GROWTH AND INCOME CONVERGENCE

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The Solow model of economic growth (Solow, 1956, Swan, 1956) concludes that poorer countries will tend to grow faster than richer ones—provided that countries share the same production function, savings rate and population growth, and labour-augmenting technology grows at the same rate in all countries. The existence of income convergence has thus been usually taken to be a test of exogenous growth model versus endogenous growth models—that do not necessarily conclude on the existence of convergence in income per capita among economies. Here we describe different concepts of convergence used in the empirical literature on economic growth and summarize the results of this literature.

The Solow model and income convergence

Total output ($Y_t$) is assumed to depend on physical capital ($K_t$), labour input ($L_t$) and (labour-augmenting) technology ($A_t$) according to a Cobb-Douglas production function with costant returns to scale on all inputs,

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha},$$

where $\alpha \in (0,1)$. Labour input and technology are assumed to grow at constant rates $n$ and $g$, respectively. Physical capital is accumulated through savings (with a constant savings rate $s$) and depreciates at a constant rate $\delta$,

$$\frac{dK_t}{dt} = \dot{K}_t = sY_t - \delta K_t.\quad(1)$$

We can write (1) in terms of effective labour as

$$\dot{k}_t = sy_t - (n + \delta + g)k_t,\quad(2)$$

where $k_t = K_t/(A_t L_t)$ and $y = Y_t/(A_t L_t) = k_t^\alpha$. The steady state level of capital per unit of effective labour ($k^*$) can be found by setting $\dot{k}_t = 0$, which leads to

$$s(k^*)^\alpha = (n + \delta + g)k^*.\quad(3)$$

Graphically, the equilibrium level of $k$ is given by the intersection point of the investment per unit of effective labour curve, $s(k)^\alpha$, with the break-even investment line, $[(n + \delta + g)k]$, as shown in Figure 1.\(^1\) Countries with levels of capital per unit of effective labour below $k^*$ (see $k_1$ in Figure 1) present positive growth in the stock of capital per unit of effective labour (see (2)), while countries to the right of $k^*$ will tend to decrease their stock of capital per unit of effective labour.

Log-linearizing around the steady state level of income per unit of effective labour,

$$\frac{d \ln(y_t)}{dt} = \lambda [\ln(y^*) - \ln(y_t)],\quad(4)$$

\(^1\)The break-even investment line represents the investment needed to avoid the capital stock from falling.
which implies that the growth rate of income per unit of effective labour (and thus income per capita) is related to the distance to the steady state level of income. This means that the Solow model concludes that (after controlling for those factors that determine differences in the steady state level of income per capita) poorer countries should grow at higher rates than richer countries.

**Unconditional and conditional β-convergence**

A natural empirical test of income convergence from (4) is based on regressing the growth rate of income per capita on initial income levels for a cross-section of countries or regions (see Barro and Sala-i-Martin, 1992). A negative (positive) correlation between these two variables indicates the existence of so-called unconditional β-convergence (-divergence). Figure 2 presents the corresponding scatterplot (growth rates of GDP per capita versus initial GDP per capita in the period 1970-2000) for all countries in the world for which Penn World Table data are available and the same scatterplot for Spanish provinces in the period 1980-2005 (source: Cambridge Econometrics). As can be seen from the scatterplots, considering more homogeneous groups of economic units (which are more likely to be successfully modelled through the theoretical setting put forward above), the empirical relevance of unconditional β convergence appears more evident.

The Solow model predicts income convergence across countries which share the same production function, investment rate, population growth, depreciation rate and common growth rate of
technology. In order to account for such differences, we can control for these (and potentially other) variables in the $\beta$-convergence regression. If a negative partial correlation between initial income and subsequent income growth appears after controlling for other covariates, conditional $\beta$-convergence is said to exist.

$\sigma$-convergence and divergence

Intuitively, convergence takes place if the dispersion of income across countries is reduced over time. This concept of convergence is known as $\sigma$-convergence. Figure 3 presents the evolution of the standard deviation of income per capita across world countries and European regions (source: Cambridge Econometrics). The results in Figure 3 give evidence of $\sigma$-divergence across countries at the world level but convergence within more homogeneous economic areas (in this case, Europe). The statistical significance of changes in the dispersion of income can be evaluated using the test put forward by Carree and Klomp (1997).

It can be easily shown that $\sigma$ convergence implies $\beta$ convergence, but the opposite does not necessarily apply (see for example Furceri, 2005).

The dynamics of the world distribution of income

The convergence analyses presented above (and most of the studies existing on the dynamics of income at the world level) take countries as the natural unit of analysis. The results concerning convergence/divergence across countries do not necessarily imply convergence/divergence across individuals at the world level. A first hint at the differences appearing from both approaches can be obtained from weighted cross-country $\beta$-convergence regressions using population as a weight. The unweighted parameter estimate corresponding to the data presented in Figure 2 for the whole world is 0.002 (standard deviation = 0.002), while the weighted estimate is -0.012 (standard deviation = 0.001), which indicates convergence once that we take into account the size of each country in terms of population.

Sala-i-Martin (2006) reconstructs the dynamics of world income across individuals by matching macroeconomic estimates of income per capita (at purchasing power parity) with estimates of income dispersion across individuals within countries. Notwithstanding the degree of uncertainty implied by the fact that within-country dispersion estimates are not available for all countries and need to be projected from neighbouring countries, Sala-i-Martin (2006) finds evidence of income convergence for individuals in the period 1970-2000. Sala-i-Martin’s (2006) approach is not without criticism. Milanovic (2003) criticises the approach heavily and pinpoints several reasons
why Sala-i-Martin’s (2006) study has some drawbacks which tend to exaggerate the decrease in
global income inequality. In particular, among other issues,

a) several nations where within-country income inequality has risen in the period under study
(and where data are available!) are omitted from the analysis,

b) the data on the distribution of income across households is treated as if they reflected
income distribution across individuals.

Nonlinearities and club convergence

Azariadis and Drazen (1990) present a theoretical model where heterogeneity in the marginal
productivity of capital across levels of the capital stock leads to multiple equilibria corresponding
to steady states at different income levels. This result implies that depending on the initial level
of income, countries may converge to different equilibria and thus may get stuck at relatively low
levels of GDP per capita, corresponding to a so-called poverty trap.

Empirically, the existence of club convergence can be tested by estimating piecewise-linear
models where the initial level of GDP per capita determines the parameters corresponding to
the other covariates in the regression equation. Durlauf and Johnson (1996) present empirical
evidence of this type of nonlinearities in cross-country growth regressions.

References

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