Economic Growth, Education, and AIDS in Kenya: A Long-run Analysis

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Abstract

The AIDS epidemic threatens Kenya with a long wave of premature adult mortality, and thus with an enduring setback to the formation of human capital and economic growth. To investigate this possibility, we develop a model with three overlapping generations, calibrate it to the demographic and economic series from 1950 until 1990, and then perform simulations for the period ending in 2050 under alternative assumptions about demographic developments, including the counterfactual in which there is no epidemic. Although AIDS does not bring about a catastrophic economic collapse, it does cause large economic costs – and very many deaths. Programs that subsidize post-primary education and combat the epidemic are both socially profitable – the latter strikingly so, due to its indirect effects on the expected returns to education – and a combination of the two interventions profits from a modest long-run synergy effect.

Keywords: HIV/AIDS, Growth, Education, Kenya, Policy Programs
JEL Classification: I10, I20, O11, O41


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*We thank Emanuel Jimenez, Mamta Murthi and participants in seminars at University of California, Berkeley, Mannheim University and the World Bank for their valuable comments and suggestions.
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1 Introduction

This paper analyses the prospects for the formation of human capital and economic growth in Kenya, even as the AIDS epidemic threatens that country with a long wave of premature adult mortality. It focuses, naturally enough, on the young. For young people in Kenya, as in most parts of the world, are confronted with a real choice between continuing their education and entering the labour market, and they usually have some say in that decision. At roughly the same time, they also become sexually active, with all the gratifications and risks that entails, including unplanned parenthood and contracting sexually transmitted diseases. Somewhat later, as young adults, they choose partners, have children and take on the responsibilities of family life.

The long wave of premature adult mortality is not the only factor imposing the time frame. What is called human capital is built up slowly, through a combination of child-rearing within the family and formal education during childhood and adolescence, and is then extended in some ways through experience on the job, especially in young adulthood. This gestation process far exceeds that connected with investments in virtually all forms of physical capital. Yet viewed statistically, the capital embodied in young adults is also strikingly durable – unless the hazards posed by accidents, violence and lethal communicable diseases are substantial.

Our approach assigns a central role, first, to the formation of human capital through a process in which child-rearing and formal education combine to produce the wellspring of long-term economic growth and, second, to premature adult mortality as the primary threat to that process. The victims of AIDS are overwhelmingly young adults or those in their early prime years, the great majority of them with children to raise and care for, so that the nature of the ensuing long-term threat to social well-being is clear. For if parents sicken and die while their children are still young, then all the means needed to raise the children so that they can become productive and capable citizens
will be greatly reduced. The affected families’ lifetime income will shrink, and hence also the means to finance the children’s education, whether in the form of school fees or taxes. On a parent’s death, moreover, the children will lose the love, knowledge and guidance which complement formal education. AIDS does much more, therefore, than destroy the existing abilities and capacities – or human capital – embodied in its victims; it also weakens or even wrecks the mechanism through which human capital is formed in the next generation and beyond. These ramifications will take decades to make themselves fully felt: like the course of the disease in individuals, they are long-drawn out and insidious. For these reasons, expectations about the future course of mortality have a potentially decisive influence on investment in education.

History plays an important role in any long-run analysis, and there is a fairly complete account for Kenya from the middle of the 20th century onwards. The salient features are the onset of rapid population growth and falling mortality (even before World War II), the acceleration, slowing and then decline of individual productivity, and the substantial but fitful expansion of education. The corresponding series provide much of the material that is indispensable, not only to the calibration of the model in Section 3, but also to the demographic projections in Section 4, the setting up of the benchmark cases in Section 5 and the analysis of policy interventions in Section 6.

The central emphasis placed here on the role of human capital is fully in keeping with Azam and Daubréé’s (1997) finding that its accumulation was the main force that drove economic growth in Kenya between 1964 - 1990. Their econometric analysis also reveals a sharp fall in the growth of total factor productivity in the second half of that period, which is consistent with the results of our calibration. It emerges from the latter that the slowdown and then subsequent decline in the economy after 1980 are attributable to two factors. First, there was a fall in the efficiency of the ‘technology’ for producing human capital young from inputs of schooling and the quality of child-rearing in the family (as measured by the parents’ own level of human capital). Second, and with a
lag, there was a fall in the productivity of existing human capital in producing aggregate output. Both bode ill for Kenya’s prospects, quite independently of the outbreak of the AIDS epidemic.

Section 5 analyses two benchmark cases for the period 1990 to 2050, namely, the counterfactual in which the epidemic never arose and the world in which breaks out. Neither involves any explicit form of intervention, though the latter rests heavily on the assumptions adopted by the researchers at the U.S. Bureau of the Census, whose projections form the basis of the benchmarks. AIDS does not bring about a catastrophic economic collapse, but it does inflict large economic costs – and very many deaths.

Independent, though indirect, support for our conclusion that the epidemic will exact a heavy economic toll is provided by micro-economic evidence that premature adult mortality in rural areas of Kenya has strongly adverse effects on school attendance and advancement from one grade to another. Yamano and Jaynes (2005) find the effect to be especially adverse among poorer households and young girls. Evans and Miguel (2005), who use a still larger and in some ways more detailed panel, albeit one restricted to a single district in which the HIV-prevalence rate is high, confirm the former finding but not the latter. They also establish that maternal death produces a bigger setback than a paternal one, and that children whose test scores were low before bereavement are more likely to be withdrawn from school. The cross-country evidence for sub-Saharan Africa yields the same general finding where schooling is concerned; see, for example, Kalemli-Ozcan (2006).

The question of how public policy can alter the course of the epidemic or mitigate its impact upon the accumulation of human capital is taken up in Section 6. Since health and education are clearly intertwined in this setting, the design of public policy should be concerned with both domains. The relationship between spending on combating the epidemic and the profile of age-specific mortality is speculative territory, for it involves the efficacy of various policy interventions that seek to reduce premature adult
mortality. There is little research that addresses this relationship as we have formulated it, so we have drawn on work for South Africa (Bell et al., 2006), which turns out to be transferable to Kenya. On this basis, we find that separate programmes to subsidise post-primary education and combat the epidemic are both socially profitable – the latter strikingly so, due to its indirect effects on the expected returns to education – and that a combination of the two profits from a modest synergy effect.

No single paper can hope to deal with all aspects of such a topic, so it will be helpful to say at the outset what will be left out. The intimate connection between risk-taking and demography in the context of AIDS is recognised, but not analysed: infection and demography are treated here as exogenous processes, in contrast to Young’s (2005) study of the South African epidemic. Nor do we go into the bargaining that takes place between the young and their parents. Since some rule for determining outcomes is unavoidable, we adopt a simple one. We also steer clear of learning on the job as a contributing factor to the accumulation of human capital, and of youth unemployment as a barrier thereto. For contributions to the macroeconomic literature on AIDS that use an OLG framework in which agents can also invest in physical capital, the reader is referred to Corrigan, Glomm and Méndez (2004, 2005).

2 The model

The model is a substantial extension of Bell, Devarajan and Gersbach (2006): there are now essentially three overlapping generations within an extended family, in which all surviving adults care for all related children. This arrangement reflects Kenya’s social structure reasonably well. There are two levels of the educational system, with a corresponding elaboration of the process through which human capital is formed. In applying the model, moreover, we choose a still finer unit time period than a generation, namely, the inter-censal interval of a decade. This permits a relatively detailed
treatment of demography, which plays a central role in the analysis.

Children are defined to be aged 5 to 14 (that is, centered on 10). This definition corresponds closely to the official span of primary education. The minimum school age in Kenya has been six for generations, and the period of primary schooling has likewise comprised eight years, with the exception of the interlude between 1966 to 1985, when it was a year shorter. Leaving out the five-year olds, assuming that many children, especially boys, will not be enrolled until their seventh birthday, and allowing for the fact that there has always been a fair amount of grade-repetition, full primary schooling according to schedule will nicely exhaust the decade in question.

An important object of this paper is to incorporate the effects, costs and benefits of human capital formation, especially among what can be called the ‘youth’, who are defined to be individuals aged 15 to 24. On reaching 15, the child has become a youth, put his primary education behind him and augmented his original endowment of human capital. A maximum of ten years is then available to build on this foundation before he settles into the duties and routines of mature adulthood.

2.1 Human capital and output

We begin by introducing some notation.

\[ N_a^t: \text{the number of individuals in the age-group } a \text{ in year } t (a = 0 \text{ for 0-4 year-olds, } a = 1 \text{ for 5-14 year-olds, } \ldots, a = 6 \text{ for 55-64 year-olds, } a = 7 \text{ for those over 64 }), \]

\[ \lambda_a^t: \text{the human capital attained by an individual in age-group } (a = 2, 3) \text{ in year } t \text{ on completing the decade corresponding to primary and higher education, respectively, } \]

\[ \gamma: \text{the human capital of a school-age child, } \]

\[ \alpha_t: \text{output per unit of human capital input in year } t, \]

\[ e_1^t: \text{the proportion of their school-age years (6-14) actually spent in primary school} \]
by the cohort of children who are aged 5-14 years in year \( t \),

\( e_i^2 \): the proportion of their youth (15-24) actually spent in secondary and tertiary education by the cohort aged 15-24 years in year \( t \),

\( z_t \): the transmission (or efficiency) parameter associated with the educational technology,

\( Y_t \): GDP in year \( t \),

\( y_t \): GDP per adult of working age (15-64) in year \( t \).

Some of these series and the associated sources have been discussed and set out in Bell et al. (2004). Those for \( \alpha_t \), \( \gamma \), \( z_t \) and \( \lambda_t \) must be estimated as part of the calibration.

Two difference equations govern the system’ dynamics. They involve the level of parents’ human capital and the educational technologies, one for primary, the other for secondary and higher education. The evidence suggests that parents in Kenya have most of their children when they are in their twenties. Since the cohort of children in primary school is centred on 10-year olds, we form the level of their parents’ human capital by taking the weighted average of the human capital of the age-groups \( a = 2, 3 \) in period \( t \). The human capital attained by a child on becoming a youth (\( a = 2 \)) in period \( t \) is given by

\[
\lambda_i^2 = 2z_{i-10}^1 f^1(e_{i-10}^1) \cdot [(N_{i-10}^2\lambda_{i-10}^2 + N_{i-10}^3\lambda_{i-10}^3)/(N_{i-10}^2 + N_{i-10}^3)] + 1, \tag{1}
\]

where \( f^1(\cdot) \) represents the educational technology at the primary school level and the transmission factor \( z_t^1 \) for the cohort of school-age children in period \( t \) is defined with respect to the parents’ combined human capital, normalised to a representative couple.

In the secondary and higher stages of the educational process, a plausible, general relationship is

\[
\lambda_i^3 = \Phi(e_{i-10}^2, \lambda_{i-10}^2, \lambda_{i-10}^3),
\]
where it is only the fully mature adults who influence the inter-generational transmission process at this stage. For the purposes of calibration, however, one needs a specification that is closely anchored to (1). It is arguable that, in the present framework, the transmission parameter $z_i^a$ does not vary much across educational levels – and anyway, there is no chance whatsoever of calibrating the model with the data available without the restriction $z_i^1 = z_i^2$. Where the educational technology is concerned, the assumption that $f^1(\cdot)$ continues to hold at higher levels of education, with argument $e_{t-20}^1 + e_{t-10}^2$, seems rather implausible for the following reasons. The cohort of school-going age in 1980 had completed, on reaching adulthood, just over six years of education on average, and earlier cohorts fewer still. It is also known that net enrollment rates in secondary and tertiary education were very low until recently (they are still modest at best). Up to 1990, therefore, the position was $e_t^1 < 1$ and $e_t^2 = 0$. Given the strong diminishing returns that set in when $f^1(\cdot)$ is strictly concave and $e_t^1$ approaches unity, this implies that the private returns to secondary education would be much lower than those to primary education. In fact, the opposite was the case in the 1990s (Appleton et al., 1999; Manda, 2002; Manda et al., 2004; Wambugu, 2003).

Faced with this difficulty, we proceed as follows. First, we note that completion of primary schooling, in the form of passing the State Examination at the end of Standard 8, is a necessary condition for entry into Form 1 in a secondary school: formally, $e_t^2 > 0$ only if $e_{t-10}^1 = 1$. Secondly, we impose two conditions on the function $f^2(\cdot)$, namely, that, given $e_{90}^1 = 1$ and the expectation of full primary education thereafter, it yield the observed net secondary enrollment rate in 2000 and that the asymptotic rate of growth of $\lambda_t^3$ when all cohorts enjoy full primary and secondary education ($e_t^1 = e_t^2 = 1 \forall t$) be the long-run rate estimated by Ndulu and O’Connell (2002), which is based on cross-country regressions (see Section 3 for the numerical derivation of the parameters).
Then, corresponding to (1), we have

\[
\lambda_t^3 = \begin{cases} 
\lambda_{t-10}^2 & \text{if } e_{t-10}^1 < 1 \\
2\zeta_{t-10}^1 \cdot f^2(e_{t-10}^2) \cdot \lambda_{t-10}^3 + \lambda_{t-10}^2 & \text{if } e_{t-10}^1 = 1
\end{cases}
\]  

(2)

where \( f^2(0) = 0 \).

At this point, a brief remark is needed on the asymptotic behaviour of \( \lambda_t^2 \) when there is full primary education but never any investment in secondary education. Observe from eqs. (1) and (2) that, starting from a sufficiently large value of the parents’ combined human capital, unbounded growth of \( \lambda_t^2 \) is possible only if \( 2\zeta_1 f^1(1) \geq 1 \) and that the growth rate then approaches \( 2\zeta_1 f^1(1) - 1 \) from above. If, however, \( 2\zeta_1 f^1(1) < 1 \), then \( \lambda_t^2 \) will approach the stationary value \( 1/(1 - 2\zeta_1 f^1(1)) \).

Turning to aggregate output, this takes the form of an aggregate consumption good. We assume its level is proportional to inputs of labor measured in efficiency units. A natural normalization is that an adult who possesses human capital in the amount \( \lambda_t^a \) is endowed with \( \lambda_t^a \) efficiency units of labor, which he or she supplies completely inelastically. Let a child supply \( (1 - e_t^1) \gamma \) efficiency units of labor when it works \( 1 - e_t^1 \) units of time, where \( \gamma \in (0, 1) \), i.e., a full-time working child is less productive than an uneducated adult. Ignoring unemployment, we then have the identity

\[
Y_t \equiv \alpha_t \left[ 0.9 N_t^1 (1 - e_t^1) \gamma + N_t^2 (1 - e_t^2) \lambda_t^2 + \sum_{a=3}^{6} N_t^a \gamma_{t+10(3-a)} \right],
\]  

(3)

where the scalar 0.9 reflects the fact that among the group \( a = 1 \), five year-olds can do very little in the way of useful work. The advantages of (3) are that it is free of any other behavioural assumptions and, if \( \alpha_t \) is specified exogenously, it is linear in \( \lambda_t^a \ (a = 2, 3) \) and \( \gamma \). It also fully reflects premature mortality among adults, whatever be its causes.
2.2 The family’s preferences and decisions

The next step is to specify the (extended) family’s preferences and its decision-making rules. Some additional notation is needed.

$c_t$: the consumption of the aggregate good by each young adult ($a = 2, 3, 4$),

$\beta$: the proportion of a young adult’s consumption received by each child under the age of 15,

$\rho$: the proportion of a young adult’s consumption received by each adult over the age of 44,

$\sigma_a^t$: the direct costs per child of each unit of full-time schooling ($a = 1, 2$),

$n_t$: the number of children born to a representative couple who survive to school age in period $t$,

$q_x t$: the probability that an individual aged $t$ will die before reaching the age of $t + x$,

$q_t$: the probability that a young adult in period $t$ will die before reaching the third phase of life,\(^1\)

$\tau_t$: the poll tax levied on each young adult.

The parameters $\beta$ and $\rho$ are viewed as contractually binding by all concerned and they are rigorously enforced by appropriate social sanctions.

A brief remark on the introduction of a poll tax is in order. Taxation and public expenditures in the period up to 1990 are already implicitly included in the calibration procedure in Section 3. For the period from 2000 onwards, we shall be concerned with measures to combat the spread of the epidemic and treat its victims, so that the means of financing them must enter into the reckoning. To keep matters simple, and to preserve consistency with the calibration in section 3, we use the time-honoured device

\(^1\)In the present structure, this statistic corresponds to $20q_{20}$, the probability that an individual will die before 40, conditional on surviving until 20.
of a poll tax, whose proceeds are used solely for the purpose of financing the measures in question. It should be noted that, by definition, the poll tax is zero not only in the historical period 1950-90, but also in the period 1990-2050 in the counterfactual case in which there is no epidemic. It is zero also in the base case in which the epidemic simply runs its course without any specific intervention by the government within the framework of the model.

The extended family’s expenditure-income identity involves expenditures on the aggregate consumption good and education, where the latter include both the direct expenditures and the opportunity costs of the students’ time. In what follows, it will be convenient to normalize the identity by the number of individuals in the first phase of adulthood in period \( t \), namely, by

\[
N^y_t \equiv \sum_{i=2}^{4} N^a_i. \tag{4}
\]

The said identity may then be written as:

\[
P_t(\beta, \rho) \cdot c_t + Q^1_t(\alpha_t, \gamma, \sigma^1_t) \cdot e^1_t + Q^2_t(\alpha_t, \lambda^2_t, \sigma^2_t) \cdot e^2_t + \tau_t \equiv \alpha_t \cdot (\Lambda_t + 0.9 N^1_t \gamma) / N^y_t, \tag{5}
\]

where

\[
P_t(\beta, \rho) \equiv [1 + \rho(N^5_t + N^6_t + N^7_t) / N^y_t + \beta(N^0_t + N^1_t) / N^y_t] \tag{6}
\]

is the ‘price’ of \( c_t \),

\[
Q^1_t \equiv (0.9\alpha_t\gamma + \sigma^1_t)N^1_t / N^y_t \tag{7}
\]

is the ‘price’ of a unit of full primary education, and

\[
Q^2_t \equiv (\alpha_t\lambda^2_t + \sigma^2_t)N^2_t / N^y_t \tag{8}
\]

can be interpreted as the ‘price’ of a unit of full secondary and tertiary education.
combined, all prices being normalised by $N^y_t$. The quantity

$$\Lambda_t \equiv N^2_t \lambda^2_t + N^3_t \lambda^3_t + \sum_{a=4}^{6} N^a_t \lambda^3_{t+10(3-a)}$$

(9)

is the aggregate human capital of all individuals over the age of 14. The RHS of (5) is the level of so-called (normalised) full income in year $t$. It is seen that the demographic structure plays a potentially important role in determining both relative prices and the level of full income.

We turn to the question of who decides how to allocate current full income. As noted above, Kenya’s parents have most of their children in their twenties and early thirties, so it is plausible that the age groups $a = 2, 3$ have the greatest say in making the decisions. Since the unit time period is a decade, the younger ones will be parties to a partial revision a decade later. To keep matters manageable, we assume that they take no heed of this fact when contributing to the current outcome. Observe also from (5) that those aged 35-44 also enjoy $c_t$ – although, by assumption, they have no say in the matter. That those aged 15-24 should have a strong say in their own education at this stage of their lives seems perfectly natural. Given these assumptions, the decision variables comprise the triple $(c_t, e^1_t, e^2_t)$, whereby the social rules governing the distribution of consumption among the three generations, as expressed by $\beta$ and $\rho$, are regarded by all participants as permanent.

The young adults’ preferences are, in principle at least, defined over four goods: the levels of consumption in young adulthood and old age, $c_t$ and $c_{t+20}$, respectively, and the human capital attained by their school-age children on attaining full adulthood ($\lambda^3_{t+20}$), which they may appreciate in both phases of their own lives. Investment in the children’s education therefore produces two kinds of pay-offs, namely, in the form of altruism, as expressed by the value directly placed on $\lambda^3_{t+20}$, and selfishly, inasmuch as an increase in $\lambda^3_{t+20}$ will also lead to an increase in $c_{t+20}$ under the said social rules.
Although the pooling arrangement implicit in the extended family structure and the law of large numbers combine to eliminate some risks, others remain. A young adult still faces uncertainty about whether he or she will actually survive into the last phase of life, and whether his or her children will do likewise, conditional on their reaching adulthood in their turn. There is also arguably uncertainty about future demographic developments, which will influence the realised levels of $c_{t+20}$ and $\lambda^3_{t+20}$.

To make the analysis tractable, we resort to a number of further assumptions. First, and plausibly, the parents’ altruistic motive makes itself felt in their preferences only when they themselves are young and actually make the sacrifices. Second, their preferences are additively separable in $c_t$, $\rho c_{t+20}$ and $\lambda^3_{t+20}$, and also conform to the expected utility hypothesis. Third, the partners making up the representative young couple in the extended family regard consumption in both phases as a private good, but the human capital attained by each of their children on attaining adulthood as a public good within the partnership. Fourth, in keeping with common practice in the macroeconomic literature, the sub-utility functions for consumption are logarithmic in form. Fifth, the sub-utility function representing altruism through $\lambda^3_{t+20}$ belongs to the iso-elastic family with parameter $\eta$:

$$\phi(\lambda^3_{t+20}) = 1 - (\lambda^3_{t+20})^{-\eta}/\eta, \quad (10)$$

where the fact that $\eta$ is to be estimated lends the system substantial flexibility, especially by allowing the two types of sub-utility functions to exhibit different degrees of curvature. The preferences of a young adult at time $t$ are then represented as follows:

$$E_t U = b_1 \ln c_t + b_2 (1 - q_t) E_t (\ln \rho c_{t+20}) + (1 - q_{t+20}) n_t \phi(\lambda^3_{t+20}), \quad (11)$$

where the taste parameters $b_1$ and $b_2$ are also to be estimated by calibration. It should be noted, first, that the ‘pay-offs’ in the event that the parent should die prematurely
(with probability $q_t$), or that the children, in their turn, should die prematurely in adulthood (with probability $q_{t+20}$), have been normalized to zero, and second, that $c_{t+20}$ is also a random variable for those who survive into old age, by virtue of the fact that its level depends on a whole variety of future economic and demographic developments.

The individual takes all features of the environment in periods $t$ and $t + 20$ as parametrically given. It will be helpful to summarize the latter in order to distinguish between what the individual knows and what he or she must forecast. The current environment is described by the vector

$$Z_t \equiv (P_t, Q_{1t}^1, Q_{1t}^2, N_t^a (a = 0, ..., 7), n_t, \alpha_t, \Lambda_t, \tau_t).$$  \hfill (12)

This assumed to be known. The future environment, as described by the vector

$$Z_t^e \equiv (N_{t+20}^a (a = 0, ..., 7), \alpha_{t+20}, q_t, q_{t+20}, \tau_{t+20}),$$  \hfill (13)

must be forecast, where it should be noted that forecasting $N_{t+20}^a$ on the basis of knowing $N_t^a$ implicitly involves forecasting cohort-specific mortality rates over the period from $t$ to $t + 20$. For simplicity, the individual’s forecasts of all elements of the future environment are assumed to be point estimates: equivalently, there is uncertainty about individual premature mortality, but none about the future mortality profile itself or future fertility. The world being what it is, this assumption is a lot to swallow; but the calibration will be virtually impossible without it.

Investments in education at the secondary and tertiary levels in period $t$ will yield pay-offs to both age groups in future periods, albeit tempered by the chances of an untimely death, and these expectations will enter into their calculations. Since implementing the whole under full rational expectations would entail a research project in
itself, we choose a (comparatively) simple procedure. As assumed above, consumption in the second phase of adult life, namely, $\rho c_{t+20}$, is determined by a social rule under which the decisions of neighbouring cohorts of young adults have a strong influence on the outcome. Given the complexity of the associated forecasting problem with which young adults are confronted, we assume that they approach it as follows.

First, $\rho c_{t+20}$ is taken to be a fixed fraction, $\zeta$, of full income per adult in the first phase of life in period $t + 20$, net of taxes:

$$\rho c_{t+20} = \zeta c_{t+20} (A_{t+20} + 0.9 N_{t+20}^1 \gamma - \tau_{t+20}) / N_{t+20}^1.$$  \hspace{1cm} (14)

Observe that the terms $N_{t+20}^2 \lambda_{t+20}^2$ and $N_{t+20}^3 \lambda_{t+20}^3$ in $A_{t+20}$ make up the greater part of the new human capital that will arise in period $t + 20$, and these depend on the decisions of young adults not only in period $t$, but also those in $t + 10$. The term $N_{t+20}^4 \lambda_{t+10}^3$, in contrast, depends only on current decisions – given those made earlier in periods $t - 10$ and $t - 20$. To bring home this point, it is useful to define

$$A_{t+20}(e_1^t, e_2^t, e_{t+10}^1, e_{t+10}^2) \equiv N_{t+20}^2 \lambda_{t+20}^2(e_1^t, e_{t+10}^1) + N_{t+20}^3 \lambda_{t+20}^3(e_1^t, e_{t+10}^2) + N_{t+20}^4 \lambda_{t+10}^3(e_1^t, e_{t+10}^2) + N_{t+2}^6 \lambda_{t-10}^3.$$  \hspace{1cm} (15)

The results of decisions made by cohorts of young adults in preceding periods are, of course, known to the young adults at $t$, but what pair $(e_{t+10}^1, e_{t+10}^2)$ the cohort that follows will choose is something that the young adults at $t$ must guess when deciding on $(e_t^1, e_t^2)$. This brings us to the second step: the young adults at time $t$ are assumed to have stationary expectations concerning $(e_{t+10}^1, e_{t+10}^2)$ – and more distant future choices, to the extent that these are relevant. That is, they employ the rule

$$E_t e_{t+10}^a = e_t^a, \quad a = 1, 2.$$  \hspace{1cm} (16)
In arriving at their decision at $t$, however, they take $E_t e_{t+10}^a$ as parametrically given, that is, they view $\Lambda_{t+20}$ as only directly influenced by $(e_t^1, e_t^2)$ when they calculate the optimum. Only after the optimum has been found does (16) come into play. Having cut this particular Gordian knot, we can then solve the model sequentially, one period at a time.

The said individual’s decision problem takes the following form:

$$\max_{((c_t, e_t^1, e_t^2), z_i)} \quad E_t U$$

subject to: $c_t \geq 0, e_t^a \in [0, 1], a = 1, 2, (1), (2), (5), (14)$. If $f^1(\cdot)$ is strictly concave, $E_t U(\cdot)$ will be strictly concave in $(c_t, e_t^1, e_t^2)$. Hence, problem (17) has a unique solution and the first-order necessary conditions are also sufficient.

Observe that the optimum always involves $c_t > 0$; but the corner solution in which the children are not educated at all can also be ruled out when $f^1(\cdot)$ satisfies the lower Inada condition, since $\partial \lambda_{t+10}^3 / \partial e_t^1$ is then unbounded at $e_t^1 = 0$. What cannot be ruled out, however, is the possibility that $e_t^2 = 0$ – indeed, as $e_t^2$ is defined on the basis of the data, this was precisely the state of affairs until 2000.

The associated Lagrangian is

$$L_t = E_t U + \mu_t [\alpha_t (\Lambda_t + 0.9 N_t^1 \gamma) / N_t^2 - P_t c_t - Q_t^1 e_t^1 - Q_t^2 e_t^2 - \tau_t].$$

The first-order conditions are

$$b_t / c_t - \mu_t P_t = 0,$$

$$b_2 \cdot \frac{1 - q_t}{c_{t+20}} \cdot \frac{\partial c_{t+20}}{\partial c_t} + \frac{(1 - q_{t+20}) n_t}{(\lambda_{t+20}^3)^{\gamma+1}} \cdot \frac{\partial \lambda_{t+20}^3}{\partial e_t^1} - \mu_t Q_t^1 \geq 0, e_t^1 \leq 1 \text{ compl.}$$

and

$$b_2 \cdot \frac{1 - q_t}{c_{t+20}} \cdot \frac{\partial c_{t+20}}{\partial c_t} + \frac{(1 - q_{t+20}) n_t}{(\lambda_{t+20}^3)^{\gamma+1}} \cdot \frac{\partial \lambda_{t+20}^3}{\partial e_t^2} - \mu_t Q_t^2 \geq 0, e_t^2 \geq 0 \text{ compl.}$$

16
As can be seen from (1), (2) and (15), the derivatives of $c_{t+20}$ and $\lambda_{t+20}^3$ involve very long and messy expressions, which are suppressed here.\(^2\)

To close this section, a remark on the (non)-uniqueness of the expectations path is called for. Although the solution to problem (18) is unique for any given set of expectations, only when the solution satisfies (16) is it admissible as an equilibrium under stationary expectations. The imposition of (16) when solving the first-order conditions therefore opens up the possibility that there is more than one pair $(e_1^t, e_2^t)$ that will satisfy them – in effect, there may be more than one equilibrium path.

3 Calibration

To determine the parameters and past variables of interest, we need to go back deep into the past century, and since censuses were carried out in 1948, 1962, 1969, 1979, 1989 and 1999, it is natural to choose the time-points, or periods, as $t = 10, 20, \ldots, 100, \ldots$, where 100 denotes the year 2000. Morbidity and mortality from AIDS became significant after 1990, so that some aspects of the calibration of the model may be restricted to the period 1910 to 1990. We proceed in two steps.

3.1 Step 1: Human Capital and Output

Eqs. (1) and (3), together with the specialisation of $f^1(\cdot)$ to the iso-elastic form $(e_1^1)^\epsilon$, provide the basis for the first stage of the calibration of the system’s parameters and the historical values of $\lambda^a_t$. The first, preliminary step is to smooth out the considerable fluctuations in the Kenyan economy. In keeping with the unit time period of a decade, the values of $Y_t$ are obtained by forming 5-year moving averages of GDP per caput from the Penn World Tables (hereafter, PWT) and multiplying them by the population.

\(^2\)They are available upon request.
series. Since the PWT series run from 1950 to 2000, the resulting estimates for those boundary years contain an element of further guesswork, which is based on inspection of the entire series. The smoothed estimates are reported in Table 1.

Starting with \( t = 50 \) in (3), we need \( \lambda_{a10}, \ldots, \lambda_{a50} \): this is the deepest foray back into the 20th Century. At \( t = 10 \), apart from a few colonial administrators, missionaries and white settlers, not even the youngest adults had any education at all (Sheffield, 1973; Thias and Carnoy, 1972). This may be termed a ‘state of economic backwardness’, in which, by definition, \( \lambda^a = 1 \) for all age-groups. Hence, we anchor the system to \( \lambda_{a10} = 1 \). The history of education in Kenya also leaves no doubt that \( \lambda_{a20} \) was scarcely above unity, so we shall set it exogenously, at a value to be determined in the light of the series for \( e_{at} (a = 1, 2) \).

To obtain \( \lambda_t^a \) for \( t = 30 \) onwards, only (1) need come into play. For the fact that \( e_{t}^2 = 0 \) up to and including 2000 implies that \( \lambda_{t+10}^3 = \lambda_t^2 \forall t \leq 100 \). Since we have set \( \lambda_{a10} \) and \( \lambda_{a20} \) exogenously and have estimates of \( e_{t}^1 \) for \( t = 20 \) onwards from the Censuses, we can employ (1) starting with \( t = 30 \) if we have estimates of the relative sizes of the age-groups 15-24 and 25-34, respectively. In 1950 and 1960, the said ratio was 57:43, which we assume to have held also in the earlier decades of the 20th century. Thus, with the above assumptions, we can indeed obtain \( \lambda_t^2 \) from \( t = 30 \) onwards by choosing the plausible value \( \lambda_{a20}^2 = 1.01 \).

Since (1) is a purely technical relationship and (3) is effectively an identity, we can employ both for the entire historical period 1950-2000, even though AIDS had begun to make itself felt by 1990. Eqs. (1) and (3) then yield 14 equations in the following 26 unknowns: \( (\lambda_{30}, \ldots, \lambda_{100}); (\alpha_{50}, \ldots, \alpha_{100}); (z_{20}^1, \ldots, z_{50}^1); \epsilon, \gamma \). Some restrictions on the latter are therefore unavoidable if we are to solve the system.

We employ the following criteria to define the restrictions and select the solution. First, we use additional information to identify the possible points in time when struc-
tural breaks in $\alpha_t$ and $z_t$ occurred. Second, we vary $\epsilon$ exogenously and the timing of a possible structural break in $z^1_t$ between 1920 and 1970, and then reject any values of $\epsilon$ that imply infeasible economic values such as negative human capital. This produces an admissible range of $\epsilon$. Third, we vary $\epsilon$ exogenously by a grid search within this range and the timing of the structural break in $z^1_t$ between 1920 and 1970 until we find solutions in which $b_1$ and $b_2$, which will be estimated in step 2, are in keeping with the results of other studies in which the pure rate of time preference plays a key role. In other words, the value of $\epsilon$ and a possible structural break are anchored by well-known time preference relationships in the literature through systematic sensitivity analysis.

Inspection of the series for the level of GDP per adult aged 15-64,

$$y_t \equiv Y_t / \sum_{a=2}^{6} N^a_t,$$  \hspace{1cm} (22)

which is set out in Table 1, reveals that Kenya suffered, first, a marked slowdown in the growth of $y_t$ in the 1980s, and then a further turn for the worse in the form of an actual fall in $y_t$ the 1990s. Since $e^1_t$ continued to rise somewhat, even up to 1990, this slowdown and subsequent decline must have arisen from one or more sharp reductions in $\alpha_t$ or $z^1_t$ after 1980. With a peak to be explained, it is hard to pin the blame on the productivity factor $\alpha_t$ alone. Rather, intuition suggests that the transmission factor $z^1_t$ may have declined first, thereby braking the growth of $\lambda_t$, with a subsequent fall in $\alpha_t$ to produce the retreat in $y_t$. There are good grounds for supposing that the quality of schooling declined substantially in the late 1970s, with a surge in enrolments, overcrowded classrooms and an influx of poorly trained teachers. As a working hypothesis, therefore, we impose the following restrictions:

$$\alpha_{50} = \alpha_{60} = \alpha_{70} = \alpha_{80} = \alpha_{90}; \text{ and } z^1_{80} = z^1_{90},$$  \hspace{1cm} (23)

with the possibility of an earlier break in $z^1_t$ between 1920 and 1970. These restrictions
leave us with the following 15 unknowns:

$\lambda_{30}, \ldots, \lambda_{100}; \alpha_{50}, \alpha_{100}; z_{20}, z_{80}; \epsilon, \gamma$; and one structural break in $z_{30}, \ldots, z_{70},$

where the superscript on $z_t$ can now be suppressed without introducing ambiguity. In the light of the results of the calibration procedure described above, we confined $\epsilon$ to the range 0.4 to 0.6. Outside this range, the solutions have unacceptable features, for example, some parameters or levels of human capital (including $\gamma$) become negative.

3.2 Step 2: Preferences

In a purely formal sense, the first stage of the calibration can be carried out quite independently of the second. It turns out, however, that the choice of $\epsilon$ and the degree of freedom conferred by the structural break in $z_{30}, \ldots, z_{70}$ combine to produce a substantial effect upon the results from the second stage, which involves the preference parameters $b_1, b_2$ and $\eta$. We continue, therefore, by deriving a condition from which $b_1, b_2$ and $\eta$ can be estimated, given the results from the first stage; only then do we choose and report some combined results.

Recalling that $e^2_t = 0$ during this period, we may drop (21) and seek an expression in $e^1_t$ alone. Some manipulation using (14), (15), (19) and (20) yields

$$\left(\frac{1}{\lambda_{t+10}^2(e^1_t)} \cdot \frac{\partial \lambda_{t+10}^2}{\partial e^1_t}\right) \cdot \left(\frac{(1 - q_{t+20})n_t}{\lambda_{t+10}^2(e^1_t)n^2} + \frac{(1 - q_t)b_2}{(A_{t+20} + 0.9N_{t+20}^1)/(N_{t+20}^3\lambda_{t+10}^2(e^1_t))}\right) \geq \frac{b_1 Q^1_t}{\alpha_t[(A_t + 0.9N_t^1\gamma)/N_t^3] - Q^1_t e^1_t - \tau_t}$$

where the first term on the LHS is obtained from (1) and, apart from a limiting case, (24) holds as a strict equality when $e^1_t < 1$. Observe that (24) is independent of the social sharing rules, as expressed by the parameters $\beta$ and $\rho$, a feature that stems from the choice of a logarithmic sub-utility function for consumption.

The first stage of the calibration yields complete descriptions of the current envi-
ronment for the years 1950, 1960, 1970, 1980 and 1990. Where the associated future environments are concerned, we assume that agents possessed perfect foresight, except for the timing of the outbreak of the epidemic and its immediate consequences. In this regard, the data in Table 1 and the demographic data up to 1990 suffice for 1950 and 1960. Thereafter, we need progressively more. In view of (11), we require $q_{70+20}$ for 1970, which involves the population pyramid for 2010; for 1980, we require the entire population pyramid for 2000 and $q_{80+20}$, whereby the latter involves $N_{120}^3$; and we require the corresponding items for 1990. All these elements are estimated in Bell et al. (2004), and $q_t$ is reported in Table 1. As for the question of whether the outbreak of the epidemic featured in expectations about the future at that time, it is surely safe to rule out this possibility – except, possibly, for 1990. The cumulative number of deaths due to AIDS was still rather small in relation to the population in 1990, and the prevalence rate at that time was also low. It seems very plausible, therefore, that the overwhelming majority of Kenyans had not, at that time, come to recognise the dimensions of the wave of mortality that was about to befall them. In any case, that is the position we adopt for the purposes of calibration.

It still remains to estimate the series $\{\sigma^1_t\}_{t=50}^{t=90}$. According to a survey of some 6000 farm households, representatively drawn from districts in six of Kenya’s eight provinces in 1981–82, no less than 20 percent of total expenditure on average was devoted to education (Evenson and Mwabu, 1995: 12-15). For the purposes of calibration, we assume that this proportion stayed constant from 1950 until the mid 1980s, though it seems a bit questionable in the early years, when school-places were rather rationed. By the mid 1990s, there is reliable evidence that this budget proportion had fallen substantially (Kenya, 1996). In calibrating the model, therefore, no use will be made of the year 1990.

The number of equivalent, full-time schoolchildren in period $t$ is $e_1^t N_1^t$. Using the budget share 0.20 and recalling that, by assumption, the whole of GDP accrues to
households in the first stage of the calibration, we have \( \sigma_1^t c_1^t N_1^t = 0.2Y_t \), or

\[
\sigma_1^t = 0.2Y_t / c_1^t N_1^t. \tag{25}
\]

Substituting for \( \sigma_1^t \) in (20) and rearranging, we obtain

\[
\left( \frac{1}{\lambda_{t+10}(c_1^t)} \cdot \frac{\partial \lambda_{t+10}^2}{\partial c_1^t} \right) \cdot \left( \frac{(1 - q_{t+20})n_t}{\lambda_{t+10}^2(c_1^t)^n} + \frac{(1 - q_t)b_2}{(\Lambda_{t+20} + 0.9N_{t+20}^1\gamma) / N_{t+20}^3 \lambda_{t+10}^2(c_1^t)} \right) \\
\cdot \left( \Lambda_t + 0.9N_1^1\gamma \left[ \frac{0.9\gamma + 0.2Y_t / (c_1^t \alpha_t)]N_t^1}{N_t^1} - c_t \right) \right) \geq b_1, \tag{26}
\]

where the poll tax \( \tau_t \) is zero throughout the period 1950-90. Since only interior solutions are observed in this period, (26) will hold as a strict equality for the purposes of calibration.

### 3.3 Selection

As noted above, the values of the three parameters \( b_1, b_2 \) and \( \eta \) are determined by the values of other parameters and \( \lambda_t \) that emerge from the first stage of the calibration. For each of a wide variety of results from the first stage, we obtained the corresponding triple \( (b_1, b_2, \eta) \) by selecting three from the four years 1950, 1960, 1970 and 1980, and then applying (26) to the years in question. The only apparent reservation about deriving the triples \( (b_1, b_2, \eta) \) from (26) is that there is some evidence that school-places were rationed somewhat in 1950 and 1960. In judging the constellation of values from both stages combined, we note that the quantity \( 1 - b_2/b_1 \) is the rate of pure impatience, for the probability of prematurely dying as an adult, \( q_t \), is already allowed for in the specification of \( E_t U \). Although we are unaware of any attempts to estimate this rate for Kenya, two well-known studies for the U.S. suggest that it lies between 0.25 and 0.5 per cent per annum (Altig et al., 1996; Fullerton and Rogers, 1993). On this basis, we reject any constellation for Kenya that yields an implied rate of pure impatience much
exceeding 1 per cent per annum. Tables 1 and 2 report two plausible constellations that meet this criterion. In the first, the implied rate is 1.23 per cent, in the second, 1.26 per cent per annum. It is also noteworthy that \( \phi(\cdot) \) is roughly midway between the logarithmic (\( \eta = 0 \)) and the Ramsey (\( \eta = 1 \)) cases, which confirms the importance of not forcing the entire preference structure into a logarithmic straitjacket.

The adverse developments in the production of human capital and output that began around 1980 express themselves in the behaviour of \( z_t \) and \( \alpha_t \). In solution 1, the first break, in 1960, is a small one; it is followed by a very sharp fall in 1980, which leaves \( z \) at barely half its previous value. The story in solution 2 is very similar, albeit with a much earlier and sharper first break, in 1940. The fall in \( z \) between 1970 and 1980 leads to a corresponding fall in \( \lambda_2 \) between 1980 and 1990, in both solutions of almost 20 per cent. These troubles are compounded by a decline in \( \alpha \) between 1990 and 2000, also by just under 20 per cent.

A particularly depressing feature of both solutions is that the value of \( z_t \) lies some way below the critical value of 0.5 from 1980 onwards. Recall from section 2 that unbounded growth through primary education alone is possible only if \( 2z^1 f^1(1) \geq 1 \). Since \( f^1(1) = 1 \) under the specialisation adopted for the purposes of calibrating the model, it follows at once that the said condition is violated in both solutions, so that a continuation of these values of \( z^1_t \) without investments in secondary education imply that \( \lambda^2_t \) will converge, at best, on the stationary values \( 1/(1 - 2 \times 0.41) = 5.56 \) and \( 1/(1 - 2 \times 0.43) = 7.14 \), respectively. This finding indicates the pressing importance of improving the performance of the primary school system and of promoting secondary schooling in order to ensure long-run growth. To complete the picture, note that the productivity of a school-aged child, \( \gamma \), is estimated to be about 70 per cent of that of an uneducated adult in both solutions. This seems a little high, perhaps, but it is not outlandish.

Summing up, there is little to choose between solutions 1 and 2, except that the
Table 1: Calibration: solution 1

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<td>0.163</td>
<td>0.154</td>
<td>0.141</td>
<td>0.127</td>
<td>0.113</td>
<td></td>
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<tr>
<td><strong>Endogenous:</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
<td>3.16</td>
</tr>
<tr>
<td>$b_2$</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
<td>2.46</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.47</td>
<td>0.47</td>
<td>0.47</td>
<td>0.47</td>
<td>0.47</td>
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<td>0.47</td>
<td>0.47</td>
<td>0.47</td>
</tr>
</tbody>
</table>
course of $z_t$ is noticeably smoother in the former, so we plump for 1.

### 3.4 Secondary Education

Recall that the function $f^2(\cdot)$ is assumed to satisfy the conditions that it yield the observed net secondary enrollment rate in 2000, namely, $e_{100}^2 = 0.065$, and that the asymptotic rate of growth of $\lambda_t^3$ when all cohorts enjoy full primary and secondary education ($e_t^1 = e_t^2 = 1 \ \forall \ t$) be the long-run rate of growth of GDP per head, as estimated by Ndulu and O’Connell (2002), namely, 2 per cent per annum. In view of this modest growth rate, it is clear that $f^2(e_t^2)$ must be concave but flexible. Let it take the following form, with two free parameters:

$$f^2(e_t^2) = a_1 \ln \left( \frac{e_t^2}{1 + a_2 e_t^2} + 1 \right).$$

Then (2) specialises to

$$\lambda_t^3 = \begin{cases} 
\frac{\lambda_{t-10}^2}{2z_{t-10}} & \text{if } e_{t-10}^1 < 1 \\
2z_{t-10}^1 \cdot \left[ a_1 \ln \left( \frac{(1+a_2)e_{t-10}^2 + 1}{1 + a_2 e_{t-10}^2} \right) \right] \cdot \lambda_{t-10}^3 + \lambda_{t-10}^2 & \text{if } e_{t-10}^1 = 1
\end{cases} \quad (27)$$

The parameters $a_1$ and $a_2$ are obtained from the two conditions as follows. The household’s first-order condition with respect to $e_{100}^2$, namely, (21) with $e_{100}^1 = 1$, holds with equality when $e_{100}^2 = 0.065$. The second condition involves finding an expression for $\lim_{t \to \infty} \lambda_{t+10}^3 / \lambda_t^3$. Noting that $N_{t}^2 \approx N_{t}^3$ in steady-state growth with a stationary population, some tedious manipulation yields

$$\lim_{t \to \infty} \frac{\lambda_{t+10}^3}{\lambda_t^3} = \frac{(2f^2(1) + 1)z_t}{1 - z_t + 2f^2(1)(z_t)^2}. \quad (28)$$

Given the fact that the system converges rather slowly to the asymptotic growth rate from above, its growth rate will exceed Ndulu and O’Connell’s estimate of 2 per cent
a year even over the later stretch of our time horizon. We have

\[
\lim_{t \to \infty} \frac{\lambda_{t+10}^3}{\lambda_t^3} = 1.02^{10} = 1.219.
\]

This value, together with the lower branch of (27) for \( e_{100}^2 = 0.065 \), completes the second condition. The resulting values of the parameters are \( a_1 = 1.522 \) and \( a_2 = 0.560 \).

4 Demographic Projections for 1990 - 2050

In order to estimate the effects of the epidemic and to evaluate public policies designed to combat it, we need the corresponding demographic projections. These include the natural counterfactual case in which the epidemic never breaks out and variations in which it does so – with or without policy interventions. In undertaking this rather speculative task, we shall draw heavily on the projections made by the U.S. Bureau of the Census (USCB) for the period 1985-2050. It should also be noted that the formal economic analysis of the period 1990-2050 requires demographic forecasts for the period 2000-2070.

In what follows, the counterfactual ‘without the epidemic’ and the actual case ‘with the epidemic’ will be denoted by \( D = 0 \) and \( D = 1 \), respectively. Both projections are based on a whole variety of assumptions, which are known to its authors at the USCB, but are not reported in the data file made available to us.\(^3\) This omission is not especially problematic when analysing \( D = 0 \) and \( D = 1 \) as benchmarks, since the formal description is clear enough. It does, however, confront us with serious difficulties when attempting to construct and analyse alternative policies, for they may be partly or fully incorporated in the USCB’s variant of \( D = 1 \) itself. In any event, the relationship between the level of spending on combating the disease and mortality that

\(^3\)This in no way reflects badly on the authors, of course, for such background material hardly belongs in tabular files of this kind.
we construct in Section 6 is to be interpreted as representing the effects of measures not contained in that variant of $D = 1$.

By 1990, it was apparent that the epidemic had started to take hold, though the number of annual deaths was still very modest and the virulence of its course was difficult to forecast. This date is therefore not only a natural ‘break point’ in the calibration of the model, but also a natural point of departure for the construction of alternative demographic projections.

4.1 The projections

In view of the differences between our ‘revised’ estimates of the demographic structure for 1990 and the USCB’s (see Bell et al. [2004]), and given the need to ensure a smooth transition into the period 2000-2050 when the unit of time is a decade, we choose the former population pyramid for both $D = 0$ and $D = 1$ for 1990 – a small inconsistency.

Beginning with $D = 0$, a comparison of the age-specific mortality rates implied by the ‘revised series’ for 1950-90 with those implied by the USCB’s estimates for 1990 and 2000 reveals that the former are so much higher than the latter that reconstruction of the USCB’s counterfactual estimates for 2000-2050 rather than mere smoothing is called for. This preliminary conclusion is reinforced by the fact that the USCB’s relatively low initial levels are also followed by steep projected declines in age-specific mortality rates out to 2050. In reconstructing the USCB’s estimates for the counterfactual in the direction of higher mortality, we err, therefore, on the side of optimism.\(^4\)

To anchor the system at the other end of the horizon, at 2070, we assume that in the absence of the epidemic, Kenya would have enjoyed the profile of age-specific mortality rates prevailing in the U.S. in 2000, as reported by the WHO (2002): that is surely a

\(^4\)The UN’s implicit estimates of mortality rates for the period 1990-2000 are even lower than the USCB’s, especially among young adults. For those aged 25 to 34 in 1990, for example, the USBC’s survival rate is 80.9 percent, whereas the UN’s is 90.8 percent (UN, 2004).
sufficiently optimistic view of the long-term prospects for good health in Kenya. The mortality profiles in the intervening period are obtained by simple linear interpolation. The pyramid for the year 2000 is generated by applying the corresponding mortality profile to the cohorts aged 5 to 54 in 1990, and then inserting the cohort sizes for children under 15 and adults over 64 taken directly from the USCB \( (D = 0) \) series itself. This process is then applied to the resulting pyramid for 2000 in order to yield the pyramid for 2010, and the procedure is repeated up to 2050. The pyramids for 2060 and 2070 are obtained by making the assumption that the cohorts of 0-4 and 5-14 year-olds, as well as those over 64, remain at their sizes for 2050, which is in keeping with the projection that the population will have almost reached a stationary configuration by that date. It is readily admitted that this mixing of cohort sizes at the top and bottom of the pyramids generated by one set of assumptions with mortality profiles generated by quite another is inconsistent, but inspection of Table 3 in relation to the USCB’s counterfactual does not reveal any departures so egregious as to call into question the implicit assumption of identical fertility.

The corresponding series for \( D = 1 \) from 2000 onwards is obtained in similar fashion, except that the USCB’s mortality profiles are taken over almost in their entirety. The sole amendment concerns \( 10q_{10} \) in 1990, which appears to be implausibly high in view of the fact that among all age groups, children of school-going age are the least likely to die of AIDS. The said statistic is set instead at its level in 1980. The pyramids for 2060 and 2070 are generated using the mortality profiles for 2040. In this connection, it should be remarked that the mortality rates for adults between 15 and 44 in the year 2040 are between 50 and 100 percent higher than those that would rule in the absence of AIDS: the USCB’s projections are based on the assumption of a continuation of the epidemic beyond the middle of this century, albeit on a greatly weakened scale.

The above procedure above somewhat understates the mortality due to AIDS, as estimated by the USCB at least. For the level of mortality in Bell et al.’s (2004) ‘revised’
Table 3: Projected population pyramids, No AIDS

<table>
<thead>
<tr>
<th></th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
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<tbody>
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<td>0-4</td>
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<tr>
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<td>8995</td>
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<td>8968</td>
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<td>8715</td>
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</tr>
<tr>
<td>15-24</td>
<td>4715</td>
<td>6875</td>
<td>8666</td>
<td>9236</td>
<td>8744</td>
<td>8759</td>
<td>8807</td>
<td>8601</td>
<td>8644</td>
</tr>
<tr>
<td>25-34</td>
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<td>4447</td>
<td>6526</td>
<td>8277</td>
<td>8878</td>
<td>8458</td>
<td>8525</td>
<td>8624</td>
<td>8475</td>
</tr>
<tr>
<td>35-44</td>
<td>1833</td>
<td>2731</td>
<td>4116</td>
<td>6098</td>
<td>7809</td>
<td>8454</td>
<td>8129</td>
<td>8269</td>
<td>8442</td>
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<tr>
<td>45-54</td>
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<td>1634</td>
<td>2463</td>
<td>3755</td>
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<td>55-64</td>
<td>698</td>
<td>911</td>
<td>1379</td>
<td>2115</td>
<td>3280</td>
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<td>6581</td>
<td>7323</td>
<td>7234</td>
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<tr>
<td>65+</td>
<td>511</td>
<td>864</td>
<td>1294</td>
<td>1969</td>
<td>3131</td>
<td>5266</td>
<td>8395</td>
<td>8395</td>
<td>8395</td>
</tr>
</tbody>
</table>

Source: Bell et al. (2004)

Table 4: Projected population pyramids, AIDS

<table>
<thead>
<tr>
<th></th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
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<td>0-4</td>
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<td>4556</td>
<td>3874</td>
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<td>5-14</td>
<td>7182</td>
<td>8612</td>
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<td>6578</td>
<td>6258</td>
<td>5971</td>
<td>5971</td>
<td>5971</td>
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<tr>
<td>15-24</td>
<td>4715</td>
<td>6839</td>
<td>8209</td>
<td>8040</td>
<td>6912</td>
<td>6429</td>
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<td>5902</td>
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<td>25-34</td>
<td>2979</td>
<td>4187</td>
<td>5780</td>
<td>6912</td>
<td>6997</td>
<td>6360</td>
<td>6182</td>
<td>5948</td>
<td>5676</td>
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<tr>
<td>35-44</td>
<td>1833</td>
<td>2410</td>
<td>3052</td>
<td>4136</td>
<td>5266</td>
<td>5873</td>
<td>5878</td>
<td>5713</td>
<td>5498</td>
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<tr>
<td>45-54</td>
<td>1099</td>
<td>1487</td>
<td>1760</td>
<td>2173</td>
<td>3125</td>
<td>4372</td>
<td>5352</td>
<td>5357</td>
<td>5207</td>
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<tr>
<td>55-64</td>
<td>698</td>
<td>906</td>
<td>1149</td>
<td>1361</td>
<td>1746</td>
<td>2666</td>
<td>3956</td>
<td>4843</td>
<td>4847</td>
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<tr>
<td>65+</td>
<td>511</td>
<td>821</td>
<td>1087</td>
<td>1362</td>
<td>1693</td>
<td>2353</td>
<td>3657</td>
<td>3657</td>
<td>3657</td>
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</tbody>
</table>

Source: Bell et al. (2004)
series, to which our modification of the USCB \((D = 0)\) series is closely anchored until 2020, is somewhat higher than that in the USCB’s \((D = 0)\) series itself. To this extent, the series for \(D = 1\) in Table 4 represents a more conservative estimate of the level of mortality in the presence of AIDS than the USCB’s \((D = 1)\), since by no means all those who die of AIDS would have died at the same age of other causes. The use of the series in Tables 3 and 4 as reference cases therefore involves a somewhat more conservative stance than that implicit in the USCB series where both the size of the initial ‘shock’ to mortality and the general level of mortality are concerned. With this reservation, the size of the shock to the level of premature adult mortality can be represented by the difference between the summary statistics \(20q_{20}(D = 1)\) and \(20q_{20}(D = 0)\). These statistics are reported in Table 5.

<table>
<thead>
<tr>
<th></th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO AIDS</td>
<td>(20q_{20})</td>
<td>0.127</td>
<td>0.113</td>
<td>0.099</td>
<td>0.085</td>
<td>0.070</td>
<td>0.056</td>
</tr>
<tr>
<td>AIDS</td>
<td>(20q_{20})</td>
<td>0.353</td>
<td>0.395</td>
<td>0.359</td>
<td>0.270</td>
<td>0.154</td>
<td>0.111</td>
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Inspection of this wave of mortality reveals that it is large indeed, but of comparatively short duration in relation to an inter-generational period of 20 years, in which, by hypothesis, young adults make decisions with a horizon of 30 years. After the initial surge in 1990 (looking forwards), \(q_t\) peaks in 2000, falls a little by 2010, then more sharply by 2020, and again by 2030, by which point, the shock has essentially passed. Since the chances that a mortality shock will bring about an economic collapse depend on its duration as well as its size, it is desirable to do some sensitivity analysis. As a variation on \(D = 1\), therefore, we prolong the shock by a full decade in the following way. In 2010, we repeat the age-specific mortality profile for 2000 and push those for 2010, 2020, \ldots out one decade into the future. The age pyramids in Table 4 are then recalculated accordingly. This variant of the base case is labelled \(D^* = 1\). Note that in
view of the fact that the calibration of $f^2(\cdot)$ is anchored to $e_{100}^2 = 0.065$, the differences in $q_t$ associated with the difference between the two ‘epidemics’ $D = 1$ and $D^* = 1$ will yield different values of $a_1$ and $a_2$.

5 The benchmark cases

The evolution of the entire system is determined by the extended household’s decisions $(c_t, e_1^t, e_2^t)$ at each point in time, which then determine, in turn, the formation of new human capital through the educational technology, as specified in (1) and (2). Put formally, given the current environment $Z_t$ and future environment $Z_t^e$, as forecast by the household’s decision-makers, and given the assumption of stationary expectations concerning future investments in education, the household solves problem (17) for period $t$. In doing so, it determines some of the elements of $Z_{t+10}$ and $Z_{t+10}^e$. The associated demographic elements are provided by the forecasts in Section 4. The values of $\alpha_t$ and $\sigma_t^a (a = 1, 2)$ are assumed to remain constant from 2000 and 1990 onwards, respectively. These complete the respective vectors, whereby the values $\sigma_{90}^1 = 185$ and $\sigma_{90}^2 = 800$ are drawn from Kenya (1996) and Appleton et al. (1999, Table 6), after conversion into the units of $Y_t$. One period later, in $t + 10$, the household solves its problem anew, and so forth. The results of this process for $D = 0$ and $D = 1$ are set out in Tables 6 and 7, starting in 1990.

A noteworthy feature of the counterfactual is that all children complete full primary schooling ($e_1^t = 1$) from 1990 onwards, a result that does not square with the actual value of $e_{90}^1 = 0.76$ (see Table 1). The main reason for this discrepancy is that the chosen value of $\sigma_{90}^1$ relates to the mid 1990s, at which time its level appears to have been markedly lower than in the mid 1980s (Appleton et al., 1999, Table 6). Unable to find any estimates for the intervening years, we choose the later of the two, thereby erring on the optimistic side. With the promise of declining mortality and no further
Table 6: Benchmark 1: No AIDS ($D = 0$)

<table>
<thead>
<tr>
<th>Year</th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_t^1$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$e_t^2$</td>
<td>0.000</td>
<td>0.220</td>
<td>0.296</td>
<td>0.419</td>
<td>0.605</td>
<td>0.748</td>
<td></td>
</tr>
<tr>
<td>$\lambda_t^2$</td>
<td>3.69</td>
<td>4.29</td>
<td>4.33</td>
<td>4.83</td>
<td>5.32</td>
<td>6.03</td>
<td>6.96</td>
</tr>
<tr>
<td>$\lambda_t^3$</td>
<td>4.52</td>
<td>3.69</td>
<td>5.12</td>
<td>5.77</td>
<td>6.93</td>
<td>8.55</td>
<td>10.55</td>
</tr>
<tr>
<td>$y_t$</td>
<td>2647</td>
<td>2183</td>
<td>2390</td>
<td>2654</td>
<td>2946</td>
<td>3403</td>
<td></td>
</tr>
<tr>
<td>$Y_t(10^9)$</td>
<td>29.97</td>
<td>36.24</td>
<td>55.32</td>
<td>78.25</td>
<td>101.15</td>
<td>129.19</td>
<td></td>
</tr>
<tr>
<td>Population ($10^3$)</td>
<td>23475</td>
<td>31164</td>
<td>38596</td>
<td>44948</td>
<td>50972</td>
<td>56591</td>
<td></td>
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</tbody>
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Table 7: Benchmark 2: AIDS ($D = 1$)

<table>
<thead>
<tr>
<th>Year</th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_t^1$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$e_t^2$</td>
<td>0.000</td>
<td>0.065</td>
<td>0.115</td>
<td>0.241</td>
<td>0.431</td>
<td>0.589</td>
<td></td>
</tr>
<tr>
<td>$\lambda_t^2$</td>
<td>3.69</td>
<td>4.29</td>
<td>4.33</td>
<td>4.63</td>
<td>4.91</td>
<td>5.40</td>
<td>6.11</td>
</tr>
<tr>
<td>$\lambda_t^3$</td>
<td>4.52</td>
<td>3.69</td>
<td>4.57</td>
<td>4.92</td>
<td>5.82</td>
<td>7.07</td>
<td>8.63</td>
</tr>
<tr>
<td>$y_t$</td>
<td>2647</td>
<td>2347</td>
<td>2475</td>
<td>2571</td>
<td>2703</td>
<td>3016</td>
<td></td>
</tr>
<tr>
<td>$Y_t(10^9)$</td>
<td>29.97</td>
<td>37.16</td>
<td>49.37</td>
<td>58.17</td>
<td>65.09</td>
<td>77.52</td>
<td></td>
</tr>
<tr>
<td>Population ($10^3$)</td>
<td>23475</td>
<td>29818</td>
<td>33327</td>
<td>34617</td>
<td>35588</td>
<td>37349</td>
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adverse shocks to $z_t$ and $\alpha_t$, and with sufficient human capital accumulated in the past, the expected pay-off to primary education rises with accumulation, and so ensures the continuation of the process.

Where secondary education is concerned, the series broadly accords with intuition. With relatively low current mortality and the prospect of falling levels in the future, $e_{100}^2$ is considerably higher than the actual level of 0.065 under $D = 1$. Thereafter, $e_t^2$ accelerates until 2030, and slows only slightly thereafter, reaching 0.748 in 2040. The factors at work here are the modest concavity of $f^2(\cdot)$, which does much to offset the rising opportunity costs of secondary education as $\lambda_t^2$ increases through primary education. The end result is hardly cause for reproach: $e_{130}^1 + e_{140}^2 = 1.748$ corresponds to about 16 completed years of education, on average, for the cohort in question.

The trajectories of $\lambda_t^2$ and $\lambda_t^3$ reflect these investments in education just as one would expect. There is an early stumble in $\lambda_t^2$, as the effects of the fall in $z$ between 1970 and 1980 work their way through the system (see Table 1). By 2040, however, a primary school graduate will be about 60 per cent more productive than her counterpart 50 years earlier, a improvement that owes not a little to her parents’ secondary education. The course of $\lambda_t^3$ is bumpier still, with a decline between 1990 and 2000 – the combined effect of the earlier fall in $z$ and the fact that the upper branch of eq. (2) applies when $e_t^2 = 0$. Only after 2020 does the trajectory settle down. A secondary school graduate in 2040 with $e_{130}^2 = 0.605$ has a productivity two-thirds greater than her counterpart in 2010 with $e_{100}^2 = 0.220$.

These various lags make the road to recovery of GDP per adult after the fall in $\alpha_t$ between 1990 and 2000 a long one. It takes about 30 years for $y_t$ to regain its level of 1990, but it then increases by almost 30 per cent in the following 20 years up to 2040.

As noted in Section 4, the AIDS epidemic is forecast to cause heavy mortality and smaller cohorts of births for at least four decades from 1990 onwards. It is striking,
therefore, that investment in primary education fully withstands this shock. Investment in secondary education, however, suffers a considerable setback in the period of peak mortality, where it should be noted that the observed net enrollment rate in 2000, \( e_{100}^2 = 0.065 \), is imposed exogenously in order to anchor the calibration of the function \( f^2(\cdot) \) and the household is assumed to have perfect foresight. The level of \( e_t^2 \) lies just under 40 per cent of that in the counterfactual in 2010, rising to almost 60 and 80 per cent, respectively, in 2020 and 2040.

This setback in secondary education is felt throughout the system. By 2040, a primary school graduate will be 10 per cent less productive than her counterpart in the counterfactual, despite \( e_t^1 = 1 \) over the whole time span, a secondary school graduate 17 per cent less, and GDP per adult will be 11 per cent less. Another way of summing up the effects of the epidemic is that it delays the attainment of the corresponding counterfactual values of these central variables by about one decade.

The effects of the prolonged shock (\( D^* = 1 \)) are not reported here. It turns out that ‘recovery’ in the sense defined in the previous paragraph is delayed by a mere additional year or two. This, at first sight, rather surprising result stems from the fact that, given the anchoring point \( e_{100}^2 = 0.065 \), the post-primary educational technology must be somewhat more productive in order to offset the higher value of \( q_t \) under \( D^* = 1 \). The additional toll of mortality results in a still lower population in 2040: about 2 million, or a little over 5 per cent, fewer.

### 6 Public policy

In this setting, there are three domains in which interventions can promote growth and welfare, namely, health, education and the conduct of economic policy. Measures designed to combat the epidemic are not only intrinsically valuable because they reduce the toll of suffering and death, but they also promote investments in education by
increasing the family’s lifetime resources and the expected returns to such investments. In contrast, measures that promote education directly, for example, school-attendance subsidies, have no demographic effects in our model – an assumption that demands great care when making comparisons of policies across these two domains. Improvements in the general conduct of economic policy manifest themselves in increases in $\alpha$ and thence in families’ full income. They have no effect on the long-run rate of growth in this framework, and are not considered here.

In what follows, the model is not ‘closed’ where public finance is concerned; for the budgetary outlays entailed by the interventions analysed below are financed not by domestic taxes, but rather by outright grants from abroad. Kenya has been quite heavily dependent on budgetary support of this kind in the past – though it should be added that its present prospects are clouded by the fact that corruption continues unabated under the latest government. Be that as it may, we adopt this simplifying assumption. Closing the model by introducing the government’s budget constraint and associated tax instruments would involve a substantial increase in complexity.\footnote{Some idea of what is involved can be obtained from the corresponding sections of Bell, Devarajan and Gersbach (2006), whose two-generation OLG model for South Africa has a much simpler structure.}

### 6.1 Public spending and mortality

Interventions aimed at altering the course of mortality in the presence of the epidemic produce variants of the benchmark $D = 1$. In order to formulate them precisely, it is necessary to establish the relationship between the level of spending on measures to contain the epidemic and treat those infected and the profile of age-specific mortality. For this purpose, we draw heavily on both the procedure and estimates in Bell, Devarajan and Gersbach (2006). In essence, this involves choosing a functional form for the relationship between the probability of premature death among adults and the level of aggregate expenditures on combating the disease. Bell, Devarajan and Gersbach argue...
that when the main source of this mortality is AIDS, the efficacy of such spending depends directly on the number of (sub-) families, \( N_t^F \), within the fictional extended family that makes up the whole society. Aggregate public spending is then written as \( N_t^F G_t \), where \( G_t \) is the level of spending per sub-family in period \( t \).

For simplicity, and erring on the side of optimism, we assume that such aggregate expenditures produce a pure public good, so that ridding the relationship of the size of the population, we write

\[
q_t(D = 1) = q_t(G_t; D = 1),
\]

where the function \( q_t(G_t; D = 1) \) is to be interpreted as the efficiency frontier of the set of all measures that can be undertaken to reduce \( q_t \) in period \( t \). We also assume – and this seems sensible – that the effects of these expenditures last for only the period in which they are made. Although very little is known about the exact shape of \( q_t(\cdot) \), our definition of the benchmark \( D = 1 \) implies that \( q_t(0; D = 1) \) should yield the estimates in Table 5. A second, plausible, condition is that arbitrarily large spending on combating the epidemic should lead to the restoration of the status quo ante, that is, \( q_t(\infty; D = 1) = q_t(D = 0) \) in all periods. It is also desirable to choose a functional form that not only possesses an asymptote, but also allows sufficient curvature over some relevant interval of \( G_t \), so the natural choice falls on the four-parameter logistic:

\[
q_t(G_t; D = 1) = d_t - \frac{1}{a_t + c_t e^{-b(G_t-t)}},
\]

With four parameters to be estimated, two additional, independent conditions are required. One way of proceeding is to pose the question: what is the marginal effect of efficient spending on \( q_t \) in high- and low-prevalence environments, respectively? Equivalently, we need estimates of the derivatives of \( q_t(G_t; D = 1) \) at \( G_t = 0 \) and
some value of $G_t$ that corresponds to heavy spending, when the scope for using cheap interventions has been exhausted. The basis of the estimation and the steps involved are set out in detail in Bell, Devarajan and Gersbach (2006). The said four conditions yield the values of the parameters $a_t$, $b_t$, $c_t$ and $d_t$ for the years 2000, 2010 and 2020 in Table 8. The associated functions $q_t(G_t; D = 1)$ are plotted in Figure 1.

Table 8: The parameter values of the functions $q_t(G_t; D = 1)$

<table>
<thead>
<tr>
<th></th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_t$</td>
<td>0.9073</td>
<td>0.9863</td>
<td>1.3852</td>
</tr>
<tr>
<td>$b_t$</td>
<td>0.0169</td>
<td>0.0187</td>
<td>0.0266</td>
</tr>
<tr>
<td>$c_t$</td>
<td>0.3124</td>
<td>0.3395</td>
<td>0.4768</td>
</tr>
<tr>
<td>$d_t$</td>
<td>1.2152</td>
<td>1.1128</td>
<td>0.8066</td>
</tr>
</tbody>
</table>

Source: Bell et al. (2004)

It should be remarked that the approach employed in deriving these results rests on a rather sharp distinction between preventive measures and treatment. Some would argue, however, that such a distinction is not always easy to draw, especially in high-prevalence environments. Granted as much, the general import for our approach is clear: the absolute slopes of the functions in Figure 1 at $G_t = 0$ are too large; for in these settings, there are many infected individuals other than prostitutes and truck-drivers, and the cost-efficient bundle of interventions at low levels of spending will also involve HAART, at least in some measure.

6.2 School-attendance subsidies

The two reference cases involve the following levels of the direct costs of education: $\sigma^1_t = 185$ and $\sigma^2_t = 800 \forall t \geq 90$. The former proves to be no deterrent to full primary education for all from 1990 onwards, even in the face of the epidemic. The size of the latter, though clearly smaller than the opportunity costs of time spent in secondary
education, is surely a contributing factor to the very modest levels of $e_t^2$ from 2000 until well towards the end of the horizon at 2040. This prompts consideration of a substantial subsidy to secondary education, say, of 50 per cent. The resulting claims upon the government’s budget follow trivially from the corresponding course of $e_t^2$.

### 6.3 Health or education?

It would be unrealistic to expect indefinite budgetary support for such programmes. Given the course of the epidemic, as expressed by $q_t(D = 1)$, the worst should be over by 2030, so that one can imagine the possibility of a corresponding stream of grants that ends in 2029. The exact stream, denoted by $(A_{100}, A_{110}, A_{120})$, is generated by the above proposal to cut $\sigma_t^2$ by a half from 2000 onwards. Since the course of demographic developments is exogenous under this particular proposal, the sums in question are arrived at by repeating the simulation $D = 1$ with $\sigma_t^2 = 400$ and calculating the
Table 9: Education policy: school-attendance subsidy ($D = 1$)

<table>
<thead>
<tr>
<th>Year</th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_t^1$</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$e_t^2$</td>
<td>0.000</td>
<td>0.127</td>
<td>0.194</td>
<td>0.329</td>
<td>0.455</td>
<td>0.605</td>
<td></td>
</tr>
<tr>
<td>$\lambda_t^2$</td>
<td>3.69</td>
<td>4.29</td>
<td>4.33</td>
<td>4.71</td>
<td>5.09</td>
<td>5.69</td>
<td>6.42</td>
</tr>
<tr>
<td>$\lambda_t^3$</td>
<td>4.52</td>
<td>3.69</td>
<td>4.81</td>
<td>5.30</td>
<td>6.34</td>
<td>7.53</td>
<td>9.19</td>
</tr>
<tr>
<td>$y_t$</td>
<td>2647</td>
<td>2277</td>
<td>2430</td>
<td>2592</td>
<td>2863</td>
<td>3218</td>
<td></td>
</tr>
<tr>
<td>$Y_t(10^9)$</td>
<td>29.97</td>
<td>36.04</td>
<td>48.49</td>
<td>58.64</td>
<td>68.94</td>
<td>82.70</td>
<td></td>
</tr>
<tr>
<td>Population ($10^3$)</td>
<td>23475</td>
<td>29818</td>
<td>33327</td>
<td>34617</td>
<td>35588</td>
<td>37349</td>
<td></td>
</tr>
<tr>
<td>Subsidy$^a$</td>
<td>0.94</td>
<td>1.29</td>
<td>1.82</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*a in % of the AIDS GDP level without any intervention.

As can be seen from a comparison of Tables 7 and 9, the programme of subsidies to secondary education produces a fairly substantial effect on $e_t^2$, and hence indirectly on $\lambda_t^2$. In 2040, the productivity of both primary and secondary school graduates will lie almost exactly half way between their respective levels in the reference cases $D = 0, 1$. The same holds for GDP per adult. Put somewhat differently, the programme reduces the delay in attaining the counterfactual levels from ten years to five.

If these funds were allocated instead to combating the epidemic, not only would there be a substantial fall in mortality – which is desirable in itself – but the associated improvement in expectations would also promote investment in secondary education. The latter effect is small at first, but then gathers pace, with a startling jump between...
Table 10: Health policy \((D = 1)\)

<table>
<thead>
<tr>
<th>Year</th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e_1^1)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(e_1^2)</td>
<td>0.000</td>
<td>0.067</td>
<td>0.133</td>
<td>0.292</td>
<td>0.550</td>
<td>0.701</td>
<td>0.701</td>
</tr>
<tr>
<td>(\lambda_1^2)</td>
<td>3.69</td>
<td>4.29</td>
<td>4.33</td>
<td>4.64</td>
<td>4.95</td>
<td>5.52</td>
<td>6.38</td>
</tr>
<tr>
<td>(\lambda_1^3)</td>
<td>4.52</td>
<td>3.69</td>
<td>4.58</td>
<td>5.00</td>
<td>6.04</td>
<td>7.59</td>
<td>9.39</td>
</tr>
<tr>
<td>(y_t)</td>
<td>2647</td>
<td>2346</td>
<td>2458</td>
<td>2554</td>
<td>2694</td>
<td>3058</td>
<td>3058</td>
</tr>
<tr>
<td>(Y_t(10^9))</td>
<td>29.97</td>
<td>37.13</td>
<td>52.24</td>
<td>66.11</td>
<td>79.28</td>
<td>89.84</td>
<td>89.84</td>
</tr>
<tr>
<td>Population ((10^3))</td>
<td>23475</td>
<td>29818</td>
<td>34633</td>
<td>37880</td>
<td>40937</td>
<td>41028</td>
<td>41028</td>
</tr>
<tr>
<td>Subsidy(^a)</td>
<td>0.94</td>
<td>1.29</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
<td>1.82</td>
</tr>
</tbody>
</table>

\(^a\) in % of the AIDS GDP level without any intervention.

2020 and 2030. The reason for the latter is the reduction in \(Q_t^2\) and the increase in full income per adult that stem from the cumulative reductions in premature adult mortality up to 2030, since the ending of both spending programmes in 2029 leaves them with identical mortality profiles thereafter. \(\lambda_1^3\) just exceeds the level attained under the educational programme, while \(\lambda_1^2\) lies just below its corresponding level (see Table 10). The lags involved are sufficiently drawn out, however, that \(y_{140}\) is still 5 per cent lower. The reductions in mortality would lead to a more numerous population in 2040 (by about 10 per cent), and in 2050 new graduates are more productive. It is this consequence of spending on measures to reduce premature adult mortality that make them socially profitable against the alternative of spending directly on education – at least when the time horizon is long enough.

Yet another alternative is to combine the two programmes. Table 11 reports the results from spending half the stream of educational subsidies paid out under the 'pure' programme \(\sigma_t^2 = 400\) (see Table 9) on measures to combat the epidemic and treat its victims, while reducing the educational subsidy from 400 to 200. This combined programme would cost 0.82 per cent of GDP in 2000, 1.20 per cent in 2010 and 1.79
Table 11: Combined Program \((D = 1)\)

<table>
<thead>
<tr>
<th>Year</th>
<th>1990</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e_1^1)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>(e_2^2)</td>
<td>0.000</td>
<td>0.096</td>
<td>0.164</td>
<td>0.314</td>
<td>0.522</td>
<td>0.674</td>
<td></td>
</tr>
<tr>
<td>(\lambda_2^2)</td>
<td>3.69</td>
<td>4.29</td>
<td>4.33</td>
<td>4.68</td>
<td>5.02</td>
<td>5.62</td>
<td>6.44</td>
</tr>
<tr>
<td>(\lambda_3^3)</td>
<td>4.52</td>
<td>3.69</td>
<td>4.69</td>
<td>5.15</td>
<td>6.20</td>
<td>7.64</td>
<td>9.42</td>
</tr>
<tr>
<td>(y_t)</td>
<td>2647</td>
<td>2313</td>
<td>2445</td>
<td>2574</td>
<td>2771</td>
<td>3138</td>
<td></td>
</tr>
<tr>
<td>(Y_t(10^9))</td>
<td>29.97</td>
<td>36.61</td>
<td>50.59</td>
<td>63.23</td>
<td>76.66</td>
<td>88.51</td>
<td></td>
</tr>
<tr>
<td>Population ((10^3))</td>
<td>23475</td>
<td>29818</td>
<td>34070</td>
<td>36562</td>
<td>39172</td>
<td>39859</td>
<td></td>
</tr>
<tr>
<td>Subsidy(^a)</td>
<td>0.82</td>
<td>1.20</td>
<td>1.79</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) in % of the AIDS GDP level without an intervention

per cent in 2020. As can be seen from comparing Tables 9, 10 and 11, the combined program requires a smaller stream of subsidies, and yet results in a slightly higher level of \(\lambda_3^3\) and then of both \(\lambda_2^2\) and \(\lambda_3^3\). This indicates a certain synergy between health and education policy.

The full results for the variants with prolonged adult mortality \(D^* = 1\) are not reported here. Suffice it to say that ‘recovery’ is only slightly delayed under all programmes. The levels of spending are somewhat lower than under \(D = 1\), in keeping with the lower levels of \(e_2^2\) \((t = 100, 110, 120)\) that ensue from grimmer expectations. The above-mentioned synergy is also evident in this variant, whereby the difference in public spending streams is still larger than under \(D = 1\).

### 6.4 Cost-benefit analysis

In view of the very different time paths of the streams of costs and benefits of these three interventions, an evaluation of their social profitability demands that they be discounted at an appropriate (real) rate. Four percent seems quite appropriate for
public sector projects in Africa, for there is always the alternative of investing in international paper of good quality. To be on the safe side, we also set the bar a bit higher – at 5 percent. Deciding on an appropriate welfare index is not at all straightforward, because one of the policy instruments – education subsidies – will not save lives, so that troubling ethical problems arise in comparing the above programmes. One crude, but defensible, way of proceeding is to use per capita and aggregate GDP, respectively, as (instantaneous) indices. Since only the latter gives any weight to the number of individuals actually alive – and then equally – these two indices arguably bracket the range of possibilities when output is the underlying measure of performance.

Table 12 reports the net present values of the streams of costs and benefits generated by the three programmes, whereby the streams of benefits after 2040 are interpolated from the differences in growth rates between 2030 and 2040 with and without the intervention, respectively, and then discounted over an infinite horizon. Linear interpolation is used to arrive at the estimates for all intervening years within each decade. All three programmes pass the test with flying colours whatever be the choice of index; but the two that involve the promotion of health have a benefit-cost ratio that is roughly four-to five-fold that involving educational subsidies alone. This striking improvement is due not only to the saving of lives, but also to the fact that the resulting reduction in expected mortality provides an additional potent incentive to invest in education.

<table>
<thead>
<tr>
<th>Intervention</th>
<th>Aggregate GDP</th>
<th></th>
<th>Per Capita GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r = 4% )</td>
<td>( r = 5% )</td>
<td>( r = 4% )</td>
</tr>
<tr>
<td>School-attendance subsidy</td>
<td>3.5</td>
<td>2.1</td>
<td>2.9</td>
</tr>
<tr>
<td>Health programme</td>
<td>16.3</td>
<td>13.5</td>
<td>14.6</td>
</tr>
<tr>
<td>Combined programme</td>
<td>13.3</td>
<td>10.6</td>
<td>11.7</td>
</tr>
</tbody>
</table>
7 Concluding discussion

From about 1980 onwards, Kenya has run into increasing difficulties in three separate, but related spheres, all of which pose serious threats to its citizens’ long-term prosperity and well-being. The first is a significant weakening of the mechanism through which human capital is transmitted and accumulated from one generation to the next – so much so, that in the absence of improvements in the primary school system and significant levels of secondary and higher education, the level of output per head may well approach a ceiling not very many times higher than that ruling at present. The second is a fall in the productivity of human capital, measured in terms of aggregate output, a fall that surely has much to do with failings in policy-making and governance during the past two decades.

The third is the outbreak of the AIDS epidemic, whose effects on mortality and morbidity became evident by the early 1990s. A useful summary measure of the epidemic’s economic effects is that, in the absence of any public intervention, it will delay by about a decade the attainment of the levels of human capital in 2040 under the counterfactual in which there is no outbreak.

The damage caused by these developments is not simply additive; rather they interact in mutually reinforcing ways. Conversely, measures designed to remedy failings in one sphere will relieve the problems in the other two, in part at least. Indeed, the results for a combined programme of measures in the spheres of health and education reveal a definite synergy among them. It is for this reason that any programme to combat the spread of the epidemic, to treat its victims and to support needy families must be complemented by reforms in the educational system and in economic policy in general. All three elements play a central role in determining the expected returns to investment in human capital, and hence the rate of growth over the long haul.

Where future work is concerned, we think it important to introduce the effects of
learning on the job where the young are concerned. Translating freshly derived educational attainment into output of goods and services is not accomplished overnight; rather it can be likened to the maturing of wine. The waiting involved will reduce the earnings, and hence the opportunity costs, of primary school graduates; but it will also delay the stream of expected returns from all education. What the net effect turns out to be is a matter of more than academic interest.

References


