WAGE RIGIDITY, INPLICIT CONTRACTS, AND ECONOMIC EFFICIENCY: ARE MARKET WAGES TOO FLEXIBLE?*

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Abstract

The analysis of implicit contracts between risk-averse workers and risk-neutral firms must recognize that (i) the contract is implicit, not explicit, (ii) it may only be conditioned on observable variables, and (iii) there are limits to contract complexity. If contracts are perfectly flexible than neither the constraint of enforceability nor observability result in unemployment. However, even with perfect enforceability and flexibility, limitations on contract flexibility may generate unemployment. Finally, even with flexible enforceable contracts and no unemployment the market equilibrium is inefficient.

We construct some simple general equilibrium models and explore the consequences of restrictions on the set of feasible contracts, at the same time commenting on the present state of implicit contract theory.
I. INTRODUCTION

The implicit contract theory of wages argues that workers are more risk averse than firms, and that firms will therefore find it less costly to hire labour if they provide some degree of income insurance against fluctuations in demand which would otherwise lead to fluctuations in the demand price of labour. Wages will be less flexible than in the Walrasian model, and, in extreme cases, will be rigid. Since there is a widespread and long standing belief that wage rigidities give rise to unemployment, it is natural to conclude that the risk shifting aspect of implicit contracts is responsible for unemployment, and that these risk shifting benefits outweigh the costs of unemployment. In the strong form of this argument, it is claimed that the resulting levels of unemployment are constrained Pareto efficient, where the constraint is that the absent income insurance markets which give rise to the demand for reduced wage fluctuations remain absent.

There are five important problems with this theory. First, if states of nature are observable to both employers and employees, then this explanation of unemployment fails, for the resulting equilibrium is identical to one in which the firm always pays Walrasian wages, and the individual purchases income insurance from an insurance company. The equilibrium would entail full employment, and the income insurance will, if anything, reduce the variability in demand. 1/

Second, the standard theory -- as well as some of the more recent developments -- fails to distinguish between implicit and explicit contracts. The fact that contracts are implicit has several important consequences. Thus explicit contracts can rely on third party enforcement through the law courts, but implicit contracts cannot. Instead, the firm must

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1/ For earlier discussions on this point, see Stiglitz (1977) and Akerlof and Miyazaki (1980). The Akerlof and Miyazaki paper, like that of much of the implicit contract literature, does not allow for any variability in the number of hours worked by individuals, and thus cannot address some of the central questions at issue.
have an incentive to honour the implicit contract, and the most obvious such
incentive is the preservation of its reputation as an employer. It is
important to recognise that there are limits to the enforceability of
contracts from both sides of the market. Firms have an incentive to cut wages
more than the workers expected in bad times, whilst workers may have an
incentive to quit in good times even though the firm had continued to pay them
throughout a slack period. (See, e.g. Holmstrom, 1983). The reputation
mechanism may work to prevent the first, but not the second.

Third, while explicit contracts can in principle contain elaborate
and complicated provisions which could be enforced, it seems unlikely that
implicit contracts relying on reputation can be very complicated. The
relationship between what the parties to the implicit contract can observe and
what consequences follow must be clear to both sides, for, by definition, they
do not have a written set of rules which specifies consequences for each
individual case. Another way to make the same point is that only very simple
relationships can be identified econometrically from past, limited,
observations, and more complex relationships would require more data to
distinguish from alternatives than is likely to be available. Thus implicit
contracts must have a simple form, and this will greatly limit their ability
to provide both full employment and adequate insurance.

Fourth, what terms of the contract are enforceable depends both on
what is observable and what the enforcement mechanism is. Clearly, the terms
of any contract can only be contingent on variables observable by both sides
of a contract, but if the contract is to be enforced by appeal to a third

2/ Myopic self-interest is another possible enforcement mechanism — workers
may respond directly to an unjustified drop in their wages in such a way
as to reduce the firm’s profits. The efficiency wage hypothesis argues
that workers’ productivity falls when wages are cut — either for
nutritional reasons (Bliss and Stern, 1978), or because of a drop in
morale. See Calvo (1979), Shapiro and Stiglitz (forthcoming), Stiglitz
For a survey of some efficiency wage models, see Stiglitz (1983).

3/ The potential importance of restrictions was also noted by Akerlof and
Miyazaki (1980). The restrictions which they focused upon, however, were
markedly different from those which we consider below.
party, the observations must also be verifiable, which is a much stronger requirement. Much information is transient, and may be readily observable by workers and their employer at the relevant moment, but because it is hard to record, may be immediately lost and hence unavailable for verification. Reputation mechanisms, on the other hand, do not require the same kind of verifiability. On the other hand, if a contract is to be enforced by a reputation mechanism then additional constraints on the set of feasible contracts are required: the firm must be better off maintaining its reputation than it would be "breaking" the implicit contract.

The earlier implicit contract literature assumed that the state of nature was both observable and verifiable, and thus contracts could be written which were contingent upon the state of nature. The more recent literature recognises that much of the information about the state of nature is neither observable nor verifiable, but fails to give a convincing account of what information is both available and can be used in the design of an enforceable implicit contract. In particular, it fails to introduce into the analysis information which is plausibly available and verifiable (like the aggregate unemployment rate, the price level, product prices charged by other firms, etc.) which would be introduced in any explicit contract which attempted to maximise the expected (utility of) profits of the firm, subject to the workers receiving a particular level of expected utility, and subject to various self-selection constraints. (But see Grossman, Hart and Maskin, 1982). In this paper, we shall argue that while the state of nature may not be directly observable, the probability distribution of wages and hours associated with any firm can be observed. While such information might be irrelevant for the structure of one period contracts which were to be enforced solely by legal means, it is critical for multi-period contracts, and the fact...
that most implicit contracts are by necessity enforced through a reputation mechanism makes these contracts, implicitly, multi-period contracts. 

Fifth, and finally, the implicit contract literature is concerned with explaining unemployment; unemployment is a macro-economic phenomenon, yet with a few exceptions, the implicit contract literature has analyzed only partial equilibrium models. It is only within a model of the whole economy that one can assess claims about the effect of alternative contractual arrangements on the equilibrium level of employment. Moreover, the frequently expressed claims about the optimality of contract equilibrium are usually nothing more than the restatement of the problem of contract equilibrium: the firm maximizes its expected (utility of) profit, subject to being able to obtain workers, i.e. giving them a given level of expected utility (subject to whatever informational or other constraints that are imposed). The solution to this problem is obviously efficient in a partial equilibrium sense, e.g. given the probability distribution of prices, the probability distribution of wages offered by other firms, etc. However, the relevant question is whether

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4/ This is a second sense in which the recent literature on implicit contracts with asymmetric information is internally inconsistent. The fact that such contracts are enforced by reputation makes them multi-period contracts, while these papers characterize the optimal one period contract. However, with asymmetric information, the optimal multi-period contract may look markedly different. For instance, if the rate of discount is zero, and there were an infinite number of periods, equilibrium will be close to that which would have prevailed with symmetric verification. See below, Section 2.4.

5/ We should emphasize that this list of deficiencies with the implicit contract literature is not meant to be exhaustive. Two other problems should be mentioned. A theory of unemployment must not only explain the aggregate number of hours worked, and why this is less than that associated with full employment, but it also needs to explain the pattern of unemployment, why, in particular, there are layoffs rather than work sharing. The standard arguments provided by the traditional implicit contract literature are consistent only with there being work sharing; they do not explain layoffs. This fact, in itself, should make these theories suspect. Second, the fact that a worker is laid off does not mean that he is unemployed. With costless labor mobility, a worker who is laid off will be immediately rehired by some other firm. Thus, a theory of unemployment must not only explain why workers are laid off, but also explain why other firms do not immediately rehire them. (Hosios, Arnott, Hosios, and Stiglitz have recently constructed such models.)

the market equilibrium is (constrained) Pareto efficient when all firms sign such contracts, and prices and wages are determined endogenously. Does there exist, for instance, a set of taxes and subsidies which can generate a Pareto improvement to the economy? We show that, in general, the equilibrium generated by implicit contracts is not Pareto efficient.

The objective of this paper then is to construct some simple, general equilibrium models within which we can explore the consequences of alternative assumptions about the nature of the set of feasible contracts. We shall examine three aspects of contract design: what is observable (which determines the events upon which the contract can be contingent); what is enforceable; and what limits the complexity of admissible contracts. While the traditional implicit contract literature assumed perfect observability, perfect enforceability, and perfect flexibility, we shall argue that plausible assumptions on observability and enforceability combined with perfect flexibility gives rise to full employment. Consequently we argue that a central explanation of unemployment is the limited flexibility allowed by implicit contracts.

Before beginning our analysis, we should explain what we mean by unemployment. The standard definition focuses on the individuals' perceptions: if the individual is actively seeking a suitable job and is unable to obtain one, then he is unemployed. The requirement that the job be suitable for the individual is obviously important, for there may be a job opening for a dishwasher, but the fact that a trained engineer, who cannot get

7/ Taking the set of insurance markets and the available information as given. For a discussion of notions of constrained optimality, see Diamond (1967), and Newbery and Stiglitz (1982).

8/ One such attempt, from a very different perspective than that reported here, is that of Hall and Lazear. They assume that workers cannot confirm the state of nature (their marginal productivity within the firm.) But they also assume that the firm cannot write a contract with wages contingent on hours or employment levels. The only feasible contracts are those with completely rigid wages. In a sense, this is an extreme case of the model that we consider in Section II, where we postulate that there is a simple relationship between the wage paid and certain macro-economic variables. (There are, of course, a number of other important differences between the two models including the fact that they model only the labor market, not the general equilibrium of the economy.)
a job as an engineer, does not take this job does not vitiate the fact that he is involuntarily unemployed, though some economists would suggest that it does. The point is that there are individuals with the same skills and qualifications who have jobs and he is willing to take an identical job but he cannot obtain one.

The difficulty with this approach is that it rests on the individual's perception of his qualifications, and an individual may have an overly inflated view of his abilities. To avoid this difficulty we employ a more technical definition: we say that an individual is (partially) unemployed if his marginal rate of substitution between consumption and leisure differs from his marginal rate of transformation; hence if the wage is equal to the marginal rate of transformation, and he would like to work more at the going wage, we say he is (partially) unemployed. The definition does not say anything about whether the level of employment, in the given situation, is greater or less than it would be in the corresponding Walrasian economy (with perfect information, perfect insurance markets, etc.) For that requires a comparison of general equilibrium solutions, which takes us beyond the scope of this paper. Nor do we directly address the philosophical question of whether an individual who signs a contract which has the provision that with some probability he will be laid off, is voluntarily or involuntarily unemployed when laid off. He has voluntarily signed a contract which has assigned to others the right to make a decision, which he would not voluntarily make himself.

Our definition does not, in itself, say anything about the efficiency of the equilibrium. Since we are describing market structures in which there are, for instance, real costs associated with information, these costs need to be taken into account. An equilibrium in an economy with costly information looks markedly different from that of an economy with perfect information; in such an environment, some unemployment might be efficient.

The fact that unemployment in such a context is "natural" says nothing about whether the level of unemployment which emerges is or is not efficient. We shall show that, in general, it will not be.
II. THE BASIC MODEL

We consider a firm with a simple, fixed coefficients technology. So long as the capital constraint is not binding, the average and marginal productivity of a worker is the same. The price must be sufficiently high not only to pay labor, but to obtain an adequate (expected) return on capital. Thus, even if in a bad state, the value of the marginal productivity of labor declines slightly, it still exceeds the wage (even if the wage does not decline). In this case, the lack of wage flexibility has no cost, but generates a clear benefit to the risk averse workers. If, on the other hand, the value of the marginal product declines enough, then with rigid wages there will be some induced employment. Given this trade-off between risk reduction and unemployment it seems plausible that the optimal contract will entail a sufficiently rigid wage that there is at least some probability of unemployment. The central question addressed in this paper is whether, or under what conditions, this hypothesis is valid.

We first formulate the general problem of the design of the implicit contract. Since we wish to explore the welfare properties of the implicit contract, we need to construct a general equilibrium model with a well defined source of uncertainty. For simplicity, we assume that there are two sectors in the economy, one of which produces an export crop by means of a constant returns to scale technology, using only labor. One unit of labor produces one unit of the export crop, which sells for a price $p$ in terms of the numeraire, which is the imported consumption good. Export price fluctuations are the source of risk: $\tilde{p}$ is a random variable. There is a second, non-traded, consumption good which is produced by means of a fixed coefficients technology:

$$C = \min (N_c h, K/v)$$ (1)

where $C$ is the output of the consumption goods industry, $N_c$ is employment in the industry, $h$ is the hours worked, $K$ is the capital stock in the industry, and $v$ is the fixed capital-output ratio in the sector. Capital must earn an expected return of $r$. 
Our concern is with the design of the optimal contract in the non-traded sector. For simplicity, we assume workers in the export sector are self-employed, and thus receive a wage of $p$ (per unit of labor supplied). At the beginning of the period workers are freely mobile, but once they have decided whether to work in the export or non-traded goods sector, they are perfectly immobile.

We shall be concerned with interior equilibria in which there are workers in both the non-traded and the exported goods sector. All individuals are identical; they have a standard utility function defined over their consumption of the imported consumption good, the non-traded consumption good, and leisure (work); it is more convenient if we represent it by the indirectly utility function, $V = V(q, w)$, giving utility as a function of the wage received, $w$, and the price of the non-traded consumption good, $q$, provided that workers are not constrained in the amount of labor which they can supply. If they are constrained, then their utility can be represented by a modified indirect utility function $V = V(q, y, h)$, giving utility as a function of the price of the non-traded consumption good, their income, $y$, and the number of hours which they work.

In equilibrium, there will be some relationship between the price of the export good (the exogenous source of uncertainty in this model) and the price of the non-traded consumption good, which we denote by $q = (p; \Omega)$, where $\Omega$ is a vector of market parameters, to be specified later.

Since for workers in the export sector (subscript $x$)

$$ w_x = \tilde{p} $$

we can write the expected utility of workers in the export sector by

$$ \bar{W}_x = E V(q (\tilde{p}; \Omega), \tilde{p}). $$

The equilibrium implicit contract in the non-traded consumption goods sector is "signed" before the state is known. How the contract is formulated depends, of course, on what assumptions concerning observability, enforceability, and flexibility we make. We begin our analysis by altering
only one of the three assumptions of the standard implicit contract literature; we shall assume that the state of nature is perfectly observable, and that the contract can be as complicated a function of the state of nature as desired. However, we shall assume that the contract is not enforceable as an explicit contract. We shall assume that the firm announces a wage as a function of the state of nature \( p \) at the beginning of the production period, and then, once \( p \) becomes known, it commits itself to pay a particular wage, though not necessarily the one implied by its earlier announcement. The firm can break its implicit contract in two different ways -- it can lay off workers when the contract calls for their retention, or pay them a lower wage than the contract implies. The problem of enforcement is different in each case. The second kind of contract breach -- paying the wrong wage -- can be avoided by the following self-enforcement mechanism: if the worker receives a wage lower than the implicitly agreed wage, then he shirks, and his labour productivity drops by more than the shortfall in wages, thus discouraging the firm from deviating from its implicit wage schedule. 9/

On the other hand, if it lays off workers that it had promised not to lay off, those laid off workers cannot retaliate by shirking. (It is, of course, possible that retained workers shirk in sympathy, but we ignore this possibility.) The implication is that the contract can specify the number of

9/ The following argument demonstrates that it is not in the interest of the firm to deviate from its wage schedule. Suppose the worker either provides the correct level of effort, \( e \), or zero effort. If the firm can subsequently observe but not verify the worker's effort, then it cannot make the current wage contingent on current effort. The firm can, however, make future employment or future wages contingent upon observed levels of effort in earlier periods, and the worker can make his level of effort contingent upon the behaviour of the firm in previous periods. There is a perfect equilibrium in which firms respond to shirking by paying a zero wage (or firing the worker), and in which workers respond to contract breach by firms by providing zero effort. Given that the worker believes that once he has shirked, the firm will pay him zero wages from then on, it pays the worker to shirk from then on; and given that the worker shirks from then on, it pays the firm to pay the worker only zero wages. Similarly, given that the firm has broken its contract and that it expects the worker to shirk in subsequent periods, the worker expects the firm will pay him a zero wage in subsequent periods; and given that the worker believes that the firm will be paying him a zero wage in subsequent periods, it pays him to shirk this period.
hours worked and income paid as a function of the state, \( p \), but that the set of (hour, income) combinations which are admissible are those for which it is in the interests of the firm to retain the worker, i.e. for which the value of the worker's marginal product exceeds the wage paid. 10/ 

There is another argument for the asymmetric treatment of the enforceability of wage promises and lay-off promises when the enforcement mechanism is reputation. A firm which loses its reputation will have to pay workers higher wages in the future in order to recruit them. But the firm will only lose its reputation if it is observed violating the implicit contract, and information about wages may be more readily obtainable than information about layoffs. Individuals are laid off for a variety of reasons. The worker may not be able to distinguish individuals who have been laid off because the firm is reducing its employment from individuals who have been laid off for some other reasons (e.g. because they are bad workers.) Since firms are always expanding some operations, contracting others, the worker would find it difficult to verify compliance for even an explicit contract provision that specified the fraction of the employees that would be laid off in some state of nature. Clearly, verifying such a provision in an implicit contract is even more difficult. In a formal sense, then, the contracts which we analyze in section 2.2, where, in some states of nature, the enforceability constraint is binding, can be thought of as the optimal 

10/ If workers can never receive a wage in excess of the value of the marginal product, and if the competitive equilibrium contract is to have the property (for risk neutral firms) that the expected wage equal the value of their expected marginal product, then does this imply that the wage must be equal to the value of the marginal product in every state of nature? The confusion arises from failing to take into account the difference between the ex ante and ex post marginal products. Before the individual is hired, we compare the profits of the firm with and without the individual, taking into account all of the ancillary expenditures required for him to be productively employed, e.g. specific training, machines, etc. After he has been hired, at least some of these ancillary costs are sunk, and thus are irrelevant for the decision about whether to lay off an individual. Although we could have couched our analysis in terms of specific training costs, we shall assume that each individual requires a fixed amount of capital to be employed; this is the sunk cost which results in there being a distinction between ex post and ex ante marginal productivities.
contract contingent on the unobservability of the layoff rate. 11/ Workers of course know that the behavior of the firm will be affected by the wages specified in the contract (this is the standard moral hazard problem) and it is this which motivates them to prefer contracts in which the wage does not fall below the value of the marginal product.

Thus, to summarize, the contract will be chosen to maximize profits, subject to two constraints: (i) it must be possible for the firm to hire workers, i.e., the expected utility of workers under the contract must be at least equal to $W_x$; and (ii) the contract must satisfy the enforceability constraints just discussed. In equilibrium the number of firms in the industry will be such that (the maximized value of expected net) profits in the industry are driven to zero.

For purposes of comparison with the constrained social optimum, it is more convenient if we analyze the dual of the problem that we have just formulated; i.e., consider the contract which maximizes the expected utility of workers, subject to a constraint on profits (which, in equilibrium, is simply the zero profit constraint), and subject to the enforceability constraint. Thus, the implicit contract 12/

$$\text{maximizes } \mathbb{E}[q(p; \Omega), y(p), h(p)]$$

subject to

$$h(p)q(p; \Omega) > y(p), \text{ (the enforceability constraint)}$$

$$K/v > N_c h(p), \text{ (production efficiency - see equation (1))}$$

and

$$\mathbb{E}[h(p)q(p; \Omega) - y(p)] > rK/N_c, \text{ (zero profit constraint)}$$

11/ Thus, it should be noted that an implicit assumption of the recent literature on implicit contracts with asymmetric information is that the lay-off rate is observable.

12/ Note that this formulation is sufficiently general to admit both layoffs and work-sharing. Layoffs entail $h = 0$, with some probability. Complete work sharing entails, in any state of nature, $h$ being the same for all individuals.
where each of the competitive firms takes as given the relationship between the state \( p \) and the market equilibrium price \( q \), which is given by the market equilibrium equation that in each state demand equal supply: 13/

\[
D(q;p,y(p),h(p),N_c) = D_c N_c + D_x(N-N_c) = N_c h(p),
\]

where \( D_c \) and \( D_x \) are per capita demands. Demand is a function of the market price, \( q \), and in addition, a vector of market parameters -- the price of the export good (equal to the income of export workers), the income of workers in the non-traded sector, the number of hours they work, and the allocation of labor between the two sectors. (In the special model of Part III, the demand depends only on some of these parameters.) The demand curve can easily be derived from the behavior of workers in each of the two industries. (See Appendix 1). Thus for workers in the export industry, the demand (per worker) of the non-traded good is given by \( D_x = -\hat{V}_q/V_I \) where \( V_I \) is the marginal utility of income. In the non-traded sector demand per worker is \( D_c = -\hat{V}_q/V_y \), where hats distinguish the non-traded sector.

Variations in the price of the export crop give rise to variations in the demand for the non-traded consumption good. These variations, in turn, lead to variations in the equilibrium price, hours, output and possibly employment in the non-traded consumption goods sector.

The (rational expectations) market equilibrium may now be analyzed. Given a function \( q(p; \omega) \) and an allocation of labor between the two sectors, \( N_c \), the maximization problem (4) yields an equilibrium contract \( \{h(p), y(p)\} \) and an equilibrium level of expected utility for consumption good workers (subscripted \( c \)), \( W_c[q(p)] \). The market equilibrium equation (8) then yields an equilibrium price function \( q^*(p; \omega) \). The (rational expectations) market equilibrium is then a function \( q(p) \) and \( N_c \), such that

\[
q(p) = q^*(p),
\]

13/ There is, of course, a balance of trade equilibrium condition: but by Walras' Law, if the demand for non-traded goods equals the supply for non-traded goods, the demand for traded goods must equal the supply of traded goods.
and

\[(11) \ W_c\{q^*(p)\} = EV\{q^*(p), p\} (\equiv \tilde{W}_x),\]

i.e., the "expected" price distribution is the actual price distribution, and the allocation of labor is such that expected utility in the two sectors is identical (the \textit{ex ante} free mobility condition).

The maximization problem (4)-(7) can be written as a Langragian to be maximized:

\[
L = E[\hat{V}(q(p), y(p), h(p)) + \lambda(p)\{h(p)q(p) - y(p)\} + \nu(p)\{K/N - N_c h(p)\} + \mu[h(p)q(p) - y(p) - rK/N_c]]
\]

where the expectation operator applies to all terms, and the vector of market parameters, \( \Omega \), has been omitted for brevity. The market equilibrium contract must satisfy the first order conditions (differentiating with respect to \( y \) and \( h \) in each state, \( p \)),

\[
\frac{\hat{V}_y}{\nu} - \lambda(p) - \mu = 0 \tag{11}
\]

\[
\frac{\hat{V}_h}{\nu} + q(p)\{\lambda(p) + \mu\} - \nu(p)\ N_c = 0. \tag{12}
\]

Dividing (12) by (11), we immediately observe that

\[
\frac{\hat{V}_h}{\hat{V}_y} = -q \frac{\lambda + \mu - \nu N_c/q}{\lambda + \mu} \tag{13}
\]

Hence, if \( \nu = 0 \), i.e., capital is not fully utilized (\( h \) is less than \( h_{\text{max}} \)): \( \hat{V}_h/\hat{V}_y = -q \). The marginal rate of substitution between leisure and consumption goods must equal the value of the marginal productivity of labor: the implicit contract entails full employment (in the sense that we have defined it above, p.), that the marginal rate of substitution equals the value of the marginal product, \( q \), even when the enforceability constraint is binding. (When \( \nu > 0 \), i.e. capital is fully employed, then \( h \) is given by
\[ h^* = h_{\text{max}} = \frac{K}{vN_c}. \] The marginal product of labor is not well defined; it is zero for increases in labor, \( q \) for decreases in labor. Equation (13) says that the marginal rate of substitution is greater than zero but less than \( q \).

One might be misled by these results into believing that the economy is efficient. After all, the labor market contracts have workers working at an efficient level and the marginal rate of substitution equals the marginal rate of transformation. But the contracts have been drawn up with each firm taking the price distributions as given; and these price distributions depend on the allocation of labor between the two sectors and on the level of investment in the non-traded sector. In the next section, we show that these decisions will not be made correctly in a market economy.

We summarize the result of this section in

**Proposition 1.** An equilibrium implicit contract with or without the enforceability constraint entails full employment. 14/

2.3 The Efficiency of Market Equilibrium

We now consider the optimal policy of a government which controls the non-traded goods sector. To make the comparison with the market equilibrium as clear as possible, we shall assume that the government faces the same enforceability constraints as does the private sector, that it cannot directly control labor allocation, and that as a result, it must equate the level of welfare in the two sectors. (There is no compelling reason why any of these constraints should be binding upon the government; if they are not, the case for the non-optimality of the market equilibrium is even stronger.)

The government's maximization problem can thus be summarized by the Lagrangian

\[
L = E \left[ N \left( V(q, y + s, h) + \alpha(N - N_c) V(q, p, - \frac{sN_c}{N - N_c}) + \lambda(p)\{hq - y\} + \nu(p)\{K/v - N_c h\} + \mu[hq - y - rK/N_c\} + \xi N_c [\hat{V} - V] \right) + \right.
\]

11/ This result parallels that of Akerlof and Miyazaki (1980) and Stiglitz (1977), but they did not allow workers to vary the number of hours that they worked, nor did they impose the enforceability constraint.
where the control variables are \{y, h, s, N_c, K\}, and where \(q\) is a function of the control variables given by (8). Differentiating (14), we obtain

\[
\frac{\partial L}{\partial y} = N_c (1 + \xi) \hat{V}_y - (\lambda + \mu) + \frac{\partial L}{\partial q} \frac{dq}{dy} = 0,
\]

(15)

\[
\frac{\partial L}{\partial h} = N_c (1 + \xi) \hat{V}_h + q(\lambda + \mu) - \nu N_c + \frac{\partial L}{\partial q} \frac{dq}{dh} = 0.
\]

(16)

The competitive market will be efficient if there exists a value of \(\alpha\) for which (15) and (16) are consistent with (11) and (12) when \(s = 0\). (As the distributional weight \(\alpha\) varies, the social optimum will trace out the set of efficient allocations. We are adopting the same strategy to explore the efficiency of competitive equilibrium that we used in Newbery and Stiglitz, 1982.) If \(\frac{\partial L}{\partial q} = 0\), then the two solutions coincide, but in general, \(\frac{\partial L}{\partial q} \neq 0\) and hence the market equilibrium will be inefficient.

It is fairly straightforward to show that essentially the same conditions are required for market efficiency as those established by Newbery and Stiglitz (1982), namely, that risk markets must be redundant. To show this we examine the conditions needed for \(\frac{\partial L}{\partial q} = 0\):

\[
\frac{\partial L}{\partial q} = N_c (1 + \xi) \hat{V}_q + \alpha(N - N_c) V_q + (\lambda + \mu)h - \xi N_c V_q.
\]

(17)

Substitute for \((\lambda + \mu)\) from (15) and use Roy’s formula and (8) to give

\[
\frac{\partial L}{\partial q} (1 - h \frac{dq}{dy}) = -N_c (1 + \xi) D_c \hat{V}_y - D_x V_I \left\{\alpha(N - N_c) - \xi N_c\right\}
\]

\[
+ (1 + 2\xi) N_c h \hat{V}_y,
\]

\[
= (N - N_c) D_x \left\{(1 + \xi) \hat{V}_y - \hat{V}_I \left\{\alpha - \xi \frac{N_c}{N - N_c}\right\}\right\}.
\]

(18)

But \(\alpha\) is chosen to make \(\frac{\partial L}{\partial s} = 0\) at \(s = 0\), i.e.

\[
\frac{\partial L}{\partial s} = E \left[N_c \hat{V}_y - \alpha N_c V_I + \xi N_c \left\{\hat{V}_y + \frac{N_c}{N - N_c} V_I\right\}\right] = 0.
\]
or

\[ \alpha = (1 + \xi) \frac{\hat{V}^y}{EV_I} + \xi \frac{N_c}{N - N_c}. \]  \hspace{1cm} (19)  

Substitute (19) into (18):

\[ \frac{3L}{3q} (1 - h \frac{3q}{3y}) = (1 + \xi) (N - N_c) D_x \left[ \hat{V}^y - \frac{V_I}{EV_I} \hat{V}^y \right]. \]  \hspace{1cm} (20)  

Equation (20) says that \( \frac{3L}{3q} = 0 \) if and only if the ratio of the marginal utilities of income in the two sectors, \( \hat{V}^y/V_I \), is constant across states of nature (i.e. as \( p \) and hence \( q \) varies). This is a very stringent requirement, equivalent to

\[ \frac{d}{dp} \log \left( \frac{\hat{V}^y}{V_I} \right) = 0 = \left( \frac{\hat{V}^y q}{V_y} - \frac{V_I q}{V_I} \right) \frac{dq}{dp} + \frac{\hat{V}^h h}{V_h} \frac{dh}{dp} - \frac{V_I p}{V_I}. \]  \hspace{1cm} (21)  

Even if \( \hat{V}^y q = V_I q = 0 \), the remaining terms will in general only be zero if the agents are risk neutral, for \( h \) affects the income of the non-traded good workers, whilst \( p = w_x \) directly affects the income of the agricultural workers.

These results can be summarized as follows.

**Proposition 2.** An equilibrium implicit contract equilibrium is inefficient unless agents are risk neutral.

### 2.4 Reputation and the Enforceability Constraint

In the preceding sections we assumed that wages were constrained to being less than or equal to the value of the marginal product of labor. Otherwise, we argued, the firm would have an incentive to renge on the (implicit) contract. This assumed that there was no penalty on renge. In fact, it is plausible to assume that any firm that reneged on its contract could only obtain workers in the future at a higher wage; there is a penalty associated with paying a wage different from the promised or of laying off a worker or making him work more than one promised. The penalty associated with
reneging on a contract ("losing one's reputation") is \( J(p, \Omega) \), where \( p \) continues to denote the state and where \( \Omega \) is a vector of market parameters. \( J \) represents the reduction in the present discounted value of profits from a loss of reputation. The firm considering laying off a worker that it has promised to pay a wage of \( w(p) \), which exceeds the value of his marginal product, has a one-time gain of \( w - qh \). Thus, enforceability requires that

\[
w - qh < J(p, \Omega) \text{ for all } p.
\]  

(22)

The analysis of the market equilibrium is identical to that discussed earlier (with the straightforward modification of the Lagrangian (10)), and we again find that there will be full employment.

It will still be true that the market equilibrium will not be Pareto efficient, but now there is an additional source of inefficiency. The magnitude of the penalty will in general be a function of the wage, and the employment policies of all other firms. If that is the case, any single firm ignores its effect on the set of enforceable contracts, but the government, in deciding on the wage, employment policy, will take this effect into account. 15/

**Imperfect observability.** So far, we have assumed that the state of nature is perfectly observable to both firms and workers. Assume, however, that the state of nature is not observable, but the probability distribution of the states of nature is known. Suppose, for example, that it is known that 50 percent of the time, \( p \) is high, and 50 percent of the time \( p \) is low. Then a contract which specified that 50 percent of the time the wage should be \( w_1 \) and 50 percent of the time the wage should be \( w_2 \), where \( w_2 \) is the wage that would have prevailed with perfect observability of the state of nature, would have the firm paying the high wage in the good state and the low wage in the bad state: the restriction on the probability distribution of states forces

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15/ For a formal analysis of this point, in a different context, see Shapiro and Stiglitz (1982).
the firm to be honest. 16/ 17/ We can thus reformulate our problem of determining the optimal implicit contract, by imposing an additional set of constraints; in the case of two equally likely states these constraints can be written 18/

\[
\begin{align*}
& h(p_1)q(p_1) - y(p_1) + h(p_2)q(p_2) - y(p_2) > 0 \\
& h(p_1)q(p_2) - y(p_1) + h(p_2)q(p_1) - y(p_2)
\end{align*}
\]

or

\[
\{h(p_1) - h(p_2)\} q(p_1) - q(p_2) > 0.
\]

Constraint (23) says that the profits when the firm announces that it is state 1 when it is state 1 (and, therefore, that it is state 2 when it is state 2) exceeds the profits which would accru if the firm announced that it were

16/ This is not quite true if there is a positive interest rate; it may pay the firm to announce that it is a bad state when it is in fact a good state, and some time later, announce that it is a good state, when it is in fact a bad state; such lies reduce the undiscounted value of profits, but since the firm may gain in the earlier period, and the subsequent losses in the later period are discounted, such lies may increase the present discounted value of the firm's profits. Thus, if there is a positive interest rate, the contracts must be chosen to impose a sufficiently large penalty on lying that lying never occur.

17/ Notice the difference in information assumptions between our analysis and that standard in the implicit contract literature with asymmetric information. In both it is assumed that the worker knows the probability distribution of the states of nature, and that the actual state of nature is not observable. But in the standard asymmetric information model, it is assumed that workers know the firm's profit function, whilst not being able to observe profits (or else the state of the world could be deduced, and we would be back in the full information, full employment case.) On the other hand, we assume that workers do not know the profit function, or the level of profits but do know the probability distribution of wages and hours. We would argue that this assumption about the observability of the profit function and the non-observability of profits is implausible, and is better replaced by the assumption that workers can observe the probability distribution of wages and hours.

18/ This condition will obviously be satisfied in the first best contract, provided only that the marginal rate of substitution between consumption and leisure does not vary greatly with the relative price of consumption goods.
state 1, when it is state 2, and conversely. The presence of this self-selection constraint leaves unaltered all of our earlier conclusions. 

III. IMPERFECT WAGE INDEXING

We noted earlier that if wages could be set as any arbitrary function of the state of nature (the price of the export good), then, even if there is an enforceability constraint limiting the losses which employers are willing to undertake on their employees, and workers are risk averse, the optimal implicit contract nevertheless involves full employment (in the sense defined above). Wages may, in fact, be partially rigid, but they are never so rigid as to give rise to unemployment.

These contracts effectively require not only that wages be indexed on the exogenous sources of uncertainty in the economy but that the index formula can take on any form. In general, the relationship between y (and h) and p will be highly nonlinear. In this section, we show that restricting the contract to a linear index function leads to an optimal degree of wage flexibility which will, in general, entail a finite degree of unemployment. Since we are only concerned to demonstrate the possibility of unemployment, the model is chosen to be the simplest analytically soluble general equilibrium model we could devise. Rather than model the particular form unemployment takes (work-sharing versus layoffs) we shall further simplify by assuming that if there is a constraint on the demand for labor, there is perfect work sharing.

---

19/ We emphasize that the kinds of contracts discussed in the recent literature on asymmetric information contracts are necessarily explicit contracts, since they are one period contracts and cannot be enforced by reputation. We should also note that if firms are risk neutral, and workers have utility functions which are separable in consumption and leisure, then the asymmetric information equilibrium will be characterized by overemployment, not underemployment. If firms are risk neutral and individuals have utility functions which are (a monotone transform) of the form \( C + b \psi(h) \), then there is neither underemployment nor overemployment. Only if firms are sufficiently risk averse, will there be underemployment. (See Azariadis and Stiglitz, 1983).

20/ If there are endogenous variables that are monotonically related to the exogenous variables, clearly the contract can be indexed on these endogenous variables.
We assume that all workers are identical and have the utility function:

\[ U = \log(2 - h) + \beta \log c + (1 - \beta) \log m - \log \left\{ g^\beta (1 - \beta)^{1-\beta} \right\} \]  \hspace{1cm} (24)

where, for simplicity, we have assumed that the individual does not consume the exported good, but does consume an imported good, \( m \), the price of which we take to be our numeraire.

If the individual’s income is \( y \) (which, in the absence of lumpsum transfers will be \( w h \)), then he will spend a fraction \( \beta \) of his income on the non-traded consumption good, and \( 1 - \beta \) on the imported good, so that his indirect utility function will be, substituting in (24):

\[ \hat{V}(q,y,h) = \log (2-h) + \log y - \beta \log q. \]  \hspace{1cm} (25)

If there are no constraints on labour supply, so that the worker can choose \( h \) freely, then he will maximize his utility when \( h = 1 \). Since his unconstrained labour supply is independent of prices and wage rates, it is easy to identify full employment in this model as \( h = 1 \). In the unconstrained case the worker’s indirect utility function is just

\[ \hat{V} = \hat{V} = \log w - \beta \log q. \]  \hspace{1cm} (26)

This utility function has properties which greatly simplify the analysis. In particular, its additive separability makes welfare analysis simple, whilst it continues to give a simple indirect utility function if the worker is constrained in the labour market. (These useful properties account for its widespread use in the analysis of constrained markets, e.g., Malinvaud [1977].) Finally, it generates unit price elastic demands which can be perfectly aggregated across individuals of differing incomes. Thus, if total wage income in the economy is \( Y \), the total demands for non-traded and traded goods are
C = \frac{\beta Y}{q}, \quad M = (1 - \beta)Y; \quad Y = \alpha x N_x + \omega h N_c. \quad (27)

The critical assumption which generates unemployment is the restriction on the set of admissible contracts. There is a single, exogenous source of randomness in our economy, and hence it is natural to index wages on \( p \). But the set of contracts is restricted to those in which wages are a linear function of price:

\[
\frac{w(s) - \bar{w}}{\bar{w}} = (1 - \rho) \frac{p(s) - \bar{p}}{\bar{p}}, \quad 0 < \rho < 1. \quad (28)
\]

Here the export price is \( p(s) \) in state \( s \), \( \bar{p} \) is its mean value, whilst \( \bar{w} \) is the mean non-traded sector (or urban) wage, and \( \rho \) is a measure of wage rigidity. (Equivalent, \( 1 - \rho \) measures wage flexibility.) If \( \rho = 0 \), then urban wages will be identical to agricultural wages (from (2)) and hence, obviously, utility levels will be equated, as required by (3). If \( \rho = 1 \), urban money wages are perfectly inflexible, and always equal to \( \bar{w} \). In Figure 1 the level of unemployment generated in state \( s \) by a given urban wage \( w(s) \) is shown to be determined by the demand schedule (which itself depends on the state \( s \)) and the supply schedule, which, given the enforceability constraint, is equal to the wage cost in low demand states. Figure 1 shows the equilibrium for a representative firm with capacity employment of one man.

We will consider how the economy equilibrates for any given choices of \( \rho \), the degree of rigidity in equation (28). There are two possibilities. Either the range of values of \( \rho \) is such that unemployment of the type shown in Figure 1 never occurs, or periodic unemployment are the same, but the form of the equations will differ. We consider first the simpler case of full employment.
3.1 Full Employment Equilibrium.

In Appendix 1, we solve explicitly for the equilibrium values of the economy. If there is full employment, we have already argued that \( h = i \).

Since the utility function of (25) is separable, the condition that workers' expected utility be the same in non-traded and export sector becomes (using 28)

\[
E \log \bar{w} \left[ \rho + (1 - \rho) \frac{p(s)}{\bar{p}} \right] = E \log w(s) = E \log p(s). \tag{29}
\]

Once \( p \) has been fixed, this fixes \( \bar{w} \) as a function of the distribution of export prices, \( p(s) \). Knowing \( \bar{w} \), one can solve for all the other variables in the model.
Welfare Analysis at Full Employment. If the possibility of unemployment can be ignored, there is an intuitive argument why the optimum degree of wage flexibility, \( 1 - \rho^* \), should be roughly \( \beta \), the share of the traded good in consumers' expenditure. The argument is that the non-traded good price responds to income, which fluctuates with the price of exports. Urban workers will therefore want higher money wages when non-traded goods are expensive. If their income rises by \( \beta \) times the rise in export prices, and if non-traded goods prices rose as much as export prices, then their real purchasing power would be maintained, and hence real income stabilized.

This argument is essentially correct, and in Appendix 1 the optimum degree of wage flexibility is shown to be

\[
1 - \rho^* \approx \beta \frac{\bar{w}(1 - \beta) + rv}{\bar{w}(1 - \beta^2) + rv} \sim \beta
\]

where \( r \) is the required average rate of return of (7), and \( v \) is the capital-output ratio of (1).

Expected welfare increases as real income becomes more stable, and a graph of expected welfare, \( W \), against wage rigidity, \( \rho \), is as shown in Figure 2, which is drawn on the assumption that there is no unemployment — an assumption which is explored below.

The form of the dependence of welfare on wage flexibility is easily analyzed once it is appreciated that since all workers enjoy the same average level of utility in equilibrium, it is only necessary to study the average welfare of an agricultural worker. From equation (25) this is

\[
W = EV = E \log p(s) - \bar{E} \log q
\]

The effect of wage rigidity on welfare can be found by differentiating (31) totally with respect to \( \rho \):
The algebraic details of evaluating equation (32) are provided in Appendix 1, which shows that

\[
\frac{dW}{dp} = \frac{dEV}{dp} = E \log p(s) - \beta E \frac{1}{q} \frac{dq}{dp} = - \beta E \frac{1}{q} \frac{dq}{dp}.
\]  

(32)

as shown. However, as \( \rho \) is increased towards 1, so the variable cost of production of the non-traded good rises relative to the demand price in the lowest demand state of the world, until at some critical value \( \hat{\rho} > 0 \), unemployment first occurs in this state, as shown in Figure 1. For \( \rho > \hat{\rho} \), the equations on which the formulae were derived become invalid, and
if \( p^* > \hat{\rho} \), it becomes necessary to ask what effect unemployment has on the desirable level of wage rigidity.

The results of this section may now be summarized:

**Proposition 3.** If the variability in price is sufficiently small that there is always full employment, then there is an optimal degree of wage flexibility, approximately equal to the share of the non-traded good in consumption.

3.2 Equilibrium with Periodic Unemployment.

Suppose that there are a finite number, \( S \), of values that the export price may take, each occurring with strictly positive probability, with \( p(1) < p(2) < \ldots < p(s) \). Assuming that \( rv > 0 \), it follows that if the degree of rigidity, \( \rho \), is high enough, but not too high, there will be unemployment in state 1, but in no other state, as shown in Figure 1. We denote by \( \hat{\rho} \) the critical value of \( \rho \) at when employment first appears. In Appendix 1 we show that

\[
\hat{\rho} = \frac{1}{1 - \beta} \frac{rv}{w} \left( \frac{1 - \sigma}{\sigma} \right), \quad \sigma = 1 - \frac{p(1)}{\hat{p}},
\]

where \( \sigma \) is defined as the proportional downside range of \( p \).

For the optimum degree of wage rigidity to imply periodic unemployment, \( \hat{\rho} < \rho^* \), where \( \rho^* \) is approximately \( 1 - \beta \). Clearly, this is possible if \( \beta \) (the consumer's expenditure share on the non-traded good), is small, and \( \sigma \) is large.

**Welfare Analysis with Unemployment.**

Suppose that, ignoring unemployment, the desired level of wage rigidity is \( \rho^* > \hat{\rho} \), the level at which unemployment first occurs, so that welfare increases as \( \rho \) is increased to \( \rho \), as shown in Figure 3 below. The crucial question is what happened to the slope \( dW/d\rho \) for values of \( \rho \) above \( \hat{\rho} \).
Two possibilities are shown. Curve A has

\[
\frac{dW}{d\rho} \bigg|_{\rho^+} > 0
\]

in which case the optimum degree of wage rigidity implies some unemployment, whilst curve B has

\[
\frac{dW}{d\rho} \bigg|_{\rho^-} > 0, \quad \frac{dW}{d\rho} \bigg|_{\rho^+} < 0,
\]

implying that the optimum degree of wage rigidity is \( \hat{\rho} \), the point at which unemployment is just about to occur. To establish which case occurs it is necessary to compute \( dW/d\rho \bigg|_{\rho^+} \). Whilst it is difficult in general to
sign \( \frac{dW}{dp} \bigg|_{\rho^+} \), it can be shown that the possibility of unemployment makes wage rigidity uniformly less attractive, in the following sense:

\[
\frac{dW}{dp} \bigg|_{\rho^-} - \frac{dW}{dp} \bigg|_{\rho^+} > 0. \tag{35}
\]

That is, angle \( \Theta \) in Figure 3 is positive, and curves A and B both lie below the dotted curve, which ignores the unemployment constraint, as shown in Figure 3.

There is one condition in which it is possible to sign \( \frac{dW}{dp} \bigg|_{\rho^+} \), for it is obvious that unemployment is never desirable in a two-state world. If there are only two states of the world, there need only be two wages; equivalently, a linear schedule which fixes the two parameters \( \rho \) and \( \bar{w} \) allows wages to be any desired function of the states. This explains the special results obtained in Newbery and Stiglitz [1981, Ch. 26], in which unemployment was never optimal.

However, the two state case is very misleading, for any realistic description of uncertainty would recognize many more than two states. The next section shows that with four states and linear contracts (which are now really restrictive) periodic unemployment may be implied by the optimal contract.

3.3 The Desirability of Intermittent Unemployment

If the implicit wage contract is constrained to be a simple linear function of the state (the export price) as in (28), then, provided there are more states than parameters in the wage function, it is possible that the best such contract will generate periodic unemployment. One such example has a symmetric four point distribution for export prices:

\begin{align*}
\rho(1) &= 1 - \sigma \\
\rho(2) &= 1 - \sigma/2 \\
\rho(3) &= 1 + \sigma/2 \\
\rho(4) &= 1 + \sigma
\end{align*}

with probability

\begin{align*}
\Pi &\quad \frac{1}{2} - \Pi \\
\Pi &\quad \frac{1}{2} - \Pi \\
\Pi &\quad 0 < \Pi < \frac{1}{2}
\end{align*}
where, for convenience, the mean export price, \( \bar{p} = E p(s) \), is taken as unity. The model can now be solved once \( \beta, rv/\bar{p}, \sigma \) and \( \Pi \) are specified, where \( \beta \) is the share of non-traded goods in consumption, and \( rv/\bar{p} \) is, roughly, the average mark up on prime costs in the non-traded good sector. Table 1 summarizes the main results, and is to be interpreted as follows. Column 5 gives the coefficient of variation (CV) of export prices implied by \( \sigma \) and \( \Pi \), and column 10 measures the welfare of the economy assuming \( \rho = 0 \), that is, with no wage rigidity in the urban sector. The measure is taken as that riskless income which yields the same utility as the expected utility, expressed as a percentage of the income which would be received if export prices were perfectly stabilized as their mean price. Thus, looking across row A, with no wage rigidity, but a CV of export prices of 35%, the economy achieves 94.81% of the riskless level; or, put another way, the cost of the export price instability is 5.19% of average income. (The method of calculating these welfare measures is explained in Appendix 2.) Column 6 gives the value of \( \rho \) at which welfare is maximized, ignoring the possibility of unemployment, while column 11 gives the welfare in that case. Again, looking across row A, welfare rises to 95.46%, or the loss is reduced to 4.54%. However, the critical value of \( \rho \) for which unemployment occurs, \( \hat{\rho} \), is given in column 7, welfare there is given in column 12, and given the parameters of Table 1, \( \hat{\rho} \) cannot be achieved without unemployment. The optimum wage rigidity allowing for unemployment, \( \rho_u \), is given in column 8, the associated level of unemployment in state 1 is given in column 9, while the welfare associated with \( \rho_u \) is given in column 13. Except for case D, where the optimum level of unemployment is zero (i.e., \( \rho_u = \hat{\rho} \)), the last column strictly exceeds column 12 even where the two figures are shown identical.

The table shows what should be intuitively clear from the argument. As the risk of unemployment falls (as \( \Pi \) falls) so the optimum degree of wage rigidity rises and unemployment rises. (Average unemployment, \( \bar{\Pi} \), may rise and then fall.) As \( \sigma \) increases, the optimum level of unemployment in state 1 also rises (compare A and E, B and F). As the profit mark up falls (\( rv/\bar{p} \)) the optimum level of unemployment rises because it is more likely that unemployment will occur for given wage rigidity (compare \( u^* \), \( \rho_u \) for F and G.) As the share of non-traded goods, \( \beta \), falls, so optimal unemployment...
## Table 1: Sensitivity Analysis of Optimal Wage Rigidity

<table>
<thead>
<tr>
<th>Inside Range of Worst State</th>
<th>Profit Mark Up</th>
<th>CV of Price, p</th>
<th>CV of C-Good Share, β</th>
<th>Critical Rigidity, ρ*</th>
<th>Optimum Rigidity, ρ</th>
<th>Unemployment, u</th>
<th>Welfare, W, as a function of rigidity, ρ</th>
<th>Welfare, W, as a function of unemployment, u</th>
<th>Equivalent Riskless Income (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.15</td>
<td>0.25</td>
<td>0.2</td>
<td>35</td>
<td>0.8</td>
<td>0.35</td>
<td>2</td>
<td>94.81</td>
<td>95.46</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>0.25</td>
<td>0.2</td>
<td>32</td>
<td>0.8</td>
<td>0.40</td>
<td>6</td>
<td>95.68</td>
<td>96.23</td>
</tr>
<tr>
<td>0.5</td>
<td>0.025</td>
<td>0.25</td>
<td>0.2</td>
<td>27</td>
<td>0.8</td>
<td>0.69</td>
<td>22</td>
<td>96.99</td>
<td>97.38</td>
</tr>
<tr>
<td>0.8</td>
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<td>0.25</td>
<td>0.2</td>
<td>59</td>
<td>0.7</td>
<td>0.08</td>
<td></td>
<td>80.49</td>
<td>82.58</td>
</tr>
<tr>
<td>0.8</td>
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<td>0.25</td>
<td>0.2</td>
<td>55</td>
<td>0.7</td>
<td>0.09</td>
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<td>83.57</td>
<td>85.37</td>
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<td>0.7</td>
<td>0.13</td>
<td>12</td>
<td>86.71</td>
<td>88.20</td>
</tr>
<tr>
<td>0.8</td>
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<td>0.1</td>
<td>0.2</td>
<td>51</td>
<td>0.7</td>
<td>0.03</td>
<td>20</td>
<td>86.71</td>
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<tr>
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<td>0.32</td>
<td>16</td>
<td>96.55</td>
<td>97.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.05</td>
<td>0.25</td>
<td>0.1</td>
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<td>0.9</td>
<td>0.29</td>
<td>20</td>
<td>96.13</td>
<td>96.40</td>
</tr>
</tbody>
</table>

Computer Simulation
rises, though the effect of the optimum rigidity \( \rho_u \), is ambiguous. The reason for this last effect becomes clear in Figure 3. As \( \beta \) falls, \( \rho^* \) rises, but \( \rho \) falls, moving \( \rho \) to a steeper part of the graph and raising \( |dW/d\rho| \).

Thus, it may be desirable to increase wage rigidity to the point at which periodic unemployment ensues, and this is the more likely the smaller the risk of low demand \( (\Pi) \), the greater the dispersion of export price \( (\sigma) \), the smaller the profit share in the non-traded good sector \( (rv/p) \), and the smaller is the fraction of income spent on the non-traded good \( (\beta) \).

**A Simple Model of Inflation and the Phillips Curve**

So far we have assumed that urban money wages are at least partially flexible. The standard Keynesian assumption is, however, that they are sticky downwards, though there is nothing to prevent urban money wages being bid up in periods of high demand. For real wages to be flexible in such an economy, it is necessary to devalue when export prices are low in order to raise the domestic price level and hence reduce real wages. If the government is willing to do this, then all traded goods prices will rise by the extent of the devaluation (for a small country), and the economy can behave exactly as before in real terms. In the simplest case, suppose there are only two states of the world, with either a high export price or low price. Then, letting \( w_h \) denote wages in the good state, \( w_m \) in the bad state, traded goods prices will rise by a factor \( w_h/w_m \) whenever world prices fall. The price level will follow a simple random walk with drift at rate

\[
\dot{\pi} = \Pi(w_h/w_m - 1)
\]

per period. The greater the ratio \( w_h/w_m \), the higher will be the average rate of inflation \( \dot{\pi} \). In other words, the more flexible real wages are, or the more responsive they are to demand, the higher the rate of inflation, but the lower the level of urban unemployment. Conversely, the more rigid real wages are, the higher the rate of unemployment, but the price level now does not need to
be raised so much when export prices fall, so inflation is lower on average. In short, there is a trade-off between the average rate of inflation, \( \bar{\phi} \), and the average rate of urban unemployment \( \bar{u} = \bar{u}_h \).

For example, if there are just two states in which \( p = 1 + \sigma \) with equal probability, then, if \( \sigma = 0.5 = \beta \), \( rv_p/\bar{p} = 0.25 \), the average rate of inflation falls from 28% p.a. at zero unemployment to 8.7% p.a. at an average level of urban unemployment of 10%, and zero inflation at the maximum feasible level of average urban unemployment, \( \bar{u} = 15\% \) (where \( w_h = w_m \)). The trade-off is essentially linear over the whole range with slope about -1.9.

IV. CONCLUSIONS

The central thesis of this paper has been that the analysis of the consequences of implicit contracts between risk averse workers and risk neutral firms must take into account three central aspects of contract design: (a) the contracts are implicit, rather than explicit, and this restricts the terms of the contract which can be enforced; (b) the terms of the contract can only be made contingent upon variables which are observable; and (c) there are limitations on the degree of complexity of contracts; the provisions of even explicit contracts are seldom contingent upon all the observable, potentially relevant, variables in the economy. In particular, in labor contracts, we seldom see the kind of complex indexing of contracts on publicly available price and quantity data that an economic theory which ignored the "complexity constraint" would predict.

Our analysis has three central results. First, with perfectly flexible contracts, neither the constraints on enforceability nor the limitations on observability lead to unemployment in long term contractual arrangements. Second, even with perfect enforceability and observability, limitations on the flexibility of contracts may lead to unemployment. Third, even when firms sign optimal contracts with their workers, i.e. contracts which, given market parameters such as wage and price distributions and the level of unemployment, maximize the expected profits of the firm, subject to the worker attaining a given level of expected utility, the market equilibrium is not Pareto efficient. The source of the inefficiency, the failure of each firm to take into account the changes in risks which face consumers and
workers in other firms, resulting from the changed probability distribution of prices and employment, is quite distinct from the kinds of inefficiencies which have been noted elsewhere in the literature. 21/

Our analysis shares one deficiency with that of most of the other literature on implicit contracts: it explains underemployment but not unemployment; it is consistent with work-sharing, but not with lay-offs. There are two sorts of arguments why firms prefer to lay off workers rather than to engage in work sharing. The first identifies some non-convexity in the production process, which implies, for instance, that it is more efficient to have one worker work 40 hours a week, rather than two workers work 20 hours each. But this is only an explanation for the time unit of work sharing: if there n machines and 2n workers, obviously, they should not rotate every 30 seconds; the optimal rotation may be a week, or a month, but it unlikely that the non-convexities are such that no rotation is desirable. 22/ The second we refer to broadly as "efficiency wage arguments". The wage (or more generally, the terms of the contract) affects the net productivity of the labor force (either by affecting the quality mix of those who apply or of those who leave; the quit rate and hence the turnover costs, or the effort level.)

The problem is that efficiency wage considerations can lead to unemployment, even without implicit contracts. Does that mean that the theory of implicit contracts is redundant? We think not. We suspect that there are circumstances in which, in the absence of implicit contracts, efficiency wage considerations by themselves would not lead to unemployment. Implicit contracts explain the presence of unemployment, efficiency wage considerations explain the form which the unemployment takes.

21/ E.g. the search externalities associated with the work of Hosios (1981) and Diamond (1981), the effort incentive externalities, associated with the work of Shapiro and Stiglitz (1982), the more general moral hazard externalities, (Arnott and Stiglitz, 1982) or the informal externalities (Greenwald and Stiglitz, 1983).

22/ An exception would arise if there were significant learning by doing effects.
Appendix 1
Derivation of Formulae of Part III

Full Employment Equilibrium

In this case efficiency (1) and the zero profit constraint (7) together with \( h = 1 \) (full employment) and \( y = w \) imply

\[
C = N_c = K/v, \tag{A1}
\]

\[
Eq - Ew = rK/N_c = rv. \tag{A2}
\]

The market clearing price for consumption goods can be derived by equating the demand schedule (27) to supply:

\[
q = \frac{\beta Y}{C} = \frac{\beta(N_x w + N_c w)}{N_c} = \beta \left( \frac{N_x}{N_c} \tilde{p} + w \right) \tag{A3}
\]

where \( Y \) is aggregate income and \( w_x = \tilde{p} \). This may be substituted in (A1) to give

\[
rv = \beta \frac{N_x}{N_c} \tilde{p} - (1 - \beta) \tilde{w}. \tag{A4}
\]

Given \( \tilde{w} \), since \( \tilde{p} \) and \( rv \) are exogenous, the allocation of labour between sectors, \( N_x/N_c \), is then determined. Equations (A3) and (A4) can be solved for \( q \) in each state, \( s \):

\[
q(s) = \alpha \tilde{w} + \left[ (1 - \alpha) \tilde{w} + \frac{rv}{p} \right] \frac{p(s)}{p}, \quad \alpha = \beta p. \tag{A5}
\]

The Optimal Degree of Wage Rigidity Assuming Full Employment

Equation (32) can be solved using (A5)

\[
\frac{dq}{dp} = \left[ \alpha + (1 - \alpha) \frac{p(s)}{p} \right] \frac{d\tilde{w}}{dp} - \beta \tilde{w} \frac{p(s)/\tilde{p} - 1}{\tilde{p}}, \quad \alpha = \beta p, \tag{A6}
\]

where, in equilibrium, \( \tilde{w} \) is a function of \( p \) from (29):
\[
\frac{d}{dp} E \log w(s) = \frac{d}{dp} E \log p(s) = 0
\]

or

\[
\frac{1}{w} \frac{d}{dp} w = -E \left[ \frac{1 - p(s)/\bar{p}}{\rho + (1 - \rho) p(s)/\bar{p}} \right] < 0 \quad \text{for } 0 < \rho < 1,
\]

(A7)

confirming the intuition that increased wage rigidity (higher \( \rho \)) generates risk benefits which permit a lower average wage, \( \bar{w} \). Equation (A7) can be substituted in (A6) and then into (32) to give the effect on average welfare, which will depend on the value of \( \rho \). At \( \rho = 0 \) we have:

\[
\frac{dW}{d\rho} = \frac{2}{\bar{p} + rv} \left[ E \left( \frac{\bar{p} - \bar{p}}{\bar{p}} \right) + \beta E \left( \frac{\bar{p} - \bar{p}}{\bar{p}} \right) \right] > 0,
\]

demonstrating that it is desirable to offer some income insurance by increasing wage rigidity starting from completely flexible wages. However, at \( \rho = 1 \), corresponding to complete rigidity,

\[
\frac{dW}{d\rho} = \frac{2}{\bar{p} + rv} \left[ \frac{\bar{p} - \bar{p}}{\bar{p}} \right] < 0.
\]

The graph of welfare against \( \rho \) thus looks like Figure 2, with the optimum degree of wage rigidity, \( \rho^* \), strictly between 0 and 1. At the optimum \( dW/d\rho = 0 \), or, from (A6)

\[
E \frac{1}{q} \frac{dq}{d\rho} + E \frac{\bar{w}}{q} \left[ \frac{1 + (1 - \alpha)v}{1 + (1 - \rho)v} + \beta v \right] = 0,
\]

(A8)

where

\[
v = \frac{\bar{p} - \bar{p}}{\bar{p}}, \quad q = \bar{q} \left[ 1 + (1 - \frac{\alpha \bar{w}}{\bar{w} + rv}) v \right].
\]
Equation (A8) can be solved for $p$ once the distribution of $p$ (or $u$) is specified, and can be approximately solved by expanding about $u = 0$ and ignoring higher terms than $u^2$ to give (30).

**Equilibrium With Periodic Unemployment**

If workers are constrained to work $h < 1$ in state 1 and there is perfect work sharing, then urban utility is given by (25). The unemployment rate is then $1 - h = u$, and the urban wage which equates expected utilities is given (compare (29)) by

$$E \log(1 - u^2)\bar{w} = E \log(1 - u^2)\bar{w}[\rho + (1 - \rho)p(s)/\bar{p}] = E \log \bar{p}. \quad (A9)$$

The allocation of labour between the sectors is found as follows. In state 1 there is unemployment and hence excess capacity, as shown in Figure 1. Hence the price falls to the variable cost, $q(1) = w(1)$. Labour input is $hN_c = (1 - u)N_c$, where $N_c$ is the number of urban workers. Equation (A3) gives the non-traded goods price in state 1 as:

$$q(1) = \beta \frac{N_x p(1)}{N_c (1 - u)} + \beta w(1) = w(1) \quad (A10)$$

or

$$\frac{\beta N_x}{N_c} = (1 - \beta)(1 - u) \frac{w(1)}{p(1)}. \quad (A11)$$

If we define the (proportional) downside range of $p$ as $\sigma$ then

$$w(1) = \bar{w}[1 - (1 - \rho)\sigma], \quad \sigma \equiv 1 - p(1)/\bar{p}. \quad (A12)$$

We can now calculate the critical value of rigidity, $\hat{p}$, at which unemployment first appears in state 1. From equations (A2) and (A11) if $u = 0$

$$\frac{\beta N_x}{N_c} = \frac{rv + (1 - \beta)\bar{w}}{\bar{p}} = (1 - \beta) \frac{\bar{w}}{\bar{p}} \frac{1 - (1 - \hat{p})\sigma}{1 - \sigma} \quad (A13)$$

which gives (34).
The average rate of profit must again be \( rv \) per unit of output but this time profit is only positive in states \( s = 2, 3, \ldots, S \). Thus (A4) becomes

\[
rv = \frac{N}{N_c} \sum \frac{1}{N} \Pi(s)p(s) - (1 - \beta) \sum \frac{S}{2} \Pi(s)\omega(s)
\]

(A14)

where \( \Pi(s) \) is the probability of state \( s \) occurring. If \( \Pi (= \Pi(1)) \) is the probability of state 1 occurring, then, from (A22) and (A23)

\[
\sum_{s=2}^{S} \Pi(s)p(s) = \bar{p} - \Pi p(1) = (1 - \Pi)\bar{p}(1 + \gamma_0), \quad \gamma = \Pi/(1 - \Pi),
\]

\[
\sum_{s=2}^{S} \Pi(s)\omega(s) = \bar{\omega} - \Pi \omega(1) = (1 - \Pi)\bar{\omega}[1 + (1 - \rho)\gamma_0].
\]

Substituting these and (A11-12) into (A14) gives

\[
rv = (1 - \Pi)(1 - \beta)\bar{w} \left[(1 - u)\left[1 - (1 - \rho)\sigma\right] \frac{1 + \gamma_0}{1 - \sigma} - \left[1 + 1 - \rho\right] \gamma_0\right].
\]

(A15)

Equation (A9) can be written

\[
\Pi \log(1 - u^2) + \log \bar{w} + \sum \log \left[\rho + (1 - \rho)p(s)/\bar{p}\right] = E \log p(s).
\]

(A16)

Equations (A15) and (A16) can then be solved for \( \bar{w} \) and \( u \) given \( \rho \) and the probability distribution of export prices, \( p(s) \). For the numerical solutions presented in Table 1, \( \hat{p} \) is first calculated from (34) and (29) (by an iterative algorithm). Unemployment, \( u \), is increased from zero in 2% steps, and for each step, \( \rho \) is found from (A15) and (A16) (again by an iterative algorithm). Finally, welfare is evaluated for each pair \( \{u, \rho(u)\} \) to locate the optimum degree of rigidity and associated unemployment rate.
Appendix 2

Welfare Measures

We would like a monetary measure of the costs of export price instability and of the benefits (or costs) of wage rigidity. One natural method is to take as benchmark the welfare enjoyed if export prices are stabilized at their mean, but workers pay a lump sum equal to a fraction $L$ of their income, $w$, to enjoy the advantages of stabilized prices. Normalize so that the (mean) export price $\bar{p}$ is unity, as are wages. $w$, then if $W$ is the expected utility we wish to measure by $L$, from equations (25) and (A2)

$$W = V(\bar{p}, \bar{q}, \bar{p}_0, -L) = 2 \log (1 - \frac{L}{2}) - \beta \log \bar{q}, \quad \bar{q} = 1 + rv/\bar{p}.$$ 

This can be written as

$$L = 2(1 - \sqrt{\psi}), \quad \psi \equiv \exp \left\{ W + \beta \log (1 + rv/\bar{p}) \right\}. \quad (A30)$$

In this form, $L$ measures the cost of risk as a fraction of original average income. Alternatively, $W$ can be measured by $1 - L$, and this is the measure adopted in Table 1.
References


More, J. (1982), "Optimal Labour Contracts when Workers have a Variety of Privately Observed Reservation Wages", (mimeo, Birkbeek College).


