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Sudan Transport Study Model

November 9, 1970

This is the seventh in the series of Transport Planning Models Study papers. The Study, directed by Messrs. Jan de Weille and Leon H. Miller, Jr. is a continuing investigation of mathematical models developed for transport planning. Existing transport models are being analyzed, and revised and extended where practical. New models will be developed where needed. Eventually, the Study will include cases of models' application in specific transport planning studies and a critical review of the methodology.

This paper is based on the three-volume consultant's report Transportation Development Through Systems Analysis prepared in 1968 for Sudan by Lockheed Aircraft International, Inc. under contract to the U.S. Agency for International Development. The Lockheed model places transportation sector and project planning in a general planning framework which facilitates comparison between transport and nontransport projects. Transport projects and projects in other sectors are ranked heuristically according to multiple objectives and criteria, political and social as well as economic. A regional economic model is used to generate surplus and deficit zones for each major transport commodity, and three transport submodels are used to determine (1) the costs and performance on each link, (2) the optimum route for each origin-destination flow, and (3) the optimum interzonal distribution of traffic flows. The implementation timing of potential transport projects may be tentatively determined by a mixed linear and integer programming routine in the route optimization model, but the implementation ultimately recommended by the model depends on the project's rank vis-a-vis projects in other sectors and on available funds. Congestion costs are not considered in the model, but there is a fixed capacity constraint for each segment of the transport network in the route assignment model.

The paper was prepared by Clell G. Harral and Suzy Henneman and draws upon an earlier review by Iona Isaac. It is expository in nature; a critical evaluation of the model will be given in a subsequent paper. Bank staff members are invited to make comments and suggestions.

Sector and Projects Studies Division

TABLE OF CONTENTS

FIGURES

TABLES

THE LOCKHEED TRANSPORTATION MODEL FOR THE SUDAN

I. INTRODUCTION

1. This paper briefly describes the methodological framework of the Lockheed transportation study for the Sudan.1/ The overall system, or what we shall call "the Lockheed model", consists in a number of parts which may be grouped in different ways. Here we find it convenient to distinguish three major components: (i) the project identification and ranking models, DEPICT and PRITI, and the macroeconomic model, MEMO, which generates the capital budget constraint; (ii) the regional income and consumption model, DANSE; and (iii) the family of transport cost and traffic simulation models, COMPAC, TRADE, and VARO. Figure 1 presents in flow chart form the Lockheed model and the interactions among the various components.

2. A primary characteristic of the Lockheed Model is its emphasis from the beginning on project identification and ranking. The planning process is initiated by assembling a list for each region of all potential projects in all economic sectors, which is entered in the DEPICT routine. In practice this list would be compiled by asking government departments, foreign consultants, major private firms, and knowledgeable individuals what their investment expectations are for the immediate future. Relatively little need be known about the investments at this stage beyond the type of product or service they will create and the regional location. Government officials are then asked to go through a rather elaborate ranking process by which each project is compared with each other project according to each of several weighted social, political and economic objectives and criteria, such as political stability, income redistribution and income growth. The result of this stage of the analysis is a list of projects for each region ranked according to order of priority as viewed from a multidimensional, rather than strictly economically oriented, objective function of the government.

3. The sum of the investment requirements generated by the proposed projects in any given time period cannot exceed the available investment funds, i.e., domestic savings plus foreign investments and grants. It is the function of the macroeconomic model, MEMO, to generate the measures of domestic savings and foreign investment funds expected to be available for each time period. Detailed information specifying an estimated schedule of expected capital outlays for each project on the DEPICT list is then submitted to the PRITI routines which calculate the time profile of total investment requirements for

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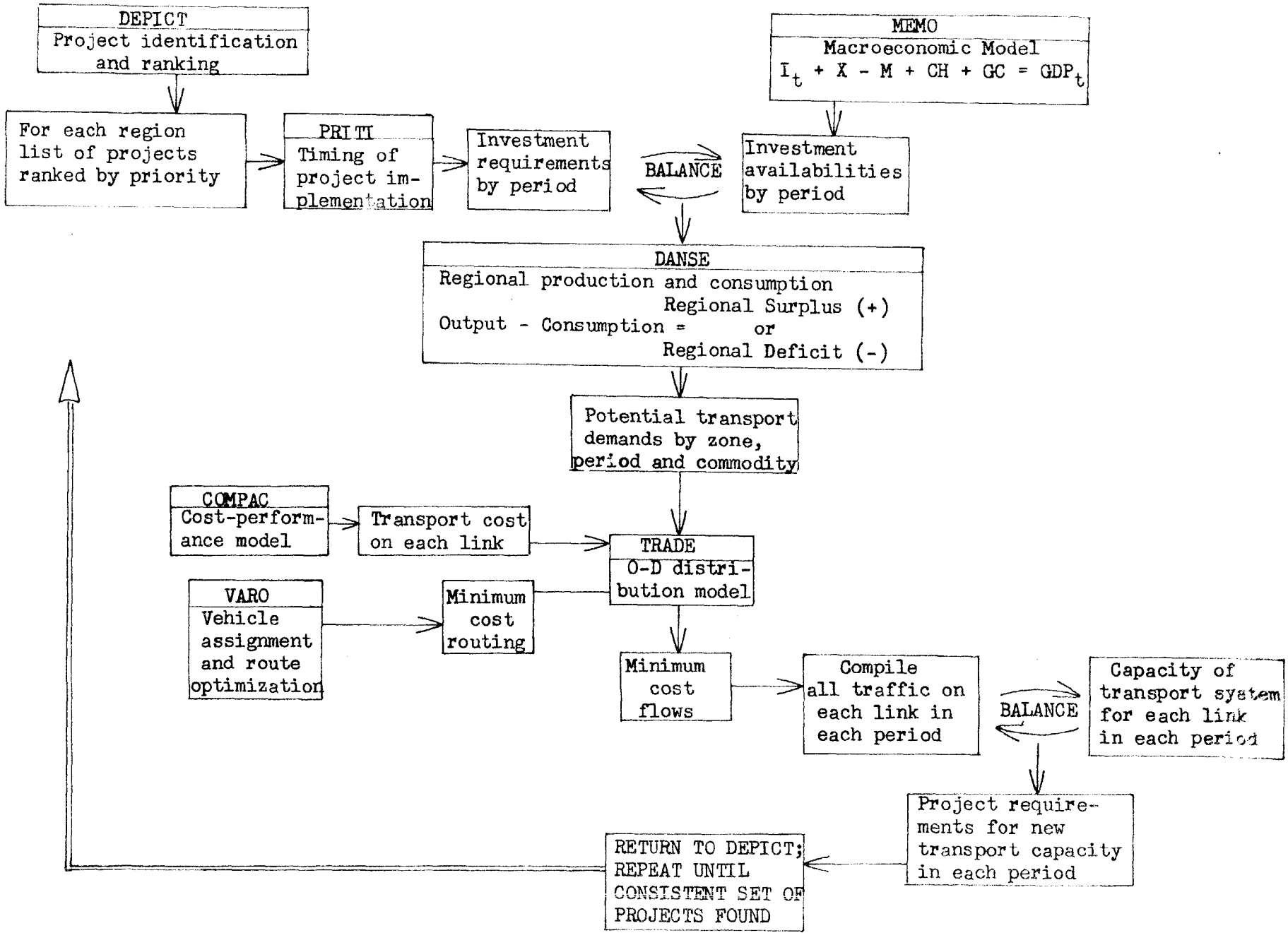
1/ Lockheed Aircraft International, Inc., Final Report U.S. AID Contract AID/afr. 359 (Los Angeles, 1968): Vol. I, Transportation Development Plan Sudan; Vol. II, Transportation Development Through Systems Analysis, Part 1: Concepts, Part 2, Applications. Acknowledgement is made to the Agency for International Development for permission to use this material.

Figure 1: Structure of the Lockheed Model for the Sudan



domestic and foreign currencies implied by the list of projects, and compare this total with the MEMO estimate of total local and foreign funds available. Initially the total proposed investments may greatly exceed the availability of funds in one or more time periods, so that some projects must be eliminated or postponed and the implementation of others is stretched out over time. It is assumed that, ultimately, a set of investment projects over time will be found which is consistent with the level of investment availabilities.

4. Once such a feasible set of investment projects is found, the effects of these investments on regional output are estimated in the regional economic model, DANSE. DANSE calculates production and consumption for each of some 20 commodities in each of 66 regions for each time period. Since the Sudan is predominantly a primary economy with quite limited demands for intermediate and manufactured products, the DANSE model concentrates on agriculture.1/ In each region, consumption of each good is subtracted from production to determine the excess supplies or excess demands, which constitute the internodal transport demands.2/

5. A family of three models is used to develop a transport plan to meet these demands. Highway, railway, river and air transport modes are considered. The COMPAC model first calculates transport costs (including vehicle operating costs, route maintenance and construction costs3/, the cost of time losses and expected cargo casualty) for each link in the transport network. The VARO model then uses this information in a linear programming routine to determine the minimum cost route between each pair of demand-supply nodes for a specified network, and the related vehicle requirements. An integer programming routine in the VARO model makes it possible to consider the effect on minimum cost routing of introducing as many as 20 new network links in a given year.

6. Using the minimum cost mode and route determined in VARO, the TRADE model (a version of the classical Hitchcok-Koopmans linear programming model) determines the pattern of internodal origin-destination commodity flows which minimizes transportation costs.

1/ Apparently for this reason, no input-output analysis of intersectoral flows (intermediate demands) was attempted.

2/ Intranodal transport demand (assumed to be carried by trucks) is calculated by multiplying the tonnage requiring transport within the given region by the average length of haul.

3/ Construction outlays are annualized by application of an amortization (capital recovery) factor.

Thus, transport investment, operating and maintenance costs are minimized for the given development plan.

7. After the minimum cost traffic flows have been determined for each commodity, the traffic flows on each link are summed and compared with the link's capacity. Where the demand on a given link exceeds the fixed capacity of a link, a "bottleneck" is identified and a potential transport investment is specified. The list of all such projects constitutes a new estimate of the transport investment program required to meet demands implied by the original plan of development, given by the DEPICT routine.

8. The new program for the transport sector is entered into a new DEPICT "scenario" and the entire procedure we have just described is repeated until a national investment program in all sectors is found which yields the minimum transport costs associated with the maximum attainable national objectives consistent with investment availabilities. Other scenarios may be performed to examine as many alternative assumptions and policies as the government and its planners may wish to consider.

II. PROJECT RANKING AND BALANCE OF REQUIRED AND AVAILABLE FUNDS OVERTIME

9. Three models are used together to translate the available information on proposed projects, on social, economic and political objectives, and on the availability of funds into a list of projects by year which constitutes the series of investments most valued in terms of several public objectives, consistent with the available funds.

10. The DEPICT model (DEvelopment Projects Interleafed by Criteria Technique) embodies a procedure for ranking projects within a single region according to national objectives and criteria. The PRITI model (PRoject Implementation TIming) combines detailed information on the investment requirements of development projects specified by DEPICT with a projection of investment availabilities as provided by MEMO (Macro Economid MOdel) to determine the year-by-year timing of all projects. The set of projects thus chosen provides a picture of the investment stream on which the calculation by DANSE of zonal production and consumption for each period, described in section III, can be based. For example, since the Sudan is primarily an agricultural country, investments to improve agricultural yields or acreage will affect zonal production.

Project Ranking (DEPICT)1/

11. The DEPICT model can handle eight objectives, fifteen criteria and fifteen projects. The model is applied separately to each region. Since the objectives and criteria may be weighted differently in each region, the final DEPICT output is a list of ranked projects, for each region.

12. No formal mechanism is provided for combining the regional lists into a country-wide list. Since such a ranking is needed as an input to PRITI, the DEPICT output is very simply regrouped: the projects with priority rank (1), of which there is one for each of the six regions, are the elements of the first priority group, called Priority I; projects with priority rank (2) constitute Priority II, and so forth, down to Priority XIII. The ranking of the projects within a priority group is not specified by the DEPICT model, that is, there is no explicit ranking or weighting of the relative importance of the different regions' projects.

13. To implement the DEPICT model, the regions, objectives and criteria must first be specified. The country is divided into five regions: the central area, in which the country's most modern economic activities take place; and four less developed areas pivoting around it, each with relatively similar resources and geography. A sixth "superregion", which refers to activities which are important to the country as a whole but are not of great importance in any single region, is also defined. For each of these

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1/ Throughout our presentation of the individual models, we shall use the same notation and acronyms as in the consultant's report; however, the equations and figures in this paper are numbered differently from those in the report.

regions, a list of candidate development projects is prepared, drawing on the government's Ten-Year Plan (1961-1971), on analysis of uniquely regional problems and resources, and on new suggestions made by individuals and agencies. The lists for the country's five regions specify projects (ports, roads, airports, rail links, bridges, plantations, etc.), while the list for the superregion comprises what might better be called programs (e.g., animal resource development, industrial development, urban development).

14. The qualitative objectives according to which candidate projects in every region are judged consist in attracting foreign investment, maintaining political and economic stability, broadening the base of production and consumption, increasing per capita income, and improving the country's physical and cultural accessibility. The criteria are more concrete than the objectives: growth in the export sector, creditworthiness, disposable income, availability and distribution of goods and services, diffusion of technology, capital-output ratio, value added, debt service ratio, and internal rate of return.

15. We summarize in the next two paragraphs the procedure which is carried out separately for each of the six regions. In the subsequent subsections, we describe in more detail each of the steps, which are schematized in Figure 2.

16. A weighting exercise must first be performed, comprised of three steps: (i) weights are assigned to each of the 15 criteria to measure its contribution to each of the eight objectives; (ii) weights are assigned to each objective to measure its importance to the region, and (iii) (i) is weighted by (ii), that is, the weight attached to each criterion to measure its contribution to each objective (a 15 by 8 matrix) is weighted by the regional weight of each objective (an 8 by 1 matrix) to yield the overall weight attached to each criterion in that region (a 15 by 1 matrix).

17. Then, the project ranking exercise may begin. Each pair of projects in the region's list is compared, taking one criterion at a time. Whichever member of the pair will contribute more according to that criterion receives as its score the whole value of the criterion's overall weight as determined in the exercise described above. The other member of the pair scores zero for that criterion. If there is no preference for one member over the other, the two members of the pair split equally the value of the weight. One project's score vis-a-vis one other project is the sum of all the criteria weights credited to that project. The sum of a project's scores vis-a-vis all other projects is its rank value; the project with the highest rank value receives rank (1), the one with the next highest, rank (2), and so on, until all the projects in that region are ranked.

Calculating Criteria Weights

18. For brevity's sake, we will limit ourselves to five criteria and four objectives, and we will illustrate the ranking of three projects in a given region. The weighting exercise can be summarized in Table 1.

Figure 2: Project Ranking and Investment Timing Models

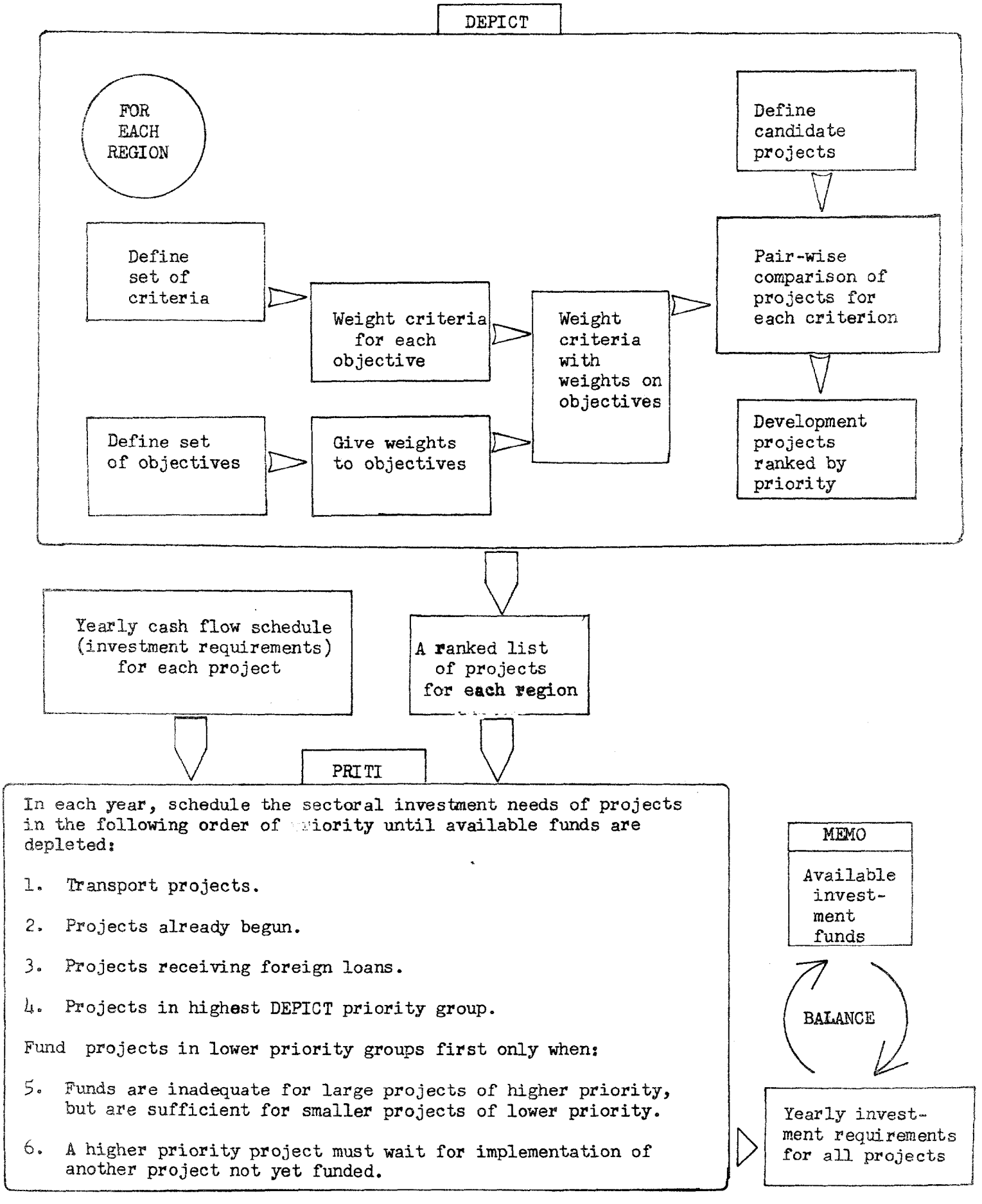


Table 1: Calculating Criteria Weights for One Region

	Weights of Objectives (V_j; $\sum_j V_j = 1$)				
	0.5	0.2	0.2	0.1	
Objectives (O_j)					Weights of Criteria (X_i)
	O_1	O_2	O_3	O_4	
Criteria (C_j)					
C_1	0.1	0.5	0.1	0	0.17
C_2	0.3	0.1	0	0.1	0.18
C_3	0.2	0.4	0.1	0	0.20
C_4	0.4	0	0.2	0	0.24
C_5	0	0	0.6	0.9	0.21
Total	1.0	1.0	1.0	1.0	1.00

19. The matrix of the C_i by the O_j, whose cells we call W_{ij}, relates the criteria and objectives. The weighting factors (W_{ij}) are assigned on a best-judgment basis to every criterion for every objective so that: 1/

$$\sum_i W_{ij} = 1, \; 0 \leq W_{ij} \leq 1.$$

The resulting matrix reflects the fact that the accomplishment of one objective is very often measured by more than one criterion and that a given criterion may give an indication of the accomplishment of more than one objective. This matrix will be the same for every region.

20. The weights of objectives (V_j) are assigned to each objective according to a procedure which formalizes decision-makers' views on the outlook and potential of the region at hand. 2/ The (W_{ij}) are then weighted by the (V_j)

1/ The notation in this subsection has been simplified considerably from the consultant's version.

2/ The "minimum discernible difference method" for weighting objectives devised by Lockheed is not described here, but is given in the Final Report, Vol. II, section 4.2, pp. 12-16.

to give the (X_i). For example, the weight of criterion C_1 in the region is:

$$X_1 = V_1 W_{11} + V_2 W_{12} + V_3 W_{13} + V_4 W_{14}$$

or, numerically:

$$\begin{aligned} X_1 &= (0.5)(0.1) + (0.2)(0.5) + (0.2)(0.1) + (0.1)(0) \\ &= 0.05 + 0.10 + 0.2 + 0 \\ &= 0.17. \end{aligned}$$

Generally, then:

(1) $X_i = \sum_i V_j W_{ij}$, and $\sum_i X_i = 1$.

Ranking Projects

21. Comparing now the region's three projects (1), (2) and (3), the decision maker (e.g. government planning board) considers them two by two and criterion by criterion. The judgments as to their relative merits are that:

for criterion C_1 = (3) is superior to (2) and (2) to (1);
C_2 = (1) is superior to (2) and (2) to (3);
C_3 = (2) is superior to both (1) and (3), which are tied in preference;
C_4 = (3) is superior to (2) and (1), which are tied in preference; and
C_5 = (3) is superior to (2) and (2) to (1).

These preferences tell us how to assign the X_i in the Table 2 below:

Table 2: Pair-Wise Preference of Projects

		Project Pairs					
C_i	X_i	(1)	(2)	(1)	(3)	(2)	(3)
C_1	0.17	0	0.17	0	0.17	0	0.17
C_2	0.18	0.18	0	0.18	0	0.18	0
C_3	0.20	0	0.20	0.10	0.10	0.20	0
C_4	0.24	0.12	0.12	0	0.24	0	0.24
C_5	0.21	0	0.21	0	0.21	0	0.21
Total	1.00	0.30	0.70	0.28	0.72	0.38	0.62

22. What we have earlier called the score of (1) vis-a-vis (2) is, then, the sum of (1)'s share of the criteria weights, or, in our example, 0.30. We enter this in the matrix which is our ultimate interest:

Table 3: Priority Summary Matrix

Project	Project (1)	Project (2)	Project (3)	Total Rank Value	Rank
(1)	*	0.30	0.28	0.58	Third
(2)	0.70	*	0.38	1.08	Second
(3)	0.72	0.62	*	1.34	First

The totals of each project's scores versus the other projects then tell us the project ranking in the region we are analyzing, as shown in the last column of Table 3.

23. Mathematically, let us call P_{mn} the score of the (m)th project vis-a-vis the (n)th project, that is:

$$(2) \qquad P_{mn} = \sum_i X_i \, E(m,n)$$

where E(m) is an existence function such that

E(m,n) = 1 when (m) is prefered over (n);
E(m,n) = 0 when (n) is prefered over (m);
and E(m,n) = ½ when (m) and (n) are tied in preference.

Then the rank value R_m of the (m)th of M projects will be:

$$(3) \qquad R_m = \sum_{n=1}^{M} P_{mn}$$

and the highest R_m takes the highest rank.

The Availability of Investment Funds (MEMO)

24. The basic function in sector planning of the macroeconomic model, MEMO, is to predict the amount of domestic and foreign funds which will be available for investment in each time period. The measure thus estimated then becomes the capital budget constraint in the project ranking and time staging process of the PRITI model. MEMO is concerned with national aggregates and is not regionally disaggregated, nor is the specific effect of individual investments on national income (or on transport demand) given by the MEMO model, but by the separate regional economic model DANSE described in section III. 1/ The MEMO model may also be used for simulating the time stream of national income in macroeconomic policy planning.

1/ Since the MEMO and DANSE models are independent, the aggregate impact on national income, production and consumption of a given set of investments as estimated by the two models will differ unless consumption coefficients and capital output ratios for each sector in the MEMO model are continuously matched with the consumption and productivity coefficients of the DANSE model. Apparently lack of data in the Sudan prevented the Lockheed team from effecting this connection.

25. There are four sources of investment funds in the MEMO model, two domestic and two foreign, which together must at least equal in time period (t) the demand for investment in the six economic sectors plus any exogenously specified increase in foreign reserves:

$$(4) \qquad DIG_t + DIP_t + FIS_t + FGS_t = \sum_{m=1}^{6} I_{mt} + (FRS_{t+1} - FRS_t)$$

where DIG_t = government domestic investment

DIP_t = private domestic investment

FIS_t = gross private foreign capital inflow

FGS_t = foreign grants

I_{mt} = investment required by sector m (exogenously specified)

FRS_{t+1} = desired level of foreign reserves

Government Investment Funds

26. Equation (5) generates government contribution to investment as the difference between government income and its expenditure on current (non-investment) account.

$$(5) \qquad DIG_t = GYS_t - GXC_t$$

where DIG = Domestic investment by government. Defined as receipts, except foreign grants and loans, less expenditure on the current budget.

GYS = Government income in total. Includes central and local tax revenue, central and local government share in earnings of government and quasi-government enterprise plus, theoretically, income from domestic debt creation through deficits on current account.

GXC = Government expenditure on the current budget, excluding capital expenditure.

27. Equation (6) relates private investment to the rate of change of GDP during the previous three-year period. It is postulated that rapid growth in GDP induces large amounts of investment, and vice versa.

$$(6) \qquad DIP_t = b_1 + n_1 \left[\frac{GDP_t - GDP_{t-3}}{GDP_{t-3}} \right]$$

where DIP = Private domestic investment. Made up of the domestic savings of individuals and private corporations invested domestically. Private savings loaned to the government are also included.

b_1 = A constant, empirically derived.

n_1 = The increase in private investment associated with a percentage point increase in the rate of growth of GDP over a three-year period.

28. Equation (7) states that foreign capital inflow is a function of the change in Gross Domestic Product plus the relending of principal payments made on indebtedness accrued in earlier time periods. Thus, if there is no change in GDP (i.e. $GDP_t - GDP_{t-1} = 0$), loans may still be forthcoming to the extent of principal payments.

(7) $$FIS_t = d_4 + d_1 (GDP_t - GDP_{t-1}) + FDP_{t-1}$$

where FIS = Foreign investment or gross foreign capital inflow.
FDP = Foreign debt principal payments which are assumed to be relent. These, together with interest payments, make up total foreign debt service. Principal payments are considered separately because of their role in determining future foreign capital inflows.
d_1 = Limit ratio of foreign debt outstanding to GDP.
d_4 = A constant representing the minimum inflow of foreign investment.

Project Timing (PRITI)

29. The PRITI routine defines development projects as investment requirements over time, and expresses them in terms of sources and allocation of funds in the different sectors of the economy. In essence the routine schedules investment outlays in accordance with the availability of funds. Each project has its investment requirements classified by source and sector. The investment requirements of each project in year (t), which depend on the point reached in the project's implementation schedule by that year, are added together until one or more types of investment funds is depleted. When no further projects may be started in year (t), the sum of investments by sector is sent as an input to the MEMO routine.1/ Timing can also be initially determined with no capital constraint, the output sent to MEMO, MEMO's subsequently generated capital budget constraint reinput into PRITI, and the cycle repeated until the two models are consistent.

30. A formal system for determining the priority and timing of project implementation in the face of the constraint on available capital is not given in the Lockheed model. A heuristic procedure is followed instead, in which the rules listed below in the order of importance are applied.

1/ Investment availabilities may be approximated for several successive years in the "fast forward" operation of the MEMO routine. These estimates are used to avoid the premature inclusion of projects that will in the future exceed the available investment funds.

(i) The investment requirements for the transport sector are given priority over all other development projects. A formal optimizing procedure (mixed linear and integer programming) for determining the minimum cost transport system is provided in the VARO model (See section IV.)

(ii) Development projects currently being implemented have priority over projects not yet initiated.

(iii) Development projects or partial projects that have received foreign loans in 1967 and after are given first priority within the DEPICT priority groups.

(iv) Development projects will be implemented according to the DEPICT priority groups.

(v) Projects of lower priority will be initiated ahead of higher priority projects only if available investment funds are adequate for the former but not the latter.

(vi) A project may be delayed in its initiation because it must follow another project for technological reasons, or because it requires more resources at a later time than will be available then.

31. The final list of projects is presented in the order of the DEPICT priority groups (Priority I is composed of the projects with rank (1), one for each region, Priority II of those with rank (2), and so forth up to Priority XIII). For each project, PRITI has calculated the period over which funding should be maintained. In general, the lower the priority, the later the implementation; however, as the heuristic rules imply, not all Priority I projects are scheduled for the earliest investment date and some projects of the middle priority groups (V, VI, and VII) receive funding several years earlier than other projects in those groups.

III. ZONAL PRODUCTION AND CONSUMPTION

32. The job of calculating the tonnages of the twenty goods produced in this primarily agricultural economy which require transport from one zone to another is performed by the regional economic model, DANSE (Demands and Availabilities for Nodal Shipments). In each of 66 zones, data on acreages, crop yields, per capita consumption, and population is used to calculate production and consumption of each good over four-year periods. The comparison of production and consumption in each zone yields the number of tons of each good available for internodal transport. If, for example, production of wheat in a zone exceeds the amount of wheat consumed there, that zone's net supply of wheat is available to be transported to another region in which wheat production does not meet its consumption requirements. Thus, the model is comprised of three families of equations: those which calculate production, those which calculate consumption, and those which calculate net demand or supply. Accessory to the first two groups are the equations which calculate data on acreages and population.

33. For all agriculturally produced goods - about half the total number - total production is the sum of four components: production on irrigated acreage brought under modern cultivation, on irrigated acreage still cultivated by traditional practices, and production on nonirrigated or rain acreage in the same two categories. Each of the four components is the product of a per acre yield factor which is characteristic of its water source and cultivation method, and the number of acres receiving that treatment. It is chiefly by increasing the acreage under modern cultivation that the investments determined by the models of the previous chapter influence the DANSE model.

34. The basic production equation is:

$$(8) \qquad F_{nct} = (A_{ncio} - \sum_{k=1}^{t} A'_{ncik}) \cdot Y_{ncit} + \sum_{k=1}^{t} A'_{ncik} \cdot Y'_{ncik}$$

$$+(A_{ncro} - \sum_{k=1}^{t} A'_{ncrk}) \cdot Y_{ncrt} + \sum_{k=1}^{t} A'_{ncrk} \cdot Y'_{ncrk}.$$

F_{nct} is total production of good (c) in zone (n) in period (t). Subscripts (i) and (r) refer throughout to irrigated and rain acreage, respectively. The A'_{nck} terms denote acreage of crop (c) taken over by modernization projects in the (k) periods following the initial period (o); acreage in the initial period devoted to (c) is A_{nco}. Yield on traditionally cultivated acreage is Y; on acreage cultivated by modern techniques, Y'. The goods transported (r) are the same as those produced (c), but only about half of these 20 goods are specified as consumption goods (g). The 66 zones (n) are the areas influenced by the 66 specified transport nodes through accessibility, geography, or soil type.

35. The production of nine goods -- tea, coffee, kerosene, gasoline, salt, industrial and craft items, personal (body) oil, oils and fats, and sugar products -- cannot be related to acreage and yields. These goods are all imported via Port Sudan either as raw materials or finished products. The model sets tons of production equal to tons of consumption (discussed next) for each of these goods, and assigns either Port Sudan or Khartoum as the production center.

36. For all goods except one - industrial and craft items (manufactures) - the basic consumption equation defines total consumption as the sum of three components: consumption of farm dwellers who farm irrigated land, of those who farm rainland, and of city dwellers. Each component is the product of the population in the designated group and a per capita consumption norm, with the consumption of the two farm-dwelling groups being weighted by a measure derived from the prooortion of land under modern cultivation.

The basic consumption equation is:

$$(9) \qquad C_{ngt} = E_{ifng} \cdot P_{nift} \cdot W_{nift} + E_{rfng} \cdot P_{nrft} \cdot W_{nrft} + E_{hng} \cdot P_{nht},$$

where C_{ngt} is total consumption in zone (n) of good (g) in period (t), E is the per capita consumption norm, and P is population. The (W) are the weighting factors. The subscript (if) refers to dwellers on irrigated farm land; (rf), to dwellers on rain-fed land, and (h), to city dwellers. For the goods category (12), industrial and craft items, consumption is calculated as a fraction of the total consumption of all other goods.

37. The equations for calculating net demand or supply are of the form:

$$(10) \qquad D_{nrt} = F_{nct} - C_{ngt},$$

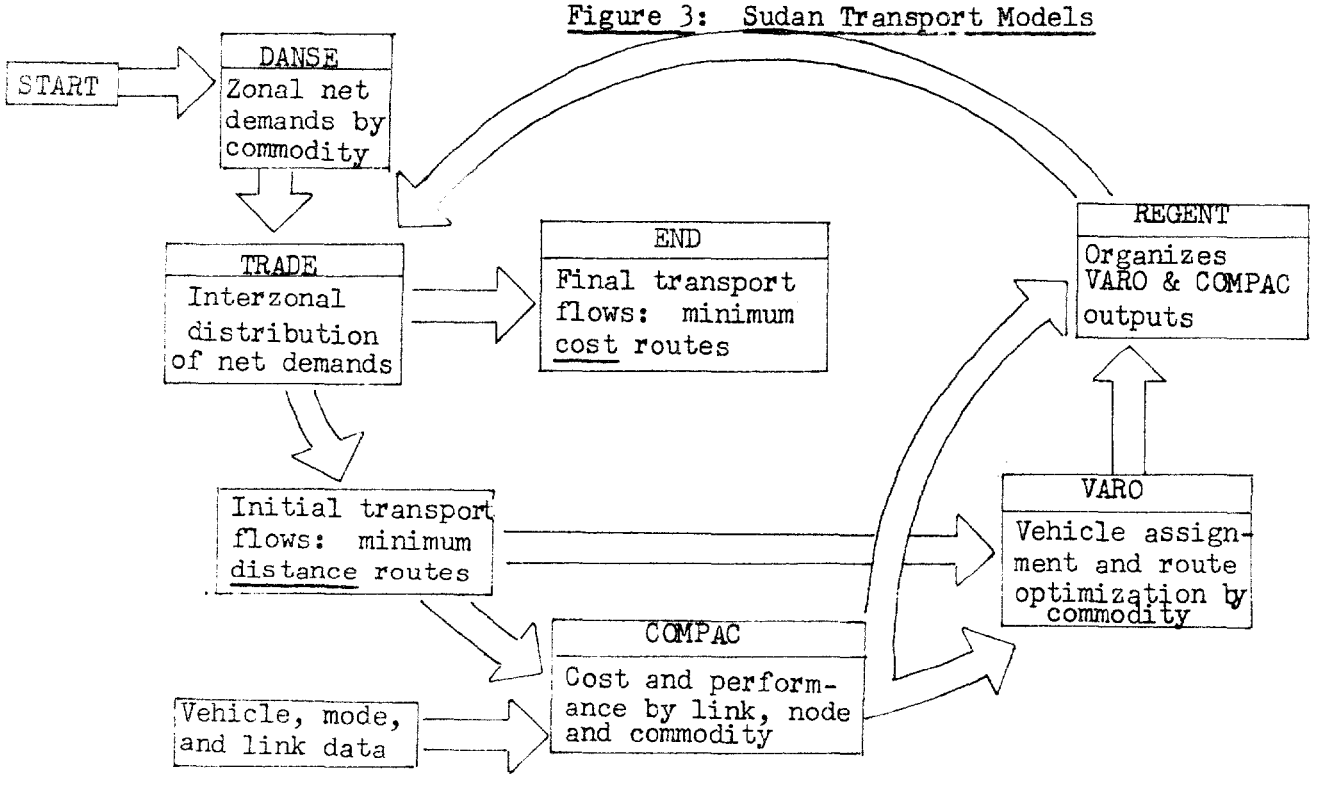
that is, in zone (n), the tonnage of a good available for interzonal transport (D_n) in period (t) equals the difference between production of that good (F_c) and consumption of the good (C_g), in the same zone in the same period.

IV. THE TRANSPORT MODELS

38. The transportation sector is simulated by three interrelated models, TRADE, COMPAC, and VARO, and an information organizing routine, REGENT. Before discussing these models individually, we would like to describe briefly how they are related. TRADE (TRansport Allocation DEvice) uses the information on net regional demands and supplies from DANSE to find the set of flows (origin-destination distribution) which satisfies all demands at the lowest total transport cost. The model is based on the classical linear programming formulation of the transportation problem. COMPAC (COmputation of Matrix Productivities And Costs) is the link cost-performance model, which calculates average vehicle productivities and operating and maintenance cost for each link or segment of every transport mode. VARO (Vehicle Assignment and Route Optimization) uses the output from COMPAC to search all possible routes (combinations of links) and select that route which, for each required shipment (O-D flow), minimizes the costs.

39. In actual usage, an initial solution for the TRADE model is found by assuming the interzonal transport demands of DANSE are distributed over the shortest distance route. Once the COMPAC and VARO model calculations, which specify routing by the minimum cost path, are complete, REGENT (REport GEnerator on National Transportation) organizes the VARO and COMPAC outputs to be fed back into TRADE. The resulting new picture of interzonal transport flows which TRADE provides is now based on minimizing total transport costs, the desired criterion. The figure below summarizes these interrelations.

Figure 3: Sudan Transport Models



40. COMPAC, VARO, and TRADE are individually described in the next three subsections.

Interzonal Distribution of Traffic Flows (TRADE)

41. Once DANSE has determined nodal zones and their respective production and consumption patterns, it is necessary to calculate the pattern of interzonal (more precisely, internodal) flows -- how much of each good flows from which supply source to which demand "sink". It is desirable to be able to distribute the traffic according to any one of a number of criteria, such as cost, distance or time.

42. Thus, the TRADE model solves transportation problems in which an optimal O-D distribution must be determined for specific commodities for which:

(i) a fixed amount is available at sources (production centers);
(ii) fixed amounts are sent directly (without transshipment) to various "sinks" (consumption centers);
(iii) the total supply is equal to the total demand;
(iv) the cost of shipment is directly proportional to the amount shipped.

43. In mathematical terms, we seek to minimize:

$$(11)\ OBJ = \sum_{j}^{n} \sum_{i}^{m} c_{ij}\, x_{ij}$$

subject to:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$\sum_{j=1}^{n} x_{ij} = s_i \qquad i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = d_j \qquad j = 1, 2, \ldots, n$$

where s_i = the supply at the ith source;

d_j = the demand at the jth "sink";

x_{ij} = the amount shipped from i to j;

c_{ij} = the cost in terms of time, money, distance, or other measure, of shipping from i to j.

Available computer programs can simulate very large networks extremely efficiently while staying well within today's computer limitations. Lockheed used the IBM SOTRC program.

Link-Cost Performance Model (COMPAC)

44. COMPAC calculates the costs of transport separately for each link and time period for each of four modes: highway, rail, river, and air. Two cost concepts are employed throughout so that two separate estimates are derived: system cost (SYSC) and shippers' cost (SHPC). The system cost consists of facility construction and maintenance costs, equipment ownership, and operating costs. Shippers' cost is the sum of transport rates charged to the shipper by the carriers, and service quality costs associated with speed, dependability and safety of service.1/ The model also calculates the productivity of vehicle classes for each mode, which is essential to the estimation of vehicle requirements by the VARO model. Since the equations for the different modes are structured in essentially the same way, we will consider the highway mode for purposes of illustration.

Transport System Cost (SYSC)

45. Facility Construction Cost. The equation below expresses total road construction cost as the product of per kilometer improvement cost on each segment of the road link, length of the segment, on an annual basis through application of an amortization factor.

1/ Neither concept (SYSC or SHPC) constitutes an estimate of economic (or social) costs as commonly defined by economic planners. If transport charges were subtracted from shippers' cost and the remainder added to system costs, the resulting measure would estimate economic (or social) costs as commonly employed. This could be approximated by a linear combination of SYSC and SHPC as discussed under the VARO model below.

(12) $$RPC_{jk} = \left(\sum_{n=1}^{N} RI_{njk} \, NDS_{nj} \right) (AF)$$

where RPC_{jk} = total improvement cost for road link (j) in period (k);

RI_{njk} = improvement cost per kilometer for the given surface type (specified in the code for each link) on segment (n) of link (j) in period (k);

NDS_{nj} = length in kilometers of segment (n);

AF = amortization (capital recovery) factor.

46. For both road and rail, the improvement cost per kilometer (RI_{njk} in the case of road) is a function of soil type, proximity to water and to crushed rock aggregate, length of haul, manpower, and equipment.

47. Vehicle-Associated Costs. The equation used to calculate cost associated with operating and owning road vehicles states that the total cost is the sum of four components: operating cost, fixed vehicle maintenance cost, cost of terminal usage adjusted for the commodity carried, and variable road maintenance cost.

(13) $$ROC_{tjk} = (VKM_{tjk})(RKC_{tjk}) + VPC_{tjk} + (RRT_{tjk})(RTF_{cjk})(RTC_{tjk}) + (RRT_{tjk})(RDS_j)(RMC_{tjk})$$

where ROC_{tjk} = total vehicle operating, ownership, and road maintenance costs for vehicle (t) on road link (j) during period (k);
VKM_{tjk} = kilometers traveled by vehicle (t) on link (j) during (k);
RKC_{tjk} = per kilometer operating cost for vehicle (t) on link (j) during (k);
VPC_{tjk} = periodic ownership cost for vehicle (t) on link (j) during (k);
RRT_{tjk} = number of round trips for vehicle (t) on link (j) during (k);
RTC_{tjk} = terminal cost per round trip for vehicle (t) on link (j) during (k);
RTF_{cjk} = cost adjustment factor for commodity (c) carried on link (j) during (k);
RDS_j = length of link (j);
RMC_{tjk} = road maintenance variable cost per kilometer per round trip for vehicle (t) on link (j) during (k).

48. Vehicle operating cost (RKC), which includes the cost of fuel, oil, tires and repairs, is drawn from Winfrey and de Weille [1/] with adjustments for the effect of different surface types on fuel consumption and tire wear based on records of three survey vehicles operated for approximately 20,000 miles. The periodic vehicle ownership cost for the existing fleet (VPC) contains allowances for depreciation, insurance, crew pay and maintenance. Tables of these costs by vehicle type and road class are generated and stored for reference in the calculation of the vehicle coefficients.

Shippers' Cost (SHPC)

49. The four major categories of shippers' service cost included in the model are: interest cost on goods in transit (S_1), inventory cost due to variability in transit time (S_3), transport rates or carrier charges (S_4), and losses caused by spoilage, breakage, and pilferage during loading, unloading and transit (S_2 and S_5). The model also includes a measure of passenger time lost in transit and of cost due to transfer operations.

50. The opportunity cost of capital tied up in goods in transit is expressed as a function of a shipment's value, the interest rate and the transit time.

(14) $S_1 = C\,[\exp(kt) - 1]$,

where S_1 = interest cost due to transit time;
C = per ton value of the cargo;
exp = the exponential function ($\exp(x) = e^x$, $e = 2.718..$);
k = daily rate of interest;
t = travel time in days.

51. A significant variability in transit time is likely to create a cost for the shipper whether he ships prematurely and incurs storage charges or ships late and risks penalty for late delivery. The following method estimates this cost (S_3) under the assumption that the shipper seeks to minimize it. We also assume that transit time is normally distributed. The general expression for expected cost due to storage and delay penalties will be:

1/ The consultants used a preliminary draft of Robley Winfrey's Economic Analysis for Highways, Scranton, Pennsylvania: International Textbook Co., 1969; Jan de Weille, Quantification of Road User Savings, World Bank Occasional Paper No. 2, Baltimore: The Johns Hopkins Press, 1966.

$$(15) \quad C(x) = C_1 \int_{-\infty}^{T} (T - z)\, N(z;\, x+t, \sigma)\, dz + C_2 \int_{T}^{+\infty} (z - T)\, N(z;\, x+t, \sigma)\, dz,$$

where C_1 = storage cost, incurred by premature arrival;
C_2 = late delivery penalty;
T = date shipment was promised;
z = date shipment actually arrives;
w = weight of shipment;
x = shipping date;
t = average transit time;
σ = standard deviation of transit time;
y = total transit time;
$N(z;\, x+t,\ )$ = normal distribution with mean at $x+t$ and variance σ^2.

52. This expression for $C(x)$ is differentiated and solved explicitly to determine the optimum time of shipment and the minimum inventory cost in a given case. If we assume that there is a minimum transit time and that after that arrival time is governed by the exponential distribution, we can derive

$$(16) \quad S_3 = C_1 \sigma \ln \left[\frac{C_1 + C_2}{C_1} \right] .$$

53. Cargo casualty may occur while in transit (S_2) or when the cargo is being loaded or unloaded (S_5). The former is estimated as a function of transit time, while the latter is directly related to the number of transfers.

$$(17) \quad S_2 = C_o \left[1 - \exp(-pt) \right]$$

$$(18) \quad S_5 = b\, n\, C_o$$

where C_o = value of the cargo;
p = spoilage loss per day;
b = average rate of loss due to breakage and pilferage;
n = number of transfers.

Vehicle Assignment and Route Optimization (VARO)

54. The VARO model is used to determine optimum vehicle routing (including intermodal allocation), vehicle requirements, and possible construction of new links in the transport network to serve a known pattern of interzonal shipments. The model is a linear program combined with a mixed integer subroutine. The O-D distribution pattern is supplied by the TRADE model (or can be exogenously specified), while the COMPAC model provides the cost and vehicle productivity coefficients for each link of the transport system upon which the cost minimization is based. Up to 20 new links may be (exogenously) proposed for consideration in each time period by specifying construction costs; the computations of the mixed integer subroutine can determine whether or not each new linkway is to be constructed.[1/]

55. The planner can optimize either system cost (SYSC) or shipper's cost (SHPC), or, by introducing a linear factor, alpha (α), any linear combination of the two. As previously described, COMPAC calculates all components of SYSC and SHPC; these cost coefficients are inputs for the VARO model. Thus, the possible objective functions are given by (19), (20) and (21).

$$(19) \quad SYSC = \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{k=1}^{K} OC_{ijk} V_{ijk} + \sum_{i=1}^{I} \sum_{k=1}^{K} IDV_{ik} IDC_{ik} + \sum_{i=1}^{I} \sum_{k=1}^{K} VC_{ik} PV_{ik} + \sum_{j=1}^{J} \sum_{k=1}^{K} A_{jk} E_{jk}$$

where SYSC = system cost;
OC_{ijk} = operating and maintenance cost of vehicle type (i) on link (j) during period (k);

1/ Apparently, the operational version of the VARO model, which handles up to 17 commodities, 5 transfers and 20 new routes, relates to the base period only. A growth version of the model, which would determine the time period in which new construction is to take place is referred to, but computational requirements for a realistic problem would exceed the capacity of the IBM 7094 with which the Lockheed team was working, e.g. for 50 internodal flows and five commodities, a maximum of three time periods could be considered; for more than ten commodity classes the model is restricted to one period. See Lockheed Final Report, Exhibit 6.3-11.

V_{ijk} = number of vehicles of type (i) operating on link (j) during (k);
IDV_{ik} = number of idle vehicles of type (i) in period (k);
IDC_{ik} = cost of an idle vehicle of type (i) during (k);
VC_{ik} = cost of purchasing a vehicle of type (i) in (k);
PV_{ik} = cumulative number of vehicles of type (i) purchased up to period (k);
A_{nk} = cumulative amortized costs of link (n) established in period (k);
E_{nk} = existence factor: equals unity if link (n) exists in (k) and zero if it does not.

(20) Alternately, $$SHPC = \sum_{i=1}^{I} \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{m=1}^{5} S_m V_{ijk}$$

where

SHPC = shippers' cost;
S_1 = capital loss due to transit time;
S_2 = expected value of loss due to spoilage, breakage, and theft while in transit;
S_3 = charges due to late or early delivery;
S_4 = tariff charges
S_5 = estimate of loss during transfers.

(21) Or, finally, $OBJ = \alpha\ SYSC + (1 - \alpha)\ SHPC$

56. The specified objective function is solved for the value of two variables: (V_{ijk}), the number of vehicles on each route (i.e. route assignment and vehicle requirements), and (E_{jk}), the existence or nonexistence of proposed new routes linking nodal pair (j) in period (k). The solution is found subject to constraints concerning (i) the fulfillment of each transport demand, (ii) the (fixed) capacity of each transport link, and (iii) the capital budget available for equipment investment.

<u>Demand Satisfaction-Vehicle Utilization Constraint</u>

(22) $$\sum_{i=1}^{I} PD_{ijk}\ V_{ijk} \geq D_{jk}$$

57. The number of vehicles of type (i) operating between the node-pair (j) in period (k), or (V_{ijk}), multiplied by its productive capacity in tons carried by each vehicle type, (PD_{ijk}), should meet the demand (D) for each commodity that must be moved on link (j) in period (k).

Link Capacity Limit

58. Capacity is treated simply as a fixed limit or discrete interval; if traffic volume is below the fixed capacity limit, then there is assumed to be no congestion. Thus, for highways the capacity constraint for segment (n) can be expressed very simply.

$$(23) \qquad \sum_{j=1}^{J} \sum_{i=1}^{I} V_{ijkn} - C_n \leq 0 \text{ for all } n,$$

where C_n is the capacity of the link in numbers of vehicles per time period, exogenously determined from engineering studies.

59. For railways, the capacity, which is expressed in numbers of trains per day, is dictated by the number of sidings and the length of time necessary for a train to traverse the given segment:

$$(24) \qquad \sum_{j=1}^{J} \sum_{i=1}^{I} \frac{PT_{ink}}{T_{ijk}} \cdot \frac{V_{ijkn}}{VT_{jkn}} - LM_{nk} \leq 0$$

where PT_{ink} = traveling time for vehicle type (i) on segment (n) in period (k);
T_{ijk} = traveling time for vehicle type (i) on node-pair (j) in period (k);
V_{ijkn} = number of rail vehicles of type (i) operating between node-pair (j) on segment (n) in period (k);
VT_{jkn} = number of rail vehicles per train traveling between node-pair (j) on segment (n) in period (k);
LM_{nk} = number of sidings.

Vehicle Availability Constraints

60. The number of vehicles of type (i) required to be purchased in time period (k), ($\overline{PV}$), is equal to the total number of vehicles required to handle the traffic, less the total number of vehicles available at the beginning of the time period, (V_{ik}), net of vehicles out of service, (IDV_{ik}). Thus:

$$(25) \qquad \overline{PV}_{ik} = \sum_{j=1}^{J} V_{ijk} - (V_{ik} - IDV_{ik})$$

The funds available for the purchase of new vehicles may, however, be constrained by a fixed capital budget, (CAP_k), or

$$(26) \qquad \sum_{i=1}^{I} \overline{VC}_{ik} \, \overline{PV}_{ik} \leq CAP_k$$

where $\overline{VC}_{ik}$ is the purchase price of a vehicle of type (i).