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The Week

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Abstract

Is a five-day workweek followed by a two-day weekend a socially optimal schedule? This paper presents a model in which labor productivity and the marginal utility of leisure evolve endogenously over the workweek. Labor productivity is shaped by two forces: restfulness, which decreases over the workweek, and memory, which improves over the workweek. The structural parameters of the model are disciplined using daily variation in electricity usage per worker. The results suggest that increases in the ratio of vacation to workdays lead to output losses. A calibration of the model suggests that a 2–3 day workweek followed by a 1 day weekend can increase welfare.

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1 Introduction

Since the 1940s, a five-day workweek followed by a two-day weekend has dictated the rhythm of economic activity in the US.[1] According to the American Time Use Survey, in 2013, 83% of employed persons worked on any given weekday, whereas only 34% of employed persons worked on a given weekend.[2] This suggests that the structure of the week has real economic consequences that are beyond the accounting of time. Yet, little is known about its normative properties: does it maximize welfare, or can we improve welfare by switching to a different labor-leisure cycle? For example, it has recently been proposed to transition to a four-day workweek, followed by a three-day weekend.[3] If adopted, how will this policy affect output and welfare?

The starting point of this paper is the observation that given the need for wide-scale coordination in both production and leisure, observing an equilibrium outcome in which the current workweek is practiced is uninformative of its optimality.[4] Given the multiple-equilibria nature of the calendar, assessing the optimality of the current workweek requires a more structural approach.

To this end, this paper develops a simple representative agent framework in which both labor productivity and the marginal utility of leisure evolve

[1] For an excellent survey on the history of the week, see Zerubavel [1989].

[2] See the American Time Use Survey Summary 2013, released on Wednesday, June 18, 2014.

[3] See, for example, "Should Thursday Be the New Friday? The Environmental and Economic Pluses of the 4-Day Workweek", *Scientific American*, July 24, 2009, by Lynne Peeples; or, "Where the Five-Day Workweek Came From: It's a relatively new invention - is it time to shave another day off?", *The Atlantic*, August 21, 2014, by Philip Sopher.

[4] For example, a factory worker will not be able to work on a weekend since his factory will be closed. Similarly, on a workday, a worker on vacation would not derive the same utility from leisure since his family and friends will be working. For a recent study in psychology on the benefits of coordinated leisure, see Hartig et al. [2013]. For further reading on the coordination aspects of production and leisure, see Alesina et al. [2006], Jenkins and Osberg [2004] and Pande and Pantano [2011].

endogenously over the workweek. Each day is designated as either a workday or a rest day, and production occurs only during workdays. To capture the effects of fatigue, I assume that productivity depends on a stock of rest that depreciates during workdays and is replenished during rest days. To capture potential productivity gains from continual, uninterrupted work, I assume that productivity depends also on a stock of knowledge that is accumulated during workdays and depreciates during vacation days.

I restrict the structural parameters of the model based on characteristics of the comovements between daily electricity usage and predetermined fluctuations in labor supply dictated by the calendar. Since direct output measures on a sufficiently high frequency are unavailable, electricity usage is used as a measure of production intensity. I assume that, on a daily frequency, electricity and labor inputs are required in fixed proportions. Fluctuations in electricity reflect proportional variation in the utilization rate of capital and in labor productivity, resulting in proportional variation in output.[5]

Additionally, I restrict the parameters of the model based on empirical estimates of the wage penalty associated with recent prolonged absence from work. In the model, absence from work is associated with a reduction in fatigue, but also a depreciation of the stock of knowledge. The reentry penalty disciplines the model by requiring that, after long vacations, the knowledge effect dominates the fatigue effect.

For the set of admissible parameters, I compute the counterfactual productivity gains associated with alternative weeks. The results suggest that, despite the reduction in fatigue, increases in the proportion of vacation to workdays tend to reduce both output and productivity. An important exception is the workweek consisting of two workdays followed by a single vacation day. This schedule generates some productivity gains associated with more and more-frequent leisure.

[5] A similar methodology for recovering variations in effective units of labor has been previously used in the context of understanding business cycle fluctuations (Basu [1996]).

To further explore the welfare implications of alternative schedules, I consider a representative household that values both final good consumption and leisure. I follow Friedman [1969], Kydland and Prescott [1982] and Hotz et al. [1988] and assume that the marginal utility of current leisure is decreasing in recent leisure. I calibrate the agent's relative preference towards leisure to match a market overtime premium of between 0-50%, consistent with the findings of Bell and Hart [2003]. The calibration suggests an optimal week consisting of a 2-3 day workweek followed by a 1-day weekend. Depending on the preference for leisure, the expected steady state welfare gains associated with these alternatives are equivalent to a permanent increase in consumption of between 3-7%. This analysis suggests that increasing the frequency of leisure may be a more efficient way to reduce worker fatigue than increasing the proportion of rest to work days.

This paper is closely related to the literature assessing the relationship between productivity and work hours. The conclusions of this literature are mixed. Denison [1962], Leslie and Wise [1980], Hanna et al. [2005] Bourles and Cette [2006], Bourles and Cette [2007], Cette et al. [2011] and Pencavel [2014] present reduced-form evidence suggesting that, at least beyond a certain threshold, productivity is decreasing in hours. Feldstein [1967], Craine [1973], Shapiro [1986] and Hart and McGregor [1988] present evidence that productivity is increasing with working hours, or that there are higher returns to hours per worker than to workers. Anxo and Bigsten [1989] find that productivity is roughly invariant to hours, and Garnero and Rycx [2014] and Lee and Lim [2014] document a non-monotone relationship between productivity and hours.[6] This paper departs from previous literature in adopting a structural approach that implies that the relationship between work duration and productivity depends not only on average hours but also on the spacing of vacation time.

[6]Similar to the analysis in this paper, Lee and Lim [2014] attribute the non-monotonicity to the combination of a fatigue effect and a leaning effect.

It is worth noting that this paper studies the optimal division of time into work and rest days, implicitly holding fixed the number of hours per workday. Pencavel [2014] presents evidence suggesting that variations in the number of workdays (and, in particular, Sunday work) have a larger effect on output than variation in total hours worked. The related issue of the optimal hours per workday is beyond the scope of this paper.

The idea that labor productivity may be affected by fatigue appears elsewhere in the literature, for example in Dixon and Freebairn [2009], Marchetti and Nucci [2001] and Pencavel [2014]. The idea that productivity is increasing with accumulated experience builds on Arrow [1962], Jovanovic [1979], Lucas [1988] and Lucas [1993] (and a vast literature that followed). Consistent with this view, there is evidence suggesting that part-time workers are less productive than full-time workers (see Aaronson and French [2004], Baffoe-Bonnie [2004] and Hirsch [2005]). The literature mostly treats learning-by-doing as a stock of knowledge that does not depreciate. However, evidence from cognitive psychology suggests that it is more difficult to retrieve memories that have not been recently retrieved (see, for example, Anderson and Schooler [2000]). In other words, workers may need time to re-familiarize themselves with their tasks after returning from vacation, and thus there may be some benefits associated with consecutive, uninterrupted work. This channel is related to the notion of human capital depreciation during unemployment explored by the labor literature (see, for example, Mincer and Ofek [1982], Keane and Wolpin [1997], and Pavoni [2009]).

Finally, this paper is related to the literature assessing the impact of religion on economic outcomes (see McCleary and Barro [2005] and references therein). This literature has mostly focused on the role of religion in providing incentives for pro-social behavior, such as charity that can serve as informal insurance. This paper contributes to this line of work by assessing the economic impact of one particular religious institution, namely the seven-day week. The results suggest that the optimal week departs from the cycle of religious observance, which prescribes a single rest day in a seven-day cycle. However, in section 5.1 I show that it is possible to generate welfare improvements under the restriction that the labor-leisure cycle is synchronized with the cycle of religious worship.

The rest of the paper is organized as follows. Section 2 presents some institutional background. Section 3 presents a model of production in which labor productivity depends on the work-rest schedule. Section 4 discusses restrictions on the parameterization of the model. Section 5 embeds the production model in a representative agent framework, and discusses welfare implications. Section 6 concludes and discusses avenues for further research.

2 Background

This section presents a brief survey of the week as an economic institution. The purpose of this survey is twofold: first, to illustrate that the adoption of the current five-day workweek was not an outcome of optimal policy; rather, it was shaped by a variety of factors that are unrelated to its productivity attributes. Second (and relatedly), the purpose of this survey is to illustrate the importance of religious observance in shaping the pattern of the workweek in order to motivate the discussion in section 5.1.

The Hebrew Bible, arguably the most influential book in history, opens with a micro foundation of the seven day week.[7] According to the Judeo-Christian tradition, after spending six days creating the world, the Lord rested on the seventh day. To honor this, Jewish religion places heavy restrictions on work during the Sabbath. While Christianity and Islam adopted the custom of weekly religious observance during Sundays and Fridays, respec-

[7]The seven-day week is also rooted in ancient traditions relating to the planetary cycle. Their primary use was the accounting of time rather than dictating the work-rest schedule. The Romans used an 8-day maket cycle, which was used as a device for coordinating on market days. The adoption of the seven-day week came with the adoption of the Julian calendar in 45 BC.

tively, there are mostly no restrictions on work. Nonetheless, most countries adopted the day of religious observance as a day of rest.

The transition to a two-day weekend took place in the United States during the beginning of the 20th century, and was legally instituted in 1940 (see Zerubavel [1989]). Prior to that, Sunday was the official day of rest and Saturday was a workday. In 1908, the New England Cotton Mill instituted a two-day weekend to accommodate the religious observance of its many Jewish workers. In 1926, Henry Ford began shutting down his factories on Saturdays, in hope that a two-day weekend would allow sufficient time for travel and increase the demand for cars.

The 1938 Fair Labor Standards Act (instituted in 1940) gave binding legal status to the two-day weekend. Against the backdrop of the Great Depression, the legislation was motivated by the idea that reducing the number of hours per worker would force companies to hire more workers and help share employment. While this reasoning may have been appropriate at the time, it is perhaps less relevant in environments with well-functioning labor markets. Despite this, the two-day weekend has persisted to the current day.

Many countries followed suit and today, most countries have a two-day weekend. Although the structure of the week is quite uniform across countries, consisting of a five-day workweek followed by a two-day weekend, there are some exceptions. In some Muslim countries such as Iran or Afghanistan, workers follow a six-day workweek with Fridays off. Brunei follows a schedule with Fridays and Sundays off and work on Saturdays. Until very recently (2013-2014), it was customary for schools in France to close on Wednesdays (making up the hours either on other school days or with Saturday classes).[8]

There are some interesting historical examples of deviations from the seven-day week cycle, which were, at least in part, explicitly targeted at combating religious practice (see Zerubavel [1989] for details). The French Revolutionary Calendar, instituted in France in 1773, consisted of a nine-

[8] See The Economist, "Weird about Wednesday", September 21st, 2013.

(a) The five-day week, 1930

(b) The six-day week, 1933

Figure 1: Calendars depicting the five-day week (1929-1931) and the six-day week (1931-1940) in the Soviet Union. Under the five-day week, each worker was assigned a color and took his day off on his color-coded day. In the six-day week, every sixth day was a day off for the entire workforce. Source: Foss [2004].

day workweek followed by a single vacation day. The conflict with religious observance and the reduction in vacation time made it unpopular, and the seven-day week was restored in 1805.

Another interesting example comes from the Soviet Union. Between 1929-1931, the Soviet Union instituted a five-day workweek in which, on every day, one-fifth of the workforce had the day off (see figure 1a). While this arrangement maximized the capital-labor ratio on any given day, and increased the proportion of vacation days from one in seven to one in five, problems resulting from lack of coordination at the workplace led to its abandonment. In 1931, the five-day week was replaced by a six-day week with a uniform day off for all workers (see figure 1b). In 1940, the seven-day week was restored, with Sunday as the day of rest. The choice of Sunday as the day of rest is interpreted by some as evidence that the failure of the six-day week was rooted in the non-compliance of religious workers, even under the strict Soviet regime.

In light of these historical experiences, it perhaps worth questioning the implementability of policy prescriptions that require deviations from a seven-day cycle. Most of the analysis in this paper ignores this constraint, and finds that the optimal week consists of shorter cycles of 2-3 workdays followed by a single vacation day. However, the paper concludes with a discussion on how the results may be used to efficiently restructure a seven-day cycle that is synchronized with religious worship.

3 Production model

Time is divided into discrete days indexed $t \in \{0, 1, 2...\}$. Each day is designated as either a "workday" or as a "rest day". It is assumed that either production or leisure (or both) require widespread coordination and that agents coordinate to work on workdays and rest on rest days. Let $\chi(t)$ be an indicator function that takes the value 1 if and only if day t is designated as a rest day.

Labor productivity. Effective units of labor are a function of the calendar, the "stock of rest", R, and the "stock of memory", M:

$$L_t = (1 - \chi(t))R_t^{\alpha_r} M_t^{\alpha_m} \tag{1}$$

The above specification of labor productivity assumes that workers work only on workdays ($1 - \chi(t) = 0$ on rest days). The parameters $\alpha_r, \alpha_m \in (0, 1)$ govern the intensity in which rest and memory affect labor productivity.

The stock of rest evolves as follows. Rest depreciates at a rate δ_r per day. The stock of rest is replenished by rest days:

$$R_{t+1} = (1 - \delta_r)R_t + \chi(t) \tag{2}$$

where $\delta_r \in (0, 1)$. Formally, rest days constitute investment in the stock of rest, helping workers reduce fatigue, ultimately making them more productive.[9]

In contrast, the stock of memory increases during workdays and depreciates during rest days. During rest days, workers are engaged in other things and forget some of their work-related memories. The depreciation rate of the stock of memory is $\delta_m \in (0, 1)$:

$$M_{t+1} = (1 - \delta_m)M_t + (1 - \chi(t)) \tag{3}$$

In this formulation, the depreciation of the stock of memory is positively related to the length of vacation. A worker returning to work after a single vacation day will have an easier time reorienting himself than a worker returning to work after a longer vacation. This model is in the spirit of the cognitive psychology literature on memory (see, for example, Anderson and Schooler [2000]), which finds that it is easier to recall information that has been more recently recalled.

It is worth emphasizing that R_t and M_t capture notions of rest and memory as they relate to labor productivity, and are unrelated to the utility from leisure. Preferences will be discussed in section 5.

Intuitively, at the beginning of the week, workers are well-rested but perhaps somewhat disoriented. As the week progresses, memory improves but fatigue increases. From a productivity standpoint, a very long workweek is unlikely to be optimal because of the fatigue effect. A very short workweek may be suboptimal as well, as consecutive work increases productivity by making work-related memories more readily available.

Steady state cycles. For a given cycle of i consecutive workdays followed by j consecutive rest days, there are unique steady state cycles for R_t and

[9] Note that this setup is a specific instance of a more general model, in which $R_{t+1} = (1 - \delta)R_t + \kappa\chi(t)$. However, as R_t is homogenous in κ (starting from $R_0 = 0$), without loss of generality I restrict attention to $\kappa = 1$.

M_t that can be solved recursively using equations 2 and 3. The subsequent analysis will focus on comparing across different steady states, ignoring the transitions from one steady state to another.

Final good production. The final good, Y_t, is produced using a Leontief production function with labor inputs and other underutilized factors, X_t:

$$Y_t = \min\{X_t, L_t\} \tag{4}$$

This specification assumes that, on a daily frequency, labor and other inputs are required in fixed proportions, and that other factors of production, such as capital, are fixed and underutilized. This assumption reflects the view that substitutability between goods or inputs is lower at higher frequencies. A given sewing machine requires fixed amounts of labor, electricity and materials to produce a garment. It would be difficult to find productive uses for more workers given a single sewing machine. Of course, depending on relative factor prices, firms will decide to install sewing machines that require different factor proportions, thus generating some substitutability between inputs on longer horizons.[10]

Another crucial feature of equation 4 is that it embeds the assumption of time-separability in production. While this assumption is commonly used in the macro literature, it is likely somewhat restrictive when applied to a daily frequency. For example, livestock that is left unfed for many days will die out, implying that there are productivity gains associated with shorter vacations. Alternatively, there are costs associated with sending and recalling long-range fishing boats, which would imply benefits associated with longer periods of

[10] See Gallaway et al. [2003] for a similar discussion regarding the higher substitutability between consumption goods at longer horizons. More closely related, Griffin and Gregory [1976] suggests that, for a given unit of installed capital, utilization is roughly proportional to energy inputs, while the decision regarding which unit of capital to install may take energy requirements into consideration. Eden and Griliches [1993] use a similar Leontief assumption related to labor and capacity utilization.

consecutive work. In these examples, the productivities of non-labor inputs depend on past values of $\chi(t)$. The assumption of time-separability is likely to apply to most forms of industrial production and services, in which, at least at high frequencies, the productivity of capital does not depend on its past use.

Consider a week of length T, in which the first $(1-\lambda)T$ days are workdays and the subsequent λT days are rest days. Let Y_d denote the steady state value of output on workday d. The following lemma illustrates how the evolution of labor productivity over the workweek changes with the model's parameters.

Lemma 1

1. *If $\alpha_r = 0$ then $Y_{i+1} - Y_i \geq 0$ for $1 \leq i < (1-\lambda)T$.*
2. *If $\alpha_m = 0$ then $Y_{i+1} - Y_i \leq 0$ for $1 \leq i < (1-\lambda)T$.*
3. *For $1 \leq i < (1-\lambda)T$, $(Y_{i+1} - Y_i)/Y_i$ is increasing in α_r, decreasing in α_m, and decreasing in δ_r.*
4. *$(Y_{(1-\lambda)T} - Y_1)/Y_1$ is decreasing in δ_m for $T = 7$ and $\lambda = 2/7$ (though not necessarily for other values of T and λ).*

The proof, together with other omitted proofs, is in the appendix. This lemma illustrates the flexibility of the production structure in generating different evolutions of labor productivity over the workweek. If productivity depends only on the stock of memory ($\alpha_r = 0$), then it is increasing over the workweek. If, in contrast, productivity depends only on the stock of rest ($\alpha_m = 0$), then it is decreasing over the workweek. In general, the productivity increase between two consecutive workdays is positively related to the intensity of memory, α_m, and negatively related to the intensity of rest, α_r, and to the depreciation rate of rest, δ_r. While the productivity increase over the workweek (Monday to Friday) is generally a non monotone function of δ_m, numerical results confirm that for the 5-2 workweek, it is decreasing in δ_m.

To develop intuition for how the optimal week varies with the model's parameters, the following lemma illustrates the comparative statics of the model with respect to different schedules:

Lemma 2 *Let* $\bar{Y}$ *denote the average output per workday given* λ *and* T*:* $\bar{Y} = \sum_{d=1}^{(1-\lambda)T} Y_d/((1-\lambda)T)$.

1. *Assume that only rest is important:* $\alpha_r > 0$ *and* $\alpha_m = 0$*. Then,* $\bar{Y}$ *is increasing in* λ *and decreasing in* T*.*

2. *Assume that only memory is important:* $\alpha_m > 0$ *and* $\alpha_r = 0$*. Then,* $\bar{Y}$ *is decreasing in* λ *and increasing in* T*.*

The fatigue effect will tend to imply productivity gains associated with more and more-frequent vacation. Vacation days increase the stock of rest, which makes labor more productive during workdays. The lemma illustrates that it is possible to reduce the adverse effects of fatigue not only by increasing the proportion of vacation days (λ), but also by increasing the frequency of vacation days: for a given λ, a lower value of T would imply productivity gains from reducing fatigue.

The memory effect works in the opposite direction. More vacation is associated with lower productivity, because the stock of memory depreciates during vacation. Moreover, for a given λ, there are productivity gains associated with "bunching" vacation days to allow for longer periods of consecutive work. Interruptions from work are costly in terms of productivity, because workers need to rebuild their stock of memory after it depreciates during the break.

This analysis illustrates that the productivity gains associated with different work patterns depend crucially on the relative importance of the fatigue and memory effects. The main result of this paper is that more-frequent leisure increases productivity and welfare, reflecting gains from reducing fatigue.

4 Parameterization

In this model, labor productivity is a function of the calendar, as well as two depreciation rates, δ_r and δ_m, and two intensity parameters, α_r and α_m. In this section, I narrow down the parameter space based on three criteria. First, the parameters must imply labor productivity that peaks during the workweek, consistent with the patterns of electricity usage documented below. Second, the variation in productivity over the workweek must be within the range of predicted variation in electricity usage per worker. Finally, the parameters must be such that a year of absence from work is associated with a reentry penalty, consistent with magnitudes estimated in the labor literature.

The first two criteria are based on predicted variation in electricity per-worker over the workweek. Variation in electricity usage is informative of variation in the utilization rate of capital, which, in the context of the model, is proportional to variations in L_t (equation 4). I use data on electricity loads from seven regional transmission organizations (RTOs): ERCOT, PJM, ISO-NE, CAISO, MISO, NYISO and SPP.[11] The aggregate generating capacity of these RTOs corresponds to roughly 60% of US totals, though coverage varies by RTO.[12] Observations range from January 1st, 2003 and February 15th, 2015. Data are available on an hourly frequency (and on a higher frequency for some RTOs) and are aggregated to a daily frequency. To remove long-run and seasonal trends, the data is de-trended using a weekly moving average.[13]

To account for potential variation in the number of workers over the workweek, I use the American Time Use Survey (ATUS) for the years 2003-

[11] Data for PJM, ERCOT and SPP are taken from their websites. Data for ISO-NE, CAISO, MISO and NYISO are taken from energyonline.com.

[12] Unfortunately, load data for the remaining RTOs is not readily available. In particular, the data does not cover most of the southeast, and parts of the northwest and the southwest of the United States.

[13] Specifically, I construct percent deviations from the weekly average as $e_t = \frac{E_t}{\sum_{t'=t-3}^{t'=t+3} E_{t'}}$, where E_t denotes the electricity load data.

2013. Survey respondents are asked to report their time usage by minute for a particular diary day. The data are aggregated to obtain daily working hours, averaging respondents by their sample weights.[14] Note that ATUS respondents are a random sample of the population aged 15 or older, and include many respondents that are not full-time employed. Thus, average daily working hours during the workweek vary between 4-4.5 hours, significantly lower than full-employment hours.

I generate predicted weekday values of electricity and hours based on a regression that includes weekday fixed effects. Days that fall within a week of a holiday are omitted, as the dynamics of labor productivity are likely different during those weeks. For example, workers are likely to be more well-rested on a Tuesday that follows a Monday holiday than on an ordinary Tuesday. Though the model does not include any anticipation effects, I omit all 7-day periods preceding holidays to allow for such effects (for example, people taking time off before a holiday), and to account for potential error resulting from the moving-average filtering of the electricity series. After omitting holidays and the days surrounding them, the predicted values of electricity and hours capture "normal" weekly variation induced by the structure of the week, provided that the dynamic effects of holidays on labor productivity are contained within a 2-week period.

Figure 2 plots the resulting weekday coefficients for working hours, and Figure 3 plots the resulting weekday coefficients for electricity usage in four out of the seven RTOs (the figures for the remaining RTOs are in Appendix D). Both hours and electricity exhibit weekly cyclicality, and, consistent with the model, are significantly higher during workdays than during weekends. Note that both hours and electricity peak during the workweek.

Not surprisingly, the results indicate positive levels of work and electricity usage during weekends, reflecting the reality that some people work on

[14] Total minutes worked correspond to the sum of categories 050101, 050103 and 050189, which represent work related activities.

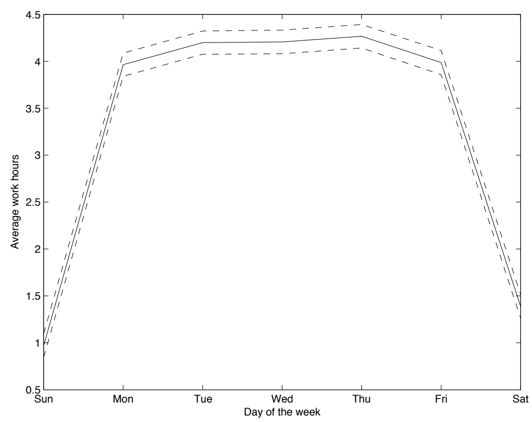


Figure 2: Predictable weekly variation in working hours over the week. Dashed lines represent the associated 95% confidence intervals.

weekends, and some electricity is used (for production or otherwise). The model can accommodate this by assuming some constant amount of hours, $\bar{H}$, and some constant amount of electricity, $\bar{e}$, that do not vary over the workweek. Using $\hat{e}_d$ and $\hat{H}_d$ to denote the predicted values of electricity and hours on weekday d, I construct the following proxy for output per worker:

$$\hat{Y}_d = \begin{cases} \frac{\hat{e}_d - \bar{e}}{\hat{H}_d - \bar{H}} & \text{if } d \text{ is a workday (Mon-Fri)} \\ 0 & \text{if } d \text{ is a weekend (Saturday-Sunday)} \end{cases} \tag{5}$$

where $\bar{e} = \min_d\{\hat{e}_d\}$ and $\bar{H} = \min_d\{\hat{H}_d\}$ are the predicted Sunday values of electricity and hours, respectively. Note that $\hat{e}_d - \bar{e}$ is the "extra" electricity used on weekday d, beyond the amount that is used during the weekend, and similarly, $\hat{H}_d - \bar{H}$ is the "extra" hours worked on weekday d, beyond the constant amount worked during the weekend. The proxies $\hat{Y}_d$ capture the variation in electricity usage per-worker over the workweek, which is interpreted as variation in labor productivity given the Leontief production structure.

To interpret the units of $\hat{Y}_d$, I consider the (log) deviations of $\hat{Y}_d$ from its

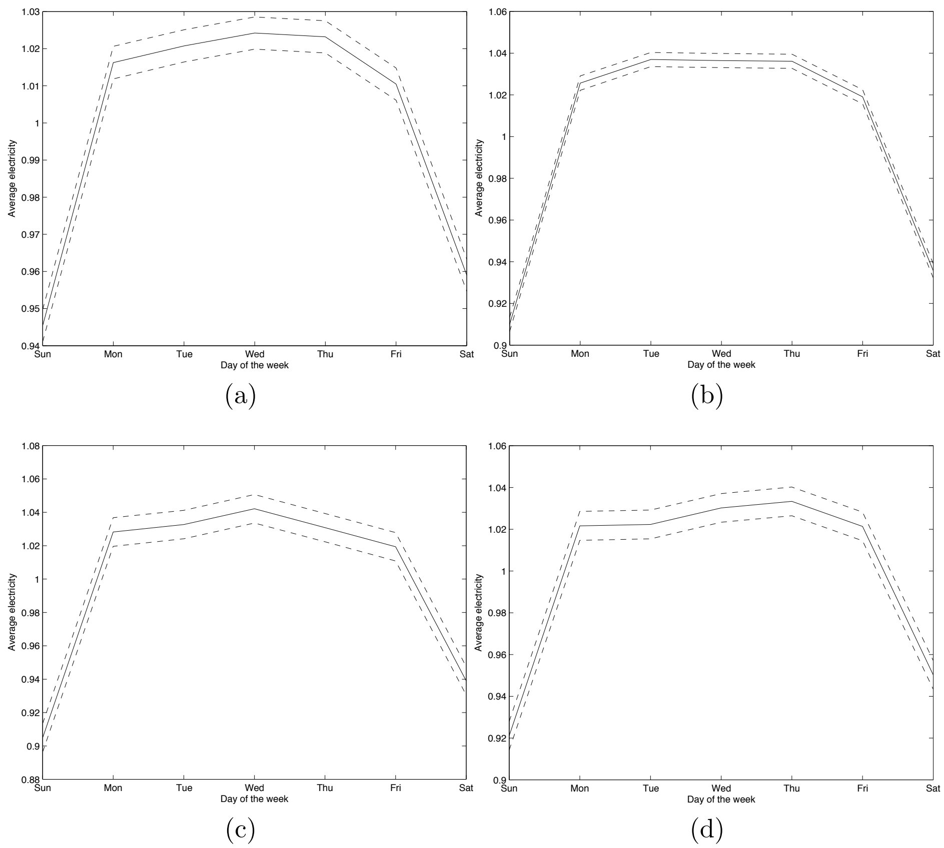


Figure 3: Average electricity usage in different RTOs over the week. Dashed lines represent the associated 95% confidence intervals. See figure 6 for the remaining three RTOs.

minimal workday value:

$$Y_d = \frac{\hat{Y}_d - \min_{d'}\{\hat{Y}_{d'} | d' \text{ is a weekday}\}}{\min_{d'}\{\hat{Y}_{d'} | d' \text{ is a workday}\}} \tag{6}$$

This normalization maps into the range of variation in Y_d. For example, $\hat{Y}_{Tue} = 0.1$ and $\hat{Y}_{Fri} = 0$ implies that electricity per hour is, on average, 10% higher on a Tuesday than on a Friday.

Figure 4 plots the resulting estimates of Y_d for four out of the seven RTOs (the figures for the remaining three RTOs are in Appendix D). The weekly patterns of Y_d seem to differ substantially across RTOs, reflecting perhaps differences in industrial composition. However, one commonality is that, in all seven RTOs, Y_d is neither monotonically increasing nor monotonically decreasing over the workweek. This suggests that, at the beginning of the workweek, there are gains from the reconstruction of M, and towards the end of the week the fatigue effect dominates. For six out of seven RTOs, Y_d peaks during the workweek. Thus, to account for the pattern in weekly variation in Y_d in most RTOs, it is necessary to consider parameters that

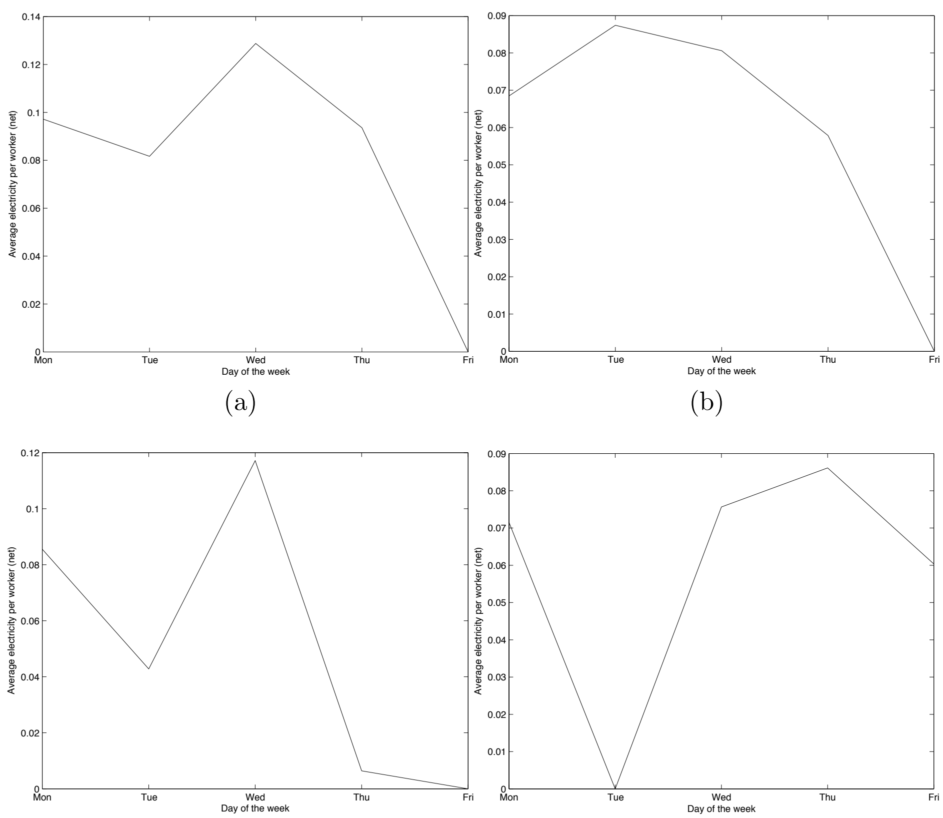


Figure 4: Net electricity per worker (Y_d) over the workweek in different RTOs (see also figure 7).

generate productivity peaks during the workweek. This motivates the first criterion, which is that the model's parameters must be consistent with an interior maximum of productivity over the workweek.

This first restriction is consistent with Brogmus [2007], who documents that the occupational injury rate in the US is minimized on Wednesdays. If the weekly variation in the occupational injury rate reflects weekly variation in fatigue and in absent-mindedness that is monotonically related to variation in productivity, this finding suggests that productivity peaks mid-week.

The second criterion relates to the range of variation, measured as the maximum level of Y_d. The estimates for different RTOs suggest that output per worker varies by between 8.6% and 22% over the workweek. This range is inconsistent with a "naive" model in which output per worker is constant over the workweek; nor is it consistent with extreme parameters which imply wild variation in output per-worker.

The last criterion is based on empirical estimates of human capital depreciation due to absence from work. Mincer and Ofek [1982] and Keane and Wolpin [1997] (among others) document that recent unemployment or non-participation in the labor market is associated with lower wages. With the usual caveats, unemployment or non-participation can be thought of as a long vacation. Their findings can therefore be interpreted as suggesting that, for relatively large increases in vacation, the "learning by doing" effect dominates the fatigue effect, resulting in an overall loss in worker productivity. The empirical estimates suggest that a year of absence is associated with a reentry wage that is 3.5-30.5% lower than the continual employment wage. These estimates are inconsistent with parameters that imply long-run gains from rest that outweigh the losses from the depreciation of the stock of memory.

To isolate the set of admissible parameters, I evaluate the criteria on 100^4 points in the parameter space $(\alpha_r, \alpha_m, \delta_r, \delta_m) \in (0,1)^4$. I compute the steady state productivity cycle implied by each vector of parameters,

$y_{sun}, ..., y_{sat}$. Consistent with the empirical counterparts, I compute the range as the maximal value of $(y_d - y_{\min})/y_{\min}$, where $y_{\min}$ is the minimal value of y during the workweek (Monday through Friday). To compute the reentry penalty, I assume that the annual wage is equalized with the net present value of output over the following year, and compare the wage of an agent at the steady state with the wage of an agent that reenters the workforce after one year of absence.[15]

Table 1 summarizes the results. While the criteria are insufficient to uniquely pin down the model's structural parameters, they are sufficient to narrow down the set of admissible parameters to 0.25% of the parameter space. The most binding criterion is the restriction that productivity peaks during the workweek, which is satisfied by only 16.5% of the parameter space. This suggests that most parameter choices imply either a dominant fatigue effect or a dominant learning-by-doing effect, generating weekly productivity patterns that peak either on Monday or on Friday. The second-most binding criteria are the upper-bound on the range of variation, and the minimal reentry penalty. This implies that most parameters generate wild variation in weekly labor productivity, and that many parameters generate only a small (or even negative) reentry penalty. The restrictions on the maximum reentry penalty and on the maximum productivity range do not seem to be very binding.

Table 1: Criteria for admissible parameters

Criterion	Percent satisfying criterion
Productivity peak during the workweek	16.5
Productivity range > 8.6%	92
Productivity range < 22%	20.9
Reentry penalty > 3.5%	20.9
Reentry penalty < 30.5%	96
All criteria	0.25

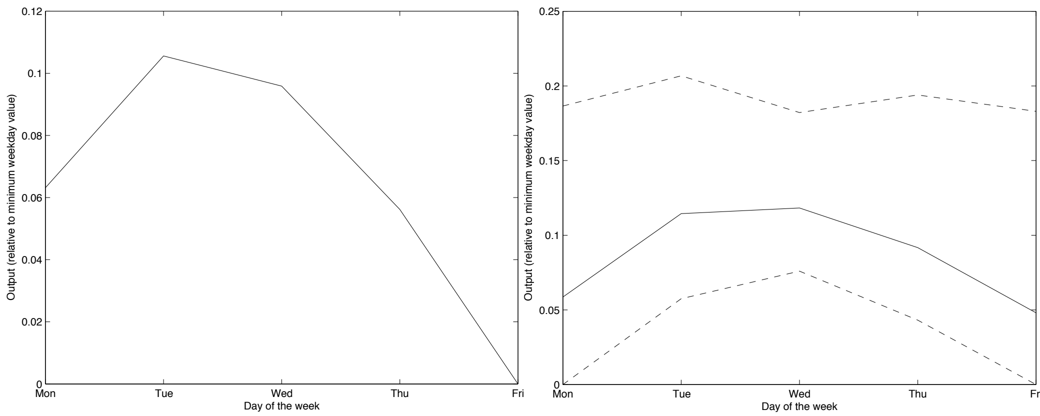
[15]To compute the net present value of output, I use an annual depreciation rate of 3 pp.

Table 2: Summary statistics of admissible parameters

	Mean	Standard deviation	Range
α_r	0.12	0.07	[0.01,0.5]
α_m	0.84	0.12	[0.37,1]
δ_r	0.55	0.2	[0.12,1]
δ_m	0.27	0.04	[0.16,0.39]
Range	13%	4%	[8.6%,22%]
Reentry penalty	5.75%	2%	[3.5%,16.6%]]

Table 2 presents some summary statistics for the set of admissible parameters. It may be interesting to note that while the criteria substantially narrow down the set of admissible parameters, there is a rather large range of individual parameters satisfying the criteria; the criteria impose relatively minor restrictions on the values of particular parameters, and relatively strong restrictions on the admissible combinations of parameters. The high mean depreciation rates should be viewed through this lens: while, on their own, these values might imply implausibly large variation in labor productivity over the workweek, the restrictions on the set of admissible parameters guarantee that any admissible *combination* of parameters implies weekly variation in labor productivity that is within the range of predicted electricity usage per-worker. The mean range of variation is substantially lower than the upper-bound of 22%, and is consistent with the magnitudes in figure 4. The average reentry penalty is 5.75%, substantially lower than the upper bound of 30.5%.

Figure 5a presents the weekly productivity cycle implied by the mean values of admissible parameters, which are reported in table 2 (the mean vector happens to constitute an admissible vector of parameters). For these parameters, output peaks on Tuesday and declines thereafter, suggesting that the fatigue effect dominates during most of the week, but that the learning effect plays a dominant role at the beginning of the week. Figure 5b presents the distribution of productivity implied by the set of admissible parameters.



(a) Mean vector

(b) Distribution

Figure 5: Figure 5a presents simulated weekly variation in output for the mean vector of admissible parameters. Output is normalized as deviations of the workday minimum (in this case, Friday): $(y_d - y_{\min})/y_{\min}$. Figure 5b presents the mean values of $(y_d - y_{\min})/y_{\min}$ and their associated 95 interpercentile ranges implied by the set of admissible parameters.

For all admissible parameters, productivity is minimized either on Monday or on Friday. The admissible parameters also include parameter values in which either Monday or Friday are highly productive days, with productivity up to 20 percent higher than the minimum.

I proceed by using the set of admissible parameters to evaluate different week structures. Table 3 illustrates the range of output gains (relative to the current workweek 5-2). The output gain associated with a week consisting of i workdays and j rest days is computed based on average daily output, including vacations days. Using $y_d(i,j)$ to denote the output on workday d given an i-j week, the reported output gains are:

$$\text{Output gain} = \frac{\frac{1}{i+j}\sum_{d=1}^{i} y_d(i,j)}{\frac{1}{7}\sum_{d=1}^{5} y_d(5,2)} \tag{7}$$

The results reveal that, unfortunately, almost any schedule that has a

Table 3: Range of output gains

		Rest days		
		1	2	3
Work days	1	[0.55,0.74]	[0.27,0.43]	[0.17,0.3]
	2	[0.91,1.07]	[0.55,0.71]	[0.37,0.53]
	3	[1.04,1.23]	[0.75,0.87]	[0.55,0.7]
	4	[1.05,1.3]	[0.9,0.96]	[0.68,0.81]
	5	[1.03,1.33]	[1,1]	[0.78,0.89]
	6	[1,1.36]	[1.01,1.07]]	[0.84,0.94]

Highlighted cells are schedules in which the ratio of rest days to work days exceed that of a 5-2 schedule. Output gains are computed as (average daily output)/(average daily output in the 5-2 week) (equation 7).

Table 4: Range of productivity gains

		Rest days		
		1	2	3
Work days	1	[0.79,1.05]	[0.58,0.92]	[0.48,0.85]
	2	[0.98,1.15]	[0.79,1.01]	[0.66,0.94]
	3	[0.99,1.18]	[0.9,1.04]	[0.78,1]
	4	[0.93,1.16]	[0.96,1.03]	[0.85,1.02]
	5	[0.88,1.14]	[1,1]	[0.89,1.02]
	6	[0.83,1.14]	[0.96,1.02]	[0.9,1.01]

Highlighted cells are schedules in which the ratio of rest days to work days exceed that of a 5-2 schedule. Productivity gains are computed as $\bar{Y}_{i,j}/\bar{Y}_{5,2}$ =(average workday output in week (i, j))/(average workday output in the 5-2 week).

higher ratio of vacation to work days generates an output loss. The one possible exception is the week 2-1 (two workdays followed by one rest day). The ratio of rest to work days in this schedule is 0.5, which is higher than 2/5=0.4. Some admissible parameters imply that this alternative is associated with up to a 7 percentage point increase in output.

While most schedules that increase leisure generate output losses, some are consistent with productivity gains at least for some admissible parameters. Table 4 presents the average workday productivities relative to the 5-2 benchmark. The 2-1 schedule may be associated with productivity gains of up to 15 percent. The productivity gains from switching to 4-3 are bounded

Table 5: Expected output gains

Work days	Rest days: 1	2	3
1	0.64 [0.58, 0.7]	0.35 [0.3, 0.41]	0.24 [0.19, 0.28]
2	0.97 [0.91, 1.04]	0.64 [0.58, 0.69]	0.46 [0.41, 0.51]
3	1.12 [1.07, 1.19]	0.82 [0.0.78, 0.86]	0.63 [0.58, 0.68]
4	1.19 [1.12, 1.26]	0.93 [0.91, 0.95]	0.75 [0.71, 0.79]
5	1.23 [1.12, 1.3]	1 [1, 1]	0.83 [0.8, 0.86]
6	1.23 [1.11, 1.32]	1.04 [1.02, 1.06]	0.89 [0.85, 0.92]

See notes in Table 3. In each cell, the first row contains the mean value generated by the set of admissible parameters, and the second row corresponds to the 95 inter-percentile range.

above by 2 percent, and bounded below by -15 percent.

Under the minimal assumption that the true vector of parameters is in the admissible set, the only informative statistics are the ranges of estimates: for example, we can rule out that the productivity gains from switching to a particular schedule are lower than the minimal gains implied by the admissible set, or higher than the maximum gains implied by the admissible set. As an alternative approach, it is possible to consider a uniform prior on the admissible set. This prior implies that, absent additional information, each vector of admissible parameters is equally likely. Under this prior, the expected gain from switching to a certain schedule is the average gain implied by the admissible set. Similarly, the 95 inter-percentile range corresponds to the 95% confidence interval.

Tables 5 and 6 present the expected output and productivity gains from switching to different schedules, together with their associated 95% confidence intervals. Note that all schedules that increase leisure, including 2-1, are associated with expected output losses. Under the uniform prior, we can

Table 6: Expected productivity gains

		Rest days 1	2	3
Work days	1	0.92 [0.83, 1]	0.76 [0.65, 0.87]	0.67 [0.55, 0.8]
	2	1.04 [0.98, 1.11]	0.91 [0.83, 0.98]	0.82 [0.73, 0.92]
	3	1.07 [1.02, 1.14]	0.97 [0.92, 1.02]	0.9 [0.83, 0.96]
	4	1.07 [1, 1.13]	1 [0.97, 1.02]	0.94 [0.89, 0.98]
	5	1.05 [0.96, 1.11]	1 [1, 1]	0.95 [0.92, 0.99]
	6	1.03 [0.93, 1.1]	0.99 [0.97, 1.01]	0.95 [0.91, 0.98]

See notes in Table 4. In each cell, the first row contains the mean value generated by the set of admissible parameters, and the second row corresponds to the 95 inter-percentile range.

reject any productivity gains associated with moving to a four-day workweek followed by a three-day weekend at the 5% level. Most schedules that increase leisure are expected to reduce productivity, while most schedules that reduce leisure are expected to increase productivity. This suggests that, for most schedules, the memory effect dominates the fatigue effect in shaping productivity patterns. However, an important exception is 2-1, which increases both leisure and expected productivity. While this schedule is associated with an increase in leisure of 25 percent,[16] the expected output loss is only 3 percent.

5 Welfare implications

The analysis in the previous section suggests that there are no "free lunches" associated with switching to different calendars, as any increase in the proportion of vacation days is likely to be met with a drop in output. Nonetheless,

[16]Under 2-1, the ratio of vacation to workdays is 0.5. Under 5-2, the ratio is 2/5=0.4. The difference between the two is $0.1 = 0.25 * 0.4$.

depending on the preferences towards final good consumption and leisure, there may be some gains associated with changing the structure of the week.

To evaluate welfare, it is necessary to take a stance on agents' preferences towards leisure. I therefore begin by discussing how to introduce utility from leisure into this framework. Consider a representative household, whose preferences are given by:

$$U(\{c_t, l_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t \frac{(c_t^{1-\phi} l_t^{\phi})^{\gamma}}{\gamma} \text{ where } l_t = f_t(\{\chi(\tau)\}_{\tau=0}^{\infty}) \tag{8}$$

In this environment, $\beta \in (0,1)$ is the discount factor; γ governs the intertemporal elasticity of substitution; and $\phi \in (0,1)$ is the preference towards leisure.

Leisure is denoted by l_t, where l_t is a function of the calendar. The conventional approach of introducing leisure as a flow (e.g., one minus the share of time spent on work) and assuming time-separable utility from leisure does not seem appropriate for evaluating leisure at a daily frequency: it does not capture the plausible notion that the marginal utility of a vacation depends on past vacation. I therefore follow Kydland and Prescott [1982] and assume the following utility function:

$$l_t = \alpha_0 \chi(t) + \alpha_1 \sum_{\tau=0}^{\infty} (1-\eta)^{\tau} \chi(t-\tau) \tag{9}$$

The variable l_t is a linear combination of current vacation time, $\chi(t)$, and a discounted sum of past vacation time, where the discount rate is $1-\eta$ (and $\eta \in (0,1)$). The parameters α_0 and α_1 capture the relative importance of current and past vacation, respectively. The case $\alpha_1 = 0$ is the time-separable specification, in which only current vacation enters into the utility function, whereas $\alpha_1 \to \infty$ implies that past vacation is more relevant for evaluating the marginal utility of leisure. Note that when $\eta \to 0$, the utility from leisure is roughly proportional to the share of rest days in the week.

This specification of preferences assumes that the marginal utility of vacation is higher after a long period of consecutive work than after a recent vacation.[17] While this concavity is commonly assumed in the macro literature, one might question its applicability at a daily frequency. For example, if a longer vacation allows for more weekend travel, agents might have a preference for leisure bunching over leisure smoothing. This preference cannot be incorporated in the functional form in equation 9. However, there are perhaps also good reasons for assuming a preference towards leisure smoothing at a daily frequency - Milton Friedman addresses this issue in the Appendix to his seminal essay "The Optimum Quantity of Money" (Friedman [1969]):

> "Expenditure on going to the movie is regarded as expenditure on the maintenance or building up of capital in the form of a stock of memories of movies seen. The stock may depreciate very rapidly, in which case, for example, for some individuals, it may require going to one movie a week to keep the stock constant, but the utility derived from the stock is regarded as not concentrated at the moment of paying for the movie ticket, or even during the time of seeing the movie, but as derived at a steady rate so long as the stock is maintained."

In equation 9, $\sum_{\tau=0}^{\infty}(1-\eta)^{\tau}\chi(t-\tau)$ can be interpreted as a stock of leisure along the lines suggested in Friedman [1969]. The stock depreciates at a daily rate of η and is maintained by vacation days, which constitute investment in the stock.

I assume that agents can smooth the consumption of the final good by buying and selling bonds (b_t). The interest rate r satisfies $\beta(1+r)=1$. This assumption abstracts from consumption smoothing motives in shaping

[17] Note that this result follows from a relabelling of Lemma 2, part 1: if output depends only on the stock of rest, there are productivity gains associated with shorter weeks. Similarly, since utility depends only on a stock of leisure (and on time-separable consumption), there are utility gains from more-frequent leisure.

the optimal calendar. Given reasonable discount rates, $\beta \to 1$ on a daily frequency, and thus $r \to 0$; the assumption $\beta(1+r) = 1$ is thus similar to assuming a storage technology for goods. In contrast, leisure is non-tradable and non-storable.

The social planner's problem takes the following form:

$$\max_{\{\{\chi(t)\}_{t=0}^{\infty}\in\Gamma, b_{t+1}, c_t, l_t, M_t, R_t, L_t, Y_t\}_{t=0}^{\infty}} U(\{c_t, l_t\}_{t=0}^{\infty}) \tag{10}$$

s.t. $b_0 = 0$, R_0, M_0, $\{\chi(-t)\}_{t=1}^{\infty}$, equations 1, 2, 3, 8 and 9, $Y_t = L_t$ and the budget constraints:

$$c_t + b_{t+1} = Y_t + (1+r)b_t \tag{11}$$

$$\sum_{t=0}^{\infty} \frac{b_t}{(1+r)^t} \geq 0 \tag{12}$$

The budget constraints (equations 11-12) imply that the net present value of consumption equals the net present value of output. Since output is assumed to be proportional to L_t ($Y_t = L_t$), this specification abstracts from costs associated with producing non-labor inputs.

To discuss the implications of this omission, it is useful to distinguish between capital and materials (which include capital utilization costs such as electricity). The omission of materials will not affect the results under the assumption that the unit cost of producing materials is constant.[18] The omission of payments to fixed capital will tend to overstate the consumption gains associated with productivity gains, as, in the model, more productive

[18] To illustrate, assume that the cost of producing electricity (e) is p, and that output is given by $Y_t = \{e_t, L_t\}$. Optimality requires that $e_t = L_t$, and thus the expenditure on electricity is proportional to output. Since ignoring these costs amounts to a proportional change in output, a percent change in output will map into a percent change in consumption. However, if there are decreasing returns in the production of materials, the price of materials will be increasing with e_t, and the expenditure share on material will be increasing with L_t. In this case, the model over-states the consumption gains associated with schedules that imply productivity gains.

labor requires more capital.[19] However, this may not be a realistic feature of the model. It is possible to think of circumstances in which the efficient level of capital is proportional to the number of workers, and is invariant to labor productivity. For example, even the most productive textile worker uses one sewing machine at a time. Highly productive researchers may use the same amount of computers as less productive researchers, etc. If the optimal capital stock depends on the number of workers rather than on labor productivity, any gains in output will map into consumption gains and will not necessitate additional investment.

The set Γ (equation 10) represents the planner's choice set of calendars. As a starting point, I assume that Γ consists of a set of weeks, where a week is defined as a calendar consisting of a recurring cycle of work and vacation days. I restrict attention to weeks that have no more than 6 consecutive work days and 3 consecutive rest days. These restrictions are made primarily for computational reasons.[20] Later, I will consider the alternative restriction that calendars must conform to a seven-day cycle.

Welfare gains, denoted $w_{i,j}$, are computed as the compensating increase in consumption that leaves the representative agent indifferent between a week consisting of i workdays and j rest days and the 5-2 week:

$$U(\{(1+w_{i,j})c_t(5,2), l_t(5,2)\}_{t=0}^{\infty}) = U(\{c_t(i,j), l_t(i,j)\}_{t=0}^{\infty}) \tag{13}$$

where $c_t(i,j)$ and $l_t(i,j)$ denote equilibrium values corresponding to an i-j week, at the steady state. Note that this exercise abstracts away from

[19] Given the Leontieff production structure, the efficient level of the capital stock is given by the maximal level of L_t: $K_t = \max\{L_t\}$. If capital depreciates at a rate δ, steady state investment is given by $\delta \max\{L_t\}$, and steady state consumption is given by $c = rNPV(Y) - \delta \max\{L_t\}$ (where $NPV(Y)$ is the net present value of output). Thus, the omission of capital accumulation would tend to overstate the consumption gains associated with schedules that increase $\max\{L_t\}$.

[20] The set of all possible calendars between dates 0 and t is 2^t, since each day is associated with a binary indicator variable. The restriction to a limited set of weeks reduces dimensionality substantially.

transitional dynamics, and compares different steady-state welfare levels.

Calibration. Table 7 summarizes the calibration of the preference parameters. I assign $\beta = 0.97^{\frac{1}{365}}$ to capture an annual discount rate of 3 percent adjusted for a daily frequency, and, following Kydland and Prescott [1982], set $\gamma = -0.5$. As a baseline specification, the parameters α_0, α_1 and η are chosen to correspond to those in Kydland and Prescott [1982]. As an alternative specification, α_0, α_1 and η are calibrated instead to match the findings in Hotz et al. [1988]. Appendix A describes the procedure for adjusting these parameters for a daily frequency (Kydland and Prescott [1982] calibrate their model for quarterly data, and Hotz et al. [1988] estimate a model using annual data). The two specifications yield highly similar results, which are distinguishable only at the third decimal point.[21] Since the reported results are rounded at the 2 decimal point level, they are consistent with both specifications.

I calibrate ϕ so that, given a 5-2 workweek, an agent is indifferent with respect to working on a Saturday given an overtime premium of s.[22] Bell and Hart [2003] estimate the overtime premium to be between 0-50%. This maps into a range of $\phi \in (0.3, 0.4)$. Within this range, I consider the two extreme values $\phi = 0.3, 0.4$.

Table 8 presents the range of welfare gains implied by the set of admissible parameters, and Table 9 presents the corresponding means with their associated 95% confidence intervals. The results suggest that the 5-2 week is not easily dominated by any other week. When $\phi = 0.3$, for any other

[21]Given $\eta \to 0$ and $\beta \to 1$, both specifications are approximately equal to a model in which l is proportional to the average ratio of rest and work days.

[22]In Kydland and Prescott [1982], the parameter ϕ is calibrated as $\frac{2}{3}$ to match the steady state fraction of time spent on work. The calculation is roughly as follows: people spend around 40 hours a week at work. Assuming that 8 hours a day are necessary for sleep, there are 16x7=112 available hours a week: $\frac{112-40}{112} \approx \frac{2}{3}$. When the unit of account is days rather than hours, the calculation changes, as the steady state number of workdays is approximately 5 days a week, or $\frac{2}{7} \approx 0.3$. This value is consistent with the lower end of the calibrated range of ϕ.

Table 7: Simulation parameters - preferences

Paramter	Value	Source	Original
β	0.9999	Kydland and Prescott [1982]	0.99
γ	-0.5	Kydland and Prescott [1982]	-0.5
ϕ	0.3,0.4	Bell and Hart [2003]	$s \in (0, 50\%)$
η	0.0011	Kydland and Prescott [1982]	0.1
α_0	0.0057	Kydland and Prescott [1982]	0.5
α_1	0.00057	Kydland and Prescott [1982]	0.05
η	0.0014	Hotz et al. [1988]	0.4
α_0	0	Hotz et al. [1988]	0
α_1	0.0035	Hotz et al. [1988]	1

Parameters are specified in a daily frequency. The original parameters are specified for a quarterly frequency in Kydland and Prescott [1982] and an annual frequency in Hotz et al. [1988].

week, there is some vector of admissible parameters that would suggest welfare losses from that week. When $\phi = 0.4$, there are two alternatives that strictly dominate the 5-2 workweek: 4-2 and 2-1.

Table 9 suggests that expected welfare is maximized by a 2 or 3 day workweek followed by a 1 day weekend. Expected steady state welfare gains vary between 3-7%. Note that 3-1 is associated with an expected 12% gain in output, while the expected output loss from 2-1 is only 3% (see Table 5). The increase in leisure is sufficient to compensate for this loss, resulting in a welfare gain. Whether the optimal schedule is 2-1 or 3-1 depends on the preference parameter ϕ. However, conditional on ϕ, the welfare gains associated with one of these alternatives are statistically significant at the 5% level.

The results are also useful for evaluating alternative schedules that have been practiced or proposed. For example, the analysis suggests that the transition from 6-1 to 5-2 during the 1940s was almost certainly welfare improving. However, current proposals to transition to a 4-3 workweek are likely to be associated with welfare losses.

Table 8: Range of welfare gains ($w_{i,j}$)

	Rest days					
	$\phi = 0.3$			$\phi = 0.4$		
Work days	1	2	3	1	2	3
1	[-0.29,-0.06]	[-0.61,-0.38]	[-0.75,-0.55]	[-0.19,0.07]	[-0.52,-0.25]	[-0.68,-0.43]
2	[-0.03,0.15]	[-0.3,-0.1]	[-0.49,-0.27]	[0.01,0.19]	[-0.2,0.03]	[-0.39,-0.13]
3	[-0.02,0.17]	[-0.13,0.01]	[-0.31,-0.11]	[-0.05,0.13]	[-0.06,0.09]	[-0.21,0.01]
4	[-0.1,0.12]	[-0.04,0.03]	[-0.19,-0.03]	[-0.17,0.03]	[0,0.06]	[-0.11,0.06]
5	[-0.18,0.06]	[0,0]	[-0.12,0]	[-0.28,-0.07]	[0,0]	[-0.06,0.07]
6	[-0.26,0.02]	[-0.04,0.01]	[-0.11,0]	[-0.37,-0.14]	[-0.07,-0.02]	[-0.07,0.04]

Highlighted cells are schedules in which the ratio of rest days to work days exceed that of a 5-2 schedule. Welfare gains are computed using equation 13.

Table 9: Expected welfare gains ($w_{i,j}$)

	Rest days					
	$\phi = 0.3$			$\phi = 0.4$		
Work days	1	2	3	1	2	3
1	-0.18	-0.49	-0.64	-0.07	-0.38	-0.55
	[-0.26,-0.11]	[-0.57,-0.41]	[-0.71,-0.58]	[-0.15,0.02]	[-0.47,-0.28]	[-0.63,-0.47]
2	0.04	-0.19	-0.37	0.07	-0.08	-0.24
	[-0.02,0.11]	[-0.26,-0.13]	[-0.44,-0.3]	[0.02,0.15]	[-0.15,0]	[-0.33,-0.16]
3	0.06	-0.06	-0.2	0.03	0.02	-0.09
	[0.01,0.13]	[-0.1,-0.01]	[-0.26,-0.14]	[-0.02,0.09]	[-0.03,0.07]	[-0.16,-0.02]
4	0.02	-0.01	-0.11	-0.06	0.03	-0.02
	[-0.04,0.08]	[-0.03,0.02]	[-0.16,-0.06]	[-0.12,0]	[0.01,0.05]	[-0.07,0.03]
5	-0.03	0	-0.07	-0.14	0	0
	[-0.11,0.03]	[0,0]	[-0.1,-0.03]	[-0.22,-0.1]	[0,0]	[-0.04,0.03]
6	-0.08	-0.02	-0.05	-0.22	-0.05	-0.02
	[-0.17,-0.02]	[-0.04,0]	[-0.09,-0.02]	[-0.3,-0.17]	[-0.07,-0.03]	[-0.05,0.02]

See notes in Table 8. In each cell, the top row is the average welfare gain and the bottom row is the 95 inter-percentile range.

5.1 The optimal seven-day cycle

The historical experience described in section 2 suggests that there may be difficulties in implementing calendars that are unsynchronized with the cycle of religious worship. In light of this, I consider the optimal calendar under the restriction of a seven-day cycle. The set of calendars that satisfy this restriction is only partially included in the previous analysis, as it includes some cycles with non-consecutive vacation time (e.g., a week with Tuesday and Saturday off). I compute output and welfare for all possible seven-day cycles beginning with a workday and ending with a rest day (there are 2^5 such cycles, many of which are equivalent).

It turns out that in all specifications, the seven-day cycle that maximizes expected welfare is 3-1-2-1: three workdays followed by a single vacation day, followed by two additional workdays and ending with a vacation day. For example, in Christian countries, this cycle could consist of a week in which Sundays and Wednesdays are days off; in Muslim countries, the cycle could consist of a week in which Fridays and Tuesdays days off; and, in Jewish countries, the cycle could consist of a week in which Saturdays and Wednesdays are days off.[23] The expected welfare gains from this cycle are equivalent to a 5.6 percent permanent increase in consumption.

The first column of Table 10 summarizes the distribution of steady state welfare gains implied by the set of admissible parameters. Given that $\eta \to 0$ in both the Kydland and Prescott [1982] and Hotz et al. [1988] specifications, utility from leisure is approximately proportional to the share of vacation days. Thus, since the proportion of vacation days in 3-1-2-1 remains 2/7, the welfare gains reflect gains in steady state output: $w = (y(3-1-2-1) - y(5-2))/y(5-2)$, where $y(\cdot)$ is the average output level induced by the calendar. Consequently, across all specifications, the steady state welfare

[23]It is easy to see that there are other equivalent options, for example, in a Christian country, having Sunday and Wednesday off is the same as having Sunday and Thursday off.

Table 10: Welfare-improving seven day cycles

	3-1-2-1	4-1-1-1
Mean output gains	1.056	1.035
Mean welfare gains	0.056	0.035
95 percent inter-percentile range: output gains	[1.01,1.12]	[1.01,1.08]
95 percent inter-percentile range: welfare gains	[0.01,0.12]	[0.01,0.08]
Range of output gains	[1,1.16]	[1,1.11]
Range of welfare gains	[0,0.16]	[0,0.11]

Output gains and welfare gains are computed relative to a 5-2 workweek. The schedule 3-1-2-1 (first column) corresponds to three workdays, followed by one vacation day, followed by two additional workdays and ending with a vacation day. The schedule 4-1-1-1 (second column) corresponds to the Brunei calendar: four workdays followed by a vacation day, a workday and a vacation day. While there are other seven-day cycles that generate welfare gains for some parameters, these two schedules are the only ones that create welfare improvements for all parameter values.

gains are the same and equal to the output gains. Further, it is worth noting that the steady state welfare gains are positive for all admissible parameters.

While 3-1-2-1 is able to accommodate weekly religious worship, it cannot simultaneously accommodate the Christian, Muslim and Jewish worship days, and is thus perhaps more feasible in religiously-homogeneous countries. However, it should be noted that the current 5-2 schedule can only accommodate two out of the three major religious worship days. For example, in the United States and in Europe, the convention of having Saturday and Sunday off accommodates Christians and Jews but not Muslims.

In many predominantly Christian countries in Europe, the Muslim population far exceeds the Jewish population. In these countries, one might consider the adoption of the Brunei calendar, that has Fridays and Sundays off (however, one should also consider the fact that the "cost" of working on the Sabbath may be larger for Jewish people, since Islam does not prohibit Friday work while Judaism places heavy restrictions on Saturday work). To evaluate the economic implications of this alternative, the second column of

Table 10 presents the welfare statistics associated with the 4-1-1-1 schedule. The analysis suggests that there are positive welfare gains from adopting the Brunei calendar, with expected welfare gains of 3.5 percent.

6 Conclusion

There are several challenges associated with assessing the efficiency properties of the 5-2 week and quantitatively evaluating the tradeoffs associated with altering its structure. This paper attempts to formalize these challenges and make preliminary steps towards addressing them.

The first challenge relates to evaluating the relationship between work duration and productivity on a daily frequency. Conceptually, increasing the duration of consecutive work periods may be associated with some productivity losses due to fatigue, or with some productivity gains due to learning-by-doing. Empirically evaluating the quantitative relevance of each of these channels presents a challenge given that daily output measures are unavailable. Under the assumption that, on a daily frequency, electricity and labor are required in fixed proportions, the structural parameters of the model can be estimated based on movements in electricity usage. The findings suggest that both channels are quantitatively relevant for the evolution of labor productivity over the workweek. However, a permanent reduction in the ratio of workdays to rest days unambiguously results in lower output.

A second challenge relates to calibrating the preference towards leisure. Evaluating alternative schedules requires taking a stance towards the preference for leisure smoothing on a relatively high frequency. Here, I follow the macro literature and assume that agents have a preference for smoothing leisure. However, the institutional practice of "long weekend" holidays (e.g. Labor Day, Presidents' Day, Memorial Day etc) suggests that there may be some preference for leisure-bunching at high frequencies, or some fixed costs associated with vacation. These plausible channels have the potential to eliminate the gains from switching to a shorter week cycle. Further analysis regarding the preference towards leisure smoothing is necessary in order to refine the welfare analysis in this paper.

Finally, there may be scope for introducing heterogeneity into this framework. Here, I consider the simplest case of a representative agent. The minimal criteria on the set of admissible parameters allows for some differences in parameters across occupations. However, the structural parameters may systematically differ across occupations, and therefore the optimal structure of the week may vary with occupational composition. For example, cognitive tasks may be more intensive in memory and less subject to fatigue than physical labor. Additionally, a richer framework may be able to incorporate the reality of many people working during weekends.

The findings of this paper suggest that there are potentially large welfare gains associated with restructuring the week. In light of this, there is scope for further research geared at meeting the above challenges.

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A Calibration details

This section documents the process of adjusting α_0, α_1 and η for an annual frequency. Let $\hat{\alpha}_0$, $\hat{\alpha}_1$ and $\hat{\eta}$ denote estimated parameters which were derived for a unit of time consisting to T days.

The discount rate η is calculated in the standard way, so that $(1-\eta)^T = 1-\hat{\eta}$. The remaining parameters, α_0 and α_1, are more problematic: their relative magnitudes governs the relative importance of "current" leisure relative to "past" leisure. When a time period consists of T days, some "past" days are regarded by the agent as "current". I therefore transform α_0 and α_1 so that the end-of-period utility is maintained. Formally, I choose α_0 and α_1 so that for every L_1 and L_2:

$$(\hat{\alpha}_0+\hat{\alpha}_1)L_1+\frac{\hat{\alpha_1}(1-\hat{\eta})}{\hat{\eta}}L_2 = (\alpha_0+\alpha_1)\sum_{t=0}^{T-1}(1-\eta)^t L_1+\alpha_1\sum_{t=T}^{\infty}(1-\eta)^t L_2 \quad (14)$$

The expressions correspond the value of l_t that is constructed from L_1 units of leisure for the past T days (including the current day), and L_2 units of leisure forever before that. This implies that:

$$(\hat{\alpha}_0+\hat{\alpha}_1) = (\alpha_0+\alpha_1)\sum_{t=0}^{T-1}(1-\eta)^t = (\alpha_0+\alpha_1)\frac{1-(1-\eta)^T}{\eta} \quad (15)$$

$$\Rightarrow \alpha_0 = (\hat{\alpha}_0+\hat{\alpha}_1)(\frac{\eta}{1-(1-\eta)^T})-\alpha_1$$

and:

$$\frac{\hat{\alpha_1}(1-\hat{\eta})}{\hat{\eta}} = \alpha_1\sum_{t=T}^{\infty}(1-\eta)^t = \alpha_1\frac{(1-\eta)^T}{\eta} \Rightarrow \alpha_1 = \frac{\hat{\alpha_1}(1-\hat{\eta})}{\hat{\eta}}(\frac{\eta}{(1-\eta)^T}) \quad (16)$$

Substituting into equation 15 generates the value for α_0.

In Hotz et al. [1988], the utility function is specified slightly differently, as:

$$l_t = \chi(t) + \alpha \sum_{\tau=1}^{\infty} (1-\eta)^{\tau-1} \chi(t-\tau) = \chi(t) + \frac{\alpha}{1-\eta} \sum_{\tau=1}^{\infty} (1-\eta)^{\tau} \chi(t-\tau) \quad (17)$$

(where, in their framework, $\chi(t)$ is a continuous variable representing leisure at time t). Rewriting the above to correspond to the specification in Kydland and Prescott [1982] yields:

$$l_t = \chi(t) - \frac{\alpha}{1-\eta}\chi(t) + \frac{\alpha}{1-\eta} \sum_{t=0}^{\infty} (1-\eta)^{t} \chi(t-\tau) \quad (18)$$

In this framework, $\hat{\alpha}_0 = 1 - \frac{\alpha}{1-\eta}$ and $\hat{\alpha}_1 = \frac{\alpha}{1-\eta}$. For the simulation, I use the estimated values of $\eta = 0.4$ and $\alpha = 0.6$.

B Proof of Lemma 1

To prove the first part of the Lemma, let $M_0 = M_T$ denote the steady state stock of memory on the last vacation day (Sunday in the current workweek). On workday $1 \leq i \leq (1-\lambda)T$, the stock of memory M_i is given by:

$$M_1 = (1-\delta_m)M_0 + 1 \quad (19)$$

$$M_2 = (1-\delta_m)M_1 + 1 = (1-\delta_m)^2 M_0 + (1-\delta_m) + 1 \quad (20)$$

$$\vdots \quad (21)$$

$$M_i = (1-\delta_m)^i M_0 + \sum_{t=0}^{i-1} (1-\delta_m)^t = (1-\delta_m)^i M_0 + \frac{1-(1-\delta_m)^i}{\delta_m} \quad (22)$$

Since there is no accumulation of memory during the weekend, $M_0 = (1 - \delta_m)^{\lambda T} M_{(1-\lambda)T}$, and hence:

$$M_0 = (1-\delta_m)^{\lambda T}((1-\delta_m)^{(1-\lambda)T} M_0 + \frac{1-(1-\delta_m)^{(1-\lambda)T}}{\delta_m}) \tag{23}$$

$$= (1-\delta_m)^T M_0 + \frac{(1-\delta_m)^{\lambda T} - (1-\delta_m)^T}{\delta_m}$$

Solving for M_0 yields:

$$M_0 = \frac{(1-\delta_m)^{\lambda T} - (1-\delta_m)^T}{\delta_m(1-(1-\delta_m)^T)} \tag{24}$$

To show that M_i is increasing in i for $1 \leq i \leq (1-\lambda)T$, note that the derivative of M_i (equation 22) with respect to i is:

$$\frac{\partial M_i}{\partial i} = \frac{\partial (1-\delta_m)^i}{\partial i}(M_0 - \frac{1}{\delta_m}) \tag{25}$$

Since $\frac{\partial (1-\delta_m)^i}{\partial i} < 0$, the above is positive if and only if:

$$M_0 < \frac{1}{\delta_m} \tag{26}$$

which follows from equation 24. It follows that $M_{(1-\lambda)T} > M_1$, and hence, if $\alpha_r = 0$ and $\alpha_m > 0$, Y_i is decreasing in i.

To prove the second part of the lemma, let $R_0 = R_T$ and note that for $1 \leq i \leq (1-\lambda)T$, $R_i = (1-\delta)^i R_0$. Thus, R_i is decreasing in i for $1 \leq i \leq (1-\lambda)T$, and hence, if $\alpha_m = 0$, Y_i is decreasing in i.

To prove the third part of the lemma, note that:

$$\frac{Y_{i+1} - Y_i}{Y_i} = \frac{R_{i+1}^{\alpha_r} M_{i+1}^{\alpha_m} - R_i^{\alpha_r} M_i^{\alpha_m}}{R_i^{\alpha_r} M_i^{\alpha_m}} \tag{27}$$

$$= (\frac{R_{i+1}}{R_i})^{\alpha_r} (\frac{M_{i+1}}{M_i})^{\alpha_m} - 1 \tag{28}$$

The derivative with respect to α_r is negative because $R_{i+1} < R_i$. The derivative with respect to α_m is positive because $M_{i+1} > M_i$.

To show that $(Y_{i+1} - Y_i)/Y_i$ is decreasing in δ_r, note that for $1 \leq i \leq (1-\lambda)T$, $R_i = (1-\delta_r)^i R_0$ and hence (for $i < (1-\lambda)T$):

$$\frac{R_{i+1}}{R_i} = \frac{(1-\delta_r)^{i+1} R_0}{(1-\delta_r)^i R_0} = (1-\delta_r) \tag{29}$$

It follows from equation 27 that $(Y_{i+1} - Y_i)/Y_i$ is decreasing in δ_r.

To prove the final part of the lemma, using equation 27, rewrite the expression as:

$$\frac{Y_{(1-\lambda)T} - Y_1}{Y_1} = (\frac{R_{(1-\lambda)T}}{R_1})^{\alpha_r} (\frac{M_{(1-\lambda)T}}{M_1})^{\alpha_m} - 1 \tag{30}$$

Thus, the derivative with respect to δ_m depends on the derivative of the ratio $M_{(1-\lambda)T}/M_1$ with respect to δ_m. Using the above derivations,

$$\frac{M_{(1-\lambda)T}}{M_1} = \frac{(1-\delta_m)^{(1-\lambda)T} M_0 + \frac{1-(1-\delta_m)^{(1-\lambda)T}}{\delta_m}}{(1-\delta_m)M_0 + 1} \tag{31}$$

Substituting for M_0 yields:

$$\frac{M_{(1-\lambda)T}}{M_1} = \frac{(1-\delta_m)^{(1-\lambda)T} \frac{(1-\delta_m)^{\lambda T} - (1-\delta_m)^T}{1-(1-\delta_m)^T} + 1 - (1-\delta_m)^{(1-\lambda)T}}{(1-\delta_m)\frac{(1-\delta_m)^{\lambda T} - (1-\delta_m)^T}{1-(1-\delta_m)^T} + \delta_m} \tag{32}$$

$$= \frac{\frac{(1-\delta_m)^T - (1-\delta_m)^{(2-\lambda)T}}{1-(1-\delta_m)^T} + 1 - (1-\delta_m)^{(1-\lambda)T}}{\frac{(1-\delta_m)^{\lambda T+1} - (1-\delta_m)^{T+1}}{1-(1-\delta_m)^T} + \delta_m}$$

$$= \frac{(1-\delta_m)^T - (1-\delta_m)^{(2-\lambda)T} + (1-(1-\delta_m)^{(1-\lambda)T})(1-(1-\delta_m)^T)}{(1-\delta_m)^{\lambda T+1} - (1-\delta_m)^{T+1} + \delta_m(1-(1-\delta_m)^T)}$$

$$= \frac{(1-\delta_m)^T + 1 + (1-\delta_m)^{(1-\lambda)T}}{(1-\delta_m)^{\lambda T+1} - (1-\delta_m)^{T+1} + \delta_m(1-(1-\delta_m)^T)}$$

It is possible to confirm numerically that this expression is decreasing in δ_m for $T = 7$ and $\lambda = 2/7$; in general, this expression is non-monotone. For example, $T = 70$ and $\lambda = 2/7$ generates a non-monotone relationship between $M_{(1-\lambda)T}/M_1$ and δ_m.

C Proof of Lemma 2

To prove the first part of the lemma, solving recursively for R_0 yields:

$$R_0 = R_T = (1-\delta_r)^T R_0 + \sum_{t=0}^{\lambda T-1}(1-\delta_r)^t = (1-\delta_r)^T R_0 + \frac{1-(1-\delta_r)^{\lambda T}}{\delta_r} \quad (33)$$

$$\Rightarrow R_0 = \frac{1-(1-\delta_r)^{\lambda T}}{\delta_r(1-(1-\delta_r)^T)}$$

Thus, R_0 is decreasing in λ. Let R' denote the stock of rest under an alternative schedule λ', T', where $\lambda' > \lambda$ and $T' = T$. Using the above, $R'_0 > R_0$, and hence $R'_i = (1-\delta_r)^i R'_0 > (1-\delta_r)^i R_0 = R_i$ for all $1 \leq i \leq (1-\lambda)T$. If $\alpha_m = 0$,

$$\bar{Y}' = \frac{1}{(1-\lambda')T}\sum_{i=1}^{(1-\lambda')T}(R'_i)^{\alpha_r} \geq \frac{1}{(1-\lambda)T}\sum_{i=1}^{(1-\lambda)T}(R'_i)^{\alpha_r} \geq \quad (34)$$

$$\frac{1}{(1-\lambda)T}\sum_{i=1}^{(1-\lambda)T}R_i^{\alpha_r} = \bar{Y}$$

where the first inequality follows from the fact that R'_i is decreasing in i, and the second inequality follows from the fact that $R'_i > R_i$. Thus $\bar{Y}$ is increasing in λ.

To show that $\bar{Y}$ is decreasing in T, note that, if $\alpha_m = 0$, $\bar{Y}$ can be written as:

$$\bar{Y} = \frac{1}{(1-\lambda)T}\sum_{d=1}^{(1-\lambda)T}R_d^{\alpha_r} = \frac{1}{(1-\lambda)T}\sum_{d=1}^{(1-\lambda)T}(1-\delta_r)^{\alpha_r d}R_0^{\alpha_r} = \quad (35)$$

$$(\frac{1-(1-\delta_r)^{\lambda T}}{\delta_r(1-(1-\delta_r)^T)})^{\alpha_r}\frac{(1-\delta_r)^{\alpha_r}-(1-\delta_r)^{\alpha_r((1-\lambda)T+1)}}{(1-\lambda)T(1-(1-\delta_r)^{\alpha_r})}$$

Note that $\bar{Y}$ is decreasing in T if and only if $\ln(\bar{Y})$ is decreasing in T. Using the above, $\ln(\bar{Y})$ can be written as:

$$\ln(\bar{Y}) = \alpha_r(\ln(1-(1-\delta_r)^{\lambda T}) - \ln(1-(1-\delta_r)^T)) \tag{36}$$

$$+\ln(1-(1-\delta_r)^{\alpha_r(1-\lambda)T}) - \ln(T) + c$$

where $c = c(\delta_r, \lambda, \alpha_r)$ is a function that does not depend on T. Taking the derivative of $\ln(\bar{Y})$ with respect to T yields:

$$\frac{\partial \ln(\bar{Y})}{\partial T} = \alpha_r(-\frac{\ln(1-\delta_r)\lambda(1-\delta_r)^{\lambda T}}{1-(1-\delta_r)^{\lambda T}} + \frac{\ln(1-\delta_r)(1-\delta_r)^T}{1-(1-\delta_r)^T}) \tag{37}$$

$$-\frac{\ln(1-\delta_r)(1-\lambda)\alpha_r(1-\delta_r)^{\alpha_r(1-\lambda)T}}{1-(1-\delta_r)^{\alpha_r(1-\lambda)T}}) - \frac{1}{T}$$

To show that the above expression is negative, I multiply it by T and establish the following inequality:

$$\alpha_r(-\frac{\ln(1-\delta_r)\lambda T(1-\delta_r)^{\lambda T}}{1-(1-\delta_r)^{\lambda T}} + \frac{\ln(1-\delta_r)T(1-\delta_r)^T}{1-(1-\delta_r)^T}) \tag{38}$$

$$-\frac{\ln(1-\delta_r)(1-\lambda)T\alpha_r(1-\delta_r)^{\alpha_r(1-\lambda)T}}{1-(1-\delta_r)^{\alpha_r(1-\lambda)T}} \leq 1$$

Introducing the notation $h = (1-\delta_r)^T$, the above inequality can be rewritten as:

$$\alpha_r(-\frac{\ln(h^\lambda)h^\lambda}{1-h^\lambda} + \frac{\ln(h)h}{1-h}) - \frac{\ln(h^{\alpha_r(1-\lambda)})h^{\alpha_r(1-\lambda)}}{1-h^{\alpha_r(1-\lambda)}} \leq 1 \tag{39}$$

I verify the above inequality numerically for all $(h, \lambda, \alpha_r) \in (0,1)^3$ (note that $h = (1-\delta_r)^T \in (0,1)$). This establishes that $\bar{Y}$ is decreasing in T, and concludes the proof of the first part of the lemma.

To prove the second part of the lemma, assume that $\alpha_r = 0$ and $\alpha_m > 0$.

Using equation 24, it is immediate that M_0 is decreasing in λ. Thus, if $\lambda' > \lambda$ and $T' = T$, then $M_0' < M_0$. Using the recursive formula for M_i, it follows that $M_i' < M_i$ for all $1 \le i \le (1-\lambda')T$. Since M_i is increasing in i, it follows that:

$$\bar{Y}' - \bar{Y} = \frac{1}{(1-\lambda')T} \sum_{i=1}^{(1-\lambda')T} (M_i')^{\alpha_m} - \frac{1}{(1-\lambda)T} \sum_{i=1}^{(1-\lambda)T} M_i^{\alpha_m} \le \tag{40}$$

$$\frac{1}{(1-\lambda')T} \sum_{i=1}^{(1-\lambda')T} ((M_i')^{\alpha_m} - M_i^{\alpha_m}) \le 0$$

Which establishes that, in this case, $\bar{Y}$ is decreasing in λ.

To show that $\bar{Y}$ is increasing in T, using equations 22 and 24, $\bar{Y}$, the stock M_i can be written as:

$$M_i(T) = (1-\delta_m)^i \left(\frac{(1-\delta_m)^{\lambda T} - (1-\delta_m)^T}{\delta_m(1-(1-\delta_m)^T)}\right) + \frac{1-(1-\delta_m)^i}{\delta_m} = \tag{41}$$

$$\frac{1}{\delta_m}\left(\frac{(1-\delta_m)^{\lambda T+i} - (1-\delta_m)^{T+i} + (1-(1-\delta_m)^i)(1-(1-\delta_m)^T)}{1-(1-\delta_m)^T}\right) =$$

$$\frac{1}{\delta_m}\left(\frac{(1-\delta_m)^{\lambda T+i} - (1-\delta_m)^{T+i} + 1 - (1-\delta_m)^T - (1-\delta_m)^i + (1-\delta_m)^{i+T}}{1-(1-\delta_m)^T}\right) =$$

$$\frac{1}{\delta_m}\left(\frac{(1-\delta_m)^{\lambda T+i} + 1 - (1-\delta_m)^T - (1-\delta_m)^i}{1-(1-\delta_m)^T}\right) = \frac{1}{\delta_m}\left(1-(1-\delta_m)^i \frac{1-(1-\delta_m)^{\lambda T}}{1-(1-\delta_m)^T}\right)$$

To show that $M_i(T)$ is increasing in T, it is enough to show that $(1-(1-\delta_m)^{\lambda T})/(1-(1-\delta_m)^T)$ is decreasing in T, or that $\ln((1-(1-\delta_m)^{\lambda T})/(1-(1-\delta_m)^T))$ is decreasing in T. To show this:

$$\frac{\partial \ln\left(\frac{1-(1-\delta_m)^{\lambda T}}{1-(1-\delta_m)^T}\right)}{\partial T} = -\frac{\ln(1-\delta_m)\lambda(1-\delta_m)^{\lambda T}}{1-(1-\delta_m)^{\lambda T}} + \frac{\ln(1-\delta_m)(1-\delta_m)^T}{1-(1-\delta_m)^T} \tag{42}$$

To show that this expression is negative, I multiply it by $T > 0$ and arrive at:

$$-\frac{\ln((1-\delta_m)^{\lambda T})(1-\delta_m)^{\lambda T}}{1-(1-\delta_m)^{\lambda T}}+\frac{\ln((1-\delta_m)^{T})(1-\delta_m)^{T}}{1-(1-\delta_m)^{T}}<0 \qquad (43)$$

where the inequality follows from the fact that $x\ln(x)/(1-x)$ is increasing in x, and $(1-\delta_m)^{\lambda T} > (1-\delta_m)^T$.

To conclude the proof, consider an alternative schedule with $T' > T$ and $\lambda' = \lambda$. If $\alpha_r = 0$,

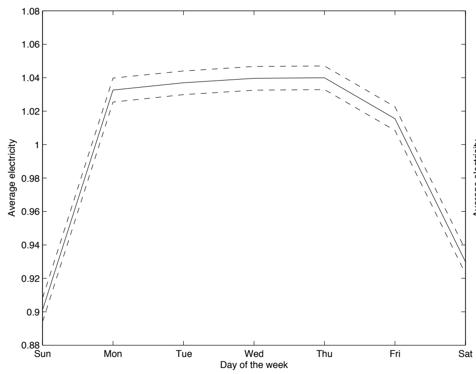
$$\bar{Y}' - \bar{Y} = \frac{1}{(1-\lambda)}\left(\frac{1}{T'}\sum_{i=1}^{(1-\lambda)T'} M_i(T') - \frac{1}{T}\sum_{i=1}^{(1-\lambda)T} M_i(T)\right) \qquad (44)$$

Since M_i is increasing in i, it follows that:

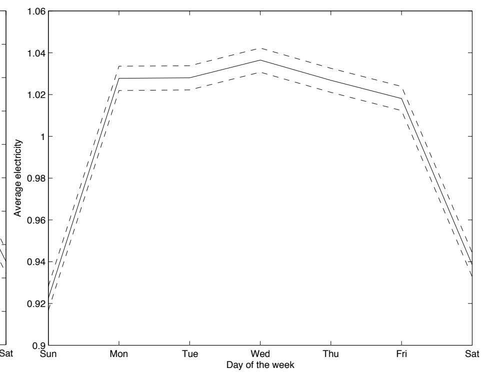
$$\bar{Y}' - \bar{Y} \geq \frac{1}{(1-\lambda)T}\sum_{i=1}^{(1-\lambda)T}(M_i(T') - M_i(T)) \geq 0 \qquad (45)$$

where the inequality follows from the fact that $M_i(T)$ is increasing in T. This establishes that $\bar{Y}$ is increasing in T when $\alpha_r = 0$ and $\alpha_m > 0$.

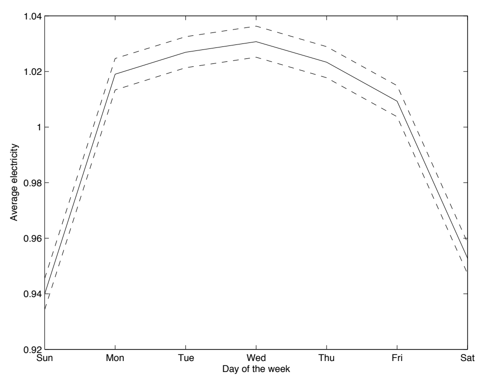
D Additional figures



(a)

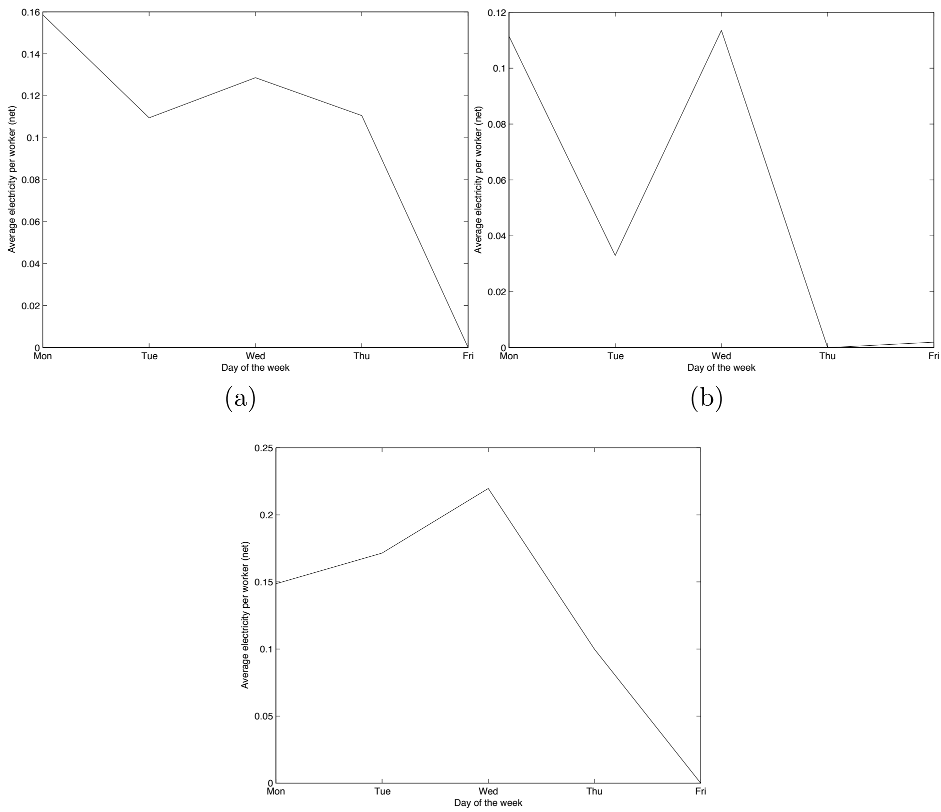


(b)



(c)

Figure 6: Average electricity usage in different RTOs over the week. Dashed lines represent the associated 95% confidence intervals. See figure 3 for the remaining four RTOs.



(a) (b)

(c)

Figure 7: Net electricity per worker (Y_d) over the workweek in different RTOs (see also figure 4).