Measuring Economic Downside Risk and Severity: Growth at Risk

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Abstract

Output collapses and crises are a fact of life. In this paper we propose a new measurement for economic downside risk and severity: Growth at Risk. It is similar to the concept of Value at Risk in finance and it summarizes the expected maximum economic downturn over a target horizon at a given confidence level. After providing a taxonomy of growth risks, we construct a panel data set on Growth at Risk for 84 countries over the period 1980-98. We then investigate the relationship between downside risks and long-term average growth in a cross-country analysis. We find that higher perceived levels of downside growth risk seem to be negatively associated with long-term growth. This result suggests that risk management in a broad sense should be a vital part of pro-growth and poverty reduction strategies.

JEL classification code: E32, N12, O40

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Measuring the Downside Risk and Severity: Growth at Risk

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All of life is the management of risk, not its elimination.

Walter Wriston
Former chairman
Citicorp

1. Introduction

The 1990s has witnessed several episodes of output collapses and severe financial crises, with grave consequences to the poor in the crisis-stricken countries. In fact, the past decade is not unique in the economic history of the world, which is replete with recessions and depressions, from the bursting of the British South Sea Bubble and the French Mississippi Bubble in 1720 (which at least one economic historian claims delayed the industrial revolution by 50 years) to the depressions of the 1870s and Great Depression of the 1930s; from the Latin American debt crisis, to the output collapse in some transitional economies. Adding to those are the collapses related to external shocks such as the AIDS epidemic, wars, hurricanes, earthquakes, floods, and internal conflicts.

Output collapses are very costly in terms of economic growth and welfare. Until recently however, not many theoretical studies had focused on economic downturns, even though volatility has been a major concern of policymakers and received much attention.1 There are possibly two main reasons for this. The first is that the widely used von Neumann-Morgenstern utility function only concerns real current consumption and future consumption. In doing so, prior downturn experience is not incorporated in the utility function.2 The second reason is that economists have been using the business cycle theory to understand output fluctuations. Lucas (1987), for example, suggested that the possible returns from eliminating business cycles seemed to be trivial. However, recent studies have found the potential welfare gains from crises avoidance to be large. Many have cast doubts on the real business cycle theory. For example, the real business cycle literature focuses on shocks to the technology, but “the absence of candidate shocks is particularly clear in the context of the Great Depression of the 1930s.” (Evans, Honkapohja and

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1 See section 2 for a literature review. Here our focus is on real sector downturns alone. Studies on growth volatility are relevant but not exactly on the same track.
2 Recently, Barberis, Huang and Santos (2001) study asset prices in an economy where investors derive direct utility not only from consumption but also from the value of their financial wealth, which are loss averse over fluctuations. And the degree of loss aversion depends on their prior investment performance.
Romer 1996, p.33). Furthermore, a growing body of literature on new Open Economy Macro (see Lane’s (2001) survey) attempts to address open economy issues in a dynamic general equilibrium with nominal rigidities and market imperfections. However, this new research direction primarily concentrates on currency and banking crises rather than output collapses.

On the empirical side, few studies have examined the relationship between downside risk of growth and the long-term growth rate, even though many have studied the lack of growth or stagnation (Easterly et al 1993, Ben-David and Papell 1997, Rodrik 1998, for example). The volatility of economic growth and its effect on growth have indeed received much attention, see for example, Kormendi and Meguire 1985; Ramey and Ramey 1995, Aizenman and Marion 1998, and Easterly, Islam and Stiglitz 1999. The standard deviation of growth rates is often used as the measurement of volatility in these studies. By definition, it contains information of both booms and recessions.

Perhaps the most important obstacle for the study of downside growth risk is the lack of an appropriate measurement for it. In this paper, we offer an alternative perspective to output fluctuation and a measurement for downside growth risk. After providing a taxonomy of risks affecting growth, we employ the concept of Value at Risk (VaR) to measure Growth at Risk (GaR), which focuses solely on the downside growth risk and the severity of economic downturns. In plain English, Growth at Risk (GaR) measures the most serious growth decline from its mean with a specified probability over a given time horizon. The benefit of growth at risk is that it measures risk in a way that most people can understand it.

After constructing a panel data set of GaR for 84 countries in 1980-98, we investigate the relationship between the downside risk and long-term output growth in the tradition of cross-country analysis. We find that higher perceived levels of downside risk seem to be negatively associated with long-term growth. An intuitive interpretation for this result is the importance of perception for growth risk: When the perceived risk is high, both domestic and foreign investors might be deterred from making long term investment and long term growth will suffer.3

This paper is organized as follows: Section 2 reviews the literature and provides a taxonomy of growth risks. Section 3 introduces the concept of Value at Risk widely used in finance and present Growth at Risk. Section 4 presents a panel data set on Growth at Risk for 84 countries during the period 1980-98. In section 5 we investigate the relationship between downside risk measured by GaR and long-term average growth in a cross-country analysis. The last section concludes.

2. A Taxonomy of Risks Affecting Growth

Business Cycle versus Growth Cycle?

3 Aizenman and Marion (1998) found a significant negative correlation between volatility measures and private investment in developing countries, even when adding the standard control variables.
The concept of a business cycle was originated by Juglar (1889), after a series episodes of economic fluctuations or crises were observed. According to Juglar, cycles are principally a feature of economies with highly developed commerce and industry, division of labor, external trade, and the use of credit. As such, economic downturns were considered as necessary stages in recurrent business cycles. This idea was accepted and developed further by many economists. However, a few economists have cast his or her doubts by asking: “Do business cycles really exist?” as Hicks (1982) did.

In respect to theoretical work, Evans, Honkapohja and Romer (1996) use “Growth Cycles” rather than business cycle to interpret economic fluctuation. They construct a rational expectations model in which aggregate growth alternates between a low growth and a high growth state. When all agents expect growth to be slow, the returns on investment would be low, and little investment takes place. This slows growth and confirms the prediction that the returns on investment will be low. But if agents expect fast growth, investment is high, returns are high, and growth is rapid. This expectational indeterminacy is induced by complementarity between different types of capital goods. In their growth cycle there are stochastic shifts between high and low growth states and agents with rational expectation take full account of these transitions. Under the dynamics implied by a correspondingly simple learning rule, there is a stable equilibrium with “Growth Cycles”. In other words, Evans, Honkapohja and Romer’s (1996) model shows that growth rate is not determined as the existing business cycle theory predicts and is also stochastic depending on agents’ expectations. Empirically, Easterly, Islam and Stiglitz (1999) find that the fact that that the length of expansion does not have a statistically significant effect on the probability of a downturn may be suggestive that there is no mechanical business cycle. This confirms work on the United States, where Furman and Stiglitz (1999), have shown that there has been no regular business cycle (no dependence of the probability of a downturn on the length of the expansion) since World War II.

The US economy is a good example of growth stability relative to many countries in the world. Yet it is not immune to output volatility and economic downturns. There is a large literature exploring the question of whether the magnitude and duration of economic fluctuations have changed after WWII [examples include De Long and Summers (1986) and Watson (1994)]. While the evidence on this issue is mixed, all studies admit the existence of year-to-year or quarter-to-quarter fluctuations in the growth rate. Figure 1 plots the growth rate of US per capita GDP over the period 1961 to 1998, which reveals several economic downturns.

[Insert Figure 1 here]

A Taxonomy of Risks Affecting Growth

Following the footsteps of authors cited above, we hereby propose an alternative perspective to understand economic fluctuations, which is growth at risk. Economic activities are inherently risky, which makes growth stochastic and unpredictable. It is
necessary to examine all the risks affecting growth and classify them. As the knowledge on growth risk grows, a clear and common taxonomy for growth risk may appear. Therefore, we adopt a generic and robust production function model to provide a taxonomy of risks affecting growth. We do not attempt to be exhaustive in defining all risks affecting growth; rather, the taxonomy is intended to be illustrative of the unpredictability of these problems that an economy may face.

Assume that all production activities conform to a general input-transformation-output model involving the real production process, financial sector, institutional framework and government policies. As such, an output downturn or growth risk is an aggregated risk of many factors in the production process of the economy. We focus first on the real production side of the economy, then move to the financial and institutional sides. Of course, the growth risk of an economy will also differ across countries according to the nature of the shocks they face, the structure of the economy, and the policy regime of the government.

♦ **Input Risks**

According to the framework of aggregate production function, inputs, and hence input risks, to an aggregate production process can be conveniently classified as follows:

- **Labor force**: demographic changes, wars and epidemics such as HIV/AIDS shocks may lead to large reduction in the quantity and quality (capability) of labor force.

- **Human capital**: Human capital risk may come from the lack of education, training and experience. China’s Culture Revolution during the period 1966-1976 when schools and universities were closed and intellectuals were persecuted, was a shock to human capital, as shown in our estimated human capital stock for China (Wang and Yao, 2001). Brain-drain is another risk associated with this.

- **Physical capital**: All physical capital including the equipment, buildings, roads and other infrastructure, information and communication systems are liable to break down. The “breakdown” may only be partial or a total and sudden cessation of output production. Some breakdowns can bring a large part of the operation to a halt. Other failures might only have a significant impact if they occur at the same time as other failures. The sources include wars, hurricanes, earthquakes, volcanoes, fires, droughts, and floods.

♦ **Production process risks**: This may include a breakdown in the chain of production process, or in the division of labor across regions/countries. An example is that the breakdown of trade links amongst the East European and Former Soviet Union countries led to a failure of original production chains in the early 1990s.

♦ **Productivity or technology shocks**: Technological progresses have led to structural adjustments and transformation: positive for some but negative for others. Examples are plentiful: sunset industries in Japan; the defense industry in the U.S. after the end of cold war; and “the new economy”, the IT bubble and the subsequent burst.

4 Much like a banker faces a collection of risks including credit risk, interest risk and etc., policymakers may understand economic fluctuation as a collection of growth risks, rather than resorting to business cycles.
Financial risks: Giving its functions of channeling savings and allocating resources, financial systems are vital parts of the production and economic process. Due to their inherent nature such as information asymmetry and market imperfection, financial systems have many risks which may lead to systemic crises and breakdowns. Many studies have stressed the linkage between financial risks and growth volatility, including Easterly, Islam and Stiglitz (1999). They found that wage rigidities, at the center of traditional Keynesian analysis, seem to have played little role in explaining output variability. By contrast, financial variables consistently turn out to be significant both in explaining variability and the likelihood of a downturn. Their results underscore the importance of financial variables in the analysis of volatility.

Macroeconomic policy risks: this may include those caused by imprudent fiscal and monetary policies and mismanagement of the economy, which put growth at risk. Ramey and Ramey 1995, Rodrik 1998 and Romer 1999 discussed policy-induced booms and recessions in developing as well as industrial countries.

Institutional and political risks: Broadly, this may include institutional arrangements in a country, such as incentive mechanisms, corporate governance, national governance, and the rule of law, level of corruption, and changes in political systems that could put growth at risk.

Global and external risks: Changes or shocks in global economic and natural environment which are beyond the controls of any one national government, or of human being.

Impact of Downside Risks on Growth

The business-cycle theory and growth theory had long been treated as unrelated areas of macroeconomics until recently. The real business cycle literature (for example, see Cooley and Prescott, 1995) views long-term growth as exogenous. Similarly, the traditional growth literature has an implicit assumption: that the growth volatility does not affect long-term economic growth.

Recently, there is an emerging research trend addressing the effect of business cycle on growth. Theoretically, the recent studies show that the relationship between output volatility and mean growth can be either positive or negative. For example, Matsuyama (1996 and 1999) develops a neo-Schumpetarian model where the economy achieves sustainable growth through cycles, perpetually moving back and forth between two phases. In contrast to this model which argues that business cycles are good for growth, Vivek and Rowe (1998) develop a 'neo-Keynesian' model, where monopolistically competitive firms set prices and produce output in advance of the realization of (stochastic) monetary velocity. In such a setting, there is an asymmetry in the effect of business cycles on income: A more severe business cycle thus reduces the expected income of a firm and the expected return to investment, which reduces the long-term growth rate of the economy.
The empirical research on the relationship between growth risk and long-term growth rate has been relatively rare so far. Using a sample of 47 countries from 1950-1977, Kormendi and Meguire (1985) regress growth rates on a group of explanatory variables, and find that the standard deviation has a significant positive effect on growth. Ramey and Ramey (1995) find, in a sample of 92 countries as well as a sample of OECD countries, that countries with higher volatility have lower growth. And they conclude, “By assuming no interaction between volatility and growth, the theoretical business cycle and growth literature omit important elements. (p.1148)”

Rodrik (1998) asks a question “where did all the growth go” and argues for a vicious circle of recession and domestic social conflicts. A recession leads to a “shrinking pie” and triggers social conflict within a country’s different groups. The resulting social conflict in turn leads to the lack of persistence in growth rates. His empirical evidence provides support for this hypothesis: Countries that experienced the sharpest drops in growth after 1975 were those with divided societies (as measured by indicators of inequality, ethnic fragmentation, and etc) and with weak institutions of conflict management (proxied by indicators of the quality of governmental institutions, rule of law, democratic rights, and social safety nets).

A big recession may lead to a long-term stagnation of an economy. A real example is the Japanese economy, which has experienced its worst postwar stagnation in the last ten years, with growth rates between zero to one percent. For developing countries, a single crisis (growth two standard deviations below mean growth) would cost the equivalent of eight years of accumulated growth. Chile, for example, had on average zero growth in GDP per worker from 1963 to 1988, largely because of two recessions: GDP per worker fell 16% between 1973 and 1975, and 19% between 1981 and 1983. In addition, it may take years or decades for a developing country to recover from an output collapse, as was evident for Latin American countries in the 1980s, and for some transitional economies in Central Asia.

**Impact of Downside Growth Risks on Welfare**

Since the Great Depression in the 1930s, government policies have been aimed at reducing business cycle fluctuations, yet some economists were uncertain whether the gains from such a reduction are worth the effort. In his celebrated 1987 book *Models of Business Cycles*, Lucas presents some simple calculations to argue that the trade-off between economic fluctuation and growth is such that a representative agent’s willingness to pay—in terms of growth rates for a more stable environment is almost zero.

However, subsequent work has challenged Lucas’ conclusion and found the welfare gains from maintaining stability to be large. For example, Storesletten, Telmer and Yaron (2000) note that there are important distributional effects associated with aggregate variation which is why individuals care about business cycles. Based on both aggregate data and microeconomic data from the Panel Study on Income Dynamics, they find that the welfare benefits of eliminating aggregate variation to be large. Moreover, growth downturns have grave consequences and long term adverse effects on the poor, as the
poor lack assets to smooth their consumption. Due to the lack of education, poor workers are less mobile than middle-income people across sectors and regions (see, for example, Lustig 1999, and World Bank 2000). The social costs associated with the crises in emerging market economies have been substantial: real wage fell, unemployment rose, and there is a sharp increase in the number of poor. School dropout rates rose among poor children with grave long term effects on the human capital of the poor (World Bank, 2000).

Finally, Chatterjee and Corbae (2000) seek to measure the potential benefit of reducing the likelihood of economic crises. Based on the observed frequency of Depression-like events, they estimate this likelihood to be approximately one in every 83 years for the U.S. The welfare gain of reducing even this small probability of crisis to zero can range between 1.05 percent and 6.59 percent of annual consumption in perpetuity. These large gains occur because although the probability of entering a Depression-like state is small, once the state is entered it is highly persistent. Athanasoulis and Wincoop 2000 also find large benefits from cross-country risk-sharing: The gains for a 35-year horizon, corresponding to a welfare equivalent permanent increase in consumption, is 6.6% when based on a set of 49 countries, and 1.5% when based on 21 OECD countries.

3. Value at Risk for Growth

Two Observations on Growth Rates

Economic growth has two important characteristics. First, growth rates have small autocorrelations and are remarkably unstable over time. “Booms” and “crashes” characterized the growth experiences of most countries. In addition, Easterly et al (1993) find that the correlation of per capita growth in 1977-1992 with per capita growth in 1960-1976 across 135 countries is only 0.08. This low persistence of growth is not just a characteristic of the postwar era. For the countries that have the data from Maddision (1995), there is a correlation of only 0.097 across 1820-1870 and 1870-1929. In terms of log per capita GDP levels, Pritchett (1998) shows several patterns of economic growth: Hills, Plateaus, Mountains, and Plains. In particular, GDP per capita in the United States is characterized by a stable exponential trend growth with modest cyclical deviations. The average annual growth rate of per capita GDP in the United States is 1.75 percent per year during the period 1870-1990 (Barro and Sala-i-Martin, 1995). For the developing countries, however, growth rates exhibit distinct patterns and highly volatile. Moreover, volatility of growth rate is enormously larger in developing than developed countries. Easterly, Islam and Stiglitz (1999) find that countries experience declining real GDP roughly 20 percent of the time. Non-OECD countries experience a downturn 22 percent of the time, while OECD countries are in a downturn just above 9 percent of the time.

Second, the observed annual growth rate distribution is markedly non-normal. There is an asymmetry of the growth cycles and it shows that growth varies systematically within upturns and downturns. It is typically skewed towards large economic downturn and has a long tail at the left hand, as shown by the figures below. This means that extreme output changes occur more frequently than implied by a normal distribution.
Due to this reason, we start looking for alternative modeling methods\(^5\). The distribution of growth rate can be estimated over a number of previous periods, assuming all observations are independently distributed.\(^6\) Normal distribution is a typical example of unconditional distribution. If it is assumed that growth rates are generated according to the normal distribution, the entire distribution of returns can be characterized by two parameters: its mean and standard deviation. Other examples of unconditional distribution models include the t-distribution, mixed-diffusion-jump model and the compound normal model. The purpose here is simply to check if the distribution can bear any resemblance of a normal distribution. A flat distribution indicates greater risk; and a tighter distribution implies lower growth risk.

Several popular tests such as Kiefer and Salmon (1983) for normality focus on measuring skewness and kurtosis. However, it is likely that they have the low power problem due to the small sample of annual growth data. Nonetheless, the values of skewness and kurtosis for 84 countries from 1961 to 1998 from the World Bank database significantly differ from 0 and 3, respectively.\(^7\)

As an alternative, we examine the distribution of growth rates of several economies from 1961-1998 by using frequency histograms. Figure 2 shows that the US economy has experienced several recessions during the period 1961-1998. The largest drop of growth rate is more than 3 percent. Figure 3 shows that Hong Kong SAR had a big decline of growth rate while its growth is relatively stable and respectable. The big recession of Hong Kong happened in 1998 with the East Asian financial crisis. Figure 4 reveals that Brazil had a “growth miracle” associated with 8 percent growth rate and “growth crash” associated with –6 percent growth. It is obvious that Brazil’s economy has been subject to more dramatic growth volatility than US and Hong Kong. These figures show the non-normality quite clearly.

**Value at Risk in Finance**

“The Daily Earning at Risk for our combined trading activities averaged approximately $15 million......”

*J. P. Morgan 1994 Annual Report*

In recent years the Value at Risk (VaR) concept for measuring downside risk has been widely studied and used in the financial industry. VaR basically is a summary statistic

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\(^5\) which can be divided into two classes: unconditional (time-independent) and conditional distributions (time-dependent). The conditional distribution of growth rates includes models such as the GARCH and Stochastic Volatility models.

\(^6\) This assumption is reasonable since growth rates are highly unstable over time, with a correlation across decades of 0.1 to 0.3 (Easterly, 1996).

\(^7\) Normal distribution has skewness of 0 and a kurtosis of 3. The table for simple testing for normality is upon request.
that quantifies the exposure of an asset or portfolio to risk including market risk, credit risk and operational risk. Essentially VaR estimates attempt to capture extreme events that occur in the lower tail of the portfolio’s loss distribution. *Value at Risk (VaR) summarizes the expected maximum loss (or worst loss) over a target horizon within a given confidence interval*” (Jorion, 1997, p.19). A distinct advantage of VaR is that it summarizes the largest possible loss, or the total exposure to risk of a financial institution *in a single number*, and easy to understand.

In the financial industry, a major purpose of using VaR is to estimate the amount of capital needed to support a bank’s risk taking activities, which is typically called “economic capital” in the banking industry. While internal systems for allocating economic capital typically encompass all forms of risk facing a bank (credit, market, and operating risks). We give an example of the VaR for credit risk resulting from the default events and also discuss the allocation of economic capital for credit risk.

Systems for allocating economic capital against credit risk are based on a bank’s estimate of the probability density function for credit losses (PDF). Banks could use its credit risk modeling system to estimate such a PDF as shown in Exhibit 1. An important property of the PDF is that the probability of losses exceeding a given amount $X$ (along the x-axis) is equal to the shaded area under the PDF to the right of $X$. A single PDF and a system PDF are expected to have a long and fat tail. The expected operational loss (shown as the left-most vertical line) shows that the amount of operational loss the bank would expect to experience on its business portfolio over the chosen time horizon. The banking industry typically focuses on the credit risk of the portfolio with a measure of unexpected loss (i.e. the amount by which actual losses exceed the expected loss). Given the shaded area (confidence level) and a chosen time horizon, $X$ is in fact the VaR relative to zero, or the absolute VaR.

Exhibit 1: The Conceptual Framework of VaR
Using information on VaR, financial institutions allocate economic capital against credit or market risks and prevent insolvency. For instance, the level of economic capital may be set to achieve a 0.03 percent chance that unexpected credit losses would exceed this level, thereby causing insolvency. The target insolvency rate will be achieved with 99.97% confidence. Financial institutions usually use loan loss reserves to cover expected credit losses, while it is the role of equity capital to cover credit risk. In Exhibit 1, therefore, the area under the PDF to the left of expected losses should be covered by the loan loss reserve, while the bank’s required economic capital is the amount of equity over and above expected losses necessary to achieve the target insolvency rate. Under this risk management framework, a bank would consider itself to be undercapitalized if its required economic capital exceeded its actual tangible shareholder equity, adjusted for any estimated surplus in the bank’s reported loan loss reserve.

The estimation procedure for credit risk VaR is as follows:

- Determine the time horizon over which we want to estimate a potential default loss. The time horizon in the banking industry arises from one year to five years.
- Determine the window length which is the length of the subsample (the observation period) used for VAR estimation. The window length choice is related to sampling issues and availability of databases.
- Select confidence level required for the estimation. With the expectation that the largest likely loss we will suffer 95 out of 100 times (95% confidence level) may not suffice for credit risk. The financial industry may need at about 99% or 99.5% confidence level.
- Work out a Probability Density Function (PDF) of likely losses for a risk component or system risk under consideration.\(^8\)
- Estimate the VaR.
- Allocated economic capital for credit risk = VaR-Expected default losses.

**Growth at Risk**

Similar to VaR in the financial industry, growth at risk (GaR) proposed by this paper summarizes the expected maximum economic downturn over a target horizon within a given confidence interval. In theory, GaR can be derived from the probability distribution of the future growth rates \(f(g)\). At a given confidence level \(c\), we wish to find the worst possible realization of growth rate, \(G^*\), such that the probability of exceeding this value is \(c\):

\[
c = \int_{g^*}^{\infty} f(\ g\ ) \ dg
\]

\(^8\) However, the major challenge in implementing VaR analysis is the specification of the probability distribution of extreme default loss used. By contrast, market risk models take the normal distribution frequently as a standard or benchmark, but this assumption has been challenged.
or such that the probability of a value lower than $G^*$, $p=P(g \leq G^*)$, is $1-c$:

$$1 - c = \int_{g^*}^{-\infty} f(g) dg = P(g \leq G^*) = p$$  \hspace{1cm} (2)

The cut off point $G^*$ is called the sample quantile of the distribution. This is the worst possible growth rate given the confidence of $1-c$. This specification is valid for any distribution of growth, fat or thin tailed. Thus, we define Growth at Risk or GaR as

$$GaR = E(G) - G^*$$  \hspace{1cm} (3)

where $E(G)$ is expected value of growth rates over a given window length (10 or 20 years), $G^*$ is the worst possible growth rate in the given window length with the confidence of $1-c$.

Using growth at risk, GaR as defined in equation (3), it is possible to analyze the possible severity of downturns over time. GaR is a sensible measure of severity that takes into account the possible size of the decline from the expected growth rate at the given confidence level. This measure shows the percentage-point of output that would be lost in a possible recession. For example, if an economy’s annual average growth rate over the certain period is 4 percent and the time horizon for GaR is one year. As such, a GaR of 10 percent implies that the economy faces a recession in which possible output loss is up to 10 percentage point below the mean of growth rates (4 percent) within one year at a given confidence level.

Growth at risk is defined using information from a growth distribution (PDF), different from the definition of business cycles. By contrast, the definition of business cycles distinguishes two different types: the “classical” cycle, which refers to peaks and troughs in the absolute levels of per capita GDP series, and the “growth” cycle, which refers to peaks and troughs in the levels of the detrended series. We also divide economic downturns into two categories: big recession and small recession. Similar to the recession definition in the “growth cycle,” our GaR for big and small recessions does not necessarily mean a negative growth rate. More specifically, the definitions are as follows:

**Definition of GaR for big recession:** a possible worst economic downturn which only happens once over a twenty-year period. This requires that the confidence level of GaR is set at 5%, i.e. $1-c=5\%$, and corresponding quantile is $G^{5\%}$ . As such, Growth at risk, $\text{GaR}_{5\%}=E(G) - G^{5\%}$, where $E(G)$ is the expected value of growth rates over the window length, 20 years.

**Definition of GaR for small recession:** a possible worst economic downturn which happens once every five-year period. This requires that the confidence level of VaR is set at 20%, i.e. $1-c=20\%$, and thus corresponding quantile is $G^{20\%}$. That is, Growth at

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9 This is the definition of GaR relative to the mean. There is another definition for GaR at zero.

10 In the press, it is claimed that three consecutive quarters of negative growth as a recession.
There are two underlying assumptions of growth at risk, GaR: (1) growth rates of one country are serially independent. This assumption implies that the growth rate in one year will not affect growth rate in any other year, which is quite reasonable. In particular, Senhadji (2000) finds that growth rate has only a weak positive autocorrelation coefficient. (2) the population distribution of growth rate is stable through time. Growth-at-risk calculation is based solely on the recent history of the growth, by construction. In other words, growth VaR assumes that the population distribution does not change over the window length. As such, it relies on an assumption that the future will be like the past and thus suffers a risk of structural breaks as discussed below.

It is important to note that growth-at-risk as a forecast measurement of economic downturns has the potential bias resulting from structural breaks. In the presence of trend breaks, there is a trade-off between forecast bias and standard error of GaR. The common practice in estimating VaR is to use a rolling window of data, which has its own shortcomings. A short window length may work well immediately after a break but will lose valuable information in the distant past. On the other hand, a long window length will delay the detection of a break and produce biased forecasts in the interim. Ideally, the window size should be large far away from the most recent break to allow for efficient estimation of GaR. The window length should be shorter, the closer to the most recent break, to avoid using too many data points prior to the occurrence of the break which will bias the estimates of GaR.


The original data used here is the annual growth rates of per capita GDP for 84 countries from 1961 to 1998 from the World Bank database. Conventionally, there are two methods of estimating VaR in the financial industry, one based on a general historical distribution, and one based on the assumption of a normal distribution. The computation can be simplified if the distribution can be assumed normal. Here we use an empirical distribution, since the distribution of growth rates is non-normal, as shown above. This means that we base our estimation of GaR on each country’s historical distribution of annual growth rates by the following steps:

1. Choose time horizon. Because we only have annual growth rate, time horizon is one year.

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11 Due to the limited sample size, there is an estimation error in GaR. The simplest method to verify the accuracy of the GaR is to record the failure rate, which gives the proportion of times GaR is exceeded in the given sample. Kendall (1994) and Kupiec (1995) discussed this error and developed some tests.

12 Ben-David and Papell (1995) use up to 130 years of annual aggregate and per capita GDP data for 16 countries to investigate whether output exhibits a trend break and whether economic growth is constant or changing over time. Their study provides empirical evidence that, for nearly every one of the countries, the years that provide the strongest evidence for a trend break are associated with a sharp decline in GDP. These breaks are associated with World War II for most of the countries and either World War I or the Great Depression for the remainder.
2. Select window length. We choose 20 years as the length of the rolling window. This means that we estimate GaR for 1980 by using the data from 1961-1980 and GaR for 1990 by using the data from 1971-1990.

3. Choose empirical distribution (Histogram) as the probability distribution of growth rates.

The cumulative distribution function is

\[ F(x) = F(x_i) + p_i \left( \frac{x - x_i}{x_{i+1} - x_i} \right) \]  

(4)

Where

\[ x_i = i \frac{\text{max} - \text{min}}{n} + \text{min}, \quad x_i \leq x \leq x_{i+1} \]

and \( \frac{n \sum p_i}{\text{max} - \text{min}} = 1 \)

Because the low frequency of annual data, however, the cumulative distribution function of the empirical distribution is based on a small number of observations for each country and thus suffers from the large standard error in estimating the shape of the tail. For the sake of simplicity, we have not used a kernel density estimator. Nonetheless, the empirical distribution is able to give a reasonably effective way to capture the left-hand-side output downturns without assuming and estimating a distribution. Also, the empirical distribution can be robust once monthly or quarterly growth data is available.

4. Estimate the \( \text{G}^* \) at the given confidence level, and

5. Calculate GaR5 (big recession) and GaR20 (small recession) as defined in the previous section.

Using a twenty-year rolling window, we estimated GaR for 84 countries from 1980 to 1998. The time series of GaR examines the continuity and change in economic downside fluctuations over time. For example, GaR in 1990 is obtained by estimation the growth performance from 1971 to 1990.

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13 In choosing an appropriate window length, two trade-offs should be considered. First, there is a trade-off between the time span of the data and the standard error of estimated VaR. For example, extreme percentiles such as the 5\textsuperscript{th} are very difficult to estimate accurately with small samples. Given the time span of the annual growth rate data is only about 40 years from 1961 to 1998, we attempt to estimate GaR for big recession at the 5\textsuperscript{th} percentile by using 20-year long observation periods and thus obtaining small standard errors. Second, there is a trade-off between the standard error of estimated VaR and the risk of structural breaks. The structure of the economies may change over time. As such, emphasis on recent growth information can be helpful in tracking the changes in growth downside risk.
First let us examine the results for the United States. In his 1959 Presidential Address to the American Economic Association, Arthur Burns (1960, p. 1) predicted, if not the end of business cycles in the United States, at least "progress towards economic stability." The advent of stabilization policy, the end of bank runs, and structural changes in the economy all seemed destined to radically reduce short-run economic fluctuations in the postwar era. In Burns's (p. 17) words, "[T]he business cycle is unlikely to be as disturbing or troublesome to our children as it once was to our fathers."

Using our GaR as the new measure for downside growth risk, we now analyze to what extent Burns's prediction of growing stability stands. Figure 5 shows that GaR for big recession remains at the same level during the period 1980-1998. This implies that large downside fluctuations have not changed in some ways over time and have remained fundamentally similar over time. What has changed between the 1980s and 1990s is the decline of the average severity of small recessions. Figure 5 shows the decline of GaR for small recession in the 1990s. Based on the GaR indicators, it appears that the risk of small recession of the U.S. economy has declined in the 1990s. One caveat to this conclusion is the possibility that a trend toward much greater stability may be becoming apparent in the last 10 to 15 years.

Now we review the pattern of GaR for some regional groups. The 84 countries are divided into six regional groups. They are OECD, South America, Sub-Saharan Africa, Middle East and North Africa, East Asia and South Asia. The Appendix gives the list of countries in each group. Figure 6 and Figure 7 draw each regional group’s time pattern of GaR for big recession and small recession over the period 1980-1998.

On average, different regional groups experience very distinct GaR patterns over time. Non-OECD countries experience a higher downturn risk, while OECD countries’ downturn risks for both big and small recessions are the lowest among the regional groups. One may expect that East Asia’s growth miracle in the 1980s and early 1990s may lead to low growth risks (low GaR) because the shocks that are required to put an economy into recession are large. Surprisingly, East Asia countries, which had been growing faster, have had a high GaR for a big downturn at around 6 percent, much higher than those for OECD countries. Whereas its GaR for small recession has been declining over the period 1980-1998, and to a level close to OECD countries. The high GaR estimate of East Asia for big recession may be considered an early warning indicator for the severe economic downturn caused by the financial crisis in 1997.

Latin American countries and Saharan African countries have maintained high GaR value for both big recessions and small recessions through the period 1980-1998. During the 1990s per-capita income in Latin America grew at an annual average rate of around 2 percent, after having fallen at an annual rate of almost 1 percent during the 1980s. Its performance in the 1990s remained below the pace of economic expansion to which the region was accustomed prior to the debt crisis. But a declining GaR for big recessions in the 1990s does give hope that the continent might be leaving the "lost decade" firmly behind. Nonetheless, the region remains gripped with a disconcerting level of economic downturn risk.
In addition, as Latin America entered the 1990s it found itself in a world of high macroeconomic volatility, driven in large part by highly volatile capital flows--or at least magnified by them. The high level of international financial integration in the region has put heavy pressure on macroeconomic policies and policymakers. Key instruments used in the past such as pegged exchange rate and interest rate have either been abolished or become ineffective. New macroeconomic policies such as inflation-targeting and other risk management instruments must be experimented and implemented. These challenges to policymakers may have exacerbated the volatility of economic outcomes both over time and across households.

Basically, GaR serves as a practical summary measure for predicting the risk for, and severity of, big and small recessions given one-year time horizon, based on the past distribution of growth within 20 years.

5. Downside Growth Risk and Long Term Growth

In this section we examine the relationship between downside growth risk and long-term growth using data on over 84 countries from 1980 to 1998 in the tradition of cross-country analysis. Following Ramey and Ramey 1995, we begin with simple correlation and followed by cross-country regressions with several controlling variables. We first calculate the correlation between the average annual growth rate and GaR5 for big recessions and GaR20 for small recessions, respectively. The result for big recession risk is

\[ g_i = 3.472 - 0.269 \text{GaR5}_i \]

\[ (3.762) \quad (-2.513) \]

\( R^2 = 0.061, \) (t statistics in parentheses), and the result for small recession risk is

\[ g_i = 2.215 - 0.228 \text{GaR20}_i \]

\[ (4.241) \quad (-2.110) \]

\( R^2 = 0.052, \) (t statistics in parentheses). As the regressions show, there is a negative and statistically significant association between long-term growth and downside growth risk.

In light of previous cross-country empirical studies of growth especially Levine and Renelt (1992), we now examine the relationship between long-term growth and GaR in models that control for other important variables. The purpose is to test whether this negative relationship is still robust. The econometric specification is given by

\[ g_i = a + bX_i + c\text{GaR}_i + \varepsilon_i \]
where $g_i$ is the average annual growth rate of per capita GDP, $X_i$ is a vector of control variables, $GaR_i$ is a vector of growth downside variables, $b$ and $c$ is the coefficient vector, $\varepsilon_i$ is the residual. Levine and Renelt’s (1992) basic information set consists of the following variables: the average investment share of GDP, initial log GDP per capita, the average growth rate of the population, trade/GDP ratio. We use this dataset as the basis and add two variables we constructed: GaR5 and GaR20.

Table 1 presents estimation results of the above models. The regression in Column 1 shows that the coefficient of GaR5 for big recession is negative and significant at 10 percent confidence. Similarly, Column 2’s regression shows that the coefficient of GaR20, for small recessions is negative and significant. These imply that there is indeed a negative association between downside growth risks and long-term growth rates. The transmission channel might be through investment. When a country’s perceived level of downside growth risk is relatively high, investors might be cautious in making long-term investment in the country. Foreign investors in particular might be deterred from investing in this country and instead they may invest somewhere else. Thus, long-term growth would suffer. Furthermore, even human capital investment might be affected as the rate of return from investing in education is low and uncertain. This association is quite plausible in countries that have had poor growth performance and political instability in the past, such as some Sub-Saharan African countries.

Column 3 includes both GaR5 and GaR20 and shows that the coefficients for both GaR5 and GaR20 are not significantly different from zero. This should not be a surprise since there is significant multi-collinearity between two risk measures. In particular, the two downside growth risk indicators are highly correlated with each other; with the Pearson correlation coefficient of 0.699 for contemporaneous correlations over 1980-1998. And by definition, GaR20 for small recessions includes all the events that were included in GaR5 for big recessions. The multi-collinearity problem in this particular regression would tend to lead to insignificant coefficients. Nonetheless, it is interesting to note that the coefficient for GaR5 is still a bigger negative number than that for GaR20. This implies that the negative effect of growth at risk for big recessions is more important, once a more complete panel data analysis has been conducted.

Also, three regressions produce the same results as Levine and Renelt (1992) for the basic variables: (1) investment rate has robust and positive effects on growth (2) the coefficients of initial per capital GDP are negative and significant; (3) the coefficients of population growth are negative and robust; (4) the coefficients of trade/GDP ratio are positive but not significant.
<table>
<thead>
<tr>
<th>Independent variable</th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
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<tr>
<td>Constant</td>
<td>-8.590** (-4.301)</td>
<td>-9.228* (1.914)</td>
<td>-8.535** (-4.265)</td>
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<td>Investment share of GDP</td>
<td>0.655** (7.686)</td>
<td>0.649** (7.610)</td>
<td>0.658** (7.703)</td>
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<td>Population Growth</td>
<td>-0.334** (-2.861)</td>
<td>-0.375** (-3.358)</td>
<td>-0.334** (-2.849)</td>
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<td>Trade/GDP ratio</td>
<td>0.052 (0.510)</td>
<td>0.064 (0.628)</td>
<td>0.063 (0.612)</td>
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<td>Initial per capita GDP</td>
<td>-0.272** (-2.105)</td>
<td>-0.290** (-2.184)</td>
<td>-0.298** (-2.246)</td>
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<tr>
<td>GaR5 for Big Recessions (1-c=0.05)</td>
<td>-0.194* (-1.970)</td>
<td></td>
<td>-0.137 (-1.160)</td>
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<tr>
<td>GaR20 for Small Recessions (1-c=0.20)</td>
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<td>-0.169* (-1.814)</td>
<td>-0.098 (-0.884)</td>
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<td>Adjusted R Square</td>
<td>0.517</td>
<td>0.514</td>
<td>0.516</td>
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Note: numbers in parentheses are t statistics.  
** indicates significance at 5 Percent confidence;  
* indicates significance at 10 percent confidence.
6. Concluding Remarks

Growth volatility is important due to its implications to welfare and long-term growth. Perhaps even more important are the severe economic downturns that occur periodically and have grave consequences to the poor. In this paper we propose a perspective of growth risk to understand economic fluctuations and a new measurement of the downside growth risk GaR. This measure enables us to quantify the perceived level of downside growth risk and severity, in a single number, thereby facilitate risk monitoring overtime and comparison across countries. However, using GaR as a predicting tool is not advisable because of the risk of structural breaks.

This paper sheds light on quantifying downside growth risks by constructing a panel dataset on GaR. Using GaR in a cross-country analysis, we find that higher perceived levels of downside growth risk seem to be negatively and strongly associated with long-term growth. When a country’s perceived level of downside growth risk is relatively high, both domestic and foreign investors might be deterred from making long term investment in this particular country and instead they may invest somewhere else. There might be a vicious circle, from one recession, to higher perceived risk and low investment, (or domestic instability) and to another recession or long term stagnation. The results seem to suggest what we already know, that prudent and consistent pursuit of socio-economic and political stability would contribute to long-term growth, and that risk management in a broader sense, as suggested in Wang (2001), should be a vital part of pro-growth and poverty reduction strategy.

Much remains to be done. This is only the first stage of our analysis on growth at risk. In the next stage we plan to conduct panel data analysis on the determinants of growth at risk, and its relationship with long term growth.
## Appendix

Table A1  The List of Countries in Different Regional Groups

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Lustig, Nora. 1999. “Crises and the Poor: Socially Responsible Macroeconomics.” Inter-American Development Bank, Sustainable Development Department, Poverty and Inequality Advisory Unit, Washington, D.C.


Figure 1

U.S. Annual Growth Rate of Per Capita GDP 1961-1998
Figure 2

Histogram of US Annual Growth Rates 1961-98

Growth rate

Frequency

Frequency

-2.95  -1.6  -0.25  1.1  2.44  3.79  More
Figure 3

Histogram of Hong Kong's Annual Growth Rates 1961-98

Frequency

-7.73 -3.93 -0.14 3.65 7.47 11.24 More
Figure 4

Histogram of Brazil's Annual Growth Rates 1961-98

Frequency

-6.48  -3.51  -0.53  2.44  5.41  8.38  More

Frequency
Figure 5

Growth at Risk for Small and Big Recessions of the United States
1980-1998

GaR (Percent)
Figure 6
Growth At Risk for Small Recessions: Regional Groups 1980-1998
Figure 7
Growth At Risk for Big Recessions: Regional Groups, 1980-1998

Growth at Risk for Big Recessions of Regional Groups 1980-1998

- OECD
- Latin America
- Sub-Saharan Africa
- MENA
- East Asia
- South Asia
- China