DISCUSSION PAPER

DRD127

PRODUCT DIFFERENTIATION AND TRADE DEPENDENCE
OF THE DOMESTIC PRICE SYSTEM IN COMPUTABLE
GENERAL EQUILIBRIUM TRADE MODELS

Jaime de Mello
The World Bank

Sherman Robinson
University of California, Berkeley

July 1985

Development Research Department
Economics and Research Staff
World Bank

The World Bank does not accept responsibility for the views expressed herein
which are those of the author(s) and should not be attributed to the World
Bank or to its affiliated organizations. The findings, interpretations, and
conclusions are the results of research supported by the Bank; they do not
necessarily represent official policy of the Bank. The designations employed,
the presentation of material, and any maps used in this document are solely
for the convenience of the reader and do not imply the expression of any
opinion whatsoever on the part of the World Bank or its affiliates concerning
the legal status of any country, territory, city, area, or of its authorities,
or concerning the delimitations of its boundaries, or national affiliation.
PRODUCT DIFFERENTIATION AND TRADE DEPENDENCE
OF THE DOMESTIC PRICE SYSTEM IN COMPUTABLE
GENERAL EQUILIBRIUM TRADE MODELS*

by
Jaime de Melo
The World Bank
Washington, D. C. 20433

and
Sherman Robinson
Department of Agricultural and Resource Economics
University of California, Berkeley, CA 94720

July 1985

The World Bank does not accept responsibility for the views expressed herein
which are those of the author(s) and should not be attributed to the World
Bank or to its affiliated organizations. The findings, interpretations, and
conclusions are the results of research supported by the Bank; they do not
necessarily represent official policy of the Bank. The designations employed,
the presentation of material, and any maps used in this document are solely
for the convenience of the reader and do not imply the expression of any
opinion whatsoever on the part of the World Bank or its affiliates concerning
the legal status of any country, territory, city, area, or of its authorities,
or concerning the delimitation of its boundaries, or national affiliation.
PRODUCT DIFFERENTIATION AND TRADE DEPENDENCE
OF THE DOMESTIC PRICE SYSTEM IN COMPUTABLE
GENERAL EQUILIBRIUM TRADE MODELS*

Abstract

This paper explores systematically the relationship between the
domestic and foreign price systems when domestically produced and foreign
produced goods are imperfect substitutes in use, which is the specification of
foreign trade most commonly adopted in computable general equilibrium trade
models. The paper shows that domestically produced goods are characterized by
a continuum of tradability rather than being arbitrarily classified as traded
or nontraded. Expressions are derived that indicate the factors affecting the
degree of autonomy of domestic prices to changes in import and export prices
and, hence, delineate the degree of trade dependence of the domestic price
system. The analysis suggests that the small magnitudes reported on the
welfare gains from tariff reduction in these models are due to this
specification since it gives great autonomy to the domestic price system.
PRODUCT DIFFERENTIATION AND TRADE DEPENDENCE OF THE DOMESTIC PRICE SYSTEM

I. Introduction

Virtually all recent multisector computable general equilibrium (CGE) trade models have used the assumption that domestically and foreign produced goods are imperfect substitutes in use. Examples are found in Srinivasan and Whalley (forthcoming) and are surveyed in Shoven and Whalley (1984). Whether the models are single-country or multicountry models, their concern is usually to estimate the welfare cost of tariff distortions. To such, they rightly emphasize a fair degree of sectoral disaggregation since it is well known (Johnson, 1960) that the costs of protection are a function not only of average tariff levels but also of dispersion across sectors. If domestic and foreign produced goods were perfect substitutes, sectoral disaggregation in the presence of few primary factors of production would lead to extreme specialization in the model economy. Thus modellers have assumed imperfect substitution between domestic and foreign goods.

The contention of this paper is that the small welfare gains reported in these numerical exercises owe much to the adoption of imperfect substitution and to the choice of restrictive functional forms that impose strong separability between goods of different origin. The reason for this contention is straightforward: the gains from a reduction in a given tariff distortion stem from a consumption gain and a production gain, both of which will be greater, the greater are substitution possibilities in consumption and the flatter is the production possibility surface (Johnson, 1965). The assumption adopted by modelers to overcome the specialization problem in
multisector models however, introduces strong curvature in preferences and, more importantly, strong curvature in the production possibility surface.

With the exception of Whalley and Young (1984) who look at the implication of alternative external closure rules in (CGE) trade models at an aggregate level, no one has raised the implications for resource allocation and the computed welfare gains of the assumption of product differentiation. The purpose of this paper is to draw the implications of this assumption. We also extend Whalley and Young's contribution by indicating an external closure rule where exports and imports are treated as differentiated products thereby avoiding the mixed-price endogenous/fixed-price formulation in their paper.

The remainder of this paper is organized as follows. Section II briefly reviews the background in favor of the foreign trade specification adopted in CGE trade models. Section III introduces the notion of product differentiation along the lines followed in applied computable general equilibrium trade models. It is assumed throughout that domestically produced goods in a given sector compete with imports but are not themselves exported. The relationship of this formulation to standard trade theory results is examined in a partial equilibrium framework. The role of exports and the implications of two-way trade are explored in section IV. The approach is to treat exports symmetrically, introducing product differentiation on the export side. This case is an extension of previous single-country specifications and corresponds closely to the treatment of foreign trade found in some global trade models used to assess the impacts of multilateral tariff reductions.
II. Background

Samuelson (1953) was the first to show that the law of one price implies extreme specialization in an economy where goods are produced under constant returns to scale and the number of commodities exceeds the number of factors of production. Specialization, following the law of comparative advantage, will generally result in only as many goods being produced under free trade as there are factors. Melvin (1968) and Pearce (1970) explored further the indeterminacy of production in the standard trade-theoretic model, while Johnson (1965) showed that the production possibility frontier is not likely to be as convex to the origin in practice as drawn in textbooks, although Vanek (1963) showed that the presence of intermediate inputs in fixed proportions along the lines adopted by most CGE modelers introduced extra curvature to the net production possibility surface. While useful, the introduction of nontraded goods—whose classification implies that their prices are always determined entirely in domestic markets—is only a partial palliative to the problem of extreme specialization.

Those working with CGE trade models have long been aware of this problem and have overcome the specialization problem by dropping the law of one price—that is, by allowing domestically produced and foreign produced goods to be imperfect substitutes in use. This assumption has often been referred to as the "Armington" assumption. Since Armington (1969) explored the nature of import demand functions when domestically produced and imported goods are imperfect substitutes in use. In all applications, an interindustry table is used and intermediates, whether of domestic or foreign origin, usually enter with a fixed coefficient (Leontief) technology. While reducing
considerably the number of parameters necessary for implementing the models, the result is a very limited scope for substitution possibilities.

Empirical evidence by Isard (1977) justifies the practice followed by modelers. His results indicate that, for the most narrowly defined domestic and foreign goods for which prices can be matched (4- and 5-digit SITC categories), disparities between the common currency prices of goods from different countries are systematically correlated with exchange rates rather than randomly fluctuating over time. Moreover, these relative price effects, indicating the presence of a wedge between domestic and import prices, were found to persist for several years. Furthermore, two-way trade has been observed in trade statistics at an extremely disaggregated level of coverage. Recent empirical evidence by Aspe and Gavazzi (1982) also suggests that producers of German machinery face different prices for domestic and foreign sales. Thus, the empirical evidence suggests that there are degrees of tradability with corresponding relative autonomy of the domestic price system.

In an earlier paper (de Melo and Robinson, 1981), we compared numerically the results from partial equilibrium and general equilibrium analysis of the impact of reductions in tariffs when domestic and foreign goods are imperfect substitutes in use. We showed numerically that resource shifts to single tariff cuts were small when capital stocks were sectorally fixed. And recent numerical exercises reported in Grais, de Melo, and Urata (1984) show that even when capital stocks were sectorally mobile across sectors, resource shifts in response to tariff cuts do not increase much. They attribute this lack of supply response to the assumption of imperfect substitution between imports and domestically produced goods, especially in the specification of intermediate input technology where the elasticity of
substitution is often assumed to be zero. Finally, Winters (1984) showed that
the two-stage budgeting assumption followed in the allocation of expenditures
between imports and domestically produced goods considerably narrowed cross-
price effects, thereby implying small resource shifts in response to a change
in tariffs policy. He went on to test the separability assumption with
aggregate data on British imports and, not surprisingly, rejected it in favor
of the more flexible functional form allowed by the "almost ideal demand
system" (AIDS) proposed by Deaton and Muellbauer (1985).

III. Product Differentiation and Imperfect Substitution
    on the Import Side

Denial of the law of one price drives a wedge between the price of a
domestically produced good, \( p^d \), and the import price, \( p^m \) (i.e., the foreign
border price times the exchange rate augmented by domestic trade taxes).
Under traditional partial equilibrium assumptions, the demand for the
domestically produced good, \( D \), will be a function of both the domestic price
of that good and the import price of the foreign-produced good of the same
category while the supply, \( S \), of the domestic good will be a function of its
own price. Thus,

\[
\begin{align*}
(1) & \quad D = D(p^d, p^m) \\
(2) & \quad S = S(p^d). 
\end{align*}
\]
The equilibrium condition in the market for the domestically produced good is $1$: 

$$(3) \quad D = S.$$ 

The central theme of this paper is the degree of trade dependence reflected by the extent of autonomy of the domestic price system. This question is investigated by asking how the domestic price is affected by a policy-induced change in the import price, e.g., a change in the tariff on imports. This is obtained by differentiating totally the equilibrium condition (3) which yields

$$(4a) \quad \frac{dP^d}{dP^m} = \frac{3D/3P^m}{3S/3P^d - 3D/3P^d},$$

Equation (4a) can be rewritten in terms of elasticities:

$$(4b) \quad E = \frac{dP^d}{dP^m} \frac{p^m}{P^d} = \frac{E^{d,m}}{\varepsilon^s - \varepsilon^d},$$

where

$$\varepsilon^{d,m} = \frac{3D}{3P^m} \frac{P^m}{D},$$

$$\varepsilon^s = \frac{3S}{3P^d} \frac{P^d}{D},$$

and

$$\varepsilon^d = \frac{3D}{3P^d} \frac{P^d}{D}.$$ 

As will be shown below, the sign of this expression for $E$, the domestic price response elasticity, is ambiguous. As soon as a wedge has been created between the domestic price and the import price, the domestic price is no longer determined entirely by the import price; it acquires a certain
degree of independence regardless of the functional forms specified in equation (4). However, to proceed further, it is necessary to formulate more specifically the demand for the domestically produced good under the presence of imports of a similar good whose price is changed exogenously. For this purpose, we introduce the Armington assumption adopted in most numerical general equilibrium trade models.

Define for each tradable commodity an aggregate or composite commodity, Q, which is a constant elasticity of substitution (CES) function of commodities produced abroad—that is, imports, M, and commodities produced domestically, D. The trade aggregation function is given by

\[
Q = \left[ 3M^{-\rho} + (1 - \beta) D^{-\rho} \right]^{-1/\rho},
\]

where \( \beta \) and \( \rho \) are parameters. The ratio in which these derived inputs are combined is determined by relative prices alone, and the sensitivity of this ratio to variations in relative prices varies directly with the elasticity of substitution, \( \sigma = 1/(1 + \rho) \), which we term the "trade-substitution" elasticity.

The trade-substitution elasticity is an elasticity in use so that it may take different values for different uses (consumption or investment), e.g., as in Grais, de Melo, and Urata (1984). As in most numerical CGE models, we ignore this possibility in the derivation below. It is customary to distinguish between varieties of a product and different products through cross-elasticities of demand and to assume that there is a discrete jump between significant and insignificant cross-elasticities. Therefore, rather than having \( n \) commodities made up of \( n^f \) foreign and \( n^d \) domestic components as in the Armington approach, one can conceive that there are \( 2n \) commodities...
characterized by strong pairwise cross-elasticities of demand ($n_i^d$ and $n_i^f$) and zero cross-elasticities between goods under a different classification ($i \neq j$). Under this formulation, the demand for imports and domestically produced goods becomes a derived demand in the same way the demand for factor inputs is a derived demand in a traditional production model.

Since total expenditure on the aggregate good must equal expenditure on its imported and domestic components, it must always be true that

\[(6) \quad PQ = P^m \cdot M + P^d \cdot D,\]

where $P$ stands for the composite commodity price. If, in addition, purchasers minimize the costs of purchasing a given amount of the composite good, one can obtain an expression for $P$ as a function of $P^d$ and $P^m$ which is the cost function derived from the CES aggregation function and is given by

\[(7) \quad P = \left[8^\sigma (P^m)^{(1-\sigma)} + (1 - \sigma)^\sigma (P^d)^{(1-\sigma)} \right]^{(1-\sigma)},\]

where $\sigma$ is the trade-substitution elasticity defined above. The first-order conditions for cost minimization also yield the derived demand for the domestic good under cost minimization, which is given by

\[(8) \quad P^d = P \frac{\partial Q}{\partial D} ;\]

that is,

\[(9) \quad D = (1 - \sigma)^\sigma \left(\frac{P}{P^d}\right)^\sigma Q.\]
Thus, the demand function for the domestically produced good can be written in general as

\[(10) \quad D(p^d, p^m) = H(p, Q, p^d), \]

where \(H\) is the right-hand side of equation (9).

We are now in a position to return to a further examination of expression (4). A change in the import price affects the demand for the domestic good through its effect on the composite price and the demand for the composite good. This effect is given by the following expression:

\[(11) \quad \frac{\partial D}{\partial p^m} = \left( \frac{\partial H}{\partial p} + \frac{\partial H}{\partial Q} \frac{\partial Q}{\partial p} \right) \frac{\partial p}{\partial p^m}. \]

The same mechanism is at work in evaluating the effect of a change in the domestic price on the demand for the domestic good, although there is now an additional term reflecting the direct effect of a change in domestic price. The resulting expression is

\[(12) \quad \frac{\partial D}{\partial p^d} = \left( \frac{\partial H}{\partial p} + \frac{\partial H}{\partial Q} \frac{\partial Q}{\partial p} \right) \frac{\partial p}{\partial p^d} + \frac{\partial H}{\partial p^d}. \]

Expressions (11) and (12) may be substituted directly into (4a).

Denote by \(\varepsilon^q\) the price elasticity of demand for the composite good and by \(\varepsilon^s\) the elasticity of supply of the domestically produced good. If changes in the import price of a sector tend to be transmitted entirely into changes in the corresponding domestic price, the sector exhibits a great deal of trade dependence. A measure of the trade dependence of a sector is provided by the domestic price response elasticity with respect to a change in the import price.
price. This elasticity can be used to measure the partial equilibrium impact of a tariff change on the price of the corresponding domestic good because a tariff affects directly the domestic price of imports.

The expression for $E$ is given by 2

\[
(13a) \quad E = \frac{(\sigma - \varepsilon q) \beta^q (p/p^m)(\sigma - 1)}{\left(\varepsilon^s + \sigma\right) - (\sigma - \varepsilon q) (1 - \varepsilon^q) (p/p^d)(\sigma - 1)}.
\]

Noting that equation (9) can be used to derive the value share of domestic goods in total purchases and that there is an analogous expression for the import share, equation (13a) can be rewritten as

\[
(13b) \quad E = \frac{(\sigma - \varepsilon q) S^m}{\varepsilon^s + \varepsilon^q + (\sigma - \varepsilon q) S^m},
\]

where $S^m = P^m M/PQ$. Note, also, that the numerator in (13b) is just the partial cross-elasticity, $E^d,m$, defined in equation (4b). Therefore, equation (13a) can be rewritten again as

\[
(13c) \quad E = \frac{E^d,m}{\varepsilon^s + \varepsilon^q + E^d,m}.
\]

These equations provide the basis for analysis of the extent of trade dependence and show that the value of $E$ depends crucially on the import share. 3 Equations (13b) and (13c) indicate that a necessary and sufficient condition for the domestic price to fall is that the domestic product and the imported product be complements ($E^d,m < 0$). Note that the small-country assumption has been kept throughout the discussion so a negative value for $E$ cannot be due to any terms-of-trade effect.
How reasonable is the assumption that imports and domestic goods are imperfect substitutes? For CGE models at a fairly high level of aggregation, the assumption can be justified as an approximation given the need to work with aggregates. However, the assumption of imperfect substitutability is also supported even at a fairly micro level. For example, as noted above, one often observes two-way trade at quite disaggregated levels. Certainly, one does not observe the degree of dependence of domestic prices on world prices that is implied by standard trade theory.

Table 1 gives the value of E for various values of initial trade shares and trade-substitution elasticities. It is especially interesting that, when the import share is low, E is very low, even when the trade-substitution elasticity is quite high. Also, when the import share is very high (90 percent), a trade-substitution elasticity of 20 yields a value for E of 0.90, which is still significantly less than the value of 1 that would arise with perfect substitutes. These results emphasize the importance of the import shares and of the share parameter in the CES trade-aggregation functions. Standard trade theory would predict that, for large countries in which import shares are relatively low, changes in world prices would still exert a strong pull on domestic prices. This specification implies that a large country with a low trade share will have substantial autonomy in its domestic price system even if trade-substitution elasticities are very high.

As the trade-substitution elasticity approaches infinity, E approaches 1, and we are back in the world of perfect substitutes and the law of one price. 4/ As the import share tends toward 0, we approach the case of a pure nontraded good. The degree of autonomy of domestic prices is also sensitive to conditions of supply and demand in the domestic market. The higher the elasticity of domestic supply, the lower is E. If the trade-
**TABLE 1**

Degree of Trade Dependence as Trade Substitution Elasticity Varies

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( S^m = 0.1 )</th>
<th>( S^m = 0.5 )</th>
<th>( S^m = 0.9 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-0.03</td>
<td>-0.14</td>
<td>-0.29</td>
</tr>
<tr>
<td>1.0</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2.0</td>
<td>0.05</td>
<td>0.20</td>
<td>0.31</td>
</tr>
<tr>
<td>5.0</td>
<td>0.16</td>
<td>0.50</td>
<td>0.64</td>
</tr>
<tr>
<td>10.0</td>
<td>0.31</td>
<td>0.69</td>
<td>0.80</td>
</tr>
<tr>
<td>20.0</td>
<td>0.49</td>
<td>0.83</td>
<td>0.90</td>
</tr>
<tr>
<td>50.0</td>
<td>0.59</td>
<td>0.93</td>
<td>0.96</td>
</tr>
<tr>
<td>100.0</td>
<td>0.83</td>
<td>0.96</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Note: The entries are tabulations of equation (13) with \( e^q = e^g = 1 \).

\( \sigma \) = trade-substitution elasticity.

\( S^m \) = import share in value of total supply.
substitution elasticity equals the price elasticity of demand for the composite good, then \( E = 0 \). In this special case, any change in import price leads to a change in composite price and a change in demand for the composite good such that demand for the domestic good remains unchanged. The typical range of parameters assumed in most studies bracket this special case, so one should be cautious in interpreting the results of exercises involving single tariff cuts.

We have seen that the assumption of imperfect substitutability of domestically produced and imported goods leads to considerable autonomy of the domestic price system. This result obtains in most CGE models used to explore the impact of tariff cuts. A relevant question is whether the Armington specification, which assumes strong separability, implies too much autonomy for the domestic price system and, hence, biases downward the resulting welfare gain estimates. From the point of view of an empirical modeler, the assumption of strong separability reduces the number of parameters that must be estimated. However, as mentioned above, Deaton and Muellbauer (1980, Chapter 5) and Winters (1984), show that the assumption of strong separability does limit considerably the extent of domestic response to a change in trade policy. The strong separability assumption assumed in multicountry models is also hard to justify on a priori grounds. Furthermore, as mentioned above, Winters reports that econometric tests reject the separability assumption for U.K. data.

IV. The Specification of Exports

The other important factor further increasing domestic price autonomy in CGE models stems from exports. In most CGE models, there is two-way trade
in most sectors. Indeed, as mentioned earlier, the fact that the Armington specification provides a theoretical justification for observing two-way trade is one of its major appeals for empirical work. The presence of exports, however, provides a second route by which world prices affect domestic prices. Below we consider the different approaches to the specification of exports in a single-country context and discuss a treatment of exports that is symmetric with that for imports.

One common approach to treating exports in this model is to assume that there is a foreign demand curve for exports with a constant elasticity of demand. This is the case considered by Whalley and Young. Working through the algebra, the expression for $E$ in equation (13) has an additional positive term in the denominator. The effect of introducing export demand is to lessen the value of $E$ for any given set of parameters on the import side. Exports provide another source of demand for domestic goods, lessening the effect of imports on the domestic price; but, more importantly, it introduces terms-of-trade effects into the welfare analysis of trade distortions. This may be undesirable and have a dominating effect as shown in Dervis, de Melo, and Robinson (1982, chapter 9). Below we introduce a treatment which we believe is generally better suited to the analysis of tariff reductions.

The alternative approach is to treat exports symmetrically with imports and to assume that goods sold on the domestic market are different from those sold on the world market. As in the case of imports, a justification for this treatment is that aggregation involves grouping quite different goods into a single sector. Recent empirical evidence consistent with this formulation is found in Aspe and Cavazzi (1982). The standard treatment of exports as perfect substitutes for goods sold on the domestic market implies a responsiveness of export supply to changes in relative prices.
that is empirically unrealistic at the level of aggregation of most applied models. Instead, it is more reasonable to assume that exports and goods sold on the domestic market within the same sector classification are imperfect substitutes. This treatment, first proposed by Powell and Gruen (1968), has been suggested for multicountry trade models but has not been used much in single-country models. 6/

Assume that the domestic producer makes a composite commodity, $X$, which is an aggregation of goods suitable for the domestic market, $X_D$, and goods suitable for the export market, $X_E$. The producer has a transformation function that determines the trade-off between producing goods with the same sectoral classification for the domestic and export markets. Following Powell and Gruen, assume that the aggregation is given by a constant elasticity of transformation (CET) function, 7/

$$X = A \left[ a_1 X_D^h + a_2 X_D^h \right]^{1/h},$$

where $A$, $a_1$, and $h$ are parameters. The transformation elasticity is given by $\alpha = 1/(h - 1)$.

Given this formulation, one can derive expressions for the derived demands for $X_E$ and $X_D$ under the assumption that producers maximize profits and, hence, equate the marginal rate of transformation between $X_E$ and $X_D$ to their price ratios. After applying algebra similar to that used in the case of imports, one obtains new expressions for the elasticity of response of domestic prices with respect to changes in both export and import prices,

$$e^m = \frac{(\sigma - \epsilon^q) S^m}{S^e \alpha + (1 - S^e) \epsilon^s + \epsilon^q + S^m (\sigma - \epsilon^q)}.$$
\begin{equation}
E^e = \frac{(\Omega - \varepsilon^e) S^e}{S^e \Omega + (1 - S^e) \varepsilon^s + \varepsilon^q + S^m (\sigma - \varepsilon^q)},
\end{equation}

where \( E^m \) and \( E^e \) are the domestic price response elasticities and \( S^e \) is the share of the value of exports in the value of production analogous to the import value share, \( S^m \), defined in equation (13b).

The new expressions for the elasticity of domestic price response are very similar to the earlier expression in equation (13). The assumption that exports and domestically produced goods are imperfect substitutes introduces a new wedge between the domestic and world markets. Models including this specification must have three distinct prices for domestic goods, export goods, and imports with the same sectoral classification. Given this approach, the small-country assumption of trade theory implies that, given the exchange rate, \( P^e \) and \( P^m \) are fixed. However, \( P^d \) is still autonomous and linked to the prices of exports and imports through the two aggregation functions. Even with high substitution and transformation elasticities, the link is rather weak and depends significantly on import and export shares as well as on demand and supply conditions in the domestic market.

V. Conclusion

The practical implication of assuming imperfect substitutability between domestic goods and both imports and exports is that estimates of the impact of trade policy on resource allocation and welfare in CCE models will yield much less dramatic results than in models in which there is perfect substitution. In particular, it is likely to have accounted for the small welfare gains from trade liberalization reported in estimates of recent tariff cutting exercises. Although the new specification is empirically much more
realistic, it raises new questions, such as how trade shares are determined in the first place. The share parameters in the CES and CET functions are exogenous, and trade policy changes cannot significantly affect comparative advantage since these share parameters are fixed. Given their importance in determining the domestic price response elasticities, it would be desirable to study how these shares are determined and how they change over time. Such change in share parameters over time is analogous to biased technical change in production functions and is clearly an important phenomenon that deserves further study.
Appendix

Derivation of Equation (13)

To derive the equations in the text, note that

\[ \frac{\partial H}{\partial p} = \sigma \frac{D}{p} \]

\[ \frac{\partial H}{\partial q} = \frac{D}{Q} \]

\[ \frac{\partial Q}{\partial p} = -\varepsilon \frac{Q}{p} \]

\[ \frac{\partial P}{\partial p} = \frac{D}{p} (\sigma - \varepsilon) \sigma \left( \frac{P}{p_m} \right)^\sigma \text{ from equation (11)} \]

\[ \frac{\partial H}{\partial p^d} = -\sigma \frac{D}{p^d} \]

\[ \frac{\partial S}{\partial p^d} = \varepsilon \frac{D}{p} \]

\[ \frac{\partial P}{\partial p^d} = (1 - \varepsilon) \sigma \left( \frac{P}{p^d} \right)^\sigma \]

and

\[ \frac{\partial D}{\partial p^d} = \frac{D}{p} (\sigma - \varepsilon) (1 - \varepsilon) \sigma \left( \frac{P}{p^d} \right)^\sigma - \sigma \frac{D}{p} \text{ from equation (12)}. \]

Direct substitution of these expressions into equation (4a) yields equation (13a).
Footnotes

1 Exports are ignored here but will be considered below.

2 The derivation of this expression is discussed in the Appendix.

3 It is easy to show that the denominator is always positive.

4 Note that the import ratio also changes as \( s \) changes. In taking this limit, we assume a fixed import ratio as we change the shape of the curve. Since \( M/D = \frac{\beta}{(1 - \beta)} s (p^d/p^m)^s \), it is convenient to pick units so that \( M/D = 1 \) and, hence, \( p^m/p^d = \frac{s}{\beta} (1 - \beta) \).

5 Whalley and Yeung show that if there is an external budget constraint, one cannot independently specify an export demand elasticity along with a trade substitution elasticity. Our partial equilibrium approach however, allows us to specify independently the two elasticities.

6 An exception is the ORANI model of Dixon et al. (1982) who used a generalization of the constant elasticity of transformation (CET) function to model agriculture. They did not, however, use this specification for exports. Their discussion of the properties of the CET function is excellent.

7 The CET function is concave to the origin for \( h < 1 \). As \( h \) approaches 1, \( \eta \) goes to infinity. With \( h > 1 \), we have the standard CES function.
References


