Loan Market Imperfections and Optimal Policy Intervention

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Abstract

The paper analyzes the effects of different types of loan market policy interventions designed to influence the flow of loans to specific sectors and uses. A direct interest subsidy is shown to be the best policy, and an interest ceiling (usually and mistakenly termed an 'interest subsidy') is shown to be the worst. In fact the latter worsens any existing market distortion, and is therefore often coupled with minimum lending requirements. This combination is most appropriately referred to as a forced lending policy. This analysis is done conditional on the assumption that a market imperfection in fact exists. The problem of correctly identifying whether an imperfection does exist is assumed to have been carried out prior to any intervention, and a formula is given for identifying such an imperfection. The difficulty involved in correctly identifying such imperfections and the need for caution is emphasized in the concluding section.
1. **Introduction**

The credit markets is one of the markets in which the governments of almost all countries have intervened at some time.\(^1\) The extent and duration of intervention has varied widely between countries and overtime; A declining trend in intervention by developed countries seems to be more than offset by the proliferation of intervention in many developing countries.\(^2\) The focus of the present paper is on government policy intervention directed at the cost and/or flow of credit to specific groups, sectors and uses.\(^3\) Examples of these are loans for housing, exports, agriculture, industry, loans to state and local governments, to large borrowers, to small producers, to foreign borrowers, and loans for medium-long term investments. Except for occupied housing almost all other types of intervention has been directed at credit flowing to firms, entrepreneurs and governments building productive infrastructure.

The virtual universality of this intervention raises two questions with more than usual force: What market imperfection could occur so widely as to justify this intervention? If such imperfections occur, which of the many policies used are effective (and which harmful) in correcting the perceived imperfection? In the case of business loans, loan repayment depends on uncertain returns to the activity for which the loan is taken. Expectations about these returns form the basis of both the borrowing and the lending decisions. Given the different experience and perspective of banks and firms, it would not be surprising to find, in a world of multiple uncertainties, differences in expectations of returns.\(^4\) There is a fundamental informational asymmetry which also underlies the problems of moral hazard and
adverse selection which have been analyzed in the loan literature. These two involve, however, an additional factor; the willful concealment or distortion of information (and actions) by the borrower. It is useful to start with the simplest case of informational asymmetry, which excludes calculated dishonesty.

The second question, the effect of various policies designed to correct loan market imperfections had not been considered in the literature till recently. Smith (1983) has considered a single policy, that of unrestricted, government loans at the post intervention market rate in an essentially aggregative or macroeconomic context. The market imperfection relates essentially to long term consumer loans, as there is no production. As already suggested the present paper will deal with group or use specific intervention, which has been most common in connection with business loans. Six different policies will be considered and their effects analyzed.

An earlier paper (author 1983) has classified and analyzed different forms of competitive equilibrium in contractual markets such as the loan market. In the present paper contract taking behavior by banks and firms, which is most akin to perfect competition in goods markets (models) will be assumed. The loan literature has also used what has been referred to (op cit) as lender or bank setting which bears on affinity to monopolistic competition. The next section presents a simple partial equilibrium model of the competitive business loan market. In section 3 an expression for ex-ante "market efficiency" is derived in the context of this model, which produces a simple measure of deviations from efficiency. It is then shown that differential expectations can lead to below "optimal" loan size and in
extreme to non-existence of the market for specific groups of borrowers. This non-existence is very similar to what Stiglitz-Weiss (1981) have defined as credit rationing. What are perceived as two different types of groups by banks may appear very similar on superficial observation, but only one receives loans under a tighter monetary policy, while both may do so under a looser one.

In section 4 the effect of six specific policies is analyzed: These are loan interest rate ceilings, loan interest rate subsidies, subsidized (below the deposit rate) rediscount rates, floors and ceilings on loan amounts and a collateral subsidy (a type of risk insurance subsidy). It is shown that if the problem as usually stated by governments has been correctly perceived, the interest and collateral subsidy policies eliminate the problem. The loan interest ceiling policy is the worst and compounds the original problem. The subsidized discount rates and loan floors can be effective under certain circumstances but may have undesirable side effects. Section 5 concludes the paper.

2. The Model

The loan market is assumed to be competitive with a sufficient number of identical banks on the lending side, and many homogeneous groups of firms on the borrowing side. Each such group of firms contains one or more identical firms. Firms and Banks are assumed to be risk neutral. The banks are assumed to obtain their loanable funds on a perfectly competitive deposit market; Banks therefore act as if the supply of deposits is perfectly elastic.
at the deposit interest rate $i$. These assumptions result in the separability of a bank's lending decision for firms from different groups. Further one can focus on a representative firm from the particular group under consideration and a representative bank.

The uncertain returns $y$ to the firm (gross of loan repayment), during the period of the loan depends on the total capital $K$, employed; This is the sum of own funds or equity $e$, and loan $L$. The uncertainty in returns is assumed to be additive;

$$y = g(K) + \theta, \quad k = e + L, \quad g^-(K) < 0, \quad g^{-}(K) \leq 0. \quad (1)$$

with $\theta$ distributed as $F(\theta, \eta)$ [density $f(\theta, \eta)$], expected value $E(\theta) = \bar{\theta}(\eta)$, and $f(\theta) = 0$ for $\theta < s$. \textsuperscript{10/} Equation (1) can be derived for expected profit maximizers given uncertainty in the production function (see Virmani 1983). In contrast a function with that multiplicative uncertainty $[y=\theta g(k)]$ often used in the loan literature,\textsuperscript{11/} does not follow from any simple form of uncertainty in production or prices (op cit) in the loan market context. In the present paper we assume (first order stochastic dominance), that

- $\eta^- > \eta$ implies $F(\theta, \eta^-) > F(\theta, \eta)$ for all $\theta$, and
- $F(\theta, \eta^-) > F(\theta, \eta)$ for some $\theta$, so that $\bar{\theta}(\eta^-) < \bar{\theta}(\eta)$.

In general banks and firms can differ in their subjective evaluation of the distribution $F$ of $\theta$. All banks are assumed to have the same
subjective distribution with regard to each member of the homogeneous group of firms. This distribution can however be different than that perceived by the firm's themselves in terms of the parameter \( \eta \) defined above. \( \eta' > \eta \)

\( \eta' < \eta \) implies that the banks are pessimistic (optimistic) relative to the firms.

A loan contract may contain certain exogenous parameters like the firms' own funds \( e \), and endogenous terms like loan amount \( L \), interest rate \( r \) and collateral \( C \). In the present paper \( C \) is assumed to be a value which the bank can recover with certainty. In general, the maximum effective collateral that a firm can give, \( C_{ma} \), depends on many factors. Among these are the firm's equity in the particular firm, other assets owned by the firm (or entrepreneur), the nature of the project, the period of the loan and the resale market for capital goods. Actual collateral \( C \) is restricted to \( C_{ma} > C > 0 \). Unless explicitly stated this constraint is assumed to be satisfied.

Most earlier loan models make repayment dependent on end of period wealth of the firm, with starting wealth taken as \( K \). If the former exceeds the due amount, full repayment is made, if not, the entire wealth is transferred to the bank. This type of repayment schedule occurs only in the case of bankruptcy, and may be reasonable for very long term loans. A large proportion of bank loans are, however, short or medium term ones. In this situation banks are extremely reluctant to demand full repayment when compliance would entail closing down the firm and declaring bankruptcy. This is quite rational when expectation of returns exceeds loan repayment obligations. Firms are of course equally anxious to ensure that their future returns are not adversely affected by a single adverse realization of what is
known by both to be a random variable. In effect repayment at the end of the period is a function of returns during the period with an expectation that remaining sums due will be cleared as soon as possible. Within the limitations of the single period model, however, we can only approximate this situation. For any loan contract \((L,r,C)\) repayment as a function of returns \(y\) is assumed to have the following form (shown in figure 1):

\[
\min \left[ (1+r)L, \max (L+y,C) \right]
\]

Figure 1: Repayment Schedule
Where the symbols have the following meanings:

\[ y = \text{Return on the production activity in which the loan is to be used.} \]

\[ r = \text{Rate of interest on funds to bank.} \]

\[ i = \text{Rate of interest on funds to the value.} \]

\[ L = \text{Value of loan.} \]

\[ C = \text{Value of collateral.} \]

\[ t = \text{Marginal transaction cost to bank if loan amount is increased.} \]

\[ T = \text{Fixed cost to bank includes administrative costs and returns to capital.} \]

\[ a' = -(L-C) = C-L, \quad b = \frac{C-L}{t} \]

\[ a = a' - g(K), \quad b = \frac{a'}{r} \]

Given this repayment function, the subjective distribution of the firm \( F(\theta) = F(\theta, 0) \), the expected repayment \( X \) of the firm and its expected profits or gain, \( P_F \), from a loan with terms \( (L, r, C) \) can be written as,

\[
X = \int_a^b \left[ \frac{a}{c} \right] \left( L + \frac{a}{c} \right) \frac{dF(\theta)}{d\theta} + \int_a^b (1 + r)L \frac{dF(\theta)}{d\theta} \\
= CF(a) + \int_a^b (L + g(K) \frac{dF(\theta)}{d\theta} + (1 + r)L [1 - F(b)] \tag{2}
\]

\[
P_F = L + g(L + e) - X \tag{3}
\]
Similarly given a subjective distribution \( F'(\theta) = F(\theta, \eta^-) \) for the bank the expected profits of the bank from the loan transaction are

\[
P_B = X^- - (1 + i + t)L - T
\]  

(4)

where \( X^- \) is obtained from (2) by replacing \( F(\theta) \) by \( F'(\theta) \).

By analogy with price taking, contract taking in the loan market involves parametric treatment of interest-collateral terms by the firms and banks. For any given terms \((r, C)\) the firm maximizes its expected profits with respect to the loan amount to obtain (assuming an interior maxima),

\[
1 + g^-(K) = X^L, \quad X^L = \frac{\partial X^-}{\partial L}, \quad g^- = \frac{\partial g}{\partial K}
\]

(5)

which implicitly defines loan demand as a function (or correspondence) of interest rate and collateral. The bank similarly maximizes expected profits with respect to the loan (subject to a non-negativity constraint on expected profits) to obtain

\[
X^L = 1 + i + t, \quad X^L = \frac{\partial X^-}{\partial L}
\]

(6)

which implicitly defines loan supply to the representative firm from a representative bank as a function of interest rate and collateral. \( X^L \) and \( X^L \) are of course the marginal change in expected repayment with loan amount, as perceived by the firm and bank respectively. Equations (5) and (6) therefore represent the usual (expected) marginal return equals (expected)
marginal cost conditions for the participants. In the present paper we will consider only cases in which an interior contract taking maxima exists for both bank and firm, and (5) and (6) can be assumed to be functions of form (5') and (6') respectively:

\[
\begin{align*}
\text{Demand:} & \quad L^D = L^D(r,C) \\
\text{Supply:} & \quad L^S = L^S(r,C)
\end{align*}
\]

In general the possibility exists that these curves are backward bending in interest rate \( r \). Only the cases in which equilibrium is on the normally sloping portion of the \( L-r \) curve will be considered in the present paper.\(^{15/} \)

Competitive equilibrium in this market is then defined by the following equations:

\[
L^D(r^*, C^*) = L^S(r^*, C^*) (= L^*) \tag{7}
\]

\[
P_B = X - (1 + i + t)L^* - T = 0 \tag{8}
\]

Both the bank making the loan to a firm, and those not making a loan to it have the same expected gain/profit, so that no competitive pressure remains for changing loan terms (i.e. \( P_B = 0 \) for all banks).
3. Existence and Efficiency of the Loan Market for Specific Groups

Before analyzing the nature of the equilibrium solution represented by equations (5) to (8) it is useful to define efficiency in this loan market. Consider a government planner who is trying to set loan terms for each group of homogeneous firms assuming other markets are efficient. In principle he would have his own subjective expectations about the distribution of returns to a representative firm from a group. These this may coincide with the expectations of either the firms or the banks, or lie in between.

The ex-ante social gains, $P_s$ is the sum of (3) and (4) (with $X' = X$) that is

$$P_s = g(L + e) - g(e) - (1 + t)L - T$$  \hspace{1cm} (9')

The planner would maximize these over all terms of the loan contract, i.e.

$$\max P_s \text{ subject to } P_s > 0,$$

which yields

$$g^*(L^* + e) = 1 + t \hspace{1cm} (9)$$

assuming an interior solution i.e. $g(L^* + e) - g(e) > T + (1 + t)L^*$.

Equation (9') and consequently (9) are independent for the planner's subjective distribution of $\theta$, and of the other loan terms $(r, C)$. This result, which is due to the additive uncertainty assumption, greatly simplifies the evaluation of market failure and the consequences of policy intervention. It is also a limitation on the generality of the conclusions.
If banks and firms have the same expectations of the distribution of returns to the latter, it follows from equation (5) and (6) that 
\[ g'(K) = i + t \]; if the market exists it must be efficient. The market equilibrium for this case can be depicted in \( L,r \) space as in figure 1.

Figure 2: Identical Expectations Equilibrium

Efficiency in the market as defined in equation (9) depends solely on the loan amount, and is consistent with any level of interest rate and collateral. In the context of the contract taking equilibrium used in this paper it is notationally convenient to refer to all the terms \( (L^*, r^*, C^*) \) prevailing in the efficient identical expectation equilibrium as the optimal ones.

In general, the bank's and firm's subjective expectations will differ. The market equilibrium is then obtained by the solution of equations.
(5) to (8). The effect of differential expectations can be illustrated graphically (figure 3) by considering an increase in bank's pessimism factor ($\eta^-$) above that of the firm's ($\eta$):

**Figure 3: Differential Expectation Equilibrium**

An increase in $\eta^-$ has two effects: It shifts the supply functions up and to the left along the demand function, at the original collateral level. It also directly reduces the expected profit of the bank. The former in turn results in a decrease in loan amount and an increase in loan interest rate. The last in turn tends to push expected bank profits up. The net effect on bank expected profits (at $C = C^*$) is ambiguous: Collateral requirements must rise (fall) if these expected profits are negative (positive), to restore equilibrium.
The precise effects can be affected by a factor, referred to as the slope factor (SF) which depends on the relative slopes (for given loan amount) of the supply and demand functions in \((r,C)\) space. We find that with a non-negative (positive) slope factor (SF) loan amount is negatively (positively) related to banks costs per loan \((T)\). As intuition suggests a negative effect of costs on loans, we can call the case of non-negative slope factor as the Conventional Case and the one with positive slope factor as the Unconventional one. We then find given a fixed \(\eta\) for the firm, the following results:

**Proposition 1: Inefficient Markets**

In the conventional case the loan size falls monotonically with the pessimism factor, but may not do so for the unconventional case. In both cases however, the equilibrium loan amount is less than "optimal" when the banks are relatively pessimistic \((\eta^- > \eta)\) and greater than optimal when the banks are relatively optimistic \((\eta^- < \eta)\).

By totally differentiating equation (5), (6) and (8) and solving, we find \(\partial L/\partial \eta^- < 0\) for \(SF \geq 0\) and \(\partial L/\partial \eta^- \leq 0\) for \(SF < 0\). We can also see from equations (5) and (6) that \(g''(L+e) - (i+t) = \lambda_L - \lambda_L < 0\) as \(\eta^- \geq \eta\), from which the last part of the proposition follows (as \(g'' < 0\)).

**Proposition 2: Loan Interest Rates**

The loan interest rate rises monotonically with \(\eta^-\). The differential of equilibrium interest rate with \(\eta^-\) is positive for all \(\eta^-\). If we compare several groups of firms which were identical in all respects, but differed only in the banks' perception of the distribution of
their returns, we would find (assuming the conventional case applies), that the more pessimistic the banks the higher the interest rate and lower the loan size. A system of varying points or upfront fees, often used in actual loan markets (e.g. housing, international loans) will accomplish this purpose even if the nominal values of the loan and the interest do not change. Note also that one of the arguments used for government intervention is that some groups of firms or some types of uses are getting insufficient loans at too high interest rates, is related directly to propositions 1 and 2.

Proposition 3: Market Failure and Borrower Profits

When the banks are relatively pessimistic $(\eta^* > \eta)$ firms' equilibrium expected profits will not rise monotonically with decreasing pessimism factor and could decline.

We can show that $\partial L/\partial \eta^* < 0$ for $\eta^* > \eta$ but has an ambiguous sign when $\eta^* < \eta$. The latter result is intuitively plausible, because the marginal expected social gain from increasing loan amount is negative when the bank is relatively optimistic. In the conventional case, therefore as $\eta^*$ falls the total social gain shrinks, and at some point the relative gain to the firm from the banks' misperception could be overwhelmed by the total loss. As the firms in a contract taking equilibrium only have the choice of setting $L$, they will continue to take the loan as long as expected profits remain above the no-loan value.
Returning to the bank pessimism case, consider a group of potential borrowers who would be indifferent between taking or not taking a loan if the identical expectations equilibrium prevailed. That is

\[ P_F = L^* + g(L^* + e) - g(e) - X(L^*, r^*, C) = 0 \] (10)

\[ P_B = X(L^*, r^*, C) - (1 + i + t)L^* - T \]

Suppose that in actuality the banks are relatively pessimistic, so that the equilibrium terms are \((L^*, r^*, C^*)\) with equilibrium equation (8) as

\[ P_B = X(L^*, r^*, C) - (1 + i + t)L^* - T = 0 \] (10')

we can the prove that,

**Corollary 3.1**

No loan market will exist for a group of firms for which the banks' expectations are pessimistic relative to the firms' expectations and which would be indifferent between taking and not taking a loan if the hypothetical identical expectations equilibrium prevailed.

**Proof:** The firm's expected profits are \(P_F\), so that

\[ P_F = L^* + g(L^* + e) - g(e) - X(L^*, r^*, C) \]

\[ < L^* + g(L^* + e) - g(e) - X(L^*, r^*, C) \] as \( X > X' \) for \( n^- > n \)
\[ g(L^* + e) - g(e) - (1 + t)L^* - T \] using (18) \[ - [g(L^* + e) - g(L^* + e)] - [(1 + t)L^* - (1 + t)L^*] \] using both equations of (10) \[ < 0 \text{ as } g^* > 0 \text{ and } L^* < L^* \text{ from proposition 3.} \]

In the general case in which the hypothetical identical expectations equilibrium would yield positive expected profits to firms, but banks are pessimistic, there are two possibilities: Either \( \lim_{n \to \infty} P^*_F(\eta_n, \eta) = a > 0 \) or there exists a \( \eta < \infty \) such that \( P^*_F(\eta, \eta) = 0 \) and \( P^*_F(\eta, \eta) < 0 \) for all \( \eta > \eta(n) \). Assuming that there are groups of firms for which \( a < 0 \) and \( \eta \) is finite we must have for these groups;

**Corollary 3.2: Non-existence**

No loan market will exist for a group of firms for which the bank’s subjective distribution of returns to the firms in the group is such that the pessimism parameter is greater than a critical value (i.e. \( \eta^* > \eta \)).

We would therefore expect to observe, in a competitive economy, groups of firms which perceive a need for loans, but cannot obtain them at any "reasonable" terms. This is one of the arguments given by governments to justify intervention, particularly in putting loan interest rate ceilings and/or minimum lending requirements for specific groups and uses.

For any set of exogenous parameters, including the deposit interest rate, all actual and potential borrower groups \((j)\) can be ranked so that there is an \( n \) such that the expected firm profits \( PF_j \).
Actual borrowers: \( P_j(n_j, n_j) > 0 \) and \( n_j > \bar{n} \) \( j = 1, \ldots, n \)

Potential borrowers: \( P_j(n_j, n_i) < 0 \) and \( n_j < \bar{n} \) \( j = n + 1, \ldots, m \)

and \( P_j > P_{j+1} \) for all \( j \).

It can then be shown that,

**Proposition 4:** Rationing Equilibrium or Competitive Equilibrium

Any government policy (like monetary expansion) or exogenous change, which reduces (increases) the supply of loanable funds (or deposits) to the banks is likely to result in an increase (decrease) in the numbers of firm-groups receiving loans.

**Proof:** In the competitive market, an increase in deposits must lead to a decline in deposit interest rates. When the banks are relatively pessimistic this will lead to an unambiguous increase in "equilibrium expected profits" of both actual and potential borrowers \( \frac{\partial P_F}{\partial i} < 0 \). One or more groups with \( j > n \) are therefore likely to have their "expected profits" raised from negative to non-negative, and will start taking and receiving loans \( \frac{\partial L^*}{\partial i} < 0, \frac{\partial r^*}{\partial i} > 0 \).

This type of market phenomenon has often been interpreted as rationing, most recently by Stiglitz-Weiss (1981). Without getting into semantic arguments, it could just as well be considered a problem of non-existence of markets as done here.
Section 4: Policy Analysis

In the present section it will be assumed that the government has correctly identified the nature of the market imperfection confronting a specific group of firms or specific uses which can be translated into specific groups of firms. In terms of the analysis of the last section the source of the problem is excessively pessimistic perceptions of the banks vis-à-vis the firms. Historically many observers of the banking industry have often commented on the 'traditional conservatism of bankers' and though it may not be the only source of imperfections it is undoubtedly one of them. The objective of this section is to analyze the effect of different policies on the specified group when this imperfection exists. The formal analysis will be carried out for each policy in isolation, to be followed by some indication of what would happen if a combination of the policies are in effect.

Consider first the commonly used policy of central bank rediscounting of loans at a subsidized rate. If the deposit side of the market is free, as assumed, non-subsidized discounting mechanism will have no effect. If rediscounting is restricted to a proportion of the loans this merely modifies the effective subsidy. Assume therefore 100% rediscounting at a subsidy \( i_s \) so that the banks unit cost of funds is \( i - i_s \) instead of \( i \) the deposit rate. The effect of an increase in subsidy (say from \( i_s = 0 \)) is similar to the effect of a decrease in the deposit interest rate. Therefore, we have, for the conventional case,
Proposition 5: Rediscount Facility

The effect of a subsidy on funds available to banks for lending to a group, is to increase the amount of loans received by the firms in the group and to reduce the interest rate they have to pay. Though the effect on collateral is ambiguous the overall loan terms improve, as there is an unambiguous increase in the expected profits of the firms.

In the conventional case, this policy moves the loan amount, loan interest and expected firm profits towards those that would prevail in an efficient market. This policy can be viewed as reversing the effects of an increase in bank pessimism factor $n^+$ above the firms factor $n$. In terms of figure 2, a fall in the loan interest rate shifts the supply function back in the southeast direction raising loan amount and lowering the interest rate, while also directly raising expected bank profits. Restoration of equilibrium profits may require a positive or negative adjustment in collateral. The only qualification in the unconventional case is the possibility of a perverse effect on loan amounts, so that efficiency deteriorates. It is not possible to rule out this perverse case even though it appears intuitively implausible.

The loan interest rate subsidy $r_s$ can be given either to the bank or the firm. In the first case the firm continues to pay an interest rate $r$, but the bank receives $r+r_s$. In the second case the firm pays the net rate $r-r_s$ while the bank continues to receive $r$. Somewhat surprisingly the effect of the two policies can be quite different, especially for groups which are not getting any loans (non-existent market situation):
Proposition 6: Loan Interest Subsidy to Bank

A loan interest rate subsidy to the banks raises loan amounts and expected firm profits (from the loan transaction) and lowers the interest rate paid by the firm and the collateral required.

The only difference in effect of this policy compared to the rediscount-one (conventional case) is an attendant reduction in collateral requirement. Though $r$ falls (as before) as a result of the subsidy the banks net interest rate $r + r_s$ does not, so that banks' expected profits are positive in the intermediate position. Collateral must therefore fall to restore equilibrium. This makes no difference in the environment, assumed so far, in which equilibrium collateral does not hit the constraint $C \leq C_{ma}$ outlined in section 2. It is however important in certain policy interventions considered below, in which income distribution constraints or objectives are relevant. Further, given the possibility of perverse effects in the unconventional case of discount policy, this policy appears to be the safer one.

Proposition 7: Interest Subsidy to the firm

An interest rate subsidy to the firm raises loan amount as well as the interest rate paid by the firm $(r - r_s)$. Though collateral requirements fall, the net effect on equilibrium expected firm profits is ambiguous.

The initial effect of the interest subsidy is to shift the demand function outwards so that the interest rate and loan amount rise. This makes
expected bank profits positive and collateral must be adjusted down to make them zero. Because of contradictory effects of interest and collateral the change in expected firms profits is ambiguous. If these profits do not change the efficiency of an existing market will be improved, but a non-existent market will not be brought into existence. This policy is therefore inferior to a bank subsidy policy, in terms of efficiency.

The effect of quantitative restrictions is somewhat more complicated. Consider first the loan interest rate ceiling when the initial equilibrium is at \((L^r, r^r, C^r)\) as depicted in figure (3).
If an effective ceiling, $\bar{r} < r^*$, is imposed on loans to this group of firms, the bank's expected profits with unchanged collateral are negative so that no loan would be made. Collateral must therefore rise. Given the constraint on the interest rate, it is clear however that a contract taking demand-supply equilibrium cannot exist; Mathematically the system of equations (6) to (8), defining the equilibrium is over-determined. A "rationing equilibrium" is possible, however, and it can be defined as the solution of bank equations (6) and (8) at $r = \bar{r}$. "Rationing" here merely means that the expected marginal profits of the firm are positive $(1 + g^* - X_i > 0)$. By the definition of contract taking, neither banks nor firms are allowed to offer a collateral-loan package which would be preferred by both. We can then show,

Proposition 8: Loan Interest Ceiling and Rationing

A loan interest ceiling, which is effective, will result in a "rationing equilibrium", characterized by lower loan amounts and higher collateral requirements. It will therefore worsen inefficiency.

When the bank is relatively pessimistic a contract taking firm actually improves its perceived terms, through the forced reduction in interest rates, despite the increased collateral requirements. Expected profits may not increase, however, because the decline in loan amount reduces the gross returns to the firm. The net effect on expected profits is ambiguous. If profits fall the target group could be eliminated from the loan market altogether. Even if this does not happen the original market failure is still unambiguously worsened by this intervention. If such a policy is
used for a large proportion of the market and for a sufficient length of time, however, the nature of the loan market could itself be affected. This is because contract setting equilibria exist (author 1983) which may be preferred by both lenders and borrowers. In the long-run therefore the contract-setting rationing equilibrium is unstable.

As shown in the author’s 1983 paper the relative collateral interest trade-off for the banks and firm is such that banks value collateral in terms of interest more than firms do. Firms therefore have an ex ante gain from the ceiling policy, when banks are relatively pessimistic.

Governments often specify that a particular sector or type of firm must be given a certain proportion or total volume of loans. Ignoring information problems, this can be thought of as a minimum lending requirement for a specific group of firms. This in turn translates into a prescription of minimum statutory loan size \( L \) for each firm in the group. The obvious choice of loan size is the socially optimal one \( L^* \). As in the previous case, forcing the firm to make larger loans than it wants results in negative expected profits if collateral is unchanged. Collateral must therefore rise. As the bank is being statutorily forced off its supply function a competitive contract taking solution can now be defined in terms of the firm’s demand function and the bank’s zero profit condition.

The general effect of this policy is even more complicated than for the interest ceiling one. The effect of the introduction of a marginal constraint at the unconstrained equilibrium, are however exactly identical. In this case too loan interest rates falls, collateral rises and expected firm
profits rise, and for the same reason. When the floor is raised further, however, interest rates may not continue to fall or expected firm profits to rise. The initial effect of the policy is to directly raise loan size to the optimal as long existence of the market is not affected; In some cases, however the market may cease to exist altogether, having the opposite of the desired result.

A **collateral subsidy** is a form of government guarantee which insures a certain amount of repayment to the bank in the event of default. It can be defined in absolute terms, say $C_s$, or as a proportion of collateral taken from the firm ($C_s = aC$): Thus the firm gives a collateral of $C$, while the firm receives $C + C_s$ in the event of default. It can be shown (see author 1982) that a collateral subsidy has exactly the same directional effects as an interest subsidy. The important effect distinguishing these two policies from other policies which also improve efficiency is that they both result in a reduction of collateral requirements. Many developing country governments have been particularly concerned about inefficiencies in loan markets for poor individuals with inadequate collateral. Examples are small urban entrepreneurs, rural small farmers and tenants. These two subsidy policies have the important side-product of unambiguously reducing equilibrium collateral requirements and therefore mitigating the effect of collateral problems.

A **loan ceiling** on particular groups has a different origin than the policies considered so far. In terms of the model of the paper, it must be due to a perception of relative optimism of banks ($\eta^* < \eta$) which results in excessive loans at low interest rates. There is an implicit assumption that
if these groups are getting "too much" everybody else will get less than they should. As before we assume that loan ceiling policy can be translated into a loan ceiling per firm for each homogeneous group of firms. If the unconstrained equilibrium is $(L^-, r^-, C^-)$, an effective loan ceiling $L < L^-$ will reduce expected bank profits below zero so that a demand-supply contract taking equilibrium does not exist. A rationing equilibrium defined by equations (6) and (8) does exist so that we have,

**Proposition 9** When the bank is relatively optimistic a loan ceiling will improve efficiency by reducing loan amount, and also raise interest rates and lower expected firm profits.

Section 5: The Government's Information Problem and Conclusions

The previous section assumed that the government had correctly identified specific groups facing imperfect markets, and the type and extent of the problem. This section briefly considers the problems that can arise when practical application is being considered. If the market environment is consistent with the assumptions of the model the government can in principle identify every group facing an imperfect market. The social optimality condition shows that the government must calculate the expected marginal product of capital of the group and the marginal cost to banks of supplying loans to the group.

One of the problems that arises in the policy context is that the groups are defined so broadly that they are far from homogeneous. There is
therefore the possibility that there will be an undesirable shift in the proportion of loans going to different sub-groups. One common form of inhomogeneity is in collateral availability, so that a policy which increase collateral requirements will tend to shift the proportion of loans going to more wealthy firms. As already suggested the loan interest subsidy and the collateral subsidy are therefore superior to both the rediscount subsidy and the loan floor policies, with the latter being unambiguously harmful in this respect.

Another problem that arises, is a misjudgement of the extent of imperfection, so that intervention is carried too far. Given the assumptions of the policy analysis, the marginal cost to the bank, also measures the marginal social cost of loans. Even a correct policy, if it is carried too far beyond the optimality point, can completely vitiate any gains from the intervention. Given the difficulties in measuring the degree of market failure, this suggests a cautious incremental application of the subsidy policy, which allows both the governments and the banks to generate and analyze new data.

The simplifying assumptions and limitations of the model have been noted in the text. The policy conclusions derived are, however likely to hold up under a more general model, as they accord with intuition from goods market models. Though the problems of dishonesty and asymmetric information, were ignored in this paper, models of moral hazard and adverse selection are presented in separate papers. Informal analysis suggests that the policy conclusions of this present paper are likely to be broadly true in these cases.
Footnotes


3/ The present paper is based on a chapter of a manuscript which was submitted for publication as a book in November 1981, and finally appeared (after review) as a Staff Working Paper in June 1982.

4/ To my knowledge this has not been formally modelled by other authors in the loan markets context. Stiglitz (1972) has, however, considered differential expectations in the context of equity bond markets.


6/ In all these cases, the persistence of informational asymmetry depends on many factors. The simplest case is one in which the markets do not exist, so that the banks receive no new information, which can be used to modify expectations. If markets exist, the maintenance of information asymmetry will depend on the term (time period) of the loan and its repetition, and the variability in the firm's operating environment. Thus we would expect this asymmetry to persist longest for medium-long term loans in a highly unstable economic environment. Another factor is the information flows in the economy; which would be superior in a developed as compared to a developing country.
The papers of Virmani (June 1982) and Smith (November 1982) appear to be the two earliest analysis.

Another type of contract setting equilibrium, firm or borrower setting has also been analyzed by the author in the same paper. The reader is referred to that paper for a comparative discussion.

Pages 394-5 points (a) and (b), particularly the latter.

There is a limit to the losses that can be sustained during the period of the loan. We also assume \( \lim_{x \to \infty} x \int dF(x) = 0. \)


The second order condition \( X_{LL} < 0 \) is assumed satisfied.

The second order condition \( g'' - X_{LL} < 0 \) is assumed satisfied. Note that an interior equilibrium cannot exist if both are risk neutral as assumed and \( g'' = 0 \). The assumption, \( g'' < 0 \) implies decreasing returns to scale in production as shown in authors (1983) paper. The alternative assumption of constant returns to scale and demand (with additive uncertainty) limited production appears to be consistent with this formulation.

Multiple loans are not considered in the paper. If such loans are from outside the banking system and banks always get repaid first, the analysis is still valid. In equilibrium only one bank (if any) will make a loan to each firm through the preceise identity of the bank-firm pair is indeterminate.

This implies \( X_{LR} > 0 \) as shown in authors (1982) paper which can be referred to for a more complete discussion of backward bending curves, and the unstable equilibrium which occurs on the backward bending portion of the curves.
16/ See for example Kane and Malkiel (1965).
References


7. Selective Credit Control in Western Europe, Association of Reserve City Bankers, Chicago, Illinois, 1976.


First and Second Order Differentials of $X$ and $(X')$, the Expected Repayment

It is useful to list these, as they are used often

$$X_L = (1 + r)(1 - F(b)) + (1 + g')(F(b) - F(a)) > 0$$

$$(X_L = 0 \text{ if } b = a = S \text{ where } S < = \text{ and is defined by } F(\theta) = 0 \text{ for all } \theta > S)$$

$$X_c = L[l - F(b)] > 0 \quad \text{in general.}$$

$$X_C = F(a) > 0 \quad \text{in general.}$$

Note: that $X_c = 0 \text{ if } a = - s \text{ i.e. } L + g(X) \geq C + s \text{ and default is not possible. In this case } C + 0, \text{ i.e. } L + g(X) > s, \text{ and we have a zero collateral solution.}$

$$X_r = - \int_{a}^{b} \frac{3F(\theta, r)}{\partial r} d\theta < 0$$

in general as $\frac{3F(\theta, r)}{\partial r} > 0 \text{ for all } \theta \text{ by definition. The unusual case in which } X_r = 0 \text{ will be ignored.}$

$$X_{LL} = g''(F(b) - F(a)) - (r - g')^2 f(b) + (1 + g')^2 f(a)$$

$$< 0 \quad \text{if } C = 0$$

$$> 0 \quad \text{if } C = (1 + r)L \text{ i.e. } a = b > - s \text{ and } r < 1 + 2g'$$
\[ X_{Lr} = 1 - F(b) - (r - g) f(b) \]

is assumed to be \( >0 \) at equilibrium.

\[ X_{LC} = -(1 + g) f(a) < 0 \quad \text{in general} \]

Note that \( X_{LC} = 0 \) if \( C = 0 \).

\[ X_{Lc} = g'(F(b) - F(a)) + (r - g) f(b) + (1 + g) f'(a) \]

\[ X_{Lc} = X_{LL} = -(1 + g) f(a) + r(r - g) f(b) \]

\[ X_{Lr} = -(r - g) \frac{3F(b,r)}{3r} - (1 + g) \frac{F(a,r)}{3r} < 0 \quad \text{for } r > g \]

Implies: \( X_{L} \lessgtr X_{c} \) as \( r^{-} \gtrsim r \) \((= 0)\)

\[ X_{rn} = -\frac{3F(b,r)}{3r} < 0 \quad \text{implies } X_{r} \lessgtr X_{r} \text{ as } r^{-} \gtrsim r \]

\[ X_{Cr} = \frac{3F(a,r)}{3r} \geq 0 \quad \text{implies } X_{C} \lessgtr X_{C} \text{ as } r^{-} \gtrsim r \]

\[ \text{and } X_{r} X_{C} - X_{r} X_{C} \geq 0 \quad \text{as } r^{-} \gtrsim r \]

Note: Strictly speaking \( \frac{3F(b,r)}{3r} = 0 \) is possible for particular values of \( b \). As the basic results are not affected we will ignore the possibility that \( \frac{3F(b,r)}{3r} \) or \( \frac{3F(a,r)}{3r} \) are zero at the interior maxima.
Totally differentiating equations (5), (6) and (8) and simplifying we have,

\[ \frac{dC}{\Delta C} = \frac{1}{X_C} \left[ -X_r^c \frac{dr}{\Delta r} - X_n^c \frac{dn}{\Delta n} + L d_i + dT \right] \]

\[ \frac{dL}{\Delta L} = \frac{X_{Lr}^c}{X_C} - \frac{X_{LC}^c}{X_C} - \frac{X_{Ln}^c}{X_C} \]

\[ \frac{dr}{\Delta r} = -X_{LL} + \frac{X_{LL}^c}{X_C} \]

\[ \frac{dT}{\Delta T} = 1 \]

\[ J = - (g" - X_{LC}) \left( \frac{X_{Lr}^c}{X_C} - \frac{X_{LC}^c}{X_C} \right) - \left( \frac{X_{Ln}^c}{X_C} - \frac{X_{LC}^c}{X_C} \right) X_{LC}^c > 0 \]

For any variable \( y = n^c \) of \( i \) or \( T \) we have,

\[ \frac{\partial P_F}{\partial y} = \frac{\partial P_F}{\partial L} \frac{\partial L}{\partial y} + \frac{\partial P_F}{\partial C} \frac{\partial C}{\partial y} - \frac{\partial P_F}{\partial n} \frac{\partial n}{\partial y} - X_r + \frac{X_{Lr}^c}{X_C} \]

where \( y = n^c \) or \( L \) or \( 1 \) as \( y = n \) or \( i \) or \( T \).

\[ \frac{\partial L}{\partial n} = \frac{X_{Ln}^c}{J} (X_{Lr} - X_{LC} + \frac{X_{Ln}^c}{JX_C} (X_{LC}X_{Lr} - X_{LC}X_{Ln}) - \]

\[ < 0 \text{ if } SF > 0 \]

\[ = \{ \begin{array}{ll} 0 & \text{as } SF < 0 \end{array} \]

where \( SF = X_{LC}X_{Lr} - X_{LC}X_{Ln} \)

\[ \frac{\partial r}{\partial n} = \frac{X_{Ln}^c (g" - X_{LL})}{J} - \frac{X_n^c}{X_C} \left[ X_{LL}X_{LC} + (g" - X_{LL})X_{LC}^c \right] > 0 \]

\[ < 0 \text{ if } n^c > n \]

\[ \frac{\partial P_F}{\partial n} = \frac{X_{Ln}^c}{X_C} + \frac{X_{Lr}^c}{X_C} - X_r \] \[ \frac{\partial r}{\partial n} = \{ \begin{array}{ll} 0 & \text{if } n^c < n \end{array} \]
\[ \frac{\partial \theta}{\partial r_s} = \frac{X_{\theta} X_{r_s}}{X_{C}} = \frac{X_{\theta} - X_{r_s}}{X_{C}} \left( X_{\theta} - X_{r_s} \right) > 0 \]

\[ \frac{\partial \theta}{\partial r_r} = -\frac{X_{\theta} X_{r_s} X_{C}}{X_{r_s}^2} \left( 1 + \frac{\partial r}{\partial r_s} \right) = -\frac{X_{\theta} X_{r_s} X_{C}}{X_{r_s}^2} \left( 1 + \frac{\partial r}{\partial r_s} \right) < 0 \]

\[ \frac{\partial \theta}{\partial r_s} = (1 + \gamma - \lambda) \frac{\partial \theta}{\partial r_s} - X_{\theta} \frac{\partial r}{\partial r_s} - X_{\theta} \frac{\partial \theta}{\partial r} = -(X_{\theta} \frac{\partial r}{\partial r_s} + X_{\theta} \frac{\partial \theta}{\partial r}) > 0 \]

\[ \frac{\partial \theta}{\partial r_r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r_s} = (1 + \gamma - \lambda) \frac{\partial \theta}{\partial r_s} - X_{\theta} \frac{\partial r}{\partial r_s} - X_{\theta} \frac{\partial \theta}{\partial r} = -(X_{\theta} \frac{\partial r}{\partial r_s} + X_{\theta} \frac{\partial \theta}{\partial r}) > 0 \]

\[ \frac{\partial \theta}{\partial r_r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r_s} = (1 + \gamma - \lambda) \frac{\partial \theta}{\partial r_s} - X_{\theta} \frac{\partial r}{\partial r_s} - X_{\theta} \frac{\partial \theta}{\partial r} = -(X_{\theta} \frac{\partial r}{\partial r_s} + X_{\theta} \frac{\partial \theta}{\partial r}) > 0 \]

\[ \frac{\partial \theta}{\partial r_r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r_s} = (1 + \gamma - \lambda) \frac{\partial \theta}{\partial r_s} - X_{\theta} \frac{\partial r}{\partial r_s} - X_{\theta} \frac{\partial \theta}{\partial r} = -(X_{\theta} \frac{\partial r}{\partial r_s} + X_{\theta} \frac{\partial \theta}{\partial r}) > 0 \]

\[ \frac{\partial \theta}{\partial r_r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r} = \frac{X_{\theta} X_{r_s}}{X_{C}} \frac{X_{r_s}^2}{X_{r_s}^2} > 0 \]

\[ \frac{\partial \theta}{\partial r_s} = (1 + \gamma - \lambda) \frac{\partial \theta}{\partial r_s} - X_{\theta} \frac{\partial r}{\partial r_s} - X_{\theta} \frac{\partial \theta}{\partial r} = -(X_{\theta} \frac{\partial r}{\partial r_s} + X_{\theta} \frac{\partial \theta}{\partial r}) > 0 \]
\[ \frac{\partial r}{\partial s} = \frac{X^r L X_L r}{J} > 0 \]

\[ \frac{\partial (r-r_s)}{\partial s} = 1 + \frac{X^r L X_L}{J} = \frac{X^r L X_{LL} - (g^r - X_{LL}) \frac{X^r L X_L - X^r L X_{LL}}{X^r C}}{J} \{ < 1 \}
\]

\[ \frac{\partial C}{\partial s} = -\frac{X^r r}{X^r C} < 0 \]

\[ \frac{\partial P_F}{\partial s} = \frac{X^r}{X^r C} \frac{\partial r}{\partial s} - 1 - X_C \frac{\partial C}{\partial s} = \left(\frac{X^r X_C}{X^r C} - X^r \right) \frac{\partial r}{\partial s} + X^r \frac{\partial C}{\partial r} \frac{X^r}{X^r C} < 0 \text{ for } n > n' \]

**Loan Interest Ceilings and Rationing Equilibrium**

Totally differentiating (8) and (6) with \( r = \bar{r} \) we find,

\[ dC = -\frac{X^r L X_L}{X^r C} dr > 0 \quad \text{for } dr < 0 \]

\[ dL = -\frac{1}{X^r L} \left( X^r L X^r X_{LL} - \frac{X^r L X_C}{X^r C} X_L \right) d\bar{r} < 0 \quad \text{for } d\bar{r} < 0 \]

\[ \frac{dP_F}{\partial r} = (1 + g^r - X_L) \frac{\partial L}{\partial r} - X_r dr - X_C \frac{\partial C}{\partial r} \]

\[ = (1 + g^r - X_L) \frac{\partial L}{\partial r} + \left( \frac{X_C X^r}{X^r C} - X_L \right) \frac{\partial X_C}{\partial r} \]

\[ \frac{\partial X_C}{\partial r} \sim 0 \text{ as } X_C X^r - X_C X^r C < 0 \text{ for } n > n'. \]

Note however that at the initial point \( r = \bar{r}' \) the first term is zero so that \( P_F \) and \( \bar{F} \) are inversely related; a marginal decrease in \( \bar{F} \) will unambiguously lead to a reduction in expected profits.
Minimum Lending Requirements

Totally differentiating (8) and (6) with \( L = L \) and simplifying, we have

\[
\frac{\partial r}{\partial L} = \frac{X_r (g - X_{LL}) - X_{LC} (1 + i + t - X_r)}{J''} > 0 \text{ in general}
\]

< 0 at the unconstrained solution point (i.e. if \( L = L^- \)) as

\[
\frac{\partial P_B}{\partial L} = X_r (1 + i + t) < 0 \text{ if banks are being forced to lend.}
\]

\[
\frac{\partial C}{\partial L} = \frac{-(g - X_{LL}) X_r + X_{LC} (1 + i + t - X_C)}{J''} > 0
\]

\[
J'' = X_r X_{LC} - X_r X_{LC}
\]

\[
\frac{\partial P_F}{\partial L} = - (X_r \frac{\partial r}{\partial L} + X_c \frac{\partial C}{\partial L}) = \frac{1}{J''} [(g - X_{LL})(X_r X_C - X_r X_C) + (1+i+t)(X_c X_{LC} - X_r X_{LC})]
\]

> 0 in general

> 0 at the unconstrained equilibrium (i.e. if \( L = L^- \)), for \( \eta^- > \eta \).

Loan Amount Ceilings

For the rationing equilibrium with loan amount equal to \( \frac{X}{X} \), we can differentiate equations (8) and (6), and simplify to obtain,

\[
dC = - \frac{X_r}{X_r} dr
\]
\[
\frac{\partial r}{\partial L} = - \frac{X_{LL}}{X_r - \frac{r}{\partial C} X_{LC}} > 0
\]

\[
\frac{\partial C}{\partial L} < 0
\]

\[
\frac{\partial P}{\partial L} = (1 + g^* - X_L) + \left(\frac{X_r}{X_C} - X_L\right) \frac{\partial r}{\partial L} > 0
\]

as \(1 + g^* - X_L > 0\) and \(X_C^* X_r - X_C X_r^* > 0\) for \(\eta^* < \eta\)

**Collateral Constrained Solution**

The "excess supply" equilibrium is defined by equations (5) and (8). Totally differentiating these and simplifying, we obtain

\[
dL = \frac{X_r^*}{r^* - X_{LR}} \frac{X_{LR} - X_{LC}}{X_r^*}
\]

\[
dC_{\text{max}} = \frac{1}{J^*}
\]

\[
dr = 1 + i + t - X_r^* - g^* - X_{LL} - X_r^*
\]

\[
J^* = X_r^*(g^* - X_{LL}) + X_{LR}(X_r^* - 1 - i - t) \geq 0 \text{ as } \frac{X_r^* - 1 - i - t}{X_r^*} \geq \frac{g^* - X_{LL}}{X_{LL}}
\]

\[
\partial_{X_r} \left|_{P = P_B} \right| \geq \frac{\partial L}{\partial r} \left( r, C_{\text{max}} \right)
\]

\[
\frac{\partial L}{\partial C_{\text{max}}} = \frac{X_r^* X_{LC} - X_r^* X_C}{J^*} \geq 0 \text{ if } J^* \geq 0
\]

\[
\frac{\partial r}{\partial C_{\text{max}}} = \frac{(1 + i + t - X_r^*)X_{LC} - (g^* - X_{LL})}{J^*} \geq 0 \text{ if } J^* \geq 0
\]
It is apparent from a visual examination (see author 1982) that for (5) and (8) to intersect, the slope of the bank is-profit curve must be more negative than that of the demand function. That is $J^* < 0$, and

$$\frac{\partial L}{\partial C_{\text{max}}} > 0, \quad \frac{\partial r}{\partial C_{\text{max}}} > 0$$

$$\frac{\partial P}{\partial C_{\text{max}}} = -x \frac{\partial r}{\partial C_{\text{max}}} < 0 \text{ in this case.}$$