{it}^{\text{Pot}}(\cdot)\}$, to capture the harvesting decision.

We shall consider two types of exogenous or non-controllable inputs. The vector $\mathbf{w}_{it}$, varies both with respect to time and place (see Hartley [1983]), and contains variables (such as indices of weather, disease and pest incidence, etc.) which affect current and subsequent values of the
endogenous variables in (4.22a)-(4.22d) above. The vector, $\mathbf{\psi}_i$, contains
variables (such as soil-type, land-slope, elevation and, possibly, certain
characteristics of the producer) which vary cross-sectionally, but not over
time. In the former case, if considering a particular age-class $k$, and if the
length of the lag-effect of past values of $\mathbf{u}_{it}$ on an endogenous variable is of
duration up to $n_3$ years, then we shall use the matrix,

$$
\mathbf{w}_{it}(n_3) = [\mathbf{w}_{it}, \mathbf{w}_{it-1}, \ldots, \mathbf{w}_{it-n_3}]
\equiv [\mathbf{w}_{it}, \mathbf{w}_{it-1}(n_3-1)],
$$

(4.24)

to denote the relevant vector sequence. Also, since the vector, $\mathbf{u}_{it}$, applies
to all age classes, we may collapse the recursion formula given in (4.19) to

$$
[w_{it}(n_3) \mathbf{w}_{it-n_3-1}] = [\mathbf{w}_{it}, \mathbf{w}_{it-1}(n_3)],
$$

(4.25)

where $\mathbf{w}_{it}$ is the innovation vector and $\mathbf{w}_{it-n_3-1}$ is irrelevant in year $t$.

Finally, $\mathbf{w}_{it-1}(n_3-1)$, defined implicitly by (4.24), denotes the exogenous
state-variables relevant to year $t$ for given $n$. 1/

We shall subsequently argue that lagged values of the observed
endogenous variables will, in part, determine their current realizations.
Such lagged endogenous variables will include the preceding land-allocation
vector, $\mathbf{a}_{it-1}$, the preceding density vector, $\mathbf{d}_{it-1}$, and, say, up to $n_2$
preceding values of the actual yield per tree,

__________________

1/ This assumes that a given realization index, $\mathbf{u}_{it}$ -- see Hartley [1983], is
appropriate for each age-class.
for \( k=1, \ldots, K_1 \). This has a corresponding matrix representation across all age-classes, \( Y_{it-1}^T(n_2-1) \), akin to (4.18) except that the first \( G \) rows are zeroes, due to the gestation period. The rotation system follows (4.19).

Finally, the set of all lagged endogenous state variables will be denoted by \( v_{it-1}^g(n_2-1) \), and is implicitly defined by

\[
\begin{bmatrix}
v_{it-1}^g(n) \\
\vdots \\
d_{it-1}(K_1) \end{bmatrix} =
\begin{bmatrix}
1 & a_{it-1} & a_{it-1}^2 & a_{it-1}^3 \\
0 & 0 & 0 & 0 \\
d_{it-1}(K_1) & v_{it-1}^T(n)
\end{bmatrix}
\]

(4.27)

With the completion of these preliminaries, we now discuss each of the types of decisions in \( x_{it}^1 \), and then consider various representations of the model, which, under appropriate simplifying assumptions, will be applicable to the perennial at issue and/or to different types of data bases encountered.

### 4.2.1 Planting Decision Variables:

Let \( x_{it}^1(k) \) denote the number of plants per unit area supplied to age-class \( k \). In the vast majority of cases, the only opportunity to supply plants is in the year of establishment of the original stand, i.e., when \( k=i \). In this case, we have the planting decision variables,

\[
x_{it}^1(k) = 0, \quad \text{otherwise.}
\]

(4.28b)

\[
x_{it}^1(k) = 0, \quad \text{if } a_{it}^1(1) > 0
\]

(4.28a)
In some exceptional cases (such as with tea), infilling becomes possible at regular intervals (when the bushes are pruned-back, permitting an opportunity for new seedlings to grow within an otherwise mature stand). In this case one has the option to supply plants up to the original stand density to eliminate any vacancies. Here, in addition to (4.28a), we would have

\[ 0 \leq x_{it}^{1,1}(k) \leq x_{it-k+1}^{1,1}(1) - d_{it-l}(k-1), \quad \text{if } k > 1 \text{ and } a_{it}^{1}(k) > 0 \]  

(4.28c)

\[ x_{it}^{1,1}(k) = 0, \quad \text{if } a_{it}^{1}(k) = 0 \text{ or infilling is not possible on age-class } k \text{ in year } t, \]  

(4.28d)

where \( x_{it}^{1,1}(1) \) is the original stand density in the year of planting, \( t-k+1 \), and \( d_{it-l}(k-1) \) is the surviving density in the previous year. In practice, the length of the pruning cycle varies with the agro-climatic zone -- and, hence, with \( u_t \). \(^2\) Also, pruning activity (as a risk-averse strategy) tends to be uniformly distributed across mature areas of a given holding, so that the resulting cyclical behavior in yields at the field level will often not be evident in aggregate (national) data, but may be evident in the sequence \( \{q_{it}^{1}\} \) at the holding level. We note that in this case the land-allocation decision would have to be amended by introducing an additional option -- viz., to permit the fraction, \( \left\{ x_{it}^{1,1}(k)/x_{it-k+1}^{1,1}(1) \right\} \), of \( a_{it-l}(k-1) \) to be rotated back to age-class 1, in addition to the replantings and new

---

1/ With tea, infilling can only be done in the year of pruning.

2/ Pruning cycles for tea in Sri Lanka vary from three to five years in duration -- depending (marginally) upon the elevation.
plantings previously accommodated. This additional complication will be ignored in subsequent discussion. 1/  

4.2.2 Cultivation Decision Variables:  

Cultivation activities and associated inputs into the production process involve both labor and materials. The weeding process may be done manually -- a direct labor input, or by chemicals -- a direct materials input decision, with associated labor requirements for application. Fertilizer application and disease and pest control activities are similar -- both involve the direct materials input of fertilizer, pesticides, fungicides, etc., with associated labor requirements. The remaining cultivation activities usually are general maintenance decisions (fencing, terracing, drainage system repairs, etc.), with associated labor and/or materials requirements. Of these, fertilizer dosage levels are generally set on a per-tree basis, while the remainder are decisions applicable on a per-acre basis, and, with knowledge of \( d_{it} \) and \( a_{it} ^{1} \), can be converted to commensurable units-per-tree (or per-acre) for each age class.

We denote the vector of all cultivation input-decisions associated with trees in age-class \( k \) by the vector \( x_{it} ^{1,2}(k) \), where

\[
x_{it} ^{1,2}(k) = 0, \quad \text{if } a_{it} ^{1}(k) = 0 \\
x_{it} ^{1,2}(k) \geq x_{min} ^{1,2}(k) \geq 0, \quad \text{if } a_{it} ^{1}(k) > 0,
\]

We denote the vector of all cultivation input-decisions associated with trees in age-class \( k \) by the vector \( x_{it} ^{1,2}(k) \), where

\[
x_{it} ^{1,2}(k) = 0, \quad \text{if } a_{it} ^{1}(k) = 0 \\
x_{it} ^{1,2}(k) \geq x_{min} ^{1,2}(k) \geq 0, \quad \text{if } a_{it} ^{1}(k) > 0,
\]

Practically speaking, infilling is a way of introducing new clonal varieties into an existing stand -- resulting in a so-called "mixed stand" of higher yield potential. This term, however, is rather imprecise, and suggests "field-splitting" to accommodate all pure varieties in existence.
in which $x_{\text{min}}^{1/2}(k)$ denotes the minimum levels of all cultivation inputs (possibly $0$) necessary for continued cultivation of each age-class, $k$. Such costs are thus fixed, given $a(k) > 0$, and can be avoided by a land-allocation decision to abandon the area. Thus, abandonment can be viewed as a rational economic decision.\(^1\)

The effect of current cultivation decisions is complex. In some cases, such as with pesticides and fungicide sprays, the response of yields and stand densities is largely restricted to the year of application. In other cases, particularly with fertilizer, a yield response is distributed over several subsequent years.

4.2.3. The Harvesting Decision Variable:

A harvesting decision involves the commitment of labor and materials to collect all, or only a part, of the available or potential perennial crop. In the majority of cases, the harvesting activity has no effect on the yield potential of the stand in subsequent years. Crop not harvested simply falls to the ground and decomposes.\(^2\) In other cases -- particularly with rubber, which is continuously tapped over the course of a year, the harvesting decision involves establishing a level of "tapping intensity," defined by an index (ATI) of the amount of tree bark which is cut away within a given

---

1/ Abandonment is often extremely important to accommodate. In Sri Lanka, approximately 100,000 acres, or about a sixth of the officially registered area under rubber, has been abandoned between 1933 and the present. Similarly, with Brazilian coffee, Arak [1969] reports massive abandonment of previously cultivated areas.

2/ For example, citrus fruits or cocoa pods. This, in fact, may increase soil fertility -- a dynamic feedback effect.
period.\textsuperscript{1} Here, raising the tapping intensity currently will adversely affect the future yield potential of the tree, as nutrients are diverted from latex production to bark renewal (see again Barlow [1978] or Peries [1970]). Indeed, in the extreme case, so-called "slaughter-tapping" will subsequently kill the tree. In short, with rubber the preceding harvesting decisions affect current potential yields -- involving a physical trade-off between current and future yields.

We denote the harvesting decision by the index, \( x_{it}^{1,3}(k) \), applicable to tree stock of age \( k \) in year \( t \), with domain,

\[
x_{it}^{1,3}(k) = \begin{cases} 0, & \text{if } a_{it}^{1}(k) = 0 \text{ or } 1 < k < G, \\ \geq 0, & \text{if } a_{it}^{1}(k) > 0 \text{ and } G < k \leq K_l. \\
\end{cases}
\]  

\hspace{1cm} (4.30a) \hspace{1cm} (4.30b)

\textbf{4.2.4 Density:} We consider the typical case of a perennial, in which the only direct control the cultivator may exercise over the density, \( d_{it}(k) \), is through the planting decision on the initial stand density, \( x_{it}^{1,1}(l) \), associated with the year of planting, \( t-k+1 \), for stock of age \( k \) in year \( t \). Given the initial stand density will be expected to decline monotonically over time, due to the effects of weather conditions, disease and pest incidence, etc., measured in \( z_{it} \); and with location-specific factors in \( u_{it} \). The only means by which the cultivator may arrest this decline is indirectly, through the use of pesticides, insecticides, fungicides, etc. -- present in the vector of

\textsuperscript{1} However, the ATI does not reflect the fact that there are many tapping systems with the same (relative) tapping intensity (Peries [1970] or Barlow [1978]).
cultivation and maintenance decision variables, \( x_{it}^{1,2}(k) \).

The simplest model for the stand density is to postulate a constant rate of depletion, \( \delta \), per year, \( 0 < \delta < 1 \), so that the surviving density follows a first-order auto-regression, except for the planting decision or a removal decision. Thus,

\[
d_{it}(k) = \begin{cases} 
0, & \text{if } a_{it}(k) = 0 \\
x_{it}^{1,1}(1), & \text{if } k = 1 \\
(1-\delta) d_{it-1}(k-1), & \text{if } 1 < k \leq K_1. 
\end{cases} 
\tag{4.31}
\]

In the event that only the sequence of initial stands, \( \{x_{it}^{1,1}(k+1)\} \), is available as observed data, one would then have to utilize the solution to (4.31) given by

\[
d_{it}(k) = (1-\delta)^{k-1} x_{it}^{1,1}(1), \text{ if } a_{it}(k) > 0, 
\tag{4.32}
\]

where \( x_{it}^{1,1}(1) \), the number of trees per unit area planted in year \( t \), is defined via the decision, (4.28a).

A more realistic model is to postulate a variable rate of depletion, depending upon \( x_{it}^{1,2}(k) \), \( u_{it} \), and \( u_t \). In this case, we have the model,

\[
d_{it}(k) = \begin{cases} 
0, & \text{if } a_{it}(k) = 0 \\
x_{it}^{1,1}(1), & \text{if } k = 1 \\
[1-D(x_{it}^{1,2}(k), u_{it}, \delta)] d_{it-1}(k-1), & \text{if } 1 < k \leq K_1, 
\end{cases} 
\tag{4.32a}
\]

where \( D \) denotes (say) a cumulative distribution function, defined over the \([0,1]\) range in order to represent the rate of depletion, and \( \delta \) is now a
vector of parameters.

In a few cases (such as tea), the cultivator also has the option of "infilling" vacancies that have occurred up to the level of the initial stand density. In such cases we would employ the infilling decision variable, \( x_{i1}^{1,1}(k) \), defined by (4.28c) and (4.28d).

The density rotation system implied by (4.33) may therefore be written as:

\[
d_{it} = \begin{bmatrix} x_{i1}^{1,1}(1) \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0' \\ \vdots \\ I - D^1(x_{i1}^{1,2}, \omega_{it}, \psi_{i1}; \delta) : 0 \end{bmatrix} \cdot d_{it-1},
\]

(4.34)

where \( D^1 \) is a \( K_1 \times 1 \) order diagonal matrix of depletion rates and \( x_{i1}^{1,2} \) is a subvector of \( x_{i1} \) of (4.15). The diagonal elements of \( D^1 \) are thus cumulative distribution functions.

In the event that only the initial stand densities, along with the sequence of intervening cultivation inputs, and weather conditions etc., \( W_{it}(k) \) of (4.24), are available, we would then have to represent the surviving densities of age \( k \) (analogous to (4.32)) as:

\[
\begin{array}{ll}
1/ & \text{To accommodate this feature within the density model (along with the change in the land rotation system previously noted) would require establishing an area-weighted average of the densities planted and infilled for } d_{it}(1). \text{ Alternatively, the area under the age-classes being infilled may be reduced and the infilled areas treated as replantings. We shall not discuss the special case of infilling further.} \\
2/ & \text{It is often the case that records on initial stand densities are available whereas the current densities -- requiring a tree-census -- are not. Indeed, the rectangular layout of most plantings permits one to infer initial stands currently in the absence of past records.}
\end{array}
\]
or, employing the matrix notation developed earlier for such lagged sequences,

\[ d_{it} = d(x_{it}^{1,1}(K_1), x_{it}^{1,2}(K_1), w_{it}(K_1), u_{it}; \delta). \]  

(4.35b)

This would then be substituted into (4.23). Thus, having data on the sequence of densities \( \{d_{it}\} \) permits the use of (4.34) and a considerable savings in data requirements. In the absence of data on either \( d_{it} \) or \( x_{it}^{1,2}(K_1) \) via (4.18), it is often possible to obtain long time series on weather conditions from neighboring reporting stations. Here, with a simplifying assumption of a constant initial stand density, the \( \{w_{it}\} \) may still be employed in (4.35a) to separate the process of declining densities from the age-profile of yields per tree.

### 4.2.5 Potential Yield Per Tree:

We now turn to the determination of the potential yield per tree, \( y_{it}^{\text{Pot}}(k) \), associated with age-class \( k \) in year \( t \). Clearly, during the gestation period, potential yields are zero, and

\[ y_{it}^{\text{Pot}}(k) = 0, \quad \text{if } a_{it}(k) = 0 \quad \text{or} \quad 1 \leq k \leq G. \]  

(4.36)

For the remaining bearing ages, \( G < k \leq K_1 \), the simplest model to accommodate the trajectory of potential yields is to assume they are solely a function of age,

\[ y_{it}^{\text{Pot}}(k) = a_{k} > 0, \quad \text{if } a_{it}(k) > 0 \text{ and } G < k \leq K_1. \]  

(4.37)
where the typical pattern for the \(a_k\) is to rise sharply during the early bearing ages, remain at a plateau during prime-bearing ages, and then steadily decline with age. A low-order polynomial in \(k\) might be entertained as a parsimonious parametrization (Smit [1976]). If stand densities and the proportion of potential crop harvested were also solely functions of age, then (4.36) and (4.37) would lead to a constant age-yield (per acre) profile, as in (4.14).

The natural extension of this model is to formulate a potential yield function, which accounts for the effects of current and preceding levels of cultivation inputs, \(X^{1,2}_{it}(k,n_{12})\); current and preceding levels of the weather and disease conditions, \(W_{it}(n_3)\); preceding levels of yield per tree, \(Y^{T}_{it-1}(k-1,n_{2}^{-1})\); the current stand density, \(d_{it}(k)\); and location-specific characteristics, \(u_i\), on the level of potential yield, \(y^{\text{pot}}_{it}(k)\). In addition, for some perennials (such as rubber), the preceding levels of harvesting inputs (tapping intensities), \(X^{1,3}_{it-1}(k-1,n_{13}^{-1})\), will influence current potential yields. In each case the constants, \(n_{2j}\) or \(n_{j}\), denote the appropriate lag-length involved. We represent this model as:

\[
y^{\text{pot}}_{it}(k) = f^1(X^{1,2}_{it}(k,n_{12}), X^{1,3}_{it-1}(k,n_{13}^{-1}), Y^{T}_{it-1}(k-1,n_{2}^{-1}), W_{it}(n_3), d_{it}(k), u_i; \theta), \tag{4.38}
\]

for \(k = G+1, \ldots, K_1\). Thus, over all age classes, we have the general form,

\[
y^{\text{pot}}_{it} = f^1(X^{1,2}_{it}(n_{12}), X^{1,3}_{it-1}(n_{13}), Y^{T}_{it-1}(n_2), W_{it}(n_3), d_{it}, u_i; \theta), \tag{4.39}
\]

where \(f^1\) is a vector of functions.

There are clearly many possible specifications for the potential yield functions and the appropriate choice will depend upon knowledge of the
perennial under consideration and the richness of the available data base. 1/

4.2.6 Actual Yield Per Tree:

The simplest approach to modeling actual yield-per-tree, \( y_{tc}(k) \), is to assume that potential yields are always harvested, i.e., 2/

\[
y_{tc}(k) = y_{tc}^{pot}(k).
\] (4.40)

A more realistic model incorporates harvesting as a decision, with associated labor and materials requirements. 2/ The level of potential yield for a perennial represents, inter alia, decisions taken in preceding years with respect to planting, cultivation and harvesting inputs, presuming that prices, would follow a particular trajectory. In the event that current crop prices, relative to labor and materials costs, differ from those formerly anticipated, the producer may revise his current harvesting decision in the light of new information. Alternatively, labor shortages at the time of harvest may preclude collection of the entire crop. Finally, weather conditions immediately surrounding harvest-time may destroy part of the crop.

1/ Ideally, we would like to employ a "flexible functional form" embodying the appropriate agronomic desiderata, by analogy with the work on imposition of "curvature restrictions" on utility, profit, cost, etc. functions (e.g., Gallant and Golub [1984], Diewert and Wales [1984]).

2/ This model is implicit in most treatments of perennial supply -- the exception being Willkens and Greenfield [1973], who postulate that the proportion of potential crop harvested is a distributed lag of current and past relative prices. However, without further restriction, there is no guarantee that the predicted values will remain within the unit interval.

3/ The reason for transforming to labor and materials within any input process is that those are the variables to which prices are attached, whereas in many cases (e.g., fertilizer) only the material input is relevant to output determination.
or interfere with its collection and distribution.

Two types of models may be formulated to incorporate these issues. In most cases of perennials with a discrete harvest-time, one may postulate a harvesting function, \( h \), and define actual yields-per-tree as

\[
y^T_{it}(k) = h(x^{1,3}_{it}(k), \omega_{it} \xi n) \leq y^\text{pot}_{it}(k),
\]

where \( x^{1,3}_{it}(k) \) would represent the labor required to collect the level of yields, \( y^T_{it}(k) \); \( \omega_{it} \) contains appropriate weather indices; and \( \xi n \) is a vector of harvesting function parameters.\(^1\) The function, \( h \), is generally monotonic increasing, with \( \frac{\partial^2 h}{\partial x^3} (x^{1,3}_{it}(k))^2 < 0 \) (reflecting the increasing cost of collecting a larger tree crop)\(^2\) and \( h(0, \omega_{it} ; \xi n) = 0 \). \(^3\) The upper bound on \( y^T_{it}(k) \) is set by the unobserved potential yield.

For other perennials (such as rubber) the harvesting decision-variable may be adjusted continuously over the crop year, and involves fixing the tapping intensity, \( x^{1,3}_{it}(k) \). This, in turn, determines both the proportion of available latex crop (apart from interference by untimely rainfall) and the tapping labor and materials requirements. In this case we would employ a model of the form,

\[\text{Note the sequential nature of the specification: Conditional upon } y^\text{pot}_{it}(k), \text{ a harvesting decision is made, where the former sets an upper bound. In cases where } y^\text{pot}_{it} \text{ is a latent (unobserved) variable, we have a further complication.}\]

\[\text{2/ Apples at the top of a tree are more difficult to harvest.}\]

\[\text{3/ This permits accommodations of the activity of "resting" a rubber stand following a period of intensive tapping by setting } x^{1,3}_{it}(k) = 0. \text{ Also, to avoid the stand falling into a state of abandonment, certain minimum maintenance activities, } x^{1,2}_{it}(k), \text{ may be required.}\]
where \( H \) is now a cumulative distribution function, defining the proportion of potential crop collected. Dynamic feedback effects of \( x_{it}^{1,3}(k) \) on future levels of potential yield are explicit in the inclusion of \( x_{it-1}^{1,3} \) in the potential yield function. Again, we choose \( H \) so that \( H(0, \omega_{it}; \pi) = 0 \).  

### 4.3 Alternative Crops

For each alternative annual crop, \( k = 1, \ldots, K_2 \), it will suffice for present purposes to define the production function,

\[
y_{it}^*(k) = H(x_{it}^{1,3}(k), \omega_{it}, \pi) y_{it}^{pot}(k),
\]

(4.42)

where \( H \) is a cumulative distribution function. Dynamic feedback effects of \( x_{it}^{1,3}(k) \) on future levels of potential yield are explicit in the inclusion of \( x_{it-1}^{1,3} \) in the potential yield function. Again, we choose \( H \) so that \( H(0, \omega_{it}; \pi) = 0 \).  

### 4.4 Labor and Materials Requirements

To complete the model of the technology of the perennial holding, we must define the total labor and materials requirements associated with the

\[
y_{it}^2(k) = r_k^2(x_{it}^{2}(k), \omega_{it}, \omega_{it}^{2}(k); \sigma_k),
\]

(4.43)

where the decision-variables satisfy

\[
x_{it}^{2}(k) = 0, \quad \text{if} \quad a_{it}^{2}(k) = 0 \quad \text{or} \quad k \text{ refers to a fallow state} \quad (4.44a)
\]

\[
x_{it}^{2}(k) > 0, \quad \text{if} \quad a_{it}^{2}(k) > 0, \quad (4.44b)
\]

and represent the total levels of planting, cultivation and harvesting decisions taken on crop \( k \), while \( \sigma_k \) denotes a parameter vector appropriate to crop \( k \).
land-allocation, perennial crop and alternative crop decisions. Certain
decisions are direct labor and/or materials inputs, while other decisions
(such as $x_{it}^0$) may require them. Further, the units of the elements in $x_{it}$
are sometimes measured on a per-tree or per-acre basis. It follows that the
labor and materials requirements associated with $x_{it}$ can be written as

$$l_{it} = Z(x_{it}, a_{it}, a_{it-1}, d_{it}; \lambda) \quad (4.46)$$

$$m_{it} = M(x_{it}, a_{it}, a_{it-1}, d_{it}; u). \quad (4.47)$$

In most cases it is reasonable to assume that the labor and materials
requirements are linear process technologies, in which the
parameters, $\lambda$ and $u$, are known from "standard agronomic practice" as in
planters manuals and extension service circulars -- see also Chenery [1963]
for a classic example.

5. A Neoclassical Model for the Supply of a Perennial

A neoclassical model for the supply of a perennial crop is based upon
the presumption of rational producer behavior. Each producer is assumed to
have an objective function, which depends upon a set of decision-variables in
the control of the producer, and a set of exogenous and other state-variables
beyond current control. A neoclassical model assumes that the observed
decision-values in any year are those which maximize the objective function,
given the exogenous and other state variables and any constraints on the
domain of the decision-variables. Substitution of the optimal decisions into
the production function, etc. defines a neoclassical supply function.
In the case of a perennial, the existence of a gestation period and the dynamic aspects of the technology and land-rotation system imply that the rational producer must have a multi-period objective function. Furthermore, the prototype model also shows that certain of the decision-variables, other endogenous and the exogenous variables determined in the current year become state-variables in the next year, which, in turn, constrains next year's decisions. In short, whatever the multi-period objective function of the producer, the determination of optimal decisions in any year -- and, hence, the determination of a neoclassical model of perennial supply -- is a complex multi-stage optimization or dynamic programming problem, for which only numerical solutions may be obtained.

It is customary in the economics literature on perennial supply (e.g., Bateman [1965], Wickens and Greenfield [1973]) to suggest that the objective of producers is to maximize the discounted present value of the current and future stream of anticipated profits over a planning horizon of (say) N future years. This implies that, in order to define the objective function, the producer must first formulate a sequence of anticipated current and N future prices, etc. based upon information available at the time of decision-making. Thus, the anticipations-formation process becomes an integral part of a model of perennial supply, along with the agronomic, technological and land-rotation system equations in the prototype model of the preceding section.

We now consider formulation of the dynamic programming problem associated with a neoclassical model of perennial supply and methods to calculate optimal decisions. We then discuss methods for calibration and/or
estimation of the various parameters. 1

5.1 Optimal Decisions and Dynamic Programming

We shall represent a neoclassical model of perennial supply in terms of four types of variables. The vector of decision-variables, representing the land-allocation proportions and input decisions associated with the perennial and alternative crops, has been denoted by

\[ \mathbf{x}_{it} = [x_{it}^0, x_{it}^1, x_{it}^2]'. \] (5.1)

The vector of endogenous non-decision variables will be denoted by 2

\[ \mathbf{y}_{it} = [y_{it}^0, y_{it}^1, y_{it}^2', y_{it}^3] = [a_{it}, d_{it}, z_{it}, \text{Pot}_{it}, \text{Lit}_{it}, m_{it}, n_{it}]'. \] (5.2)

and the vector of exogenous variables, beyond the control of the producer, will be denoted by

\[ \mathbf{z}_{it} = [z_{it}^0, z_{it}^1, u_{it}]'. \] (5.3)

Finally, the vector of state-variables will consist of those lagged decision, endogenous non-decision and exogenous variables which influence current decisions and other endogenous variables, denoted in general by

---

1/ See Hartley [1984b] for a discussion of the distinction between calibration and estimation.

2/ This supercedes previous use of the symbol \( y_{it} \).
\[ s_{it} = \text{vec}(X_{it-1}(n_1-1), V_{it-1}(n_2-1), Z_{it-1}(n_3-1)). \]  

(5.4)

The maximum lags, \((n_1, n_2, n_3)\), associated with each type of state variable, will be termed the lag-structure of the model and characterize "the dynamics" of a perennial supply model.

The particular simplified version of the prototype model employed in a particular application will depend upon data availability and the perennial under consideration. All such models, however, should conform to the basic structure to be discussed below. It will be convenient, therefore, to illustrate in terms of the following model:

\[ v_{it} = a_{it} = [a_{1it}, a_{2it}, a_{3it}]' = x_{it}^0 \cdot a_{it-1}. \]  

(5.5a)

\[ v_{it}^{11} = d_{it} = d(x_{it}, x_{it-1}^{12}, w_{it}, d_{it-1}; a). \]  

(5.5b)

\[ h(x_{it}^{13}, v_{it}; a) \leq \text{Pot}, \quad \text{or} \]

\[ h(x_{it}^{13}, v_{it}; a) \cdot \text{Pot} \]

(5.5c)

\[ v_{it}^{12} = x_{it}^T = h(x_{it}^{13}, v_{it}; a) \cdot \text{Pot}, \]

(5.5d)

\[ v_{it}^{13} = \text{Pot} = \xi(x_{it}^{12}, w_{it}, x_{it-1}^{12}(n_2-1), x_{it-1}^{13}(n_3-1), d_{it}, u_{it}, 2), \]

(5.5e)

\[ v_{it}^{14} = z_{it} = x_{it}^T \cdot d_{it}. \]

(5.5f)

\[ v_{it}^{15} = q_{it} = a_{it} \cdot x_{it}. \]  

(5.5g)
\[ \nu_{it}^2 = \eta_{it}^2 = \xi_{it}^2 (x_{it}, a_{it}, u_{it}, \alpha_{it}; \alpha), \]  
(5.5g)

\[ \nu_{it}^{31} = \xi_{it} = g(x_{it}, a_{it}, a_{it-l}, d_{it}; \lambda), \]  
(5.5h)

\[ \nu_{it}^{33} = m_{it} = m(x_{it}, a_{it}, a_{it-l}, d_{it}; u), \]  
(5.5i)

\[ \nu_{it}^{33} = \eta_{it} = \rho_{it}^1 q_{it}^1 + \rho_{it}^2 q_{it}^2 - \rho_{it}^3 \xi_{it} - \rho_{it}^4 m_{it}, \]  
(5.5j)

where \( \xi_{it} \) is a latent or unobserved endogenous vector.

We may represent the structural form of the model consisting of equations, (5.5a)-(5.5j), more compactly as follows:

\[ \nu_{it} = g(\nu_{it}, x_{it}, z_{it}, \theta_2), \]  
(5.6)

where \( g \) is a vector of functions and \( \theta_2 \) is a vector of all parameters in the technology of the model -- in the present case,

\[ \theta_2 = [\lambda', u', \eta', \xi', a]. \]  
(5.7)

Inspection of equations (5.5a)-(5.5j) reveals that (5.6) is a recursive system in the \( \nu_{it} \) given \( x_{it} \), and may be solved for \( \nu_{it} \) by successive substitution to obtain the reduced form,\(^1\)

\(^1\) This is not necessary since (5.6) could otherwise be solved by computer for any particular value of \( \theta_2 \) (see Hartley [1984b] for the general simultaneous equations model).
\[ v_{\text{lt}} = v(x_{\text{lt}}, z_{\text{lt}}, s_{\text{lt}}; \beta) \]  

(5.8)

where \( v \) is a vector of reduced form functions.

It should also be stressed that the domain of the decision variables, \( x_{\text{lt}} \), is restricted. All elements in \( x_{\text{lt}} \) are non-negative. Also certain elements (the land-use proportions in \( x_{\text{lt}}^0 \)) must satisfy equality constraints, in that their sum is equal to one. Finally, the vector, \( x_{\text{lt}}^0 \), contained in the state-vector, \( s_{\text{lt}} \), conditions the admissible decisions within \( x_{\text{lt}}^0 \) -- i.e., in order for the elements in \( x_{\text{lt}}^0 \) associated with the allocation of \( a_{\text{lt}}^0(k') \) to \( a_{\text{lt}}(k) \) with \( s^0(k,k') = 1 \), the area \( a_{\text{lt}}^0(k') \) must be positive.

Otherwise, all such elements, \( x_{\text{lt}}^0(k',k) \), are identically zero, and no decision is required. A similar argument applies to the elements, \( x_{\text{lt}}^j(k), j = 1, 2, \) which will be identically zero, if \( a_{\text{lt}}^j(k) \) is zero, as a result of (5.5a). To represent the fact that the state-vector, \( s_{\text{lt}} \), conditions the domain of the decision-vector, \( x_{\text{lt}} \), we write,

\[ x_{\text{lt}} \in \mathbf{R}(s_{\text{lt}}), \]

(5.9)
where \( R \) denotes the domain of \( x_{it} \). 

\[ R \]

Finally, we note that in the model, (5.5a)-(5.5j), the state-vector is the special case of (5.4) consisting of

\[ s_{it} = \text{vec} \left( X_{it}^{1,2} (n_{12}+1)X_{it}^{1,3} (n_{13}+1) \right) \eta_{it}^{-1} \omega_{it+1} (n_{2}+1) \eta_{it}^{-1} (n_{3}+1) \], (5.10)

where, for notational convenience, we have used \( n_{1} = \max(n_{12}, n_{13}) \).

We now turn to the behavior of the producer. Under the maintained hypothesis, each producer is assumed to have a planning horizon of \( N \) future years beyond the current year \( t \). Over that planning period the producer must then formulate a trajectory of current and future actions for all of the exogenous variables in the model -- prices, \( p_{it} \), and states of nature, \( \omega_{it} \).

1/ The constraints on \( x_{it} \) are recursive. First, given \( a_{it-1} \), we determine \( x_{it-1} \), and hence \( a_{it} \), via (4.10a)-(4.10c). This, in turn, determines the feasible domain of \( x_{it}^{1} \) and \( x_{it}^{2} \), via (4.28a)-(4.30b) and (4.44a)-(4.44b).

2/ In the present methodology, involving the use of dynamic programming, the more constraints on \( x_{it} \), the better -- as this reduces the size of the region of decision space over which one must search to locate the optimal decision. This is in contrast to the traditional (regression-based) econometric approach for "limited dependent variables", where inequality constraints cause a proliferation of multiple numerical integrations.
which vary with time.\textsuperscript{1} Given this trajectory of anticipations, the producer must then formulate a plan or policy (with respect to the sequence of current and future decision variables) such that the present value of the discounted stream of current and future expected profits is maximized. From that optimal decision sequence, only the current year's optimal decision vector, \( x_{it}^* \), is then implemented.

In formulating the above problem we shall assume, for the present, that all structural parameters in \( \theta_2 \) of (5.7) are known constants, and that the sequence, \( \{ z_{it}, t \in T \} \), of anticipations of the exogenous vector, \( z_{it} \), for producer \( i \) for year \( t \), of the planning period, based on information available at the start of year \( t \), is given.\textsuperscript{2} The present problem, however, is to determine the optimal decision sequence \( \{ x_{it}, t \in T \} \), where \( x_{it}^* \) denotes the optimal decision planned for year \( t \), given the parameter vector, \( \theta_2 \); given the sequence \( \{ z_{it}, t \in T \} \) and given the constraints represented by (5.8) and (5.9). We then obtain \( x_{it}^* \equiv x_{it}^* \) as the optimal current decision, represented by the function,

\textsuperscript{1} In the next section we consider calibration of the elements in \( \theta_1 \) of modeling the anticipations-formation process governing \( \{ z_{it}, t \in T \} \) (with parameters \( \theta_1 \)) as joint problems.

\textsuperscript{2} In a global general equilibrium model, in which all supply and demand decisions determine price(s), which, in turn, feed back to determine supply decisions, these are no longer exogenous. Rather, prices are then non-decision endogenous variables. Thus, we are implicitly considering a partial equilibrium model of a single (small) producing country.
We define the anticipated profit of the producer in year $t+\tau$, within the plan commencing in year $t$, as the function,

$$\pi_{it,t+\tau} = \pi(\mathbf{x}_{it,t+\tau}^*, \hat{z}_{it,t+\tau}^*; \theta_2), \quad \tau = 0, 1, \ldots, N, \quad (5.12)$$

where here the $\{\hat{z}_{it,t+\tau}^*: \tau = 0, 1, \ldots, N\}$ and $\theta_2$ are assumed known; where the sequence of state-vectors, $\{\mathbf{s}_{it,t+\tau}^*\}$, obeys the recursion relation,

$$\mathbf{s}_{it,t+\tau}^* = \mathbf{g}(\mathbf{x}_{it,t+\tau}, \hat{z}_{it,t+\tau}^*, \mathbf{s}_{it,t+\tau-1}^*; \theta_2) \quad (5.13a)$$

for $\tau = 1, \ldots, N$, with the initial condition for the state-vector in year $t$, when $\tau = 0$,

$$\mathbf{s}_{it,t}^* = \mathbf{s}_{it}; \quad (5.13b)$$

and where the endogenous vector in (5.13a) is defined by

$$\mathbf{v}_{it,t+\tau-1}^* = \mathbf{v}(\mathbf{x}_{it,t+\tau-1}, \hat{z}_{it,t+\tau-1}^*, \mathbf{s}_{it,t+\tau-1}^*; \theta_2). \quad (5.14)$$

The function, $\mathbf{v}_+$, in (5.13a) "picks out" only the relevant elements of the vectors in (5.10) associated with the rotation systems discussed in section 4 defining $X_{it-1}^a(n-1), Y_{it-1}^a(n-1)$ and $Z_{it-1}^a(n-1)$. It follows from (5.12)-(5.14) that the objective function of the producer -- here, maximizing the present value of the discounted profit stream
over an $N$ year future planning horizon — may be represented by the function,

$$Q^N(X_{it}, t^{N}(N), Z_{it}, t^{N}(N), z_{it}^*; \theta_1, \theta_2) = \sum_{t=0}^{N} \frac{1}{(1+r_c)^t} \pi(x_{it}, t^{\tau}, z_{it}, t^{\tau}; z_{it}^*, t^{\tau}, z_{it}^*; \theta_2).$$

This is to be maximized with respect to the decision-sequence,

$$X_{it}, t^{N}(N) = [x_{it}, t^{N} \cdots x_{it}, t^{1} x_{it}, t] = [X_{it}, t^{N-1}(N-1) x_{it}, t],$$

for a given initial state vector, $s_{it}$, and a given sequence of exogenous vectors,

$$Z_{it}, t^{N}(N) = [z_{it}, t^{N} \cdots z_{it}, t^{1} z_{it}, t] = [Z_{it}, t^{N-1}(N-1) z_{it}, t],$$

subject to the constraint $x_{it}, t^{\tau} \in R(z_{it}, t^{\tau})$ for each $\tau = 0, 1, \ldots, N$, using (5.13a) and (5.14) recursively to define the succession of state-variables, $s_{it}, t^{\tau}$.

Let $X_{it}, t^{N}(N)$ denote the solution to the $N$-stage maximization problem defined above, where the optimal decision sequence is given by

$$X_{it}, t^{N}(N) = [x_{it}, t^{N}(N) \cdots x_{it}, t^{1}(N) x_{it}, t],$$

and each decision-vector is a function of the form,

$$x_{it}, t^{\tau}(N) = x_{it}, t^{\tau}(N) Z_{it}, t^{N}(N), z_{it}, t^{\tau-2}, \theta_1, \theta_2, \tau = 0, 1, \ldots, N.$$
state vector, \( z \), the \textbf{maximum} discounted present value over an \( N \)-stage planning horizon can be represented by the function, \( ^{1/} \)

\[
Q^N(z_{it}, t+N(N), s; \theta_2) = Q^N(X_{it}, t+N(N), Z_{it}, t+N(N), s; \theta_2)
\]

\[
= \max_{x \in \mathbb{R}(z)} \left[ s(z, x_{it}, t, s; \theta_2) + \frac{1}{(1+r_t)}Q^{N-1}(Z_{it}, t+N(N-1), s(x, x_{it}, t, s; \theta_2), Z_{it}, t, s; \theta_2) \right]
\]

where the last expression follows from (5.12)-(5.15). It will be noted that (5.20) defines a recursion formula, in which the \textbf{maximum} discounted present value, \( Q^N \), for an \( N \)-stage problem can be constructed from the \textbf{optimal} return function, \( Q^{N-1} \), for an \( (N-1) \)-stage decision problem. This reflects the familiar \textbf{Principle of Optimality} associated with multi-stage decision processes (Bellman [1957]) -- viz., an optimal decision sequence,

\[
\{x_{it}, t, t+1, \ldots, N\}
\]

has the property that whatever the initial state of the system, \( z \), and whatever the initial decision, \( x = x_{it}, t \), the remaining decisions, \( x_{it}, t+N(N-1) \), must constitute an optimal decision-sequence (or policy) with respect to the state, \( z_{it}, t+1 \), resulting from the initial decision -- here, via (5.13a)-(5.14).

The recursion formula (5.20) thus provides a procedure to recursively calculate the values of the optimal decision sequence, \( X^*(N) \) of (5.18) and (5.19), as well as the optimal return function, \( Q^N(z_{it}, t+N(N), s; \theta_2) \) associated.

\( ^{1/} \) Note that we do not assume a constant discount rate, \( r_t \), associated with each decision-making year, \( t \). Thus, one can generalize the model to include interest rate anticipations, \( r_{it} \), in the same manner as prices.
with any initial state vector $s$ -- and, hence, for the particular case, $s = g_{i0}$.

In the present problem the presence of the $\{z_{it}, t^*>$ in determining the optimal decisions suggests that we begin with the terminal year, $t+N$, and work backwards, successively determining the optimal decision functions,

$$x_{it, t+N-\tau}^*(\tau) = x_{it, t+N-\tau}^*\left(Z_{it, t+N}^*(\tau), s; \theta_2\right); \tau = 0, 1, \ldots, N, \tag{5.21}$$

associated with any state-vector, $s$, in the initial year of the plan, $t+N-\tau$, and employing the appropriate sequence of exogenous vectors, $Z_{it, t+N}^*(\tau)$. 1/

Let $S$ denote a multi-dimensional grid spanning the domain of the admissible state-vector values, $s$. Associated with every state-vector $s \in S$, there is an admissible domain for the decision vector, $x \in R(s)$, which will also be taken to be a multi-dimensional grid. For every choice $s \in S$ we must evaluate the designated function at all $x$-values within $R(s)$.

We begin by calculating and storing the maximum profit associated with the O-stage optimization problem in year $t+N$, defined by the function,

$$Q^0_s(z_{it, t+N}, s; \theta_2) = \max_{x \in R(s)} \{\pi(x, z_{it, t+N}, s; \theta_2)\}, \tag{5.22}$$

for every $s \in S$. In addition, we store the optimal O-stage decision function, $x_{t+N}^0(z_{it, t+N}, s; \theta_2)$, for every $s \in S$.

1/ A "forward solution", involving starting with construction of $Q^0_s(z_{it, t+N}, s; \theta_2)$, and proceeding directly through the sequence $\{Q^\tau(z_{it, t+N}^*(t), s; \theta_2); \tau = 1, \ldots, N\}$ is not practical, since the values, $Z_{it, t+N-\tau}^*(t)$ at which $Q^\tau$ is evaluated are not the same as those required to calculate $Q^\tau$ via (5.20).
Next, consider the 1-stage problem in year $t+N-1$, where we must calculate and store

$$Q^*_{i_t, t+N-1}(Z_{it}, t+N-1; s, \theta_2) = \max_{x \in R(s)} \left( \pi(x, z_{it}, t+N-1; s, \theta_2) + \frac{1}{(1+r_t)^r} \right),$$

(5.23)

$$Q^0_{it, t+N-1}(x, x_{it}, t+N-1; s, \theta_2),$$

as well as the optimal decision function $x^*_{it, t+N-1}(Z_{it}, t+N-1; s, \theta_2)$, for every $s \in S$. Here, we use the stored function $Q^\prime_{it, t+N-1}(x, x_{it}, t+N-1; s, \theta_2)$ of (5.22), evaluated at $s^* = s^*(x, x_{it}, t+N-1; s, \theta_2)$ to obtain $Q^*_{it, t+N-1}(Z_{it}, t+N-1; s, \theta_2)$ by interpolation and/or extrapolation in (5.23), using the grid points available for $s \in S$ in (5.22) — see Bellman and Dreyfus (1962).

The same procedure is then followed recursively to determine and store the sequences, $\{Q^\prime_{it, t+N-1}(Z_{it}, t+N-1; s, \theta_2)\}$ and $\{x^*_{it, t+N-1}(s, \theta_2)\}$, for $r = 2, 3, \ldots, N-1$. For $r = N$ we need only determine the optimum for $s = s_{it}$, the actual state vector relevant to year $t$.

Note that having determined the optimal current decision, $x^*_{it, t}(N)$, for an $N$-stage problem commencing in year $t$ given by (5.11) or (5.21) with $r = N$ and $s = s_{it}$, then, if desired, we may proceed forward through the sequence of stored functions, $\{x^*_{it, t+N-r}(\tau) : \tau = N-1, N-2, \ldots, 0\}$ to recover the complete optimal decision-sequence or policy. This requires iterative use of (5.13a)-(5.14) to calculate the optimal state-vector, $\{x^*_{it, t+N-r}(\tau)\}$, at which to evaluate $x^*_{it, t+N-r}(\tau)$ of (5.21). Thus, if there is no need to recover the remainder of the optimal decision-sequence beyond the optimal current decision -- as in the present calibration problem, then the storage of the sequence of
functions \( \{x_{it}^\ast, t+N-\tau(t)\} \) above is unnecessary. Finally, note that the method only requires the previous function, \( Q^{\tau-1} \), to calculate the present function, \( Q^{\tau} \), so that once \( Q^{\tau} \) is determined, \( Q^{\tau-1} \), may be eliminated from storage.

5.2 Direct Estimation/Calibration of the Technology Parameters:

Given a complete sample of data on the variables \( x_{it}^\ast, z_{it} \) and \( s_{it} \), the first step is to utilize the structural form of equations (5.5b), (5.5c), (5.5d), (5.5g), (5.5h) and (5.5j) to estimate the parameter vector, \( \theta_2 \), of (5.7) directly. We consider a simple stochastic analogue of the above model, obtained by adding a normal error vector with zero mean and spherical covariance matrix to the right hand side of each of these structural equations. Consistent estimates of \( \lambda, \mu, \delta \) and \( \sigma \) can be obtained via maximum likelihood or minimum distance estimators applied jointly to the system of structural equations. 1/

Estimation of \( \phi \) and \( \eta \) in the potential yield and harvesting
equations depend upon which version of (5.5c) is adopted. In the latter case, substitution of the latent vector, \( y^\text{Pot}_{il} \), into the multiplicative form of the harvesting equation still permits application of nonlinear least squares to minimize \( E (y^\text{Pot}_{il} - H_{it} \cdot e_{it})\cdot(y^\text{Pot}_{il} - H_{it} \cdot e_{it}) \) with respect to \( c \) and \( t \). In the former case, where \( y^\text{Pot} \) sets an upper-bound on \( h_{it} \), the model may be formulated as:

---

\[ x_{it}^* = \min \{ h_{it} + e_{it}^h, f_{it}^1 + e_{it}^f \}, \] (5.24)

where the functions, \( h_{it} \) and \( f_{it}^1 \), are defined in (5.5c) and (5.5d). This model will be recognized as a nonlinear vector-analogue of the "markets in disequilibrium" model (Fair and Jaffee [1972], Madalla and Nelson [1974], Goldfeld and Quandt [1975], Hartley [1977] or Hartley and Mallela [1977]), and maximum likelihood estimates of both \( \theta \) and \( n \) can be calculated.

Alternatively, we may formulate a deterministic structural model, as given already in (5.5b), (5.5c), (5.5d), (5.5g), (5.5h) and (5.5j), and define a quadratic loss-function, \( Q(\theta_2) \), involving the actual \( y_{it}^* \)'s relative to their within-sample predicted values, \( (y_{it}^*(\theta_2)) \), creating the decision variables; \( (x_{it}^*) \), as given. This is to be minimized with respect to \( \theta_2 \), as in Hartley [1984b]. This "systems approach," based on the methods of quasilinearization and identification (Bellman and Roth [1983]), searches for the parameter values which permit the sequence of model solution values to approximate the actual data as "closely" as possible. Here, all of the elements of \( \theta_2 \) are jointly calibrated.

A major advantage of this approach is that it does not require complete data. In most applications on perennials, this is likely to be the case.\(^1\) In addition, by using a Singular Value Decomposition (SVD) (Golub [1968]) of the appropriate Jacobian matrix, the computer can determine numerically whether the parameters, \( \theta_2 \), can be uniquely determined, relative to the available configuration of data -- except for some modifications

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\(^1\) See Hartley [1982] for a comprehensive description of the data base we have collected for natural rubber in Sri Lanka, where both micro panel-data samples and macro time-series exhibit essentially arbitrary configurations of missing observations.
required by the lack of differentiability at certain points, as in the case of
the model (5.24).\textsuperscript{1} If $\hat{\beta}_2$ is not unique, then the investigator must simplify
the model specification, relative to the given data base -- as discussed in
Hartley (1984b).

5.3 The Anticipations-Formation Process: \textsuperscript{2}

In section 5.1 we have shown how dynamic programming may be employed
to calculate the numerical solution,

$$\hat{x}_{it}^* = \hat{x}_{it}^*(N) = \hat{x}_{it}^*(z_{it,N}(N), s_{it}, \hat{\beta}_2),$$

(5.25)

to the N-stage problem of maximizing the discounted present value of the
anticipated future profit stream, given the values of the parameter vector,
$\theta_2$, the initial state vector, $s_{it}$, and the trajectory of current and future
exogenous variables, $z_{it,N}(N)$.

In practice, however, $z_{it,N}(N)$, while presumably known to the
decision-maker, will not be reported. The problem for the investigator is to
infer the process by which producers generate their anticipations trajectories
on the basis of the information contained in the observed decisions, $x_{it}$,
and the observed (non-decision) endogenous variables, $v_{it}$, given knowledge of
the state-vector, $s_{it}$, including the past values of the exogenous variables,

\textsuperscript{1} We shall defer further elaboration on this point to a subsequent paper.
The basic idea is that we may use approximation theory.

\textsuperscript{2} We use the term "anticipations" to avoid confusion associated with the use
of the more common term "expectations" -- the latter to be reserved for
the first-moment of a probability distribution, i.e., the mathematical
expectation. Thus we may refer to expectations and variances of future
anticipations, if the latter are treated as stochastic (see Hartley
(1983)).
This requires that the investigator formulate a maintained hypothesis as to how producers generate their anticipations trajectories.

We shall illustrate by considering the case in which the anticipations, \( z_{it,t+\tau}^* \), of \( z_{it,t+\tau} \) are generated by a deterministic, finite-order, vector autoregressive process involving (say) the \( n_3 \) preceding values, i.e.,

\[
z_{it,t+\tau}^* = \mathcal{Z}(z_{it,t+\tau-1}^{n_3-1}; \theta_1)
\]

for \( \tau = 0, 1, \ldots, N \), where the actual lagged values, \( z_{it,t-1}^{n_3-1} \), are employed to initialize the process and \( \theta_1 \) denotes a vector of anticipations-formation parameters. It follows that the complete vector sequence, \( z_{it,t+N}^* \) of (5.17), is the matrix function,

\[
z_{it,t+N}^* = \mathcal{Z}(z_{it,t-1}^{n_3-1}; \theta_1),
\]

depending only on the initial conditions, \( z_{it,t-1}^{n_3-1} \), and parameters, \( \theta_1 \).

Clearly, many other formulations of the anticipations-formation process could be employed within the present framework — e.g., the hypothesis of "rational expectations," due to Muth [1961] and Lucas and Sargent [1981a, 1981b] and the review of certain other stochastic alternatives, particularly so-called "pseudo rational expectations," in Nerlove, Grether and Carvalho [1979, ch. XIII].

In terms of our specific model, ease of implementation and parsimony could be achieved by postulating separate, functionally-independent, deterministic autoregressive models for each of the crop producer prices, wage rates, materials prices and interest rates, whilst, in the absence of secular
trends in weather conditions, the elements of \( w_{it, t+1} \) are probably viewed by farmers as having constant anticipations over the planning horizon — though they may be more influenced by more recent events, as memory is short. These would represent special cases of (5.27).\(^1\) Whatever particular formulation is adopted by the investigator, however, the problem which remains is how to calibrate \( \tilde{\theta}_1 \). We thus consider (5.27) as an illustration.

5.4 Model Calibration:

We shall thus consider only a deterministic version of the model along the lines of Hartley (1984b, 1985). The complexity of the (sequential) domain restrictions on the decision vector, \( \{x_{it}\} \), with their attendant high-order multiple numerical integrations, would appear to defy treatment by traditional econometric methods for "limited dependent variables" under a stochastic analogue.

The question of calibrating perennial supply models may be addressed at two levels of complexity: Either

(a) one takes the direct estimates of the technology parameters, \( \tilde{\theta}_2 \), as given constants, say \( \tilde{\theta}_2^* \), and attempts to calibrate only the anticipations-formation parameters, \( \tilde{\theta}_1 \), or

\(^1\) The problem of developing neoclassical econometric methods for decision-making under uncertainty with respect to future weather conditions and prices has been treated for the case of an annual crop in Hartley (1983). Generalization of this approach to the case of perennials is a difficult task due to the order of numerical integrations which may be involved — see, e.g., Haber (1970) and Quandt (1983) — and the length of the planning horizon. Also, in the present case, perennial prices are exogenous, and development of a general equilibrium model is a considerable undertaking. This motivates our preference for deterministic anticipations-formation processes in the present context.
(b) one calibrates $\theta_1$ and $\theta_2$ jointly, making use of all available data.

From a computational standpoint, (a) is a simpler problem. However, the algorithm required to calibrate either $\theta \equiv [\theta_1 \theta_2]$ or $\tilde{\theta} \equiv [\tilde{\theta}_1 \tilde{\theta}_2]$ with respect to the "unstarred" elements of $\theta$ in (a) and (b) is the same, though the latter employs all of the information within the available data set. We therefore treat the calibration of $\theta$ in general.

The model associated with unit $i$ and year $t$ under the neoclassical maintained hypothesis involves:

1. maximization of the discounted profit stream,

$$Q^N_{i,t,N}(N), Z^N_{i,t,N}(N), S_{i,t} \theta, Z_{i,t} \theta$$,

of (5.15); subject to:

2. the sequential domain restrictions,

$$\{x \in \mathbb{R}(g)\}$$,

3. the non-linear technology,

$$v_{i,t} = g(v_{i,t}, x_{i,t}, z_{i,t}, S_{i,t}, \theta)$$,

of (5.8), and

4. the definition of the anticipations-formation process, say,

$$\{z_{i,t,N}(n)\}$$ of (5.26) and (5.27) -- a dynamic vector-autoregressive system involving the parameter subvector, $\tilde{\theta}_1$. 
In section 5.1, we showed how dynamic programming (Bellman [1957]) may be employed recursively to calculate the optimal decision-vectors, \( \{x_{it}^*\} \), given \( \theta_1 \) (the anticipations process parameters) and \( \theta_2 \) (the technology parameters). These, in turn, determine the solutions \( \{y_{it}^*\} \). The problem at hand is the inverse problem, (see also Bellman and Kalaba [1963]), viz., determination of numerical values for

\[
\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}
\]

(5.28)
given the configuration of available data within \( X \equiv (x_{it}) \), \( V \equiv (v_{it}) \), \( Z \equiv (z_{it}) \), and \( S \equiv (s_{it}) \).

This model is analogous to the calibration problem discussed in Hartley [1984b], where the endogenous decision and non-decision variables satisfy domain constraints, and the parameters, \( \theta \), may also be constrained. Let \( y_{it} \equiv [x_{it}^* \ v_{it}^*]' \) and \( y_{it}^* \equiv [x_{it}^*' \ v_{it}^*]' \) denote the (say) Jx1 vectors of actual and model solution values (relative to any particular \( \theta \)) for the decision and non-decision endogenous variables, respectively. Then we use (say) a quadratic loss function,

\[
L(\theta) = \frac{1}{2} \sum_{i,t} (y_{it} - y_{it}^*(\theta))'W(\theta)(y_{it} - y_{it}^*(\theta))
\]

(5.29a)
as a criterion for the "closeness" of our model-approximation, with

\[
W(\theta) = \frac{1}{P \cdot T} \sum_{i,t} (y_{it} - y_{it}^*(\theta))' (y_{it} - y_{it}^*(\theta))^{-1}
\]

(5.29b)

for \( i = 1, \ldots, P \) producers in \( t = 1, \ldots, T \) time periods.
This defines the basic systems approach in Hartley (1984b) (and its modifications for incomplete data sets) to determine the parameter values which "best approximate" the observed data. It involves iterating between a series of "inner" dynamic programming optimization problems -- one for each \((i,t)\) -- given a value, (say) \(q^{(n)}(i,t)\) in iteration \(n\); and an "outer" parameter-updating algorithm, which locates a new parameter vector, \(q^{(n+1)}\), such that the loss-function, \(L(q^{(n+1)})\), is reduced.

Finally, while at this juncture, our calibration methods have been presented solely in terms of a (possibly stratified) random sample of panel data on individual (micro-level) producers, it is straightforward to incorporate macro-level time-series data. For example, suppose we have, in addition, macro data on total output \((q^1_{i,t} \text{ and } q^2_{i,t})\), total area under cultivation by land-use \((a^1_{i,t} \text{ and } a^2_{i,t})\); etc.) and total labor \((e^1_{i,t}; e^2_{i,t}(k):k=1,\ldots,K)\) and materials \((e^1_{i,t}; e^2_{i,t}(k):k=1,\ldots,K)\) inputs by crop -- as is not uncommon. Then, to exploit this additional macro-level information, we would simply augment the model specification to include a set of aggregation identities in the vector of non-decision endogenous variables. Thus, we introduce, e.g.,

\[
q^{1*}_{i,t} = \sum_{i=1}^{N} \omega_{it} q^{1*}_{it} \tag{5.30a}
\]

\[
q^{2*}_{i,t} = \sum_{i=1}^{N} \omega_{it} q^{2*}_{it} \tag{5.30b}
\]

\[
a^{1*}_{i,t} = \sum_{i=1}^{N} \omega_{it} a^{1*}_{it} \tag{5.30c}
\]

\[
a^{2*}_{i,t} = \sum_{i=1}^{N} \omega_{it} a^{2*}_{it} \tag{5.30d}
\]

etc., where the vector of data on macro-aggregates,
and its corresponding model-solution values,

$$\mathbf{y}_t^A \equiv [q_1, q_2, \ldots, q_n]'$$

(5.31a)

$$\mathbf{y}_t^{A*} \equiv [q_1, q_2, \ldots, q_n]'$$

(5.31b)

augment the non-decision vectors, \( \mathbf{v}_t = \text{vec}(\mathbf{v}_{it}) \) and \( \mathbf{v}_t^{*} = \text{vec}(\mathbf{v}_{it}^{*}) \), in the loss function, \( L(\theta) \), of (5.29a), with corresponding adjustment in the weighting matrix, \( \mathbf{W}(\theta) \) of (5.29b). The \( \{\omega_{it}\} \) in (5.30a)-(5.30d) are known weights, derived from classical statistical sampling theory (e.g., Cochran [1977]), to define the population variable from a stratified random sample.

In short, with micro/macro time-series/cross-section/panel data, for a deterministic model specification aggregation is simply a matter of adding up.\(^1\)

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\(^1\) The reader may reflect upon the difficulty of defining the probability density function of the aggregate variables, \( \mathbf{v}_t^A \), obtained by aggregation over the micro-level random variables, \( \mathbf{v}_{it} \), in a stochastic analogue, \( \mathbf{v}_t^{A*} \), given the complex structure of the present class of problems. This would be required to apply (say) maximum likelihood estimation methods.
6. Conclusions

The purpose of this paper has been to develop a realistic general prototype model for the land-allocation and input-demand decisions and the output-supply of a perennial crop, based upon the neoclassical maintained hypothesis that producers maximize the discounted present value of profits over a finite planning horizon. This is shown to be a dynamic programming problem, for which only numerical solutions may be calculated, given values for the parameters in the technology and anticipations-formation process. The problem confronting the investigator is the inverse -- i.e., given a set of micro-level panel and/or macro time-series data on the decisions taken by individual producers and/or their aggregate in various observed "states of nature", can we determine the parameter values which permit the model solutions to approximate the actual values of the endogenous variables as closely as possible? This is seen to involve an interface between the systems approach for calibration of dynamic economic models with incomplete data (Hartley [1984b, 1985]) and neoclassical econometric methods (Hartley [1981a, 1981b, 1983]). The latter involves embedding a series of dynamic programming algorithms (Bellman [1957]) -- one for each unit and/or time-period within the data set -- within an outer algorithm to update the parameter values.

Given the length of the economic life of most perennials, it will often be impossible to obtain reliable direct estimates of the parameters in the dynamic technology based solely on macro time series data without making extreme simplifying assumptions. This highlights the importance of obtaining samples of micro-level panel data, in order to capture the vintage aspects of

1/ E.g., the economic life span of a rubber stand is upwards of thirty years.
the problem. Here, direct calibration of the technology parameters (using data on "overlapping" sections of the complete life span) is quite feasible. Both micro and macro-level data can then be combined to calibrate the remaining parameters in the anticipations-formation process.

Such methods are highly computer-intensive and, before the advent of supercomputers, implementation of the second stage would not have been feasible. Simplifying model assumptions and approximations (e.g., Bellman [1969]) may still have to be invoked to reduce the number of decision variables, state variables, and other aspects of the dimensionality of the problem. However, with continued rapid advances in the state of computer technology, it is only a matter of time before such methods will be both feasible and cost effective.

The detail embodied in our prototype model was motivated by the nature of the data base we have collected for rubber in Sri Lanka (see Hartley [1982]). Macro-level time-series data on output, area under cultivation -- both unselected seedling and clonal varieties, new plantings, replantings, the age-distribution of existing stock, producer prices, export duties, replanting subsidies, export quotas, employment, fertilizer imports by nutrient, wage rates, fertilizer prices, other crop output, area and prices, rainfall, temperature, etc. have been collected. These include both monthly and annual series, and span various years from 1933 to 1979. In addition, an extraordinarily rich set of micro-level panel data covering the years 1970-1979 at the level of individual fields within a sample of 49 estates will

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1/ These data were collected by staff of the Central Bank of Ceylon, the Rubber Research Institute of Sri Lanka, the Janatha Estates Development Board and the Sri Lanka State Plantations Corporation as part of a collaborative research effort between the Government of Sri Lanka and the World Bank under the Bank-sponsored research project RPO 672-02.
permit direct calibration of the dynamic production technology, as well as application of our methods to a version of our complete prototype model of perennial supply. We look forward to obtaining access to a supercomputer, and hope to be able to report on attempts at implementation in subsequent papers.
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