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(continued on inside back cover)
STORAGE WITH PRICE UNCERTAINTY IN INTERNATIONAL TRADE*

BY GERSHON FEDER, RICHARD JUST AND ANDREW SCHMITZ

1. INTRODUCTION

Perhaps now, more than ever before, countries having to import such important commodities as food grains and oil realize the importance not only of trading internationally in order to meet immediate consumption needs but also of storing part of the traded goods for future consumption. Storage is important in a world of price uncertainty since, through an optimal storage policy, not only can governments and/or private traders reduce price instability, but also in times of severe commodity shortages—which may be either man-made (e.g., the formation of oil cartels) or created by extreme droughts or flooding—countries are not critically dependent on exporters for supplies. The danger of relying heavily on exporters for basic commodities such as food without having reserves on hand, at least to meet emergency situations, is that they can use export controls to limit supply availability as has been the recent case, for example, with respect to the United States exports of food grains.

Despite the fact that international trade is one of the riskiest economic activities, only recently have attempts been made to incorporate uncertainty into formal trade models. Among the recent works have been Turnovsky [14], Kemp and Liviatan [7], Ruffin [11, 12], Nsouli [8], Batra [2], and Batra and Russell [3]. With the exception of the paper by Turnovsky, the studies considered either uncertainty in prices or uncertainty in production. However, in all of these studies the possibility of storage was not explicitly considered.

In this paper an international trade model is constructed which incorporates both price uncertainty and storage. A major emphasis is on important world commodities such as food in that, for a given country, a government planning agency makes decisions concerning (1) “target” needs of local consumers for that commodity (e.g., target levels specified in India’s five-year plans); (2) imports; (3) levels of storage; (4) exports; and (5) the amount of speculative activity (e.g., the recent action of the Soviet Union in buying grain from the United States...
at a relatively low price and reselling it to other countries at a much higher price). A model with price uncertainty and storage included is constructed for food importers, food exporters, and countries which speculate by importing food for export sale at a later date.

2. THE MODEL

The economy consists of two production sectors where, like the model presented by Turnovsky [14], a single factor of production $K$ is used to produce both goods. Production is assumed separable with respect to the two goods with production functions defined by

$$F = F(K_f); \quad F' > 0; \quad F'' < 0$$

and

$$G = G(K_g); \quad G' > 0; \quad G'' < 0,$$

respectively, where $K_f$ and $K_g$ are the respective allocations of $K$ among the two sectors. The total amount of $K$ is assumed fixed with

$$K_s + K_g + K_f = K$$

where $K_s$ is the amount of $K$ required for the storage of good $F$. Only $F$ (food) is stored.

Consistent with a planner's objective of determining target-level food demands, it is assumed that good $F$ has to be consumed at a fixed amount $F$ while good $G$ can be consumed at various levels. This assumption fits the case of many importers and exporters of food. For example, as already pointed out, a country such as India has a five-year plan which specifies target consumption levels of food. In food-exporting countries — especially those of a low-income nature — targets are often set for local residents in order to calculate the residual which is sold abroad in order to generate foreign exchange earnings. Since the consumption of $F$ is fixed, the community's welfare can be represented as a function only of $C$ where $C$ is the volume of $G$ consumed,

$$U = U(C); \quad U' > 0; \quad U'' \leq 0,$$

where $U$ is the country's welfare function as perceived by the planner.5

Because of the possibility of storing, the planner in an importing country, for example, can import $F$ in period $t$ and store it for consumption in period $t+1$. It is assumed that the need to store $F$ arises because of extreme uncertainty with

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3 For simplicity, food is used to denote the commodity for which target consumption is specified. However, other commodities, such as oil, could be considered as well.

4 Actually, the analysis could be extended to include two factors. However, this would complicate the notation and the algebra without essentially changing the results.

5 Note that risk aversion implies $U'' < 0$, while $U'' = 0$ corresponds to risk neutrality.
respect to future production and price of $F$ (at time $t+1$). In such a case the
price of other goods is thus relatively stable and, particularly where $G$ is taken
as the aggregate of all other goods, its price may be considered as nonrandom to
simplify the analysis. However, the current price of $F$ is assumed known at the
time of decision making (at time $t$). For the purposes of notation, the prices of
$F$ and $G$ are denoted by $P_f$ and $P_g$, respectively, where $P_f$ is a random variable with

$$E(P_f) = \bar{P}_f$$

and $P_g$ is nonstochastic. If the planner decides to import and store $F$, two types
of costs are incurred: (1) the storage costs and (2) the purchasing costs of the
imports. Storage costs are represented by the transfer of resources from other
sectors. Thus, if $F$ is stored, a smaller quantity of $K$ is available for producing
goods $F$ and $G$. For simplicity, let the amount of resources $K$ required for $S$
(storing) be proportional to $S$, i.e.,

$$K_s = \alpha S$$

where $\alpha > 0$ and $K_s$ represents resources used for storage. The total purchasing
cost of the amount to be stored is given by $P^*_o S$ and, since the decision as
to whether or not to store is made in period $t$, the price $P^*_o$ is known. Also, it is
assumed that the country's import decisions affect neither current price nor the
probability distribution of future prices (i.e., a small country assumption).

To complete the model, the following balance-of-payments constraint is im-
posed:

$$P_o [G(K_o) - C] = - [F(K_f) + S - \bar{F}] P_f + P^*_o S$$

which, by using (3) and (6), may be rearranged as follows:

$$C = G[\bar{K} - K_f - \alpha S] + [F(K_f) + S - \bar{F}] P - P_o S$$

where

$$P = \frac{P_f}{P_g}$$

$$P_o = \frac{P^*_o}{P_g}$$

and

To arrive at the purchase cost $P_o S$, suppose that in time $t$ imports are purchased with credit
or by floating bonds in the international market at interest rate $r$. This is to be repaid in period
$t+1$ with exports. Of course, in the case with interest charges, the effective price relative for
repayment in period $t+1$ should be

$$P_o = (1 + r) \frac{P^*_o}{P_g}$$

but this change would lead to no substantive alteration in the results of this paper.
Equation (8) implies that consumption of G is determined by domestic output of G minus purchases of F for storage and minus the value of required F imports (the latter will be positive in the case of food exporters). Since P (or \(P_f\)) is not known at the time of decision making, consumption of G is a random variable. The planner, given uncertain future prices, has to determine both the amount of storage in period t which will then be available for period \(t+1\) and the production of F and G for period \(t+1\), recognizing that the stocks purchased in period t have to be paid for in \(t+1\) by export sales.\(^7\) It is thus implicitly assumed that current consumption and trade decisions have already been made and carried out.

Specifically, the planner’s problem is to maximize expected utility by choosing optimal levels of \(K_f\) and S (which also determine the optimal level of \(K_g\))

\[
\begin{align*}
\text{Max} \ E U(C) & \equiv \pi = E \{ G(\bar{K} - K_f - \alpha S) + [F(K_f) + S - F]P - P_0 S \} \\
\text{subject to} \ K_g & = \bar{K} - K_f - \alpha S \geq 0.
\end{align*}
\]

(9)

First-order conditions for maximization of \(\pi\) (assuming \(K_f, K_g, \text{and } S\) are positive at the optimal point) are\(^8\)

\[
\begin{align*}
\pi_k & = E[U' \cdot (PP' - G')] = F'E(U'P) - G'E(U') = 0 \\
\pi_s & = E[U' \cdot (P - \alpha G' - P_0)] = E(U'P) - (\alpha G' + P_0)E(U') = 0.
\end{align*}
\]

(10) (11)

Second-order conditions are satisfied when the Hessian matrix \(H\) defined by

\[
H = \begin{bmatrix}
\pi_{kk} & \pi_{ks} \\
\pi_{sk} & \pi_{ss}
\end{bmatrix}
\]

is negative definite, i.e., \(\pi_{kk} < 0, \pi_{ks} > 0\). Using (10) and (11), one ob-

\(^7\) The length of the planning horizon could be extended but only at the cost of greatly complicating the model. Anyway, because of the recent increase in short-run price instability for basic food items, the assumption of a relatively short planning horizon is realistic. Also, most international grain sales have used relatively short-term credit arrangements in recent years. This further supports our assumption that loans are usually repaid within the planning horizon.\(^8\) Actually, the full Kuhn-Tucker conditions for the maximization problem in (9) are

\[
\begin{align*}
\pi_k - \lambda & \leq 0, \quad K_f(\pi_k - \lambda) = 0 \\
\pi_s - \alpha \lambda & \leq 0, \quad S(\pi_s - \alpha \lambda) = 0 \\
\bar{K} - K_f - \alpha S & \geq 0, \quad \lambda(\bar{K} - K_f - \alpha S) = 0
\end{align*}
\]

where the Lagrangian is \(L = \pi + \lambda(\bar{K} - K_f - \alpha S)\). The case with \(K_g = 0\) or \(K_f = 0\) is not interesting. If \(K_g, K_f > 0\), then the Kuhn-Tucker conditions imply \(\pi_k = 0, \pi_s = 0\) or \(S = 0\). The former case is treated in the body of the paper; the latter is included in the Appendix. Strictly speaking, one should also restrict G consumption to be nonnegative \((G \geq 0)\). However, the possibility that \(C < 0\) is somewhat unrealistic and may be eliminated by suitably restricting the production functions and/or price distributions.
To simplify (13)-(15), let \( A = (P - xG' - P_0) \) and note that (10) and (11) imply that
\[
1 - xF' = F'P_0 > 0
\]
and
\[
P'F' - G' = F' \cdot (P - xG' - P_0) = F' A.
\]
Substituting (17) into (13)-(15) and using (16), second-order conditions are verified since
\[
\Delta = \pi_{xx} - \pi_{xs} = (1 - xF')G'E(U')E(U''A^2) + F'E(U')E(U''A^2) + \alpha^2 F''G''E(U')E(U'P) > 0.
\]
To facilitate the derivation of results in following sections, a number of lemmas will be useful. For convenience, a country with \( X = K + S - F > 0 \) at the optimum is called an exporter of \( F \); and a country with \( X < 0 \) is called an importer of \( F \). Following Arrow [1] and Pratt [9], a country is called risk averse if absolute risk aversion, \( R_s = -U''/U' \), is positive. Also, throughout the analysis, it will be assumed that \( R \) is nonincreasing in \( C \) as argued by Arrow [1] (see also Batra [2]).

**Lemma 1.** Assuming risk aversion, it follows that \( \sigma = \text{Cov}(U', P) > 0 \) for an exporter of \( F(X > 0) \), that \( \sigma < 0 \) for an importer of \( F(X < 0) \), and that \( \sigma = 0 \) when \( F \) is not traded in the uncertain stage \( (X = 0) \). Hence, \( \sigma X < 0 \) for both importers and exporters.

**Proof.** Where \( U(P) \) is used in place of \( U(G + XP - P_0 S) \) for notational convenience, it follows that
\[
U'(P) < U'(P') \quad \text{for} \quad P > P', X > 0
\]
\[
U'(P) > U'(P') \quad \text{for} \quad P < P', X > 0
\]
\[
U'(P) > U'(P') \quad \text{for} \quad P > P', X < 0
\]
\[
U'(P) < U'(P') \quad \text{for} \quad P < P', X < 0
\]
since \( U'' < 0 \). Hence, multiplying both sides by \( P - P' \), taking expectations, and noting that \( U'(P') \) is a constant, one finds \( \sigma = E[(P - P')U'(P)] < 0 \) for \( X > 0 \) and

\[\text{The technique used to prove the lemmas is similar to that used by Sandmo [13].}\]
\[ \sigma > 0 \text{ for } X < 0. \] The case with \( X = 0 \) follows trivially.

**Lemma 2.** Assuming nonincreasing risk aversion, it follows that \( E(U''A) \geq 0 \) for exporters of \( F(X > 0) \) and \( E(U''A) \leq 0 \) for importers of \( F(X < 0) \) where strict equality implies either that \( X = 0 \) or that risk aversion is also nondecreasing (\( R \) is constant). Hence, \( X E(U''A) \geq 0 \) where strict equality implies \( X = 0 \) or \( R \) constant.

**Proof.** Note that \( E(U''A) = E[(P - \alpha G' - P_0) \cdot U''(P)] = E[(P - P^*) \cdot U''] \) where \( P^* = \alpha G' + P_0 \). But with nonincreasing risk aversion, it follows that

\[
\begin{align*}
(19) \quad & R(P) \leq R(P^*) \quad \text{for } P > P^*, \quad X > 0 \\
(20) \quad & R(P) \geq R(P^*) \quad \text{for } P < P^*, \quad X > 0 
\end{align*}
\]

where \( R(P) = -U''(P)/U'(P) \). Hence

\[
\begin{align*}
& -U''(P) \leq R(P^*)U'(P) \quad \text{for } P > P^*, \quad X > 0 \\
& -U''(P) \geq R(P^*)U'(P) \quad \text{for } P < P^*, \quad X > 0.
\end{align*}
\]

Multiplying both sides by \((P - P^*)\) and taking expectations thus implies

\[
(21) \quad E[-(P - P^*)U''(P)] \leq E[(P - \alpha G' - P_0)U'(P)]R(P^*) \quad \text{for } X > 0.
\]

Using (11), however, obtains \( E[(P - \alpha G' - P_0) \cdot U''(P)] = 0 \) and, hence, \( E(U''A) \geq 0 \) for \( X > 0 \) by definition of \( P^* \). The case of an importer is proven similarly. Finally, note that strict equality is obtained in (21) only when \( R \) is constant with respect to \( P \) and, hence, strict equality occurs in (19) and (20).

**Lemma 3.** With nonincreasing risk aversion, both importers and exporters satisfy \( E[U''A(P - P)] < 0 \).

**Proof.** Note that from (11) and the definition of \( \sigma \),

\[ P - \alpha G' - P_0 = P - \frac{E(U'P)}{E(U')} = P - \bar{P} - \frac{\sigma}{E(U')} \]

and, hence, the definition of \( A \) implies

\[
E[U''A(P - \bar{P})] = E[U''(P - \bar{P} - \frac{\sigma}{E(U')})(P - \bar{P})]
\]

\[
= E[U''(P - \bar{P} - \frac{\sigma}{E(U')})^2 + \frac{\sigma}{E(U')}U''\left( P - \bar{P} - \frac{\sigma}{E(U')} \right)].
\]

Obviously, from (4),

\[
E[U''(P - \bar{P} - \frac{\sigma}{E(U')})^2] < 0.
\]

But using Lemmas 1 and 2 implies that
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$$-\frac{\sigma}{E(U')} E\left[U''\left(P - \bar{P} - \frac{\sigma}{E(U')}\right)\right] = -\frac{\sigma}{E(U')} E(U') \leq 0$$

for both importers and exporters; hence, $E[U'(P - \bar{P})] < 0$.

**Lemma 4.** From risk aversion $(U'' < 0)$, it follows that $E[U'A \ln U] > 0$ for exporters of $F$ and $E[U'A \ln U] < 0$ for importers of $F$.

**Proof.** Note that

$$\ln U(P) > \ln U(P^+) \quad \text{for } P > P^+, X > 0 \quad \ln U(P) < \ln U(P^+) \quad \text{for } P < P^+, X > 0.$$

Hence, multiplying both sides by $U'A$ (evaluated at $P$) implies $U'A \ln U(P) > U'A \ln U(P^+)$ when $X > 0$ since $A > 0$ for $P > P^+$ and $A < 0$ for $P < P^+$. But $U(P^+)$ is constant $(P^+ = \bar{g} + P^0)$, so taking expectations obtains $E[U'A \ln U] > U(P^+)E(U') = 0$ for exporters $(X > 0)$. An analogous procedure yields $E[U'A \ln U] < 0$ for importers.

3. **IMPLICATIONS OF UNCERTAINTY FOR STORAGE, TRADE, AND SOCIAL WELFARE**

In this section the effects of changing uncertainty on optimal storage and trade policies are examined. Following the approach of Sandmo [13], suppose the effective future price for the stored good is defined by

$$P_f^* = \gamma P_f + \theta_f.$$

Since this is formally equivalent to defining

$$P^* = \gamma P + \theta$$

where $\theta = \theta_f/P_f$, changes in the degree of uncertainty can be examined by changing $\gamma$ in such a way that either $E(dP_f^*) = 0$ or $E(dP^*) = 0$ which implies $d\theta_f/d\gamma = -\bar{P}_f$ or $d\theta/d\gamma = -P$. This fits the Rothschild-Stiglitz [10] notion of a mean-preserving spread. Changes in future price expectations for the stored good can be investigated by changing $\theta$ (or $\theta_f$).

For certain problems where the choice of the numeraire is arbitrary, Flemming, Turnovsky, and Kemp [6] have recently shown that the choice of the numeraire can affect the results when a mean-preserving spread indicated by (22) is used. That is, a mean-preserving spread in $P$ implies a change in the mean of $P^{-1}$. In our case, however, the choice of the price ratio is determined by the model specification. Furthermore, $P_g$ need not necessarily be interpreted as a numeraire. Although it may be of interest to examine changes in the distribution of $P_g$ as well as for $P$, our concern is primarily with $P_f$ because storage is not usually important for goods with stable prices. If $P_g$ is constant compared with $P_f$, then a mean-preserving spread in the price ratio $P$ (where $P_g$ is the denominator) pre-
serves also the mean of $P_f$ as indicated above. Hence, the problem which this paper is addressing is one in which instability in only one price (the price of the stored good) is of interest.\footnote{Flemming, Turnovsky, and Kemp [6] have shown that for some cases the results for spreading the distribution of $P$ can apply equally well for spreading the distribution of $P^{-1}$ if a geometric rather than arithmetic mean-preserving spread is investigated. Hence, the effects of changing the distribution of the denominator price can be examined at the same time the numerator price distribution is changed. However, such an approach is not possible in the framework of this paper because the argument of the utility function in (9) is not homogeneous with respect to the price relative $P$ (future prices); when storage is considered, prices in other time periods are also important. With storage, the effects of changing the distribution of $P_e$ are thus more difficult to determine. One would not want to investigate the effects of a mean-preserving spread in $P^{-1}$ alone. For example, this result would be of interest when changing only the distribution of $P_e$ rather than $P_f$. But if the distribution of $P_e$ is changed, then changes in the distribution of $P_f$ (recall $P_f=P^e/P$) also occur and, of course, those effects should be considered as well. It would be interesting, however, to know the effects of changing the distribution of $P_e$ (which could be examined by investigating a mean-preserving spread in $P^{-1}$ and $P^e$ simultaneously. Unfortunately, in this case only indeterminant results are obtained for the problem addressed by this paper.}

**Theorem 1.** With nonincreasing risk aversion, an increase in price uncertainty leads to an increase in production of both stored and nonstored commodities and a decrease in storage for exporters of the stored commodity; importers of the stored commodity decrease production of both commodities and increase storage.

**Proof:** Differentiating (10) and (11) one obtains

$$dK_f \quad dS$$

$$\frac{d}{d\gamma} H = - F'\{\sigma + XE[U'' A(P - \bar{P})]\} - \{\sigma + XE[U'' A(P - \bar{P})]\} \quad \frac{d}{d\gamma}$$

where $H$ is defined by (12). Solving the system in (23) by Cramer's rule and evaluating at $a=1$ and $\theta=0$, one obtains

$$dK_f \quad d\gamma \quad \frac{d}{d\gamma} H = - F'\{\sigma + XE[U'' A(P - \bar{P})]\} + F''E(U'P)$$

$$\quad \frac{d}{d\gamma} = \frac{dK_f}{d\gamma} = \frac{dK_f}{d\gamma} = \frac{dS}{d\gamma} \frac{dS}{d\gamma}$$

Using (3) and (6), this implies

$$\frac{dK_f}{d\gamma} = \frac{dK_f}{d\gamma} - \frac{dK_f}{d\gamma} = - \frac{dK_f}{d\gamma} = \frac{dK_f}{d\gamma} = \frac{dK_f}{d\gamma} \frac{dK_f}{d\gamma}$$

Using (2), (4), (16), and (18) to note that $G''(1-aF')E(U')/a < 0$ and observing that $F''E(U'P)<0$ implies by Lemmas 1 and 3 that

$$\frac{dK_f}{d\gamma} = \frac{dK_f}{d\gamma} - \frac{dK_f}{d\gamma} \frac{dK_f}{d\gamma}$$
With nonincreasing risk aversion and increased price uncertainty, both importers and exporters of the stored commodity tend to limit their involvement in future trade and, in fact, will cease to trade in the limit ($\gamma \to \infty$).\footnote{Turnovsky [14] obtained a similar result (see his proposition 9).}

PROOF. Where trade in the future period is represented by $X = F(K_f) + S - F$, the proof related to Theorem 1 leads to

$$
\frac{dX}{dy} = \left[1 - \alpha \gamma \{U'[(P - \bar{P}) dS - X(P - \bar{P}) + (F'P - G) dK_f - dE(U) d\gamma]} \right]
$$

and implies $dX/dy < 0$ for exporters of $F$ and $dX/dy > 0$ for importers of $F$. To show that $X \to 0$ as $\gamma \to \infty$, observe from (27) and Lemma 1 that $X = 0$ is a stationary solution, i.e., $dX/\gamma = 0$ at $X = 0$. Then, using the results above, the existence of any other limit can be easily refuted.

REMARK 1. It is interesting to note that the results of Theorem 2 with respect to the stored commodity are contrary to those obtained when storage is not possible. Imposing $S = 0$, it may be shown that $dK_f/d\gamma < 0$ for exporters of $F$, i.e., $(F - F > 0)$ and $dK_f/d\gamma > 0$ for importers of $F$, i.e., $(F - F < 0)$ (see Appendix). However, the results without storage possibilities are consistent with the broader scope of Theorem 2; namely, as uncertainty increases, there is a tendency with both importers and exporters to revert to autarky in the uncertain stage.

THEOREM 3. With increased future price uncertainty, expected social welfare is reduced for all risk-averse importers and exporters.

PROOF. Simply note that $dS/\gamma$ and $dK_f/\gamma$ are nonstochastic and, hence,

$$
\frac{dE(U)}{d\gamma} = E[U' \{ (P - \alpha G' - P_0) \frac{dS}{d\gamma} + X(P - \bar{P}) + (F'P - G') \frac{dK_f}{d\gamma} \}]
$$

$$
= E[U' \{ (P - \alpha G' - P_0) \frac{dS}{d\gamma} + E[(F'P - G')U'] \frac{dK_f}{d\gamma} + XE[U'(P - \bar{P})] \}.
$$

Using (10), (11), and Lemma 1, one obtains

$$
\frac{dE(U)}{d\gamma} = XE[U'(P - \bar{P})] = X\sigma \leq 0
$$
where strict equality holds when no trade takes place in the uncertain stage 
\( X=0 \).

All of the results in Theorems 1 through 3 are derived in the context of a risk-
averse country. However, for the case of a risk-neutral country, inspection of the 
results in (24)–(29) also reveals the following corollary.

**Corollary 1.** An increase in uncertainty has no effect on resource alloca-
tion or expected social welfare for a risk-neutral country.

To investigate further the effects of shifts in the distribution of \( P \), consider the 
possibility of a change in the future price expectation.

**Theorem 4.** With a rise in expected future price (of the stored commodity), 
all trading countries tend to shift resources into the storage activity and away 
from production of both stored and nonstored goods. But storage expands by 
more than production is reduced so that total future supply of the stored good 
increases. Hence, countries plan to import less (export more) of the stored 
good.

**Proof.** Differentiating (10) and (11), one obtains

\[
\begin{bmatrix}
\frac{dK_f}{dP} \\
\frac{dS}{dP} \\
\end{bmatrix}
= H
\begin{bmatrix}
- F' E(U') - X F'' E(U'' A) \\
- E(U') - X E(U'' A) \\
\end{bmatrix},
\]

Solving the system in (28) and using (1), (2), (4), (16), and Lemma 2 thus implies

\[
\frac{dK_f}{dP} = \alpha (1 - \alpha F') G'' E(U') \left[ E(U') + X E(U'' A) \right] \frac{\Delta}{\Delta} < 0
\]

\[
\frac{dS}{dP} = - \left[ E(U') + X E(U'' A) \right] \left[ (1 - \alpha F') G'' E(U' P) \right] E(U') + F'' > 0.
\]

Using (3) and (6), one obtains

\[
\frac{dK_a}{dP} = \frac{\alpha F'' E(U' P) \left[ E(U') + X E(U'' A) \right]}{\Delta} \leq 0
\]

where strict equality holds only for nontrading countries \( X=0 \). Finally, to ob-
serve the effect of a change in \( \bar{P} \) on import and export plans, note that

\[
\frac{d}{d\bar{P}} \left[ F(K_f) + S \right] = \frac{dX}{d\bar{P}} = F' \frac{dK_f}{d\bar{P}} + \frac{dS}{d\bar{P}}
\]

\[= - \left[ (1 - \alpha F') G'' E(U') + F'' E(U' P) \right] \left[ E(U') + X E(U'' A) \right] > 0.
\]

**Remark 2.** Interestingly, the results of Theorem 4 relating to (29) are con-
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Contrary to standard trade theory which indicates that a rise in the price of a good should lead to an increase in its production. One can also show that a rise in the expected price of a good leads to an increase in its production in the case of price uncertainty when storage is not possible, i.e., $dK_f/dP > 0$ when $S = 0$ (see Appendix and Nsouli [8]). It is true, however, that the total amount of $F$ available for future consumption and trade is increased for both importers and exporters. The introduction of storage thus leads to substitution between domestic production of $F$ and its importation for the purposes of storage. But, obviously, the possibility of storage can never lead to a loss in expected social welfare since a country can always continue with no storage.

4. EFFECTS OF CHANGES IN THE CURRENT PRICE ON STORAGE AND PRODUCTION PLANS

More information on the role of storage as a substitute for domestic production can be obtained by examining the effects of an increase in current price $P_0$. Since storage thus becomes more expensive, one would expect production to be substituted for storage at the margin. Theorem 5 indicates that such is indeed the case with exporters. The importers' case, however, may be slightly more complicated as suggested in Remark 3.

**Theorem 5.** With an increase in current price of the stored commodity, exporters increase planned future production of the stored commodity but reduce storage. In the general case, importers' behavior is not clear; but if absolute risk aversion is constant, they will do likewise. Planned exports are reduced for exporters, but importers' import plans increase if (but not only if) absolute risk aversion is constant.

**Proof.** Solving the system

$$
\begin{bmatrix}
\frac{dK_f}{dP_0} \\
\frac{dS}{dP_0}
\end{bmatrix} = \begin{bmatrix}
F'SE(U''A) \\
E(U') + SE(U''A)
\end{bmatrix}
$$

obtained from (10) and (11) yields

$$
\frac{dK_f}{dP_0} = -\left[\alpha SG''(1 - \alpha F')E(U')E(U''A) + \alpha G''E(U')^2 + F'E(U')E(U''A^2)\right] / \Delta
$$

(30)

$$
\frac{dS}{dP_0} = \left[(1 - \alpha F')SG'E(U')E(U''A) + E(U')E[PF'' + G']U']\right] / \Delta
$$

(31)

$$
+ (F')^2E(U')E(U''A) + F''SE(U'P)E(U''A).$$

Using (16) and Lemma 2 in addition to (1), (2), and (6) this yields
\[
\frac{dK_f}{dP_0} > 0, \quad \frac{dS}{dP_0} < 0
\]
for \( X \geq 0 \) or \( X < 0 \) and \( R \) constant. Finally, note that
\[
d\left[ F(K_f) + S - F \right] - dX = \frac{dX}{dP_0}
\]
\[
= \frac{\left[ E(U') + SE(U''A) \right] \left[ (1 - xF')G'E(U') + F'E(U'P) \right]}{A}
\]
Hence, similar reasoning yields \( dX/dP_0 < 0 \) when \( X \geq 0 \) or when \( X < 0 \) and \( R \) is constant.

**Remark 3.** From the proof of Theorem 5, it follows that the possibility that an importer will increase purchases for storage when \( P_0 \) increases (in particular, when the distribution of future price is held constant) cannot be ruled out. This result seems counterintuitive since there is an obvious incentive (as with exporters) to substitute domestic production for storage at the margin—the latter has become more expensive. However, note that only the possibility of decreasing absolute risk aversion for importers contributes a storage-increasing term in (31) and a production-decreasing term in (30). Interestingly, this is in line with the results of the next section which indicate that higher risk aversion is associated with heavier reliance on storage; that is, as \( P_0 \) increases, the importer’s possible levels of consumption decreases; thus, risk aversion increases (with decreasing absolute risk aversion), and there is incentive to increase storage.\(^{12}\) But since purchasing for storage becomes more expensive, there is a conflicting incentive to substitute domestic production for storage. The net result is not obvious.

### 5. Changes in Risk Preferences

The impact of changes in attitudes toward risk on the allocation of resources and the volume of storage cannot be investigated without further specification of the welfare function in (4) and the form in which changes in risk preferences may be represented.\(^{13}\) A useful choice for the purposes of this section is the constant elasticity function
\[
U(C) = C^{1-\varepsilon}, \quad 0 < \varepsilon < 1
\]
(note that \( U' > 0, U'' < 0 \)). Here the elasticity is \( 1 - \varepsilon \) (assumed constant) but,

\(^{12}\) Note that an increase in absolute risk aversion implies an increase in relative risk aversion and vice versa for any given level of \( C \).

\(^{13}\) For a rigorous analysis of the general relationship between risk aversion and a change in risk, see Diamond and Stiglitz [5]. Their results deal only with one control variable and are not directly applicable.
in particular, the measure of relative risk aversion is (Arrow [1])

\[ \varepsilon = -\frac{U'}{U} \cdot C. \]

Hence, changes in risk aversion can be examined easily using the parameter \( \varepsilon \). The following theorem is thus obtained.

**Theorem 6.** With a constant elasticity welfare function, an increase in relative risk aversion has the same qualitative effects as an increase in risk, namely, exporters (of the stored commodity) increase planned production of both stored and nonstored commodities while storage is reduced; importers reduce production of both commodities while increasing storage. Planned trade is reduced in either case.

**Proof.** Differentiating (10) and (11) obtains

\[
\begin{bmatrix}
\frac{dK_f}{de} \\
\frac{dS}{de}
\end{bmatrix} = H^{-1}
\begin{bmatrix}
F' \\
1 - \varepsilon E(U' A \ln U)
\end{bmatrix},
\]

the solution of which is

\[
\frac{dK_f}{de} = -\alpha G'' \cdot \frac{(1 - \alpha F')E(U' A \ln U)}{(1 - \varepsilon)A},
\]

\[
\frac{dS}{de} = \frac{[(1 - \alpha F')G'' + F'E(U' P)]E(U' A \ln U)}{(1 - \varepsilon)A}.
\]

And using (32) and (33) obtains

\[
\frac{dK_x}{de} = -\alpha F'' E(U' P)E(U' A \ln U) \\
(1 - \varepsilon)A
\]

since \( K \) is fixed as in (3). Thus, (1), (2), (6), (16), and Lemma 4 imply

\[
\frac{dK_f}{de} > 0, \quad \frac{dS}{de} < 0, \quad \frac{dK_x}{de} > 0 \quad \text{for } X > 0
\]

\[
\frac{dK_f}{de} < 0, \quad \frac{dS}{de} > 0, \quad \frac{dK_x}{de} < 0 \quad \text{for } X < 0.
\]

Finally, note that

\[
\frac{d[F(K_f) + S - F]}{de} = \frac{dX}{de} = \frac{\left[(1 - \alpha F')^2 G'' + F'' E(U' P)\right] E(U' A \ln U)}{(1 - \varepsilon)A}.
\]

Hence, \( dX/de < 0 \) for \( X > 0 \) and \( dX/de > 0 \) for \( X < 0 \) using the same results.
6. CONCLUSION

This paper has presented some propositions describing the effects of price uncertainty in a two-good trade model where the possibility of storage is taken into account. Once storage is allowed, one has the possibility that an increase in price uncertainty affects not only output but also the storage of this output for future use. Consequently, the general conclusions of standard trade theory do not necessarily hold. For example, the standard result is that an increase in the expected price of a certain good should increase the production of that good by both exporters and importers. With the possibility of storage, however, resources will be shifted into storage and away from production of both stored and non-stored goods.

In the case of increase in price uncertainty, both importers and exporters tend to decrease their planned involvement in uncertain trade via appropriate substitutions between storage and production. The substitution is made such that resources are diverted from the uncertain activity (namely, future trade) into activities which are not risky. In other words, exporters reduce storage (which, in their case, is for a speculative purpose) while importers increase storage (which, in their case, is preferred to dependence on the uncertain future market).

Historically, importers of basic commodities such as food have been reluctant to undertake large storage programs in order to have these available during periods of extreme world shortages. Our model suggests, for example, that, given the recent increase in future commodity price uncertainty around the world, importers should seriously consider expanded storage as a vital economic activity.

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APPENDIX

The Appendix deals with the case where $S=0$. An exporter is thus defined by $Z=F(K_f)-F>0$ and an exporter by $Z<0$. The maximization problem in this case is

$$\text{Max } \pi = EU\{G(K_f-K_f) + [F(K_f)-F]P\}$$

subject to

$$K_f, K_g = \bar{K} - K_f \geq 0.$$ 

Assuming $K_f, K_g > 0$ at the optimum, first-order conditions for maximization imply

$$\pi_k = E[U'(PF'-G')] = 0.$$ 

Since $\pi_{kk} = E[U''(PF'-G')^2 + U'PF'' + U'G] < 0$, second-order conditions are satis-
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fied. Lemmas 1 through 4 can be shown to hold if $A$ is replaced by $B=F'P-G'$. Using procedures similar to those used in the proofs of Theorems 1 through 6, the following results are obtained:

\[
\begin{align*}
\frac{dK_f}{d\gamma} &= -\frac{dK_g}{d\gamma} = \alpha F' + ZE[U'B(P - \overline{P})] - \pi_{kk} \\
\frac{dK_f}{dP} &= -\frac{dK_g}{dP} = F'E(U') + ZE(U''B) - \pi_{kk} \\
\frac{dK_f}{d\epsilon} &= -\frac{dK_g}{d\epsilon} = \frac{E(U'B \ln U)}{(1 - \epsilon)\pi_{kk}}
\end{align*}
\]

(assuming $U=C^{1-\epsilon}$). Using (A1)-(A3) and Lemmas 1 through 4, the following theorem is immediate.

**THEOREM.** Assuming storage is not possible, the following results are obtained: (1) an increase in uncertainty leads to a decrease in $F$ production by an exporter and an increase in $F$ production for an importer (conversely, for $G$ production), (2) an increase in expected price for the uncertain stage leads to an increase in $F$ production and a decrease in $G$ production for both exporters and importers, and (3) assuming a constant elasticity utility function, an increase in risk aversion leads to a decrease in $F$ production for exporters and an increase in $F$ production for importers (conversely, for $G$ production).

**REMARK.** As in the case with storage possible, a change in uncertainty and a change in risk aversion lead to the same qualitative conclusions. Interestingly, using a completely different dynamic Ricardian model, Nsouli [8, (246)] obtained results similar to case (1) (our other cases were not investigated).

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