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ON THE PROGRESSIVITY OF COMMODITY TAXATION

by

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December 1986

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ON THE PROGRESSIVITY OF COMMODITY TAXATION*

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This paper presents a simple method for analyzing the distributional effects of marginal changes in commodity taxes. The distributional effect of a tax is measured by its impact on the Gini coefficient of the overall distribution of real income. Then a decomposition of the Gini coefficient is carried out in a way which resembles the analysis of variance. As a result of this decomposition, non-parametric estimates of income elasticities of the Engel curves emerges. The magnitudes of these income elasticities determine the desired changes in the taxes. The method is illustrated with data from Israel.
Subsidies for basic commodities play an important role in income maintenance programs of many developing countries. Other commodity taxes are an important source of revenue. These subsidies and taxes also affect the consumer price index. Hence in periods of rapid inflation there is a tendency to increase these taxes (or subsidies) beyond the original targets, in order to curtail the increase in the consumer price index (subsidies) or as a mean for decreasing the deficit in the budget. Sometimes the changes in the subsidies and the taxes are not planned. Since administered prices are quoted in nominal terms, inflation or changes in prices abroad affect the magnitude of the subsidy or the tax. Moreover since economic growth and changes in relative prices affect consumption patterns, it may happen that the subsidies do not achieve their intended distributional goals.

For these reasons a reexamination of the structure of commodity taxation is called for. Economic theory teaches us that we have to examine three aspects of the problem: the effect on revenue, the distributional effect and the inefficiency caused by the tax system. The revenue effect, of the taxes (subsidies) can be easily evaluated, because it is written in the budget or, in the case of administered prices, it can be derived using data on total expenditure. The distributional effects are harder to evaluate because data on consumption patterns of different income groups are needed (a cross-section sample). Finally an evaluation of the efficiency cost requires knowledge of detailed price elasticities that hardly exist even in developed countries (a sample with variation in prices is needed, see Deaton (1984) for a discussion of the econometric issues). Since the availability of data differs among countries, a method of analysis that can be implemented in stages, has an advantage over a general method that requires unavailable data.
The aim of this paper is to suggest a method for analysis of indirect tax reforms that can be implemented in developing countries. It is based on analyzing the effect of small changes in taxes on the inequality in the distribution of real income in the society. The inequality is measured by the Gini or by the extended Gini coefficients. The use of several measures of inequality enables us to examine the robustness of the results with respect to different quantifications of inequality.

In the next section the literature on the analysis of commodity taxation is surveyed. In the second section the decomposition of the Gini coefficient according to income sources is presented, while in the third section the technique is adapted to handle changes in commodity taxes. The forth section suggests alternative interpretations to the main coefficients while in the fifth and sixth sections the method is illustrated with data from Israel.

I. Related Methods

There are three basic methods which are available for the analysis of commodity tax reforms. The first is referred to here as the textbook approach, the second is Deaton's method for calculating optimal commodity tax-rates and the third is the Shoven-Whalley general equilibrium approach.

The textbook approach can be summarized by the following common wisdom: if the income elasticity of a commodity is greater than one then a subsidy is regressive while a tax is progressive and the higher the income elasticity the more regressive is the subsidy. Furthermore, the lower the
price elasticity the lower will be the efficiency cost. This approach encounters several problems: First of all, the income elasticity may vary along the Engel curve so that it is not clear at what point should we calculate the income elasticity. A common procedure is to calculate the income elasticity at the mean income. While this may be an appropriate procedure in situations where the main interest is in estimating changes in demand, it is not clear what are the justifications for such a procedure in welfare economics. Assume that income elasticity is declining with income. Then an elasticity which is calculated at the median income, overestimates the elasticity for fifty percent of the population and underestimates it for the other half of the population. To use an elasticity at the mean income means that a higher weight is given to rich individuals. In a typical income distribution, mean income lies in the sixth or seventh decile, hence the income elasticity at the mean income is an understimation of the income elasticity for more than sixty percent of the population. It seems to me that an equal treatment approach would suggest calculation of the elasticity at median income (or at the mode) rather than at the mean income. Moreover, even if we have agreed on the magnitude of income elasticities, it is not clear how to evaluate the effect of the specific tax rate. In many developing countries consumption consists both of home produced commodities and of commodities that are purchased in the market. Taxes only affect commodities purchased in the market, hence the relevant income elasticity, the elasticity of the money-expenditure, need not be the same as the elasticity of consumption. In sum, this method provides us intuitive guidelines rather than a specific formula to work with.

The second method that can deal empirically with the problem of commodity taxation is the Shoven-Whalley general equilibrium approach. In this
method, the overall population is divided into several representative consumers' and producers' groups, and the equilibrium prices are calculated. When taxes are changed, equilibrium prices are recalculated and changes in the distribution of utility determined. This approach seems to be the best approach, provided that the required data exist and that the parameters representing each group are reliable. It seems to me that these requirements exclude successful implementation of this method for most less developing and developed countries.

The third method is presented in Deaton (1977). Deaton assumes that the Engel curves are linear while the social welfare function is of the Atkinson (1970) type. These two assumptions enable Deaton to present simple formulas for deriving optimal commodity tax rates. Although Deaton's approach is elegant, its weakness is the strong assumptions that are required. One of the basic empirical findings, is the law of declining marginal propensity to spend, especially when dealing with basic commodities. When analyzing consumer demand systems, Deaton and Muellbauer (1980) preferred the semi logarithmic function to represent the Engel curve while De-Witte and Cramer (1986), Aasness and Rodseth (1983) argue that more complicated forms for modeling the Engel curve are required. The argument that linear Engel curves are just a first order approximation does not carry much weight here because the global behavior of the Engel curve is essential to the determination of the income elasticity and hence to the optimal tax rates. Therefore, even if we accept the social welfare function as the one which represents the country we are dealing with, we still may end up having calculated the optimal tax rates for a different economy than the one we are interested in, and since the assumption of linearity of the Engel curve is essential for obtaining a
solution, there is no way to find out what is the impact of our assumptions on the optimal tax rates.

Ahmed and Stern (1984) suggested a simpler version of Deaton's approach. Instead of calculating optimal taxes they restricted their interest to finding out the effect of marginal changes in taxes and subsidies on welfare and efficiency. Alternatively, they are interested in the direction of the tax reform and not in its magnitude. This simplification enables them to abandon the assumption on linear Engel curves.

The approach taken in this paper is to suggest a method with modest objectives. Its goal is not the determination of optimal commodity taxes, but in finding the effect of a change in commodity taxes (subsidies) on the distribution of real income. This procedure can be viewed as one component of the problem of optimal taxation. The specific question to be answered is the following: If the government changes one or several commodity taxes, what will be the distributional impact? This question is relevant whenever there is a need to increase revenue and the question posed is which commodity or commodities should be taxed, or which subsidy should be reduced if we care about the distributional aspects of the tax change. In this sense the method
can be viewed as adding operational content into the textbook approach. Provided that we have the required data, the approach can be extended to include efficiency considerations, and furthermore, to calculate optimal commodity taxes. However, this line will not be pursued in this paper. The basic idea is to evaluate the distributional impact of an additional dollar of revenue raised by commodity tax. The distributional impact is evaluated by the resultant change in the Gini coefficient (or the extended Gini coefficient) of real income. The Gini coefficient is a popular measure of inequality, and its use may also be justified by two additional arguments. The first is that it is a quantitative measure which may represent two alternative approaches: the theory of Relative Deprivation and Yaari's (1986) principle of just taxation. The second kind of arguments are based on statistical properties of the estimates. As we show later the effects of taxes on the Gini coefficient are based on nonparametric estimates of the income elasticity of the monetary expenditure on the commodity. Moreover, calculations of the effects of taxes on the extended Gini can be viewed as a sensitivity analysis of the nonparametric income elasticity estimators.

The method enables us to estimate the effect of taxes on the Gini coefficient or alternatively, to find out changes in taxes which keep the (extended) Gini unchanged, while raising revenue. The method is applicable to all kinds of indirect taxes. Hence we present first the application to taxes on incomes sources and later we adapt the method to commodity taxation. It is worth noting that there is a similarity between Ahmed and Stern (1984) approach and the method presented in this paper. The basic idea is identical, and the main difference is that the Gini coefficient is used instead of the Atkinson type welfare function. As we show later, this substitution enables
us to interpret the main coefficients in terms of textbook approach and at the same time simplifies the calculations.

II. Decomposition of the Gini Coefficient

In this section we rely heavily on the decomposition of the Gini coefficient as presented in Lerman and Yitzhaki (1985) and in Stark, Taylor and Yitzhaki (1986). Readers who are familiar with these papers can skip this section.

Let $y$ denotes total after tax family income while $x_i$ ($i=1, \ldots, n$) represent income sources such as labor income, capital income, direct taxes and so on. Assume that we have a cross-section sample so that these variables can be found for each family. Then, the extended Gini coefficient of total income can be calculated using the following formula.

$$G_y(v) = -v \frac{\text{cov}(y, [1-F_y(y)]^{v-1})}{m_y}$$

where $G_y$ is the extended Gini coefficient of total income $y$, $m_y$ represents mean income while $F_y(y)$ is the cumulative distribution of $y$ and $v>1$ is a constant which represents inequality aversion. (For derivation of this formula and a geometrical interpretation, see Lerman and Yitzhaki (1984). The properties of the extended Gini coefficient are described in Yitzhaki (1983)).

The role of $v$ can be seen by concentrating on the following extreme cases:
a: If \( v=1 \), then the extended Gini represents indifference to inequality.

b: If \( v=\infty \) then the extended Gini represents attitude toward inequality as depicted by the Rawlsian criteria, that is it represents the attitude toward inequality of someone who is interested in maximizing the income of the poorest in the society.

c: If \( v=2 \), then the extended Gini represents the ordinary Gini coefficient; which can be written as

\[
G_y = \frac{2 \operatorname{cov}(y, F_y(y))}{m_y}
\]

It can be shown that the difference between members of the extended Gini family is the weight which is attached to different segments of the income distribution. In this sense all members are similar in their properties. In order to simplify the presentation, only the decomposition of the Gini is presented. The decomposition of the extended Gini follows similar lines.

Utilizing the properties of the covariance, multiplying and dividing by \( m_i \) and \( \operatorname{cov}(x_i, F_i(x_i)) \) yields a decomposition of the overall Gini into:

\[
G_y = \sum_i S_i R_i G_i
\]

where \( G_i \) is the Gini coefficient of income component \( i \)

\( S_i \) is the share of income component \( i \) in total income.

\( R_i \) is the (Gini) correlation between income component \( i \) and total income, where \( R_i = \frac{\operatorname{cov}(x_i, F_y(y))}{\operatorname{cov}(x_i, F_i(x_i))} \)

The statistical and large sample properties of the term \( R_i \) are discussed in Schechtman and Yitzhaki (1987). For our purposes it is sufficient to mention the following properties:
a: $R_i$ is equal to 1 (-1) if $x_i$ is an increasing (decreasing) monotonic transformation of $y$.

b: $R_i$ is equal to zero if (in general) $x_i$ and $y$ are independent, or if $x_i$ is a constant.

c: $R_i$ is equal to Pearson's correlation coefficient if $x_i$ and $y$ are normally distributed variables. 2/

As can be seen from the properties mentioned above, the properties of $R_i$ are a mixture of Pearson's and Spearman's correlation coefficients.

Equation (3) states that the contribution of each income component to overall inequality depends on three parameters, the share of the component in total income, the Gini coefficient of the component and the Gini correlation of the component with total income. 3/

Our interest is in finding out the effect of small changes in income components on overall inequality. For this purpose let me assume that income component $i$ is multiplied by $(1+e)$ where $e$ can be interpreted as a tax or subsidy. Then we can write

$$x_i(e) = (1+e)x_i$$

and the overall Gini will be a function of $e$.

Then one can show (see Stark, Taylor and Yitzhaki (1986)) that

$$\frac{\partial G_y}{\partial e} = S_i R_i G_i - S_i G_y$$

and by measuring the change in relative terms:

$$\frac{(\partial G_y / \partial e)}{G_y} = S_i R_i G_i / G_y - S_i$$
Equation (4) states that the percentage change in the overall Gini caused by a small change in the subsidy (tax) on income component \(i\) is equal to the contribution of the component to overall inequality minus the contribution to overall income.

Equation (4) enables us to compare the effect on inequality of subsidies (taxes) with equal percentage rates that are imposed on different income sources. However, a fair comparison between subsidies should take into account the revenue effect on the government. Hence, we may be interested in the effect of a dollar spent on (collected from) income source \(i\). Since the effect on the budget is a linear function of the tax base all we have to do is to divide equation (4) by the tax base, \(S_i\). The effect on inequality of a dollar collected from source \(i\) is thus

\[
(5) \quad \left( \frac{\partial \bar{G}_i}{\partial x} / \bar{G}_i \right) S_i = R_i G_i / \bar{G}_y - 1
\]

Equation (5) says a tax on income source \(i\) is progressive or regressive depending on whether \(R_i G_i / \bar{G}_y\) is higher or lower than one. As we show later, this term can be interpreted as the income elasticity of income source \(i\), an interpretation which is consistent with the textbook approach of taxation.

III. Commodity Taxation

In the preceding section, we were interested in the change in inequality that is caused by raising a dollar of revenue from one income source. Our aim in this section is to apply the same methodology to commodity taxation. As will be shown, the effects are the same, although, the directions
are different: a commodity tax is equivalent to a subsidy on an income source and a tax on an income source is equivalent to a subsidy on a commodity.

Let $P_i, x_i$ represent prices and consumption of commodities while $y$ represents total net income. The budget constraint is:

$$y = \sum P_i x_i$$

Assume that the government imposes a small tax (or alternatively increases the tax) on commodity $i$. Then, the expenditure needed to buy the same quantity of the commodity is $P_i x_i (1+e)$ where $e$ is the ad-valorem tax. Since the change in the tax is small we are allowed to use first order approximations. Our interest is to calculate the effect of the tax on inequality. In order to answer this question let us consider the compensation needed to preserve the level of well being that each family enjoyed before the tax was imposed. If the compensation needed is progressive (it increases with income), then the tax affects the rich more than it affects the poor, so that the tax is progressive. This exercise is exactly the same as in the previous section excepts that the signs reverse. A tax on a commodity is equivalent (sign wise) to a subsidy to a component of income. Therefore we could repeat the derivation in the previous section.

Note that because we are looking at the derivative at a point at which the consumer is at an optimum, the Hicksian demand curve is the same as the Marshallian demand curve. So that for this case it does not matter whether we are looking at the Hicksian demand function (in order to compensate the individual so he will be at the same utility level), or whether a Slutsky compensation is proposed. 4/
IV. Alternative Interpretations of the Coefficients

Equation (5) shows the effect of a small change in a subsidy on real income inequality as reflected in the Gini coefficient. In this section we would like to show that one can interpret the coefficient in equation (5) as the elasticity of the Engel curve with respect to income, so that the distributional effect of a subsidy is related to the textbook argument.

Rewriting equation (5) in terms of covariances we get:

\[(6) \quad \frac{\partial G_y}{\partial e} / \frac{S_i \cdot C_y}{G_y} = b_i \cdot m_y / m_i - 1\]

where \(m_i\) is the mean of \(x_i\) and

\[(7) \quad b_i = \text{cov}(x_i, F_y(y))/\text{cov}(y, F_y(y)).\]

The term \(b\) is the key variable in equation (7). Some of its properties are presented in Yitzhaki and Olkin (1986), where it is termed the Gini regression coefficient. The rest of this section is devoted to several alternative interpretations of \(b\).

Let us start with a geometric presentation of \(b\). Consider a population which is ordered according to income and assume that the population is positioned in equi-distance spaces as in an honor guard. Each person is represented by his income and by his expenditure on the \(i\)th commodity. Then \(b_i\) is the ratio of slopes of two regression lines. The numerator is the (average) increase in expenditure for an increase in the rank, while the denominator is the increase in income for an increase in rank. That is \(b_i\) represents the marginal propensity to spend for an increase in rank.
Alternatively, $b_i$ can be interpreted as a weighted average of the marginal propensity to spend. As shown in the appendix,

$$b_i = \int w(F(y)) X'_i(y) \, dy,$$

where $X'_i(y)$ is the marginal propensity to spend on commodity $i$ and $w$ is the weight attached to this marginal propensity to spend. Formally

$$w(F(y)) = F(y)(1-F(y)) \int \frac{F_y(z)(1-F_y(z))}{f_y(z)} \, dz$$

Equation (9) shows that the highest weight is given to the median individual, and that the weights are symmetric in the individual's rank in the population. For example the weight given to the first quantile is equal to the weight attached to the third quantile. $\$\$

A third way of interpreting $b_i$ is based on nonparametric estimation of a regression coefficient as developed by Sievers (1983). Consider the following linear model

$$x = a + b \, y + e$$

and assume that the errors are independent with zero mean and a finite variance. However, the distribution of the error term is unknown. Sievers suggests and investigates large sample properties of the following estimator:

$$b = \sum_i \sum_j W_{ij} \left| \frac{x_i - x_j}{y_i - y_j} \right|$$

where

$$W_{ij} = \frac{|y_i - y_j|}{\sum_i \sum_j |y_i - y_j|}$$

is the weight attached to the marginal propensity to spend. That is the estimator of $b$ is the sum of all possible slopes calculated for any pair of $((x_i, y_i), (x_j, y_j))$ weighted by the distance between them. As shown in Stark and Taylor and Yitzhaki (1986) this is the discrete version of (8) for the
standard Gini. Therefore we can view b as a nonparametric estimator of the marginal propensity to consume. It is worth to note that there are major differences among the various interpretations of the coefficient. However, these differences are important only if we try to statistically test the significance of the estimates, a subject which is beyond the scope of this paper.

Having interpreted $b_i$ as a weighted sum of the marginal propensity to spend, the rest of the interpretation of equation (6) is quite simple: the first term on the right hand side of equation (6) is marginal propensity to spend on commodity $i$ divided by the average propensity to spend -- that is the income elasticity of commodity $i$. Therefore, the progressivity (regressivity) of a change in the tax on commodity $i$ depends on whether this elasticity is greater or lower than one. In this sense, we can argue that the proposed method gives operational meaning to the textbook approach. Note, however that while the textbook approach refers to the effect of taxes, the approach in this paper deals with marginal changes in taxes. One can ask what is meant by marginal changes? It is hard to answer such a question but the rule of thumb seems to be that marginal changes are changes that a first order approximation is sufficient in evaluating them.

Finally, two comments are worth mentioning: the suggested methodology can be carried out by using a simple regression program, one must be able to calculate the covariance and to find out the ranks of the households. In this sense, it can be viewed as a simple method that can substitute for simulation models in certain cases when the investigator is mainly
interested in marginal changes in the income distribution. Moreover, in order to carry out the suggested methodology, a knowledge of the existing tax system is not required (although, it will not hurt either). However, if one wants to carry out other simulations, such as finding out the effect of changing income tax function in a particular way then the distribution of the changes in the tax for each family should be known.

V. An Illustration with Israeli Data

Our aim in this section is to illustrate the methodology by using Israeli data. The data set is the Survey of Family Expenditure (1979/80) conducted by the Central Bureau of Statistics. This Survey consists of a sample of 2,271 households, and it covers the urban population in the country. Since we are interested in level of economic well-being, the concept of income per standard adult is used. 7/ The households are ordered according to net total income per standard adult, where net total income is defined as monetary income plus imputed income from ownership of housing and vehicles minus income and social security taxes. 8/

Table 1 presents the decomposition of the Gini coefficient with respect to to several income sources. As expected, the Gini coefficient of self employment and capital income is higher than the overall Gini, while the Gini of spouses' income is surprisingly high. 9/ The reason for this outcome is that nonworking spouses are considered as spouses with zero income. There is a simple formula which connects the overall Gini with a subgroup Gini when some of the population receives zero income. The formula is
\[ G = (1 - P) + P G^* \]

where \( G \) is the overall Gini, \( P \) is the proportion of population with nonzero income, and \( G^* \) is the Gini of the population with nonzero income. Using this formula, the Gini coefficient among working spouses is .281, lower than the overall Gini. Note also that the contribution of social security allowances is low relative to the contribution of the personal income tax.

Table 2 presents the effects of marginal changes in taxes on inequality. The column on the left hand side presents the effect of increasing (decreasing) a tax while the column on the right is the effect of equal revenue increases. A minus sign in these entries indicates that inequality declines if the income source is increased. By adding one to this column we get a nonparametric estimate of the income elasticity. The income elasticity of spouse wages is the highest (1.22) while the income tax elasticity is 0.8. It is worth noting that the effect of social security allowances on inequality is more than twice the effect of the income tax.

Table 3 presents the distributional effect of commodity taxes (subsidies). It is interesting to note that the (Gini) correlation between cooking oil and income, as well as between bread and income, are negative, while the correlation of the expenditure on other commodities is positively correlated with income. This implies that these commodities are inferior goods. The estimated income elasticities can be calculated by adding one to the last column. Two inferior goods can be identified, cooking oil and bread. The income elasticity of public transportation (.15) is relatively high although is lower than the income elasticity of eggs and milk (.25).
In table 4 the effect of a one dollar change in taxes on the above commodities is presented for various extended Gini parameters. These coefficients can be interpreted in two ways. The first one is assume that inequality is measured by the appropriate extended Gini. As we noticed earlier the higher $v$ the more we care about low income groups. The second way, is to view the extended Gini as a different scheme of weighting the marginal propensity to spend, and again the higher $v$ the higher the weight given to low income groups.

As can be seen from the table the income elasticities do not change in a dramatic way and we can argue that for a first approximation the income elasticities are robust. Note, however, that there is a tendency for the estimators to increase with $v$. For example, bread is more an inferior good the more we concentrate on low income groups. This is a reflection of the nonlinearity of the Engel curve. Note, on the other hand that the income elasticity of cooking oil does not change in a monotonic way which means that the marginal propensity to spend is not always declining with income.

VI. Policy Analysis

The previous section presented a detailed analysis of the interpretation that can be attributed to the coefficients. Economists who intend to use the method for approximating the effect of tax reform may ignore the different interpretations and concentrate the derivatives of inequality with respect to a dollar change in the tax revenue. He can approach the evaluation
in one of two ways: the first is to assume that we are interested in preserving the inequality in the society, and to deduct the revenue implications of the tax reform. The second is to assume that revenue should be kept constant and to deduce the implications for inequality. Since both methods are identical we will illustrate the first approach. Assume that we want to keep inequality constant, then we want to move along the equal inequality curve, the slope of which is:

\[
\frac{\partial e_i}{\partial e_j} = \left. \frac{\partial G}{\partial e_i s_i} \right|_{y = c^*} = - \frac{\partial G}{\partial e_j s_j}
\]

where the terms on the right hand are the derivative of the Gini with respect to a dollar change in each tax. Then we can compare the effect of subsidies and taxes on the margin. For the sake of convenience, the tax on the head of the household wage is chosen as a numeraire. Table 5 presents the amount of taxes on other sources of income of the household that will have the same effect on inequality. The second line in the table shows that 24 cents of tax on the spouses' income achieve the same effect on inequality as one dollar of tax on the wage income of the household's head. Alternatively, as a first order approximation we can say that a tax on spouses' income that raises four dollars and a subsidy on the head of household income which costs one dollar, would leave inequality unaffected. It is worth mentioning that such a policy would have other implications too. It will affect horizontal equity (since not all families have the same share of spouses' and head of households income) and it will affect the deadweight loss of the tax system. However, these considerations are not addressed in this paper. The rest of table 5 presents the effect of other instruments in the Israeli tax system.
Several important observations can be seen from the table: First, subsidy for cooking oil is the most important subsidy. Its effect is equivalent to the social security allowance to the second child in the family. This subsidy was the first subsidy to be removed in the reform of 1985. Another important allowance is the allowance for the elderly. As can be seen it is approximately equivalent to the allowance for the third child in the family. The reduction of this allowance was another target of the treasury in 1985.

The derivatives presented in table 5 can be used in order to evaluate distributional effects of tax reforms which involve several changes. All we have to do is to add up the effects of the different changes and we will get a first order approximation of the effect of the reform on inequality.

VII. Concluding Comments

In this paper we have presented a simple method for analyzing the distributional impact of small changes in commodity taxes. The data and the assumptions needed to carry out the analysis are minimal hence the method suits developing countries. The method is similar to the one suggested by Ahmed & Stern (1984). The main difference is that the Gini coefficient is used instead of an Atkinson type welfare function. The main advantage of our approach over the A&S approach is a result of the Gini coefficient. Since on one hand it can be used as a normative index which represents the welfare function (Yaari (1986)), while on the other hand it is a statistical index of
variability, the estimation of the key parameters and the interpretation of the results are based on the same procedure. This property implies the analysis.
Table 1: The Contribution of Income Components to Inequality
(Survey of Family Expenditure, Israel, 1979)

<table>
<thead>
<tr>
<th>Income Components</th>
<th>Correlation with Income $R_i$</th>
<th>Gini at Source $G_i$</th>
<th>Share $S_i$</th>
<th>$S^* G^* R$</th>
<th>Percent of $G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage of Head of Household</td>
<td>.6152</td>
<td>.5741</td>
<td>.5921</td>
<td>.2088</td>
<td>69.4</td>
</tr>
<tr>
<td>Spouses' Wage</td>
<td>.6643</td>
<td>.7844</td>
<td>.1549</td>
<td>.0807</td>
<td>26.9</td>
</tr>
<tr>
<td>Self Emp. and Capital Inc.</td>
<td>.6029</td>
<td>.6703</td>
<td>.3859</td>
<td>.1560</td>
<td>51.9</td>
</tr>
<tr>
<td>Other Incomes</td>
<td>.3031</td>
<td>.8274</td>
<td>.1298</td>
<td>.0326</td>
<td>10.9</td>
</tr>
<tr>
<td>Social Security Allowances</td>
<td>-.3589</td>
<td>.5734</td>
<td>.0770</td>
<td>-.0150</td>
<td>-5.3</td>
</tr>
<tr>
<td>Income Tax</td>
<td>.8617</td>
<td>.5855</td>
<td>-.2924</td>
<td>-.1475</td>
<td>-49.1</td>
</tr>
<tr>
<td>Social Security Tax</td>
<td>.6505</td>
<td>.4692</td>
<td>-.0464</td>
<td>-.0142</td>
<td>-4.7</td>
</tr>
<tr>
<td>Net Income</td>
<td>1.0</td>
<td>.3005</td>
<td>1.0</td>
<td>.3005</td>
<td>100.0</td>
</tr>
</tbody>
</table>

* Income is defined as Total income (including imputed income from owned housing and vehicle) minus taxes per standard adult.

Table 2: The Effect of Marginal Changes in Income Components

<table>
<thead>
<tr>
<th>Income Component</th>
<th>Marginal* Effect</th>
<th>Marginal Effect Per $</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage of Head of Household</td>
<td>.0311</td>
<td>.053</td>
</tr>
<tr>
<td>Spouses' Wage</td>
<td>.0342</td>
<td>.221</td>
</tr>
<tr>
<td>Self Employment and Capital Income</td>
<td>.0400</td>
<td>.104</td>
</tr>
<tr>
<td>Other Incomes</td>
<td>-.0065</td>
<td>-.050</td>
</tr>
<tr>
<td>Social Security Allowances</td>
<td>-.0390</td>
<td>-.507</td>
</tr>
<tr>
<td>Income Tax</td>
<td>-.5065</td>
<td>-.204</td>
</tr>
<tr>
<td>Social Security Tax</td>
<td>-.0002</td>
<td>-.004</td>
</tr>
<tr>
<td>Net Income</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

* A positive sign means that an increase in the income source increases income inequality.
Table 3: The Effect of Subsidies on Inequality in Standard of Living
Israel, 1979

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread</td>
<td>2.0</td>
<td>-.08</td>
<td>.289</td>
<td>-.02</td>
</tr>
<tr>
<td>Cooking Oil</td>
<td>0.7</td>
<td>-.10</td>
<td>.461</td>
<td>-.009</td>
</tr>
<tr>
<td>Eggs and Milk</td>
<td>3.4</td>
<td>0.30</td>
<td>.266</td>
<td>-.025</td>
</tr>
<tr>
<td>Sugar</td>
<td>1.0</td>
<td>0.05</td>
<td>.433</td>
<td>-.009</td>
</tr>
<tr>
<td>Water</td>
<td>0.5</td>
<td>0.20</td>
<td>.501</td>
<td>-.003</td>
</tr>
<tr>
<td>Public Transportation</td>
<td>1.4</td>
<td>0.08</td>
<td>.611</td>
<td>-.012</td>
</tr>
<tr>
<td>Other Expenditures</td>
<td>91.19</td>
<td>.998</td>
<td>.352</td>
<td>.079</td>
</tr>
</tbody>
</table>

* Income is defined as net total income per standard adult.

Table 4: The Effect of Subsidies on the Extended Gini

<table>
<thead>
<tr>
<th>Effect Per $, v=1.5</th>
<th>Effect Per $, v=2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bread</td>
<td>-1.057</td>
</tr>
<tr>
<td>Cooking Oil</td>
<td>-1.127</td>
</tr>
<tr>
<td>Milk and Eggs</td>
<td>-.780</td>
</tr>
<tr>
<td>Sugar</td>
<td>-.931</td>
</tr>
<tr>
<td>Water</td>
<td>-.690</td>
</tr>
<tr>
<td>Public Transportation</td>
<td>-.879</td>
</tr>
<tr>
<td>Other Consumption</td>
<td>.088</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5: The Magnitude of Taxes or Subsidies with Equal Effect on Inequality

<table>
<thead>
<tr>
<th>Tax or Subsidy Base</th>
<th>Magnitude of Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a. General Taxes</strong></td>
<td></td>
</tr>
<tr>
<td>Head of household salary</td>
<td>1.0</td>
</tr>
<tr>
<td>Spouses' wage</td>
<td>.24</td>
</tr>
<tr>
<td>Self employed and capital income</td>
<td>.51</td>
</tr>
<tr>
<td>Income tax</td>
<td>.26</td>
</tr>
<tr>
<td><strong>b. Social Security Taxes</strong></td>
<td></td>
</tr>
<tr>
<td>Social security tax</td>
<td>12.23</td>
</tr>
<tr>
<td>All allowances paid by social security</td>
<td>.10</td>
</tr>
<tr>
<td>Allowance for first child in the family</td>
<td>.06</td>
</tr>
<tr>
<td>second</td>
<td>.04</td>
</tr>
<tr>
<td>third</td>
<td>.03</td>
</tr>
<tr>
<td>fourth and more</td>
<td>.02</td>
</tr>
<tr>
<td>Allowances for the elderly</td>
<td>.04</td>
</tr>
<tr>
<td><strong>c. Subsidies on Commodities</strong></td>
<td></td>
</tr>
<tr>
<td>Bread</td>
<td>.049</td>
</tr>
<tr>
<td>Cooking oil</td>
<td>.046</td>
</tr>
<tr>
<td>Eggs and milk</td>
<td>.070</td>
</tr>
<tr>
<td>Sugar</td>
<td>.056</td>
</tr>
<tr>
<td>Water</td>
<td>.075</td>
</tr>
<tr>
<td>Public transportation</td>
<td>.062</td>
</tr>
</tbody>
</table>

a. The sign of the tax change is suppressed. In general, taxes on income source have the same sign as subsidies on expenditures.
ENDNOTES


2. In the case of the extended Gini then the extended Gini correlation is defined as \[ \frac{\text{cov} (x_i, [1-F_y]^{\nu-1})}{\text{cov} (x_i, [1-F_\nu(x_i)]^{\nu-1})} \]. Properties a, b and c hold in this case too, see Stark, Taylor and Yitzhaki (1986b).

3. Note that the share and the Gini correlation can be negative (e.g. taxes) and of course the Gini correlation.

4. Note, however, that one way to introduce efficiency consideration into the analysis, is to take into account only the change that will keep the consumer at the same utility level. In this case the higher the compensated elasticity of demand, the lower the compensation needed.

5. It is worth noting that the use of ranks in order to avoid the assumption of Normality implicit in the analysis of variance is an old one. Friedman (1937) suggested it in connection with analysis of family expenditure. The difference between Friedman approach and the approach in this paper, is that Friedman uses ranks to represent both the income and the expenditure on a specific item while in this paper only income is represented by its rank.
6. In the case of the extended Gini the weights will be:

\[
\frac{[1-F_y(y)] - [1-F_y(y)]^v}{\int ([1-F_y(z)] - [1-F_y(z)]^v) \, dz}
\]

In this case the weight function is asymmetric and the weights given to the marginal propensity to spend of low income groups is an increasing function of \( v \). For a detailed description of the weight function see the appendix.

7. The concept of standard adult is intended to take into account the effect of the size of the household on needs. The scale that is used is the following: single 1.25 standard adults, a couple without children -- 2.0 standard adults, a couple with one child 2.65, with two children 3.2, with three children 3.75, with four children 4.2, and .4 for each additional child.

8. The sample is a weighted sample, which means that each household in the sample represents a different number of population's households. The equations given in this paper, should be corrected to take care of this situation. The type of correction needed is discussed in Lerman and Yitzhaki (1986).

9. See Lerman and Yitzhaki (1985) for a similar result when decomposing U. S. data. However, as Lerman and Lerman (1986) show this results does not hold if capital gains are included in income.

10. For the sake of convenience, it is referred to as a "dollar" change in revenue.
11. Note that this is not the elasticity of the income tax with respect to before tax income (marginal tax divided by average tax). This is the income elasticity of a proportional surcharge on the income tax with respect to after tax income.

12. This point is discussed at a greater length in the next section.

13. A comparison of these income elasticities with other conventional estimation methods can be found in Yitzhaki (1986).

14. Note, that all derivations are written as positive numbers. It is assumed that when dealing with sources of income the derivative represents a decrease in taxes while when dealing with commodities they represent a decline in subsidy.

15. A detailed analysis of the implications of this results is in Yitzhaki (1986).

16. Another difference between Ahmed and Stern approach and the one presented in this paper is that A&S take into account the efficiency cost of the tax changes. This can be taken care of in a similar way. However, this kind of correction requires the knowledge of the price elasticities of the commodities.
REFERENCES


Our aim is to derive the weighting scheme of the (Gini) regression coefficient. Let $X,Y$ represent two continuous random variables with a density function $f(x,y)$. Assume that $x$ represents expenditure on a consumption good while $y$ represents income.

The extended (Gini) regression coefficient of $x$ on $y$ is

\begin{equation}
(A.1) \quad b = \frac{-v \text{cov}(x,(1-F(y))^{v-1})}{-v \text{cov}(y,(1-F(y))^{v-1})} \\
v > 1
\end{equation}

Let $X(y) = E_{X} \{X|Y=y\}$ be the Engel curve. First note that

\begin{equation}
(A.2) \quad E_{y} \{[1-F(y)]^{v-1} f(y) dy = \int_{0}^{\infty} [1-F(y)]^{v-1} f(y) dy
\end{equation}

and by transformation of variables

\begin{equation}
= \int_{0}^{1} (1-F)^{v-1} dF = \frac{1}{v}
\end{equation}

By utilizing (A.2) and eliminating zeros, the nominator in equation (A.1) can be written as

\begin{equation}
(A.3) \quad -v \text{cov}(x,[1-F(y)]^{v-1}) = E_{y} E_{X} \{x [1-v(1-F(y))]^{v-1}\}
\end{equation}

= $E_{y} \{X(y)[1-v(1-F(y))]^{v-1}\} =$

$= \int X(y) [1-v(1-F(y))^{v-1}] f(y) dy$
using integration by parts where

\[ v(y) = X(y) \]

\[ u(y) = -[1-F(y)] + [1-F(y)]^v \]

then

\[ = X(y) \cdot \left[ [1-F(y)]^v - [1-F(y)] \right] \bigg|_0^\infty \]

\[ + \int X'(y)\left[ [1-F(y)] - [1-F(y)]^v \right] dy \]

The first term is equal to zero, hence

(A.4) \[-v \text{ cov}[x,(1-F(y))^{v-1}] = \]

\[ = \int X'(y)\left[ (1-F(y) - [1-F(y)]^v \right] dy \]

while the denominator in (A.1) can be developed in a similar way to be

(A.5) \[-v \text{ cov}[y,(1-F(y))^{v-1}] = \int [(1-F(y)) - (1-F(y))^v] dy \]

That is it can be thought of as a normalization procedure of the weights

\[ w = [1-F(y)] - (1-F(y))^v \]

In the case of the Gini \( v=2 \), then
\[ [1-F(y)] - [1-F(y)]^2 = F(y) [1-F(y)] \] which is

a symmetric weighting scheme in F.

In general, the weight attached to each marginal propensity to consume is

(A.7) \[ w[F(y)] = \frac{[1-F(y)] - [1-F(y)]}{\int ([1-F(z)] - [1-F(z)]^v) \, dz} \]

where the denominator is a normalizing factor.

Let us investigate this weighting scheme. The maximum weight is a function of \( v \). In order to find out the maximum weight, let us ignore the denominator. Then we can write

\[ w(F) = (1-F) - (1-F)^v \]

where \( F \) is the cumulative income distribution since

(A.8) \[ \frac{\partial w}{\partial F} = -1 + v (1-F)^{v-1} \]

(A.9) \[ \frac{\partial^2 w}{\partial F^2} = -v (v-1)(1-F)^{v-2} \]

the second derivative is negative for \( v>1 \). The first derivative is equal to zero for

(A.10) \[ F^* = 1 - \left( \frac{1}{v} \right)^{v-1} \]
This means that the weights increase with the rank in the income distribution, reach a maximum and then decline. Table A.1 presents the rank at which the weights reach the maximum value (equation A.10).

Table A1: The Rank With the Highest Weight

<table>
<thead>
<tr>
<th>( v )</th>
<th>1.1</th>
<th>1.2</th>
<th>1.5</th>
<th>2.0</th>
<th>3.0</th>
<th>6.0</th>
<th>10.0</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F^* )</td>
<td>.61</td>
<td>.598</td>
<td>.556</td>
<td>.5</td>
<td>.423</td>
<td>.30</td>
<td>.22</td>
<td>.045</td>
</tr>
</tbody>
</table>

As can be seen from the table, the rank at which the weights reach the maximum value is a decreasing function of \( v \), and at \( v=2 \), the maximum weight is given to the median income. As \( v \) increases, the maximum weight is given to someone with a lower rank.

An additional insight on the weighting scheme can be gained by looking at the weight attached to different quantile of the income distribution by different \( v \).

Table A.2 presents the weights in terms of the weight attached to the median income. As can be seen from the table choosing \( v=2 \), means that the weighting scheme is symmetric in the distance of the rank from the median rank. The higher \( v \), the higher the weight attached to lower income individuals. The use of different weighting schemes, enables the user to analyze the sensitivity of the estimated coefficient.
Table A.2: Weights Attached to Different Quantile of the Distribution (Relative to the Weight Attached to Median Income)

<table>
<thead>
<tr>
<th>F/V</th>
<th>V=1.3</th>
<th>V=1.6</th>
<th>V=2</th>
<th>V=3</th>
<th>V=5</th>
<th>V=6</th>
</tr>
</thead>
<tbody>
<tr>
<td>.1</td>
<td>.30</td>
<td>.32</td>
<td>.36</td>
<td>.46</td>
<td>.56</td>
<td>.76</td>
</tr>
<tr>
<td>.3</td>
<td>.76</td>
<td>.79</td>
<td>.84</td>
<td>.95</td>
<td>1.05</td>
<td>1.2</td>
</tr>
<tr>
<td>.7</td>
<td>.97</td>
<td>.90</td>
<td>.84</td>
<td>.73</td>
<td>.66</td>
<td>.62</td>
</tr>
<tr>
<td>.9</td>
<td>.53</td>
<td>.44</td>
<td>.36</td>
<td>.26</td>
<td>.23</td>
<td>.20</td>
</tr>
</tbody>
</table>
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