The Management of the Developing Countries’ Debt: Guidelines and Applications to Brazil

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Some principles which may be useful in formulating a debt management strategy are articulated. There are four major policy recommendations: (1) stretch out the repayment of the debt; (2) monitor both exports and GDP; (3) ignore the capital loss of the creditors; and (4) watch the domestic deficit. The goal of the recommendations is the maximization of the intertemporal welfare of the country under the constraint that it services its debt.

The financial crisis of 1982 sent a simple message: the developing countries’ debt could not be rolled over according to the previous rules of the financial game. In order to service their debts, debtors were required to reduce their imports and boost their exports. At that time, the main uncertainty was: would a debtor country default rather than adjust? Today, this question has been partially answered. Brazil, for example, chose to generate a $10 billion (billion is 1,000 million) (noninterest) surplus rather than default on its external debt, an action which can help to change the view that some may have had of the developing countries’ debt problem. Furthermore, the shocks which now seem to affect the world economy are more idiosyncratic than the earlier worldwide recession and high real interest rates. They arise from terms of trade uncertainty, which penalizes some and helps others, or from domestic financial difficulties, which call for country-specific remedies. In brief, the developing countries’ debt management may have gained, in the aggregate, some margin for action.

In this article, some principles which may be useful in formulating a debt management strategy for the coming years are articulated. There are four major policy recommendations which draw on an analysis presented in the appendixes: (1) stretch out the repayment of the debt; (2) monitor both exports and gross domestic product (GDP); (3) ignore the capital loss of the creditors; and (4)
watch the domestic deficit. The goal of the recommendations is the maximization of the intertemporal welfare of the country under the constraint that the debt is serviced.

**Stretching Out the Repayment of the Debt**

As emphasized in previous work (Cohen 1985), a country need repay neither the principal nor even all the interest falling due on its external debt in order to remain solvent. Solvency requires only that the debt grow less rapidly than the interest rate. To be feasible, this requires that the debt grow no faster than the country's revenues. Yet this is certainly not a sufficient guide for monitoring the debt. It is not only the capability of the country to service its debt which matters but also its willingness to do so. Credit ceilings are required which are low enough to keep the country from defaulting (at least under some set of events). Out of a simple deterministic intertemporal framework, drawing on Eaton and Gersovitz (1981) and Cohen and Sachs (1985), I show how these credit ceilings can be calculated. The basic result is as follows: the credit ceiling must be low enough that the country would prefer to keep its debt-over-"resource" ratio a constant "on average" rather than default. ("On average" and "resource" are defined below; here exports can serve as a proxy for "resource.")

From this, one can see that a country which is believed to have reached its credit ceiling should not be asked to reduce its debt-to-export ratio; it would default rather than so do. Instead, it should be asked to stretch out the service of its debt so as to keep its debt-to-resource ratio constant (on average).

**Watching Both GDP and Exports**

Which measure of resources should one take? Exports are too narrow, GDP too broad. Even if factors of production could be inexpensively transferred from one sector of the economy to another, GDP would still be an inadequate measure upon which to base reschedulings. In effect, if the country were to understand that its new loans were to depend upon its GDP growth (in dollars), then it would have an incentive to overvalue its currency so as to inflate the dollar value of its GDP artificially. Conversely, if exports were taken as the sole measure of resources, this would create a bias in the opposite direction. What is needed is a measure of resources which is invariant with respect to the real exchange rate, a version of the "invariant measure of wealth" (the "standard commodity") calculated by Sraffa (1960) for wage-profit sharing. In a simple framework, I show that such a measure can be derived from GDP and exports. A numerical estimate is provided for the case of Brazil: it is 10 percent of GDP plus 90 percent of exports.

**Ignoring the Creditors' Capital Loss**

In view of the repayment strategy sketched above, one could expect lenders to be more than happy with Brazil's recent trade balance as the debt-to-export ratio has declined substantially. Yet, they are not: on the secondary markets, Brazil's
debt is discounted. To see why this is so, consider an oil exporting country.
Assume it is servicing its debt with interest being paid when due. If this were
expected to last forever, then the debt should be priced at its face value. Now,
instead, assume that there is always a 20 percent probability that the price of oil
will fall to zero. Whatever the face value of the country's debt, however good its
servicing intentions, there will always be a 20 percent discount on its debt. If the
banks were to grant a 20 percent moratorium, a 20 percent discount would still
apply to the newly written-down debt. What this example suggests is that the
market value of the debt is of little help in assessing the repayment strategy of an
indebted country. In a framework in which all uncertainties are assumed to be
perfectly assessed, I show that the simple maxim indicated in the first point,
"stretch out the repayment of the debt," should continue to be applied to the face
value of the debt, even when it does not coincide with its current market value.

Watching the Domestic Government Deficit

Assume that a country does achieve the external adjustment expected of it.
This does not imply that the country's government has undertaken the appropri-
ate adjustment. The rationing of imports may create a trade balance surplus, but
does not necessarily bring the government the income it needs to repay its
debt. If the government postponed its own adjustment, then a secondary burden
would exist: the raising of taxes to repay the government's domestic debt.

To investigate this problem, I proceed as follows. Assume, as a first approxi-
mation, that all external debt is government debt. Then any transfer of resources
abroad can only be accompanied by a combination of the following three
changes: (a) an increase in the government budget surplus; (b) an increase in
government domestic debt; (c) money creation. I decompose the respective share
of these three terms in the case of Brazil (for 1983-85) and find that 71 percent
of the trade surplus was financed by an increase in domestic debt. As a result,
domestic debt increased by 84 percent and domestic real interest rates rose
sharply, to more than 20 percent in 1985. In contrast, I show that the repayment
scheme outlined above would have allowed the domestic debt to rise by just 22
percent. I conclude from this analysis that Brazil should allow its external debt
to grow faster and should slow the growth of its domestic debt.

Each of the four policy recommendations is elaborated on in the remainder of
the article. Mathematical derivations are presented in the corresponding appen-
dixes.

I. STRETCHING OUT THE REPAYMENT OF THE DEBT

In this section, I assume that the economy is managed by a social planner; this
assumption avoids the issue of taxation and domestic finance, which is the topic
of section IV. The social planner can be viewed as an economic agent whose
income is equal to all the resources of the country. To simplify the discussion, I
postulate that these resources can be unambiguously defined and assume that the
economy produces only one good which can be traded freely on the world market. This good is the numeraire of all international transactions. This assumption is relaxed in the next section, in which the issue of defining a measure of wealth based on GDP and/or exports is tackled. Finally, I assume that the future can be predicted perfectly, that is, I assume away any uncertainty. The issue of uncertainty is dealt with in section III.

Assuming No Threat of Debt Repudiation

Consider a debtor-to-be country at some initial time, before it has started to borrow on the world financial markets. The future can be characterized by a sequence of fluctuating expected rates of growth of the country’s income. The social planner discounts the future at some fixed rate. How is his decision to borrow (or to lend) taken? Contrary to the intuition derived from the analysis of a stationary economy, it is not solely the comparison of the world interest rates to the domestic discount factor which matters. If the country expects a very sharp decrease in its income in the future, it will not be induced to borrow, but rather it will lend now so as to match the shortfall in its future income. The decision to borrow will depend upon three factors: the domestic discount factor, expected interest rates, and expected growth rates. This problem is examined in appendix 1. Under a suitable set of assumptions, I show that the problem boils down to a comparison between the domestic discount factor and an “average” difference between expected interest and growth rates. The thinner this difference, the larger the incentive to borrow. The “average” difference is defined as follows: it is that constant difference which would yield the same wealth to the country as the actual (fluctuating) difference.

Once the initial decision to borrow has been taken, the law of compound interest takes over and there must come a point when the country starts “servicing” its debt, that is, generating a sequence of trade balance surpluses with a discounted sum equal to the country’s external debt. Assuming that the country does not repudiate its debt, it can very well be the case that the social planner drives the country to a long-run equilibrium in which all the resources of the economy are channeled to its creditors. (This will be the case if the planner’s discount factor is always larger than the difference between interest rates and growth.) In such a long run, the ratio of the trade balance to GDP would converge toward one. Debt crises start far before such a point is reached: lenders usually worry about the creditworthiness of their client long before he drives himself to asymptotic starvation.

The Option to Repudiate Debt

Following Eaton and Gersovitz (1981) and Cohen and Sachs (1985), I now assume that a country has the option of repudiating its debt if the burden of its repayment becomes “too heavy.” “Too heavy” means that the cost of servicing the debt (measured in terms of foregone utility) is larger than the cost that the country would bear if it were to default. This latter cost is extremely complex to
evaluate in practice, but it seems natural to represent it as some fraction of the country's income. This fraction will measure all the retaliatory devices that the lenders may inflict upon the defaulting debtor. In the analysis in appendix 1, the cost of default is not assumed to be a constant fraction of the country's income over time. It might very well be the case that most of the costs are up-front, being much higher just after default than ten years later.

Once the option to default is acknowledged, lenders will wish to ensure that the country will not be in a situation in which its postdefault real income is greater than if it were to service its debt. (As before, "service" the debt means to generate a sequence of trade balance surpluses with a discounted sum equal to the value of the debt.) This leads lenders to impose a credit ceiling. How can it be calculated? In the framework examined in appendix 1, I show that the country will not be willing to repay more than some fixed fraction $b^*$ of its resources each period, the fraction depending only upon the costs of default. Therefore, once the cost of default has been established, the credit ceiling is readily obtained: it is this same fraction $b^*$ applied to the wealth of the country (when the wealth is measured as the sum of all future resources discounted at the world rate of interest).

The credit ceiling amounts to a fixed upper bound to the "average" debt-over-resources ratio. In other words, a country that has reached its credit ceiling would be asked by its creditors to transfer abroad enough income so as to keep its debt-to-resources ratio from rising above the level it reached when the credit ceiling started to bind. In the framework analyzed in the appendix, this requirement will be exactly equivalent to asking the country at its credit ceiling to devote the fraction $b^*$ of its income to its debt service every period. This fraction $b^*$ will satisfy the two requirements: (1) it generates a sequence of trade balance surpluses with a discounted sum equal to the face value of the debt; and (2) it forces the country to forgo, for debt service, a stream of consumption no larger than it would expect to forgo by defaulting.

A Numerical Exercise and Some Economic Policy Implications

Accepting this framework as a benchmark, one sees that the first step in assessing whether a country would repudiate its debt rather than service it is to calculate the maximum fraction of its resources which should be devoted to debt service. In earlier work (Cohen 1985), I did exactly that, taking the exports of the country as a proxy for measuring its resources. I found that most countries should devote no more than 15 percent of their exports to debt service. This calculation was based upon pessimistic assumptions with regard to future interest rates and growth. In order to see the intuition behind this result, consider the case where the "average" difference between interest rates and growth is 5 percent in real terms. Then a country such as Brazil, which has net debt-to-exports ratio of 3, should devote 15 percent of its exports to external debt service. In other words (under these assumptions), 15 percent of the country's exports are sufficient: (1) to keep its debt-to-export ratio a constant on average; and (2) to
meet the solvency condition that the sum of the discounted flows of income transferred abroad is equal to the face value of the debt. If a country were to generate a surplus greater than or equal to 15 percent, rather than repudiating its debt, its debt-to-exports ratio would decline or be constant. Under the assumptions spelled out in appendix 1, the country will never repudiate its debt rather than service it according to the 15 percent rule.

The policy implications which stem from this analysis are clear. When designing the adjustment program of a country which is believed to have reached its credit ceiling, the World Bank or the International Monetary Fund (IMF) should not aim to reduce the debt-to-export ratio but rather should aim at stabilizing it. The idea that the debt-to-export ratio should be brought down to a point at which "voluntary lending" could resume is an invalid notion in the context of this analysis. If voluntary lending is to resume, it means that the debt-to-export ratio might rise again. It is hard to conceive of any optimizing framework which would prescribe a U-turn policy according to which the ratio should go down and then up.

II. WATCHING BOTH EXPORTS AND GDP

For external creditors, the resources that matter are those which can be channeled to the world markets and exchanged against "hard" currencies. From this aspect, total GDP is certainly too broad a measure and exports too narrow. Traditionally, the "tradeable" goods sector is regarded as an appropriate measure, but this is very much a short-run concept. In the long run, human and technological resources can be shifted from nontraded to other activities. From this point of view, using GDP is less inappropriate than it might at first seem. Yet even in the case when all resources can be inexpensively shifted from one sector to another, it would be a policy mistake to base one's lending strategy on GDP, for the borrowing country might be induced to overappreciate its currency so as to inflate the dollar value of its GDP artificially. Conversely, if bankers were to base their lending strategy upon exports only, they might induce the country to overvalue its currency, and again the measurement of wealth would be distorted. The two biases which are introduced by taking GDP or exports as a measure of a country's income are of opposite signs: one lending strategy induces too few resources to be channeled to the traded sector, the other too many.

A measure of wealth which is invariant with regard to the real rate of exchange is derived in appendix 2. Exports are positively related to a real depreciation of the currency; GDP negatively related. The invariant measure of wealth is that linear combination which fails to depend upon the real exchange rate. In the case of Brazil, this analysis yields the result that 10 percent of GDP plus 90 percent of exports is an invariant measure of Brazil's resources. Why is it that exports carry a much greater weight than GDP? First, it must be recalled that exports amount to 10 percent of GDP. Therefore, an increase of GDP by 1 percent counts almost as much, in the definition of the invariant measure of wealth, as a
1 percent increase of exports. Yet it is true that one more cruzado in the export sector counts much more than one more cruzado in the rest of the economy. Why is that? Think of a country the exports of which consist of raw materials only. If there is no substitutability between exports and the production of other goods, then the invariant measure of wealth should consist of exports only: it is the only source of foreign currency earnings (which is what matters for our purpose). Obviously, Brazil is not in this situation, and some substitution is possible. What our weighting indicates is that such substitution is costly, so that 1 percent of GDP counts as much as 1 percent of exports in the definition of the country's resources. Obviously this is an area where much more work is needed before we can reach a conclusive answer.

Figure 1 shows the ratio of Brazil's net debt to its exports, GDP, and "resources." Although the net debt-to-exports ratio has fallen significantly in recent years, the debt-to-GDP ratio decline has been much more modest. This is the result of the large real depreciation of the currency which was undertaken in 1983. The debt-to-resources ratio is a weighted average of the other two ratios.

Figure 1. Brazil: Stock of Net Government Debt to Exports, GDP, and "Resources"

Key: --- debt/exports --- debt/GDP --- debt/"resources"

Note: "Resources" are the sum of GDP × 0.10 and exports × 0.90.

Sources: World Bank estimates; International Monetary Fund, International Financial Statistics, various years.
III. Ignoring the Creditors' Capital Loss

I have assumed so far that interest and growth rates can fluctuate, but along perfectly predicted paths. Let us now introduce uncertainty with regard to both rates. Uncertainty might be thought of as the sum of two components: a continuous component, which only creates smooth deviations around a deterministic path, and a discontinuous component, which allows for an abrupt swing of the overall trend. The first kind of uncertainty (say a stochastic diffusion process) would not yield substantially different results from those examined previously. Because unforeseen variations of this type take place continuously, the risk of default cannot take lenders by surprise, and the previous principles of debt management would continue to be valid. Lenders should impose a credit ceiling which is a function of the expected average difference between growth and interest rates, and the ceiling would be adjusted continuously to allow for stochastic deviations. In this section and in the following, I limit the analysis to the second kind of uncertainty, that which is associated with abrupt changes in the economic environment.

To simplify the analysis, first consider the case in which there are only two different states of nature. State 1 is a "good" state, characterized by low real interest rates and fast growth. State 2 is a "bad" state, characterized by high real interest rates and slow growth. In state 1, there is a probability $p_1$ of staying there and a probability $p_2 = 1 - p_1$ of going to state 2. Once in state 2, there is a probability $q_1$ of returning to state 1 and a probability $q_2 = 1 - q_1$ of staying in state 2. The variables $p_1$ and $q_1$ need not be equal. If $p_1$ and $1 - q_1$ are both close to one, the economy is likely to stay in the state it is in, whatever this state might be.

How can we characterize the lenders' strategy? In the previous deterministic setting there was no ambiguity. Lenders would provide any amount of lending below the credit ceiling at the riskless rate of interest. Provided that the credit ceiling was accurately calculated, the absence of uncertainty made the loans riskless. Furthermore, despite fluctuating growth rates, the country did not need any further insurance. It could spread out its consumption over time up to a credit ceiling. In the uncertainty case, the borrower would now like to insure itself against uncertainty regarding both the interest rate and the growth rate. As for the interest rate, it would rather borrow at a fixed rate and let the international lenders take the risk of fluctuations. As for the growth rate, the borrower would like to protect itself against an uncertain future and find some form of insurance against a drop in its income. Surprisingly, despite the obvious relevance of this problem for developing countries, very little risk diversification either on interest rates or on growth rates appears to have been undertaken.

**Bank Lending under Uncertainty**

I now assume that the borrowing country does not protect itself against fluctuating interest or growth rates. Assume also that the country starts to borrow in
the good state of nature, when interest rates are low and growth fast, and that
during this state 1, interest rates are lower than growth rates. Then, as long as
the world stays in state 1, any level of debt is sustainable: refinancing both the
principal and interest falling due does not affect the solvency of the country. Its
debt-to-resources ratio would still decline, even when the country makes no real
transfer of resources to the rest of the world. Therefore, any credit ceiling
imposed on the country can only be derived from the possibility of it moving to
state 2. Lenders would seek to impose, in state 1, a credit ceiling such that the
country would not default were the economy to shift abruptly to state 2. As
before, the ceiling would be imposed as a maximum bound to the debt-to-
resources ratio (see appendix 3). Once this ceiling has been reached, the country
can keep borrowing net new amounts (in excess of principal and interest pay-
ments falling due) as long as the economy stays in state 1. This situation fits the
happy state of affairs of the 1970s. Thus from 1977 to 1981, Brazil could keep
its borrowings in excess of its debt service obligations and yet stabilize the debt-
to-exports ratio at about 3. Now, assume that the state of nature shifts from
state 1 to state 2 when the credit ceiling is already binding. In order to keep its
debt-to-resources ratio at a constant level, the country must now generate a
trade surplus. The principles which guide how this surplus should be determined
are the same as in section 1, except that the future must be projected as if the
economy were to stay forever in state 2. Relief will only come if state 1 returns.

What this analysis shows, at this point, is that one cannot argue that the large
interest rate rise during the 1980s makes the debt-to-exports ratio of the 1970s
unsustainable. Because interest rates were lower than growth rates during that
decade, any ceiling on the debt-to-exports ratio imposed according to the princi-
pies of this article must have been imposed by reference to another state of
nature.

Banks' Misperceptions or Risk Sharing? A View of the
Discount on Developing Countries' Debt

This two-states-of-nature view of the world fits well the process we have
witnessed during these past years in many countries, including Brazil, except for
one thing: since the crisis of 1982, and despite the country's outstanding success in
generating trade surpluses, Brazil's debt has been trading at a substantial
discount on the secondary markets. (In August 1986 the discount was 25 per-
cent.) If only states 1 and 2 could occur, this should not be so. Once a country
has shown that it would stabilize its debt-to-resources ratio rather than default,
it reveals that the debt ceiling imposed in the 1970s was indeed safe enough to
absorb the shock of the 1980s. Why then is the debt traded at a discount?
Depending upon the optimism that one wishes to display with respect to the
financial market, two answers might be given: risk sharing or risk mispercep-
tions.

Let us start with the latter. Assume that lenders forecast only two possible
states of affairs, but as the borrowing country moves from the good state 1 to the
bad state 2, lenders suddenly see that an ugly state 3 is also possible: a state where things would be even worse and in which the borrowing country would certainly default or have its debt partially written off. Given this risk, debt would then be traded at a discount on the secondary markets. Yet lenders cannot incorporate this risk into the rate charged to the borrower: if the country has indeed reached its credit ceiling, it would certainly default rather than pay a higher interest rate.

A more optimistic story might be told, however. Assume that the possibility of state 3 was indeed foreknown and that the transition from one state to another comes as follows: the economy can go from state 1 to state 2 or stay in state 1, but it cannot go directly from state 1 to state 3. Once it has moved from state 1 to state 2, however, there is now a positive probability of passing to either of the other two states. Let us assume that state 3 is so bad that any borrower would default rather than repay its debt. If banks decide to lend in state 1, they know that they face a positive probability of the debt being written down. If one assumes (as I have) that the banks are not ready to take a risk that they cannot diversify, this is only relevant if state 3 is confined to the country in question or a limited group of countries. A sudden and permanent fall of the country's terms of trade would be a good example of a state 3.

One can think of two polar ways in which to deal with this risk. In the first case, the banks could charge the riskless rate in state 1 and the risk-adjusted rate in state 2. As the economy goes from state 1 to state 2, they would not take a capital loss. However, the credit ceiling would have to be low enough to keep the country from defaulting in state 2, when it has to pay risk-adjusted interest rate on existing, as well as new, debt. Another way to deal with the state 3 problem is to charge the same spread in state 1 as in state 2. In this case, the spread measures the risk of default from a state 1 point of view: it is therefore relatively low because it measures the probability of going from state 1 to state 2 multiplied by the probability of going from state 2 on to state 3. Under this scheme, the credit ceiling is higher than before, for although the country must still be kept from defaulting in state 2, it is being charged a lower spread. If the borrowing country is risk averse, it will prefer this latter option.

What are the policy implications of this analysis? Whether it is a result of risk sharing or risk misperception makes little difference. Once the economy has moved into state 2, the lenders are faced with a capital loss on their assets. The outstanding debt will be quoted on the secondary market at a discount. The repayment scheme which should be undertaken is unambiguous. It is that which I set out in section I: the repayment should be stretched out so as to stabilize the debt-to-resources ratio. Again, this should be done under the assumption that the economy will stay in state 2. In other words, neither the possibility of going into state 3 nor that of returning to state 1 should be taken into account. The lenders' capital loss should be simply ignored. It is only when the state of nature has indeed shifted to state 3 that the debt should be written down: in state 3, the country would certainly default rather than service the face value of the debt,
contrary to what happens in state 2. (See Sachs 1986 on why an explicit write-off is superior to an implicit one.)

A more troublesome question arises if an agency such as the World Bank or the IMF has some leverage to raise the country's efficiency and push the debt below the credit ceiling (while the economy is still in state 2). In this case, the lending banks will have an incentive to ask for faster repayment, and their aim would be to push the debt down to the point at which even the occurrence of state 3 would not induce default.

One sees from this analysis that a discrepancy between the market value and the face value of the debt should not necessarily imply that the debt should be written off. If the country is at its credit ceiling in state 2, it is the face value of the debt which must be rescheduled when it moves to state 3 so as to keep the debt-to-resources ratio a constant (on "average"). However, the discrepancy between the face and the market values does explain why the banks are so impatient to be repaid. And my analysis, thus far, suggests that they may indeed succeed in bringing to zero (if they want) the debt-to-exports ratio of a country which has "revealed" that it would bring the ratio down rather than default. I now argue, however, that this need not be so, once the domestic finance aspect of the external debt has been taken into account.

IV. WATCHING THE DOMESTIC DEFICIT

Domestic Finance with Distortionary Taxes

Until now, I have assumed that the government had at its free disposal all the country's economic resources. This is equivalent to assuming that an extremely efficient tax system could perform the desired transfer of resources underlying the design of the government's policy. Obviously, this assumption fits no country, developed or developing. In this section, I develop a framework in which the government faces a domestic financial problem in addition to the external one. I assume (as a first approximation) that only the government can borrow or lend abroad so that the private sector's only net financial wealth is the government's domestic debt. If taxes were not distortionary, this restriction would not modify my previous analysis, and the government would face the external debt ceiling as its only relevant constraint. Yet if taxes are distortionary, the government really faces two constraints: a domestic and an external one.

Consider a world economy with the two states of nature described in the previous section (see appendix 4 for details). Assume that the constraint on external debt is binding while the economy is in the good state, state 1. As long as the world economy stays in this state, the country can borrow a new net amount every period. This allows the government to maintain low domestic interest rates and low taxes. If the economy moves to state 2, all the bad news comes together. A trade surplus is required: this pushes the domestic interest rate up so that (distortionary) taxes must be raised to keep the domestic debt from exploding. This rise in domestic interest rates may be thought of as a version of
the transfer problem, the source of so many controversies. In the traditional literature, when a country that can affect its own terms of trade is forced to accomplish a transfer of income to the rest of the world, it must bear a terms of trade deterioration in order to obtain a trade surplus. This creates a secondary burden which adds to the primary burden of forgoing consumption at the pre-transfer relative prices. In the case we discuss, a government which is forced into a trade surplus incurs the secondary burden of repaying its own domestic debt at a higher interest cost, depending on the duration and terms of existing domestic debt. This analysis supports the IMF’s emphasis on the necessity of domestic fiscal accommodation of an external adjustment. I now argue that, at least until 1986, this adjustment had not been undertaken by Brazil.

An Analysis of the Domestic Counterpart to Brazil’s External Adjustment

Brazil’s external surplus (excluding interest payments) reached more than $10 billion in each of the three years 1984–86 as a result of a large cut in imports and a boost in exports following a large real depreciation. If one takes as a first approximation that all foreign debt is public debt, this surplus must have been reflected in a combination of: (1) money creation; (2) domestic debt increase; and (3) a government budget surplus. The arithmetic of this relationship is as follows. The increase in the sum of domestic and external debt is equal to the government budget deficit net of money creation. If only the government borrows abroad, then the decrease in the government’s external debt is equal to the current account surplus. Therefore, the current account surplus is the sum of the increases in domestic government debt, the budget surplus, and money creation.

This relationship can be written as follows: Call $B(t)$ the domestic debt, $B^*(t)$ the foreign debt; $\rho_i B(t)$ the interest falling due domestically; $r^* B^*(t)$ the interest falling due abroad; $Z_0(t)$ the non-interest-payment government surplus; and $S(t)$ new real money creation (the seignorage tax). (The variable $t$ represents either the situation at the end of period $t$ or the flow in period $t$, as appropriate.) The total budget deficit of the government is the sum of the primary deficit and of interest payments, that is,

$$-Z_0(t) + \rho_i B^*(t - 1) + \rho_t B(t - 1).$$

It must be financed either by money creation or by an increase in domestic or external debt. Thus:

$$-Z_0(t) + \rho_i B^*(t - 1) + \rho_t B(t - 1) = B(t) - B(t - 1) + B^*(t) - B^*(t - 1) + S(t)$$

This equation can be rewritten as:

$$-Z_0(t) + \rho_i B^*(t - 1) + \rho_t B(t - 1) = S(t) + Z_0(t) + B(t) - (1 + \rho_t) B(t - 1)$$

The left-hand side is the non-interest-payment current account surplus; the right-hand side is the sum of seignorage tax, the government surplus (excluding interest payments), and new domestic debt. Even though the current account surplus might have been obtained by a real devaluation or a rationing of im-
ports, it must be reflected in one of the three latter items. Money creation is triggered when the government buys the dollars earned by exporters. (Recall the assumption that only the government holds foreign assets and liabilities.) If a (non-interest-payment) budget surplus can match the (non-interest-payment) external surplus, then this source of money creation can be offset. Alternatively, an open market operation might reduce the growth of the money supply brought by an external surplus.

How did Brazil manage its external surplus domestically? Figure 2 plots the non-interest-payment current account surplus, seignorage due to money creation (in real terms), and the domestic debt increase. Except for the last quarter of 1983, domestic debt has been steadily increasing, following a pattern which is very similar to the current account surplus. As a result the real value of domestic debt increased by 84 percent between December 1982 and December 1985 (see figure 3). Not surprisingly, the soaring domestic debt put substantial pressure on domestic interest rates, as shown in figure 4.

Figure 2. Brazil: Noninterest Current Account Counterparts

Key: __ change in domestic debt ___ noninterest current account ___ seignorage

Note: “Seigniorage” is defined as the variation of central bank liabilities in real terms.

Sources: International Monetary Fund, *International Financial Statistics*, various years; Getulio Vargas Foundation; World Bank estimates; Central Bank of Brazil, *Brazil—Economic Program: Internal and External Adjustment*, various issues.
Figure 3. Brazil: Stock of Quarterly Net Domestic Debt

Source: Central Bank of Brazil, Brazil—Economic Program: Internal and External Adjustment, various issues.

Allowing for private capital transactions with foreigners, the contribution of each of these factors to the current account surplus is as follows:

\[ TB(t) = S(t) + Z(t) + B(t) - (1 + p_t) B(t - 1) \]

\( TB(t) \) is the non-interest-payment current account surplus, \( S(t) \) is the seignorage tax, and \( B(t) - (1 + p_t) B(t - 1) \) is the amount of net new resources channeled to the government through the issuance of domestic debt. \( Z(t) \) is calculated as a residual. It measures the non-interest-payment government surplus \( (Z_0) \) plus the net external debt decrease of the private sector inclusive of capital flight. \( Z \) is likely to overstate the government surplus. \( TB, S \) and \( Z \) are shown in figure 5A; figure 5B plots \( TB \) and \( S + Z \). If \( Z \) is taken as a proxy of the government non-interest-payment surplus, \( S + Z \) measures its net income excluding interest payments but including seignorage from money creation. As figure 5B shows, the gap between \( TB \) and \( Z + S \) has been substantial. To derive a quantitative measure of this discrepancy, we proceed as follows. We write (1') as:

\[
\frac{\sum_{i=1}^{T} \frac{TB(t)}{\prod_{i=1}^{i} (1 + \rho_i)}}{\sum_{i=1}^{T} \frac{S(t)}{\prod_{i=1}^{i} (1 + \rho_i)}} + \frac{\sum_{i=1}^{T} \frac{Z(t)}{\prod_{i=1}^{i} (1 + \rho_i)}}{\sum_{i=1}^{T} \frac{1}{\prod_{i=1}^{i} (1 + \rho_i)}}
\]

(2)
Figure 4. Brazil: Ex Ante and Ex Post Real Interest Rates

Key: ——— ex ante interest rate ——— ex post interest rate

Note: "Ex ante" rates refer to the indexation scheme ex ante premised to the buyer of government bonds. "Ex post" rates are actual rates of return.

Sources: World Bank estimates; Getulio Vargas Foundation.

\[ + B(0) - \frac{B(T)}{\prod_{i=1}^{t} (1 + \rho_i)} \]

(2) is simply \(1'\) written in discounted terms (from a time zero point of view) and summed over time.

On the basis of this decomposition, the seignorage tax amounted to 57.5 percent of the current account surplus (excluding interest payments), the domestic debt increase amounted to 71.0 percent, and the variable Z to minus 28.6 percent. Taking \(Z + S\) as a rough proxy of the government's net income, one sees that only 29 percent of the external surplus was really paid for by the government and that 71 percent was financed by domestic debt. In other words, one sees that the secondary burden of raising taxes in order to service the government's external debt has yet to be paid.
Figure 5. Brazil: External and Domestic Surpluses

Key: --- noninterest government surplus (z) --- noninterest current account --- seigniorage

Note: "Seigniorage" is defined as the variation of central bank liabilities in real terms.

Sources: World Bank estimates; International Monetary Fund, International Financial Statistics, various years; Getulio Vargas Foundation; Central Bank of Brazil, Brazil—Economic Program: Internal and External Adjustment, various issues.
This decomposition also reveals the importance of the seignorage tax in total government revenue. In the balance of evils, inflation must be carefully weighed against rising domestic debt and rising real interest rates. Here, any use of the monetary approach to the balance of payments might be very harmful if it were to imply that monetary restraint is the appropriate tool for managing the external debt. (This argument is clearly an open economy version of the Sargent and Wallace [1981] "unpleasant monetarist arithmetic.")

One argument, however, against the seignorage tax is that it is becoming less and less efficient. As is well known, there is a maximum inflation rate above which increased money creation brings in less real income. Casual evidence seems to indicate that Brazil might have come near that point. The inflation rate doubled after the 1982 crisis and yet seignorage tax receipts rose by only 17 percent (see figure 6). Econometric evidence with regard to the elasticity of money demand vis-à-vis inflation seems to confirm this. By the beginning of 1986, more inflation did not necessarily mean more income to the government (see appendix 4, where it is also shown that the seignorage tax displays a low

Figure 6. *Brazil: Annualized Inflation and Seigniorage*

![Brazil: Annualized Inflation and Seigniorage](image)

*Key:* — seigniorage, - inflation

*Note:* "Inflation" is defined as the December to December change in the consumer price index.

covariance with the total government deficit). If this view is warranted, it would support the analysis by Kharas (1984), according to which a country's solvency must be judged both from its ability to generate a trade surplus and from the ability of its government to raise taxes. Roughly speaking, it is the minimum of the two bounds which should constrain the external debt. If the seignorage tax indeed reached an upper limit in 1986, it would make the repayment of external debt at the previous speed a dangerous exercise, one which might end by domestic default.

On the basis of this analysis, one might want to conclude that although Brazil's external adjustment has been successful, its internal adjustment has not. Yet, based on the analysis in section I, one might also want to argue that the external adjustment was too successful. In order to illustrate this point, I calculate what would have been the level of the domestic debt by the end of 1985 if the trade surplus has been held to the level implied by the analysis in section I, 15 percent of exports. The answer is that the domestic debt would have risen by 22 percent (in real terms) above its December 1982 level. Econometric estimates (shown in appendix 4) indicate this might have reduced 1985 real interest rates by 5 percentage points.

To conclude, we see that the Brazilian adjustment has not been completed. From the government's viewpoint, domestic debt has replaced external debt. Yet the former is dearer than the latter. In these circumstances, pushing the external debt-to-exports ratio down further would seriously damage the Brazilian economy and need not be in the creditors' self-interest. What would be preferable for Brazil is a relaxation of the scheme for servicing its external debt and a tougher fiscal stance.

V. Conclusion

I have argued in this article that a country which was believed to have reached its credit ceiling should not be asked to bring down its debt-to-resources ratio but instead should be asked to stabilize it. I have indicated that a trade surplus equal to roughly 15 percent of its exports should achieve that purpose. This analysis was shown to be valid even for cases in which the current market value of the debt does not coincide with its face value. Furthermore, I have indicated why the same ceiling to the debt-to-exports ratio might be applied both in the 1970s and in the 1980s. Because interest rates were lower than growth rates in the 1970s (and as it is the difference between the two rates that matters), it must

1. Obtaining a trade surplus is a slow and painful process, and it would certainly not be wise to try to reduce it blindly. However, all that is needed to take the pressure off the domestic debt is a reduction in the speed of repayment of the external debt. This would allow the government to reduce its domestic debt. Indirectly, it would exchange its external debt against its domestic debt, selling dollars to buy cruzados and selling cruzados to rebuy its domestic debt (leaving the money supply unchanged).
have been the case that the ceiling imposed during the 1970s was aimed at absorbing some shock such as that which occurred during the 1980s.

Conversely, I have indicated that the ability of a country to stabilize its debt should apply both to its external and to its domestic debt. Yet in the case of Brazil, domestic debt rose as a result of foreign debt payments. If continued, this process may seriously damage the working of Brazil's financial system. Under these circumstances, a slowdown of external debt payments coupled with a tougher fiscal policy stance might yield a superior outcome for both the debtor and the creditors.

**APPENDIX 1**

Assume that the resources of the country can be unambiguously measured by a good \( \Omega \), with which the country is exogenously endowed according to a law of motion (a dot superscript means a time derivative):

\[
\dot{\Omega}_t = n_t \Omega_t
\]

The country has access to a world market on which this good is traded. A world financial market, whose rates of interest are \( r_t, t \geq 0 \), allows the country to spread its consumption intertemporally. Assume that the country's representative consumer (or its social planner) has the objective:

\[
\text{maximize } \int_0^\infty e^{-\delta t} \log C_t \, dt
\]

in which \( C_t \) is consumption (of the numeraire) at time \( t \) and \( \delta \) is the rate of time preference.

Following the work by Eaton and Gersovitz (1981) and my own research with J. Sachs, assume that a country may repudiate its debt if the cost of repaying it becomes "excessive." The threshold with which this "excessive" burden is compared is an "autarkic" utility level which measures the capital to which the country has access after it has defaulted on its debt. (This is autarky with regard to trade in capital but not trade in goods.) The autarky utility threshold is defined as follows. Take a country which defaults at time \( t^* \). Assume that its resources are reduced by a factor \( \lambda_{s-t} \) at time \( s \), and that it is forced to financial autarky after time \( t^* \). In other words, a country defaulting at time \( t^* \) only receives the endowments \( [1 - \lambda_{s-t}] \Omega_s \) at time \( s \) and cannot spread out its pattern of consumption. A defaulting country, therefore, necessarily consumes:

\[
C_{t^*+s} = (1 - \lambda_s) \Omega_{s+t^*}, s \geq 0
\]

The variable \( \lambda_s, s \geq 0 \), measures the cost of debt repudiation, which need not be a constant—for instance, we could assume that it is a decreasing function of time. Lenders will then limit their exposure to the country so that repayment never becomes so heavy as to make debt repudiation a superior alternative. Associated with this equilibrium lending strategy, the borrower can reach a
utility level, which we can write \( U(D, \Omega, F) \) in which \( D_t \) = external borrowing and \( F_t = \{r_t, n_t\} \), \( s \geq t \) measures the prospects of a future strategy such that the country never finds it profitable to repudiate its debt. Call \( U_a(\Omega, F) \) the autarky utility level.

Lenders will seek to lend so that the country will never find that

\[
(A1-4) \quad U(D, \Omega, F) < U_a(\Omega, F)
\]

that is, that it will never prefer autarky to repaying the debt. To design this strategy, we first show:

**Proposition:** On any interval of time during which the constraint \( A1-4 \) is binding, the country repays its creditors a fixed fraction of its resources.

The proof is obtained by time differentiating the equation \( U = U_a \). Variable \( b \) is shown to be a scalar which solves:

\[
\log (1 - b) = \int_0^\infty \log (1 - \lambda_s) e^{-b \lambda_s} ds
\]

In the simple case where \( \lambda_t \) is a constant \( \lambda \), \( b \) equals \( \lambda \). In that case, the interpretation of the above proposition is straightforward: lenders restrict their lending so that the country reaches a trade surplus which drives the country’s consumption to its default level. What the proposition tells is that there exists a fixed scalar \( b \) such that the country must obtain a trade surplus equal to \( b\Omega \), whenever \( A1-4 \) is binding.

From this proposition, one may define the supply of credit. To simplify, assume that the constraint on the debt, once it binds, keeps binding. Maximum lending is defined by the inequality:

\[
(A1-5) \quad D_t \leq b \int_0^\infty (\exp - \int_0^t r_u du) \Omega_s ds
\]

Call \( h^*_t \) the maximum debt-to-resources ratio. From this inequality, we can write:

\[
(A1-5) \quad h^*_t = b \left[ \int_0^\infty \exp - \int_0^t (r_u - n_u) du \right] ds
\]

The term in parentheses may be interpreted as an “average” difference between interest and growth rates. Call this average difference \( \theta_t \), where

\[
(A1-6) \quad \theta_t = \int_0^\infty \exp [- \int_0^t (r_u - n_u) du] ds
\]

One can show that the maximum debt-to-resources ratio \( h^*_t \) follows a law of motion:

\[
(A1-7) \quad \frac{\dot{h}^*_t}{h^*_t} = (r_t - n_t) - \theta_t^*
\]

Equation \( A1-7 \) shows that the maximum value of the debt-to-resources ratio may increase, decrease, or stay constant depending upon whether the difference between interest and growth rate is above, below, or equal to its average value. In a stationary environment, the debt-to-resources ratio is constant when the
constraint starts binding. Otherwise, when \( r_t - n \) oscillates, the maximum debt-to-resources ratio may be allowed to oscillate countercyclically.

APPENDIX 2

In appendix 1 resources of the country are measured by a numeraire, \( \Omega_t \). Assume now that \( \Omega_t \) is not observable, but exports and GDP are. Assume that the country produces two goods: a home good (1) and an export good (2). Whereas residents consume only the home good, imports are required to produce it, so exports are required.

Let \( Q_1 = M^{1-\alpha} \Omega_t \), \( Q_2 = \Omega_t \), and \( \Omega_t + \Omega_t \leq \Omega \).

Variable \( \Omega \) is the endowment of the country, and \( M \) is the imports which enter into the production of good 1. Imports are the numeraire. Let \( P_2 \) be the terms of trade so that: \( M = P_2 Q_2 - P \), where \( P \) is the trade balance. Assume that the cost of default is a penalty \( \lambda_c \) imposed on exports. A country choosing to default would therefore import \( M_{t+1} = P_2 (1 - \lambda_c) Q_2_{t+1} \), in exchange for its exports \( Q_2_{t+1} \), the cost of default being the worsening of the terms of trade. Lenders will lend so that \( U(D, \Omega, F) \geq U_0(\Omega, F) \), and as before we can check that \( U(zD, z\Omega, F) \geq U_0(z\Omega, F) \) holds for all \( z \) whenever \( U(D, \Omega, F) \geq U_0(\Omega, F) \) holds.

The optimal borrowing strategy should therefore be set as previously:

\[
(A2-1) \quad \frac{D_t}{\Omega_t} \leq b_i
\]

As \( \Omega_t \) is not observable, which of GDP and exports, \( X \), should proxy the resources of the country in equation (A2-1)? Assume that the pricing mechanism is efficient. Then:

\[
P_1 = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} P_2 \quad \text{and} \quad GDP = P_1 Q_1 + P_2 Q_2 - M = P_2 \Omega
\]

\[
X = P_2 (1-\alpha) \Omega + \alpha P
\]

We see that GDP provides a direct measure of \( \Omega \) and \( X \) an indirect one.

An invariant measure of resources is now devised. Assume that the country adopts a rationing or a subsidy scheme on imports or on exports, respectively, which we shall measure by the shadow price, \( \gamma \), of imports. When \( \gamma = 1 \), imports are priced at their world level, and we are back to the equilibrium pricing case examined above. When \( \gamma > 1 \), imports are rationed and their shadow price (the black market premium) is above world price. When \( \gamma < 1 \), however, exports are subsidized above their world level. Given this measurement of price distortion, GDP and exports become
\[
\text{GDP} = \frac{1}{1+\alpha(\gamma-1)} [\gamma P_2 \Omega - (1-\alpha) (\gamma-1) P] \\
X = \frac{1}{1+\alpha(\gamma-1)} [(1-\alpha)P_2 \Omega + \Omega \gamma P]
\]

GDP increases and exports decrease with \(\gamma\). Now consider the following alternative measure of a country’s wealth, \(W_t\).

\[
W_t = \frac{\alpha}{1+\alpha} \text{GDP}_t + \frac{1}{1+\alpha} X_t
\]

\(W_t\) is designed so as to be independent of \(\gamma\). Because of our various “Cobb-Douglas linearities,” the definition of \(W_t\) is simple. Curvature of the production possibility frontiers would change the coefficient \(\alpha\), however. For instance, assume that the resource constraint is written as \(\Omega_1 + \phi (\Omega_2/\Omega_1) \Omega_2 \leq \Omega\), in which \(\phi\) is a decreasing function which measures the cost of shifting resources from sector 1 to sector 2. Now the coefficient \(\alpha\) should be changed to \(\alpha^1 = \alpha/\rho\), in which \(\rho \geq 1\) is a measure of the curvature of the production possibility frontier. In the extreme case when \(\rho = \infty\), \((\phi = 1\ if\ \Omega_2/\Omega_1 = \infty,\ 0\ otherwise)\) exports become the sole measure of wealth. Otherwise, the more difficult it is to shift resources, the more weight will be given to exports in the measurement of wealth.

In a numerical application of the above to Brazil, for an explicit estimate of the invariant measure of wealth we need to find a scalar \(\kappa\) such that \(W_t = \kappa \text{GDP}_t + (1-\kappa) X_t\) fails to depend upon the real exchange rate. For Brazil, exports are shown to depend upon the real exchange rate as follows:

\[
\log X(t) = 5.75 + 0.08 \text{ time} + 0.88 \log z(t-1) \\
(4.6) \quad (11.9) \quad (2.6)
\]

\(R^2 = 0.97\quad DW = 1.3\quad (t\text{-statistics in parentheses})\)

with \(z(t-1)\) the lagged value of the real exchange rate and \(X(t)\) the constant dollar value of exports. Exports respond significantly to real depreciation, the volume elasticity being 0.88. The constant dollar value of \(\text{GDP}_t\) depends upon the real exchange rate:

\[
\log \text{GDP}_t = 14.6 + 0.06 \text{ time} - 0.78 \log z(t-1) \\
(11.5) \quad (9.9) \quad (-2.3)
\]

\(R^2 = 0.93\quad DW = 0.97\)

The real exchange rate and the dollar value of \(\text{GDP}\) are negatively related. We then look for a value \(\kappa\) which makes \(\log W_t = \log [\kappa \text{GDP}_t + (1-\kappa) X(t)]\) independent of \(z(t-1)\). The sign of the relationship between \(\log W_t\) and \(z(t-1)\) changes for values of \(\kappa\) between 0.095 and 0.1. Therefore, for Brazil, \(W(t) = 0.1 \text{ GDP}_t + 0.9 X(t)\).
A Two-State Uncertainty Model

Assume there are two possible states of nature \( i (i = 1, 2) \), in which the world real interest rate is \( r_i \) and the rate of growth of the country’s economy is \( n_i \).

As in appendix 1, assume that the resource can be unambiguously identified with GDP. In state 1, the economy faces a probability \( p_1 \) of staying there and \( p_2 = 1 - p_1 \) of moving to state 2. In state 2, the probability of moving to state 1 is \( q_1 \), that of remaining in state 2 is \( q_2 = 1 - q_1 \). Contrary to the structure in appendix 1, I now assume a discrete time economy. The subjective discount factor of the social planner is \( \beta = 1/1+\delta \) and \( \delta \) is the rate of time preference. The objective is to maximize the expected value of \( J = E_0 \sum \beta^t \log C_t \). The social planner’s welfare function can be written as \( J_1(D, Q) \) in which \( Q = GDP \) and \( D \) is the debt accumulated by the country at the beginning of the period.

When the economy is in state \( i \), GDP during that period is \( Q(1 + n_i) \) and the debt at the end of the period is \( D_i = (1 + r_i)D + C - Q(1 + n_i) \), where \( C \) is the consumption during that period. I assume that the decision to consume is made after the country has learned which state of nature it is in. \( J_1 \) and \( J_2 \) are the solutions to the following system:

\[
\begin{align*}
J_1(D, Q) &= \max_C \{ \log C + \beta P_1 J_1 [(1 + r_1)D + C - Q(1 + n_1), Q(1 + n_1)] \\
&\quad + \beta (1 - P_1) J_2 [(1 + r_1)D + C - Q(1 + n_1)] \\
J_2(D, Q) &= \max_C \{ \log C + \rho \beta q_1 J_1 [(1 + r_2)D + C - Q(1 + r_2), Q(1 + n_2)] \\
&\quad + \beta q_2 J_2 [(1 + r_2)D + C - Q(1 + n_2), Q(1 + n_2)] \}
\end{align*}
\]

To prevent default, lenders must ensure that \( J_1(D, Q) \geq J_{d,1} \) and \( J_2(D, Q) \geq J_{d,2} \) when \( J_{d,i} \) is the expected postdefault utility in state \( i \). Now, given the assumption that

\[
(A3-2) \quad r_1 \leq n_1 \text{ and } r_2 \geq n_2
\]

it is clear that \( J_2(D, Q) \geq J_{d,2} \) is the only inequality to watch. Following the same argument as in appendix 1, this inequality yields a credit ceiling for both states:

\[
(A3-3) \quad D/Q \leq h^* \]

As in appendix 1, the country’s decision to borrow will depend on \( [\delta, r_1, r_2, n_1, n_2] \). Assume \( \delta \geq r_2 - n_2 \); even in state 2 the country would still like to borrow. Under this hypothesis, the country will borrow until the credit ceiling \( (A3-3) \) binds, but even then the country will borrow if it is in state 1. In effect assume that the country’s initial debt and endowment satisfy \( D = h^*Q \). If it is in state 1, it can still borrow net new debt (over and above all debt service payments): \( C - Q_1 = (n_1 - r_1)D \). Its end-of-period debt is then \( D_1 = (1 + r_1)D + (n_1 - r_1)D = (1 + n_1)D \) so that its end-of-period debt-to-GDP ratio \( D_1/Q_1 \) is
indeed unchanged. However, if it is in state 2, it must pay \( Q_2 - C = (r_2 - n_2)D \) so as to keep its debt-to-GDP ratio unchanged.

A Three-State Uncertainty Model

Assume now that a third state of nature might arise once the economy has moved to state 2. As before, \( P_1 \) and \( P_2 = 1 - P_1 \) are the probabilities of staying in state 1 and passing to state 2, respectively. But now let \( (q_1, q_2, q_3 = 1 - q_1 - q_2) \) be the probabilities of passing from state 2 to state 1, remaining in state 2 or passing into state 3, respectively. Assume that state 3 is a state that one cannot leave. It is characterized by \( (r_3, n_3) \) such that \( r_3 - n_3 \) is very large. Let \( J_3 (D, Q) \) be the welfare that is obtained by the country when it is (forever) in state 3, assuming that it services its debt. The condition for obtaining \( J_3 (D, Q) \geq J_{3,a} \) yields a very low debt ceiling \( D \leq h_3 Q \) (with \( h_3 \) much less than \( h_2 \)). If the lenders want to take no risk of debt repudiation at all, it is this ceiling which they should impose. Now assume that state 3 is a state of nature which is idiosyncratic to the debtor country; for instance, assume that \( r_3 = r_2 \) but \( n_3 \) is much less than \( n_2 \). Contrary to states 1 and 2, which were worldwide phenomena, the risk of state 3 can now be diversified away by the lenders. In other words, we may assume that they will act as risk-neutral lenders with respect to all stochastic shocks related to the occurrence of state 3.

Let us assume that the country starts borrowing in state 1. A continuum of loans can be offered to the debtor. At one extreme, consider the following: all loans offered in state 1 are priced at the riskless rate. Yet as the economy moves to state 2, the lenders offer a risk-adjusted interest rate whenever \( D \geq h_2 Q \) so as to reflect the probability of going to state 3 and having the debt defaulted upon. In this case, as soon as \( D \geq h_2 Q \), the law of motion of the country's welfare can be written as:

\[
\begin{align*}
(A3-4) & \quad J_1, \text{ as in equation } A3-1 \\
(A3-5) & \quad J_2, \text{ which solves:}
\end{align*}
\]

\[
J_2 (D, Q) = \max \{ \log C + \beta q_1 J_1 [\left( \frac{1 + r_2}{q_1 + q_2} \right) D + C - Q(1 + n_2), Q(1 + n_2)] \\
+ \beta q_2 J_2 [\left( \frac{1 + r_2}{q_1 + q_2} \right) D + C - Q(1 + n_2), Q(1 + n_2)] \\
+ \beta q_3 J_3 [Q(1 + n_2)]
\}
\]

In order to prevent default in state 2, lenders impose \( J_1 (D, Q) \geq J_{2a} (Q) \) which is equivalent to a new credit ceiling \( D \leq h_2 Q \), the same credit ceiling that applied in both states 1 and 2.

Now there is another lending strategy available. The lenders charge a risk premium \( \epsilon \) in both state 1 and state 2. But this risk premium now reflects the probability of default from a state 1 point of view, that is \( \epsilon = (1 - p_1) q_3 \). If \( q_3 = 20 \) percent and \( (1 - p_1) = 5 \) percent, then \( \epsilon \) is 1 percent. Under this new lending
strategy, the borrower pays a spread in state 1 as well as in state 2, but the new credit ceiling is larger since, in state 2, the debt is rolled over at the rate $1+r_2/(1-p_1)(1-q)$. The lender is indifferent between the two schemes, but the borrower will certainly take the latter (if $1-p_1$ is low enough). As indicated in the text, in state 2 the debt will be traded at a discount and the lenders will take a capital loss on the debt. Yet the same simple maxim for servicing the debt should apply: keep the debt-to-resources ratio at its ceiling.

Appendix 4

Theory

Again assume two possible states for the world economy, but assume now that the government is distinct from the private sector and that only the government has access to the rest of the world’s financial market.

Assume for simplicity that the government’s only action is to transfer payments to or collect taxes from the private sector. Call $B^*$ the foreign debt. The law of motion of external debt is given by:

$$
(A4-1) \quad B^*_i(t) = (1 + r_i) B^*(t - 1) + C - Q (1 + n_i) \quad i = 1, 2
$$

Depending upon which state of nature prevails, domestic debt $B(t)$ follows the law of motion:

$$
(A4-2) \quad B(t) = (1 + \rho_i) B(t - 1) + Q (1 + n_i) - C - T \quad i = 1, 2
$$

where $T$ is the budget surplus and $\rho_i$ is the domestic rate of interest in each of the two states.

It can be seen by checking that equations A4-1 plus A4-2 yield the result that the government deficit ($-T$) is financed by domestic and foreign debt.

Empirical Estimates of Money Demand and Real Interest in Brazil

Decreasing returns to money creation. Two estimates of the following money demand equation have been made where $M$ is money demand and $\pi(t)$ is the inflation rate:

$$
\log \frac{M(t)}{GDP(t)} = a - \mu \pi(t).
$$

Estimation on a yearly basis for 1971–85 yields $a = -2.39 (t = -46.6)$, $\mu = 0.765 (t = 10.1)$ ($R^2 = 0.886, DW = 1.4$). Thus the maximum seignorage tax is obtained for a monthly inflation rate $\pi^* = 10.9$ percent (in logarithm; 11.5 percent in rate of growth).

Estimating the same equation on a quarterly basis over the subsample 1983–85 yields $\mu = 3.6 (t = 9.3)$ and implies a maximum monthly inflation rate $\pi^*$ of 9.2 percent, very much in line with the previous result. In 1985, the inflation rate has reached 10.0 percent per month on average, leaving little doubt that Brazil has nearly exhausted its capability to expand its seignorage tax receipts.
Other evidence of this can be obtained by analyzing the covariance of the government's financial need, \( U \), with government debt increase and seignorage tax \( s \). We write \( U = TB - Z \) and \( U = S + VB \), in which \( TB \) is the non-interest payment current account, \( Z \) is non-interest government surplus, and \( VB = B_t - (1 + \rho_t) B_{t-1} \) measures the net new resources from bond holders. It must be matched either by money creation or an increase in domestic debt. We write this relation as follows:

\[
1 = \frac{\text{cov}(S, U)}{\text{var} U} + \frac{\text{cov}(VB, U)}{\text{var} U}
\]

We find that the two terms on the right-hand side are 10 percent and 90 percent, respectively. This supports other evidence showing the low responsiveness of the seignorage tax to government needs.

**Domestic interest rate.** The influence of mounting domestic debt upon real interest rates has been estimated on a quarterly basis from 1983 to 1985 in the form \( r_t = \alpha + \beta \log B(t) \), in which \( B(t) \) is the real value of domestic debt. The results are \( \alpha = -0.707 \) \( (t = -2.04) \), \( \beta = 0.118 \) \( (t = 1.98) \) \( (R^2 = 0.28, DW = 1.15) \). Even though it is not a very satisfactory relationship, it does support the responsiveness of interest rates to domestic debt. Furthermore, the estimate of \( \beta \) seems consistent with that which results from a larger-sample analysis. [On a yearly basis, 1971–85, we find \( \beta = 0.178 \) \( (t = 1.17) \); this is a hint (only) that the past three years were not abnormal.]

Finally, consider the exercise presented in the text. If the repayment of external debt had followed the scheme described at the beginning of the article (pay 15 percent of Brazil's exports to its creditors), domestic debt would have increased by 22 percent instead of the 84 percent rise which has been observed. If one trusts the first relationship obtained above, our proposed repayment scheme would have reduced real interest rates by 5 percentage points. This would have amounted to a reduction of 1.8 percent of GDP for the government deficit (in 1985 the overall government deficit, after monetary correction, was 3 percent of GDP).

**References**


