Policy Research Working Paper 7423

Value of Improved Information about Forest Protection Values, with Application to Rainforest Valuation

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WORLD BANK GROUP
Development Research Group
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September 2015

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Abstract

What is the utility from obtaining more precise values of natural resource objects (rainforests), through surveys or other similar information gathering? In the value of information problems studied here, a principal who wishes to preserve a resource sets a price to offer to a seller without knowing precisely the protection value or opportunity value, to the seller. The value of information related to more precise information about the protection value for the principal is a key issue in environmental and natural resource valuation, but it is in most cases implicit and not analyzed. More precise resource values reduce the frequency of two types of mistakes (saving the resource when it should not be saved, and not saving the resource when it should be saved), and increases the principal's ex ante expected utility value. This paper applies the model to Amazon rainforest protection and considers the hypothetical value of perfect information. The analysis finds that the value of perfect information can easily exceed realistic information costs, thus perhaps justifying significant expenditures for valuation studies, given that all available information is used efficiently for conservation decision purposes. The value of perfect information also depends on the nature of buyer-seller interactions, and is higher in the altruistic case, where the principal has full concern for the outcome for the seller.

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Value of Improved Information about Forest Protection Values, with Application to Rainforest Valuation

By

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JEL classification: C11; C72; Q23; Q51.

Key words: Value of information; asymmetric information; Bayesian analysis; rainforests; natural resource valuation.

Value of Improved Information about Forest Protection Values, with Application to Rainforest Valuation[1]

1. Introduction

This paper analyzes, mathematically and through simulations, "value of information" (VOI) problems with applications to natural resource valuation. The objects we more concretely have in mind are rainforests and their possible preservation. The values of rainforests are inherently uncertain. It can then be relevant to expend economic resources to obtain more precise assessments of their values, through investigative research. It may however turn out that the (expected ex ante) gain from obtaining more precise valuation estimates is not "worth it" as it may cost more than this expected value gain. This paper seeks to answer the following basic question: What is the additional value to a decision maker (principal; denoted P in the following) of more precise information (VOI) about the value of a specific forest, given that P can influence the decision to either save or convert (deforest) this forest; and this decision is made optimally on the basis of all current available information? We will, for reasons of tractability, focus on the additional ex ante expected value of *perfect* information about the resource (called VOPI); which can be simulated in a straightforward way. In such cases, the additional information provided by any survey or other information-gathering process is assumed to fully reveal the true protection value of the object (although not necessarily its opportunity value). We will below also discuss relationships between VOPI and VOI in practically relevant cases, where some uncertainty will always remain after a valuation study has been carried out.

This line of research on explicit VOI analysis has little tradition in the environmental economics literature, but somewhat more in other fields including the medical risk literature (see references in section 2 below). Such interest is here often influenced by business perspectives including the development of new drugs and medical procedures, which can involve substantial investments and equity values. Small changes in precision can then easily have values multiple times that of carrying out a study. Our analysis provides direct information on the value of perfect information about tropical rainforest protection values. We also discuss the clues that our analysis gives to the value of improved but not perfect information. An aim of this study is to stimulate an increased use of such tools, in valuing the environment and natural resources more widely.

We give this problem two distinct considerations. The first is to find a value of perfect information given that decisions taken on the basis of such information are always globally

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efficient. Most environmental valuation studies are, by contrast, carried out not knowing the value of the information that they could provide; and often also without knowing whether the information will actually be used to make better decisions.

A second and more novel perspective in our analysis is to combine a VOI analysis with explicit modeling of the decisions by which the improved information is used to save or convert (not save) the resource, and where fully optimal outcomes are not necessarily realized. We argue that this would tend to be the norm for many natural resources, including rainforests. One class of reasons for such inefficiency is asymmetric information. Assume that a (small) part of the rainforest has an uncertain gross preservation value about which we seek more precise information through forest valuation. This forest and its land also have value in alternative uses (its opportunity value; e.g., as a combination of the net timber value that can be cashed out immediately, and as a source of agricultural production), also imprecisely known to the principal or buyer, but often more precisely known to the seller who decides to convert or not convert this forest to agricultural uses (such as a local farmer). We assume, for simplicity and clarity of our argument, that the study whose value we consider does not affect our estimate of the object's opportunity value. This can be relevant when the two values are uncorrelated (as we assume), and the information provided by the valuation study is directed specifically at the relevant natural resource object. To implement an efficient decision (to save or convert this piece of forest), P must construct a "revelation mechanism" which is incentive compatible for the agent or seller, S; see Myerson (1979); Myerson and Satterthwaite (1980); Gibbons (1992). This will often lead to a less than fully optimal decision, at least socially as the decision maker's (P's) objective function then does not reflect all social gains. Similar mechanisms for implementing solutions for rainforest protection have recently been considered by Mason and Plantinga (2013), and Benthem and Kerr (2013).

A key problem is that P (with preferences for saving the forest) and S (who has an interest to use the forest in alternative activities such as agriculture) may be in different countries, and this may make it difficult to fully align or aggregate their forest preservation preferences. This "mechanism design" problem interacts with the valuation problem as the VOI will depend on how large a part of the theoretical surplus from additional information can be realized in practice.

When P does not fully incorporate all the interests related to preserving the resource (including that of the agent who has the power to convert the forest), we show that less than an optimal amount of the forest will be preserved. This also leads to a different, and usually lower, VOI value, as decisions are then inefficient in more cases (both before and after new information is provided).

In the simple solution characterized below, P offers a given price to S if the forest is not converted. S may then either accept this price, or convert the forest. Section 2 is introductory, while section 3 sets up our asymmetric information model where only S knows the exact opportunity value of the resource. We also illustrate certain aspects of the solution(s) through simulations of particular, simple, parameterized cases given independent and normal distributions. Appendix B derives probabilities of the two types of errors inevitably made by the buyer under imperfect information, namely a) saving the resource when the opportunity value is higher than the protection value; and b) allowing the resource to be converted when the opportunity value is lower than the protection value.

Section 4 studies the VOI, the central topic of the paper. We here focus on simulations in simplified cases where the valuation study in question is assumed to fully reveal the gross resource protection value of the resource for the buyer, and where distributions are normal and independent. This leads to the value of perfect information (VOPI), which is a useful upper bound on the achievable informational value. It is an over-estimation as perfect information will never be realistically achieved from any valuation study (or set of such studies), but still useful as it provides an upper bound to the VOI. We also discuss clues to how we can use our results to derive (the more practically relevant) VOI values when only partial information is provided through a valuation study. Section 5 concludes.

2. The imperfect information VOI model: Basics and background

Consider individual plots of forested land controlled by a seller or agent (denoted S), who may either preserve the forest, or convert it to other uses (such as agriculture). An interested outside party, buyer or principal (denoted P), has an interest in the forest being saved by offering a payment H to S, for the event that the resource is saved. The true net value to S of converting the forest is called V, which could be positive or negative.[2] Only if V is positive will S have an interest in converting the resource (deforesting). P has a true (gross) protection value of B related to preserving the land area as forest, which could also be positive or negative.[3]

Consider first briefly the solution under perfect information, where P knows B precisely, and both P and S know V precisely. P could then select to make a payment V (+ ε; where ε is small) to S, given a) $V > 0$ (since we assume that, otherwise, the land will not be converted even with no payments), and b) $B > V$ (since otherwise the opportunity value to the seller would be so high that the buyer could not profitably make the transaction). This would result in an efficient solution with the forest being preserved if and only if its protection value exceeds its opportunity value. Note also that all net rent associated with the transaction goes to P in this case.

Two sets of problems may here arise. The first is related to the opportunity cost V, which may or may not be observed correctly, and possibly observed differently by buyer and seller. P may not

[2] Note that when $V < 0$, there is no management problem as the forest will always be saved. A problem for P, discussed below, could still be to determine whether this condition is actually fulfilled, as V may not be observable.

[3] We here assume that S behaves non-strategically, by acting optimally taking P's decision as given and not caring about the value of B. This might not hold when S is large (possibly, a government). Analysis of such cases will be left for future research.

observe the true V. The second, essential here, is that the buyer may not observe the true gross protection value of the resource, B.

Under asymmetric information, on which we focus in the continuation, when S but not P observes the seller's valuation V of the resource, solving the problem of protecting or not protecting this resource is a mechanism design problem similar to that initially studied by Myerson (1979), Myerson and Satterthwaite (1980); more recently Montero (2008); and (in the more specific context of forest protection) Salas and Roe (2011), Mason and Plantinga (2013), and Benthem and Kerr (2013). We will here not go deeply into the issue of finding P's optimal mechanism. We instead propose a very simple mechanism where P maximizes the expected value of any "project" to protect or not protect a given piece of forest, given risk neutrality of all parties involved. This drastically simplifies the technical discussion by allowing us to focus on VOI aspects.

This is a problem based on Bayesian decision theory; see related work by DeGroot (1970), Berger (1985), Pratt et al (1995), Robert (2001), Gollier (2001); see also Bergland (2005, 2014), Eeckhoudt et al (2011), and Peck et al (1999).[4] We are rather interested in studying the effect on "precision" of the decision to protect and not protect the resource, following from reduced uncertainty; parameterized here by the standard deviation on B which is initially stochastic for P. Our focus will be on VOI: in our context, the utility gains from avoiding, or reducing the statistical prevalence of, two types of mistakes that occur under imperfect information: to convert the resource to other uses when it ought to be saved; and to save it when it ought to be converted. In our context, more precise information will reduce the statistical prevalence of both types of mistakes. Under perfect information for P, there will be no such mistakes. Under improved but still imperfect information, the number of mistakes will be reduced, but not all mistakes will be eliminated.

An interest to avoid similar mistakes (or reduce their prevalence and/or seriousness) lies, implicitly or explicitly, behind all work on benefit valuation, notably in the environmental and natural resource economics literature to which our work is most closely tied. Surprisingly little has however been done to make the benefits of greater such precision concrete and precise. Few attempt have been made to measure them; although a few studies exist that are somewhat related, including Polasky and Solow (2001) considering the VOI for biological reserves; and Peck et al (1999) and Yohe (1996) for elements entering calculations for the social cost of carbon, in the context of optimal policies to mitigate climate change.[5] Such clarification and measurement should, ideally, be the starting point for any (costly) benefit valuation study. Considering this as a research topic has so far been more common, and with more applications, in the medical risk literature; see Claxton (1999), Ades et al (2004), Yokota and Thompson (2004), and Eckermann

[4] For a recent survey of a wider set of VOI studies, see Keisler et al (2014).

[5] For the latter issue, one might argue, the problem of non-optimal decisions would interact severely with the true VOI, much in the same way as it does in the context discussed here.

and Willan (2007) for overviews and applications; and related analyses in Gold et al (1996), Samson et al (1989); Dakins et al (1996); and Tanner and Wong (1987). See also Costello, Polasky and Solow (2001), and Kennedy and Barbier (2013), for value of information analyses in the context of renewable resource management. The number of applications is however even here not very large, in part due to the perceived analytical and computational complexities involved.

As however noted, this paper also recognizes that practical decisions to save or protect a natural resource object are often not globally optimal. The value of additional information then needs to be considered in that light.

3. The model

Assume in the following that P (the principal or buyer) initially observes not the true value of B, but instead an indicator variable b. From the point of view of P, B has distribution F(B), with expectation E(B) and standard deviation σ_B.

S (the potential seller of the resource) is assumed to observe the true opportunity value of the resource in question, V. P however only knows a (subjective and continuous) distribution function G(V) with density g(V). This is thus where asymmetric information enters. Both P and S are assumed to be risk neutral.

Asymmetric information about V is likely the typical case, studied here. It takes the same basic form as in recent related work by Benthem and Kerr (2013) and Mason and Plantinga (2013), who introduce asymmetric information about the opportunity value of the resource, V. S knows the true V, while P does not. Our model is however more general than theirs in assuming that the "gross protection value" of the resource, B, is not known but stochastic for the buyer. We still assume that S has no specific information about B to be extracted, and that B and V are generally independent.

Assume that B and V are uncorrelated. For B, V has a distribution function G(V) defined on (-∞, ∞), assumed perfectly continuous.[6] For a given payment H from P to S, the probability that the resource is saved is then G(H).

The objective function for P can be written as

$$(1) \qquad ER = G(H)(EB - H) + \alpha[G(H)H - \int_{-\infty}^{H} Vg(V)dV],$$

where α is a weight given by P to the expected outcome (or net gain) of B. The resource is here purchased with probability less than unity, G(H); so that the cost H, while always offered from P

[6] We consider both B and V as generally not possessing bounds, mainly with a view to later simulations where normality will be assumed.

to S, is incurred by P only with such probability. The reason is that H is offered to some sellers whose value of converting the resource exceeds the offered H (with probability 1-G(H)), and will thus not accept the offer.

Two useful special (limiting) cases are in the following:

a) $\alpha = 0$, the utility of S plays no role for P; and

b) $\alpha = 1$, the outcome for S is fully internalized by P.[7] In this case we might say that P maximizes "social welfare" related to protecting the resource, considering S and P as the only participants in this market that have preferences for the resource.[8]

Cases where P considers a "development impact" of its forest-saving policy is likely to involve $\alpha > 0$. $\alpha = 1$ is an extreme case, but still potentially relevant. In particular, when P also is a donor (as is often the case in a North-South context) and with also other funding provided to the country where the resource is located, we could have $\alpha = 1$.[9]

Maximizing (1) with respect to the payment H now yields

$$\frac{dER}{dH} = g(H)(EB - H) - (1-\alpha)G(H) = 0 \,. \tag{2}$$

The optimal solution in this case depends on the weight α given by the buyer to the utility of the seller (or formally here, to the net utility of the seller associated with conserving the forest land). We focus on two special extreme cases, namely a) $\alpha = 0$ (the buyer has no concern for the welfare of the seller, and b) $\alpha = 1$ (the buyer has equal concern at the margin for the seller's as for own welfare).

In case a) $\alpha = 0$ we have the solution

$$EB - H = \frac{G(H)}{g(H)} > 0 \,, \tag{3}$$

where the right-hand side of (3) is the "inverse hazard" rate, G(H)/g(H). Note that a necessary second-order condition for maximum of (1) must here be fulfilled, namely:

[7] This corresponds to what in the literature is described as paternalistic altruism; see e g Jones-Lee (1992), Strand (2007).

[8] There may in practice be other parties interested in protecting the resource, and whose preferences are not internalized via the procedures discussed here. (1) would then not represent global welfare related to the resource, and would typically under-assess the protection value from a global point of view.

[9] Note here that $\alpha = 1$ does not necessarily mean that (1) will represent global welfare. The reason is that other potential donors (or "principals") are likely to have preferences with respect to protecting this forest. In this sense, B is likely to represent an under-valuation of true global welfare of protecting the resource, outside the region.

(4) $$\frac{d^2 ER}{(dH)^2} = g'(H)(EB - H) - 2g(H) < 0 .$$

A sufficient (but not necessary) condition for this to hold is g'(H) $\leq$ 0.

From (3), the optimal solution for P in this case is to select H < EB. P is here averse to making payments to S, and acts as a monopsonist in purchasing "offsets" from S. This is in similar fashion to other recent analyses of offset markets such as Strand (2013), and Rosendahl and Strand (2014).

In case b) α = 1, the last term drops out of (2), which simplifies to

(5) $$EB = H .$$

(5) represents a constrained efficient solution, overall the best feasible under the information constraints given. The reason for (constrained) efficiency is that when α = 1, P in effect incorporates S's utility in his own objective function, and maximizes "global" utility with respect to H. This is not the case when α = 0.

Consider $\alpha \in [0,1]$, and an associated payment H from P to B. The cumulative distribution function (cdf) for V is in this case conditional on V $\leq$ H; and given by

(6) $$G_C(V) = \frac{G(V)}{G(H)}, V \in (-\infty, H],$$

with probability density function (pdf) given by g(V)/G(H) on the same domain.

Consider effects on welfare in this case. Using (2), we can write (1) as

(7) $$ER = G(H)\left[(1-\alpha)\frac{G(H)}{g(H)} + \alpha(EB - E_C V)\right],$$

where $E_C V$ is the conditional expectation of V in this case. Potentially, simulations may help us to identify the relevant features such as probabilities of incorrect decisions (so that the resource is saved when B-V < 0, and not saved when B-V > 0), and how these are affected by changes in "risk parameters" (standard deviation of B and V).

In Appendix A we derive the mathematical distribution for B-V = Z, and simulate this distribution. Appendix B derives expressions for four relevant and important probabilities: 1)-2) saving the resource when this is correct, and incorrect; and 3)-4) not saving the resource when this is correct, and incorrect. This distribution and these probabilities are there also simulated for cases where B and V are independent and normal, and for alternative standard deviations of B and V. We find, as expected, that a higher standard deviation on B increases the probability of errors; and that the last bit of improvement in precision (to perfect information) has a particularly large impact. Note that with independent B and V, EZ = EB – EV, and var Z = var B + var V.

Improved decisions to protect the resource can occur either in terms of more precise B, or more precise V given that B and V are uncorrelated. In either case, only the total variance on Z matters, and this is affected in symmetric fashion by each. It also follows that the most valuable information improvements are those that most reduce this total variance.

These probabilities serve as a backdrop to the analysis of VOI, to which we now turn.

4. The Value of Perfect Information about B

4.1 Analytical framework

This section attempts to answer the following basic question: What is the additional value of perfect information about B to P, in the context of the decision problem studied here (optimal forest protection), when information is asymmetric, only S knowing the resource opportunity value, B, for certain? Under imperfect and asymmetric information about B, and given that P perfectly incorporates the outcome for S (α=1 above), two types of mistakes are made: the resource can be converted when it should not be; and the resource can be saved when it should not be.[10] This leads to welfare losses (to P and the world), in an ex ante expected sense. These losses can, at least in some cases, be avoided when information is perfect and P acts optimally upon this information.

Collecting more information about the value of B will lead to less uncertainty about this value for P, and, plausibly, to a smaller welfare loss due to incorrect decisions: thus, to higher overall expected welfare. We will here formalize this welfare improvement, as evaluated ex ante, in the (admittedly, special) case where all uncertainty is resolved ex post.

Define the standard deviations of B and V by σ_B, and σ_V, respectively. The perfect information case with $\sigma_B = 0$ and thus no uncertainty about B, is useful as it provides us with an appropriate benchmark for the study of informational values in intermediate and more realistic, but also more complex, cases where the initial uncertainty is only partly resolved.

From the point of view of an individual decision maker or buyer, deriving the optimal solution is now straightforward. In the objective function (4) one now only needs to replace EB by the now known B, and the optimal solution is (5), also only replacing EB by B. Define also D = B-H = the net realized value for the buyer related to this particular unit. We can in this case express the expected gain to P, for given B and α, as:

[10] Appendix B derives expressions for these probabilities, and simulates them in some stylized cases.

(8) $$ER(B;\alpha) = G(B-D)D + \alpha[G(B-D)(B-D) - \int_{-\infty}^{B-D} Vg(V)dV]$$

This solution is particularly simple when α = 1. The buyer then sets H = B, and thus D = 0 (giving all the surplus to S, making sure that all resource units with B>V are saved). Since the seller knows V perfectly, the solution is now always Pareto optimal.

When α < 1, we find that, generally, H will be set below B, and that H will vary with B, and in such a way that B-H also generally varies. From (3), only when G(H)/g(H) is constant will B-H be constant, independent of B.

Consider however this special case. For the market as a whole, a relevant consideration is the distribution for the difference Z = B-V. We denote the distribution function for Z by M, and its density by m. At a social optimum, the resource ought to be saved when Z > 0, and converted in the opposite case. It is then clear that given H = B, the social optimum will be realized.

Consider first a general value of α. The expected value of a random "project" (across the entire distribution of possible projects) can then, for the simplified case where the difference B-H only depends on α, be written as:[11]

(9) $$ER(\alpha) = [1 - M(D(\alpha)]D(\alpha) + \alpha\left[\int_{B-V=D(\alpha)}^{\infty} (B-V-D(\alpha))m(B-V-D(\alpha))d(B-V)\right].$$

The resource will here be saved with probability 1-M((B-H)(α)), which is generally an increasing function of α.

This model is simplest and most instructive for α = 1 (optimal buyer behavior). The solution then implements an optimal (first-best) allocation in terms of saving the resource, subject to the given informational constraints. From the modified relation (4), H then always equals B, and D(1) = 0. (9) then simplifies to:

(10) $$ER(1) = \int_{B-V=0}^{\infty} (B-V)m(B-V)d(B-V),$$

which expresses the (maximized) ex ante expected social surplus of a random unit of the resource in this case (drawn randomly from the universe of possible B-V values), evaluated before perfect information about B is achieved, but assuming that the actual decision (about H) will be taken under perfect information. Note that if B and V are independent normal random variables, M = B-V is a normal random variable with mean E(B) – E(V).

[11] Note that when D is a function also of B, which is the case generally, (12) becomes more complicated as, e.g., the integration limits will also depend on B. We leave this for future research.

Rewriting (7) by setting α = 1 and EB = H, we get the value of incomplete information in this case which we call *ER*(1;*I*):

$$(11) \qquad ER(1;I) = \left[G(H)H - \int_{-\infty}^{H} Vg(V)dV \right].$$

Thus, VOPI, the increase in the value of full information over partial information about B (or rather, about a set of objects each with value B, and where all their collective distribution is described by the distribution F), is under these conditions given by

$$(12) \qquad ER(1) - ER(1;I) = \left[\int_{B-V=0}^{\infty} (B-V)m(B-V)d(B-V) \right] - \left[G(H)H - \int_{-\infty}^{H} Vg(V)dV \right].$$

This solution is still extreme as P, in implementing a first-best solution, collects no net benefit, as the purchase price H equals his entire surplus. Thus while useful as a benchmark, this solution is not particularly realistic.

The VOPI value in the case of α = 0 is similarly given by (9) minus (7), inserted for α = 0.

4.2 Simulations for α = 1

The following Figure 1 illustrates how changing the levels of EV influences on the realized additional value of perfect information about B in our case simulated (with both B and V normal and independent). We note that if EV differs more from EB, then the additional value of full information is reduced. Intuitively, a reason is that when it is already known (with a high degree of confidence) that 0 << EV < < EB, the resource ought to be protected by the principal in most cases. Having more precise information about B will make little difference for the decision to convert or save the resource in such cases, given that the saving decision is made optimally subject to given information. A similar situation would hold when 0 << EB << EV given α = 1: then most of the resource will converted also under uncertainty, and this is socially optimal in a vast majority of statistical cases. By contrast, when EV and EB are close together, the decision on how to optimally manage the resource is highly sensitive to the more precise information about the resource value. It then matters greatly for such decisions whether there is full or imperfect information; this feature increases the value of perfect information in this case.

Figure 1 also illustrates that a higher initial σ_B leads to increased value of fully resolving the uncertainty (reducing σ_B to zero). This is intuitive: a high σ_B leads to many mistakes in saving and not saving the resource B; resolving this uncertainty completely will, in the case we here study (α = 1), eliminate all such mistakes.

Ex ante distributions for B and V are in all the following simulations assumed to be independent and normal; which could be a good approximation in many cases.[12]

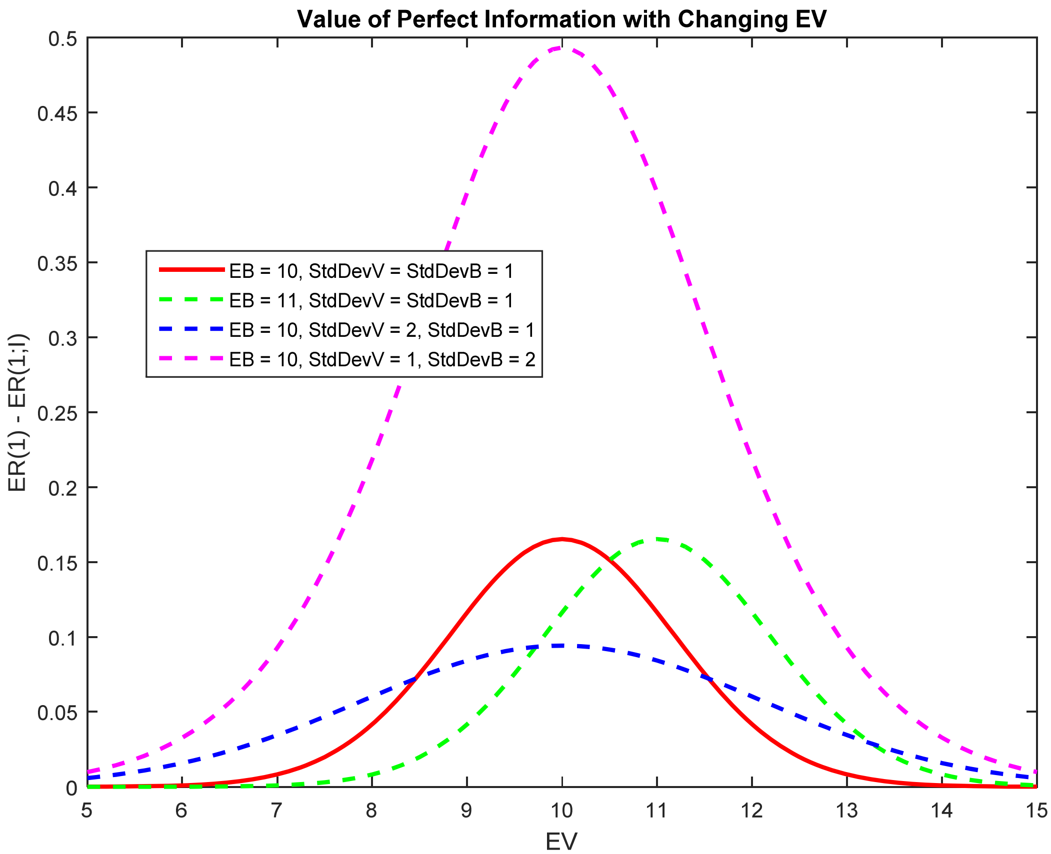

Figure 1: Graph showing the VOPI about B as function of the expected value of V

When we increase σv, as pictured in Figure 2, the displayed simulated results are different: VOPI about B is reduced, as compared to the higher-standard deviation imperfect information case for EV = EB. This might immediately appear as surprising. The intuition is that when V is more dispersed, the agent's decision to convert or not convert the resource will be less affected by a more precise assessment of B by P. Put otherwise, in such cases most of the decision is left for S; there is not much P can do by to provide more precise information. A lower standard deviation of V would leave more of the real decision in the hands of P, and increase the usefulness of perfect information. From Figure 2, we see that this holds when expectations are the same (here EV = EB = 10).

[12] The independence assumption may sometimes be unrealistic as gross forest values and opportunity values are often likely to be positively correlated (as e. g. when better road access increases both values). Such cases must be left for future research. We conjecture that this will tend to reduce the informational problem for P as V can in part be inferred from P's own prior assessment of B (for cases where $\alpha < 1$).

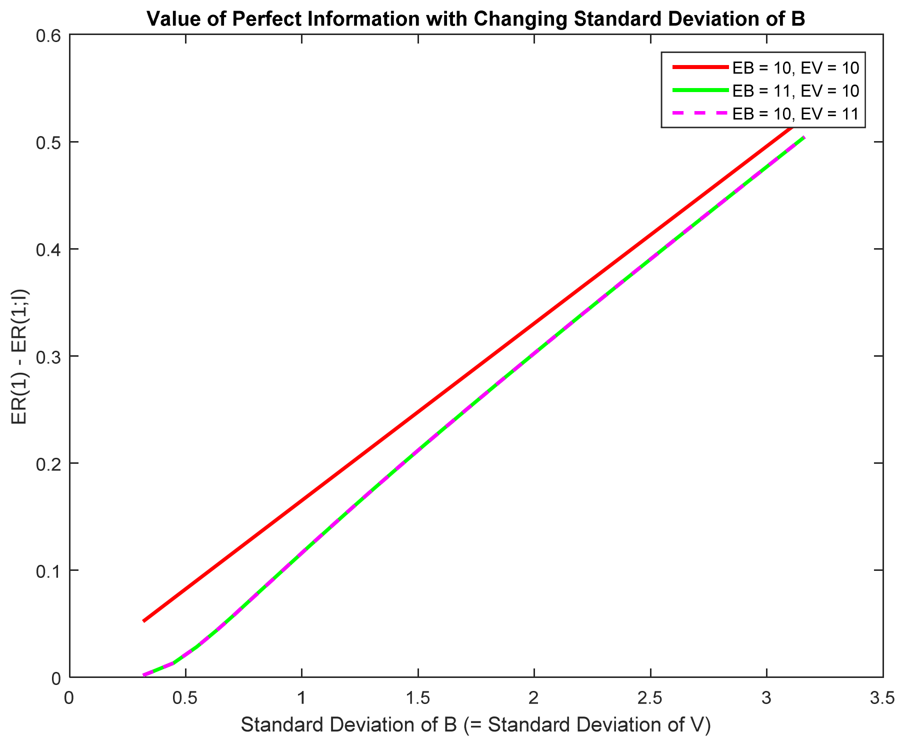

Figure 2: Graph showing the VOPI about B as function of σ_B (= σ_V)

It needs to be remarked that the functional shape of the VOPI as described by the unbroken red line in Figure 2 is specific to the case of equal expectations for V and B (where EV = EB = 10); together with both distributions being normal and independent. Consider instead an example where EV and EB differ; an example in Figure 2 is EV = 10, and EB = 11. The VOPI then displays other functional relations with the relevant parameters. VOPI is still always positive, but changes non-monotonically with standard deviation. Two factors now work in opposite directions: When the standard deviations σ_B and σ_V are small, protection is likely to be chosen regardless of the actual value of V; it is unlikely that the decision to protect the resource will be affected by additional information about B. When standard deviations increase, it becomes more likely that this decision is affected by more information about B. This tends to increase the VOPI value. We see that this value approaches the value under the case of EV = EB in the limit as standard deviations grow.

Further illuminating features are shown in Figure 3. Here, we increase σ_B keeping σ_V, EB and EV fixed, which increases the VOPI. We find a strictly convex relationship with σ_V (which however converges to a linear relationship as σ_B increases). It is here more valuable for the principal to gain perfect information, as the number of mistakes is then reduced by more. We note here similarly that a partial increase in σ_V reduces the VOPI value about B (albeit only slightly), and the same holds when the expected values of B and V differ (two cases considered by us, EV=11, EB=10; and EV=10, EB=11, are here very similar).

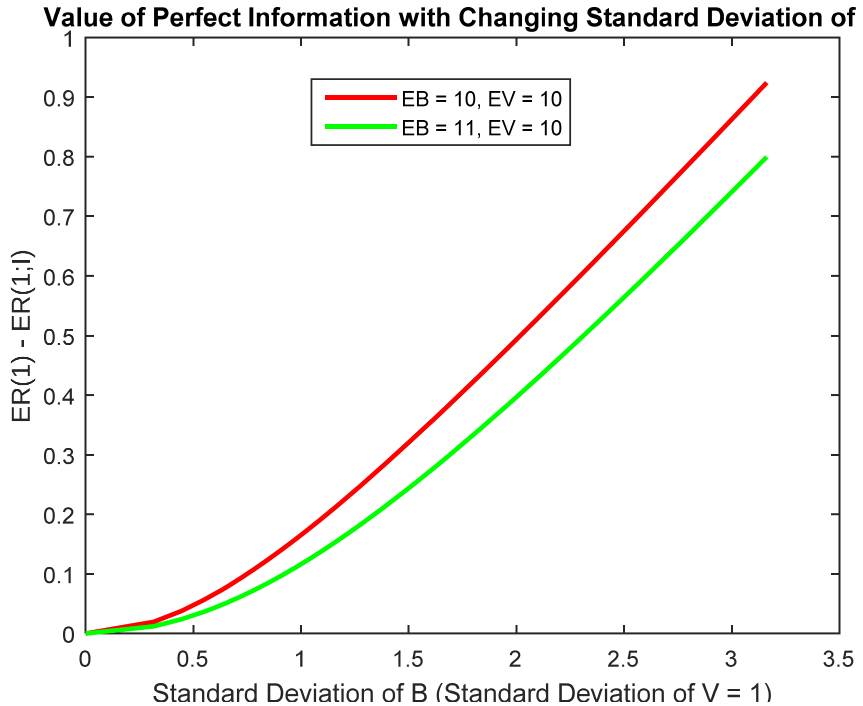

Figure 3: Graph showing the VOPI about B as function of σ_B with $\sigma_V = 1$

4.3 Simulations for α = 0

We now simulate the other extreme specification of P's objective function, namely the case of $\alpha = 0$. P now considers the outcome only for himself, and ignores the outcome for S. Through simulations we derive approximate VOPI values for this case. Given the assumed prior distributions for V and B, we can compute the improvement in (ex ante expected) payoff, when P chooses H after having first observed B (under full information ex post), as compared to when only knowing the prior distribution for B. We retain our basic assumptions that P's prior distributions for B and V are both normal and independent.

P now chooses H to maximize his payoff conditional on the realized value of B. From (6), P will select a value for H below B. Remember that in the case with uncertain B, H is in the uncertainty case chosen lower than EB; but not necessarily lower than any one actual realization of B.

To calculate the value of full over partial information, we need an expression similar to (15). However, obtaining such an expression when $\alpha = 0$ is not straightforward. H is here chosen to maximize P's payoff, at a value lower than EB in the imperfect information case, and lower than B in the perfect information case. Note that when $\alpha = 1$, D (= B – H, P's net gain from saving a given unit of forest) is zero; while when $\alpha < 1$, D is always positive. A problem here is that D will take a whole set of values depending on the ex post realization of the initially uncertain B, when B is revealed following new valuation information. As we cannot derive a closed-form solution for H, from (6) and (7), we cannot either find closed-form solutions for D, and will need to resort to simulations.

We plot, in Figure 4, EH as a function of EB for the perfect information case, for fixed EV (= 10) and fixed standard deviations $\sigma_B = \sigma_V = 1$. EH is always lower than EB whenever this standard deviation is positive, and increases in the standard deviation. In the imperfect information case, by contrast, P will need to set a fixed H, from (6). On the same graphic, we show the expected value of H, EH, in the full information case. In this case, H will differ depending on the realized value of B; the red curve in Figure 4 shows how EH (taken over the whole distribution of H values for a given prior distribution for B) varies with the prior standard deviation of B. The greater is this standard deviation, the lower is EH. Intuitively, it is more favorable for P to reduce H more when B is low, than it is to increase H when B is high. As the means of the two distributions diverge more (to about 3-4 standard deviations), then this advantage is reduced. This is similar to Jensen's inequality, as H is a concave function of B so that $\int B(H)db \leq H\left(\int Bdb\right)$.

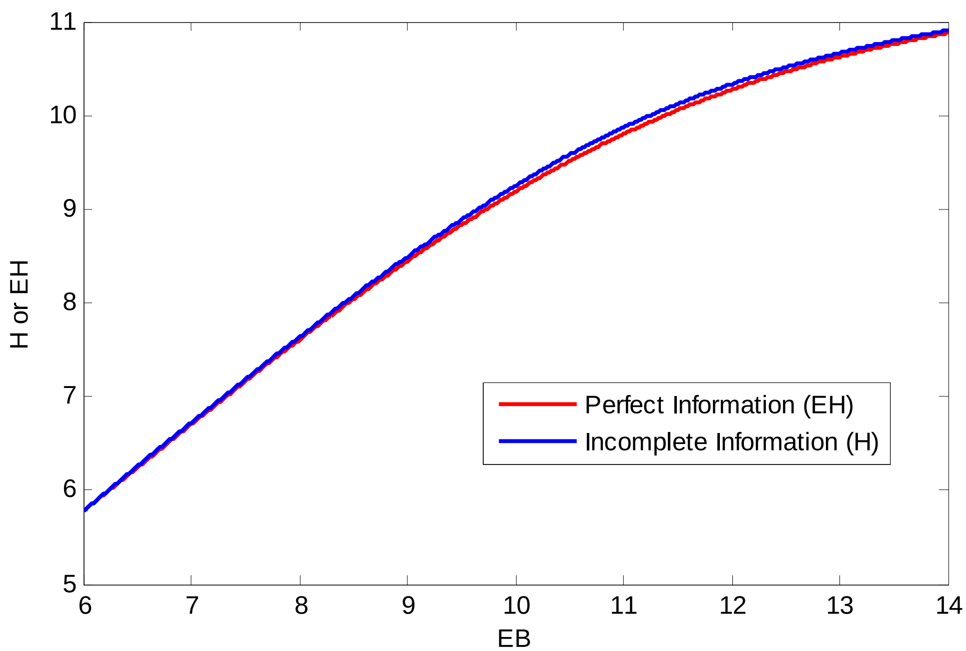

Figure 4: Payment (H) and Expected Payment (EH) from P to S with changing EB (given EV = 10, $\sigma_B = \sigma_V = 1$), in the case of $\alpha = 0$

We are interested in finding how D (in the incomplete information case), respectively the ex ante value of ED (in the ex post perfect information case), both representing P's "net gain per transaction", vary with background parameters. Figure 5 shows simulations for D = EB-H and ED as functions of (ex ante) EB in these cases. We see that D (or ED) generally increases partially in EB, as one should expect. We see that ED is greater in the perfect information case, than the fixed D in the incomplete information case. The difference is however small.

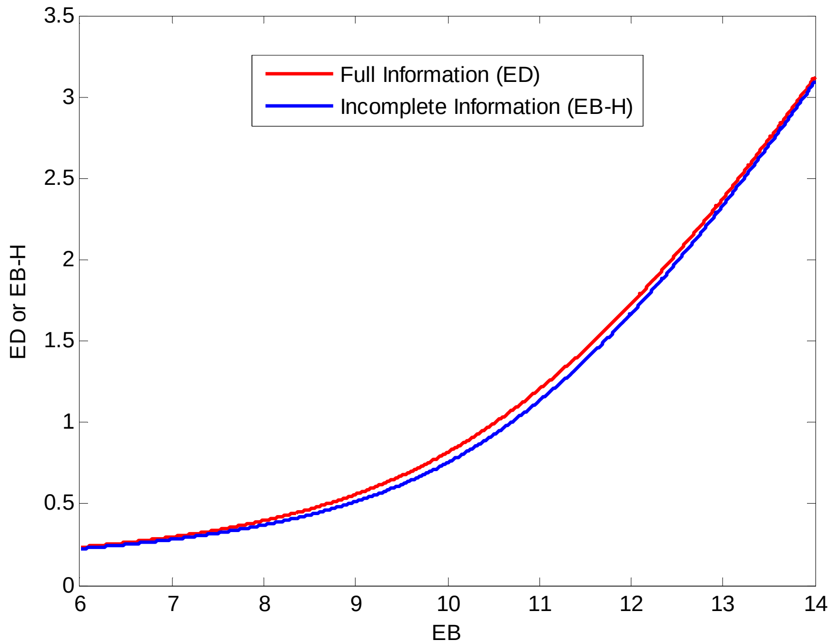

Figure 5: ED as function of EB with $\alpha = 0$ (EV = 10, $\sigma_B = \sigma_V = 1$) in the case of $\alpha = 0$

These simulations thus indicate that when the standard deviation of B is "relatively small" (as in Figures 4-5), P's main gain from better information is to a relatively small degree in terms of lower average payments to S, and more in terms of better resource allocation (as mistakes are fully avoided in the VOPI case). Although it is not so easy to see from Figure 5, the gains for P are larger the closer EV is to EB. This is intuitive, as the resource is more "contested" when the two expectations are close; a correct decision then depends heavily on the precise value information about B.

When σ_B is larger, the expected change in outcome for P, from the imperfect to the perfect information case, is more dramatic. This is illustrated in Figure 6, where the gain is simulated now as a function of the standard deviation of B, σ_B, and focusing on an example where $\sigma_V = 1$, and EB = 11, EV = 10 (where again the essential feature is the difference EB – EV). The reason for the relatively drastic change in outcomes between the incomplete and complete information cases, as depicted in Figure 6, is that the informational value of perfect information increases (at least) linearly with the ex ante standard deviation σ_B: the ex post adjustments in payments from P to S will be greater. In particular, these payments will be adjusted down when the ex post observed B is high. Overall, this results in ex ante expected savings for P in terms of reduced payments to S, that are more substantial in the perfect information case.

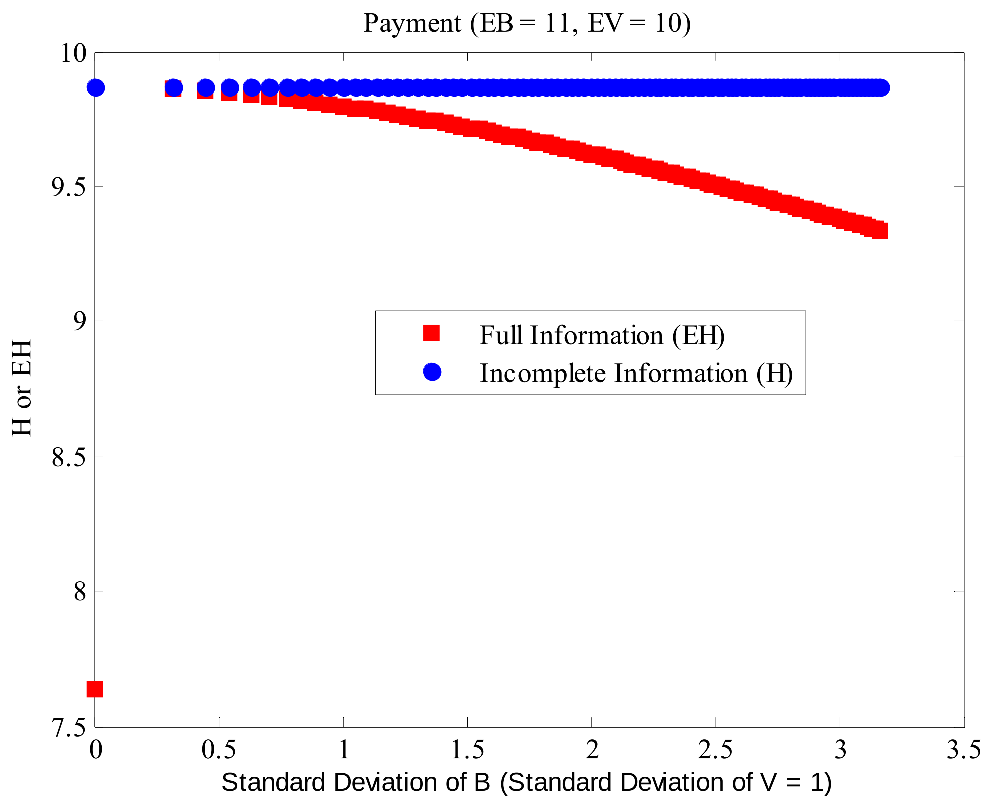

Figure 6: Expected payment (EH) from P to B as function of σ_B (given $\sigma_V = 1$)

We here also derive the VOPI values for the case of α = 0, to be compared with those for α = 1. The simulations are done, in similar way as for α = 1 in sub-section 4.2, by first generating a set of values of B, which corresponds to drawing the values from the prior normal distribution over possible B values. We then calculate, for each draw, H for the corresponding full-information case, considering the realized value of B. Note again that in the case with uncertain B, H is chosen at a given level. In the full information case, H is chosen after the realization of B, and thus differs by realized B.

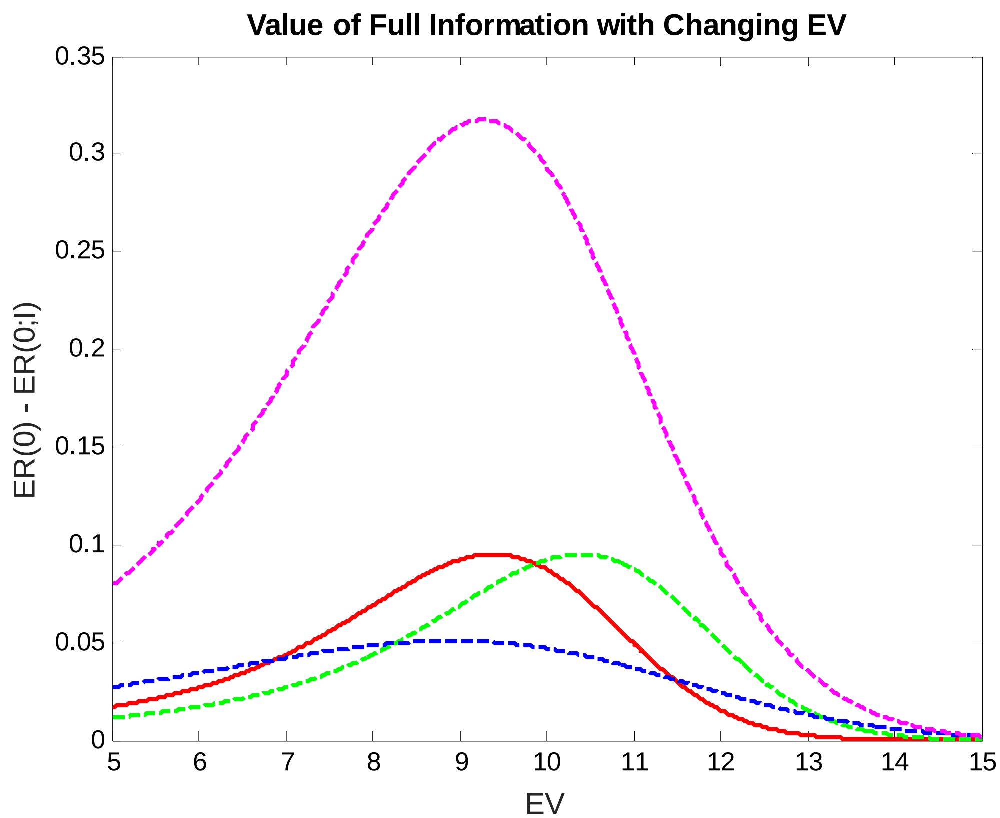

Figure 7: The VOPI about B as function of EV for α = 0

The results are shown in Figures 7-9. We see that several of the results are qualitatively similar to those found for α = 1, in Figures 1-3. In Figure 7, VOPI is found to be generally lower-valued when α = 0 than when α = 1 (in which case it was given from Figure 1). Similar comparative results are found when comparing Figures 2-3 (for α = 1), to Figures 8-9 (for α = 0). From all diagrams, we find the VOPI in the case of α = 0 to be, typically, between one half and two thirds of its value for α = 1. This shows that when P always makes a globally optimal resource protection decision (as when α = 1), instead of always making a "selfish" decision (as when α = 0), not only is the overall expected resource value reduced; the value of adding more information (here of perfect information, VOPI) is also reduced.

The intuitive reason for the reduced values is that overall realized values are lower when the decision to save or develop the resource is socially inefficient (as when α = 0), as compared to efficient (α = 1). Both values before and after improved information, as well as the value of information as such (VOPI), are higher in the latter case, and by similar margins.

Note also that when α = 0, VOPI is no longer necessarily maximized for EB = EV, as was the case when α = 1. The reason is that H is set below B and EB in this case: maximizing VOPI here instead requires H (or EH) to be close to EV; which requires EB > EV.

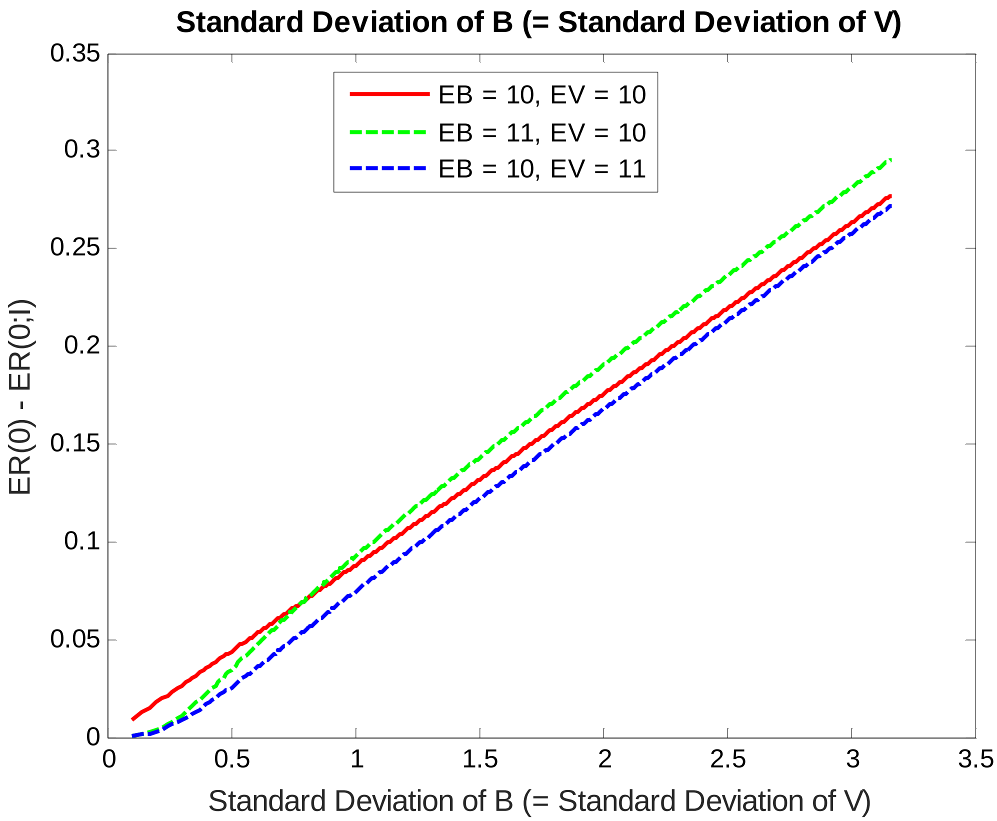

Figure 8: The VOPI about B as function of σ_B (= σ_V), for α = 0

4.4 Potential empirical magnitudes of VOPI

How significant is the VOPI; and is it likely to matter economically? We discuss this issue by considering further parameterized examples, focusing initially on the case of a fully altruistic donor (α = 1, so that the preferences of forest sellers are fully incorporated in P's utility function; simulated in subsection 4.2). From Figures 1 and 3 we find that gains from perfect information are highly related to two magnitudes: a) It is higher the more similar the relative expectations of B and V are; and it is higher the larger the standard deviation for the initial (prior) distribution of B is. The intuitive reason for a) is that when the two expectations are similar, the resource is "highly contested" in the sense of being subject to a high level of competition, and the decision to save and not save the resource is highly sensitive to detailed information about B. More information about B is then highly valuable. The intuition behind b) is that resolving the uncertainty about B (which is done by the exercise behind VOPI) is more valuable, the more uncertain B is at the outset.

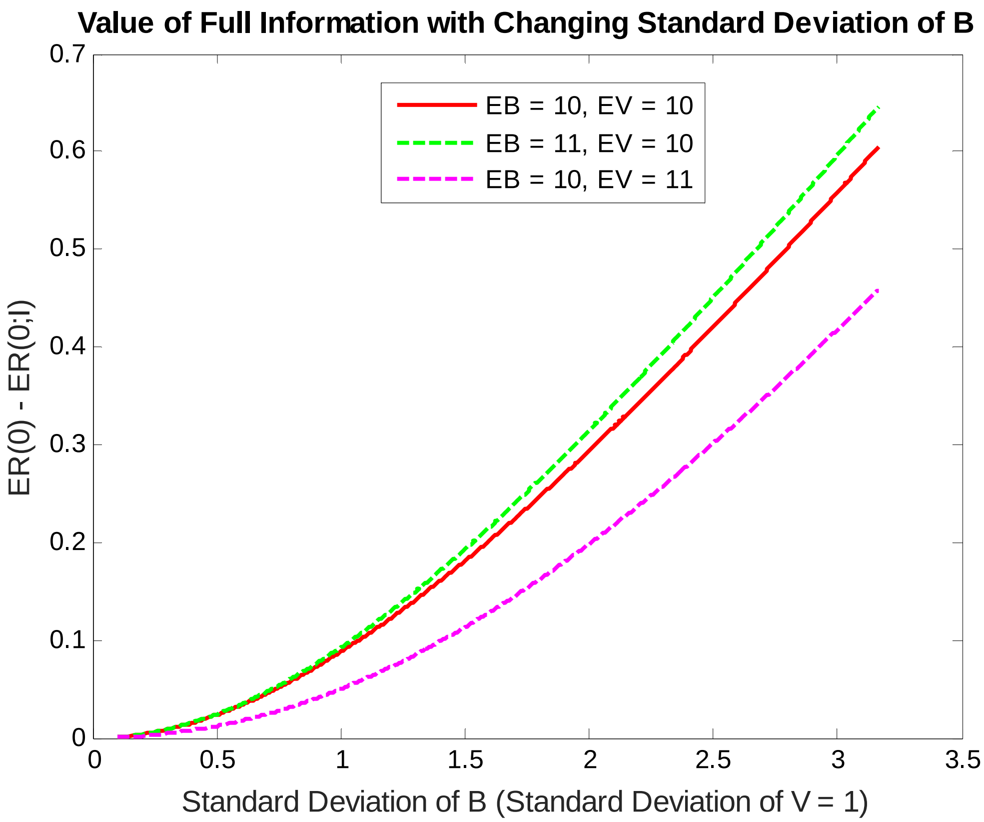

Figure 9: The VOPI about B as function of σ_B given $\sigma_V = 1$, for $\alpha = 0$

Figures 1-3 provide us some notion of the value of information in relation to the standard variation of the prior distribution for B in this case.

Consider first EV = EB = 1; the forest is then highly contested as the expected opportunity value equals the prior expected protection value, both set at unity. Assume a standard deviation for the prior distribution of B at half of its expected value: high uncertainty but not unreasonable. Assume also that the standard deviation of B and V are the same (so that $\sigma_B = \sigma_V$). Since VOPI values are found to be basically proportional to these standard deviations when their levels are identical, we find the VOPI value as approximately 0.08 times EB in this case.[13]

This is a not insignificant number. To get a notion of how large this value could conceivably be, suppose that B and V are interpreted as the values of a relatively small area of Amazon forest. Then consider a case where 10% of the forest will be similarly "contested" over the period in

[13] Note that when instead σ_B is larger than σ_V (which is arguably realistic in particular when agricultural yields are widely known) VOPI will be larger. Note secondly that for this example we have set EB = EV = 1, and not both = 10 as in the figure. Identical shifts in EB and EV however do not affect VOPI values in this case, both B and V following normal distributions.

question (for the analysis in Strand et al (2014), up to 2050); meaning that protection values and opportunity values are similar, and that saving or not saving this part of the forest will depend highly on its precise assessed value in relation to its opportunity value (and such that actual saving decisions are optimal contingent of available information).[14] Let us also assume that the parameter values are otherwise as in the last example presented; importantly in particular we assume that EB = EV. Then VOPI for this part of the forest would be 8% of its gross protection value.

How large could such values be for the Amazon rainforest? In a gross value sense, the numbers could be very large. Consider a gross (non-carbon) protection value of $5,000 per hectare; this can be viewed as conservative.[15] 8% of this value is $400 per hectare, which will then be the VOPI for the relevant forest area. With more than 500 million hectares of Amazon rainforest, 10% of the area means more than 50 million hectares of contested rainforest, leading to a VOPI of at least $20 billion. This would correspond to the (present discounted) value of providing perfect and comprehensive valuation data for this extent of the forest, for cases where decisions about saving and not saving the forest are taken optimally subject to given available information (which is improved in the VOPI case); and where the informational background for savings decisions must also be updated over time as various new areas become threatened with conversion. Even if only 1/10 of the area considered above is subject to explicit evaluation, then using the lower σ_B value, VOPI would still be $500 million.

There are several reasons why the actual value of providing more information about forest protection values will be lower in practice than this astronomical figure. First, information will not be used optimally in practice as assumed in the example above. Much forest conversion may take place with no detailed analysis of its value for any one particular piece of land. Secondly, the uncertainties to be resolved through valuation studies could be smaller than assumed here. If instead the standard deviation for the prior distribution of B was ¼ EB, instead of ½ EB, this would by itself reduce the VOPI value by half, though $10 billion is still a very high number. Thirdly, no study will resolve all uncertainty; there will always be left some residual uncertainty. Fourthly, and quite importantly here, the protection values and opportunity values are often less similar. In Figure 1, the more dissimilar EB and EV are, the more will the VOPI curve be shifted (to the right or left), and the lower will VOPI be. In particular, when EB and EV differ by one instead of being equal (so that expectations differ by two standard deviations on either of the variables), the VOPI value from Figure 3 in this case is reduced by a factor of about three. We also see in the figure that if EB – EV is greater than 4, VOPI virtually vanishes.

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[14] This argument assumes that losses of value are roughly proportional to forest losses over this loss range; which presumes that no major tipping points are reached over such a range.

[15] Note that a recent SP study of WTP for Amazon rainforest preservation in the US and Canada, Siikamäki et al (2015), indicates an annual (non-carbon) WTP, in these countries alone, of about $10,000/ha. Note also that we may here think of separating out the carbon values as our hypothetical valuation studies, reducing uncertainty about B, would not be expected to resolve any significant uncertainty about the carbon values; only other sources of uncertainty.

Consider an even more limited case where we only include the actual deforestation taking place in the Amazon in a given year. Let us assume initially that this loss is 1 million ha annually.[16] Assume that 10% of this area is contested in the sense that ex ante protection and opportunity values are similar, and the decision to deforest or not can be affected by precision of information. In this case, and with parameter values otherwise as in the example above, VOPI would amount to \$4 million per year.

It may here be of interest to discuss also the other extreme case where P has no concern whatever for S, and thus $\alpha = 0$. Comparing Figures 4-7 to Figures 1-3, VOPI values for $\alpha = 0$ are found to be approximately 2/3 of VOPI values for $\alpha = 1$. If we expect decisions to save or not save contested forest areas to be driven by only the preferences of the donor/principal (while preferences of the forest-managing country have no weight in this decision), VOPI values would, in the otherwise same examples as considered above, need to be scaled down to about 2/3 of the values we found for $\alpha = 1$. Under the parametric assumptions made here (independent normal distributions for B and V) this appears to be a quite robust result.

We will finally point out that our analysis might provide clues to VOI values also from studies that do not resolve all uncertainty, which is the relevant assumption in practice. Such clues can be found from Figures 3 ($\alpha = 1$) and 9 ($\alpha = 0$) showing VOPI values as functions of the standard deviation of B for given standard deviation of V. A conjecture (so far not formally substantiated) is that the value gain from achieving a lower uncertainty estimate can be found by considering the difference in the VOPI value gains for different initial standard deviations on B. These gains would seem likely to be close to proportional to the reduction in the standard deviation for small risk changes; but less than proportional as the remaining risk drops toward zero. If so, a valuation study that reduces σ_B to half of its pre-study value will provide (perhaps somewhat more than) half of the VOPI value. We intend to further analyze, and simulate, such cases in follow-up work.

In this context, Figures 3 and 9 indicate that eliminating the last bit of uncertainty (in the figures, going from $\sigma_B = 0.5$ to $\sigma_B = 0$, when $\sigma_V = 1$) is likely to not be worthwhile; the utility value is likely to be low, and the cost of eliminating the last remaining uncertainty likely high.

5. Discussion and final comments

This paper has analyzed certain "value of information" (VOI) problems, in the context of valuing environmental and natural resources through surveying or other costly valuation and data-acquisition methodologies, when achieving a high degree of certainty about such values may be costly. The resource focused on is tropical rainforests, considering decisions to deforest or save a

[16] This corresponds approximately to the average rate of tropical rainforest deforestation in Brazil over the last 20 years; although the rate has been lower in the most recent years.

given part of the resource being taken by local agents, whose decisions may be influenced by outside parties with an interest in preserving such resources.

Our analysis deals with aspects of a cost-benefit analysis of environmental valuation projects, in comparing the costs of such valuation work (which can be easy to measure), with its benefits (often much more difficult to measure). Perhaps most important, we make precise what can be meant by such value and how it can be measured, with a further objective to communicate such concepts to environmental valuation practitioners and sponsors. When an environmental valuation study is carried out, there is usually no explicit consideration of the costs of implementing the study, in relation to the benefits of so doing, which are typically not made explicit nor clear, much less monetized. Environmental valuation project funders or executors have usually no or only a vague idea of the value of the information provided through the valuation study.

Another novel aspect of our analysis is to relate this VOI problem to a mechanism design problem involving constrained optimal decisions in response to improved information. The decision to save or convert (not save) the resource in question is taken by a principal, P, who collects information about the resource's value subject to private information of an executing agent, or seller, S. In our canonical case, a forest resource is saved or converted by S, and P influences this decision through a payment to S conditional on saving the resource. Consider a rainforest resource with preservation value B, and opportunity value V. In analyzing such cases in section 3, S is assumed to know V perfectly, while P only knows prior (Bayesian) probability distributions over both B and V. Our simulations, in Section 4, are restricted to cases where the prior distributions of B and V (for P) are normal and independent. In deriving a (monetized) VOI, related to providing more information about the resource's protection value, B, we also focus on cases where uncertainty is eliminated completely. While the latter case is unrealistic, it is useful in indicating a potential informational value (in more realistic cases where some uncertainty about B remains after a valuation study is performed, the VOI will be lower). The resulting VOI is the value of perfect information, VOPI.

In our simulations, we consider two cases: when P fully cares for S (subsection 4.2); and when P is completely selfish (subsection 4.3). We compare two stylized sets of cases: an initial (pre-information) case where P is uncertain about B; and a final (post-information) case where all uncertainty about B has been resolved for P. The actual welfare improvement will be smaller in practical cases, both because information as such will never be perfect, but also as we can expect that, often, such additional information (nor, perhaps, any explicit valuation information) will not be used in the actual decision to save or convert the resource, optimally or at all.

We show how the VOPI value depends on uncertainties about the true (gross) value, B, and about the opportunity value, V; and on relative expected values, EB and EV. More information about B is particularly helpful when EB and EV are similar, as the resource is then highly contested and the decision to save it affected in many cases. Informational value is also higher when B is more uncertain. This is reasonable as P then makes many mistakes when acting upon only prior information, which are corrected when this uncertainty is resolved. The informational value is higher when the standard deviation of V is smaller (within certain bounds): With a large standard deviation of V, the decision to save the resource is less affected by B's precision.

Our simulated VOPI values as fraction of the gross protection value of the resource depends heavily on the prior uncertainty about B (its standard deviation), in relation to gross protection value. Some simple back-of-envelope calculations, in subsection 4.4, illustrate the potential values of providing more precise resource value information. The VOPI values can be quite large, in seemingly reasonable cases. The values could significant expenditures for providing the added information, depending on several factors.. Details, or more realistic cases, have not been pursued vigorously here, and must be left for future research.

Much of our analysis presumes that P (the buyer) has full consideration for the welfare of S. This is often unrealistic. Subsection 4.3 simulates an extreme opposite case where P is completely selfish. The natural resource is then saved in fewer cases; the payment from P to S is lower; and overall efficiency is reduced. VOI and VOPI values are also lower. Saving the resource becomes less frequent both under imperfect information about B, and under prefect information. In all the cases simulated, the VOPI is between one-half and one-third lower when P is self-centered. This demonstrates a novel, but intuitive, result that the value of information depends on the way in which information is used: when decisions made upon new information are less efficient, informational value is lower.[17]

In Appendix A we supply this analysis by deriving the probabilities of the two types of mistakes made by P under imperfect information: a) to convert the resource when it ought to be saved; and b) to save the resource when it ought to be converted. We find how probabilities of correct action depend on precision, and e g show that the last bit of precision is particularly important for the probabilities of correct decisions. We also find that when P does not incorporate the preferences of S, type a) mistakes (saving the resource in fewer cases) will dominate more as payments from P to S will be lower.

Our analysis has several limitations. All examples are based on standard normal and independent distributions for both B and V, which can thus take both positive and negative values, both variables principally extending to plus and minus infinity. Negative gross rainforest values are of course unrealistic. On the other hand only the difference Z = B-V matters, at least in terms of efficiency; which could in principle be (perhaps large) negative.

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[17] This may of course have additional, sometimes unwanted, consequences. One is where the cost of providing the required information is close to the benefit, we might find that an investigation to provide such information would be worth-while given that P is altruistic, but not given that P is self-centered. Such issues must be left for future consideration.

Further work will also follow up with analyses of VOI when some degree of uncertainty always remains after a valuation study, touched upon at the end of section 4; this is the practically more relevant and interesting case. We conjecture that the VOI will be roughly inversely proportional to the remaining standard deviation of B; proofs are here clearly required.

A further issue needing follow-up work is to discuss more precise information about V, the resource opportunity value, which is here assumed uncertain for P, but certain for S. In practice, S is likely to be uncertain about V. The information about V for P could then be improved through similar valuation efforts to those studied for B. A more precise assessment of V is useful for P only when P is not fully altruistic, and S knows V for certain. When V is not known for certain by any party, additional work to find a more precise V value would be useful.

We finally mention that Appendix C studies related simulations assuming that the distributions of B and V are both log-normal (instead of normal). We find many simulation results to be similar. A major difference is that the probability of a “type 4” decision (not saving the resource when it ought to be saved) now is lower in all cases considered.

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Appendix A: Derivation of the distribution function for B-V

To derive the distribution for $B - V = Z$, call the cdf of the random variable B, $F(B)$, and denote its pdf by $f(B)$. Considering the cdf for realized values of Z (thus for cases where the forest is saved), conditional on H, this is found as the convolution

$$Q(Z|H) = \int_{-\infty}^{\infty}\int_{B-Z}^{H} f(B)\frac{g(V)}{G(H)}dVdB = \frac{1}{G(H)}\int_{-\infty}^{\infty}\int_{B-Z}^{H} f(B)g(V)dVdB. \quad \text{(A1)}$$

Substituting B = Z+V in (11) and taking its derivative with respect to Z, we get the conditional pdf for Z for given H, as follows:

$$q(Z|H) = \frac{1}{G(H)}\int_{-\infty}^{H} f(Z+V)g(V)dV. \quad \text{(A2)}$$

Figure 1 shows a simulation of the pdf given that B and V are both independent and normal, and given EB = 1 = H, EV = 0.5, $\sigma_B = 1$, $\sigma_V = 1$.

The mean (first moment) of this distribution is given by

(A3) $$E[B-V \mid H] = \int_{-\infty}^{H}\int_{-\infty}^{\infty}(B-V)\, f(B)\frac{g(V)}{G(H)}\, dBdV$$

which can be simplified as follows (assuming *f*, *g* are continuous and integrable):

$$E[B-V \mid H] = \int_{-\infty}^{H}\int_{-\infty}^{\infty}(B-V)\, f(B)\frac{g(V)}{G(H)}\, dBdV = \frac{1}{G(H)}\int_{-\infty}^{H}\int_{-\infty}^{\infty}(B-V)\, f(B)g(V)dBdV$$
$$= \frac{1}{G(H)}\int_{-\infty}^{H}\int_{-\infty}^{\infty}Bf(B)g(V)dBdV - \frac{1}{G(H)}\int_{-\infty}^{H}\int_{-\infty}^{\infty}Vf(B)g(V)dBdV$$
$$= \frac{1}{G(H)}\int_{-\infty}^{H}g(V)dV\int_{-\infty}^{\infty}Bf(B)dB - \frac{1}{G(H)}\int_{-\infty}^{H}Vg(V)dV\int_{-\infty}^{\infty}f(B)dBdV$$

leading to

(A3a) $$E[B-V \mid H] = \frac{1}{G(H)}G(H)E[B] - \frac{1}{G(H)}\int_{-\infty}^{H}Vg(V)dV = E[B] - \int_{-\infty}^{H}V\frac{g(V)}{G(H)}dV\ .$$

Note that

(A4) $$\int_{-\infty}^{H}V\frac{g(V)}{G(H)}dV = E[V \mid H]$$

is simply the expected value of V, conditional on H. We then also have from (A3a):

(A3b) $$E[B-V \mid H] = EB - E[V \mid H]\ .$$

$E[V \mid H]$ is here an increasing function of H. Intuitively, when the payment H from P to S increases, additional forest-saving projects are induced with gradually higher V levels (constrained only by V $\leq$ H), thus increasing $E[V \mid H]$, but bounded above by its unconditional expectation, EV. Thus $E[B-V \mid H]$ must be a decreasing function of H, and must always have value less than the unconditional expectation of *B*, *EB*; and value higher than EB – EV (both unconditional).

By a similar procedure we can derive the conditional variance of B-V for given H, with result:

(A5) $$E[(B-V)^2 \mid H] = E[B^2] - \left(\int_{-\infty}^{H}V^2\frac{g(V)}{G(H)}dV - (E[V \mid H])^2\right) = E[B^2] - E[V^2 \mid H].$$

Figure A1: Simulation of the pdf for Z

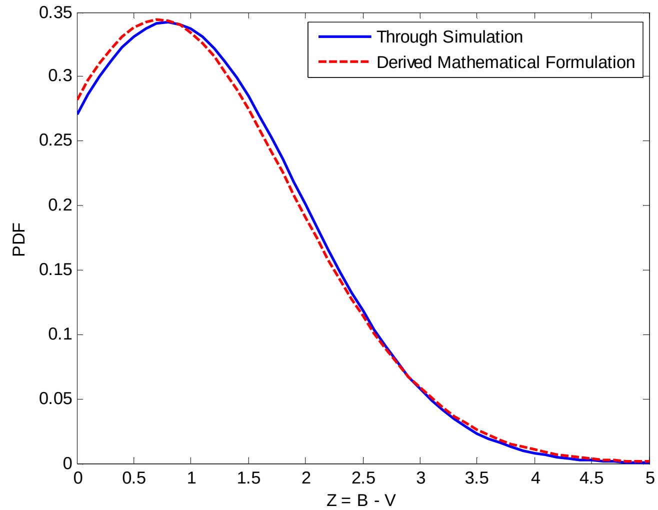


Figure A2: Plots of E(B-V) under parametric variation in H, when B and V are independent normal

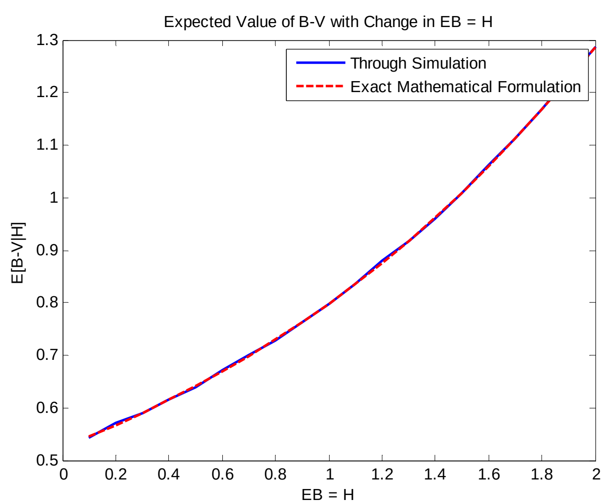
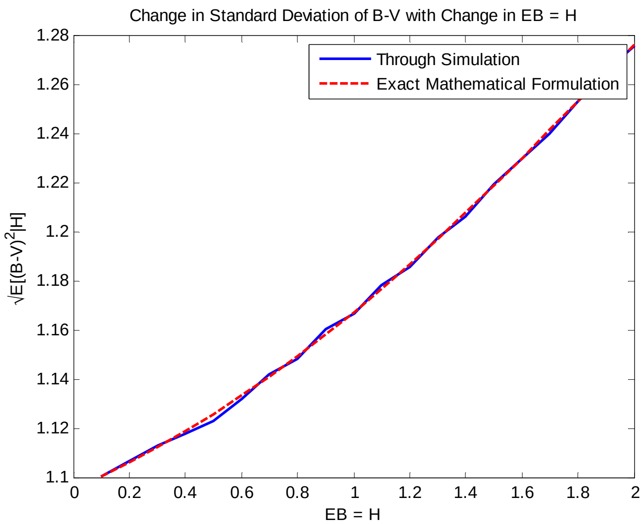
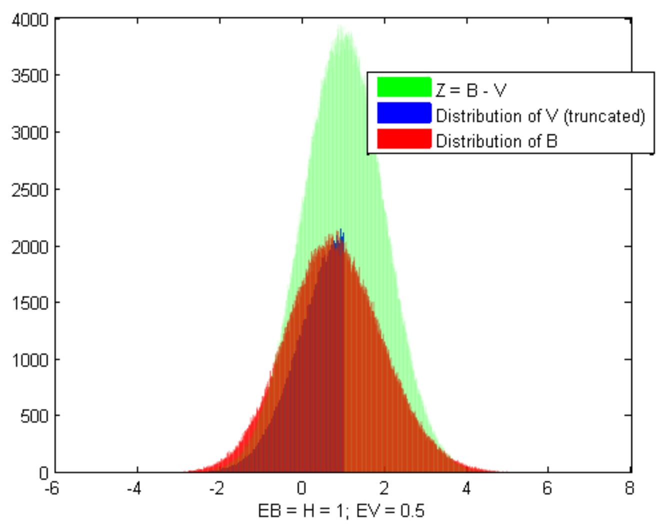


Figure A3: Plots of Standard Deviation of (B-V) under parametric variation in H, when B and V are independent normal



Further analysis can now be performed using these results. Some simulations can be done to get some notion of our formulations. In the simulations in Figures A2-A3 above, H is varied parametrically from 0 to 2 (and thus not necessarily optimal, from (A5), for given other specifications), while we assume that both B and V are independent and normally distributed, with EB = 1, EV = 0.5, $\boldsymbol{\sigma_V} = 1$, $\boldsymbol{\sigma_B} = 1$. The first graph shows the mean $E[B-V \mid H]$ and the second graph shows the standard deviation $\sqrt{E[(B-V)^2 \mid H]}$.

Figure A4: Density functions for B, V, and B-V = Z



Another nice property of this specification is that B-V will also be distributed normally if B and V are both normal (even if V is truncated). As an example, the simulation in Figure A4 above shows distributions for B, V, and Z = B-V for a numerical case where H = 0.1.

If B and V are instead lognormal, Z = B-V is no longer lognormal. For such cases our analysis could shift to analyzing B/V.

Appendix B: Derivation and simulations of probabilities of alternative outcomes

In this appendix we define and characterize four alternative outcomes of P's decision to save or not save the resource. The resource may be either saved or not saved; this occurs with probabilities 1-G(H) and G(H) respectively from the point of view of P. In each of these two cases, saving the resource may be correct (as is the case where the net social value of saving the resource is positive, thus Z > 0), or it may be incorrect (as when the net social value of saving the resource is negative, thus Z < 0).

We thus wish to find the probabilities associated with each of the following four events:

1) The resource is saved, and this is the correct (or globally efficient) decision. This occurs when H > V (with probability G(H)), and at the same time Z = B-V > 0. The probability of the latter outcome, denoted by $P_{S,E}(H) = G(H)[1-Q(0 \mid H)]$, using (11):

(B1) $$P_{S,E}(H) = G(H)\left[1 - \frac{1}{G(H)} \int_{-\infty}^{\infty} \int_{B}^{H} f(B)g(V)dVdB\right] = G(H) - \int_{-\infty}^{\infty} \int_{B}^{H} f(B)g(V)dVdB .$$

2) The resource is saved, and this is incorrect (the resource should not be saved). Again, we need to have H > V, but in this case Z < 0. The probability of this outcome, denoted $P_{S,N}(H)$, is similarly given by G(H) $Q(0 \mid H)$, and thus

(B2) $$P_{S,N}(H) = G(H)\left[\frac{1}{G(H)} \int_{-\infty}^{\infty} \int_{B}^{H} f(B)g(V)dVdB\right] = \int_{-\infty}^{\infty} \int_{B}^{H} f(B)g(V)dVdB .$$

3) The resource is not saved, and this is the correct decision. Then V > H, and Z < 0, occurring with probability [1-G(H)] $Q(0 \mid H)$. This probability, denoted $P_{N,E}(H)$, is found as

(B3) $$P_{N,E}(H) = (1 - G(H))\left[\frac{1}{G(H)} \int_{-\infty}^{\infty} \int_{B}^{H} f(B)g(V)dVdB\right] = \frac{1 - G(H)}{G(H)} \int_{-\infty}^{\infty} \int_{B}^{H} f(B)g(V)dVdB .$$

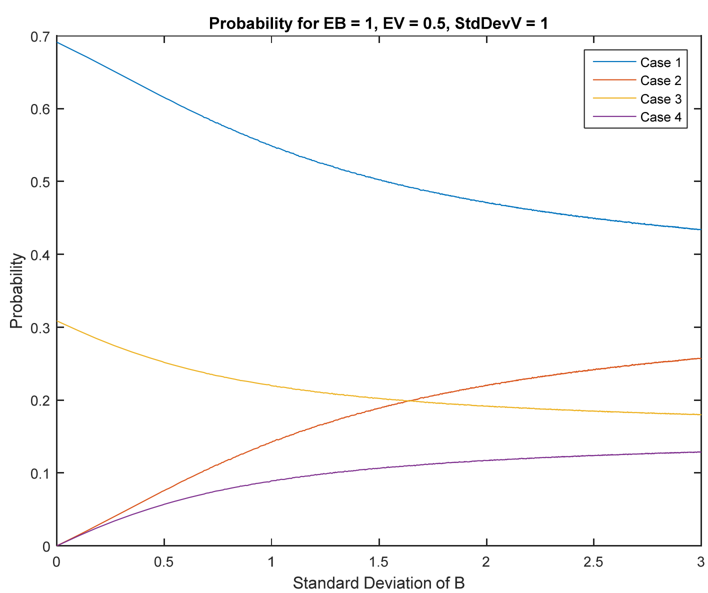
4) The resource is not saved, and this is incorrect (the resource should be saved). Then V > H, and Z > 0 with probability [1-G(H)][1-$Q(0 \mid H)$]. This probability is denoted by $P_{N,N}(H)$, and given by

(B4) $$P_{N,N}(H) = (1-G(H))\left[1-\frac{1}{G(H)}\int_{-\infty}^{\infty}\int_{B}^{H} f(B)g(V)dVdB\right] = 1-G(H)-\frac{1-G(H)}{G(H)}\int_{-\infty}^{\infty}\int_{B}^{H} f(B)g(V)dVdB\,.$$

We are here particularly interested in finding how the probabilities (B1)-(B4) depend on "precision", here represented by the standard deviation of B (over which the buyer may have some control through the collecting of more information). Note that these probabilities will also be functions of H. We find that when H is increased, the probability of saving the resource is increased; and the probability of converting the resource correspondingly reduced. Importantly here, the probabilities of both correctly and incorrectly saving the resource are increased; while the probabilities of both correctly and incorrectly exploiting it are reduced. Without further distributional assumptions, we cannot say much more about these probabilities; e g whether the overall probability of a correct decision is increased or reduced when H increases.

We here concentrate on the (simplest) case of α = 1 so that H = EB. As shown in Figures B1-B3 below, we have simulated, given that B and V are both normally distributed and independent, how improvements in information about B change the probabilities of correct and incorrect decisions related to the four possible outcomes 1) - 4) described above. Among these outcomes, 1 and 3 are "correct", while 2 and 4 are "incorrect". In all examples we assume α = 1 in (5), so that H = EB: the buyer fully incorporates the preferences of the seller in setting its optimal compensation level, H.

Figure B1: Probabilities of outcomes 1) - 4) as functions of standard deviation of B, given H = EB = 1, EV = 0.5, σ_V = 1, given independent normal distributions of B and V



A key assumption is that the distribution for B, including its expectation EB, are formed in Bayesian fashion on the basis of a subjective distribution across "true" possible values of B (only one of which is actually correct).

A way to study this, numerically and analytically, is to consider the correct value of B as drawn from the relevant (prior) distribution over B, F(B).

We present three such simulations of how solutions change σ_B is changed. For the two first simulations, we normalize by setting EB = σ_V = 1. In the first case we assume EV = 0.5 (so that, in "most" cases, the resource ought to be saved). In the second case, we assume EV = 1 (so that the numbers of cases where the resource ought and ought not to be saved are evenly distributed). The third case reverses the first by assuming EV = 1, while EB = 0.5. In this case, the resource ought to be saved in "few" cases. In each figure, the probabilities of outcomes 1)-4) are plotted as functions of σ_B.

From Figure B1, the probabilities of "correct" outcomes (cases 1 and 3) increase as we move leftward in the figure, to a more precise assessment (lower standard deviation) of B. As σ_B tends to zero, the overall probability of a correct outcome (the sum of probabilities under cases 1 and 3) converges to one. We see that most of the reduction occurs for type 1 decisions, namely, the probability of saving the resource when it should be saved grows, from a low level of about 0.43 when $\sigma_B = 3$, to a limit near 0.7 for $\sigma_B = 0$. Correspondingly, the probability of incorrectly saving the resource (so that it ought not to be saved; type 2 decisions) is then reduced almost proportionately to the increase in type 1 decisions. When σ_B moves from 3 to 0, the probability of mistakenly saving the resource is reduced from about 0.25 to zero. Most of the reduction in the probability of a faulty decision (from about 0.15 to zero) is accomplished with the "last" piece of informational improvement, as σ_B goes from unity to zero.

Case 4, not saving the resource when it ought to be saved, is also made substantially less likely with better precision: from about 0.13 at $\sigma_B = 3$ to zero for $\sigma_B = 0$; and also here with most of the improvement in precision occurring for the last bit of reduction in the standard deviation (moving from about 0.09 at $\sigma_B = 1$, to zero at $\sigma_B = 0$).

Figure B2 indicates similar probabilities for a case where expectations of B and V are equal (= 1) but where distributions are otherwise as in Figure 5. The probabilities of the two "correct" cases (1 and 3) are now always equal, as are the probabilities of the two "wrong" cases (2 and 4); the two latter being each 0.2 when $\sigma_B = 3$; and each approximately 0.13 when σ_B (= σ_V) = 1. Probabilities of each of the two types of correct decisions now however move from 0.3 to 0.37 as σ_B is reduced from 3 to 1; and from 0.37 to 0.5 as σ_B is further reduced to zero. Probabilities of each of the two incorrect decisions move first from 0.2 to 0.13, and then from 0.13 to zero. The overall improvement in the probability of correct assessment is now somewhat greater; from 0.6 when $\sigma_B = 3$, compared to about 0.66 when $\sigma_B = 1$.

Figure B2: Probabilities of outcomes 1) - 4) as functions of the standard deviation of B, given H = EB = EV = σ_V = 1, given independent normal distributions of B an

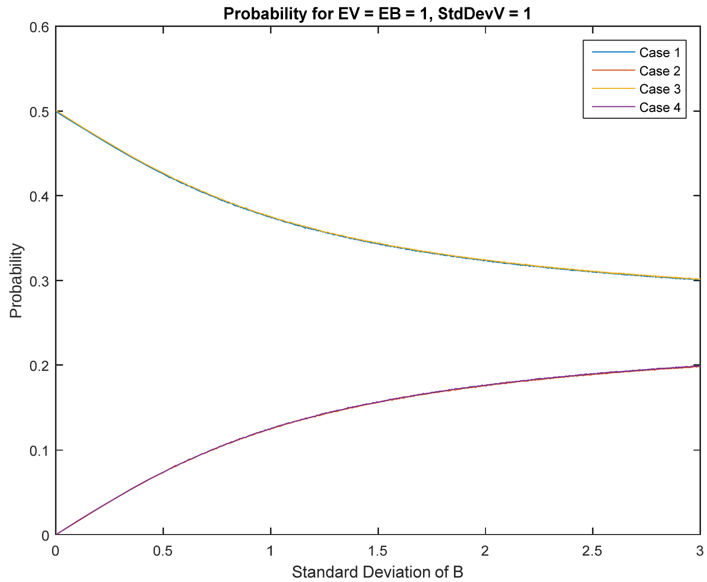
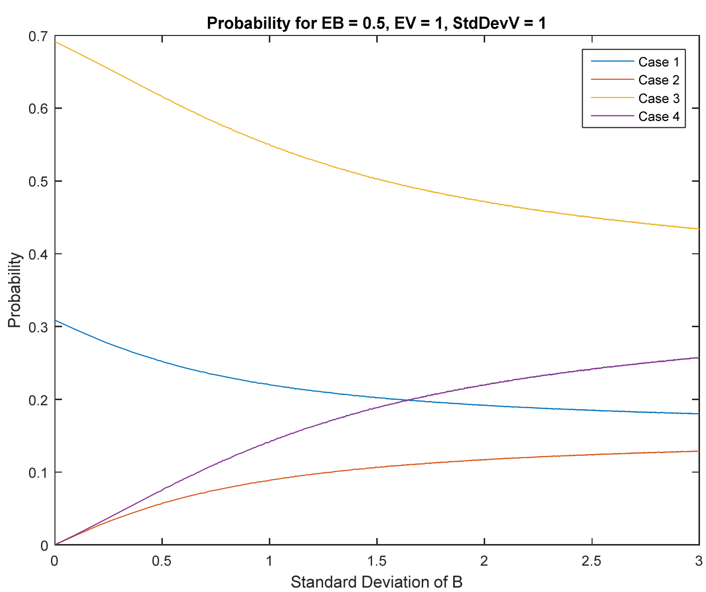


Figure B3: Probabilities of outcomes 1) - 4) as functions of the standard deviation of B, given H = EB = 0.5, EV = 1, σ_V = 1, given independent normal distributions of B and V



In our last example, in Figure B3, the expected gross protection value of the resource is below the expected opportunity value as EB = 0.5, and EV = 1; thus the opposite as compared to figure 1. We see that the probabilities as a consequence are shifted relative to figure 1, in a simple way:

the two correct probabilities of cases 1 and 3 are shifted; as are the two incorrect probabilities for cases 2 and 4. Thus in particular, the probability of incorrectly not saving the resource is now reduced from about 0.26 for $\sigma_B = 3$, to about 0.15 for $\sigma_B = 1$, and to zero for $\sigma_B = 0$.

Appendix C: Probabilities of different outcomes given log-normal distributions

The following three figures C1-C3 represent simulations parallell to figures B1-B3 except with log-normal distributions instead of normal. Remember then that the log-normal distribution has the "advantage" relative to the normal of taking only positive values; but the "disadvantage" of being skew (while the normal is symmetric) thus e g making intuitive interpretations more difficult.

Figure C1: Probability outcomes 1) - 4) as functions of the standard deviation of B, given EB = 1, EB = 0.5, and $\sigma_V = 1$, given independent log-normal distributions of B and V

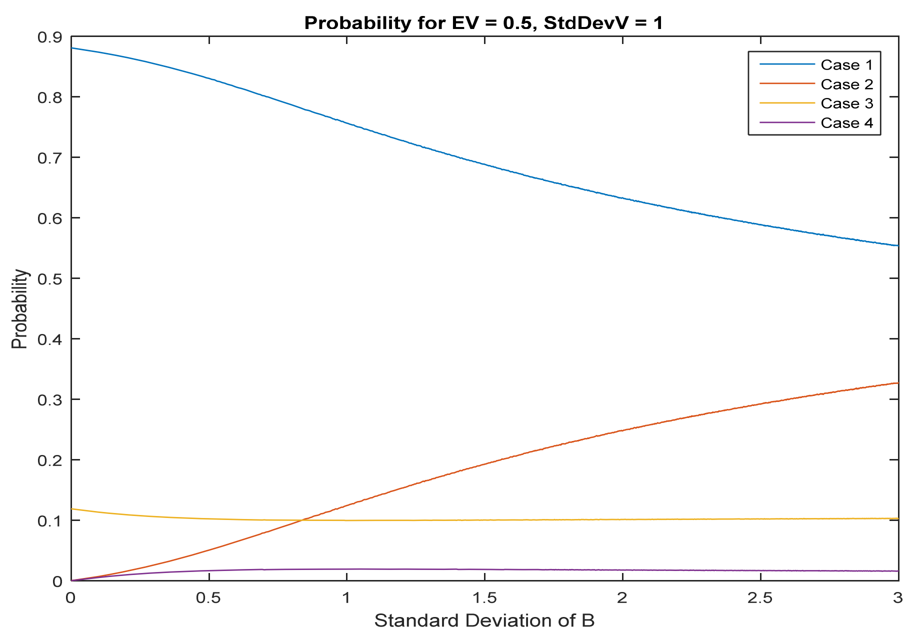


Figure C2: Probability outcomes 1) - 4) as functions of the standard deviation of B, given EB = EV = σ_V = 1, given independent log-normal distributions of B and V

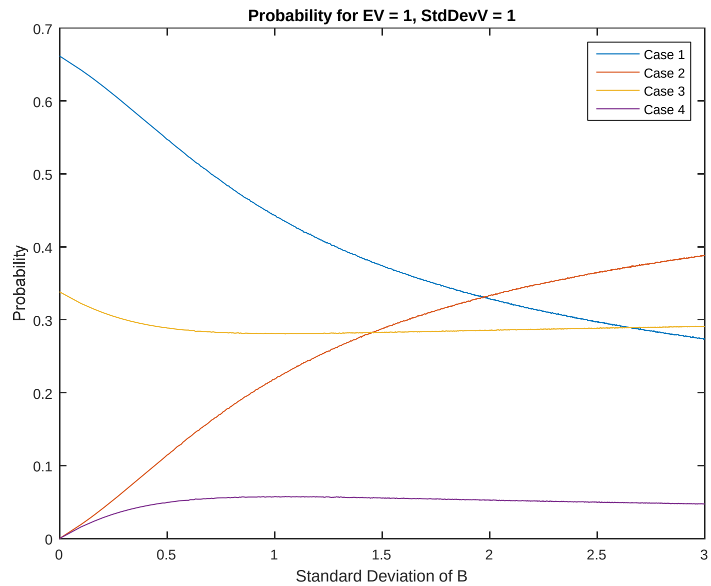


Figure C3: Probabilities of outcomes 1) - 4) as functions of the standard deviation of B, given EB = 0.5, EV = σ_V = 1, given independent log-normal distributions of B and V