Budget Rules and Resource Booms and Busts: A Dynamic Stochastic General Equilibrium Analysis

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Abstract

This paper develops a dynamic, stochastic, general-equilibrium model to analyze and derive simple budget rules in the face of volatile public revenue from natural resources in a low-income country like Niger. The simulation results suggest three policy lessons or rules of thumb. When a resource price change is positive and temporary, the best strategy is to save the revenue windfall in a sovereign fund and use the interest income from the fund to raise citizens' consumption over time. This strategy is preferred to investing in public capital domestically, even when private investment benefits from an enhanced public capital stock. Domestic investment raises the prices of domestic goods, leaving less money for government to transfer to households; public investment is not 100 percent effective in raising output. In the presence of a negative temporary resource price change, however, the best strategy is to cut public investment. This strategy dominates other methods, such as trimming government transfers to households, which reduces consumption directly, or borrowing, which incurs an interest premium as debt rises. In the presence of persistent (positive and negative) shocks, the best strategy is a mix of public investment and saving abroad in a balanced regime that provides a natural insurance against both types of price shocks. The combination of interest income from the sovereign fund, transfers to households, and output growth brought about by public investment provides the best protective mechanism to smooth consumption over time in response to changing resource prices.

JEL classification: C68, E17, E62, F41, Q32

Key words: DSGE modeling, open-economy macroeconomics, budget rules, natural resources

I. Introduction

The discovery of oil and gas reserves in several low-income African countries has revived interest in the management of resource windfalls. On one hand, the experience of Africa's traditional resource exporters has been sobering. Despite hundreds of billions of dollars in oil revenues, countries such as Nigeria, Gabon, and Angola continue to suffer from high levels of poverty, low human development, and a
capital stock that is scarcely adequate for the post-oil era. On the other hand, countries such as Norway and Chile, which used strict fiscal rules to ensure that resource windfalls are saved and not subject to the irresistible temptation to spend, have achieved great progress. Yet, some argue that poor countries, being capital-scarce, should be investing their resource windfalls domestically rather than saving them abroad.

In this paper, we attempt to resolve these issues by simulating the impact of resource windfalls and policy responses in a model that captures both the implications of current decisions on future growth and welfare, and the uncertainty that is intrinsic to resource prices. We ask: What rules-of-thumb does dynamic stochastic general equilibrium (DSGE) analysis provide for public spending from uncertain and fluctuating resource windfalls in a low-income and newly resource-rich developing country like Niger? In particular, we analyze how budget rules should be set in countries that face spending pressures for social needs in the context of highly volatile mineral revenue. Drawing on the experience of Norway and Chile, many low-income developing countries with limited capacity to hedge or stabilize revenue are looking at budget spending rules to manage and allocate mineral revenue over time. In 2004, Nigeria adopted a fiscal rule for oil revenue, whereby any revenues in excess of the benchmark level for budgetary use are transferred into the “Excess Crude Account (ECA)” for future use. The Democratic Republic of Timor-Leste’s natural resource fiscal rule, the Petroleum Fund Law, defines the estimated sustainable income (ESI) as 3 percent of the country’s petroleum wealth based on an assumed reference price.

Two features of budget rules are potentially appealing in their applications to developing countries: a well-defined and publicly monitored budget price or spending rule may substitute for weak institutions; and transparent mechanisms may make it easier to achieve the necessary savings for future generations and guard against unsustainable expenditures in the present.

Even so, when mineral revenue is volatile, reserves finite, and spending pressures high, it is still not easy to set the optimal threshold price for a budget rule. When the resource price is high and on an upward trend, it is easy to define a threshold price that is below the market price to generate savings for future use. However, when the resource price falls, setting the threshold price becomes difficult and contentious. Are there simpler alternative budget rules when the resource price is very uncertain and volatile?

The problems associated with resource windfalls, variously known as the Dutch disease or the paradox of plenty, have been examined by several authors, such as Corden and Neary (1982) and Collier and Hoeffler (2004). Several recent studies point to pathways by which resources can have positive effects on long-term growth. These include: (i) good governance (Mehlum, Moene, and Torvik 2006), (ii) openness to international trade (Arezki and van der Ploeg 2012 and Sachs and Warner 1997), and (iii) better management of the resource revenue (Collier, van der Ploeg, Spence, and Venables 2010, Benigno and Fornaro 2014, and Arezki and van der Ploeg 2008, who embrace strategies relating to the permanent income hypothesis and sovereign wealth funds).

Among low-income countries with newfound mineral resources, Niger is a good example to examine. Over the next two decades, Niger’s exports of oil and uranium are expected to account for 20 percent or more of GDP beginning in 2015 (from less than 8 percent in 2009), and total public revenue will reach at least 20 percent of GDP in 2015 (from less than 15 percent in 2009). However, its historical growth record has been very weak due to decades of political instability and turmoil. Per capita income growth was negative in the 1990s and close to zero in the 2000s, albeit more positive in recent years. The country currently ranks at the bottom of UNDP’s human development index. Its poverty rate is high—about 44 percent of the population lives below $1.25 a day. With a terrain that is harsh and dry, and more than 80 percent in desert conditions, the road and infrastructure network in this landlocked economy is very limited, and lack of water is a major problem.1 In such a setting, Collier et al. (2010) argue for scaling up present consumption and investment where the marginal social value of consumption is high in the near term because the country is poor and where the social returns to investment are high because capital is scarce. But given Niger’s weak capacity and

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1 See Go et al. (2013) below for a more detailed description of Niger and its resource revenue.
institutions and the danger of potential boom-bust cycles from overspending, Go, Robinson, Thierfelder, and Utz (2013) caution against complicated policy rules and recommend the more conservative bird-in-hand (BIH) strategy, essentially spending only the interest income from banked and accumulated mineral revenue. Their analysis, however, uses a CGE model where decisions may not be intertemporally optimal, nor does it account for revenue volatility and uncertainty, both of which may alter the prescription.

In this study, we examine in a dynamic, stochastic, general-equilibrium model the implications of various budget rules for resource windfalls in a developing country in an uncertain environment. Instead of explicitly defining a threshold or optimal “budget price” of the natural resource, we explore alternative budget rules when the resource price is volatile and uncertain. We focus on simple decision rules that are applicable to countries with capacity constraints. As a case study, we apply the analysis to the economy of Niger, which has recently joined the club of low-income mineral exporter countries.

We build upon recent contributions to the vast literature of DSGE models that have been applied mainly to developed economies and upon the modeling lessons learned from the analysis of the Dutch disease in the static and dynamic CGE literature. More recently, DSGE models have also been applied to policy issues of developing countries, including resource-rich countries (see, e.g., Berg, Portillo, Yang, and Zanna 2013 and Melina, Yang, and Zanna 2014). In our study, we extend the well-established 1-2-3 CGE model of a small, open economy to a dynamic setting, incorporating uncertainty. The 1-2-3 CGE model, which is a one-country, two-producing sectors, and three-goods model, was initially developed in a static framework and has been used to analyze several issues in small open economies by Devarajan, Lewis, and Robinson (1993) and Devarajan, Go, Lewis, Robinson, and Sinko (1997). It was later extended to incorporate dynamics in a deterministic setting by Devarajan and Go (1998). Our methodology incorporates DSGE analysis to the 1-2-3 model, with the following features: it keeps both the microeconomic foundations of the dynamic analysis and the consistency in the circular flow of incomes and expenditures at every point in time; and it uses a solution strategy that avoids linearization. In addition to introducing uncertainty in the model, we consider productive government spending, whereby public capital affects the productivity of private inputs. We are then able to assess the macroeconomic dynamics of stochastic shocks to resource windfalls and policy responses to them and trace the impulse or time responses of macroeconomic aggregates such as output, consumption, investment, exports, imports, government revenue and balances, real exchange rate, and debt.

The remainder of the paper is organized as follows. We present the model specification in the next section and discuss the data, calibration, and solution strategy in the third section. The simulation results are presented in the fourth section, followed by the conclusions in the last section.

II. The Model

Overview
The model extends the dynamic 1-2-3 (D123) model of Devarajan and Go (1998), where the two sectors were the domestic and export sectors, and three goods, the domestic good, exports, and imports. We introduce three new features that make it possible to analyze the impact of uncertainty on the price of natural resources in the economy. First, we split the exports into two goods: traditional exports and natural resource exports, increasing the number of goods to four. Second, we distinguish two types of capital – private and public. And third, we introduce uncertainty in the model by incorporating the stochastic process of the world price of the natural resource in the decision-making of economic agents. We call the model the DS1234 model: a dynamic and stochastic model of one country, with two types of capital, three producing sectors, and four goods. The separation of public and private capital permits the government to use mineral revenues to raise current consumption through transfers to households or increase future output through public investment in infrastructure. The three sectors produce the domestic good, which is consumed domestically, and the traditional export good and the natural resource, which are not. The fourth good is represented by the import good, which is not produced domestically.
This small open economy has four economic agents: the representative household, the representative firm, the government, and the rest of the world (ROW). Households and firms exhibit forward-looking behavior: their current decisions are affected by expectations of future period variables or policy parameters. The representative household’s portfolio consists of two assets: shares of domestic assets and foreign assets. It values leisure, supplies labor to firms, receives dividends from domestic firms, transfers from the government, and net transfers from the rest of the world. It pays income taxes, consumes goods, and saves.

The representative firm produces output for final demand purposes only. There are no intermediate inputs in the model; as such, gross output is identical to GDP. The production of output uses labor and private physical capital. In contrast to the original D123 model, we assume that public capital generates an externality in production and its level affects the productivity of private capital and labor. The government levies taxes on economic activities, on factor incomes, and on transactions; it consumes goods, invests in public capital, enacts transfers to households, and has the option to invest part of its revenue in a sovereign wealth fund that yields interest income. The government invests in public infrastructure, whose stock affects the productivity of private inputs. Both firms and households take government policies and the stock of public capital as given. The government does not issue domestic or foreign bonds, and it is constrained to follow some fiscal rules that will be discussed later. Since we abstract from government debt to finance its spending, we assume that the private sector (the representative household) is responsible for the foreign debt.

The economy is small in the sense that it takes world prices of imports and exports as given. Domestic economic agents can borrow on the international market with a debt-elastic interest rate premium as in Schmitt-Grohé and Uribe (2003). The main reason that the interest rate is sensitive to the debt level is to ensure that the steady-state solution will be independent of initial conditions.

We abstract from the exogenous long-run growth rate of the economy; we make the model stationary by detrending all quantity variables by expressing them in efficiency units.

**Households**

We consider a small open economy inhabited by a continuum of identical households with unit mass. Following microeconomic foundations and the specifications in DSGE or intertemporal models, the representative consumer has preferences over consumption and leisure. In each period, he has one unit of time that can be devoted to work \(H_t\) and leisure. His preferences are represented by a time-separable utility function \(U_0\):

\[
U_0 = E_0 \sum_{t=0}^{\infty} \left( \frac{1}{1+\rho} \right)^t u(C_t, H_t)
\]

where \(C_t\) is aggregate consumption, \(H_t\) labor supply, and \(\rho\) the pure rate of time preference. We specify a CRRA (constant relative risk aversion) within-period utility function \(u(C_t, H_t)\) with the following representation:

\[
u(C_t, H_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} + \frac{\mu}{1-\psi} (1-H_t)^{1-\psi}
\]

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2 It is possible that liquidity constraints in a poor country like Niger would limit consumption-smoothing, making consumption more directly responsive to current income. However, the emergence of resource wealth also presents a significant departure and opportunity to conduct optimal intertemporal decisions through a sovereign wealth fund even in a low-income country. With more income or in high-income countries, the permanent income hypothesis is generally supported by studies such as Campbell and Mankiw (1989); Shea (1995) also found that liquidity constraints do not seem to affect consumption behavior in households with no liquid assets relative to those with liquid assets.
where \( \sigma \) is the inverse of the intertemporal elasticity of substitution, \( \psi \) is a parameter linked to the Frisch labor supply elasticity,\(^3\) and \( \mu \) is the leisure weight in the utility function. The representative household derives income from wages, returns on assets, government transfers \( TR_t \), and foreign remittances \( FR_t \). It pays taxes on consumption, on labor income, and on dividends received from domestic firms.

The representative household maximizes expected intertemporal utility (1) subject to a sequence of period budget constraints (3) while respecting a transversality condition (4).

\[
F_{t+1} = (1 + r_t)F_t + (1 - \tau_{YL})W_tH_t + ER_tFR_t - \tau_k Div_t - (1 + \tau_c)PC_tC_t
\]

\[
\lim_{E_t} \frac{F_{t+1} - \epsilon}{\prod_{s=1}^t (1 + r_s)} \geq 0
\]

The parameters \( \tau_{YL}, \tau_c, \) and \( \tau_k \) are, respectively, the labor income tax rate, consumption tax rate, and the tax rate on dividends. \( W_t, PC_t, ER_t, \) and \( Div_t \) are, respectively, the wage rate, the price of the consumption good, the currency conversion factor (nominal exchange rate), and the dividends received from firms; \( r_t \) is the debt-elastic interest rate and \( F_t \) the value of the domestic firm less the net liabilities of the private sector to foreigners (private foreign debt). The currency conversion factor is in units of domestic currency per unit of foreign currency.

Solving the optimization problem makes it possible to determine the expected optimal paths of consumption and leisure. The first-order conditions of the household optimization problem are the consumption Euler equation (5), the period budget constraints (equation (3) above), and the contemporaneous arbitrage condition between leisure and consumption (6).

\[
\frac{1}{C_t} = E_t \left[ \frac{1}{C_{t+1} (1 + \rho)} \frac{PC_t}{PC_{t+1}} \right] \quad (5)
\]

\[
\mu(1 + \tau_c)PC_tC_t = (1 - \tau_{YL})W_t(1 - H_t)\psi \quad (6)
\]

The representative household determines the optimal path of consumption spending, which is then allocated within each period between the domestic good and the import good depending on their relative price. At the margin, the representative household allocates aggregate consumption between two consecutive periods such that an increase in the expected real interest rate in comparison to the rate of time preference penalizes current period’s consumption to the benefit of the expected value of next period’s consumption. The expected real interest rate depends on the exogenous interest rate and the change in the price of aggregate consumption, which is a composite of the domestic good price and the import good price. Following a terms-of-trade shock, changes in the price of the domestic good, for example, will have an impact on the expected real interest rate and induce a change in the path of aggregate consumption. Moreover, a change in the debt-elastic interest rate will induce the same effect. In each period, the level of aggregate consumption depends on the household’s expected total wealth, which is the sum of its expected financial wealth and its expected human wealth. In this set-up, the financial wealth of the representative household equals the value of domestic firms less the value of net liabilities of the private sector to foreigners.

In each period, aggregate consumption is allocated between the domestic good and the import good through expenditure minimization. Without loss of generality, we assume that the composition of aggregate consumption is identical to those in other components of domestic absorption, the sum of household and government consumption, private and public investment. In particular, domestic absorption is a constant elasticity of substitution (CES) aggregate of the domestic good and the import good. At the margin, an increase in the relative price of the domestic good will reduce the ratio of the demand for the domestic good to the import good.

\(^3\) In our specification, \( \psi \) is not exactly equal to the inverse of the Frisch labor supply elasticity.
The Representative Firm

We consider a joint production technology of output, a composite of the domestic good and the two export goods. The composite good, $\text{XTS}_t$, is produced by means of a Cobb-Douglas production function by combining private capital, $K_t$, and labor, $LD_t$. The level of public capital, $KD_t$, generates an externality in production by affecting the productivity of both private inputs. As mentioned earlier, firms take the level of public capital as given in their decisions.

$$\text{XTS}_t = AV(KG_t)^{a_G}(K_t)^{a_V}(LD_t)^{1-a_V}$$  \hspace{1cm} (7)

where $AV$ is total factor productivity parameter, $0 < a_G < 1$ is the output elasticity of public capital, and $0 < a_V < 1$ is the input share of private capital in the production function.

In each period, the level of capital stock is predetermined by past investment decisions, through the capital accumulation equation, which is characterized by a constant rate of depreciation of capital, $\delta$.

$$K_{t+1} = (1-\delta)K_t + I_t$$  \hspace{1cm} (8)

The representative firm funds its investment expenditures out of retained earnings and pays dividends to the representative household. Total investment, $J_t$, includes installation costs, since private capital accumulation is subject to adjustment costs that are foregone output linked to investment decisions. We stipulate a quadratic adjustment cost function that is linearly homogeneous in $I_t$ and $K_t$ and hence the following specification for total investment function:

$$J_t = I_t \left[1 + \frac{\beta I_t}{2K_t}\right]$$  \hspace{1cm} (9)

where $\beta$ is a positive adjustment cost parameter. The installation costs help to capture the fact that the capital stock cannot instantaneously reach its desired level. It follows that labor input and investment decisions are the two decision variables that must be determined by the representative firm in each period to determine the aggregate level of output.

The economy is subject to a stochastic shock to the world export price of natural resources. We assume that this price follows an autoregressive $AR(1)$ process with a persistence parameter $\rho_Z$ and with identically, independently distributed disturbances $\varepsilon_t$:

$$\text{PWEXR}_t = \overline{\text{PWEXR}} e^{\rho_Z Z_t}$$  \hspace{1cm} (10)

$$Z_{t+1} = \rho_Z Z_t + \varepsilon_{t+1}; \quad \varepsilon_{t+1} \sim NID(0, \sigma_e^2); \quad t \geq 0$$  \hspace{1cm} (11)

where $\overline{\text{PWEXR}}$ is the steady-state level of the resource export price. These shocks affect the representative firm’s decisions and hence factor prices and domestic good price, with some repercussions on household and government decisions. After determining the expected optimal time profile of its output, the representative firm decides upon its allocation in each of the three markets depending on their relative price.

Intertemporal Decisions

The objective of the representative firm is to maximize the expected value of the discounted sum of current and future cash-flows subject to a capital accumulation equation in the presence of adjustment cost and a transversality condition, in order to determine the optimal path of decision variables (labor and investment).

$$\max V_0 = E_0 \sum_{t=0}^{\infty} \prod_{s=0}^{t-1} \left(1 + r_s \right)^{1-s} \text{Div}_t$$  \hspace{1cm} (12)
where $Div_t$ is the dividends paid to households. The first-order conditions of the firm’s problems (including the capital accumulation (8) or in the constraint (12)) are as follows:

$$W_t LD_t = (1 - \sigma_v) PXTS_t XTS_t$$

(13)

$$Q_t = \left[1 + \beta \frac{I_t}{K_t}\right] PC_t$$

(14)

$$Q_t (1 + r_t) = E_t[(1 - \tau_p) PXTS_t RK_{t+1}] + E_t(1 - \delta) Q_{t+1} + E_t \left[PC_{t+1} \left(\frac{\beta}{2} \left(\frac{I_{t+1}}{K_{t+1}}\right)^2\right)\right]$$

(15)

$$RK_t = \frac{\sigma V XTS_t}{K_t}$$

(16)

$$Div_t = (1 - \tau_p) PXTS_t RK_t - PC_t f_t$$

(17)

where $Q_t$, $RK_t$ and $\tau_p$ are, respectively, the shadow price, the marginal productivity of private physical capital, and the output tax rate. $PXTS_t$ is the price of the composite good. At the margin, the optimal level of labor will be determined in each period by setting its marginal product equal to the wage rate (13). Current period investment is determined using the $Q$-theoretic rule, that is, the optimal level of investment is determined so as to equalize the marginal cost of investing to the shadow price of capital (14). Expressions (15), (16), and (17) are, respectively, the motion equation of the shadow price of private capital, the marginal product of private capital, and the dividend paid to the representative household. From the optimal levels of labor, investment (hence the level of capital), the optimal level of the composite output can be determined.

### Intratemporal Decisions

We assume that shifting production between the domestic good and the two export goods is costly, in the sense that there exists a concave transformation curve of the composite good into its three components. Specifically, we consider a constant elasticity of transformation (CET) function between the composite output $XTS_t$ and sales in the domestic market, $XDS_t$, traditional exports, $EXT_t$, and exports of natural resources, $EXR_t$. Once the optimal level of the composite output is known, its allocation in the three markets can be determined through revenue maximization subject to the technological constraint. The first-order conditions of this maximization problem are the following:

$$PXTS_t = \left[\left(\delta_{XT}\right)^{-\sigma_x} (PEXT_t)^{1+\sigma_x} + (\delta_{XR})^{-\sigma_x} (PEXR_t)^{1+\sigma_x}\right]^\frac{1}{1+\sigma_x}$$

(18)

$$EXT_t = (AX)^{-1-\sigma_x} XTS_t \left(\frac{PEXT_t}{\delta_{XT} PXTS_t}\right)^{\sigma_x}$$

(19)

$$EXR_t = (AX)^{-1-\sigma_x} XTS_t \left(\frac{PEXR_t}{\delta_{XR} PXTS_t}\right)^{\sigma_x}$$

(20)

$$XSD_t = (AX)^{-1-\sigma_x} XTS_t \left(\frac{PD_t}{(1 - \delta_{XT} - \delta_{XR}) PXTS_t}\right)^{\sigma_x}$$

(21)

where $PEXT_t$, $PEXR_t$, and $PD_t$ are, respectively, the prices of the traditional export good, the natural resource export good, and the domestic good. $AX$, $\sigma_x$, $\delta_{XT}$, and $\delta_{XR}$ are, respectively, the shift parameter, the elasticity of substitution, and the weights of traditional exports and resource exports in the CET function. At the margin, an increase in the relative price of any of the components of the composite output induces an increase of its supply at the expense of the other components.

Hence, in our model, changes in the export price of the natural resource will also affect its export supply, which is endogenous (not fixed) and responsive to relative prices in our CET function. The supply of
the other two goods are likewise affected by the changes in the relative prices along the process. As the resource export price is volatile, so too is the export volume of the mineral resource, which adds another source of uncertainty in the stochastic dynamics.

The Government
In each period, the government derives revenue, \( YG_t \), from current taxes on output, consumption, dividends, labor income, imports, from transfers received from the ROW, and from royalties on the exports of resources (23). The royalties are on an ad valorem basis, and as in Berg et al. (2013), we assume that for its regular revenue, \( YG_t \), the government does not consider the current value of the royalties but their steady-state value. Hence, the difference between the current and the steady-state values of the royalties is considered a windfall, \( WF_t \). We consider four regimes described below for the use of the windfall by the government. It is important to note that in a given year the windfalls could be negative if the current value of the royalties is lower than their steady-state level. With the small-country assumption, the royalties create a wedge between the world price and the price received by the producer, as shown in the following equation:

\[
P_{EXR}(1 + \tau_{EXR}) = ER_t \cdot P_{WXEXR_t}
\]

where \( \tau_{EXR} \) is the royalty rate and \( P_{EXR} \) is the producer price of the resource export good.

The expressions of government revenue, \( YG_t \), and of the windfall, \( WF_t \), are as follows:

\[
YG_t = \tau_p PXTS_t XTS_t + \tau_C PC_t C_t + \tau_d Div_t + \tau_Y W_t LD_t + \tau_w PM_t M_t + \tau_{EXR} P_{EXR} \cdot EXR + ER_t TROWG_t
\]

\[
WF_t = \tau_{EXR}(P_{EXR} - P_{EXR} \cdot EXR)
\]

Regular government expenditures in each period consist of the current value of its initial steady-state real total expenditures on consumption and investment goods \( (\overline{G}_t) \). Its value changes from year to year because of the changes in prices. We assume that \( \overline{G}_t \) is a Leontief function of government steady-state consumption, \( \overline{G}_c \), and investment in public infrastructure, \( \overline{G}_i \), (25). In addition to the steady-state level of investment in public infrastructure, depending on the fiscal regime considered, the government may use part of the windfall, \( WF_{t^{inv}} \), to fund additional investment in public infrastructure as described in equation (27). The evolution of the stock of public capital, which appears in the production function, is described in equation (30). The parameter \( 0 < \xi < 1 \) captures the efficiency of translating public investment into effective public capital.

\[
\overline{G}_t = \min \left[ \frac{G_t}{x_t \cdot \overline{G}_t} \right]
\]

\[
INVG_t = \overline{G}_t + \frac{WF_{t^{inv}}}{PC_t}
\]

\[
KG_{t+1} = \xi \cdot INVG_t + (1 - \delta_G)KG_t
\]

In each period, the government transfers to households, \( TR_t \), the balance between its revenue and its expenditures (28). The right-hand side of the expression in (28) has a third component, \( WF_{t^{tr}} \), which may differ from zero depending on the fiscal rule used, such as when the government increases the transfer part of the windfalls to households.

\[
TR_t = YG_t - PC_t \overline{G}_t + WF_{t^{tr}}
\]
government, and investment demands as in (29). As discussed earlier, we assume that all components of the domestic absorption have the same preferences over $XDD_t$ and $M_t$. We assume that domestic absorption is a CES aggregate of $XDD_t$ and $M_t$. A cost-minimization rule makes it possible to determine the optimal allocation of domestic absorption between its two components. The first-order conditions of this problem give the price of the composite consumption good (30), the demand for domestic good (33), and the demand for import (32), where $\delta_M$, $\sigma_M$ and $AM$ are, respectively, imports weight, substitution elasticity, and shift parameter in the CES aggregate of domestic good and imports. $PM_t$, $PWM_t$, and $\tau_M$ are, respectively, the price of imports gross of the import taxes in the domestic currency, the world price of imports in foreign currency, and the import tariff rate.

$$XT_t = C_t + G_t + INV_t + J_t$$ (29)

$$PC_t = \frac{1}{AM}[(\delta_M)^{\sigma_M} + (PM_t)^{1-\sigma_M} + (1 - \delta_M)^{\sigma_M}(PD_t)^{1-\sigma_M}]^{\frac{1}{1-\sigma_M}}$$ (30)

$$M_t = AM^{\sigma_M-1}XT_t\left[\frac{\delta_MP_t}{PM_t}\right]^{\sigma_M}$$ (31)

$$PM_t = ER_tPWM_t(1+\tau_M)$$ (32)

$$XDD_t = AM^{\sigma_M-1}XT_t\left[\frac{(1 - \sigma_M)PC_t}{PD_t}\right]^{\sigma_M}$$ (33)

$$FSAV_t = PWM_tM_t - PWEXT_tEXT_t - PWEXR_tEXR_t - (FR_t + TROW_tG_t) + WF^{faw}_t$$ (34)

$$BF_{t+1} = (1 + r_t)BF_t + FSAV_t$$ (35)

$$r_t = r^* + \gamma(e^{BF_t-BF} - 1)$$ (36)

Foreign saving, which is the current account deficit (34), is the difference between the trade deficit and the transfers received from the rest of the world by households and the government. A third component, $WF^{faw}_t$, related to the use of the windfalls is added to foreign saving. Its value depends on the fiscal rule considered. Foreign savings or the current account deficit contributes to the country’s external debt (35), whose interest rate is sensitive to the level of debt above some threshold (36).

Fiscal Rules

Four different fiscal regimes are considered for the use of the windfall, $WF_t$:

Regime 1 - (All-consuming approach): the resource windfall is completely transferred to households for consumption. This is technically an all-transferring case as households will save a portion for investment, but the effect is to raise consumption.

Regime 2 - (All investing approach): the resource windfall is completely used for public investment in addition to the steady-state public investment.

Regime 3 - (All savings approach): the resource windfall is entirely invested abroad in a sovereign wealth fund, $SWF_t$; the interest generated by the fund is entirely transferred to households for consumption. Note that a negative value for the sovereign wealth fund is equivalent to a debt held by the government.

Regime 4 - (The balanced approach): A fixed share $\phi$ of the windfall is invested in the sovereign wealth fund ($SWF_t$), and the remainder $(1-\phi)$ is invested in public infrastructure. The interest generated by the sovereign fund is returned to households.

Regimes 1 and 2, all-consuming and all-investing, are the more aggressive spending strategies to raise consumption and investment, respectively. They support the argument in Collier et al. (2010) that the social discount rate is likely high in low-income countries because of the low level of consumption and the scarcity of capital; hence, spending should be higher in the present. Regime 3 is the conservative spending strategy, similar to the bird-in-hand rule in the recent literature, where the windfall is not valued
until it is banked, and only the interest income is spent, in this case only for consumption through household transfers. Regime 4 is a compromise between regimes 2 and 3.

The government regulates the values of $WF_{t}^{\text{inv}}$, $WF_{t}^{r}$, $WF_{t}^{\text{sav}}$, and $SWF_{t}$ according to the fiscal rule it wants to promote as presented in table 1. $WF_{t}^{\text{inv}}$ is the additional public investment resulting from the use of the windfalls; $WF_{t}^{r}$ is the extra transfers to households resulting from the use of the windfalls; $SWF_{t}$ is the stock value of the sovereign fund resulting from the use of the windfalls; and $WF_{t}^{\text{sav}}$ is the additional component of foreign saving resulting from the use of the windfalls.

A competitive equilibrium of this economy is represented by a set of prices and quantities such that the representative household and firm maximize their respective objective functions; the government satisfies its budget constraint and all markets clear in each period. Equations (37, 38) represent the equilibrium conditions in the domestic good market and the labor market. Equation (39) is an identity that represents the stock version of a saving-investment identity. Household financial wealth is the difference between the value of the firm, $WK_{t}$, and foreign debt, $BF_{t}$. As shown in Hayashi (1982), the value of the firm is defined as the expected value of the product of the shadow price of capital in period $t$ and the capital stock in period $t + 1$, (40).

\[ XSD_{t} = XDD_{t} \]  
\[ LD_{t} = H_{t} \]  
\[ F_{t} = WK_{t} - BF_{t} \]  
\[ WK_{t} = EtQ_{t}K_{t+1} \]

Steady-State Conditions
In the steady state, all prices and quantity variables expressed in per efficiency units of labor are constant. Equations (41)–(45) represent the steady-state conditions for the shadow price of private capital, household financial wealth, firm value, and foreign debt.

\[ Q_{t}(\delta_{K} + r_{t}) = PXST_{t}R_{K_{t}} + PC_{t} \left( \frac{R}{2} \right) \left( \frac{I_{t}}{K_{t}} \right)^{2} \]  
\[ I_{t} = \delta_{K}K_{t} \]  
\[ (-r_{t})F_{t} = (1 - \tau_{YL})W_{t}H_{t} + TR_{t} + ER_{t}TROW_{t} - PC_{t}C_{t} - \tau_{k}Div_{t} \]  
\[ WK_{t} = Q_{t}K_{t} \]  
\[ (-r_{t})BF_{t} = FS\text{AV}_{t} \]

The dynamics of the model are represented by the evolution of state variables whose current values are determined by past conditions and by jumping variables whose current values are determined by future conditions.
Data, Calibration, and Solution Strategy
As is common in the DSGE literature, we calibrate the model’s parameters to match the first moments of variables of the economy that are assumed to be in a steady state. All quantity variables are presented in per efficiency units of labor, and the time endowment adjusted for technological progress is normalized to unity. Without loss of generality, the exogenous rate of growth of the economy is set to zero. In the deterministic steady state, all real variables and prices are thus constant. Table 2 presents some key characteristics of the economy in the initial steady state.

In the steady state, the share of the resource sector in GDP is only 6.2%, while that of traditional exports is 13.8%. Still, according to various outlooks, the resource sector is expected to increase its share in GDP in the future. The calibration approach amounts to using the first-order conditions and the variable means to recover the values of some behavioral parameters to reproduce the deterministic steady state. Yet, because of the functional forms used, not all parameters can be recovered. The values of some parameters, like elasticities, must be supplied externally. Following the common practice, by appropriate choice of unit, we set the consumption price, the price of the domestic good, the producer prices of traditional exports and resource exports, and the price of imports gross of taxes to one. Hence, the volume of the variables associated to these prices can be consequently computed using data on the first moments. The various tax rates are computed using actual tax revenue and the tax bases. The other prices that are associated with taxes are thus calibrated using the computed tax rates.

The subjective discount factor in the utility function is set to 0.945. This implies a value of 5.8% for the premium-free world interest rate in the steady state, which is comparable to the values between 4–6% used in many studies. We assume that the representative households devote 30% of their time to work, which leads to a value of 0.3 for labor supply in the steady state. This value is in line with those used in other studies on African countries. There is not much data on capital depreciation and investment function parameters in developing countries. We elect to use the value of 0.1 for the depreciation rates of private and public capital. This value is higher than the common value of 0.05 found in the literature on developing countries.

Moreover, as in most studies, we set the adjustment cost parameter, \( b \), in equation (11), which is the sensitivity of investment to Tobin’s \( Q \), to 2. There is no consensus on the exact value of the output elasticity of

<table>
<thead>
<tr>
<th>Selected macroeconomic variables</th>
<th>Ratio to GDP at market prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP (market prices)</td>
<td>100.0%</td>
</tr>
<tr>
<td>Private consumption</td>
<td>81.3%</td>
</tr>
<tr>
<td>Private investment</td>
<td>13.0%</td>
</tr>
<tr>
<td>Public investment</td>
<td>8.1%</td>
</tr>
<tr>
<td>Public consumption</td>
<td>9.1%</td>
</tr>
<tr>
<td>Traditional exports</td>
<td>13.8%</td>
</tr>
<tr>
<td>Resources exports</td>
<td>6.2%</td>
</tr>
<tr>
<td>Imports</td>
<td>31.5%</td>
</tr>
<tr>
<td>Taxes on consumption</td>
<td>3.2%</td>
</tr>
<tr>
<td>Taxes on resources</td>
<td>1.2%</td>
</tr>
<tr>
<td>Taxes imports</td>
<td>4.8%</td>
</tr>
<tr>
<td>Income taxes</td>
<td>2.0%</td>
</tr>
<tr>
<td>Gov transf to households</td>
<td>2.3%</td>
</tr>
</tbody>
</table>


4 Like other newly resource-rich developing countries, it is not possible to calibrate parameter values by econometric estimation or by a “back-casting” exercise in order to replicate historical trends due to the lack of historical observations. In the future, we will explore Bayesian approaches, as in Go et al. (2015).
public capital or the efficiency parameter of public investment. We follow Berg, et al. (2013) in their study on African countries and set the value of the output elasticity of public capital to 0.11 and the efficiency parameter of public investment to 0.5. The efficiency parameter is close to the high side of the estimates found in the literature for that parameter, which vary between 0.35 and 0.5, as mentioned by Pritchett (2000) for sub-Saharan countries. Using 3SLS system approach, Agénor (2013) recently estimated the direct elasticity of output with respect to public infrastructure to be around 0.1; however, accounting for indirect transmission channels will raise the general equilibrium value to only 0.25. Given these studies, 0.1 seems reasonable for the output elasticity of public capital while 0.5 is on the optimistic side of the effectiveness of public investment.

There is no precise figure for the value of the intertemporal elasticity of substitution of consumption. Referring to the econometric estimate for Niger provided in Ogaki et al. (1996), we use the value of 0.36. For the parameter $\psi$ in the utility function, we use the value of 0.25 for the Frisch labor elasticity and calibrate the value of $\psi$ to 9.33. Using the first-order condition arbitrage between consumption and leisure, we calibrate the value of leisure’s weight in the utility function, $\mu$ to 0.103. This would be consistent with a relatively low opportunity cost of labor in this poor, African country. From the value of private investment demand in final demand (gross of adjustment cost), we use the steady-state relationship to derive the value of investment $INV_t$, that effectively increases private capital stock. We then use the steady-state relation between investment and private capital to find the value of the stock of private capital $K_t$. We use a similar method, that is, the steady-state relationship, to calibrate public capital stock from the level of public investment in final demand.

Using the calibrated values of private investment and capital stock, we then find the value of the shadow price of capital, which makes it possible to compute the value of the firm. We make use of equation (15) and calibrate the value of marginal product of capital, which we then multiply by the value of the volume of capital stock to compute the return to capital. We subtract the return to capital from GDP to find the return to labor, which we use to calibrate the wage rate given the above-defined value of labor supply. Given the level of foreign saving, we compute the level of private foreign debt that is compatible with the steady-state condition, which we use in combination with the value of the firm to calibrate household financial wealth. The dividend received by households is computed using equation (17). Domestic sales are computed as the difference between the value of output and the total value of exports.

As in Devarajan and Go (1998), we set the elasticity of substitution in the Armington and CET functions to, respectively, 0.5 and 0.6, which are consistent with the estimates for many low-income countries in Devarajan, Go, and Li (1999). The weight of imports in domestic absorption is calibrated to 0.202. There are no precise figures on the sensitivity of interest rates to the deviation of foreign debt from its steady-state level; we set it to 0.0045 to generate reasonable changes to the interest rate following shocks to resource prices. Schmidt-Grohe and Uribe (2003) use a smaller value (0.000742) so that their model generates the observed volatility in the current-account-to-GDP ratio following term-of-trade shocks. The list of the model parameters and their values are provided in table 3.

Finally, we set the persistence parameter of the shock to resource export price to 0.88 and its standard deviation to 0.13. Recall here that the resource export encompasses mainly the oil and uranium exports of Niger. We could not find in the literature any estimates on the parameters of the stochastic processes of the combined exports of oil and uranium. Since the individual estimates of persistence parameters found in the literature vary significantly, the value chosen here is within the realistic range.

Due to its complexity, there is no analytical solution to the model. We resort to numerical solution. In reality, the model is a system of nonlinear equations containing difference equations. We use the extended path method for solving DSGE models (as described in the paragraph) using the General Algebraic Modeling System (GAMS), which is well suited for large-scale numerical simulations. The model, containing close to 19500 equations and variables, is solved using the CONOPT algorithm; it can also be solved using any CNS (constrained nonlinear system) solver since the system, with equal number of equations and variables, is square.
deterministic extended path method of Fair and Taylor (1983) as suggested in Gagnon (1990). We make use of the static and dynamic first-order conditions discussed earlier and solve the model as a two-boundary-value problem. Referring to Adjemian and Juillard (2010), the main strategy of the extended path method is to use a solver designed for perfect foresight models to solve stochastic forward-looking models by treating, in each period, contemporaneous innovations as surprise shocks and setting their expected value to zero in all future periods. In that respect, the method neglects Jensen’s inequality and introduces, therefore, some inaccuracies, which are however less dramatic than those introduced through linearization that neglect the deterministic non-linearities present in the model. The model is solved for 150 periods to ensure that the economy returns to its steady state after a shock. Most of the deviations from the steady state occur in the first twenty-five periods.

In a comparison of numerical methods for solving standard business cycle models, Heer and Maußner (2008) find that the extended path approach is an accurate method. In another interesting comparison of methods to solve stochastic forward-looking models, Love (2010) finds that the relative performance of the deterministic extended path is superior to that of log-linearization methods often found in the literature. Its other advantage is that it can handle large models very well in comparison to other methods. See Dissou and Didic (2013) and Dissou and Karnizova (2012) for further discussion.

III. Simulations

We examine four spending regimes corresponding to the four fiscal rules described above. In order to look at the implications of the stochastic behavior of the resource price, none of the four regimes is associated with a specific forecast or deterministic path of the price of the resource export. Instead, we analyze and emphasize the impact of a temporary shock to that price.

The stochastic revenue windfalls in the simulations emanate from two sources of uncertainty – the export price path and the export volume of the mineral resource. Export volume in our approach is endogenous and also volatile, following from the CET supply function above. As such, export revenue and its components do not adhere to any official projections. Simulations are therefore presented as illustrations for the purpose of deriving some general lessons for managing resource windfalls in a dynamic and stochastic context. Furthermore, the presence of a windfall in any period implies a positive deviation of the mineral export price from its steady-state (or long-run growth) path. In particular, we assume a one-standard-deviation innovation to the world price of the resource in the first period. Because of its

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity of interest rate to debt</td>
<td>0.0045</td>
</tr>
<tr>
<td>World real interest rate</td>
<td>0.058</td>
</tr>
<tr>
<td>Output elasticity of public capital</td>
<td>0.11</td>
</tr>
<tr>
<td>Depreciation rate of public capital</td>
<td>0.1</td>
</tr>
<tr>
<td>Depreciation rate of physical capital</td>
<td>0.1</td>
</tr>
<tr>
<td>Weight of imports in the Armington function</td>
<td>0.202</td>
</tr>
<tr>
<td>Efficiency of government investment in public capital accumulation</td>
<td>0.5</td>
</tr>
<tr>
<td>Leisure weight in the utility function</td>
<td>0.103</td>
</tr>
<tr>
<td>Persistence parameter</td>
<td>0.88</td>
</tr>
<tr>
<td>Standard deviation of shocks</td>
<td>0.13</td>
</tr>
<tr>
<td>Substitution elasticity in the Armington function</td>
<td>0.5</td>
</tr>
<tr>
<td>Substitution elasticity in the CET function</td>
<td>0.6</td>
</tr>
<tr>
<td>Inverse of the intertemporal elasticity of substitution for consumption</td>
<td>0.36</td>
</tr>
<tr>
<td>Frisch labour elasticity</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Figure 1. Impulse Response Periods of Selected Variables following a Positive One-Standard Deviation Shock to the Price of Resource Exports: All-Consumption Regime
autoregressive property, the shock will persist over time, but the resource price will eventually return to its steady-state level in the long run, as shown in figure 1. In what follows, we first discuss in detail the transmission mechanisms of a positive price shock in the all-consuming regime. Using this as a reference point, we then compare and contrast the results in the other regimes. In order to understand the budget rules in the stochastic case, we also briefly look at the effects of a negative temporary price shock, concentrating mainly on the behavior of consumption and investment.

Impulse Responses of a Positive Temporary Resource Price Shock

All-Consuming Regime

Since all variables eventually return to their initial steady-state levels, we emphasize the transitional dynamics of the impact of the price shock.

In the results shown by the graphs of figure 1, it is evident that the transitory increase in the resource price will lead to a favorable transitory increase in resource exports, with the impulse responses of both variables mirroring one another closely. Although the resource price changes only in the first period and gradually returns to its initial steady state, there are economy-wide repercussions that will lead to different transitional dynamics in some variables.

The government receives a windfall that comes from the rise in the export price and also from the export volume, as we shall see. In the all-consuming regime, the windfall is transferred to households via endogenous government transfers. However, transfers to households do not increase by the same amount (in percent deviation) as the windfalls because of the effect on domestic prices in the economy. Indeed, even though the volumes of government consumption and investment are kept to their initial steady-state levels, their values will increase because of the rise in the consumption price induced by the increase in the domestic price or appreciation of the real exchange rate. As a result, part of the additional revenue is used to offset the nominal increase in government expenditures, reducing some of the transfers to households.

The increase in household income from the government transfers will induce an immediate transitory upswing in household consumption and savings. Government spending on public investment is kept at the level needed to replace obsolete public capital so that the public capital stock does not increase. Private investment will then rise from the higher savings, and the volume of gross output will respond positively. This is amplified by further rounds of interacting effects.

Higher consumer demand and investment demand imply an increase in domestic absorption, which translates into increases in domestic sales in terms of the demand for domestic good and the demand for imports. The increase in the demand for domestic good puts a pressure on its price, which is the Dutch disease phenomenon. As a consequence of the increase in the price of the domestic good, the consumption price, which is a composite of the price of the domestic good and the price of imports, rises and only slowly moves back towards its original level. This explains the increase in government expenditures alluded to above.

Gross output will increase not just because of the higher investment, but it also reacts to increases in labor. As labor demand initially rises with output, the wage rate will increase. This in turn induces households to increase their labor supply, which increases production further. The increase in both the producer price and volume of gross output are beneficial to dividends paid to households. The changes in both labor income and dividend income fuel household wealth, which will additionally boost household consumption.

All told and in contrast to the short-term burst of resource export price, the supply of the domestic goods increases in most periods before returning to its long-run level. Similar patterns are noted in consumption, domestic absorption, domestic sales, and the wage rate.

It is worth noticing that the appreciation of the real exchange rate eventually hurts the resource sector itself, whose exports fall below their steady-state level before going back to it in the long run. The main reason for this is that the price of the domestic good falls at a slower pace in comparison to the price of
resource exports. Hence, producers find it profitable to immediately start reducing the production of the resource good in favor of the domestic good. In an environment characterized by a rise in the costs of capital and labor and no change in the price of traditional exports, resources move out of the latter sector, shrinking it in early periods but gradually returning it to its steady-state value after the exhaustion of the initial shock to the resource price and the eventual decline of the price of the domestic good.

Despite the small initial share of resource exports in total exports and the fall in traditional exports, total exports increase as a result of the offsetting movements in the two types of exports. This persists for the first ten periods and then falls below their steady-state level, before returning to that level in the long run.

The combined profiles of imports and total exports have a positive impact on the current account balance in the initial periods; it becomes negative later and eventually returns to its steady-state level. The initial positive impact on current account makes it possible to increase foreign debt in the first few periods, which ultimately decreases before returning to its long-term level. In the meantime, the rise in indebtedness vis-à-vis the rest of the world increases slightly the interest premium, which eventually disappears in the long run.

In a nutshell, we observe the traditional dynamics of an increase in the world price of resource exports on the economy, where the traditional export sector shrinks to the benefit of the resource sector and the nontradable sector. The exhaustion of the temporary shock and its effects on the shifting relative prices will eventually return the economy to its steady state.

All-Investing Regime

In general, similar transmission mechanisms are at play in the three other regimes that make use of the revenue windfalls differently from the first regime. Graphs in figure 2 offer a comparison of the shock on selected variables in all four regimes.

In the all-investing regime, the resource windfalls are used to increase government expenditures in public investment. As expected, the profile of the change in public investment follows that of the resource price. The stock of public capital increases and has a positive impact on the productivity of private inputs, which induces firms to increase their demand for labor and investment in comparison to the situation in the all-consuming regime.

Even if all variables return to their steady-state levels, the impact on private investment in the all-investing regime is higher than in the all-consuming regime for many periods. The higher impact on private capital stock and on labor is beneficial to gross output, which expands more with a stronger impact on resource exports.

Similarly, the higher impact on gross output is also beneficial to the sales in the other two markets. Domestic supply increases more and traditional exports fall less in comparison to the all-consuming regime.

The same pattern generally applies to consumption, whose pathway is higher in this regime in comparison to the previous regime, despite the additional transfers received by households from the windfalls in the all-consuming regime. In contrast, transfers to households fall in the all-investing regime because of the increase in government expenditures induced by the increase in consumption price, which reduces the amount available for transfers. The rise in government revenue stemming from the rise of economic activities is not sufficient to compensate for the increase in government expenditures. This decrease of transfers to households explains the fall during the first periods of household consumption in the all-investing regime before its rise, thanks to the ensuing output and income effects of public investment.

The beneficial impact of rising public investment can also be seen on imports, which increase more than in the all-consuming regime. This result is largely due to the growth in all components of domestic absorption, which is stronger in this regime.

The impact on current account balance is similar to the one observed earlier as shown in figure 2.
Figure 2. Impulse Responses of Selected Variables following a Positive One-Standard Deviation Shock to the Price of Resource Exports: Comparison Across Regimes
All-Savings Regime
In this regime, the windfall is invested abroad in a sovereign wealth fund that generates interest. Interest incomes accrue to the government but are ultimately given to the households through transfers. The sovereign fund is remunerated at the same interest rate paid on private debt. This regime is thus similar to the all-consuming regime, but the profile and timing of transfers to households will be different, and so will its impact on the other variables.

While transfers in the early periods are significantly lower in the all-savings regime in comparison to the all-consuming regime, the situation reverses after the tenth period, as shown in figure 2. In the medium and long runs, transfers to households in the all-savings regime are higher than the ones in all-consuming regime, where resource windfalls are entirely consumed instead of being invested in a sovereign fund. The main reason is that the interest generated by the sovereign fund, which is transferred to households, increases over time despite the later decline in the windfalls. As transfers to households rise, households are able to consume relatively more over time in comparison to the first regime.

The impact on investment in this regime is almost identical to the one in the all-consuming regime. It is interesting to note that the transfer of resources from the sovereign fund to the economy reaches such a critical point in the future that it becomes harmful to the export sector. This is because the additional resources spent by households will eventually lead to appreciation of the real exchange rate, affecting traditional exports more negatively in comparison to the all-consuming regime. The path of imports, on the other hand, is relatively higher in this case.

The Balanced Regime
This regime is a combination of all-investing and all-savings regimes, in the sense that half of the windfalls are invested in a sovereign fund and the other half in public infrastructure. In the absence of better information about the effectiveness of public investment in any country, this is a prudent Bayesian approach. To some extent, it combines the medium- and long-run benefits stemming from increasing public investment, and the short-term benefits for consumers arising from augmented transfers to households due to the windfalls.

Although the transitional dynamics and patterns are similar across regimes, the results suggest that the impact on consumption in the balanced regime is generally higher than the ones observed in the all-consuming and all-investing regimes and lower than the one in the all-savings regime. The superiority of the all-savings regime to the balanced regime stems from two factors—investing domestically raises prices, leaving less money for government to transfer to households; any amount of immediate investment in public infrastructure is also not 100 percent effective in raising output and does not generate a return sufficiently high as the one generated by the sovereign fund.6

In contrast, the Dutch-disease-like effect generated by the returns to the investment in the sovereign fund is less pronounced in this regime since only half the windfalls are invested overseas. As a consequence, traditional exports are less hurt in comparison to the all-savings regime. Moreover, traditional exports benefit from the increased investment in infrastructure in this regime.

Impact of a Negative Temporary Price Shock
What if the resource price declines, which is entirely possible? We briefly examine the impact of a negative temporary price shock of the mineral resource. To be clear about the adjustment in this negative scenario, the all-consuming case now means reducing government transfers to households, thus cutting consumption directly. In the all-investing case, this means cutting public investment in infrastructure, which affects future output and income negatively. In the all-savings case, this implies borrowing, paying

6 For the results of sensitivity tests on the effectiveness of public investment, see the relevant section below.
Figure 3. Impulse Responses of Selected Variables following a Negative One-Standard Deviation Shock to the Price of Resource Exports: All-Consumption Regime
interest income throughout. And the balanced approach distributes the adjustment to cuts in public investment and increases in borrowing.

Figure 3 presents the impulse responses of consumption and investment under the four fiscal regimes due to a negative temporary negative resource price shock. The results are mirror images of consumption and investment in figure 2, due to the reverse pattern of effects and the depreciation of the real exchange rate. As expected, consumption in each of the four regimes declines throughout the periods. With no windfall to bank into a sovereign fund and no interest income to draw, the consumption path of the all-savings rule is now generally lower than the others; its investment path also has the highest initial decline. The consumption path of the balanced regime is somewhat in the middle, lower than all-investing and all-consuming. The all-investing regime has the greatest peak decline in consumption, however. The investment paths generally decline in all the regimes.

Figure 4(a). Welfare Impact of a Temporary Positive One-Standard Deviation of the Resource Price Figure 4(b). Welfare Impact of a Temporary Negative One-Standard Deviation of the Resource Price

Note: The welfare measure is the percentage increase in the deterministic steady-state consumption that gives the same level of utility brought about by the shock to resource price.
To compare across regimes more carefully, we look at welfare in each case, defined as the present discounted value of the utility of consumption.

Welfare Comparison of Fiscal Rules for a Temporary Price Shock

Figure 4a presents the welfare impact of a positive temporary shock to the resource price in all regimes. The measure of welfare is the percentage increase in consumption from the steady-state level, which generates the same utility as after the shock. A positive number, therefore, connotes welfare improvement.

Welfare rises in all regimes from the resource revenue in a positive shock. Even so, putting aside the resource windfalls in a resource fund generates the highest welfare gain. This result is at first sight surprising given that the consumption profiles presented in figure 2 show the highest pick in consumption is in the all-investing regime. Two points explain this relative welfare ranking. First, despite the highest peak in consumption in the all-investing regime, it also has the most negative impact on consumption in the early periods, largely because of the initial Dutch-disease effects on prices. Second, the peak actually occurs in later periods; because of the discount factor in the computation of utility, the welfare measure in the all-investing regime is lower than the three others since the benefits of future higher consumption are less important than near-term consumption for welfare. In contrast to the impact of the shock in the other regimes, the highest welfare impact belongs to the all-savings regime, which can easily be understood in light of the continually rising consumption afforded by the interest income over time.

Exactly the opposite pattern occurs in the case of a temporary negative shock (figure 4b). The all-savings regime (i.e., borrowing in a negative shock) now has the lowest welfare, followed by the all-consuming regime (i.e., reducing consumption), the balanced case, and the all-investing case (i.e., cutting infrastructure) in that order. The pattern reminds us that the Dutch disease effects are just one possible side of the movements of resource price, that price contraction and its opposite effects are just as likely.

With some simplification, the following are therefore the policy rules of thumb. When there is a temporary increase in the natural resource price, the best strategy suggested by the DSGE analysis in this paper is to bank the revenue windfall entirely in a sovereign fund and to draw on the interest income to smooth consumption through government transfers to households. When there is a temporary price decline, however, the best strategy is to cut public investment in order to protect consumption as much as possible. This is due to two reasons: investment raises the prices of domestic goods, leaving less money for government to transfer to households; and public investment is nowhere near 100 percent effective in raising output (see discussion of the output elasticity of public capital). Thus, reducing investment is better than cutting transfers to households, which reduces consumption directly, or dissaving from the sovereign fund or borrowing, which incurs an interest premium when debt rises. The DSGE analysis in this paper therefore extends prudent budgetary rules to resource price shocks that can be in either direction: during positive price shocks, the analysis confirms a conservative bird-in-hand budget rule similar to the deterministic case (such as Go et al. 2013), in order to avoid the long-term negative effects of Dutch disease and of the capacity in a poor country going so low that there is the possibility of a boom-bust cycle from wasteful spending; during negative price shocks, however, cutting investment is preferred to rules that reduce consumption directly.

Fiscal Rule against Enduring Price Volatility and Uncertainty

What if the price shocks occur in all periods and are volatile so that they cannot be anticipated? We consider unpredictable shocks to the resource export prices in all periods. These innovations to the prices are

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7 Noting that several countries in West Africa have cut public investment in the wake of a negative shock, Dessus and Varoudakis (2013) caution against this strategy, even if it is intertemporally optimal. They argue that the stop-and-go costs and the asymmetric response of investment to shocks (higher during bad times) could be a reason for the relatively low levels of infrastructure in the WAEMU countries. Consideration of stop-and-go costs, beyond the standard adjustment costs of investment, which are already in the model, are beyond the scope of the paper.
drawn randomly in each period in the presence of the AR (1) process. The model is run one hundred times, and we compute the unconditional second moment of selected variables of interest. The data in table 4 suggest that the volatility of most quantity variables is the highest in the all-investing regime. The reason for this is that in addition to the volatility of the resource price, the economy is subject to the volatility of public capital stock in the all-investing regime. Indeed, because the resource windfalls are used to fund public investment that has an impact on output in that regime, the productivity of private factors and GDP are more volatile in comparison to the other regimes. This phenomenon is also partially observed in the balanced approach in which half of the resource windfalls are used to fund public investment. The all-consuming and all-savings regimes offer the lowest volatilities of quantity variables because the public capital stock is fixed in those regimes. As illustrations, the two panels in figure 5 show the consumption and investment in the first draw of productivity shocks under the all-consuming case. In conformity with results found in most DSGE models, investment is more volatile than consumption. Thus, among the alternative uses of the windfall revenue, using the resource windfalls to fund public investment would bring greater volatility to the economy when the resource price itself is also volatile and changing.

Still, as far welfare is concerned, data in figure 6 suggest that the average welfare gains of the shock under the four regimes are not significantly different from one another. Yet, the welfare gain in the balanced regime is the largest, while the all-consuming regime offers the lowest welfare improvement. This result seems surprising in the sense that it does not follow the relative ranking of volatility of the four regimes. Everything equal, welfare should normally be lower the higher the volatility.

Clearly, welfare does not depend on volatility alone but also on the level of consumption and its reaction to the price shocks that occur in all periods, which encompass both random increases and decreases. The lessons from the temporary price shocks, therefore, offer the clue: during price increases, one should bank the windfall revenue; during price declines, one should cut public investment. When the direction of the price changes is stochastic and uncertain, the best strategy is to combine both strategies, that is, a balanced regime that provides a natural insurance against both types of price shocks that are occurring persistently.

<table>
<thead>
<tr>
<th>Table 4. Standard Deviation of Selected Variables (Average of 100 Simulations Based on Different Draws of the Resource Price Shocks)</th>
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</thead>
<tbody>
<tr>
<td>All-consuming</td>
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<tr>
<td>GDP at market prices</td>
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<tr>
<td>Consumption</td>
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<tr>
<td>Private investment (inc. adjustment cost)</td>
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<tr>
<td>Traditional exports</td>
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<tr>
<td>Imports</td>
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<tr>
<td>Domestic absorption</td>
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<tr>
<td>Domestic sales</td>
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<tr>
<td>Output</td>
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<tr>
<td>Private investment (w/o adjustment cost)</td>
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<tr>
<td>Private capital stock</td>
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<tr>
<td>Labor supply</td>
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<td>Foreign savings</td>
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<tr>
<td>Foreign debt</td>
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<tr>
<td>Consumption price</td>
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<tr>
<td>Price of domestic good</td>
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<tr>
<td>Price of resource export</td>
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<tr>
<td>Wage rate</td>
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<td>Government revenue</td>
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and randomly over time. The combination of interest income from the sovereign fund, transfers to households, and output growth brought about by public investment presents the best protective mechanism to smooth consumption over time in response to changing resources. It accentuates not just present consumption but also future consumption. It follows that the balanced regime is the preferred case in the face of volatility in the resource price and the economy. This is a significant third lesson from the DSGE analysis.

Sensitivity Tests of Efficiency of Public Investment

One of the key parameters in the model is the efficiency of public investment, which with a base value 0.5 means only half of public investment is converted to public capital stock. We conduct sensitivity experiments where the alternative values of 0.75 and 0.25 are used for the efficiency parameter of public investment. The results (the welfare graph and ranking in figure 6 in particular) are exactly the same as the ones in the base case with 0.5. This seemingly surprising outcome, the insensitivity of the results to
the efficiency parameter, can be explained as follows. With a higher efficiency parameter, the public capital stock will necessarily be calibrated higher, resulting in a lower calibrated TFP parameter to generate the same base year output in the base run. Hence, public investment in the experiments will not raise more output. Of course, it works both ways. A lower value of the efficiency parameter does not mean that public investment will lower output and welfare. The model and its results are therefore robust to this parameter.

In economic terms, this simply means that Niger is poor for many reasons. Changing the efficiency of public investment (or other factors) will likely alter other values, such as TFP, in an interrelated economic system, and there is nothing in Niger’s historical data to suggest otherwise. In a recent study, Berg, Buffie, Patillo, Portillo, Presbitero, and Zanna (2015) found that cross-country differences in the efficiency of public investment do not matter for the growth impact of increases in public investment because public capital scarcity and inefficiency are likely to be inversely related. There is therefore no magic bullet about the impact of public investment; much depends on other factors. If, for example, policy reforms lead to a significant increase in the efficiency parameter for all new public investment, a rise in the externality effect of public capital on private capital, or a higher level of TFP, then the results will tilt towards the all investment strategy, which now becomes the most preferable. These are, however, strong assumptions.

We also conducted sensitivity analyses of other factors. For example, changing the interest rate will also change the rate of consumer time preference (as in any intertemporal model), which is directly related to the social discount rate that will drive the results. A higher discount rate makes it less likely for investment projects to be funded, favoring the savings or the balanced approach, which have the greater overall consumption or welfare impact; lowering the interest rate favors more investment of the additional mineral revenue. We also tested a different depreciation rate of capital stock, from 0.1 to 0.05. The all-savings approach is the most preferable followed by all-consuming. This suggests that if depreciation is lower, the capital stock can last longer and less investment is needed; the economy can, therefore, afford to save abroad or consume more.
IV. Conclusions

In this paper, we analyzed the economic implications of various budget rules to manage public revenue from natural resources in a low-income country. We developed a 1-2-3-4 (for one country, two types of capital, three sectors, and four goods) model by introducing uncertainty in the dynamic 1-2-3 model. We captured the externality of public capital on firms’ technology. After defining as resource windfalls the difference between current and the steady-state levels of royalties on resource exports, we considered four regimes for the use of the windfalls: all-consuming, all-investing, all-savings, and balanced regimes. We then analyzed the economic implications of the volatility of the resource export price in the case of a temporary shock and in the case of persistent but uncertain shocks in all periods.

Our simulation results suggest three policy lessons or rules-of-thumb. When a resource price change is positive and temporary, one should bank the revenue windfall in a sovereign fund during the temporary price increase. Drawing interest income from the sovereign fund to raise consumption over time through government transfers to households will provide the largest welfare gains. This is because in the all-investing case, public investment is only partially effective (50 percent) in raising output, as shown in prior empirical estimates. In the presence of a negative temporary resource price change however, the best strategy is to cut public investment rather than trim government transfers to households, which reduces consumption directly, or draw down the sovereign fund or borrow, which incurs an interest premium as debt rises. Protecting public investment keeps domestic prices high, crowding out government spending on household transfers. In the presence of shocks that occur in all periods, the best strategy is to combine the all-savings and all-investing strategies, that is, a balanced regime that provides a natural insurance against both types of price shocks, positive and negative, which are occurring over time. Combining interest income from the sovereign fund, transfers to households, and output growth brought about by public investment presents the best protective mechanism to smooth consumption over time in response to changing resource prices.

We have three suggestions for future research. In the face of uncertainty and volatility of price shocks, a balanced approach seems ideal as a simple budget rule to follow. However, the optimal distribution between all-savings and all-investment will likely be different from the 50–50 split simulated here under different values of parameters such as the elasticity of output to public capital, trade elasticities, the interest rate premium for rising foreign debt, etc. Hence, determining the optimal mix for a range or combination of these parameters is one avenue for analysis. A second area of research is to model the stop-and-go costs of public investment so that the high price of stopping and restarting public investment is fully taken into account. Finally, a third research suggestion is to widen the budgetary rule to include government consumption, which is kept constant in real terms in the present paper. To do this properly, the links between two key components of current government expenditures—education and health—and human capital will need to be defined and explored. The effectiveness of public service delivery in education and health is often hard to model by itself, let alone in a dynamic stochastic framework.

References