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Farm Size and the Diffusion of Green Revolution Technology

On Information and Innovation Diffusion: A Bayesian Approach

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Farm Size and the Diffusion of Green Revolution Technology*

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I. Introduction

The introduction of high-yield-crop varieties (HYV) is one of the more significant technological changes taking place in less developed countries' agricultural sectors. This so-called Green Revolution technology consists of hybrid seeds, chemical inputs (fertilizers, pesticides), and special cultivation practices. The yield potential and the income-generating capacity of the modern variety are significantly superior to those of the traditional crops, when properly cultivated. Furthermore, the technology is divisible and neutral to scale. Thus, one may expect that adoption patterns will not be affected by farm size. But the facts are, as the record of diffusion experience in a substantial number of regions throughout the world shows, that adoption rates and the time pattern of adoption are related to farm size.

One obvious reason for differential adoption rates in many regions is the credit constraint. Working capital requirements associated with the new technology are substantially higher (fertilizers, pesticides, and hybrid seeds are cash inputs). Thus, where credit for smaller farmers is severely limited, they may not be able to adopt HYV at the same rate as larger farmers.[1]

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[1] For instance, in a study of Indian agriculture, Bhalla (1979) reported that small and large farmers differed in the reasons offered for not using fertilizer in 1970–71. Lack of credit was a major constraint for 48% of small farmers and only 6% of large farmers. Bhalla concludes that "access to credit may be responsible for the gain in income (and HYV area) made by the large farmers." Similarly, in a study of HYV adoption by Pakistani farmers, Lowdermilk (1972) found that a majority of small farmers reported shortage of funds as a major constraint on fertilizer use. Khan (1975) found that small farmers in Pakistan faced difficulties in obtaining fertilizer and seeds because they had no way to pay the premium prices during the sowing season for these inputs. Both Rochin

A number of other obstacles discriminating against smaller farmers are mentioned in the literature,[2] but the most frequently cited one is the interaction between the uncertainty associated with the innovation and the prevalence of risk aversion among farmers. Unlike the credit constraint, which is exogenous to the farmers, risk aversion is an endogenous factor and, thus, the implications of risk aversion in terms of farmers' decisions may change if farmers' perceptions change. Since the new technology is perceived as (and sometimes it indeed is) more risky, and provided that smaller farmers are more risk averse, it is argued that smaller farmers will be less inclined to adopt the innovation. However, when more rigorous analysis is applied to the relation between risk aversion and farm size, the argument is not as straightforward as it seems at first glance. Assuming that (i) no fixed adoption costs are required; (ii) exogenous constraints (such as credit) are not effective; and (iii) that farmers are aware of the superiority, on the average, of the HYV, one can show that some adoption is beneficial for all risk-averse farmers, irrespective of the degree of uncertainty or of the size of the farm. Furthermore, it is quite possible theoretically that even though smaller farmers are less willing to undertake risks in terms of the absolute size of income involved, they may demonstrate a higher rate of adoption (in terms of the relative share of land they allocate to HYV) compared to larger farmers.[3] Thus, when the assumptions above are valid, the risk-aversion argument cannot explain

(1973) and Bose (1974) found that small farmers in Bangladesh were at a disadvantage in obtaining scarce credit. Similarly, Frankel (1971), Rajpurohit (1972), and Wills (1972) found evidence of difficulty of access to credit by small farmers in India, with adverse consequences on their choice of new technology. See S. S. Bhalla, "Farm Size, Productivity, and Technical Change in Indian Agriculture," in *Agrarian Structure and Productivity in Developing Countries*, ed. R. Berry and W. Cline (Baltimore: Johns Hopkins University Press, 1979); Max Lowdermilk, "Diffusion of Dwarf Wheat Production Technology in Pakistan's Punjab" (Ph.D. diss., Cornell University, 1972); M. H. Khan, *The Economics of the Green Revolution in Pakistan* (New York: Praeger Publishers, 1975); S. W. Bose, "The Comilla Co-operative Approach and the Prospects for Broad-based Green Revolution in Bangladesh," *World Development* 2, no. 8 (August 1974): 21–28; R. I. Rochin, "A Study of Bangladesh Farmers' Experiences with IR-20 Rice Variety and Complementary Production Inputs," *Bangladesh Economic Review* 1, no. 1 (January 1973): 71–94; F. R. Frankel, *India's Green Revolution—Economic Gains and Political Costs* (Princeton, N.J.: Princeton University Press, 1971); A. R. Rajpurohit, "Study of the HYV Programme in Mysore (1967–68): Mexican Wheat in Bijapur District," mimeographed series no. 17 (Poona: Gokhale Institute, 1972); and I. R. Wills, "Projection of Effects on Modern Inputs on Agricultural Income and Employment in a C.D. Block, U.P., India," *American Journal of Agricultural Economics* 54, no. 3 (1972): 452–60.

[2] See Wayne Schutjer and Harlin G. Van Der Veen, *Economic Constraints on Agricultural Technology Adoption in Developing Nations*, USAID, Occasional Paper no. 5 (Washington, D.C.: USAID 1977); and Michael Lipton, "Inter-Farm, Inter-regional and Farm–Non-Farm Income Distribution: The Impact of the New Cereal Varieties," *World Development* 6 (March 1978): 319–37.

[3] These results depend on the properties of the underlying utility function with respect to the notions of absolute and relative risk aversion. See Gershon Feder, "Farm Size, Risk Aversion and the Adoption of New Technology under Uncertainty," *Oxford Economic Papers* 32, no. 2 (1980): 263–83.

the prevalence of nonadoption, at least in the early years, by smaller farmers; nor does it provide an explanation for the higher share of land planted to HYV observed on larger farms in many regions.

Should it thus be concluded that small farmers' higher risk aversion per se is not a valid explanation (or a complementary explanatory factor) of observed adoption pattern? "Not so," is the conclusion propounded in the present paper. The nature of new innovations, and the limitations of the rural environment are such that fixed adoption costs do exist, in which case higher risk aversion among smaller farmers is a factor which can explain, by itself, the differential, farm-size dependent pattern of technology adoption observed, from both static and dynamic perspectives.

In order to isolate the role of risk and risk aversion in generating differential behavior by farm size, all other factors which may cause differentiation will be held identical: All farmers are assumed to possess identical utility functions and to share the same perceived notions of risks involved with the new technology. Similarly, all farmers face the same prices (for both outputs and inputs), have the same quality of land, and none are affected by exogenous constraints such as credit or labor constraints, etc. Thus, the only thing differing among farmers is, in the framework of the present paper, the size of land holding. These assumptions are just analytical devices which will allow a proper assessment of risk aversion and its role as a factor inhibiting diffusion of innovations among smaller farmers. Furthermore, the dynamic pattern of adoption as implied by the present analysis can be compared with the diffusion process as observed in reality, so as to reveal the extent to which the latter can (or cannot) be explained by risk aversion and land distribution alone.

The program of the paper is as follows: The next section presents the theoretical decision model and its implications for optimal farmers' behaviors. These are followed by a specific example which serves as a basis for numerical situations tracing the pattern of adoption in a hypothetical rural region over time. The simulations provide insights regarding the individual and the aggregate adoption performance and demonstrate the income distribution effects of the innovation at any point in time and over time.

II. The Model and Its Implications

It is assumed that two distinct technologies are available to farmers: The traditional technology requires no specialized inputs and yields a given net return per acre with certainty. The new technology is characterized by superior average performance per acre if certain specialized inputs (e.g., fertilizers, improved seeds) are applied, and appropriate cultivation practices (e.g., proper timing of planting and weeding) are observed. However, the yield from an acre allocated to the modern

crop[4] is associated with some degree of uncertainty. This uncertainty stems from both objective and subjective factors: In the first instance, the modern, high-yielding variety may be more susceptible to diseases, pest damage, and climatic variation. The dependence on timely availability of the specialized inputs introduces another element of objective uncertainty. Subjective uncertainty is almost inevitable with any innovation being introduced into a system where farmers have for many years known only the traditional technology. It is likely, however, that over time the subjective uncertainty will be reduced as information regarding the innovation spreads, information and marketing channels are established, and the experience gained by the early adopters affects the perceptions of other farmers.[5] Also, research aimed at reducing pest and weather susceptibility may eliminate some of the objective uncertainty.

While per acre input costs are scale neutral, one can conceive of several factors which may introduce a fixed-cost (independent of farm size) element, even though production itself is highly divisible and scale neutral.[6]

One such factor is the monetary and time cost involved in acquiring the essential technical information related to the cultivation of the new, high-yielding varieties. Part of the cost is an investment, and it is possible that the cost declines over time; but nonetheless this fixed cost, which is independent of the intended scale of operation, may be incurred at least in the initial states. A similar view, based on the results of a case study in El Salvador, is expressed by Cutie: ". . . There seem to be fixed technological-transitional costs associated with adoption."[7] In a study of small Mexican farmers, O'Mara notes that "the set-up cost nature of information acquisition should tend to create a greater responsiveness to technical change by larger farmers. This greater responsiveness for larger farmers seems to hold even for comparisons between quite small farmers."[8] Schutjer and Van Der Veen[9] emphasize the fact that HYV practices continue to change, thus implying further adjustments (and fixed information-acquisition costs) even after the first year of adoption.[10]

[4] The terms "new technology," "modern crop," "innovation," and "high-yielding variety—HYV" will be used interchangeably.

[5] For an empirical study of the variation with experience in the subjective uncertainty of small farmers with respect to an HYV innovation, see Gerald T. O'Mara, "The Microeconomics of Technique Adoption by Smallholding Mexican Farmers," in *Programming Studies for Mexican Agricultural Policy*, ed. R. D. Norton and L. Solis (Washington, D.C.: World Bank, in press).

[6] Schutjer and Van Der Veen, p. 4.

[7] Jesus Cutie. *Diffusion of Hybrid Corn Technology: The Case of El Salvador* (Mexico D.F.: International Center for Improvement of Rice and Wheat, 1976). p. 9.

[8] O'Mara, p. 24.

[9] Schutjer and Van Der Veen, p. 6.

[10] Other fixed costs may be related to the purchase of specialized tools (fertilizer applicators and pesticide sprayers). While these are investments, they may be viewed

As it is very likely that working capital will be borrowed if adoption takes place (since the cash requirements of the new technology are considerably higher than those of traditional crops), fixed transaction costs are incurred in relation to loan application and the time required to handle the request within the official system which is characterized by "red tape." These costs are likely to be lower if the informal (moneylender) credit market is used, but then the cost of credit is much higher.

Another element of fixed costs may be incurred due to the time needed to obtain the specific inputs associated with the new technology (seeds, fertilizers, pesticides). These costs are independent of the amount required and are likely to be higher in the initial introduction period when distribution channels are not yet well organized.

In the following it is assumed that a fixed cost totaling C is required if the new technology is adopted. While part of C is, in fact, a one-time investment, it is assumed that farmers are myopic (at least at the introductory state of the new technology), so that they assign most of the investment cost to the first period. The dynamics of the adoption process will be affected, however, by the fact that once adopted, the fixed cost related to the new technology is reduced.

As the amount of land available to each farmer is fixed, maximization of expected utility (assuming that farmers are risk averse) is obtained by selecting optimal levels of the proportion of land allocated to the modern crop and optimal intensity of variable input per acre. The following results are implied by the model (the derivations are presented in the Appendix).

i) The optimal level of per acre variable input is independent of both farm size and the fixed cost, C. (ii) There exists a critical landholding size, say $\hat{L}$, such that farmers with holdings smaller than $\hat{L}$ will not adopt the modern variety, while larger farmers will plant at HYV on at least a portion of their land. (iii) On those farms larger than $\hat{L}$ the share of area allocated to HYV increases with farm size, if relative risk aversion is constant or decreasing with income, while absolute risk aversion is decreasing.[11] Even if relative risk aversion is increasing, the share of HYV may increase with farm size if the fixed cost is sufficiently high. (iv) For a given farm larger than $\hat{L}$, the share of HYV declines

as a fixed cost if the farmer adopts a myopic planning horizon. This may well be the case in view of the high uncertainty about the innovation in the first few years since introduction.

[11] Absolute risk aversion measures the insistence of a risk-averse individual for more-than-fair odds when faced with a bet whereby he can win or lose a given sum of money. Relative risk aversion measures the same insistence when the bet is such that a given proportion of wealth or income can be won or lost. It is generally accepted that absolute risk aversion declines as wealth increases. See Kenneth J. Arrow, "The Theory of Risk Aversion," in *Essays in the Theory of Risk Bearing* (Chicago: Markham Publishing Co., 1971).

when fixed costs are higher or when the degree of uncertainty regarding the new variety is higher. (v) The critical landholding size $\hat{L}$ becomes lower when the fixed costs C and/or the degree of uncertainty decline.

Results (iv) and (v) have important dynamic implications: Over time uncertainty declines (e.g., due to learning and dissemination of information), and the fixed costs may be reduced. Thus, farms which may have been initially too small to adopt HYV even partially will find it attractive, and farmers who have previously been partial adopters will allocate increasing proportions of their land to HYV. Eventually, when the degree of uncertainty and the fixed costs are sufficiently low, the farmer will switch into full adoption, that is, all of his land will be planted to the modern crop. Indeed, the evidence cited by Schutjer and Van Der Veen[12] confirms a pattern of initial experimentation which is expanded in subsequent periods until full adoption is reached.

The role of fixed adoption costs in the model is better understood if it is noted that, in the absence of such costs, nonadoption cannot be an optimal policy, given that the mean net return under the new technology is higher than the return from the traditional technology (see Appendix). However, even if these fixed costs remain high over time, the decline in uncertainty reduces the range of holding sizes which will opt for no adoption. It is quite possible, given the discrete pattern of crop seasons, that smaller farms find it advantageous to start adoption when uncertainty is sufficiently low to induce a very high initial level of adoption or even full adoption. Thus, while larger farmers (who are also early adopters) start with experimentation and gradually shift to full adoption, the smaller farmers can skip the experimentation stage. Obviously, this is not done without a cost, as the period of waiting entails the loss of the average higher income associated with the modern technology.

III. A Simulation of the Model

To demonstrate the implications of the preceding analysis, and in order to gain further insights on the dynamic aspects of innovation diffusion, an example is presented below.

The utility function is specified as a logarithmic transformation of income,[13] that is, $U = \ln \Pi$, where Π denotes income. This specification implies that absolute risk aversion is decreasing, while relative risk aversion is constant at unity. Per acre yield is assumed to take the

[12] Schutjer and Van Der Veen, p. 5.

[13] The approximation of utility by a logarithmic function dates back to Bernoulli (see D. Bernoulli, "Specimen theoriae novae de mensura sortis," *Commentarii academiae scientiarum imperiales Petro politanae* 5 [1738]: 175–92, translated by L. Sommer as "Exposition of a New Theory on the Measurement of Risk," *Econometrica* 12 [1954]: 23–36) and was more recently advocated by Arrow.

values $(1 + \sigma) \cdot y$ or $(1 - \sigma) \cdot y$ with equal probabilities, where y is average yield. It is easy to see that σ reflects the degree of uncertainty, as $\sigma^2 \cdot y^2$ is the variance of yield. Using the results presented in the Appendix (eq. [A2]), one obtains

$$l^* = \frac{[R - (C/L)] \cdot (y - m - R)}{\sigma^2 \cdot y^2 - (y - m - R)^2}, \tag{1}$$

where l^* is the share of land allocated to HYV, m is the level of variable input costs per acre, R is the per acre return from the traditional crop, L is the holding size, and C is the fixed cost. Equation (1) holds only for farmers with a holding beyond the critical size.

As is evident from (1), in the absence of fixed costs, the same share of land will be allocated to the modern crop, irrespective of farm size. However, with $C > 0$, larger farmers will have higher adoption rates. The signs of the derivatives $\partial l^*/\partial\sigma^2$ and $\partial l^*/\partial C$ can easily be verified to conform with our previous results.

Using the relevant results in the Appendix, one can calculate the critical landholding size $\hat{L}$:

$$\hat{L} = (C/R)/(1 - \phi), \tag{2}$$

where $\phi \equiv \{1 - [(y - m - R)/(\sigma \cdot y)]^2\}^{1/2} > 0$. It is noted that $\phi < 1$, thus the critical landholding size is larger than C/R. In view of this result, together with equation (1), it follows that at the switching point from nonadoption to adoption there is a discontinuity such that new adopters do not start with minuscule experimental plots. Simple derivation of (2) verifies that the signs of $\partial\hat{L}/\partial C$ and $\partial\hat{L}/\partial\sigma^2$ are positive, as predicted earlier.

For a given land distribution and specific numerical values of the parameters R, m, σ, y, and C it is easy to calculate the adoption rates at a point of time, using equations (1) and (2). In addition, dynamic elements may be introduced in both C and σ so that the process of adoption may be simulated over time. Accordingly, table 1 describes

TABLE 1

LAND DISTRIBUTION

	Farm Size (Acres)														
	1	2	3	4	5	6	7	8	9	10	20	30	40	50	90
Families (N)	26	24	22	20	18	16	14	12	10	8	5	4	3	2	1

NOTE.—Total land area: 1,300 acres; total N of families: 185; Gini coefficient of land distribution: .519.

a distribution of land which is quite typical for many rural regions in developing countries.

The Gini coefficient of the distribution of land (which applies also to the distribution of income prior to the introduction of the innovation, given that yield per acre is R for all farms) is .52, reflecting significant inequality.

The following values are assigned to the parameters: $y = 1.0$; $R = .4$; $m = .2$. Thus, on the average, an acre of the modern crop yields twice the income generated by traditional technology (exclusive of fixed costs). The fixed cost is assumed to be .7 for the first-time adopters, dropping to .35 in the subsequent year, due to the fact that a portion of the fixed cost is incurred only once. Thus, if full certainty prevailed, the new technology would be profitable for all farms with 2 acres of land or more, justifying a complete and immediate shift.

The specification of σ^2 needs to take into account the accumulation of experience and information dissemination over time. Essentially, two dynamic processes affect the degree of uncertainty. The first one is endogenous and cumulative in nature: As more acres are cultivated using the new technology, more experience is gained and uncertainty regarding the performance of the innovation is reduced. The speed of this communal "learning-by-doing" process depends on the efficiency of communication channels among the different farmers.[14] It is natural to specify the index of this experience factor by the time-cumulative acreage (since the time of initial introduction of the innovation) planted to the modern crop.[15]

The other process causing a reduction in uncertainty is exogenous and is brought about by the improvement in extension services, advances in research reducing crop susceptibility, the gradual elimination of input supply uncertainties through a more efficient distribution system, and improved information from outside sources (radio, written materials, etc.). The effect of this factor is best captured by assuming that the progress of time, by itself, reduces uncertainty.

Following this discussion, the variance of yield is specified as

$$\sigma_t^2 = \sigma_0^2 \cdot \exp\left(-\alpha \cdot t - \beta \cdot \sum_{i=0}^{t-1} Q_i\right), \tag{3}$$

[14] For instance, P. K. Voon found that in Malaysia innovations in rubber processing were adopted by Chinese growers much faster than the rate observed for other ethnic groups. This can be explained, in part, as the result of more effective information dissemination in the Chinese community, due to the structure of their settlements. See Phin-Keong Voon, "The Adoption of Technological Innovations in Rubber Processing: The Case of Malaysian Smallholders," *Malayan Economic Review* 22 (October 1977): 33–51.

[15] This procedure was used in the learning-by-doing literature by several authors, e.g., Simone Clemhout and Henry Wan, "Learning by Doing and Infant Industry Protection," *Review of Economic Studies* 37 (January 1970): 33–56.

dent of farm size, uncertainty, or risk aversion. In addition, one can show that the optimal x is independent of the fixed cost, C. Thus, treating x as given at its optimal level, and denoting $c \cdot x + w \equiv m$, the optimality condition with respect to l is

$$E\{U' \cdot [\varepsilon \cdot y - m - R]\} = 0, \tag{A2}$$

where E is the expectations operator. Second-order conditions are satisfied by the concavity of U.

In addition it must hold (if HYV is adopted) that

$$EU\{L \cdot [l^*(\varepsilon \cdot y - m - R) + R] - C\} \geqslant U(L \cdot R)\ , \tag{A3}$$

where l^* is the value of l which satisfies equation (A2). The relation between l^* and farm size is obtained by differentiation of (A3), using the notation $B \equiv \varepsilon \cdot y - m - R$:

$$\frac{\partial l^*}{\partial L} = \frac{E\{U'' \cdot B \cdot (\Pi_1 + C)\}}{-E\{U'' \cdot B^2\} \cdot L^2}\ . \tag{A4}$$

If absolute risk aversion is decreasing (a plausible assumption), then $E(U'' \cdot B) > 0$; however, $E\{U'' \cdot B \cdot \Pi_1\}$ is negative, zero, or positive, depending on whether relative risk aversion is increasing, constant, or decreasing in Π.[23] The denominator is positive, and the sign is thus determined by the numerator. It follows that the existence of fixed costs contributes toward a positive relation between L and l^*. If relative risk aversion is approximately constant, then (A4) implies $\partial l^*/\partial L > 0$.

It can be easily verified that $\partial l^*/\partial C < 0$, and that a higher degree of uncertainty (represented by a mean preserving spread of the distribution of the random variable ε) will induce lower levels of l^*.[24]

The circumstances underlying a decision of nonadoption ($l = 0$) can be identified from (A3). It is important first to note that, in the absence of fixed costs, nonadoption cannot be an optimal decision. To see that, suppose to the contrary $l^* = 0$. Then (A2) becomes $U' \cdot (y - m - R) < 0$, where U' is nonrandom. But this contradicts our prior assumption of HYV's superiority, on average, over the traditional technology (which implies $y - m > R$). Thus, it must be concluded $(C = 0) \rightarrow (l^* > 0)$. With $C > 0$, one can define a function ψ such that

$$\begin{aligned}\psi(L, C, \gamma) &\equiv EU\{[l^* \cdot (\varepsilon \cdot y - m - R) + R] \cdot L - C\} - U(R \cdot L) \\ &\equiv E\{U(\Pi_1)\} - U(\Pi_2)\,,\end{aligned} \tag{A5}$$

where γ is a parameter reflecting the degree of uncertainty as will be explained below, and l^* satisfies (A2). For given values of γ and C, the function ψ is

[23] Ibid.

[24] Gershon Feder, "The Impact of Uncertainty in a Class of Objective Functions," *Journal of Economic Theory* 16 (December 1977): 504–12.

increasing in L (i.e., $\partial\psi/\partial L > 0$). This implies that there is a unique value of L, say $\hat{L}$, which maintains the equality $\psi(\hat{L}, C, \gamma) = 0$. In view of condition (A3), it thus follows that farmers with holdings smaller than $\hat{L}$ will opt for nonadoption, given the values of C and γ, while larger farms will be at least partial adopters. A differentiation of the implicit function $\psi = 0$ yields

$$\frac{\partial \hat{L}}{\partial C} = -\frac{\partial\psi/\partial C}{\partial\psi/\partial \hat{L}} = \frac{EU'}{\partial\psi/\partial L} > 0 . \tag{A6}$$

In order to investigate the relation between $\hat{L}$ and the degree of uncertainty, a specific formulation of the random term is needed. Following Feder,[25] define $\varepsilon = \tilde{\varepsilon} + \gamma \cdot (\tilde{\varepsilon} - 1)$, where $\tilde{\varepsilon}$ is a random variable with mean equal to 1 and γ is a positive parameter. An increase in γ implies a mean preserving spread of the distribution of ε and, thus, reflects an increase in the degree of uncertainty. Differentiating ψ yields

$$\frac{\partial \hat{L}}{\partial \gamma} = -\frac{\partial\psi/\partial\gamma}{\partial\psi/\partial L} = -\frac{l^* \cdot \hat{L} \cdot y \cdot E\{U' \cdot (\tilde{\varepsilon} - 1)\}}{\partial\psi/\partial L} . \tag{A7}$$

The term $E[U'(\tilde{\varepsilon} - 1)]$ has the same sign as the covariance between U' and ε and, therefore, is negative. It is thus concluded that $\partial\hat{L}/\partial\gamma > 0$.

[25] Ibid.

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Learning and information accumulation are hypothesized to play a major role in innovation diffusion. For instance, Hiebert argues that the probability distributions of new (and unfamiliar) technological parameters, as perceived by farmers, will shift over time due to learning and experience. Probabilities will be redistributed from lower to higher payoffs. This induces farmers to increase their use of the innovation, which was new seed varieties in the Hiebert model.

The model presented by Kislev and Shchori-Bachrach introduces in the new technology production function an efficiency factor which is positively related to learning. It is approximated by the cumulative (over time) output produced with the innovative technology. As learning increases, the innovation becomes advantageous for more and more producers who then adopt it.

In a similar vein, Feder and O'Mara construct a diffusion process where uncertainty about an innovation (high-yielding varieties [HYV]) depends on cumulative area allocated to HYV. This represents experience. With the accumulation of experience, uncertainty declines, and the innovation is adopted by an increasing proportion of producers.

These models produce sensible (and empirically valid) hypotheses about the dynamics of innovation diffusion. However, the accumulation of information, which is the basic driving element in these models, is not treated explicitly. Thus, the use of cumulative output, or a cumulative input as an index of learning, while plausible, still requires formal justification. One possibility is a Bayesian learning process. Indeed, the work by O'Mara (1971, 1981) and more recently by Lindner, Fischer, and Pardey have utilized Bayes theorem to characterize an individual farmer's adoption behavior.

The purpose of this note is to formulate an aggregative innovation diffusion model based on the assumption that individual farmers revise their beliefs in a Bayesian fashion.

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Views expressed in this paper are the authors' and do not necessarily reflect those of the World Bank.

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The Model

The traditional technology provides an average profit of R dollars per acre. If the technology is familiar to farmers and has been available for some time, it is reasonable to assume that farmers know the true mean of R.

Assume that the new technology is risky. Profits per acre cultivated with the new technology, say π, are normally distributed with mean μ and variance σ^2, and $\mu > R$. Farmers are assumed to be risk-neutral and believe the mean profit from the new technology is normally distributed with mean m and variance δ^2. Also assume that farmers know the variance of profits, σ^2, with the new technology, and that they modify their beliefs about μ based on new information generated by observed outcomes on areas cultivated with the new technology by adopters in accord with Bayesian procedures. The assumption that farmers' beliefs about μ are normally distributed is not at all restrictive if π is normally distributed. It is easily shown that if farmers' initial beliefs are completely diffuse (i.e., uniformly distributed), and the underlying process is normal, then an application of Bayes Theorem in computing a posterior distribution, using new information, will produce beliefs about μ that are normal.

Since σ^2 is known, the initial beliefs about the variance of μ, (δ_0^2), imply an "equivalent sample size," n_0, associated with prior information, i.e.,

$$\delta_0^2 = \sigma^2/n_0. \tag{1}$$

Using the properties of conjugate prior distributions (Raiffa and Schlaifer, chap. 3) it can be shown that the mean and variance of μ at time t are given respectively by

$$m_t = \frac{\sigma^2 m_{t-1} + \delta^2_{t-1} N A_{t-1} \bar{\pi}_{t-1}}{\sigma^2 + \delta^2_{t-1} N A_{t-1}}, \tag{2}$$

$$\delta^2_t = \frac{\sigma^2 \delta^2_{t-1}}{\sigma^2 + \delta^2_{t-1} N A_{t-1}}, \tag{3}$$

where $\bar{\pi}_{t-1}$ is the sample mean of observations on the new technology generated by the adopters and observed in period $t-1$, N is the total number of farmers, and A_{t-1} is the proportion of farmers who have adopted the new technology in period $t-1$.

For simplicity, assume that all farmers are the same size and that the information content of a farmer's own trial is the same as that of another farmer's. Also, all farms in the population are observable by all farmers. Hence, each farmer acquires the same information about the new technology in each time period, whether or not he is an adopter. These strong assumptions limit the generality of our results. Define $y_t \equiv 1/\delta_t^2$, and substitution in (2) yields

$$(4) \qquad y_t = y_{t-1} + \frac{NA_{t-1}}{\sigma^2},$$

which upon recursive summation becomes

$$(5) \qquad y_t = y_o + \frac{N}{\sigma^2}\sum_{i=0}^{t-1} A_i,$$

or equivalently,

$$(6) \qquad \delta_t^2 = 1\Big/\left(\frac{1}{\delta_o^2} + \frac{N}{\sigma^2}\sum_{i=0}^{t-1} A_i\right).$$

Similarly, (1) may be written as

$$(7) \qquad \frac{m_t}{\delta_t^2} = \frac{m_{t-1}}{\delta^2_{t-1}} + \frac{N}{\sigma^2}\bar{\pi}_{t-1}A_{t-1}.$$

Defining $z_t \equiv m_t/\delta_t^2$ and substituting in (6), one obtains after recursive summation,

$$(8) \qquad z_t = z_o + \frac{N}{\sigma^2}\sum_{i=0}^{t-1} \bar{\pi}_i A_i.$$

Combining (6), (8), and (1) yields

$$(9) \qquad m_t = \frac{n_o m_o + N\sum_{i=0}^{t-1} \bar{\pi}_i A_i}{n_o + N\sum_{i=0}^{t-1} A_i},$$

or in a continuous time formulation,

$$(10) \qquad m(t) = \frac{n_o m_o + N\int_0^t \bar{\pi}(\tau)A(\tau)d\tau}{n_o + N\int_0^t A(\tau)d\tau},$$

Given the assumption of risk neutrality, the condition for adoption becomes

$$(11) \qquad E_t(\pi_t) = E_t(\mu) = m(t) \geq R.$$

That is, the time of adoption for a given farmer occurs when the mean of his beliefs about μ, the expected value of profits from the new technology, is at least as great as the return from the old technology. However, $m(t)$ depends not only on initial subjective beliefs (m_o, n_o) but also upon a stochastic component from the realized observations on the process $\pi(t)$, which is distributed $N(\mu, \sigma^2)$ for all t. Hence, the realization $\bar{\pi}(t)$ is also normally distributed with expected value μ for all t. Accordingly, the expected time of adoption for a given farmer is given by the condition,

$$(21) \quad E_t m(t) = \frac{n_o m_o + N\mu\int_0^t A(\tau)d\tau}{n_o + N\int_0^t A(\tau)d\tau} \geq R.$$

This condition may be written, after some manipulation, as

$$(13) \qquad m_o \geq R - \frac{N}{n_o}(\mu - R)\int_0^t A(\tau)d\tau = m^*_o(t).$$

Thus, if a farmer is an adopter at time t, his initial beliefs about the expected value of μ must be at least as large as $m^*_o(t)$. Assuming that initial beliefs about the expected value of μ (i.e., m_0) are randomly distributed over all farmers with cumulative density function $F(\cdot)$, the expected aggregate adoption rate at time t is

$$(14) \quad E_t A(t) = \text{Prob}[m_o \geq m^*_o(t)] = 1 - F[m^*_o(t)].$$

For simplicity, denote $P(t) \equiv E_t A(t)$. It can be shown by straightforward derivation (utilizing equations [13] and [14]) that

$$(15) \qquad (i)\ \frac{\partial P(t)}{\partial \mu} > 0, \qquad (ii)\ \frac{\partial P(t)}{\partial R} < 0.$$

As one would expect, the expected rate of adoption is higher at any time if the innovation is more profitable, *ceteris paribus*, and lower if the traditional technology is more profitable.

The expected rate of adoption at time zero is given by

$$(16) \qquad P(0) = 1 - F(R).$$

It follows, therefore, that the expected rate of adoption at the initial stage does not depend on the innovation's profitability (represented by μ) but rather on the profitability of the old technology and the parameters of the distribution, F. Now, the distribution of initial beliefs is probably strongly affected by sources of information other than observed fields, such as the farmer's education and innate ability, sales promotion, word of mouth reports, extension, etc. The distribution of initial beliefs will also affect the expected rate of adoption later, but then there will be an additional factor—the difference in average profitability between the new and the old technology $(\mu - R)$.

Inspection of equations (14) and (13) verifies that the Bayesian learning process introduces the cumulative adoption rate $\left(\int_0^t A(\tau)d\tau\right)$ as an element in the aggregate expected adoption function. Within the model's context, this is equivalent to the inclusion of cumulative acreage (or output) with the new technology in the adoption function. Previous diffusion models have simply assumed such a relationship. These results serve as a theoretical justification for such an assumption. By differentiating $P(t)$ twice with respect to time, one obtains, after some manipulation,

$$(17) \quad \frac{d^2P(t)}{dt^2} = \ddot{P}(t)$$

$$= \left[\frac{N}{n_0}(\mu - R)\right]^2 \cdot A(t)[(F')^2 - F''A(t)],$$

where $A(t)$ is the actual adoption rate at time t. Clearly,

$$(18) \qquad \ddot{P}(t) \gtreqless 0 \text{ as } F'' \gtreqless (F')^2/A(t).$$

Now, $\ddot{P}$ provides significant information about the curvature of the expected adoption rate function, $P(t)$. If $\ddot{P} > 0$ initially, then $P(t)$ will be strictly convex over that range. Similarly, if $\ddot{P} < 0$ after a certain time $\hat{t}$, then $P(t)$ will be strictly concave after that time. Thus, $P(t)$ will have the sigmoid shape of a typical successful innovation diffusion process if $\ddot{P}$ is first positive and then becomes negative after some time $\hat{t}$ (Griliches). The key to the change in sign of $\ddot{P}$ is the behavior of F'', the latter being the slope of the probability density function, $F'(m_0) \equiv f(m_0)$. Note that if F' has a smooth unimodal shape, as in figure 1, then F'' causes $\ddot{P}$ to be at first positive and then negative. That is, $m^*_0(t)$ will move from the right to the left in figure 1. High values of m_0 will be the first to meet the condition $m_0 \geq m^*_0(t)$, insuring that $\ddot{P}$ is first positive and then negative. Clearly, the behavior of F'' implies constraints on the distribution of initial beliefs if the adoption curve is to be sigmoid. For example, if F is a uniform distribution, then $F'' = 0$ and the adoption curve is exponential.

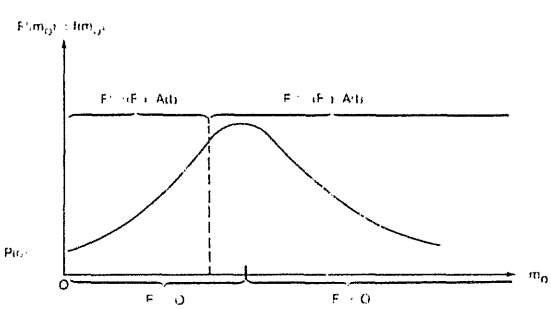

Figure 1. Probability density function of mean profits

Conclusions

This note has shown that, given our assumptions, the Bayesian learning process introduces the cumulative adoption rate as an element in the aggregate expected adoption function. This is equivalent to the inclusion of cumulative acreage (or cumulative output) in the adoption function. Previous models of diffusion have assumed such a relationship. These results serve as a justification for such an assumption. In addition, it was shown that Bayesian learning can generate a characteristic sigmoid-shaped adoption function for a dominant innovation. Although strong assumptions were made, this analysis constitutes a first step toward a more general theory of innovation diffusion.

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