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Modelling Global Demand for Fertilizer

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I. SUMMARY AND INTRODUCTION

1. This paper describes the recently-completed fertilizer model which is used by the Commodity Studies and Projections Division. The fertilizers included in the model are nitrogen, phosphate and potash. The model is a partial equilibrium model of production and consumption. A separate module was built for each of the three fertilizers, but cross linkages between the different types of fertilizers within a country or a region were established. On the demand side 24 regions are distinguished; 16 of these are individual countries while the remaining countries are modeled as 8 regions based on the geographic location of the countries.

2. The fertilizer supply model was built in a more aggregated way. For nitrogen and phosphate there are four country groups--Europe and Japan, North America, Developing Countries and Centrally Planned Economies (CPEs). The production of potash is specified only on a world basis.

3. The fertilizer demand model is linked to the Division's grains and soybean model in the sense that the two models are solved iteratively, i.e., the solutions from the grains model are entered into the fertilizer model, the results of which are then entered into the grains model. Model simulations are continued until the results become stable. 1/ Therefore, the production and prices of the four major commodities (wheat, rice, coarse grain and soybeans) in the grains and soybeans model are endogenous to the fertilizer model.

4. Major changes have occurred in the last 20 years in agricultural production. These changes have also changed the nature of the demand for

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1/ See Mitchell (1985).

fertilizers. Growth in fertilizer consumption is especially evident in the developing countries where there is generally a rapidly-increasing demand for farm inputs. But major macroeconomic changes, fiscal and monetary, have also taken place and these have strongly influenced the fertilizer market. In this study we have tried to capture the most important of these historical changes and to give an idea of what to expect in the next decade or so.

5. The format of the paper is as follows. The specification of the demand side of the model is introduced in Section II followed by the specification of the supply side in Section III. Section IV is devoted to the price equation specification while Section V presents some results of the estimated equations. An ex-post simulation of the model carried out over the historical period is presented in Section VI with summary statistics as well as graphical representation of the behavior of some of the major variables in the model. Short- and long-term projections for both production and consumption, run over the 1985-2000 period, are presented in Section VII. Section VIII contains the conclusions and suggestions for further work.

II. DEMAND FOR FERTILIZER

6. The demand for fertilizers is derived from the demand for agricultural products. In turn, the demand for products such as food and fiber are partially determined by the growth in income and population. In this study these macro-variables are assumed to influence the demand for fertilizers only through the demand for crops and other agricultural products.

7. Several approaches to modeling fertilizer demand have been used in the past and each has its advantages and disadvantages. The most popular approach has been the pure maximization approach as applied to an agronomic fertilizer response function. Although the calculus is straightforward it involves two critical elements, as has been mentioned by Timmer (1974). First, there is the necessity to assume some form of maximizing behavior on the part of the farmers and second, knowledge of the relevant agronomic function is required. One concern with this approach is the difficulty of accepting the underlying assumption of profit maximization by farmers the world over. Very few farmers equate marginal costs with marginal revenues for an input without regard to risk, uncertainty, knowledge and other constraints. Timmer has estimated that during the period 1960-80 the marginal return for the last dollar spent on fertilizer varied throughout the United States over the range $1.6 to $7.3. For developing countries, even higher marginal returns could be expected because of the widespread constraints on supplies.

8. Timmer mentions the additional problem of the difference in the use of fertilizer per hectare in various places in the world at the same price level (implying a different agronomic response function with respect to fertilizer use). The optimal level of fertilizer use varies significantly even within a country, depending on the location, the crop for which it is used,

the degree of water control, the availability of fertilizer, and other factors.

9. A second approach to analyzing fertilizer demand is the prescriptive approach; but although it is useful in understanding the nature of fertilizer use, it is somewhat irrelevant to our study. As noted by Barker (1972) "The analysis of fertilizer response can be useful. But the major problem is not necessarily one of determining the optimum level of fertilizer input, but rather of identifying the factors that constrain yield response on farmers' fields..." In short, neither the indirect demand functions derived from agronomic response functions nor the prescriptive approach to fertilizer demand give any idea of the impact of the different factors which affect fertilizer demand.

10. This paper adopts the direct approach following the work done by Griliches (1958, 1959), Timmer (1974) and others. The approach is a simple version of Nerlove's distributed lag technique, where the demand for fertilizer is specified and estimated directly.

11. The specification consists of two parts: a long-run demand function and an adjustment equation. The demand function assumes that the use of fertilizer is a function of the relevant product and input prices. Prices of agricultural products and fertilizers are affected by the aggregate response of both fertilizer consumption and product production, as well as other factors. The functional form of the demand equation is assumed to be linear in the variables, or in some cases, especially in the developing countries, linear in the logarithms of the variables.

12. The adjustment equation is based on the idea that changes in prices (both input and output) and other independent variables take time to be fully

reflected in demand. It is very clear, especially in the case where lower prices make fertilizer available to new users, that it takes time to introduce a new factor into the production process.

13. The partial adjustment method which was introduced by Griliches and others, consists basically of distinguishing between actual and "desired" levels of use. The demand function determines the long-run "desired" level of use. However, in the short run, the desired level may not be fulfilled and the actual level changes only by some fraction of the difference between the desired use and the current use. The adjustment equation takes into account the direction of the "desired" level but does not permit an instantaneous change. The structural form of the adjustment equation is a function of the difference between the "desired" and actual use of the fertilizer.

14. If we let lower case letters denote the logarithms of the variables, then:

(1) $$tc^*_{it} = \alpha_{01} + \beta_1 p_{it} + \beta_2' \underset{\sim}{ha}_t + \beta_3' \underset{\sim}{p}_{jt} + \varepsilon_t$$

represents the demand for fertilizer, where tc^*_{it} is the desired fertilizer consumption of fertilizer i at time t. p_{it} is the real price of fertilizer (deflated by the CPI). A vector, $\underset{\sim}{ha}_t$ is harvested areas of various crops planted. A vector, $\underset{\sim}{p}_{jt}$ consists of all other relevant prices in the production process such as other fertilizers and agricultural products.

The adjustment equation is:

(2) $$tc_{it} - tc_{it-1} = \gamma(tc^*_{it} - tc_{i\ t-1})$$

where γ is the adjustment coefficient.

In original units we would then have

(2') $$\left(\frac{tc_{it}}{tc_{it-1}}\right) = \left(\frac{tc^*_{it}}{tc_{t-1}}\right)^{\gamma}$$

The percentage change in actual consumption is a power function of the percentage difference between the "desired" and actual consumption. Substituting (1) into (2) and solving for tc_{it} gives:

(3) $$tc_{it} = \gamma\alpha_0 + \gamma\beta_1 p_{it} + \gamma\beta_2' \underset{\sim}{ha}_t + \gamma\beta_3' \underset{\sim}{p}_{jt} + (1-\gamma)tc_{it-1} + \gamma\varepsilon_t$$

This is the basic equation estimated in the model and the results are given in the following section.

15. Since the variables are in logarithms, the short-run elasticity of fertilizer demand with respect to its own price is given as $\gamma\beta$ and the long run elasticity is given as $\frac{\beta_1\gamma}{1-(1-\gamma)}$. Some estimates of these long- and short-run elasticities are given in the discussion of the model results. Although such a model usually performs quite well in the sense of tracking historical observations, this good performance is partially due to the fact that a lagged dependent variable is included (although with good reason). The interpretation of the size of γ may be disputed because the lagged variable captures some of the effects of excluded variables which therefore biases the estimation of $(1-\gamma)$ upward.

16. The demand side as a whole was constructed in a way that allows every region to change its demand, while the demand for the world is the sum of its 24 regions. The list of the regions and countries in each is shown in Table 2.1.

TABLE 2.1: MODEL REGIONS

Country/Region	Countries
Industrial Countries	
Australia	Australia
Canada	Canada
EC-10	Belgium, France, Italy, Luxembourg, Netherlands, W. Germany, United Kingdom, Ireland, Denmark, Greece
Japan	Japan
Other Industrial Countries	Austria, Finland, Iceland, Malta, Norway, Portugal, Spain, Sweden Switzerland, New Zealand
United States	United States
Centrally Planned Economies	
Eastern Europe	Albania, Bulgaria, Czechoslovakia, East Germany, Hungary, Poland, Romania, Yugoslavia
U.S.S.R.	Union of Soviet Socialist Republic
Developing Countries	
Argentina	Argentina
Brazil	Brazil
Central Africa	Botswana, Lesotho, Namibia, Swaziland, Kenya, Malagasy Republic, Malawi, Mozambique, Tanzania, Uganda, Zambia, Angola, Burundi, Cameroon, Central African Republic, Chad, Congo, Ethiopia, Djibouti, Benin, Gabon, Gambia, Ghana, Guinea, Equatorial Guinea, Guinea-Bissau, Ivory Coast, Liberia, Mali, Mauritania, Mauritius, Niger, Reunion, Rwanda, Senegal, Sierra Leone, Somalia, Sudan, Togo, Upper Volta, Zaire, Zimbabwe

.../continued

.../Table 2.1 continued

China	China
East Asia	Burma, Kampuchea, Laos, Vietnam, Hong Kong, Singapore, South Korea, Brunei, Malaysia, Philippines, North Korea, Mongolia,Pacific Islands, Papua New Guinea, Fiji Islands
Egypt	Egypt
India	India
Indonesia	Indonesia
Latin America and Caribbean	Bahamas, Barbados, Bermuda, Belize, Other Caribbean Islands, Cuba, Dominica, Dominican Republic, Jamaica, Trinidad and Tobago, Honduras, Nicaragua, Panama, Costa Rice, El Salvador, Guatemala, Haiti, Bolivia, Chile, Colombia, Ecuador, French Guiana, Guyana, Paraguay, Peru, Surinam, Uruguay, Venezuela
Mexico	Mexico
Nigeria	Nigeria
North Africa & Middle East	Algeria, Bahrain, Cyprus, Iran, Iraq, Israel, Kuwait, Libya, Oman, Qatar, Saudi Arabia, United Arab Emirates, Jordan, Lebanon, Morocco, Syria, Tunisia, Turkey, Yemen A.R., Yemen D.M.
Pakistan	Pakistan
South Asia	Afghanistan, Bangladesh, Bhutan, Nepal, Sri Lanka
Thailand	Thailand

III. FERTILIZER SUPPLY SPECIFICATION

17. The fertilizer supply model is based on work done by Choe (1986). It is presented here for completeness, though it was not part of this study.

18. The fertilizer industry is assumed to be competitive and is represented by a risk-neutral profit-maximizing firm. Further, fertilizer production is assumed to require two factors of production, a variable factor called material (M) and a quasi-fixed factor capital (K) or production capacity. 1/ It is assumed that fertilizer production technology is represented by the following quadratic production function:

$$Q_t = F(M_t, K_t ; t)$$

$$= \alpha_0 + \alpha_{0t} t + \alpha_K K_t + \alpha_M M_t + 1/2\ \alpha_{KK} K_t^2$$

$$+ 1/2\ \alpha_{MM} M_t^2 + \alpha_{KM} K_t M_t. \qquad (1)$$

The first- and second-order conditions for profit maximization require:

$$\alpha_K > 0,\ \alpha_M > 0,\ \alpha_{KK} < 0, \text{ and } \alpha_{KK}\alpha_{MM} - \alpha_{KM}^2 > 0.$$

1/ The labor input is ignored because it is a relatively small part of the cost of fertilizer production. The most important material input for ammonia production is natural gas. Phosphate rock is the main raw material for phosphate fertilizers. Relatively simple processing is required to transform run-of-mine potash into potash fertilizers, involving little material input.

19. Under uncertainties of future output and input prices, the firm's problem is to choose a contingency plan for inputs that maximizes the present value of expected future profits:

$$\text{Max } E_T \sum_{t=T}^{\infty} \rho^{t-T}[F(K_t, M_t ; t) - w_t M_t - v_t K_t - 1/2\ \beta\ I_t^2], \qquad (2)$$

subject to given initial capacity, K_T, and

$$I_t = K_t - (1-\delta)K_{t-1}, \qquad (3)$$

where E_T is the mathematical expectation operator conditional on information available at time T; the term $\rho = (1+r)^{-1}$ is the discount factor where ρ is the interest rate; w_t is the price of material input and v_t is the service price of capital, each normalized by the output price; I_t is the gross investment, and δ is the constant depreciation rate.

20. The last term in the objective function $(1/2\ \beta\ I_t^2)$ reflects the costs of adjusting the quasi-fixed factor. There are two ways of interpreting the adjustment cost term. One is to consider it as an expenditure, i.e., as a factor cost. The other is to consider it in the form of foregone output. In the latter case, the adjustment cost term can be included as an argument of the production function. The implications of this different treatment for the derivation of the model will be seen later.

21. It has been shown by Sargent (1979) that the first-order necessary conditions for the stochastic optimization problem can be derived by substituting (1) and (3) into (2) with respect to M_t and K_t for $t=T, T+1, \ldots, \infty$:

$$\alpha_M + \alpha_{MM} M_t + \alpha_{KM} K_t - w_t = 0, \tag{4}$$

$$\alpha_K + \alpha_{KK} K_t + \alpha_{KM} M_t - v_t - \beta\,[K_t - (1-\delta)K_{t-1}]$$

$$+ E_t\{\rho\beta(1-\delta)[K_{t+1} - (1-\delta)K_t]\} = 0, \tag{5}$$

where the certainty equivalence property of the quadratic specification is used. The condition (4) states that the marginal value product of material input should equal its price. The condition (5) is the "Euler equation" which states that the marginal value product of capital plus the expected future savings in adjustment cost by investing now rather than later should equal the service price of capital plus the current adjustment cost. Equation (4) yields the familiar demand equation for the variable factor; demand for the variable factor depends only on the current period values of the relevant variables.

22. Equation (5) is a second-order stochastic difference equation, the solution of which requires two boundary conditions. One of them is provided by the initial capacity, K_t, and the other is the transversality (terminal) condition:

$$\lim_{t \to \infty} \rho^{t-T-1} \{\alpha_K + \alpha_{KK} K_t + \alpha_{KM} M_t - v_t - \beta\,[K_t - (1-\delta)K_{t-1}]\} = 0, \tag{6}$$

which states that the adjustment cost should not affect the optimal capacity level in the very long run. To solve the difference equation, substitute (4) into (5) and rearrange terms to get:

$$\rho\, E_T\, K_{t+1} + \phi\, K_t + K_{t-1} = B + C\, w_t + D\, v_t \tag{7}$$

where

$$\phi = D\ \{A - \beta[1 + \rho(1-\delta)^2]\}, \tag{8a}$$

$$A = (\alpha_{KK}\alpha_{MM} - \alpha_{KM}^2)/\alpha_{MM}, \tag{8b}$$

$$D = \beta^{-1}(1-\delta)^{-1}, \tag{8c}$$

$$C = D(\alpha_{KM}/\alpha_{MM}, \tag{8d}$$

$$B = (\alpha_M\alpha_{KM} - \alpha_K\ \alpha_{MM})/\alpha_{MM}, \tag{8e}$$

Sargent (1979) showed that the stochastic difference equation (7) has two characteristic roots, λ and $(1/\rho\lambda)$, such that $\lambda < 1 < 1/\rho\lambda$. To satisfy the transversality condition (6), the difference equation should be solved backward with the stable root (λ) and forward with the unstable root $(1/\rho\lambda)$, to get:

$$K_t\ \lambda\ K_{t-1} - \lambda \sum_{i=0}^{\beta} (\rho\lambda)^i\ E_t\ (B + C\ w_{t+1} + D\ v_{t+1}), \tag{9}$$

where

$$\lambda = [-\phi + \phi^2 - 4\rho)^{\frac{1}{2}}]/(2\rho). \tag{10}$$

23. It is postulated that the firm's expectations about future prices are formed according to the following two univariate autoregressive processes: 1/

$$E_{t-1}\ w_t = \sum_{i=1}^{m} \theta_{1i}\ w_{t-i} + u_{1t}, \tag{11a}$$

$$E_{t-1}\ v_t = \sum_{i=1}^{m} \theta_{2i}\ v_{t-i} + u_{2t}, \tag{11b}$$

where u_{1t} and u_{2t} are independently distributed white noise. The exponential order of the above processes are assumed to be less than $\rho^{-\frac{1}{2}}$ in order to

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1/ This formulation is chosen instead of the more general vector autoregressive representation to avoid unnecessary complication. The markets for the capital and raw material inputs for fertilizers are quite separate and one is not expected to have significant influence on the other.

ensure their stability. It has been shown by Hansen and Sargent (1980) that if (11) is the expectations formation process, then the dynamics of investment is as follows.

$$\sum_{i=0}^{\infty} (\rho\lambda)^i \; E_t \; w_{t+1} = \sum_{i=0}^{m-1} \mu_{1i} \; w_{t-1}, \tag{12a}$$

$$\sum_{i=0}^{\infty} (\rho\lambda) \quad E_t \; v_{t+i}^{i} = \sum_{i=0}^{m-1} \mu_{2i} v_{t-1}, \tag{12b}$$

where

$\Pi = \rho\lambda$ and

$$\mu_{10} = (1 - \theta_{11}\Pi - \theta_{12} \; \Pi^2 - \ldots - \theta_{1m} \; \Pi^m)^{-1},$$

$$\mu_{11} = \mu_{10} \; (\theta_{12} \; \Pi + \theta_{13} \; \Pi^2 + \ldots + \theta_{1m}\Pi^{m-1}),$$

$$\mu_{12} = \mu_{10} \; (\theta_{13} \; \Pi = \theta_{14} \; \Pi^2 + \ldots \; \theta_{1m} \; \Pi^{m-2}),$$

. . .

$$\mu_{1m-1} = \mu_{10} \; (\theta_{1m} \; \Pi), \tag{13}$$

and likewise for μ_{2i}. Substituting (12) into (9), leads to

$$K_t \; \lambda \; K_{t-1} - \frac{\rho B \lambda^2}{1-\rho\lambda} + \lambda \; C \sum_{i=0}^{m-1} \mu_{1i} \; w_{t-i} - \lambda \; D \sum_{i=0}^{m-1} \mu_{2i} v_{t-i},$$

24. Following the suggestion of Epstein and Yatchew (1985), the following set of equations are estimated:

$$Q_t = \alpha_0 + \alpha_{0t} t + \alpha_K K_t + \alpha_M M_t + 1/2\, \alpha_{KK}^2 K_t + 1/2 \alpha_{MM} M_t^2$$

$$+ \alpha_{KM} K_t M_t + \varepsilon_{1t}, \tag{14}$$

$$K_t = \lambda K_{t-1} - \frac{\lambda}{\rho} B\lambda^2/(1-\delta) + C \sum_{i=0}^{m-1} \mu_{1i} w_{t-i}$$

$$- D \sum_{i=0}^{m-1} \mu_{2i} v_{t-1} + \varepsilon_{2t}, \tag{15}$$

$$w_t = \sum_{i=1}^{m} \hat{\theta}_{1i} w_{t-1} + \varepsilon_{3t}, \tag{16}$$

$$v_t = \sum_{i=1}^{m} \hat{\theta}_{2i} v_{t-1} + \varepsilon_{4t}, \tag{17}$$

where the parameters satisfy the restrictions in (8), (10) and (13), and ε_{it}'s are random variables assumed to be jointly normally distributed with zero mean and constant non-singular covariance matrix, and are serially independent. The inclusion of the production function in the specification has the advantage of permitting identification of both technology and expectations, as suggested by Epstein and Yatchew. The system of equations yields closed-form solutions for all the parameters of the model (α_{0t}, α_K, α_M, α_{KK}, α_{MM}, α_{KM}, β, ρ, δ, θ_{1i}, θ_{2i}, $\hat{\theta}_{1i}$, $\hat{\theta}_{2i}$), i=1, 2, ..., m), If ρ is treated as a variable, then equation (15) becomes the flexible accelerator investment equation.

25. The autoregressive structure in (11) postulated for the firm's expectations formation and embedded in equation (15) is more general than the rational expectations hypothesis. The equations (16) and (17) describe the evolution of actual prices. If expectations are rational, the actual prices will evolve as expected and the equations (11) and (16)-(17) will be identical

$\theta_{\ell i} = \hat{\theta}_{\ell i}$, $\ell = 1, 2$; $i=1, 2, \ldots, m$). If the rational expectations hypothesis holds, the parameter restrictions in (13) will apply with θ_{1i} and θ_{2i} replaced by $\hat{\theta}_{1i}$ and $\hat{\theta}_{2i}$. One can also experiment with the static expectations hypothesis adopted in earlier dynamic factor demand studies such as Berndt, Fuss and Waverman (1979). In the case of static expectations, $\theta_{11}=\theta_{12}=1$ and $\theta_{1i}=\theta_{2i} = 0$, for $i=2, \ldots, m$, which greatly simplifies the restrictions in (13).

26. The hypotheses associated with the restrictions can be tested by progressively imposing restrictions on the parameters. Three different versions of the model have been estimated--the unrestricted model and the rational and static expectations models; they differ only in the way the investment equation (15) is parameterized. It is useful at this point to set out the investment equations for the three cases.

Unrestricted Model

$$K_t = \beta_0 + \lambda K_{t-1} + \gamma_1 w_t + \gamma_2 v_t + \varepsilon_{2t} \tag{18}$$

Rational Expectations Model

$$K_t = -\rho B\gamma^2/(1-\rho\lambda) + \lambda K_{t-1} + \lambda C w_t/(1-\hat{\theta}_{11}\rho\lambda) - \lambda D v_t/(1-\hat{\theta}_{21}\rho\lambda) + \varepsilon_{2t} \tag{19}$$

Static Expectations Model

$$K_t = -\rho B\lambda^2/(1-\rho\lambda) + \lambda K_{t-1} + \lambda C w_t/(1-\rho\lambda) - \lambda D v_t/(1-\rho\lambda) + \varepsilon_{2t} \tag{20}$$

Note that the "unrestricted" model is equivalent to the general autoregressive model in equation (15), except that the intercept term is free rather than constrained to take on a certain value implied by the adjustment cost and the autoregressive expectations. Rational and static expectations models are self explanatory.

27. The three models are estimated with and without the adjustment cost term ($1/2\beta\ I^2$) in the production function (14). This corresponds to the alternative interpretations of the adjustment cost mentioned earlier.

IV. SPECIFICATION OF PRICE FORMATION

World Price Specification

28. The model is solved for the "world price" which equates world demand with the world supply. How the price is determined is a complex problem that involves questions about the structure of the market in which the commodity is traded. In the fertilizer market it would not be too extreme to assume that the market is competitive and therefore the "world price" equation is such that it corresponds to the law of supply and demand

29. Following this idea it is straightforward to assume that three variables are to be determined by the model: supply, demand and the market-clearing price. The equilibrium condition of demand being equal to supply is achieved by a price that will determine such a condition and will take the short-term differences between demand and supply into account. However, in such a case it must be assumed that any excess of production over supply would be willingly held. Therefore, the price equation was constructed to give weight to the level of production (or capacity), the level of consumption and the desire to hold inventories. It still remains to be determined what path prices will take from one equilibrium situation to another and this relates to what one is willing to assume about the expected behavior of the participants in the market.

30. In order to simplify the price determination as well as the formula of the reduced form for the estimated price equation, rational expectations in price behavior is assumed. Under this assumption, a simple linear model of world demand and supply consists of the following equations:

(1) $$q_t^s = \alpha_0 + \alpha_1 p_t + \alpha_2 p_t^e + \phi_s X_{st} + u_{st}$$ (supply)

(2) $$q_t^d = \beta_0 - \beta_1 \beta_t + \phi_d X_{dt} + u_{dt}$$ (demand)

(3) $$p_t^e = E(p_t/\Omega_{t-1})$$ (expectations)

(4) $$q_t^s = q_t^d$$ (market clearing conditions)

where

q_t^d quantity desired for consumption and inventories

q_t^s quantity supplied

p_t price vector

X_{st}, X_{dt} exogenous variables of supply and demand respectively

Ω_{t-1} the set of information available at time t-1

and u_{st}, u_{dt} coming from a joint normal distribution.

31. Substituting (1) - (3) into (4) and solving for p_t will give

$$p_t (\alpha_1 + \beta_1) = (\beta_0 - \alpha_1) + (\phi_d X_{dt} - \phi_s X_{st}) - \alpha_2 p_t^e + (u_{dt} - u_{st})$$

(5) $$p_t = \left(\frac{\beta_0 - \alpha_0}{\alpha_1 + \beta_1}\right) + \left(\frac{1}{\alpha_1 + \beta_1}\right) (\phi_d X_{dt} - \phi_s X_{st})$$

$$- \frac{\alpha_2}{(\alpha_1 - \beta_1)} p_t^e \frac{1}{(\alpha_1 + \beta_1)} + (u_{dt} - u_{st})$$

32. The set of equations (1)-(4) is consistent with the world production and consumption equations which have been previously discussed. Nevertheless, to ensure the homogeneity property of demand and supply, the relevant deflators such as the world price level and the exchange rate should be used to deflate the price wherever the price is used in both the supply and demand equations. That is to say, the price used should be the "real" price. Rewriting (5) gives:

$$(5') \quad p_t = \mu_1 + \mu_2 X_{dt} - \mu_3 X_{st} - \mu_4 p_t^e + \zeta_t$$

where μ_1, μ_2, μ_3, μ_4 are defined as:

$$\mu_1 = \frac{\beta_0 - \alpha_0}{A}$$

$$\mu_2 = \frac{\phi_d}{A}$$

$$\mu_3 = \frac{\phi_s}{A}$$

$$\mu_4 = \frac{\alpha_2}{A}$$

$$A = \alpha_1 + \beta_1$$

$$\xi_t = U_{dt} - U_{st}$$

However, (5') is not an estimable equation since the expected price formation, p_t^e, should be estimated first. If the expected price formation has the following simple rule

$$\hat{p}_t = \rho \frac{p_{t-1}}{p_{t-2}} + \upsilon_t$$

where υ_t is white noise,

then $p_t^e = E(p_t/\Omega_{t-1}) = \rho \frac{p_{t-1}}{p_{t-2}} + E(\upsilon_t/\Omega_{t-1}) = \rho \frac{p_{t-1}}{p_{t-2}}$ without dependency on the future values of the exogenous variables. It can now be introduced into equation (5') without further complication, obtaining:

$$(6) \qquad p_t = \delta_0 + \delta_1 K_t + \delta_2 \hat{TC} + \delta_3 \frac{p_{t-1}}{p_{t-2}} + \delta_4 \, EXR_t + \delta_5 \, INTR_t$$

where K_t is the capacity and represents all the exogenous variables determining the potential production at a given year t. $\hat{TC}$ represents the exogenous and endogenous variables determining the consumption and is estimated as a behavioral equation at a first-stage estimation. EXR_t is the effective exchange rate index, reflecting strength of the dollar relative to other currencies since the price is in US$ per metric ton. INTR is the real short-term interest rate representing the cost of inventory holding since implicitly it was assumed that the demand q_t^d is both for inventories as well as for actual consumption at time t.

Domestic Price Linkages

33. It is necessary to use domestic prices, or prices at the farmgate, in order to measure the correct price effect on fertilizer consumption. However, it is not always easy, and sometimes impossible, to obtain the relevant data. In cases where fertilizer prices paid by farmers were available, this price was linked to the world price through a simple equation as follows. If the

world price at time t be p_t expressed in US$ per metric ton, p_{it} is domestic prices in local currency per metric ton in region i at time t, and XRT_{it} is the exchange rate of region i at time t expressed in local currency per one dollar. Then

$$p^{\$}_{it} = p_{it}/XRT_{it}$$

and

$$p^{\$}_{it} = f(p^{\$}_{i,\ t-1}, p_t)$$

The error term of this equation is assumed to be autoregressive of order 1, i.e. $\varepsilon_t = \lambda\varepsilon_{t-1} + \zeta_t$ where ζ_t is white noise.

34. In cases where we did not have domestic prices the international price (the so-called world price) was used as a border price, i.e., the deflated price in region i at time t, defined as:

$$p_{it} = p_t * XRT_{it}/CPI_{it}$$

This is a good approximation in a situation where the extent of subsidies is low. It is not a very good approximation where taxes and/or subsidies are high and therefore it does not reflect the prices farmers are facing. Nevertheless, it is often the only measure for fertilizer prices obtainable.

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V. ESTIMATED EQUATIONS

Consumption Equations

35. Demand equations were estimated for each of the fertilizers and for each of the regions included in the model. A sample of equations are presented below.

NITROGEN CONSUMPTION

BRAZIL

```
NTTCBRA  =  -  13841.2695 DFNTPFBRA2 +  0.2817 NTTCBRA(-1)
                (-1.3904)               (1.3258)

         +  0.0537 SBHABRA(-1)  +  0.0074 CRHABRA(-1)
            (3.0789)    (1.9308)

         R-SQUARED(CORR.): 0.914    SEE:  77.707      DW: 2.50
         PERIOD OF FIT:   1966   1982
         F(  4, 13):    43.012
```

MEXICO

```
NTTCMEX  =  -186.3516  -  1573.0941 DFNTPFMEX +  0.9041 NTTCMEX(-1)
             (-1.7955)     (-2.3193)            (14.5988)

         +  0.4632 WHHAMEX
            (3.6758)

         R-SQUARED(CORR.): 0.981    SEE:  33.877      DW: 2.75
         PERIOD OF FIT:   1963   1981
         F(  3, 15):   304.630
```

LATIN AMERICA

```
NTTCLAC  =  -875.3987  -  0.3399 DFNTPFLAC +  0.4876 NTTCLAC(-1)
             (-1.7955)  (-2.2754)     (2.6188)

         +  0.1141 CGHALAC  +  0.4619 RPHALAC
            (1.4579)           (2.6498)

         R-SQUARED(CORR.): 0.961    SEE:  46.552      DW: 1.86
         PERIOD OF FIT:   1963   1981
         F(  4, 14):   110.788
```

NORTH AFRICA

```
NTTCNAF  =  -  0.5514 DFNTPFNAF +  1.0202 NTTCNAF(-1) +  0.0090 WHHANAF(-1)

            (-3.6210)           (28.5250)              (3.9674)

            R-SQUARED(CORR.): 0.981    SEE:  69.933      DW: 2.27
            PERIOD OF FIT:   1963   1982
            F(  3, 17):   335.732
```

EAST ASIA

```
NTTCEAS  =  - 699.0209  -  0.4754 DFNTPFEAS +  0.8492 NTTCEAS(-1)
              (-1.7633)  (-2.0930)            (9.5204)

            +  0.2162 CGHAEAS
              (2.0118)

            R-SQUARED(CORR.): 0.979    SEE:  64.051      DW: 2.26
            PERIOD OF FIT:   1963   1981
            F(  3, 15):   278.050
```

INDIA

```
NTTCIND  =  - 4494.4146  -  54.2922 DFNTPFIND +  0.7916 NTTCIND(-1)
              (-2.5018)   (-2.0202)            (7.7784)

            +  0.1469 RPHAIND
              (2.7202)

            R-SQUARED(CORR.): 0.975    SEE:  158.88      DW: 1.85
            PERIOD OF FIT:   1963   1981
            F(  3, 15):   230.459
```

CHINA

```
NTTCPRC  =  - 9484.2725  -  238.7235 DFNTPFPRC2 +  1.0598 NTTCPRC(-1)
              (-2.7834)   (-2.5707)             (13.5603)

            +  0.3927 WHHAPRC
              (2.8177)

            R-SQUARED(CORR.): 0.965    SEE:  625.36      DW: 2.26
            RHO(1): -0.388
            PERIOD OF FIT:   1968   1981
            F(  3,  9):   111.938
```

EEC - 10

```
NTTCEEC  =  - 1335.2217  +  1.0898 NTTCEEC(-1)  -  0.7083 DFNTPFEEC2
               (-0.6214)   (8.0771)              (-1.7949)

            +  654.3002 DFWHPFEEC
                (0.8960)

            R-SQUARED(CORR.): 0.986    SEE:  189.80      DW: 1.95
            RHO(1): -0.078
            PERIOD OF FIT:   1963   1983
            F(  3, 16):   446.935
```

JAPAN

```
NTTCJPN  =  306.5961  -  346.5631 DFNTPFJPN +  0.2306 NTTCJPN(-1)
             (3.2033)    (-4.1251)            (1.5600)

         +  0.2224 TCHAJPN(-1)  -  118.5227 D76
           (5.1712)               (-2.2892)

            R-SQUARED(CORR.): 0.761    SEE:  47.977      DW: 1.93
            RHO(1): -0.263
            PERIOD OF FIT:   1965   1982
            F(  4, 12):    13.767
```

UNITED STATES

```
NTTCUSA  =  - 2718.2876  -  71336.3594 DFNTPFUSA +  0.7461 NTTCUSA(-1)
               (-2.1586)      (-4.2600)            (11.4621)

            +  1374.0574 DFWHPFUSA(-1)  +  0.0972 WHCGHAUSA
   (3.0336)         (3.5109)

            R-SQUARED(CORR.): 0.976    SEE:  325.18      DW: 2.15
            PERIOD OF FIT:   1964   1981
            F(  4, 13):   171.974
```

OTHER DEVELOPED

```
NTTCODC  =  -  0.2455 DFNTPFODC +  0.8334 NTTCODC(-1)  +  0.0432 CGHAODC
             (-2.4411)            (14.8798)               (3.7322)

            R-SQUARED(CORR.): 0.982    SEE:  50.037      DW: 2.58
            PERIOD OF FIT:   1963   1981
            F(  3, 16):   331.550
```

SOVIET UNION

```
NTTCUSR  =  -1266.5438  +  1.0020 NTTCUSR(-1)  +  0.0258 WHHAUSR(-1)
              (-0.6161)  (21.6761)                (0.8819)

            R-SQUARED(CORR.): 0.988    SEE:  260.35      DW: 1.95
            PERIOD OF FIT:   1964   1982
            F(  2, 16):   726.317
```

PHOSPHATE CONSUMPTION

BRAZIL

```
POTCBRA  =  -  16538.3965 DFPOPFBRA2  +  0.8707 POTCBRA(-1)
                (-1.4680)               (6.9253)

            +  0.0059 CGHABRA  +  0.0420 SBHABRA
              (1.2927)           (1.8473)

            R-SQUARED(CORR.): 0.981    SEE:  86.454      DW: 2.10
            PERIOD OF FIT:   1964   1981
            F(  4, 14):   220.226
```

MEXICO

```
POTCMEX  =  - 680.0370  +  0.7097 POTCMEX(-1)  +  0.0678 CGHAMEX(-1)
              (-4.4882)   (4.9484)                (4.0144)

            +  0.1613 CRHAMEX(-1)
              (2.7782)

            R-SQUARED(CORR.): 0.912    SEE:  25.872      DW: 2.51
            PERIOD OF FIT:   1965   1982
            F(  3, 14):    59.961
```

LATIN AMERICA

```
POTCLAC  =  - 537.5496  -  0.0956 DFPOPFLAC +  0.5800 POTCLAC(-1)
              (-2.1729)  (-1.1809)            (3.4754)

            +  0.1130 CGHALAC  +  0.0572 RPHALAC(-1)
              (2.4313)           (1.1726)

            R-SQUARED(CORR.): 0.862    SEE:  28.562      DW: 2.23
            PERIOD OF FIT:   1963   1981
            F(  4, 14):    29.160
```

EAST ASIA

```
POTCEAS  =  1363.8975  +  0.4404 POTCEAS(-1)  +  0.2141 CGHAEAS(-1)
              (2.4859)   (2.6501)                (2.9932)

         -  0.1046 CRHAEAS(-1)
           (-3.2853)

            R-SQUARED(CORR.): 0.840    SEE:  50.477      DW: 1.70
            PERIOD OF FIT:   1966   1982
            F(  3, 13):    28.931
```

INDIA

```
POTCIND  =  - 2040.3231  -  12.8347 DFPOPFIND +  0.6659 POTCIND(-1)
              (-2.2091)   (-2.0693)             (4.0522)

            +  0.0631 RPHAIND
              (2.3660)

            R-SQUARED(CORR.): 0.906    SEE:  94.627      DW: 1.77
            PERIOD OF FIT:   1963   1981
            F(  3, 15):    58.769
```

CHINA

```
POTCPRC  =  - 2283.5125  +  0.4789 POTCPRC(-1)  +  0.1990 WHHAPRC(-1)
              (-0.4841)   (1.2484)                (1.6929)

            -  0.0206 TCHAPRC(-1)  -  932.9116 D79
             (-0.7458)               (-2.5857)

            R-SQUARED(CORR.): 0.734    SEE:  311.51      DW: 1.74
            PERIOD OF FIT:   1967   1981
            F(  4, 10):    10.669
```

NORTH AFRICA

```
POTCNAF  =  - 1006.3026  +  0.8673 POTCNAF(-1)  +  0.0536 WHHANAF(-1)
              (-2.5444)  (12.9925)                (2.6840)

            R-SQUARED(CORR.): 0.964    SEE:  75.153      DW: 2.61
            PERIOD OF FIT:   1963   1982
            F(  2, 17):   256.973
```

SOUTH AFRICA

```
POTCSAF  =  +  0.9867 POTCSAF(-1)  +  0.0039 CGHASAF
             (29.6900)               (2.1106)

            R-SQUARED(CORR.): 0.978    SEE:  13.278      DW: 2.26
            PERIOD OF FIT:   1963   1981
            F(  2, 17):   397.888
```

AUSTRALIA

```
POTCAUS  =  361.7017  -  346.5064 DFPOPFAUS +  0.0833 WHHAAUS
             (2.4417)   (-11.9181)            (9.7422)

         -  0.0107 CRHAAUS  +  365.1413 DFWHPFAUS(-1)
           (-0.4126)            (5.1420)

            R-SQUARED(CORR.): 0.903    SEE:  50.468      DW: 2.48
            RHO(1): -0.494
            PERIOD OF FIT:   1968   1981
            F(  4,  8):    28.841
```

EEC - 10

```
POTCEEC  =  - 2824.4636  -  1.9086 DFPOPFEEC2 +  0.5923 POTCEEC(-1)
              (-2.2849)  (-7.7600)              (6.8121)

            +  10.2253 RPHAEEC(-1)  +  0.1773 CRHAEEC2(-1)
               (3.1205)               (1.9750)

            R-SQUARED(CORR.): 0.932    SEE:  108.51      DW: 1.94
            PERIOD OF FIT:   1966   1982
            F(  4, 12):    56.031
```

JAPAN

```
POTCJPN  =  379.3159  -  146.7041 DFPOPFJPN +  0.4396 POTCJPN(-1)
             (2.7608)   (-2.7613)             (3.4093)

         +  143.0723 DFRIPFJPN(-1)
             (2.7370)

            R-SQUARED(CORR.): 0.813    SEE:  42.387      DW: 2.03
            PERIOD OF FIT:   1963   1983
            F(  3, 17):    29.935
```

UNITED STATES

```
POTCUSA  =  -  20750.6699 DFPOPFUSA +  0.8018 POTCUSA(-1)
                (-2.1707)            (8.4024)

            +  0.0185 CGHAUSA  +  0.0478 WHHAUSA
               (2.0024)           (2.0648)

            R-SQUARED(CORR.): 0.874    SEE:  244.85      DW: 2.55
            PERIOD OF FIT:   1963   1981
            F(  4, 15):    31.825
```

SOVIET UNION

```
POTCUSR  =  +  0.9895 POTCUSR(-1)  +  0.0024 WHCGHAUSR(-1)
             (31.7278)                (2.5487)

            R-SQUARED(CORR.): 0.982    SEE:  217.18      DW: 1.98
            PERIOD OF FIT:   1963   1982
            F(  2, 18):   519.137
```

POTASH CONSUMPTION

BRAZIL

```
KOTCBRA  =  - 59.6716  -  50557.2891 DFKOPFBRA2  +  1.1130 KOTCBRA(-1)
             (-0.2668)     (-1.5536)              (10.8048)

            +  0.0157 CGHABRA
               (0.6999)

            R-SQUARED(CORR.): 0.978    SEE:  57.339      DW: 2.39
            PERIOD OF FIT:   1963   1981
            F(  3, 15):   272.733
```

LATIN AMERICA

```
KOTCLAC  =  - 591.4120  +  0.4049 KOTCLAC(-1)  +  0.0884 CGHALAC
              (-2.7673)   (2.9938)               (2.2461)

            +  0.2328 RPHALAC  -  0.7364 DFKOPFLAC(-1)
               (4.0846)          (-1.7081)

            R-SQUARED(CORR.): 0.925    SEE:  27.094      DW: 1.99
            PERIOD OF FIT:   1963   1981
            F(  4, 14):    56.238
```

EAST ASIA

```
KOTCEAS  =  -  0.7448 DFKOPFEAS +  0.8292 KOTCEAS(-1)
             (-1.2163)            (7.6451)

            +  0.1033 CGHAEAS  -  0.0172 CRHAEAS
              (1.9167)          (-1.4081)

            R-SQUARED(CORR.): 0.945    SEE:  37.753      DW: 2.68
            PERIOD OF FIT:   1965   1981
            F(  4, 13):    69.186
```

INDIA

```
KOTCIND  =  - 699.1591  +  0.6560 KOTCIND(-1)  -  60.7125 DFKOPFIND
              (-3.0497)  (11.8951)                (-11.0945)

            +  0.0211 RPHAIND  +  0.0164 WHHAIND  +  142.1651 D68
              (2.8522)           (4.7196)             (6.0442)

            R-SQUARED(CORR.): 0.984    SEE:  23.695      DW: 2.28
            RHO(1): -0.595
            PERIOD OF FIT:   1963   1981
            F(  5, 12):   207.012
```

CHINA

```
KOTCPRC  =  - 96.1513  -  87.7067 DFKOPFPRC2 +  1.3255 KOTCPRC(-1)
             (-0.1618)   (-1.0436)             (4.0079)

            +  180.9829 RWHHAPRC  +  161.2131 D74
               (0.3002)   (3.2311)

            R-SQUARED(CORR.): 0.853    SEE:  50.338      DW: 1.63
            RHO(1):  0.387
            PERIOD OF FIT:   1967   1981
            F(  4,  9):    19.870
```

SOUTH AFRICA

```
OTCSAF   =  1.1384  -  40.2878 DFKOPFSAF2  +  0.6829 KOTCSAF(-1)
           (0.1036)   (-1.1015)              (5.9301)

          +  0.0116 WHHASAF  +  47.0672 DFWHPFSAF(-1)
            (1.7826)           (3.0767)

            R-SQUARED(CORR.): 0.941    SEE:  5.1252      DW: 2.22
            PERIOD OF FIT:   1966   1981
            F(  4, 11):    61.294
```

EEC - 10

```
KOTCEEC  =  - 1731.2428  -  6.1897 DFKOPFEEC2 +  0.6652 KOTCEEC(-1)
              (-0.8629)  (-2.1381)             (5.8578)

            +  8.2492 RPHAEEC  +  0.1659 WHHAEEC
              (1.8124)            (1.5372)

            R-SQUARED(CORR.): 0.819    SEE:  181.33      DW: 1.96
            PERIOD OF FIT:   1964   1981
            F(  4, 13):    20.290
```

JAPAN

```
KOTCJPN  =  1060.8259  -  669.9056 DFKOPFJPN -  0.2224 KOTCJPN(-1)
              (8.1666)   (-8.3149)            (-1.7281)

          +  0.0676 RPHAJPN
            (3.9068)

            R-SQUARED(CORR.): 0.798    SEE:  27.968      DW: 2.14
            RHO(1): -0.591
            PERIOD OF FIT:   1963   1981
            F(  3, 14):    23.326
```

UNITED STATES

```
KOTCUSA  =  -  90627.1797 DFKOPFUSA +  0.7780 KOTCUSA(-1)
                (-1.9009)            (7.4630)

            +  0.0636 WHHAUSA  +  0.0261 CGHAUSA
              (2.0077)           (1.6687)

            R-SQUARED(CORR.): 0.936    SEE:  278.92      DW: 2.56
            PERIOD OF FIT:   1963   1981
            F(  4, 15):    66.380
```

OTHER DEVELOPED

```
KOTCODC  =  -  0.5512 DFKOPFODC(-1)  +  0.6538 KOTCODC(-1)  +  0.0461 CGHAODC
             (-1.1200)                (4.0962)               (2.1613)

            R-SQUARED(CORR.): 0.899    SEE:  54.490      DW: 1.82
            PERIOD OF FIT:   1963   1981
            F(  3, 16):    53.995
```

SOVIET UNION

```
KOTCUSR  =  - 11674.7217  +  0.2489 KOTCUSR(-1)  +  0.1004 WHCGHAUSR(-1)
               (-3.0529)    (1.2016)                (3.0984)

            -  19.8792 KOPRWRD  +  253.3810 TIME
              (-3.1345)            (3.5287)

            R-SQUARED(CORR.): 0.948    SEE:  326.43      DW: 1.46
            PERIOD OF FIT:    1965   1982
            F(  4, 13):     79.176
```

Variables and Country Codes for Demand Equations

36. Every region, whether it is an individual country or a set of countries, was given a name consisting of three letters. The regions and their abbreviations are listed below.

Australia	AUS
Canada	CAN
EEC 10	EEC
Japan	JPN
United States	USA
Other Industrial Countries	ODC
Eastern Europe	EEU
Union of Soviet Socialist Republics	USR
People's Republic of China	PRC
Argentina	ARG
Brazil	BRA
Central Africa	CAF
East Asia	EAS
Egypt	EGY
India	IND
Indonesia	INO
Latin America	LAC
Mexico	MEX
Nigeria	NIG
North Africa	NAF
Pakistan	PAK
South Africa	SAF
South Asia	SAS
Thailand	THA

Two letters were used to identify each of the commodities used in the model: wheat-WH; coarse grains-CG; rice paddy-RP; rice-RI; total crop-TC; soybean-SB; nitrogen-NT; phosphate-PO; potash-KO. The type of variable is also indicated by two letters: total consumption-TC; production-PD; price paid by farmers-PF; harvested area-HA. If a variable is a price variable deflated by the consumer price index it is preceded by DF. For example, WHHATHA = harvested area of wheat in Thailand and DFNTPFNAF = deflated price of nitrogen paid by farmers in North Africa. The macro variables i.e. population, exchange rate and consumer price index are indicated by the letters XRT-exchange rate; CPI-consumer price index; and POP-population.

Capacity and Production Equations

37. Capacity equations were estimated for nitrogen and phosphate in Europe & Japan, North America, Developing Countries and Centrally Planned Economies. For those same fertilizers a production function was estimated which takes into account the potential capacity available and other factors (as explained in Section III). The capacity and production of potash were estimated only on a world basis. The capacity equation and the production equation are presented below for each of the fertilizers.

NITROGEN

EUROPE AND JAPAN

CAPACITY :

$$NKEJ = 1092.7333 + 0.9444\ NKEJ(-1) - 851.9091\ PMNEJ(-2) + 98.5891\ PMNEJ(-3)$$
$$(0.9701)\quad (16.8222)\quad (-1.8262)\quad (0.1520)$$

$$+\ 605.5921\ PMNEJ(-4) + 5132.2632\ PKN(-2) - 4455.6479\ PKN(-3)$$
$$(1.3453)\quad (2.6953)\quad (-1.9352)$$

R-SQUARED(CORR.): 0.971 SEE: 665.12 DW: 1.67
PERIOD OF FIT: 1964 1984
F(6, 14): 113.276

PRODUCTION :

$$NQEJ = 1651.7440 + 305.1215\ TIME + 1.0929\ NKEJ - 0.0001\ NKEJ2$$
$$(1.2979)\quad (4.1638)\quad (4.0492)\quad (-3.9179)$$

$$-\ 159.5117\ PMNEJ2$$
$$(-3.3772)$$

R-SQUARED(CORR.): 0.900 SEE: 510.76 DW: 1.84
PERIOD OF FIT: 1964 1984
F(4, 16): 45.986

NORTH AMERICA

CAPACITY :

```
NKNA  =  - 9107.6074  +  0.1905 NKNA(-1)  -  1472.6921 PMNNA(-2)
            (-2.8662)   (0.9268)              (-1.1750)

         -  1334.7931 PMNNA(-3)  +  2991.6907 PKN(-2)
             (-0.7784)                (1.1870)

         -  2703.5776 PKN(-3)  +  8412.0430 LOGTIME
             (-0.9423)              (3.7616)

            R-SQUARED(CORR.): 0.974    SEE:  657.14 DW: 2.26
            PERIOD OF FIT:   1964   1984
            F(  6, 14):   127.323
```

PRODUCTION :

```
NQNA  =  806.4847  +  477.1568 TIME  +  0.2891 NKNA  -  0.0000 NKNA2
          (0.7828)     (5.5900)        (1.6130)       (-1.0864)

      -  372.6502 PMNNA2
         (-4.1807)

            R-SQUARED(CORR.): 0.958    SEE:  541.93 DW: 1.12
            PERIOD OF FIT:   1964   1984
            F(  4, 16):   115.808
```

CENTRALLY PLANNED COUNTRIES (EUROPE)

CAPACITY :

```
NKCP  =  3873.6460  +  1.0764 NKCP(-1)  -  535.5579 PMNAVG(-2)
          (4.3202)  (53.1602)              (-0.5565)

         +  2303.0950 PMNAVG(-3)  -  4099.2563 PMNAVG(-4)
             (1.8900)                (-5.3783)

         -  3333.0867 PKN(-2)  +  2480.0168 PKN(-3)
             (-1.2685)              (0.8624)

            R-SQUARED(CORR.): 0.997    SEE:  772.73 DW: 2.87
            PERIOD OF FIT:   1964   1984
            F(  6, 14):   956.701
```

PRODUCTION :

```
NQCP = - 2010.2239 + 800.0512 TIME + 0.2156 NKCP  +  0.0000 NKCP2
         (-4.9521)   (5.9929)      (1.6269)        (1.4277)

       -  129.5176 PMNAVG2
          (-2.3784)

            R-SQUARED(CORR.): 0.998    SEE:  392.18 DW: 2.27
            PERIOD OF FIT:   1964   1984
            F(  4, 16):  2579.396
```

DEVELOPING COUNTRIES

CAPACITY :

```
NKDC  =  1871.8712  +  1.0828 NKDC(-1)  +  523.7126 PMNAVG(-2)
          (2.5632)  (33.2701)              (0.7547)

     -  1658.8002 PMNAVG(-3)  +  372.4431 PMNAVG(-4)
         (-1.3297)               (0.3870)

     -  3266.5479 PKN(-2)  +  6106.0317 PKN(-3)  -  4161.8843 PKN(-4)
         (-1.5149)             (1.6976)             (-1.4882)

            R-SQUARED(CORR.): 0.994    SEE:  554.21      DW: 2.40
            PERIOD OF FIT:   1964   1984
            F(  7, 13):   440.768
```

PRODUCTION :

```
NQDC  =  - 1158.9550  +  476.2791 TIME  -  0.2157 NKDC  +  0.0000 NKDC2
            (-2.2905)     (3.6606)       (-1.1917)        (4.8512)

         -  385176.9063 PUREA-1-2
               (-0.8410)

            R-SQUARED(CORR.): 0.996    SEE:  234.21      DW: 1.63
            PERIOD OF FIT:   1964   1984
            F(  4, 16):  1315.560
```

PHOSPHATE

EUROPE AND JAPAN

CAPACITY :

```
PKEJ  =  1864.7880  +  0.8114 PKEJ(-1)  -  2626.2107 PMP(-2)
           (2.7224)   (6.5994)              (-1.5258)

      -  752.3315 PMP(-3)  +  444.9250 PKP(-2)  -  82.6618 PKP(-3)
         (-0.3918)            (0.8768)            (-0.1200)

      -  409.5767 PMP(-4)  -  320.0896 PKP(-4)  +  282.6097 RPKPQEJ
         (-0.3596)            (-0.6041)             (0.3592)

            R-SQUARED(CORR.): 0.981    SEE:  146.22      DW: 2.13
            PERIOD OF FIT:   1965   1984
            F(  8, 11):   121.050
```

PRODUCTION :

```
PQEJ  =  2890.5840  -  131.9156 TIME  +  2.4341 PKEJ
           (3.8594)    (-4.2505)        (6.7581)

      -  0.0004 PKEJ2  -  11792.9004 PMP2
       (-3.6958)            (-4.5292)

            R-SQUARED(CORR.): 0.903    SEE:  318.38      DW: 1.94
            PERIOD OF FIT:   1960   1984
            F(  4, 20):    56.882
```

NORTH AMERICA

CAPACITY :

```
PKNA  =  1723.0560  +  0.9122 PKNA(-1)  -  5093.8765 PMP(-2)
           (0.9182)  (16.2011)              (-1.1255)

      -  533.9583 PMP(-3)  +  1791.9712 PKP(-2)  -  2181.9109 PKP(-3)
         (-0.1385)             (1.1500)             (-1.3106)

      +  1303.3057 RPKPQNA
           (0.8942)

            R-SQUARED(CORR.): 0.976    SEE:  511.74      DW: 2.73
            PERIOD OF FIT:   1963   1984
            F(  6, 15):   142.926
```

PRODUCTION :

```
PQNA  =  3900.0164  -  204.3221 TIME  +  0.6859 PKNA  +  0.0000 PKNA2
         (4.1158)     (-1.4101)        (2.5568)        (1.2218)

         -  9637.8809 PMP2
            (-2.1383)

            R-SQUARED(CORR.): 0.926    SEE:  537.57      DW: 1.66
            PERIOD OF FIT:   1963   1984
            F(  4, 17):    66.914
```

CENTRALLY PLANNED COUNTRIES

CAPACITY :

```
PKCP  =  - 705.5914  +  0.7881 PKCP(-1)  -  978.4176 PMP(-2)
         (-0.7353)     (8.8050)            (-0.3588)

         -  451.1628 PMP(-3)  +  2417.7961 PMP(-4)  +  130.2768 PKP(-2)
            (-0.2383)            (1.5051)              (0.1628)

         -  304.9286 PKP(-3)  +  3769.4492 RPKPQCP
            (-0.3178)            (2.7382)

            R-SQUARED(CORR.): 0.992    SEE:  248.89      DW: 2.11
            PERIOD OF FIT:   1964   1984
            F(  7, 13):   365.032
```

PRODUCTION :

```
PQCP  =  953.1170  +  339.4733 TIME  +  0.7812 PKCP  -  0.0001 PKCP2
         (1.7337)     (3.8371)         (2.4829)       (-3.0594)

      -  2484.3335 PMP2
         (-0.7578)

            R-SQUARED(CORR.): 0.990    SEE:  376.94      DW: 2.03
            PERIOD OF FIT:   1960   1984
            F(  4, 20):   577.122
```

DEVELOPING COUNTRIES

CAPACITY :

```
PKDC  =  524.4303  +  1.0669 PKDC(-1)  +  874.6616 PMP(-2)
          (0.5674)  (46.1274)              (0.3483)

      -  2506.5728 PMP(-3)  +  1929.1949 PMP(-4)  -  505.3241 PKP(-2)
          (-0.9386)              (0.8486)            (-0.4841)

      +  148.0381 PKP(-3)  -  119.5401 RPKPQDC
          (0.1346)            (-0.2125)

            R-SQUARED(CORR.): 0.988    SEE:  367.74      DW: 2.16
            RHO(1): -0.590
            PERIOD OF FIT:   1964   1984
            F(  7, 12):   233.153
```

PRODUCTION :

```
PQDC  =  221.1357  +  114.6447 TIME  +  0.3252 PKDC  +  0.0000 PKDC2
          (0.4170)     (1.1738)        (1.0382)        (0.1952)

      -  830.7317 PMP2
         (-0.4069)

        R-SQUARED(CORR.): 0.985    SEE:  238.70      DW: 2.15
        PERIOD OF FIT:   1963   1984
        F(  4, 17):   341.358
```

POTASH WORLD CAPACITY AND PRODUCTION

CAPACITY :

```
KKWD  =  1208.8502  +  0.8716 KKWD(-1)  +  100.5937 PPOT(-2)
          (0.4275)   (9.1955)               (2.0775)

      -  73.3431 PPOT(-3)  +  3164.8545 PKK(-2)  -  1784.6998 PKK(-3)
        (-1.4199)              (0.8557)             (-0.4504)

            R-SQUARED(CORR.): 0.952    SEE:  1552.9      DW: 1.90
            PERIOD OF FIT:   1963   1984
            F(  5, 16):    85.216
```

PRODUCTION :

```
KQWD = 504.9231 + 818.6190 TIME + 0.6357 KKWD - 0.0000 KKWD2 + 14.7639 PPOT
       (0.1158)   (3.1746)      (1.6762)     (-2.1525)        (0.6578)

            R-SQUARED(CORR.): 0.980    SEE:  953.33      DW: 1.75
            RHO(1):  0.570
            PERIOD OF FIT:   1960   1984
            F(  4, 19):   278.578
```

Codes for the Capacity and Production Equations

38. Four regions are differentiated for the supply side of the model. These regions are denoted by two letters.

Europe and Japan and other industrial countries	EJ
North America	NA
Developing Countries	DC
Centrally Planned Economies	CP

Capacity is denoted by the letter K and production by the letter Q. Other variables appear in the capacity and production equations and are defined as follows.

PKN	=	MUV/Nitrogen price where MUV is the manufacturing unit value index
PMNNA	=	natural gas price in North America/ nitrogen price) *100.0
PMNAVG	=	(PMNNA + PMNEJ)/2.0
PUREA-1-2	=	[nitrogen price]^2
PMP	=	(phosphate rock price/phosphate price)
PKP	=	MUV/phosphate price
RPKPQ		is the ratio of capacity to production that was added to the phosphate production equations in order to adjust for the different process that had been used throughout the period of the sample. It is a predetermined exogenous variable.
PKK	=	MUV/potash price
PPOT	=	potash price

Price Equations

39. Nitrogen and phosphate prices are estimated as a ratio of the price of the fertilizer and its most important input. Therefore the estimated equations for prices these two fertilizers' are:

$$\frac{\text{Natural gas price}}{\text{Nitrogen price}} = f\ (\underset{\sim}{x}_1,\ \underset{\sim}{y}_1)$$

and

$$\frac{\text{Phosphate rock price}}{\text{Phosphate price}} = f\ (\underset{\sim}{x}_2,\ \underset{\sim}{y}_2)$$

where $\underset{\sim}{x}_1$, $\underset{\sim}{x}_2$ are vectors of variables exogenous to the model, and $\underset{\sim}{y}_1$, $\underset{\sim}{y}_2$ are vectors of variables endogenous to the model. The potash price is estimated directly i.e., potash price = f $(\underset{\sim}{x}_3,\ \underset{\sim}{y}_3)$ where $\underset{\sim}{x}_3$ and $\underset{\sim}{x}_4$ are defined similarly to the equations above.

40. Regression estimates of the fertilizer price equations are shown below.

NITROGEN WORLD PRICE

```
PMNAVG = -2.1948 + 0.0000525 NKWD - 0.0000636 FNTTCWRD + 0.1084 INTR
        (-3.6691) (2.7977)        (-2.0598)             (4.3768)

         + 0.0227 EXR + 0.1840  PMNAVGRISK - 1.0418 D7483
          (4.8804)     (1.7224)             (-5.4594)

         R-SQUARED(CORR.): 0.939    SEE: 0.19741      DW: 2.47
         PERIOD OF FIT:   1962   1984
         F(  6, 16):    57.501
```

PHOSPHATE WORLD PRICE

```
PMP = 0.3780 + 0.0000123 PKWD - 0.0000203 FPOTCWRD + 0.0027 DINTR
     (4.7683) (2.8185)         (-2.9676)             (0.6873)

      + 0.1200 PMPRISK - 0.0995 D72
       (4.5439)         (-3.5759)

         R-SQUARED(CORR.): 0.796    SEE: 0.26303E-01  DW: 2.22
         PERIOD OF FIT:   1963   1984
         F(  6, 15):    14.671
```

POTASH WORLD PRICE

```
PPOT = 1.3275 - 0.0057 KKWD + 0.0096 FKOTCWRD + 0.6153 PPOT(-1)
      (0.1796) (-5.0617)     (5.9448) (5.5601)
     - 3.6374 DINTR - 20.6276 D77
     (-2.4745)       (-2.1589)

         R-SQUARED(CORR.): 0.903    SEE:  8.8564      DW: 1.59
         PERIOD OF FIT:   1962   1984
         F(  5, 17):    41.854
```

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VI. AN EX-POST MODEL SIMULATION

41. In order to validate the specification of the equations as well as the overall performance of the model an ex-post simulation was run dynamically, i.e., where the lagged variables in any equation are the simulated values, except for the first year for which the model is solved.

42. The ex-post simulation was carried out for the period 1970-1984 and the simulated series were compared with the actual series to derive statistics to evaluate the performance of each variable. The summary statistics for simulations of nitrogen, phosphate and potash consumption are shown in Tables 6.1, through 6.3, respectively. 1/

43. As Tables 6.1 and 6.2 show, the overall performance of the model is quite good; the RMSPE 2/ are low for almost all of the variables. For the major economic regions it can be seen that the RMSPE for nitrogen consumption does not exceed 7.403 (for the developing countries), and it is even lower for phosphate consumption.

44. The fluctuations in potash consumption through the period of this simulation were much greater than for nitrogen and phosphate. Even so, the diagnostics of the potash consumption variables are satisfactory (Table 6.3). The RMSPE for the world is slightly higher than 4.2, which is not as low as for nitrogen and phosphate but is still creditable. Special attention should be given to the inequality proportions. For nitrogen the covariance proportion is very close to 1.0 for all of the major economic regions. The covariance proportion factor for the centrally planned economies is slightly lower,

---

1/ The definitions of each of the statistics used in the following tables are provided in Appendix A.

2/ Root mean square percentage error.

0.569, but in the other regions the proportions are very high (developing 0.986, industrial 0.992 and for the world 0.999). The results for phosphate and potash are generally not as good as those for nitrogen. Graphic displays of the tracking performance for the consumption variables can be found in Figures 6.1 through 6.16 for nitrogen, 6.17 through 6.33 for phosphates and 6.34 through 6.48 for potash.

45. Tables 6.4 through 6.6 summarize the tracking statistics for the ex-post simulation of capacity and production. As can be seen, the performance of the supply side of the model is at least as good as the demand side. For nitrogen the highest RMSPE is about 6.6%, which was for capacity in developing countries, but even so the RMSPE for production of the developing countries is less than 2%. The fact that the covariance inequality proportion for capacity in the developing countries is 0.998 and the inequality proportion for production in that region is 0.673--which is the lowest among all the nitrogen capacity and production equations--is indicative of the good performance. There are mixed results concerning phosphate. Generally, the results are not nearly as good as for nitrogen. However, in absolute terms the results are reasonably good. The bias inequality proportions are very small for all of the regions for both fertilizers and the covariance inequality proportions are relatively high in most cases.

46. Equations for potash world capacity and production have been estimated, as previously mentioned, on a world basis. Given the difficulties of estimating a "world equation" the results are encouraging. The RMSPE of the capacity equation is 6.5 and the bias inequality proportion is almost zero. The production equation produced a RMSPE which is only 4.7 with a bias inequality proportions which is effectively zero. For both of these variables, the covariance inequality proportion is reasonably high--0.531 for capacities

and 0.548 for production. Graphical displays of the tracking performance of production by the model are presented in Figures 6.49-6.70.

47. Finally, we illustrate the performance of the most important variables in the model--prices. It should be remembered that this is a dynamic ex-post simulation; only the first year simulated uses actual lagged variables. The lagged values for periods thereafter are the simulated values. A forecast error of up to 15% can therefore be considered a reasonably good result.

48. As can be seen from Table 6.7 the RMSPE for nitrogen prices is 15.29. A large part of the reason for this result is because the model overshot in predicting the price level in 1974--the highest price peak during the simulation period. The contribution of this single year to the RMSPE statistic is very high (25%). Otherwise, the RMSPE would be no more than (9%). The same problem occurs with the phosphate price. The RMSPE for this equation is 13.18, but if the unusual case of 1974 is taken into consideration it would be less than 8% on the average.

49. It appears from our experience in estimating the potash price that the variables we were using did not fully explain the events of 1980-84 at all well. Even so, the RMSPE is only 15.86.

50. Clearly the simulated series of all three world prices do closely follow the actual series, especially in the cases of nitrogen and phosphate. The covariance inequality proportions prove this point. They are 0.69 for both nitrogen and phosphate while the bias inequality proportions are very close to zero. The coefficients for these two statistics can make the argument that there is no systematic deviation from the actual series for any of these prices and that the equations used do have the capability of duplicating the degree of variability of any of the world fertilizer prices.

51. Figures 6.71-6.73 provide the visual backup to the above discussion.

TABLE 6.1: SUMMARY STATISTICS FOR EX-POST SIMULATION OF NITROGEN CONSUMPTION

REGION NAME	MEAN OF ACTUAL	MEAN ERROR	ROOT MEAN SQUARED ERROR	MEAN % ERROR	ROOT MEAN SQUARED % ERROR	THEIL ERROR	BIAS	VARIANCE	COVARIANCE
							--INEQUALITY PROPORTIONS--		
ARGENTINA	46.870	4.471	5.499	9.438	11.170	0.523	0.047	0.121	0.832
BRAZIL	515.694	55.576	70.260	11.498	16.073	0.566	0.015	0.291	0.694
CENTRAL AFRICA	330.826	29.313	33.026	8.994	10.081	0.908	0.000	0.283	0.717
EAST ASIA	1310.141	107.485	119.488	8.810	9.994	0.912	0.011	0.001	0.987
EGYPT	457.749	43.990	49.303	9.918	11.484	0.710	0.011	0.365	0.624
INDIA	2586.100	147.726	173.061	8.246	11.875	0.538	0.082	0.345	0.572
INDONESIA	523.546	71.942	103.314	12.080	14.915	0.549	0.004	0.096	0.900
LATIN AMERICA	833.914	46.286	58.506	5.640	7.048	0.650	0.000	0.197	0.802
MEXICO	739.433	55.868	74.977	7.246	8.420	0.771	0.001	0.077	0.922
NORTH AFRICA	1167.168	57.501	74.969	5.755	7.766	0.755	0.003	0.313	0.684
NIGERIA	42.562	6.524	7.427	71.927	125.495	0.732	0.037	0.413	0.551
PAKISTAN	564.727	61.605	78.146	10.800	13.078	0.814	0.001	0.407	0.593
PEOPLE'S REP. OF CHINA	7190.067	478.105	585.850	8.146	10.361	0.716	0.004	0.002	0.994
SOUTH AFRICA	321.048	82.421	103.688	22.731	24.936	0.740	0.057	0.052	0.891
SOUTH ASIA	290.626	41.273	45.179	17.739	21.863	0.735	0.016	0.360	0.624
THAILAND	121.294	20.457	23.862	22.479	27.790	0.845	0.008	0.467	0.525
TOTAL DEVELOPING	17041.764	581.460	777.922	3.073	3.663	0.326	0.005	0.010	0.986
AUSTRALIA	197.382	11.827	14.448	6.702	8.505	0.718	0.006	0.467	0.527
CANADA	665.471	39.474	47.735	7.294	9.073	0.522	0.002	0.055	0.943
EEC-10	6500.139	221.564	271.979	3.403	4.060	0.531	0.027	0.132	0.841
JAPAN	726.540	90.895	103.118	12.703	14.544	0.547	0.011	0.001	0.988
OTHER INDUSTRIAL	2032.683	66.982	79.431	3.291	3.840	0.818	0.020	0.065	0.915
UNITED STATES	8826.339	*******	*******	11.626	14.852	1.017	0.004	0.002	0.995
TOTAL INDUSTRIAL	18948.555	*******	*******	5.909	7.403	0.874	0.002	0.006	0.992
EASTERN EUROPE	3961.564	129.067	144.975	3.410	3.870	0.625	0.000	0.162	0.837
USSR	7021.600	426.632	486.031	6.096	6.783	0.543	0.007	0.589	0.404
TOTAL CENTRALLY PLANNED	10983.165	400.928	498.424	3.504	4.176	0.476	0.006	0.425	0.569
TOTAL WORLD	46973.488	*******	*******	2.910	3.869	0.458	0.000	0.000	0.999

SOURCE: WORLD BANK, ECONOMIC ANALYSIS AND PROJECTIONS DEPARTMENT.

TABLE 6.2: SUMMARY STATISTICS FOR EX-POST SIMULATION OF PHOSPHATE CONSUMPTION

REGION NAME	MEAN OF ACTUAL	MEAN ERROR	ROOT MEAN SQUARED ERROR	MEAN % ERROR	ROOT MEAN SQUARED % ERROR	THEIL ERROR	BIAS	VARIANCE	COVARIANCE
							--INEQUALITY PROPORTIONS --		
ARGENTINA	38.185	3.648	4.915	9.545	12.042	0.698	0.006	0.294	0.699
BRAZIL	1082.971	181.086	237.104	18.653	24.821	0.605	0.001	0.508	0.491
CENTRAL AFRICA	173.774	12.792	15.744	7.232	8.810	0.875	0.003	0.173	0.824
EAST ASIA	523.030	55.321	66.246	10.653	12.596	0.970	0.011	0.375	0.614
EGYPT	81.569	13.038	15.373	19.358	24.877	0.921	0.018	0.250	0.732
INDIA	746.600	69.904	89.973	15.930	25.947	0.843	0.032	0.550	0.418
INDONESIA	154.614	19.609	23.947	28.905	54.185	1.024	0.003	0.276	0.721
LATIN AMERICA	429.648	14.630	18.163	3.458	4.292	0.684	0.001	0.070	0.930
MEXICO	248.314	20.341	24.416	8.755	10.399	0.813	0.000	0.076	0.924
NORTH AFRICA	887.036	69.002	86.638	9.844	12.680	0.726	0.009	0.428	0.563
NIGERIA	30.474	6.198	7.579	41.223	59.178	0.946	0.000	0.429	0.571
PAKISTAN	134.477	6.631	8.356	8.147	12.320	0.786	0.000	0.156	0.844
PEOPLE'S REP. OF CHINA	1891.240	153.617	206.250	8.908	12.202	0.480	0.002	0.055	0.942
SOUTH AFRICA	382.458	17.446	29.406	4.289	7.311	0.959	0.029	0.001	0.970
SOUTH ASIA	115.707	17.071	18.747	19.336	23.031	0.732	0.024	0.410	0.566
THAILAND	82.763	5.543	7.136	8.281	10.832	0.597	0.000	0.439	0.561
TOTAL DEVELOPING	7002.860	287.521	324.785	4.583	5.565	0.449	0.006	0.081	0.913
AUSTRALIA	786.786	56.036	74.550	7.974	11.285	0.402	0.000	0.526	0.474
CANADA	523.420	32.456	42.435	6.468	8.163	0.692	0.015	0.454	0.532
EEC-10	4400.975	113.026	150.904	2.615	3.578	0.544	0.000	0.450	0.550
JAPAN	721.720	58.544	69.861	8.484	10.332	0.941	0.004	0.334	0.662
OTHER INDUSTRIAL	1609.107	74.892	81.634	4.691	5.161	0.778	0.031	0.262	0.707
UNITED STATES	4547.938	255.723	318.356	5.740	7.417	0.874	0.005	0.125	0.869
TOTAL INDUSTRIAL	12589.946	409.947	511.737	3.314	4.234	0.657	0.003	0.506	0.491
EASTERN EUROPE	2713.859	171.544	203.762	6.246	7.211	0.863	0.004	0.770	0.225
USSR	4679.733	113.036	128.914	2.465	2.765	0.385	0.002	0.505	0.493
TOTAL CENTRALLY PLANNED	7393.592	242.846	298.696	3.221	3.867	0.511	0.006	0.526	0.468
TOTAL WORLD	26986.400	632.366	799.498	2.308	2.836	0.527	0.000	0.395	0.605

SOURCE: WORLD BANK, ECONOMIC ANALYSIS AND PROJECTIONS DEPARTMENT.

TABLE 6.3: SUMMARY STATISTICS FOR EX-POST SIMULATION OF POTASH CONSUMPTION

REGION NAME	MEAN OF ACTUAL	MEAN ERROR	ROOT MEAN SQUARED ERROR	MEAN % ERROR	ROOT MEAN SQUARED % ERROR	THEIL ERROR	BIAS	VARIANCE	COVARIANCE
							--INEQUALITY PROPORTIONS --		
ARGENTINA	7.446	3.438	4.085	59.301	79.611	0.841	0.004	0.323	0.673
BRAZIL	691.582	102.806	122.357	17.367	23.538	0.693	0.019	0.106	0.875
CENTRAL AFRICA	127.568	10.353	12.842	8.225	10.433	1.028	0.000	0.140	0.860
EAST ASIA	420.698	44.542	55.631	9.898	11.634	0.916	0.001	0.271	0.728
EGYPT	4.720	1.037	1.637	19.354	25.725	0.977	0.000	0.002	0.998
INDIA	421.747	48.801	55.868	12.810	14.572	0.679	0.009	0.211	0.780
INDONESIA	52.828	8.706	10.483	50.295	86.910	0.733	0.007	0.573	0.419
LATIN AMERICA	366.453	18.695	21.793	5.122	5.959	0.771	0.019	0.007	0.973
MEXICO	50.114	7.401	8.613	17.503	22.280	0.826	0.008	0.161	0.831
NORTH AFRICA	117.192	16.001	18.501	14.555	17.122	0.889	0.015	0.124	0.860
NIGERIA	15.624	2.660	3.736	40.956	67.206	0.631	0.093	0.452	0.455
PAKISTAN	8.156	2.355	2.978	86.692	112.490	0.737	0.040	0.266	0.694
PEOPLE'S REP. OF CHINA	305.533	73.255	90.265	46.767	65.438	0.873	0.010	0.635	0.355
SOUTH AFRICA	125.806	9.961	11.314	7.882	8.867	0.992	0.001	0.082	0.917
SOUTH ASIA	54.444	5.562	7.155	11.512	16.430	0.922	0.002	0.399	0.599
THAILAND	36.048	12.679	14.843	42.247	51.373	N.A.	N.A.	N.A.	N.A.
TOTAL DEVELOPING	2805.959	156.899	181.810	6.044	7.266	0.621	0.008	0.178	0.813
AUSTRALIA	107.223	8.577	11.181	8.402	11.281	0.699	0.010	0.599	0.392
CANADA	267.203	11.617	13.602	4.172	4.791	0.721	0.004	0.055	0.942
EEC-10	4148.763	157.626	202.107	3.734	4.746	0.759	0.004	0.235	0.761
JAPAN	620.420	47.979	65.465	8.149	11.556	0.905	0.000	0.105	0.894
OTHER INDUSTRIAL	1156.871	57.086	78.935	4.968	6.759	0.849	0.004	0.646	0.351
UNITED STATES	4740.339	240.141	288.765	5.105	6.305	0.810	0.000	0.213	0.787
TOTAL INDUSTRIAL	11040.819	435.839	510.520	3.904	4.521	0.839	0.001	0.296	0.703
EASTERN EUROPE	3065.786	162.299	206.787	5.054	6.232	0.888	0.009	0.459	0.532
USSR	4357.733	460.552	553.433	9.830	11.436	0.925	0.003	0.112	0.885
TOTAL CENTRALLY PLANNED	7423.520	499.293	639.466	6.320	7.768	0.860	0.001	0.198	0.801
TOTAL WORLD	21270.301	724.013	937.516	3.342	4.232	0.726	0.002	0.269	0.729

N.A.: NOT APPLICABLE.

SOURCE: WORLD BANK, ECONOMIC ANALYSIS AND PROJECTIONS DEPARTMENT.

TABLE 6.4: SUMMARY STATISTICS FOR EX-POST SIMULATION OF NITROGEN CAPACITY AND PRODUCTION

REGION NAME	MEAN OF ACTUAL	MEAN ERROR	ROOT MEAN SQUARED ERROR	MEAN % ERROR	ROOT MEAN SQUARED % ERROR	THEIL ERROR	BIAS	VARIANCE	COVARIANCE
							--INEQUALITY PROPORTIONS --		
CAPACITY:									
DEVELOPING COUNTRIES	13698.667	651.101	723.844	5.591	6.656	0.465	0.010	0.002	0.988
EUROPE & JAPAN	14494.333	700.830	806.388	4.780	5.492	0.684	0.000	0.014	0.986
NORTH AMERICA	15556.733	537.054	704.891	3.298	4.226	1.058	0.000	0.032	0.967
CENTRALLY PLANNED	26922.400	858.964	974.346	3.930	4.868	0.434	0.011	0.097	0.893
WORLD	70672.133	N.A.	N.A.	3.112	3.683	0.401	0.005	0.140	0.855
PRODUCTION:									
DEVELOPING COUNTRIES	7452.000	145.955	210.052	1.886	2.643	0.306	0.000	0.327	0.673
EUROPE & JAPAN	11035.733	509.074	580.708	4.662	5.368	1.069	0.003	0.007	0.990
NORTH AMERICA	10654.000	421.473	578.665	3.894	5.223	0.802	0.023	0.021	0.957
CENTRALLY PLANNED	19703.066	371.926	499.543	2.155	2.787	0.380	0.008	0.141	0.852
WORLD	48934.801	792.745	N.A.	1.417	2.037	0.433	0.003	0.007	0.990

N.A. - NOT APPLICABLE.

SOURCE: WORLD BANK, ECONOMIC ANALYSIS AND PROJECTIONS DEPARTMENT.

TABLE 6.5: SUMMARY STATISTICS FOR EX-POST SIMULATION OF PHOSPHATE CAPACITY AND PRODUCTION

REGION NAME	MEAN OF ACTUAL	MEAN ERROR	ROOT MEAN SQUARED ERROR	MEAN % ERROR	ROOT MEAN SQUARED % ERROR	THEIL ERROR	BIAS	VARIANCE	COVARIANCE
							--INEQUALITY PROPORTIONS --		
CAPACITY:									
DEVELOPING COUNTRIES	6034.200	198.236	224.057	4.543	6.562	0.565	0.031	0.322	0.647
EUROPE & JAPAN	5271.000	107.859	136.518	2.076	2.621	0.659	0.018	0.036	0.946
NORTH AMERICA	9381.200	351.527	434.322	4.187	5.464	0.646	0.000	0.190	0.810
CENTRALLY PLANNED	5492.533	193.958	243.179	5.419	8.269	0.395	0.031	0.333	0.636
WORLD	26178.934	512.034	598.251	2.413	3.131	0.245	0.025	0.171	0.804
PRODUCTION:									
DEVELOPING COUNTRIES	4274.267	170.463	227.677	4.473	5.786	0.576	0.013	0.591	0.396
EUROPE & JAPAN	7191.000	347.752	447.150	4.832	6.245	0.935	0.001	0.355	0.644
NORTH AMERICA	7720.667	583.506	630.664	7.631	8.193	0.812	0.006	0.416	0.578
CENTRALLY PLANNED	9526.400	252.610	330.226	3.038	4.261	0.735	0.015	0.545	0.440
WORLD	28712.334	866.467	N.A.	3.132	3.736	0.639	0.017	0.428	0.555

N.A. - NOT APPLICABLE.

SOURCE: WORLD BANK, ECONOMIC ANALYSIS AND PROJECTIONS DEPARTMENT.

TABLE 6.6: SUMMARY STATISTICS FOR EX-POST SIMULATION OF POTASH CAPACITY AND PRODUCTION

REGION NAME	MEAN OF ACTUAL	MEAN ERROR	ROOT MEAN SQUARED ERROR	MEAN % ERROR	ROOT MEAN SQUARED % ERROR	THEIL ERROR	BIAS	VARIANCE	COVARIANCE
							--INEQUALITY PROPORTIONS --		
CAPACITY:									
WORLD	30838.045	N.A.	N.A.	5.320	6.487	0.858	0.001	0.468	0.531
PRODUCTION:									
WORLD	23317.400	855.499	N.A.	3.536	4.679	0.792	0.000	0.452	0.548

N.A. - NOT APPLICABLE.

SOURCE: WORLD BANK, ECONOMIC ANALYSIS AND PROJECTIONS DEPARTMENT.

TABLE 6.7: SUMMARY STATISTICS FOR EX-POST SIMULATION OF WORLD FERTILIZER PRICES

REGION NAME	MEAN OF ACTUAL	MEAN ERROR	ROOT MEAN SQUARED ERROR	MEAN % ERROR	ROOT MEAN SQUARED % ERROR	THEIL ERROR	BIAS	VARIANCE	COVARIANCE
							--INEQUALITY PROPORTIONS --		
NITROGEN	148.127	18.234	32.926	11.564	15.286	0.470	0.002	0.305	0.693
PHOSPHATE	128.920	16.687	26.192	11.212	13.184	0.397	0.001	0.312	0.687
POTASH	65.987	9.344	12.508	13.765	15.863	0.824	0.002	0.014	0.983

SOURCE: WORLD BANK, ECONOMIC ANALYSIS AND PROJECTIONS DEPARTMENT.

NITROGEN CONSUMPTION: DYNAMIC SOLUTION TRACKING

FIGURE 6.1: BRAZIL

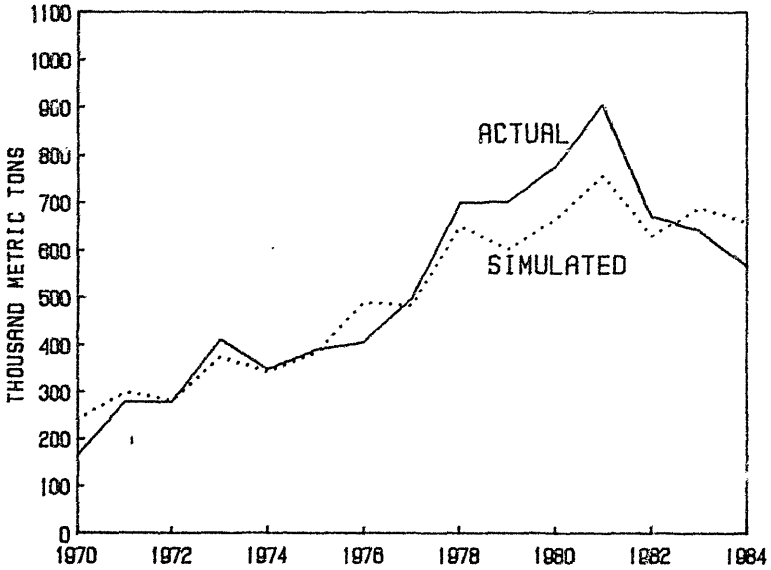


FIGURE 6.2: MEXICO

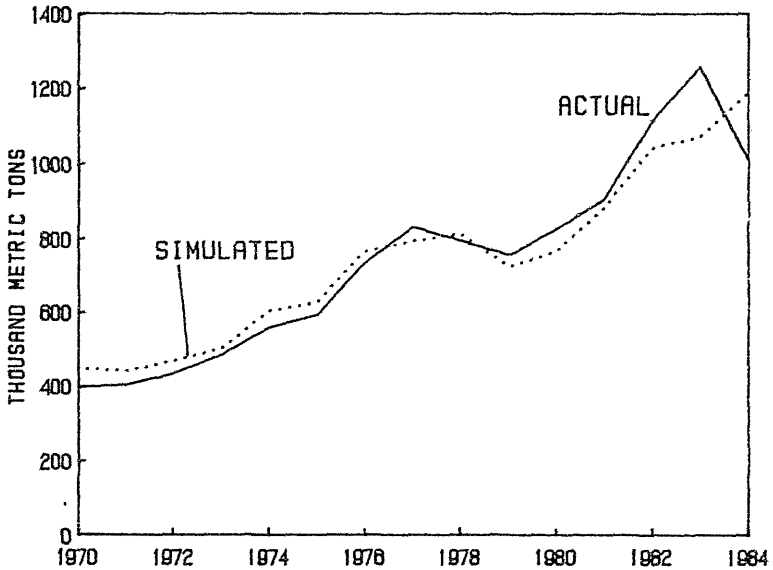


FIGURE 6.3: LATIN AMERICA & CARRIBEAN

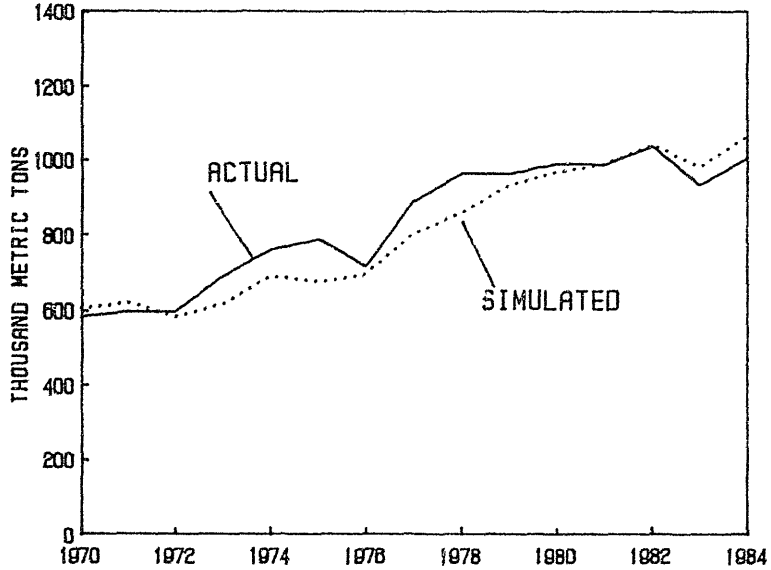
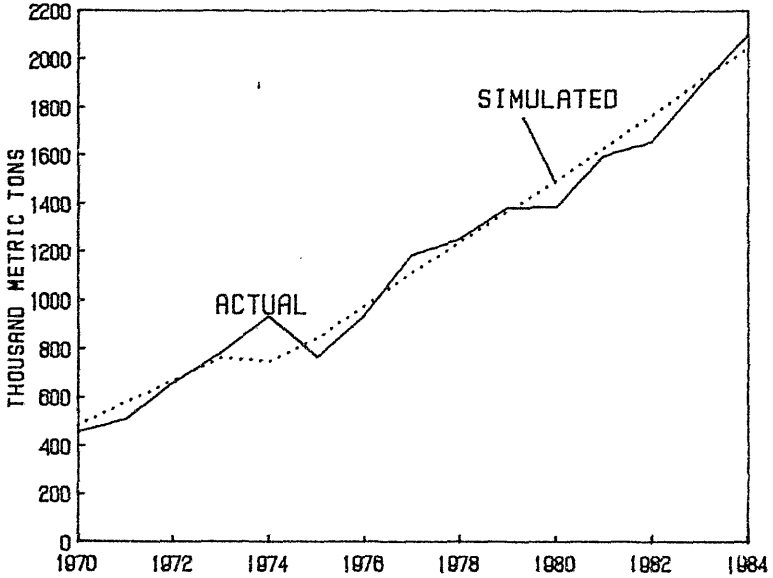


FIGURE 6.4: NORTH AFRICA & MIDDLE EAST



NITROGEN CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.5: EAST ASIA

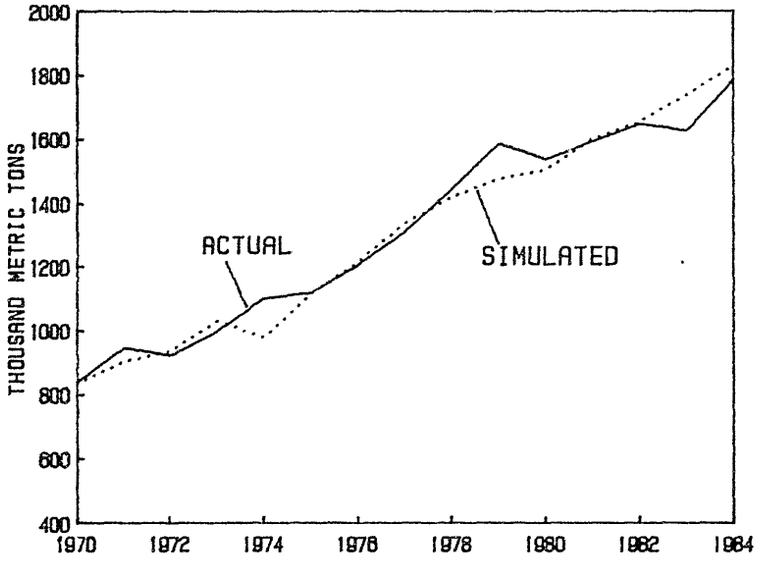


FIGURE 6.6: INDIA

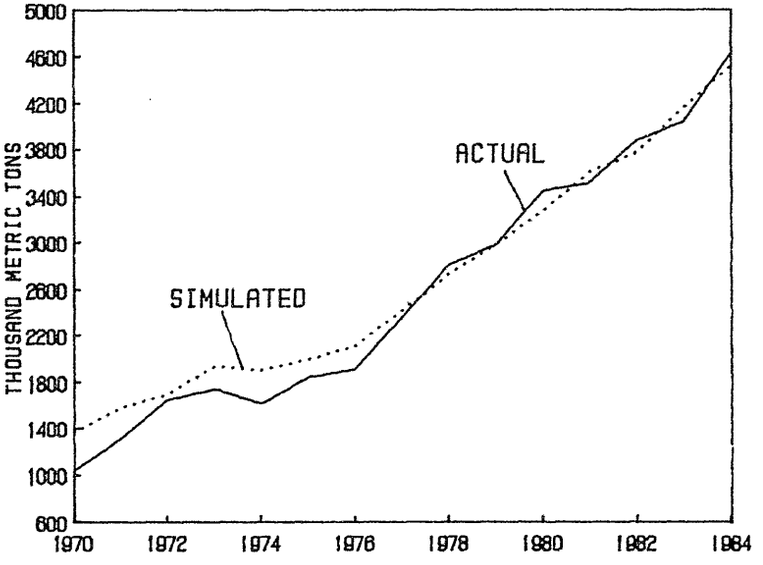


FIGURE 6.7: CHINA

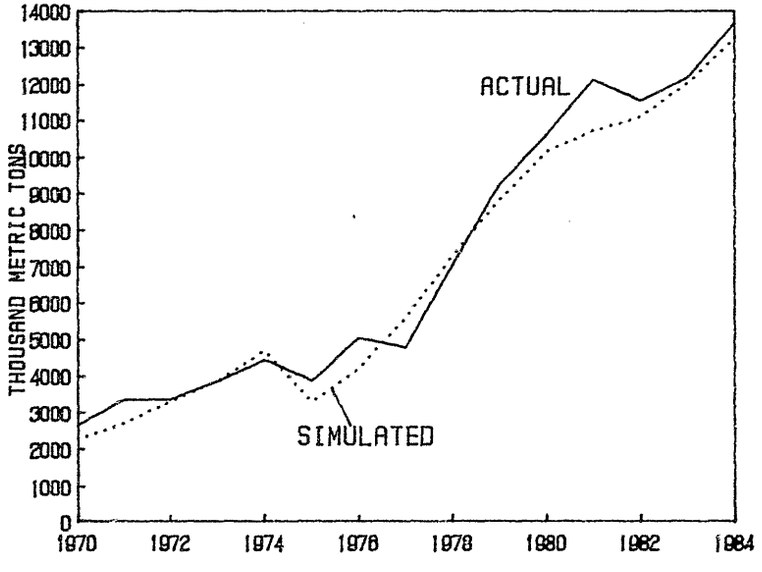
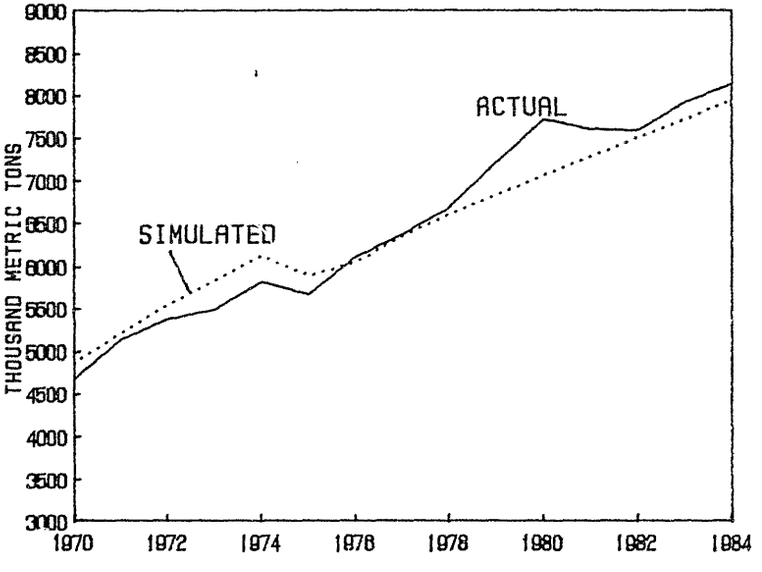


FIGURE 6.8: EEC-10



NITROGEN CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.9: JAPAN

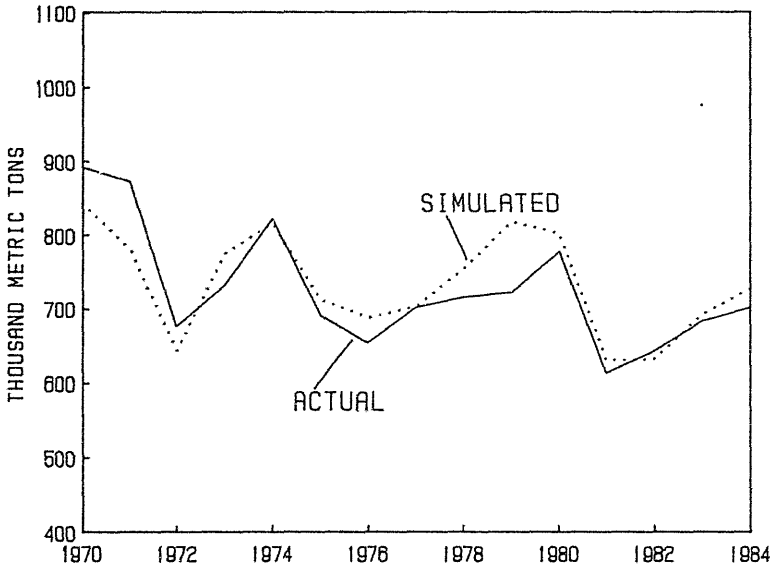


FIGURE 6.10: UNITED STATES

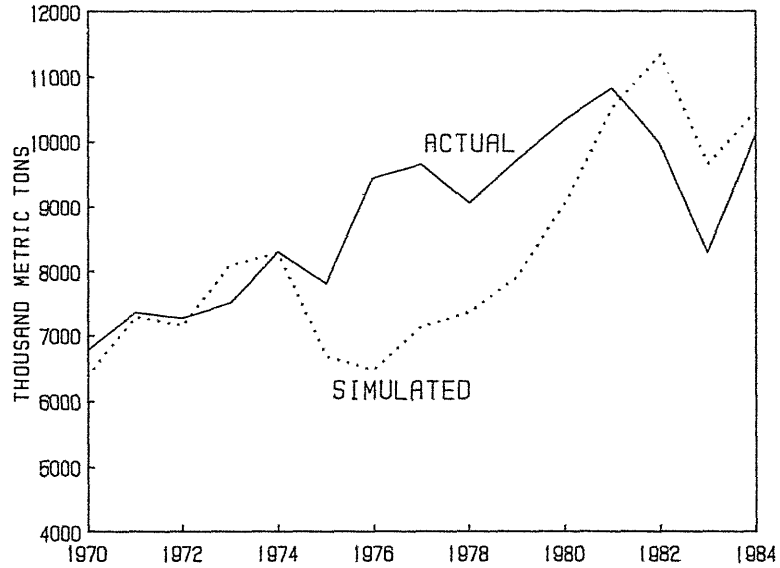


FIGURE 6.11: OTHER INDUSTRIAL COUNTRIES

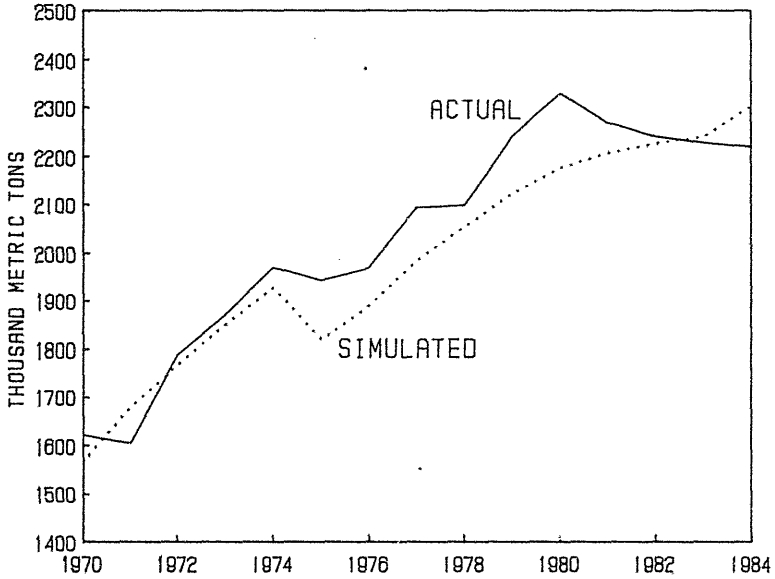
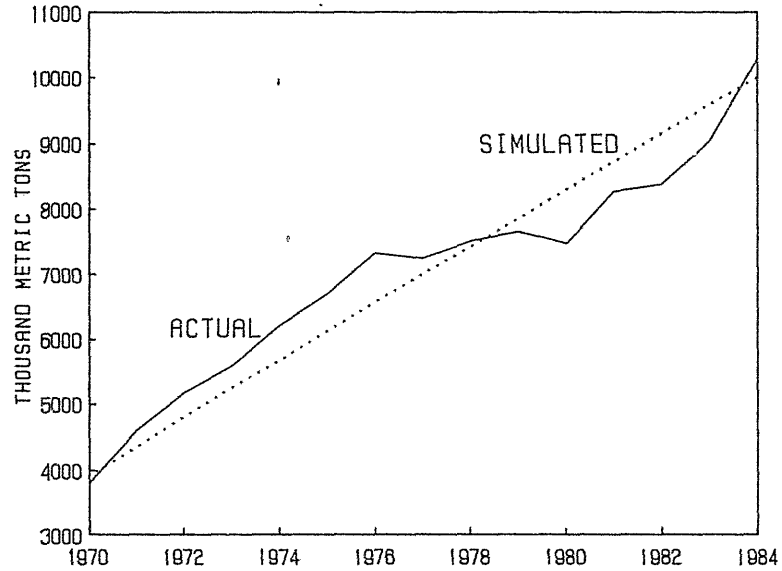


FIGURE 6.12: U.S.S.R.



NITROGEN CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.13: DEVELOPING COUNTRIES

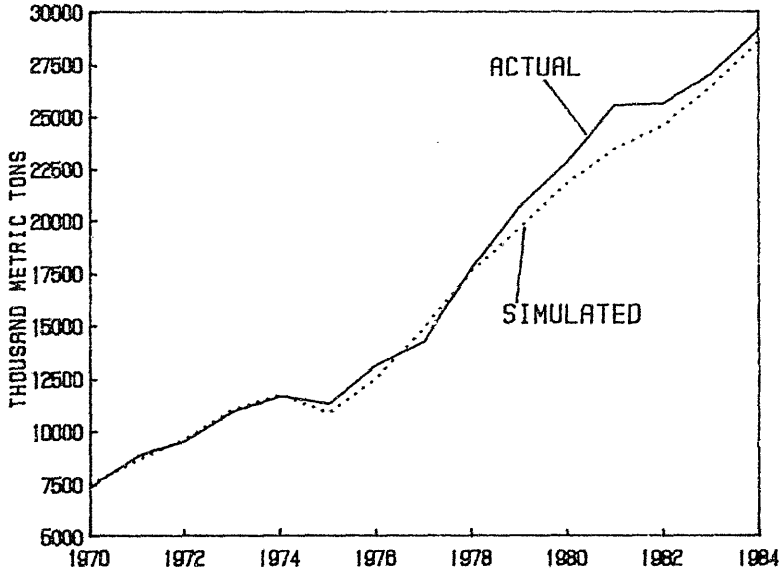


FIGURE 6.14: INDUSTRIAL COUNTRIES

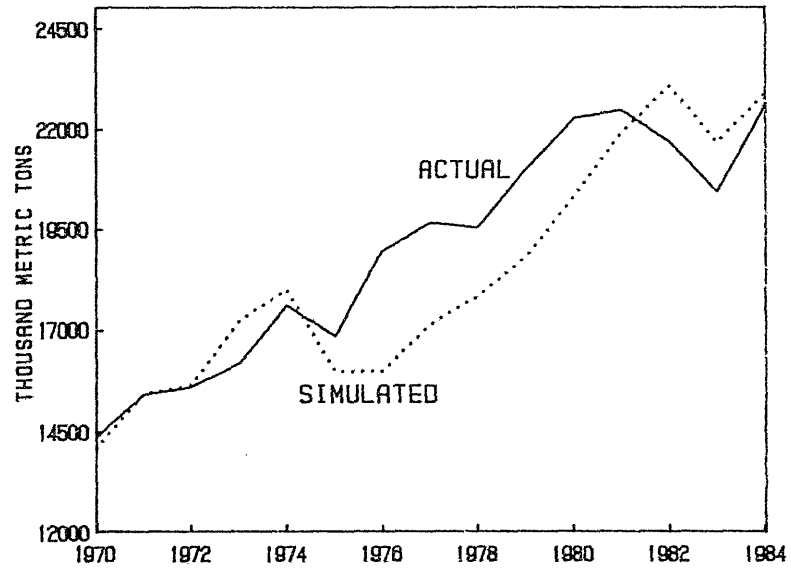


FIGURE 6.15: CETRALLY PLANNED ECONOMIES

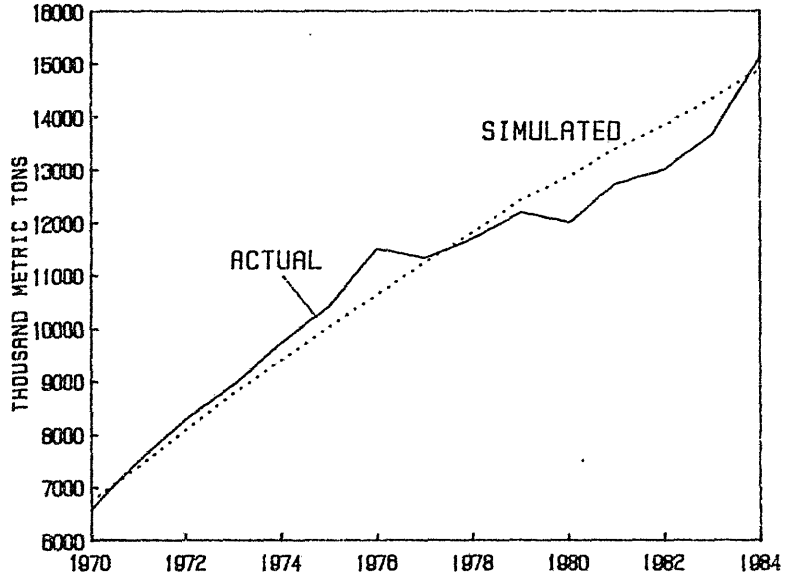
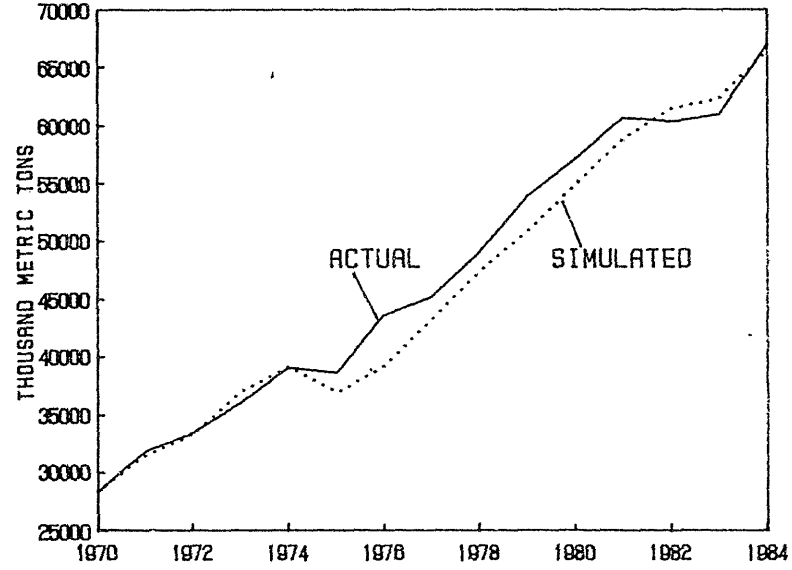


FIGURE 6.16: WORLD



PHOSPHATE CONSUMPTION: DYNAMIC SOLUTION TRACKING

FIGURE 6.17: BRAZIL

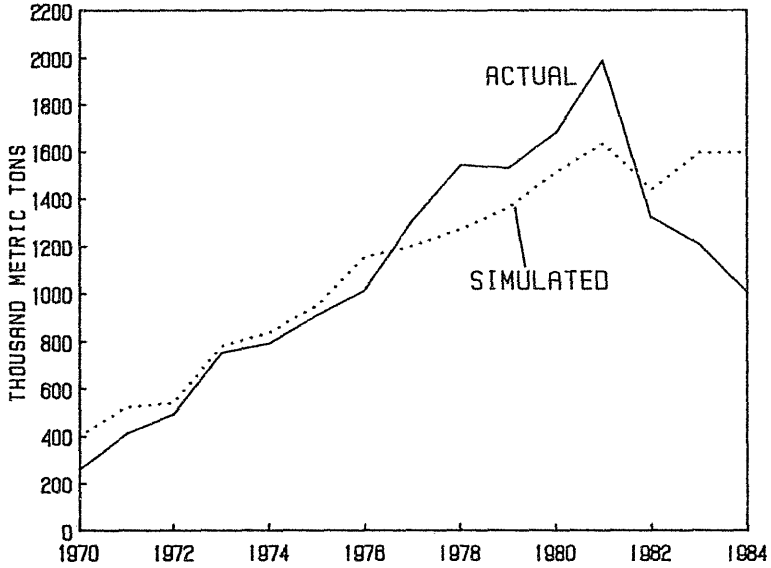


FIGURE 6.18: MEXICO

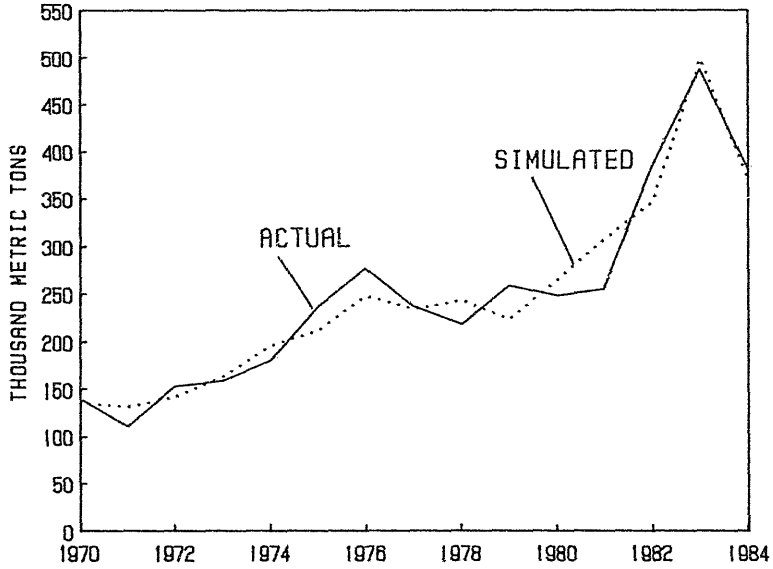


FIGURE 6.19: LATIN AMERICA & CARIBBEAN

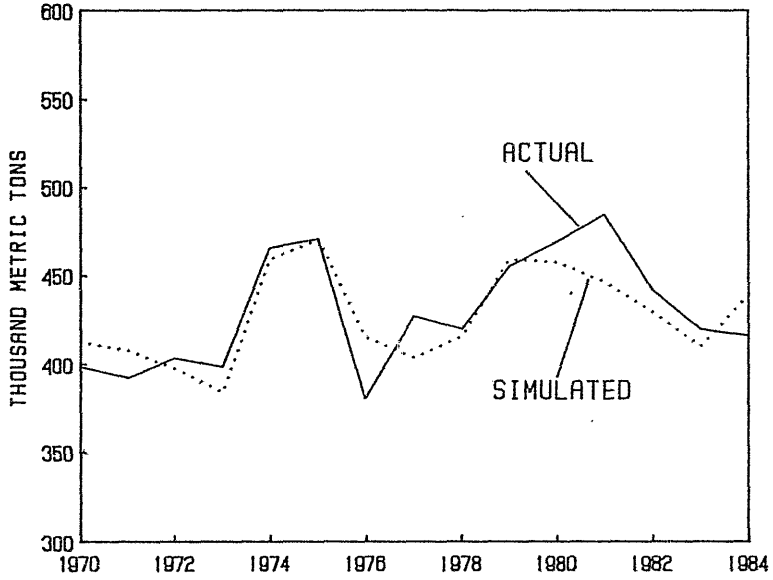
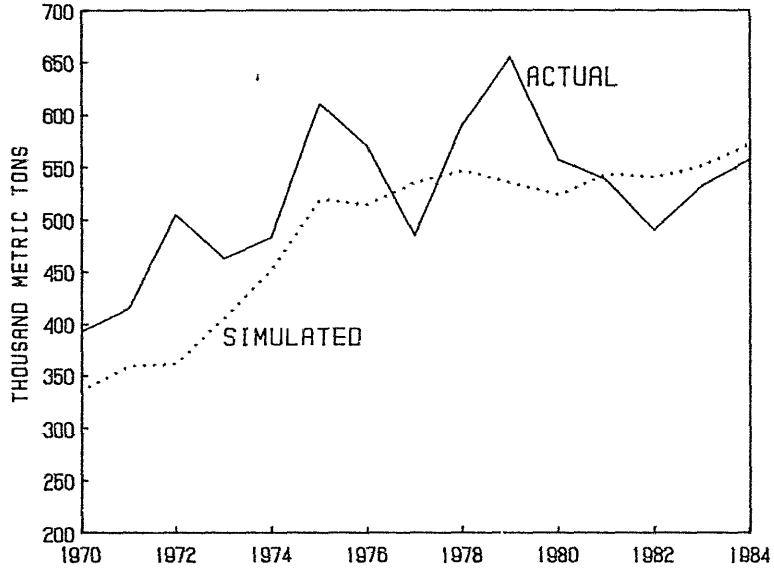


FIGURE 6.20: EAST ASIA



PHOSPHATE CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.21: INDIA

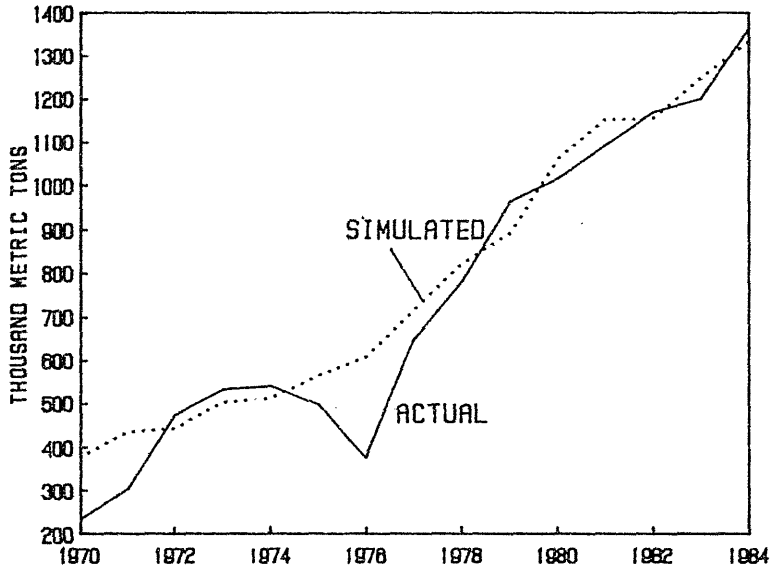


FIGURE 6.22: CHINA

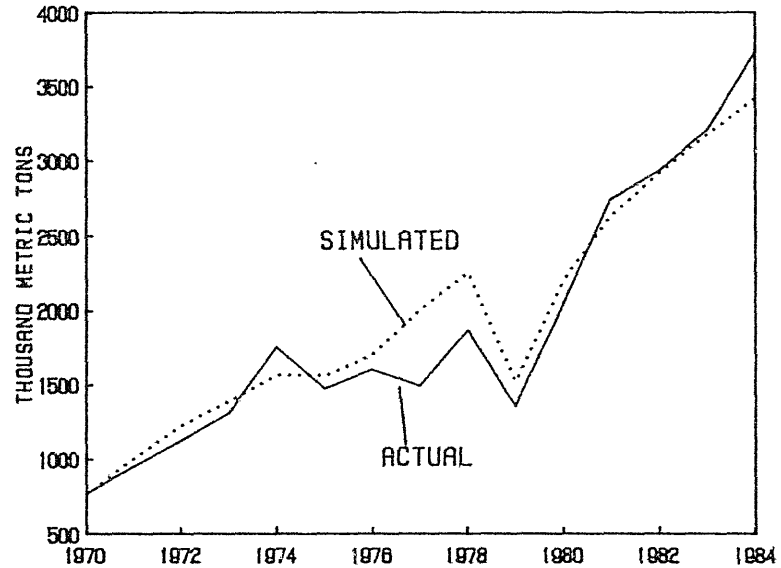


FIGURE 6.23: NORTH AFRICA & MIDDLE EAST

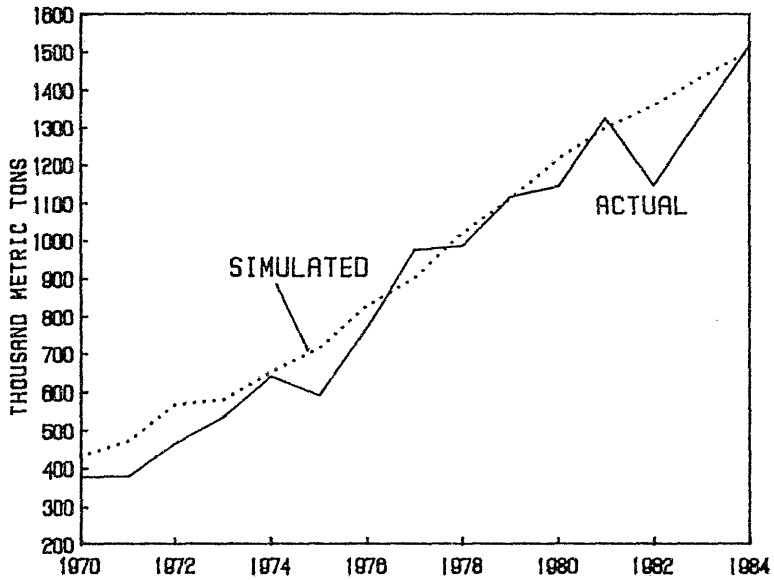
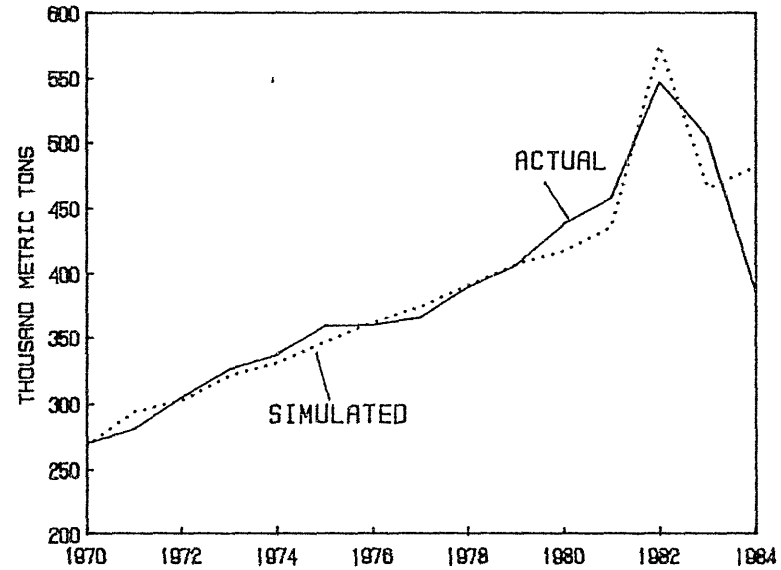


FIGURE 6.24: SOUTH AFRICA



PHOSPHATE CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.25: AUSTRALIA

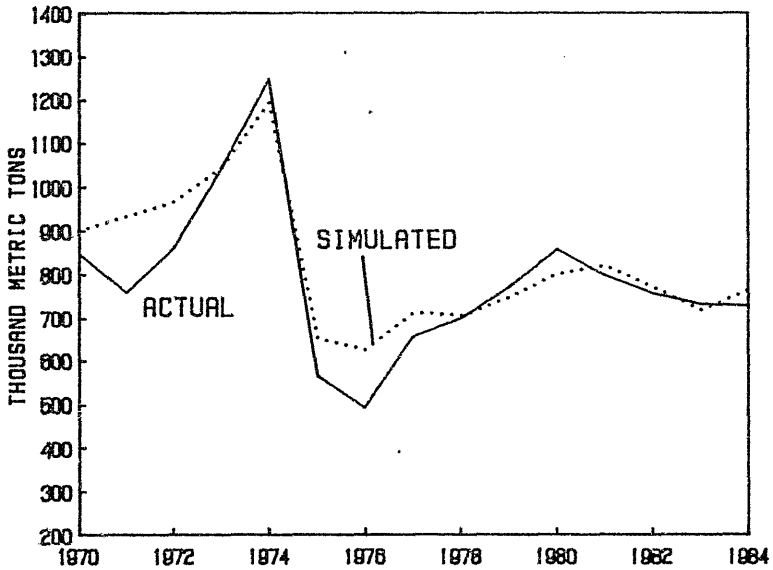


FIGURE 6.26: EEC-10

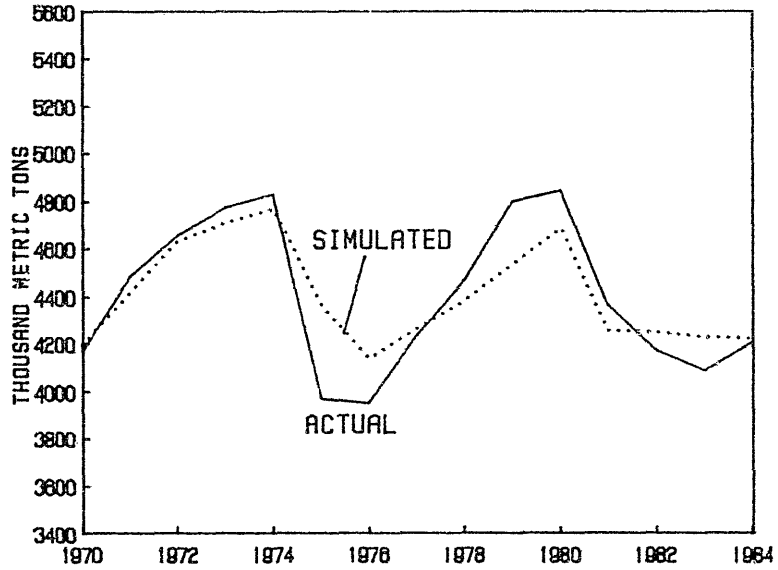


FIGURE 6.27: JAPAN

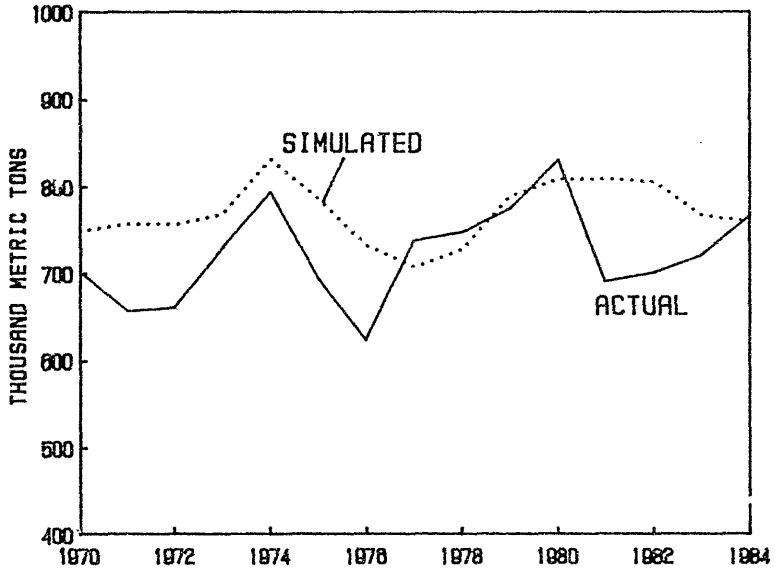
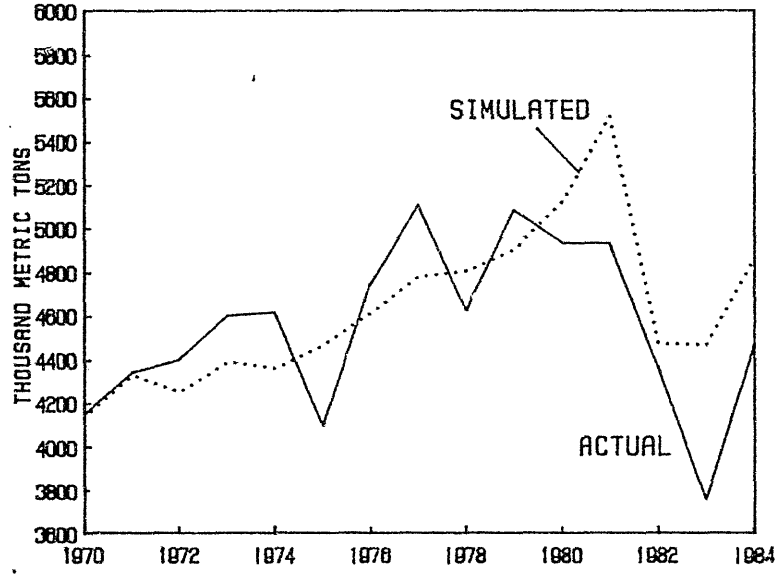
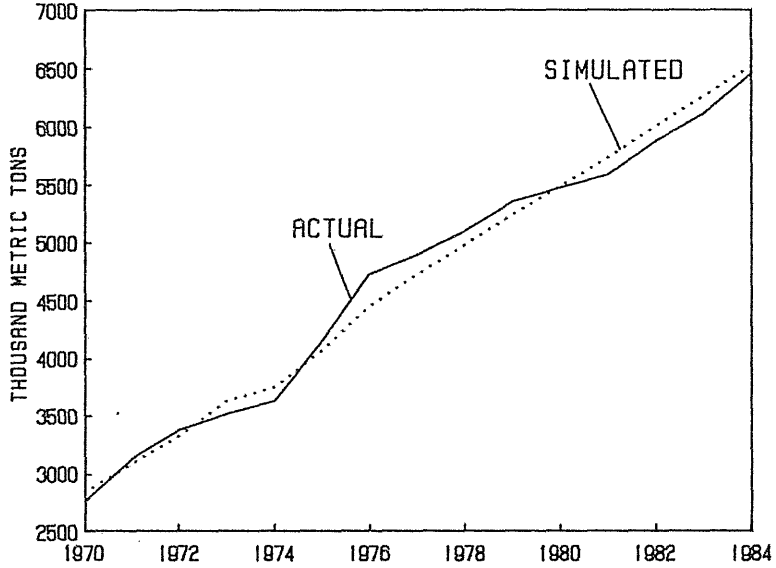


FIGURE 6.28: UNITED STATES



PHOSPHATE CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.29: U.S.S.R.



PHOSPHATE CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.30: DEVELOPING COUNTRIES

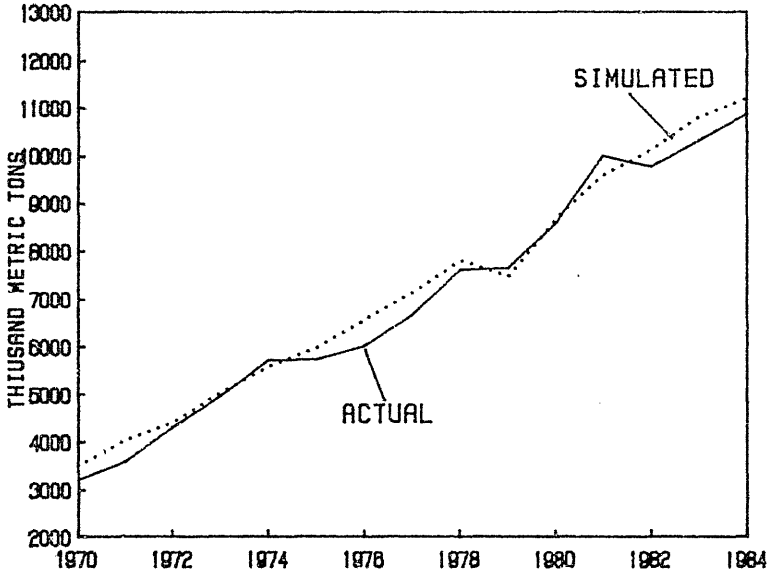


FIGURE 6.31: INDUSTRIAL COUNTRIES

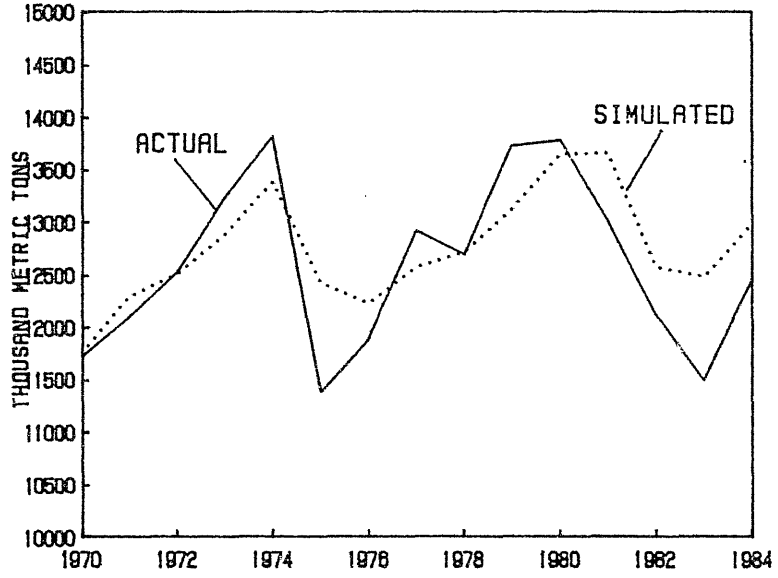


FIGURE 6.32: CENTRALLY PLANNED ECONOMIES

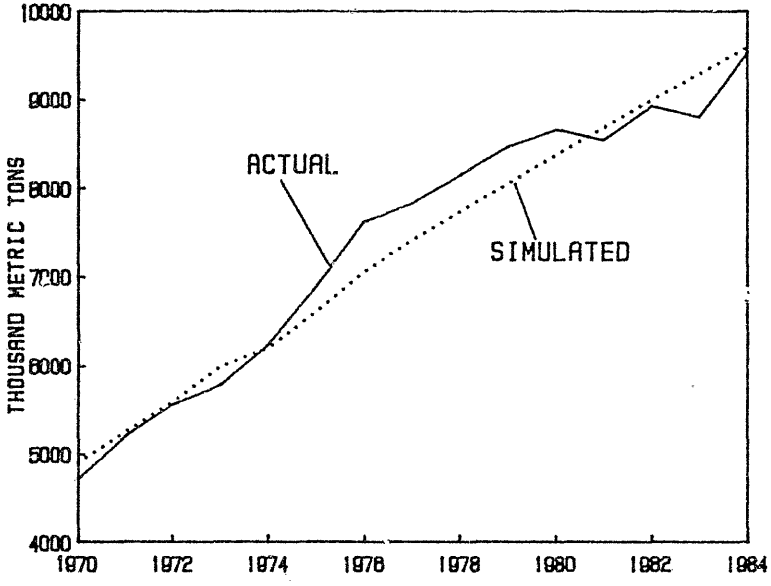
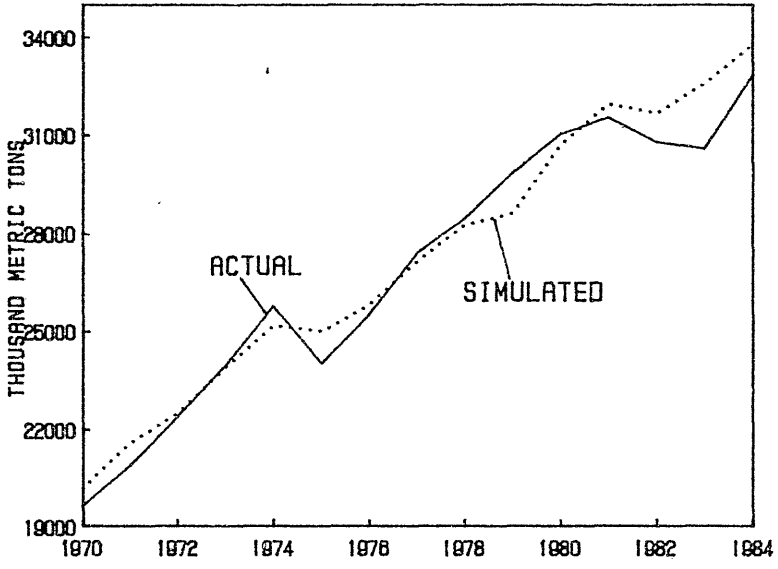


FIGURE 6.33: WORLD



POTASH CONSUMPTION: DYNAMIC SOLUTION TRACKING

FIGURE 6.34: BRAZIL

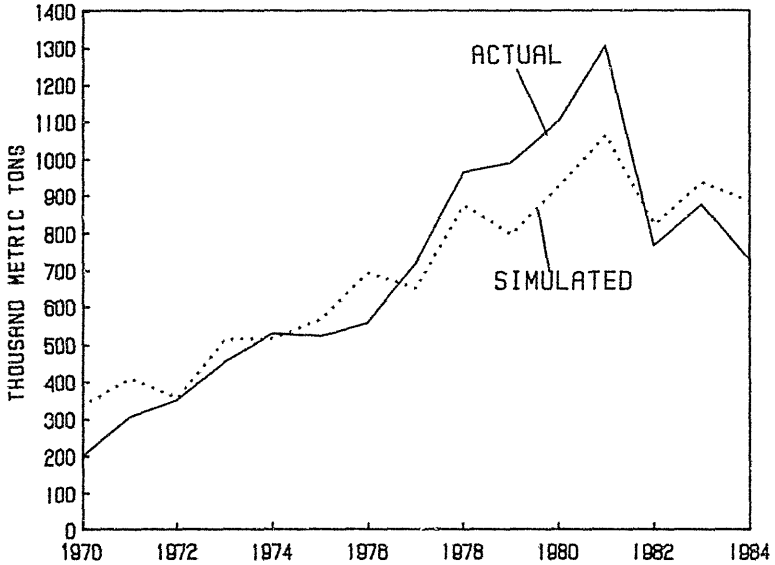


FIGURE 6.35: LATIN AMERICA & CARIBBEAN

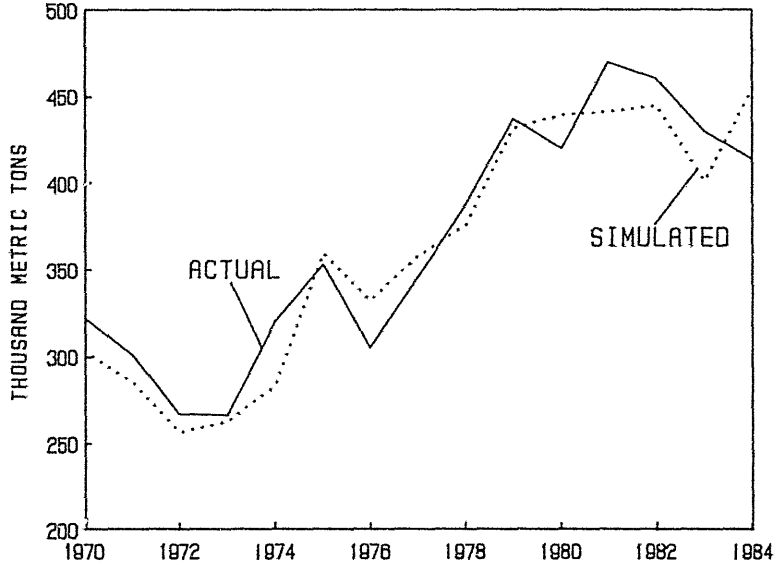


FIGURE 6.36: EAST ASIA

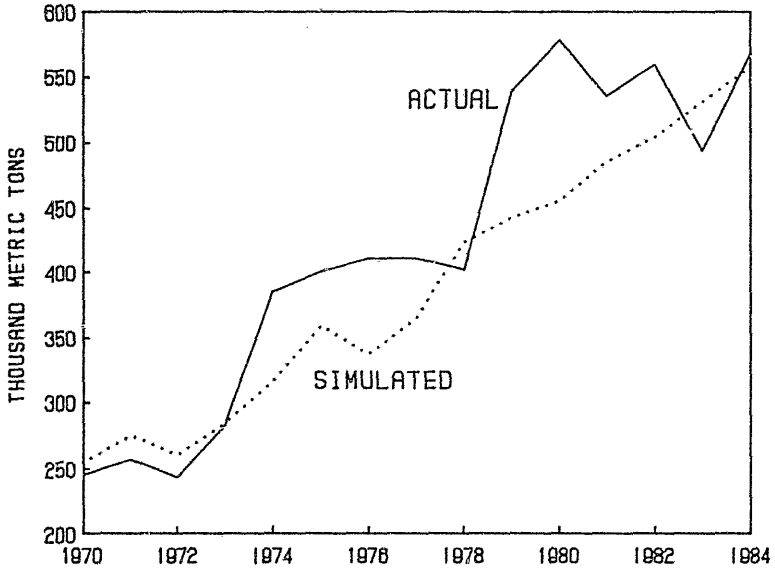
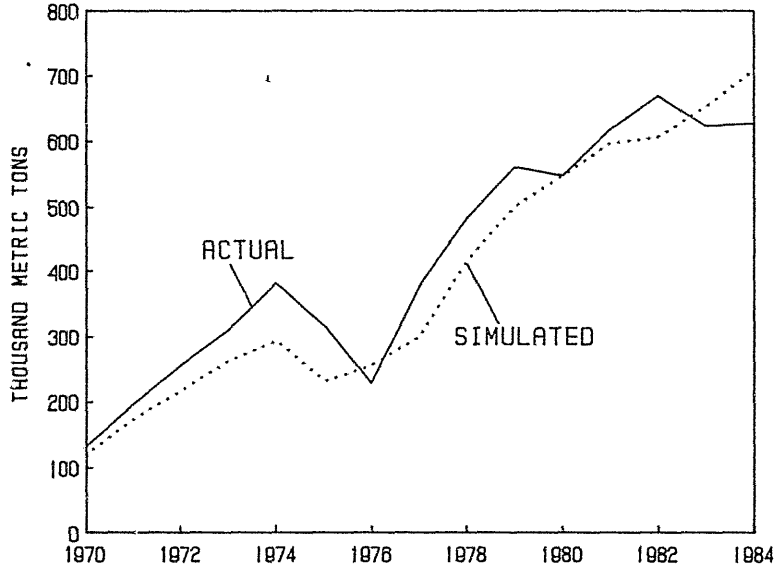


FIGURE 6.37: INDIA



POTASH CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.38: CHINA

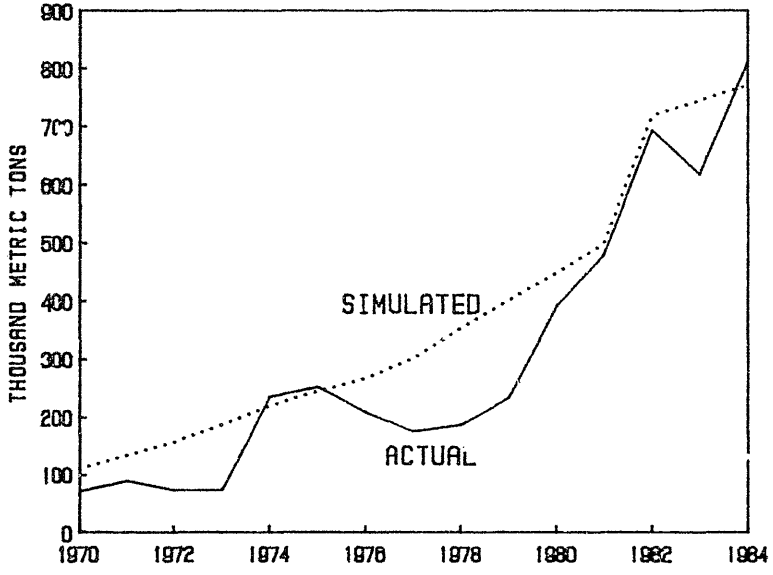


FIGURE 6.39: SOUTH AFRICA

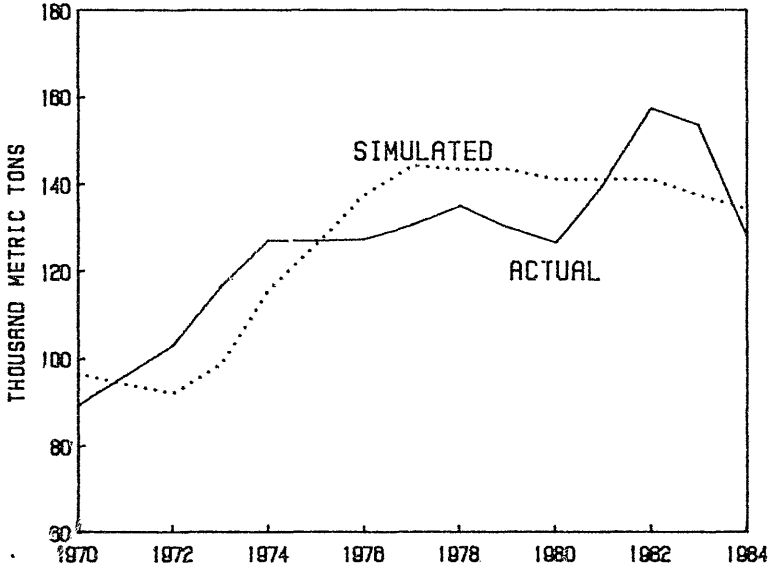


FIGURE 6.40: EEC-10

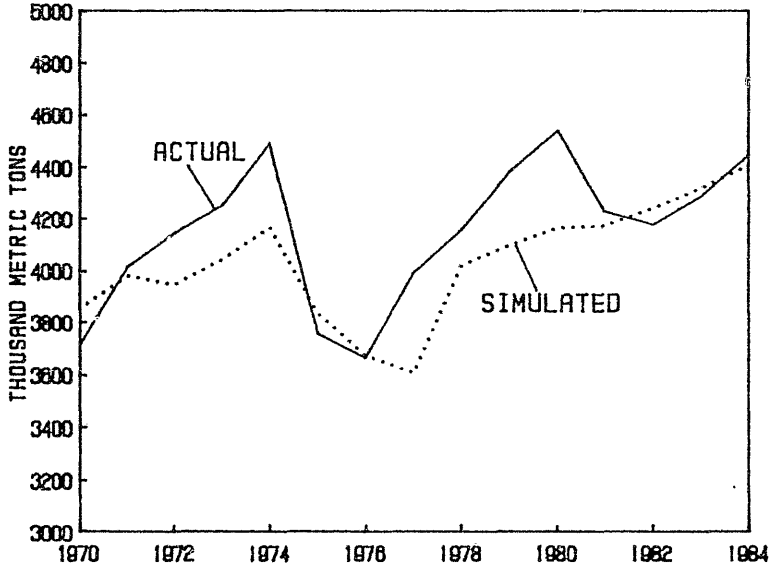
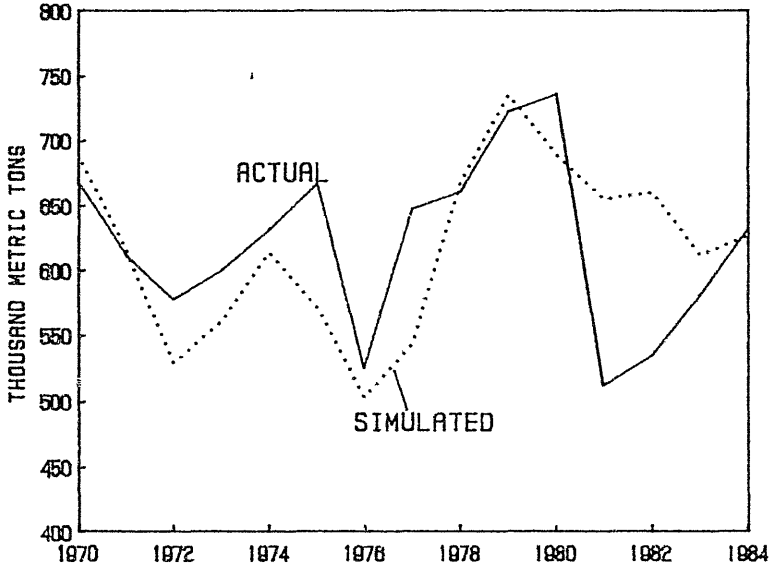


FIGURE 6.41: JAPAN



POTASH CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.42: UNITED STATES

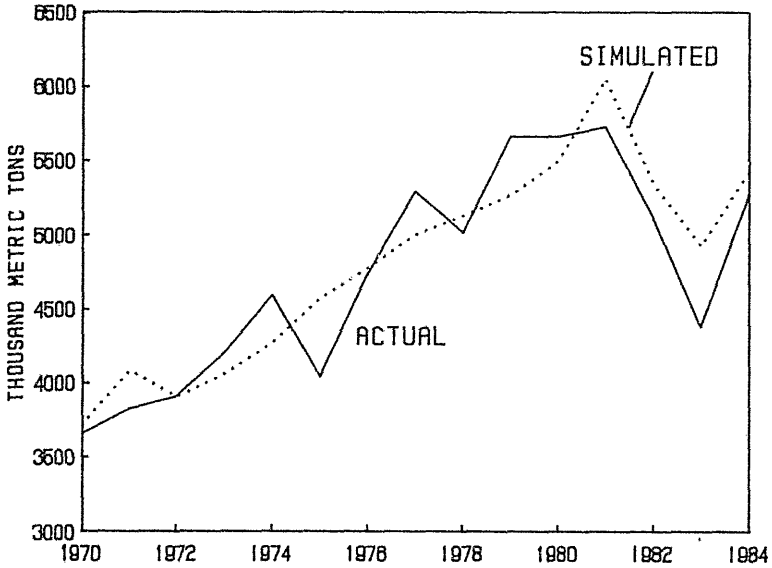


FIGURE 6.43: OTHER INDUSTRIAL COUNTRIES

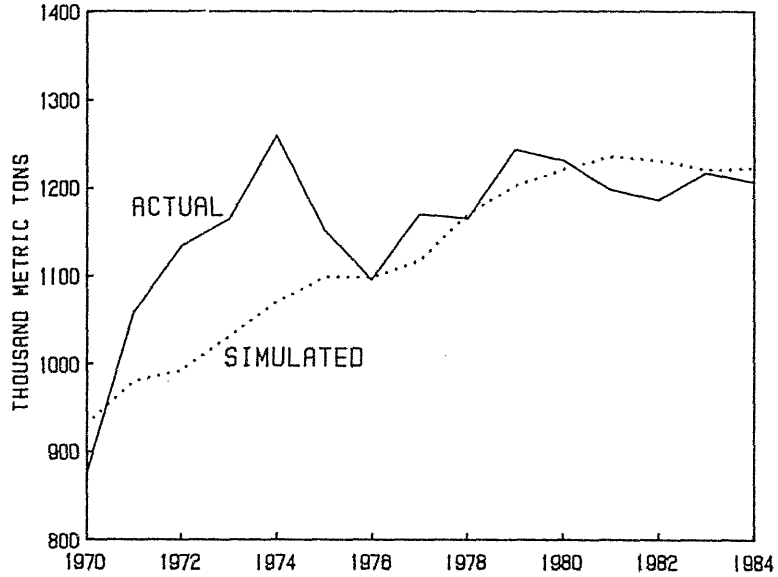
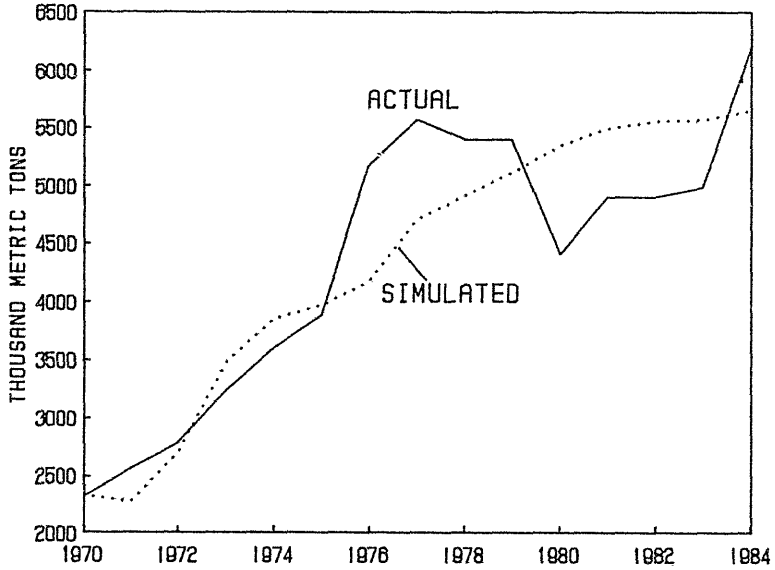


FIGURE 6.44: U.S.S.R.



POTASH CONSUMPTION: DYNAMIC SOLUTION TRACKING (CONTINUED)

FIGURE 6.45: DEVELOPING COUNTRIES

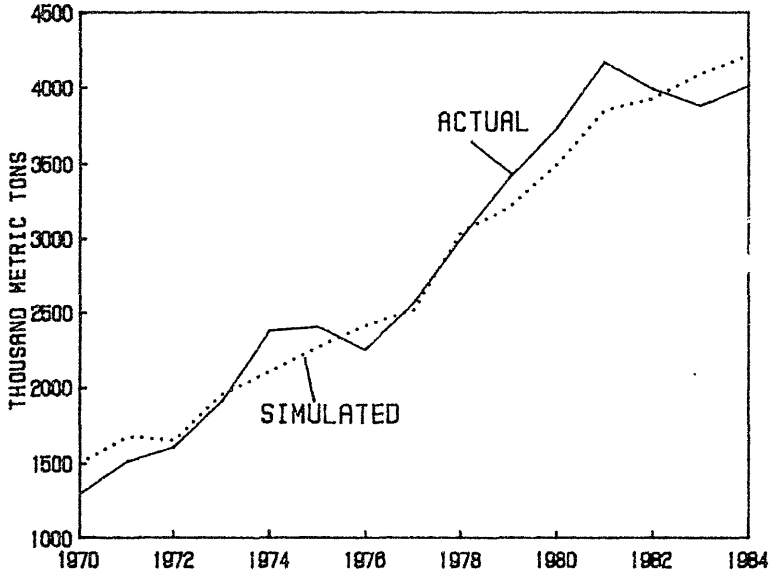


FIGURE 6.46: INDUSTRIAL COUNTRIES

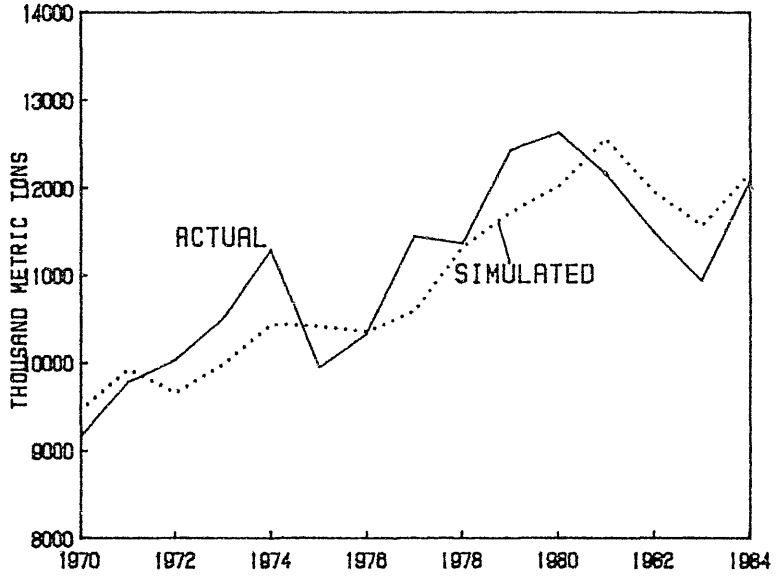


FIGURE 6.47: CENTRALLY PLANNED ECONOMIES

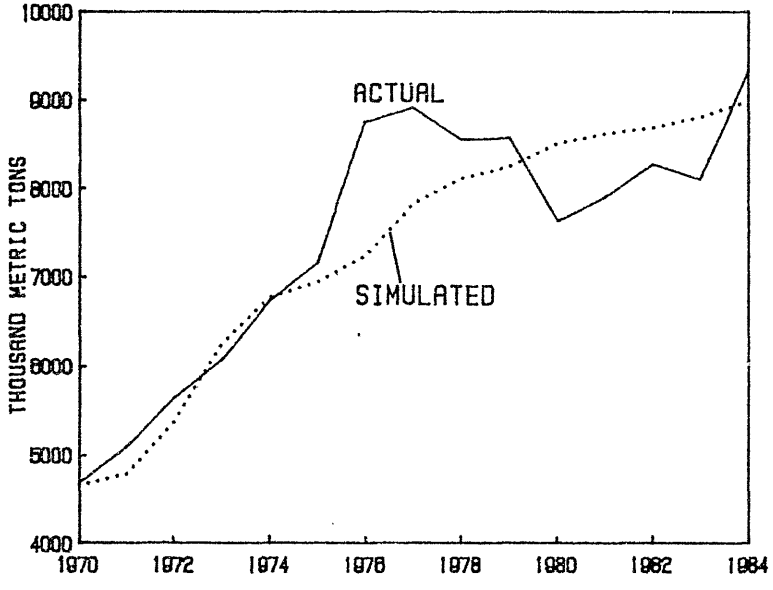
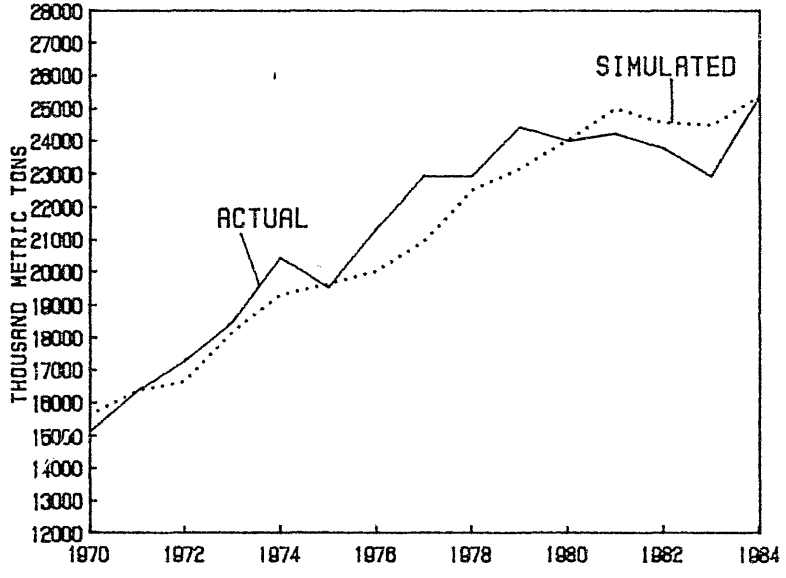


FIGURE 6.48: WORLD



NITROGEN PRODUCTION CAPACITY: DYNAMIC SOLUTION TRACKING

FIGURE 6.49: EUROPE & JAPAN

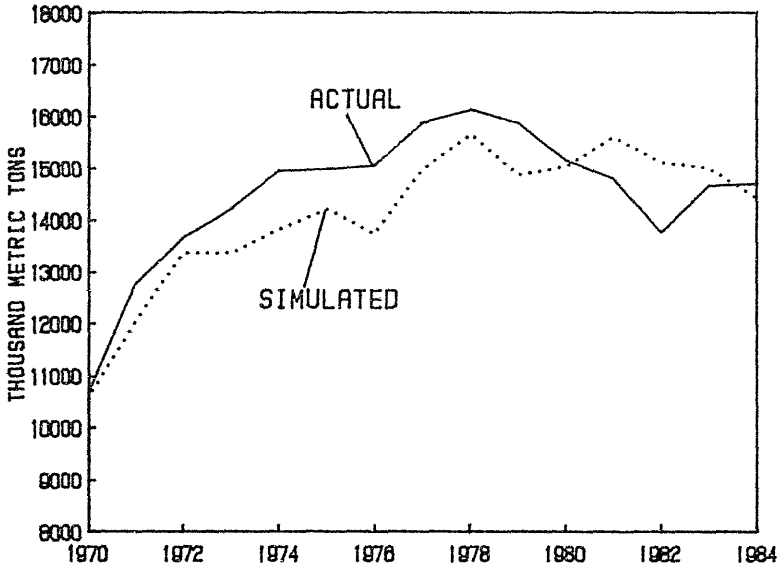


FIGURE 6.50: NORTH AMERICA

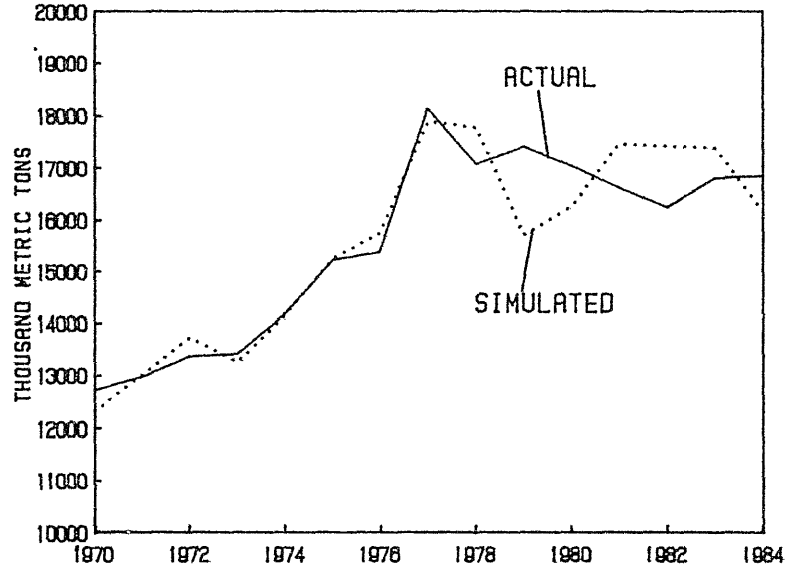


FIGURE 6.51: DEVELOPING COUNTRIES

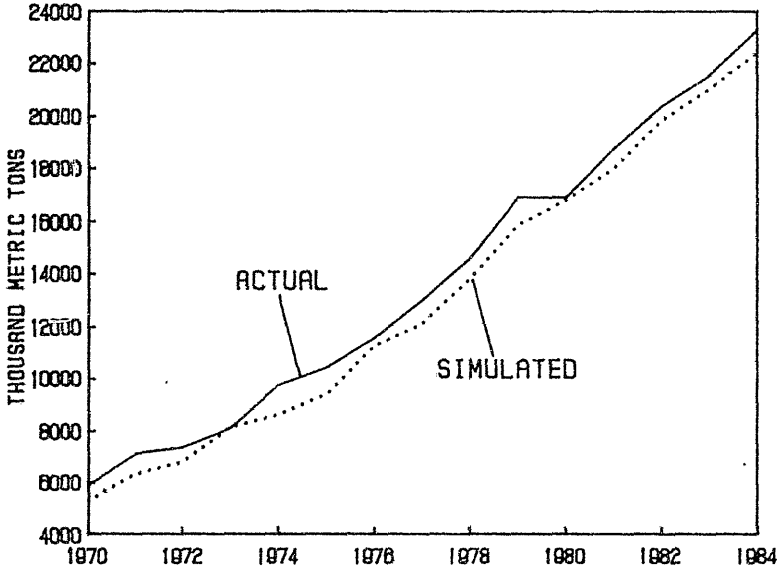
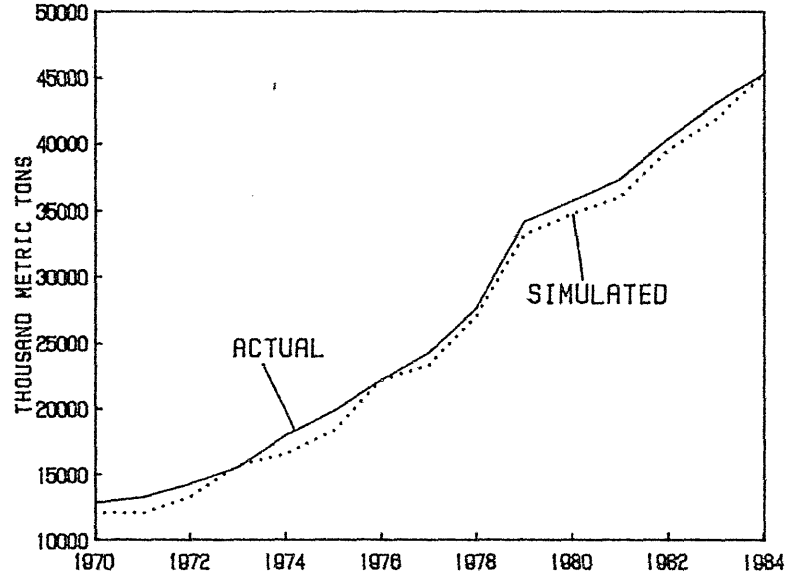


FIGURE 6.52: CENTRALLY PLANNED ECONOMIES



PHOSPHATE PRODUCTION CAPACITY: DYNAMIC SOLUTION TRACKING

FIGURE 6.53: EUROPE & JAPAN

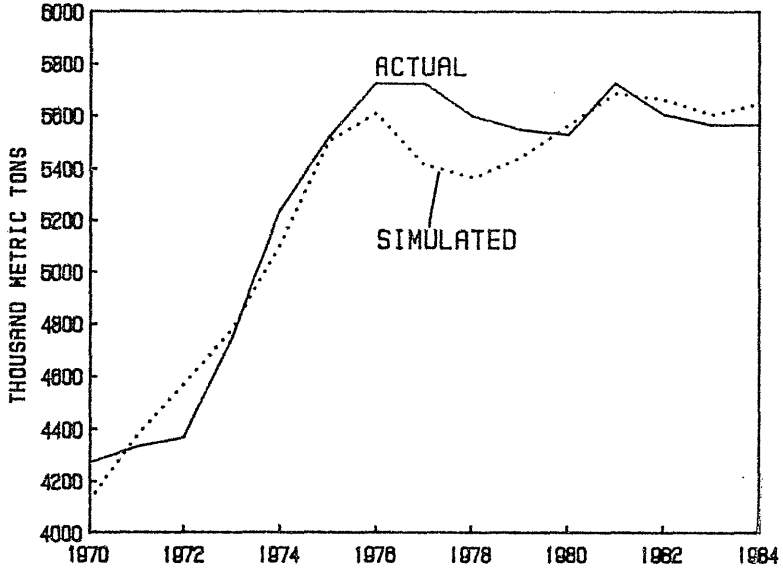


FIGURE 6.54: NORTH AMERICA

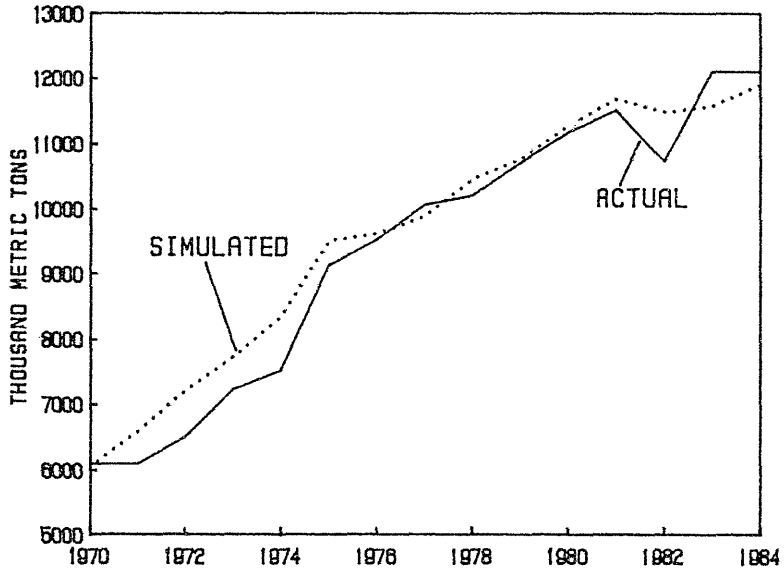


FIGURE 6.55: DEVELOPING COUNTRIES

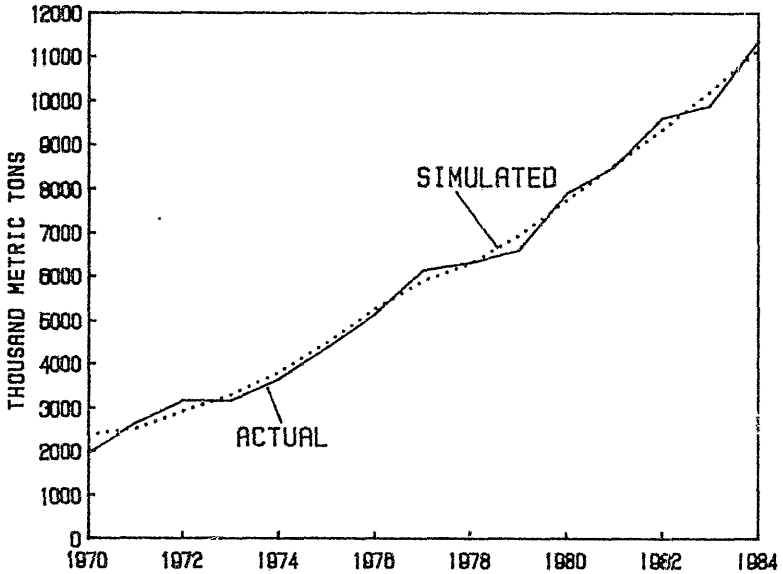


FIGURE 6.56: CENTRALLY PLANNED ECONOMIES

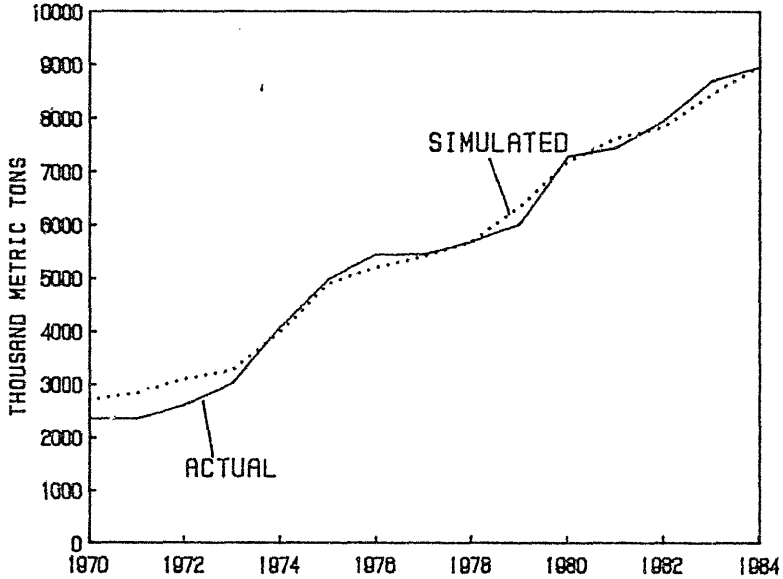


FIGURE 6.57: WORLD NITROGEN PRODUCTION CAPACITY

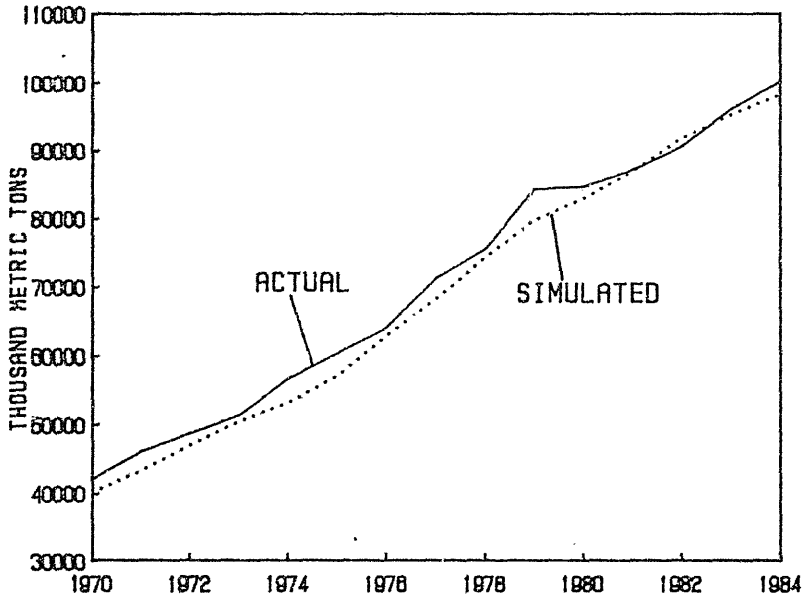


FIGURE 6.58: WORLD PHOSPHATE PRODUCTION CAPACITY

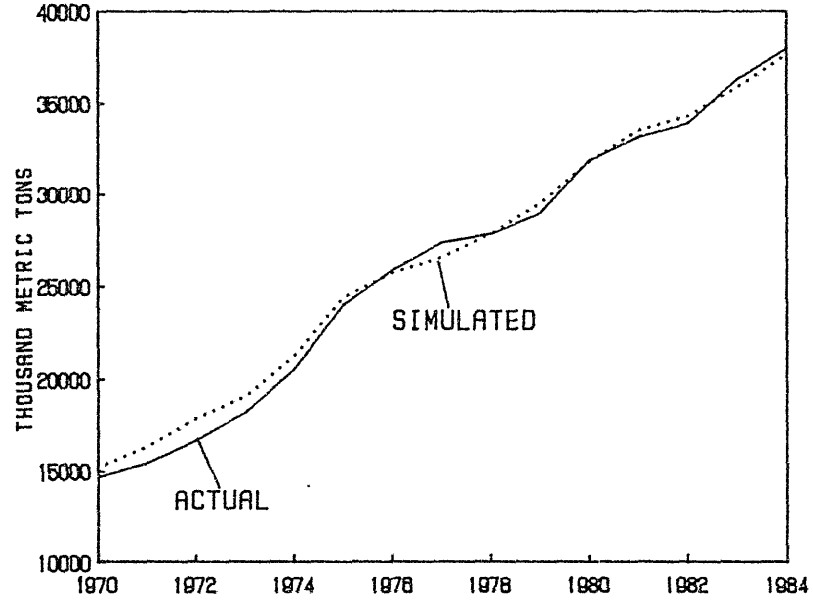
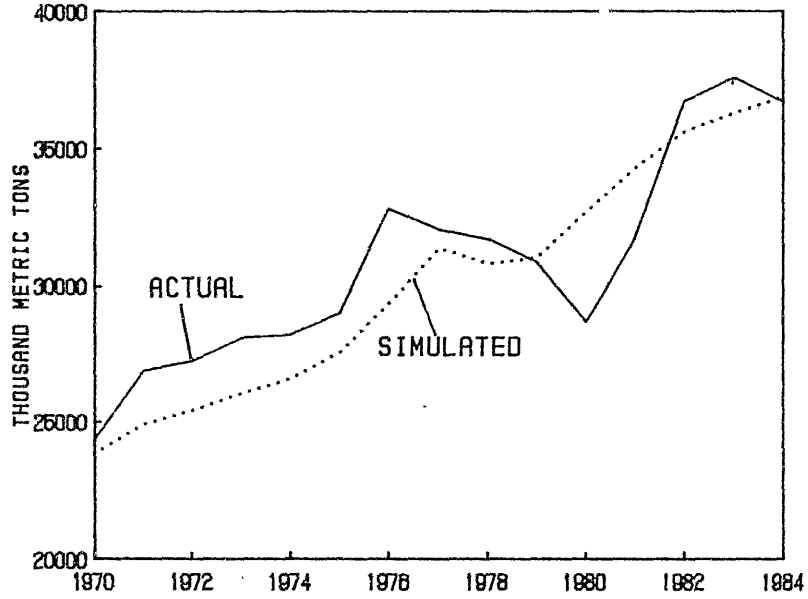


FIGURE 6.59: WORLD POTASH PRODUCTION CAPACITY



NITROGEN PRODUCTION: DYNAMIC SOLUTION TRACKING

FIGURE 6.60: EUROPE & JAPAN

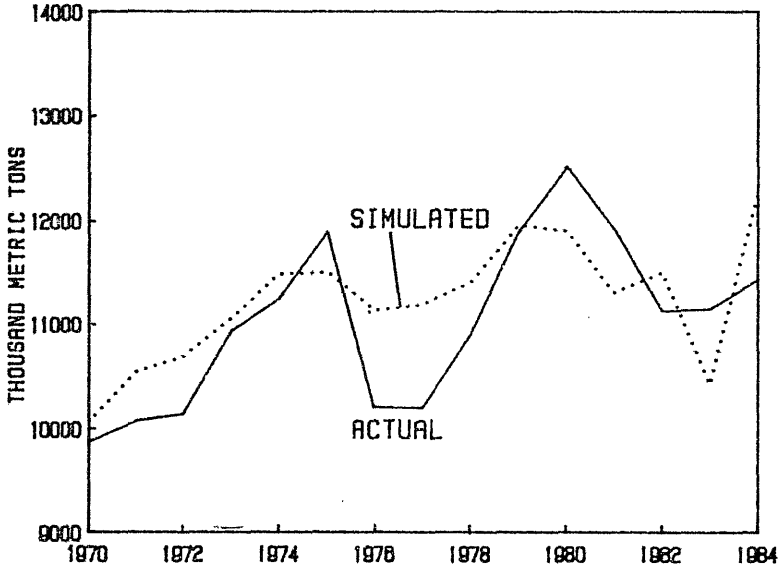


FIGURE 6.61: NORTH AMERICA

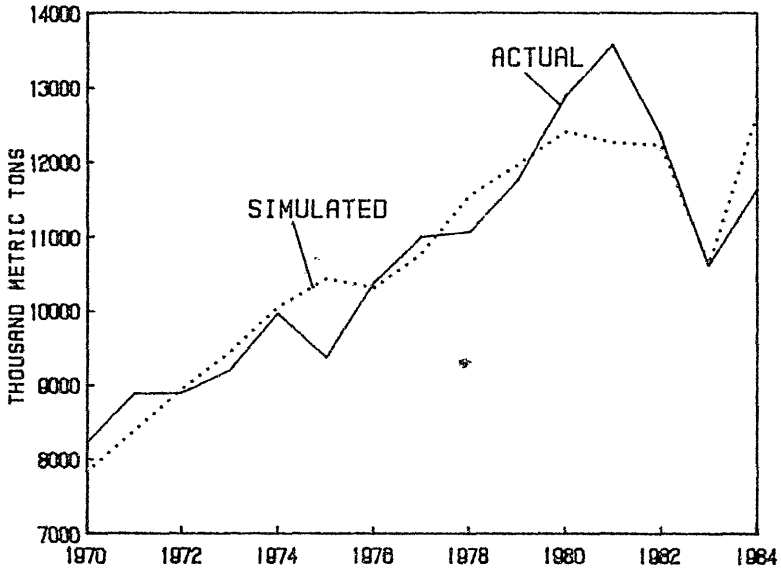


FIGURE 6.62: DEVELOPING COUNTRIES

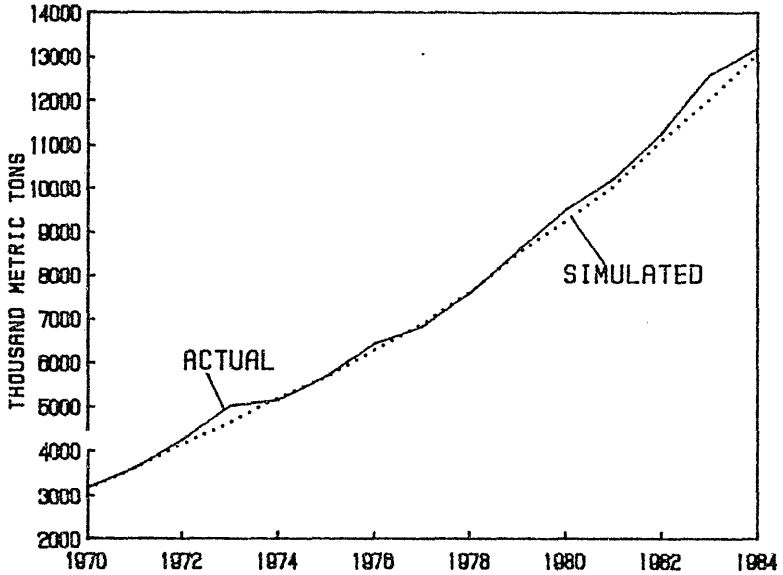
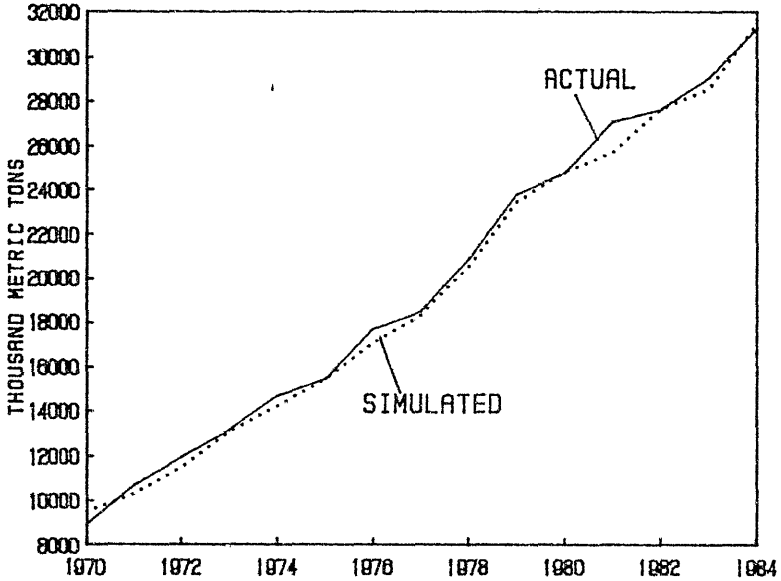


FIGURE: 6.63: CENTRALLY PLANNED ECONOMIES



PHOSPHATE PRODUCTION: DYNAMIC SOLUTION TRACKING

FIGURE 6.64: EUROPE & JAPAN

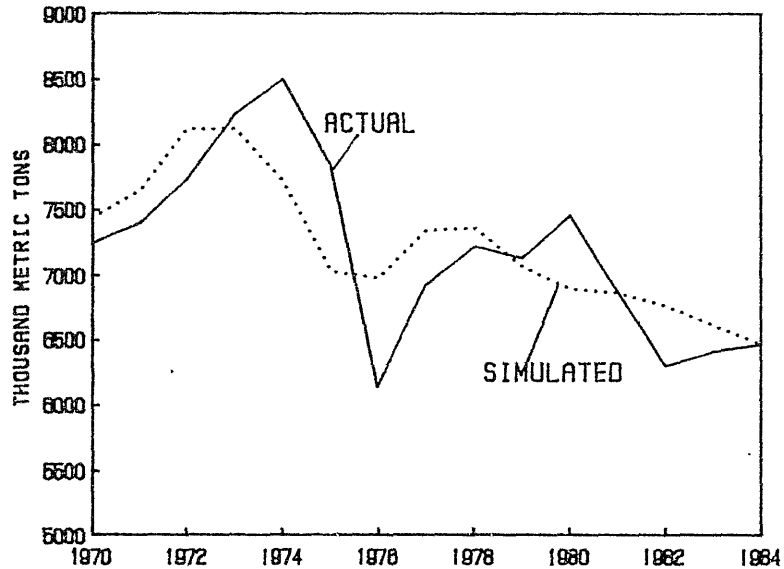


FIGURE 6.65: NORTH AMERICA

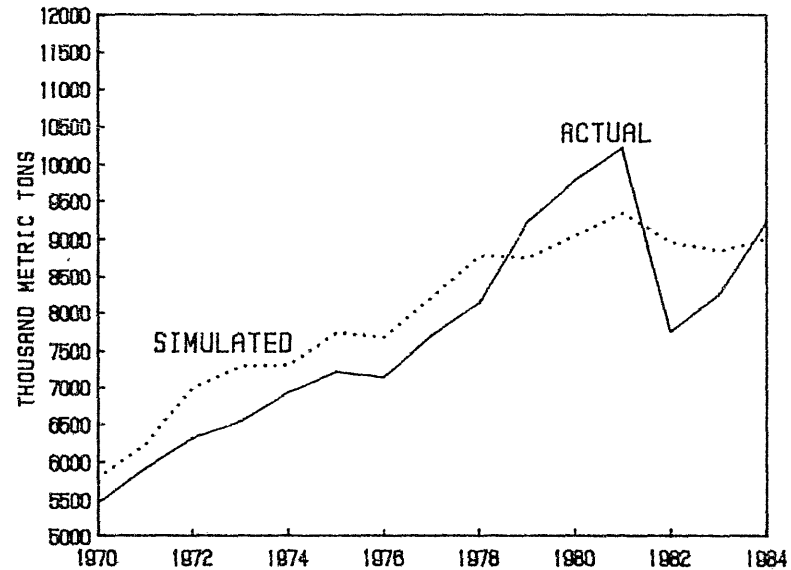


FIGURE 6.66: DEVELOPING COUNTRIES

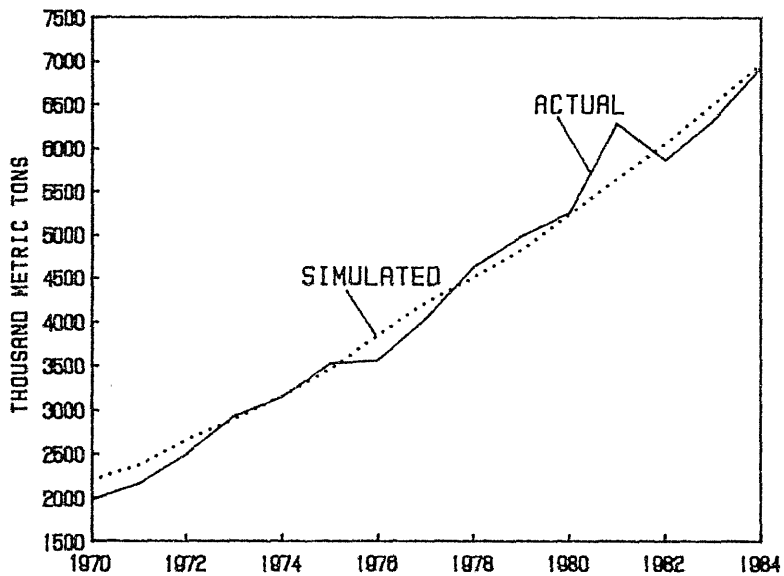


FIGURE 6.67: CENTRALLY PLANNED ECONOMIES

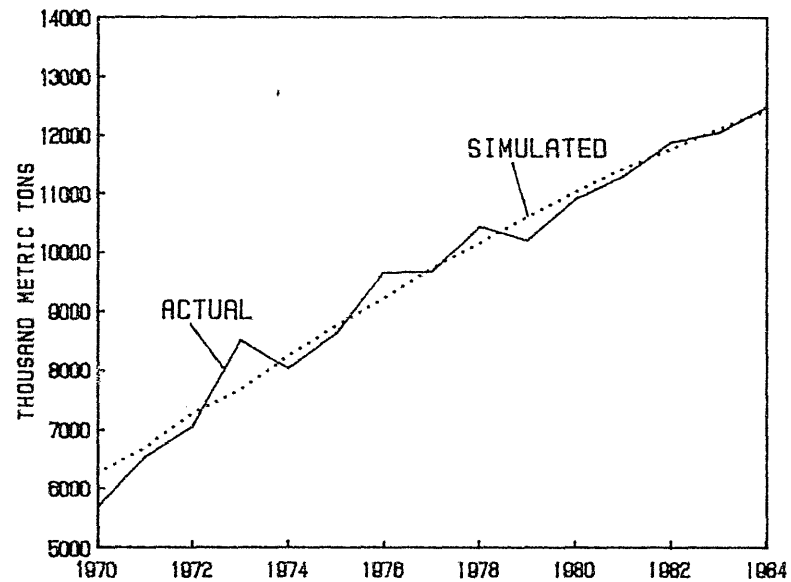


FIGURE 6.68: WORLD NITROGEN PRODUCTION

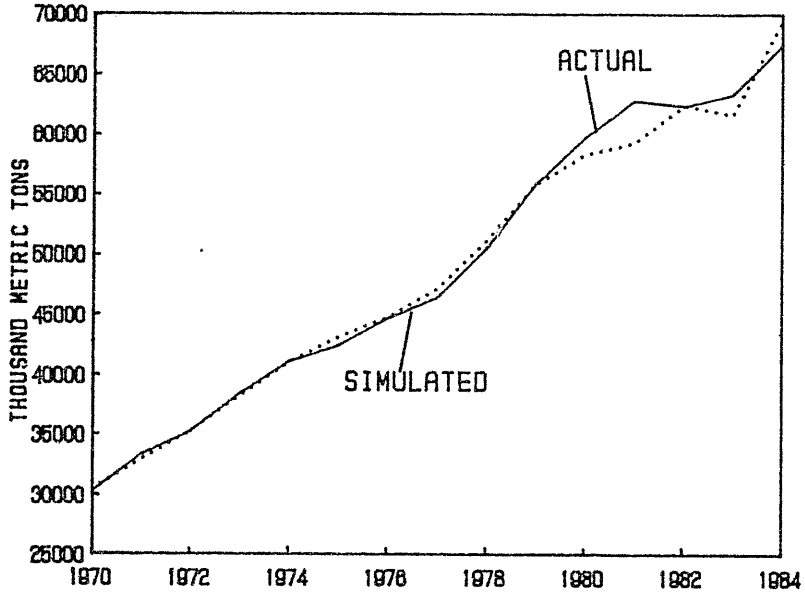


FIGURE 6.69: WORLD PHOSPHATE PRODUCTION

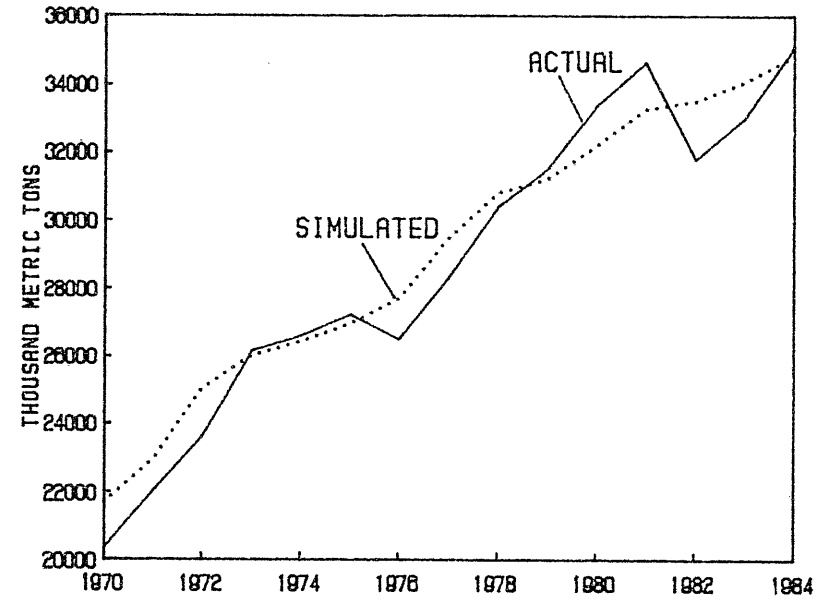


FIGURE 6.70: WORLD POTASH PRODUCTION

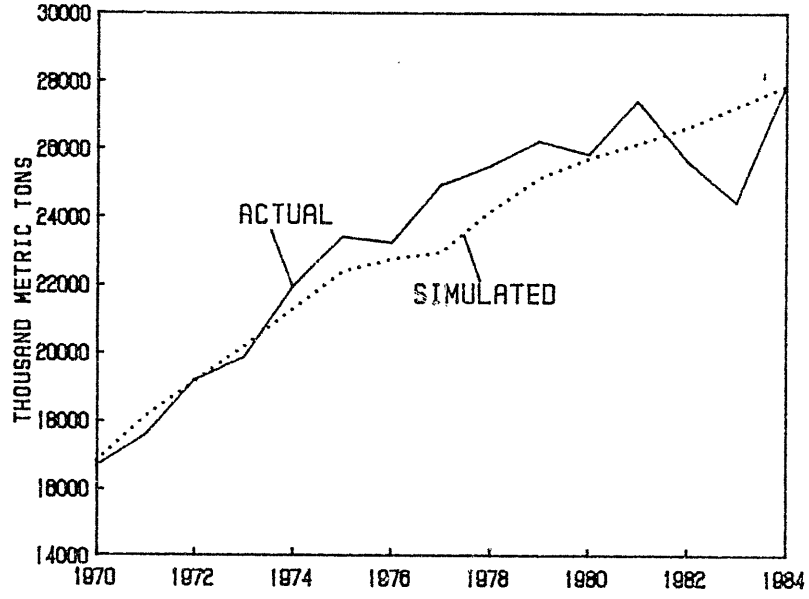


FIGURE 6.71: NITROGEN WORLD PRICE
DYNAMIC SOLUTION TRACKING

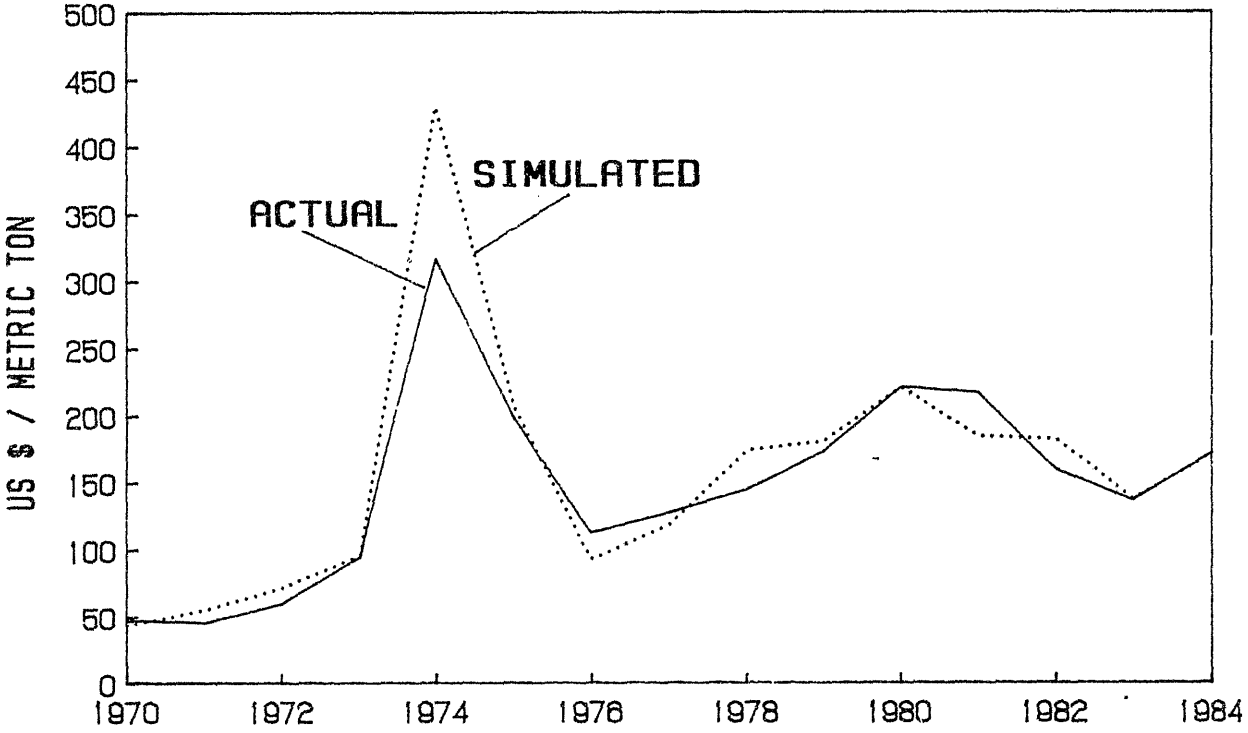


FIGURE 6.72: PHOSPHATE WORLD PRICE
DYNAMIC SOLUTION TRACKING

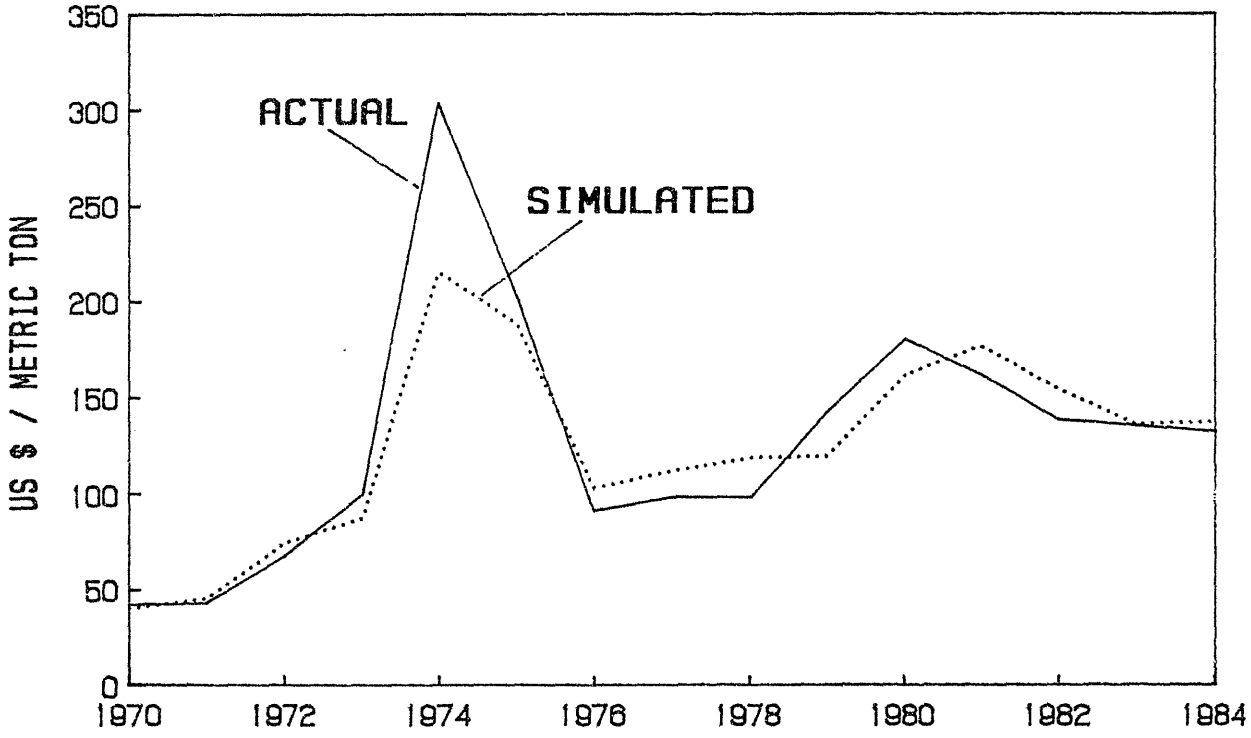
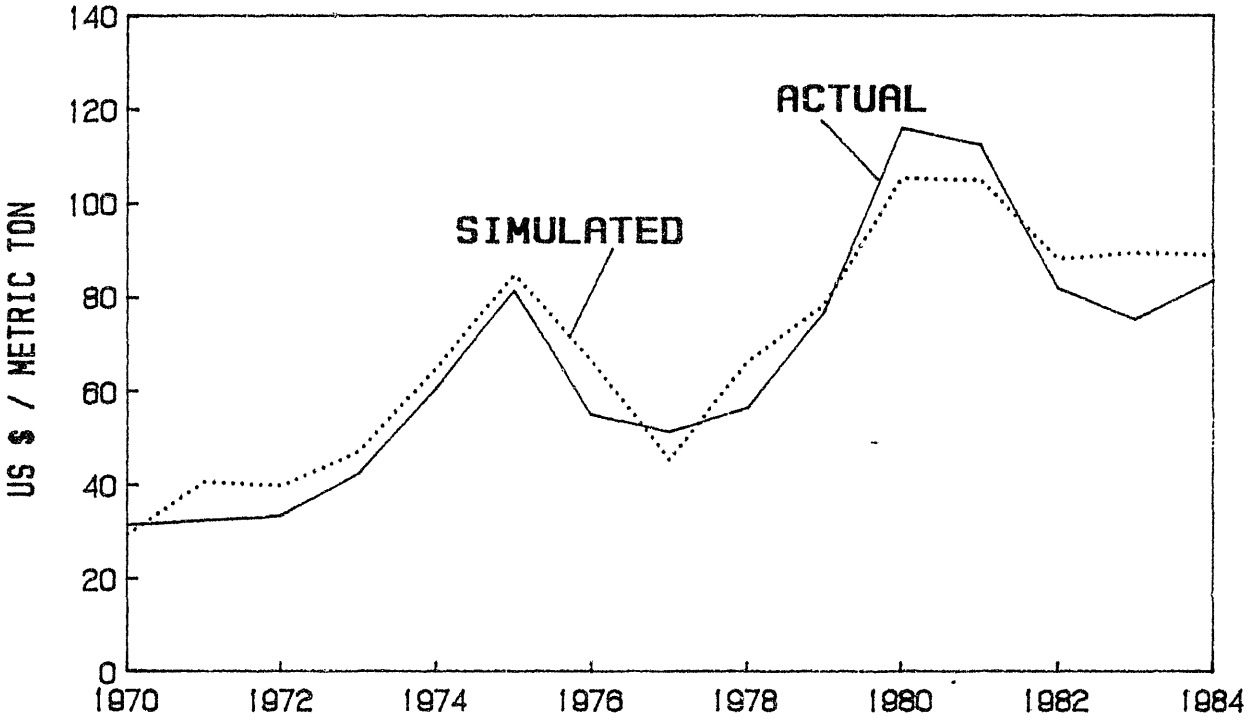


FIGURE 6.73: POTASH WORLD PRICE
DYNAMIC SOLUTION TRACKING



VII. MODEL PROJECTIONS FOR THE PERIOD 1985-2000

49. The ex-ante simulation was carried out for the period 1985-2000. Most variables were solved endogenously except for the macro-economic variables i.e., GDP, exchange rates and consumer price index. Assumptions for these macro-economic variables were taken from forecasts provided by the World Bank and Wharton Econometric Forecasting Associates. The projections for grains prices, planted area and yields were generated by the Division's grains model to which the fertilizer model is linked.

50. The projections of fertilizer production, consumption and price are in line with those reported recently in the Division's Report No. 814/86, Price Prospects for Major Primary Commodities. Graphs showing forecast changes in the market shares held by the industrial, developing and centrally planned country groups are presented in Figures 7.1 through 7.8.

51. The share of nitrogen consumption held by the industrial countries constantly declined throughout the 1960s and 1970s--from about 68% in 1962 to 51% in 1970 and 39% in 1980. This decline was mainly due to the "green revolution" which significantly increased fertilizer use in the developing countries. The developing countries went from a nitrogen consumption share of 16% in 1962 to 40% in 1980. It appears that the trend will continue in the next decade with projected shares of 48% in 1990 and 52% in 2000 for the developing countries. Much the same applies to phosphate and potash consumption, with the share of the developing countries in the phosphate market projected to increase gradually to about 39% by the year 2000, and to grow to 18% of total potash consumption.

52. The centrally planned economies of Europe increased sharply their consumption of fertilizers during the 1960s and 1970s. As a result their world

shares also increased sharply. However, due to the expected slowdown in their agricultural production, their share of world consumption is forecast to decline somewhat over the 1985-2000 period.

53. Nitrogen was produced almost exclusively in the industrial countries only 20 years ago. In 1962, they accounted for a market share of 74%. Their current share of the market has decreased to only 34% and is projected to decrease to less than 25% by the year 2000. The same appears to be happening in the phosphate market. The industrial countries' share of the market fell from 77% in 1962 to 43% in 1985 and is projected to decline to about 26% by the year 2000. The share of the developing countries in the production of these fertilizers increased sharply from about 8% in 1962 to about 20% in 1985. According to the projections, by the year 2000 they will produce about 35% of world nitrogen output and about 40% of world phosphate output.

FIGURE 7.1: NITROGEN CONSUMPTION SHARE OF WORLD TOTAL BY MAIN ECONOMIC REGIONS, 1962-2000

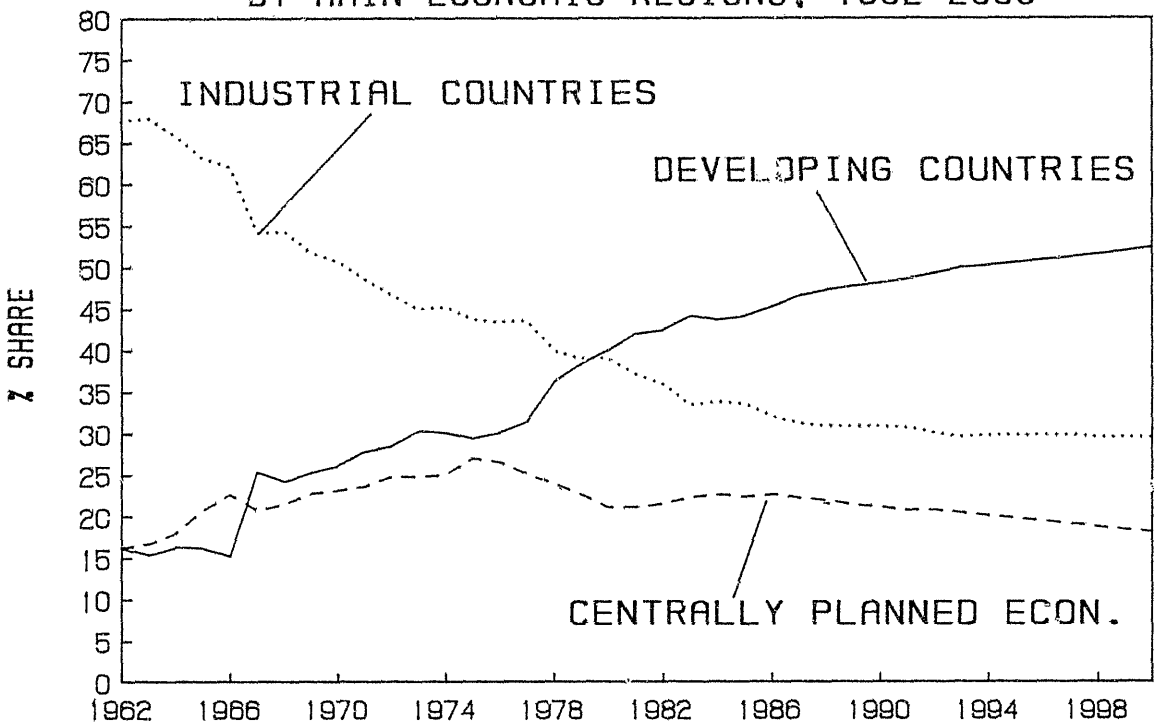


FIGURE 7.2: PHOSPHATE CONSUMPTION SHARE OF WORLD TOTAL BY MAIN ECONOMIC REGIONS, 1962-2000

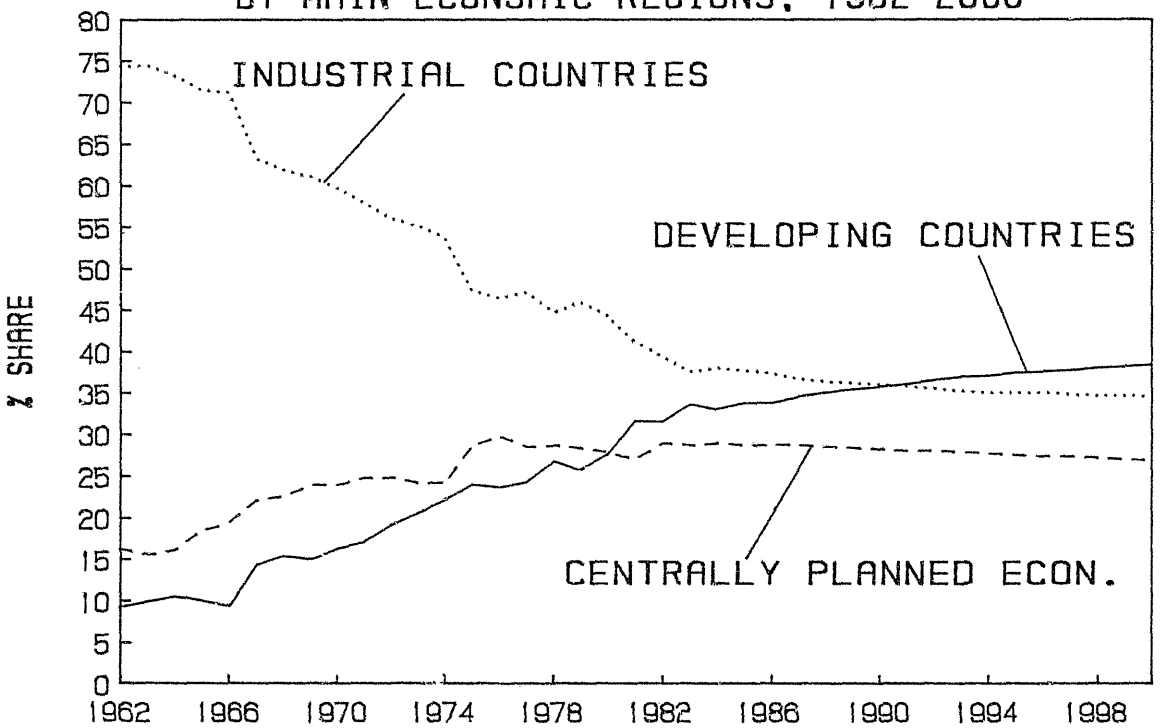


FIGURE 7.3: POTASH CONSUMPTION SHARE OF WORLD TOTAL BY MAIN ECONOMIC REGIONS, 1962-2000

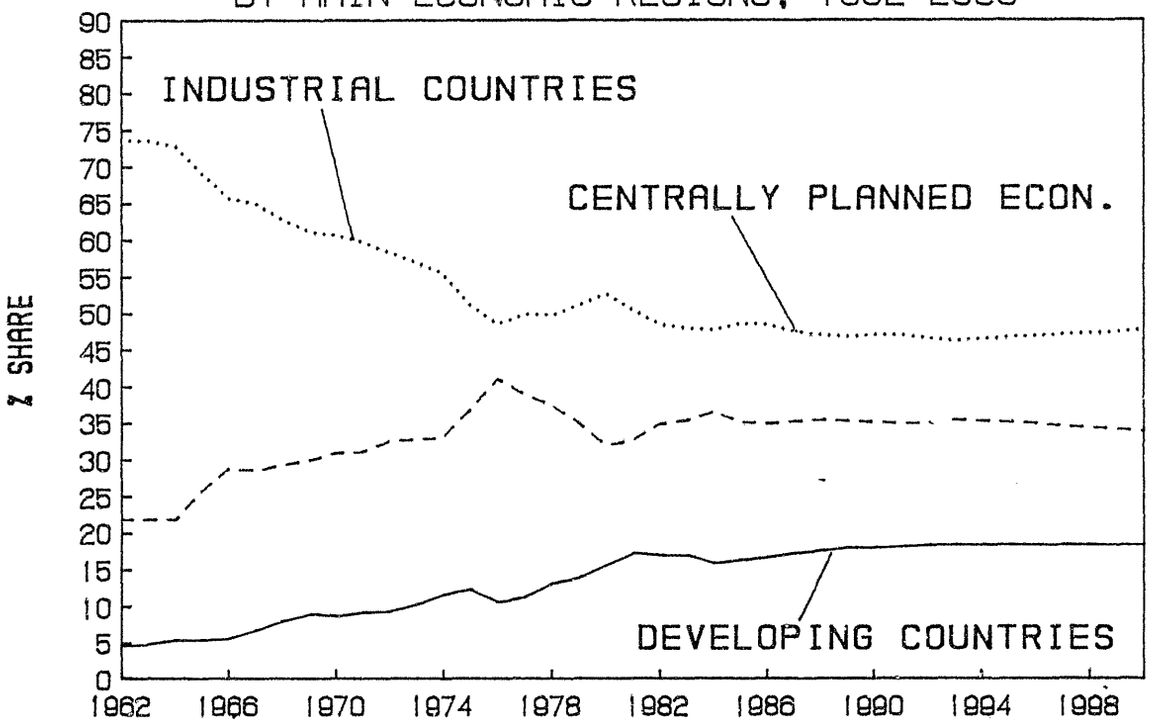


FIGURE 7.4: NITROGEN PRODUCTION SHARE OF WORLD TOTAL BY MAIN ECONOMIC REGIONS, 1962-2000

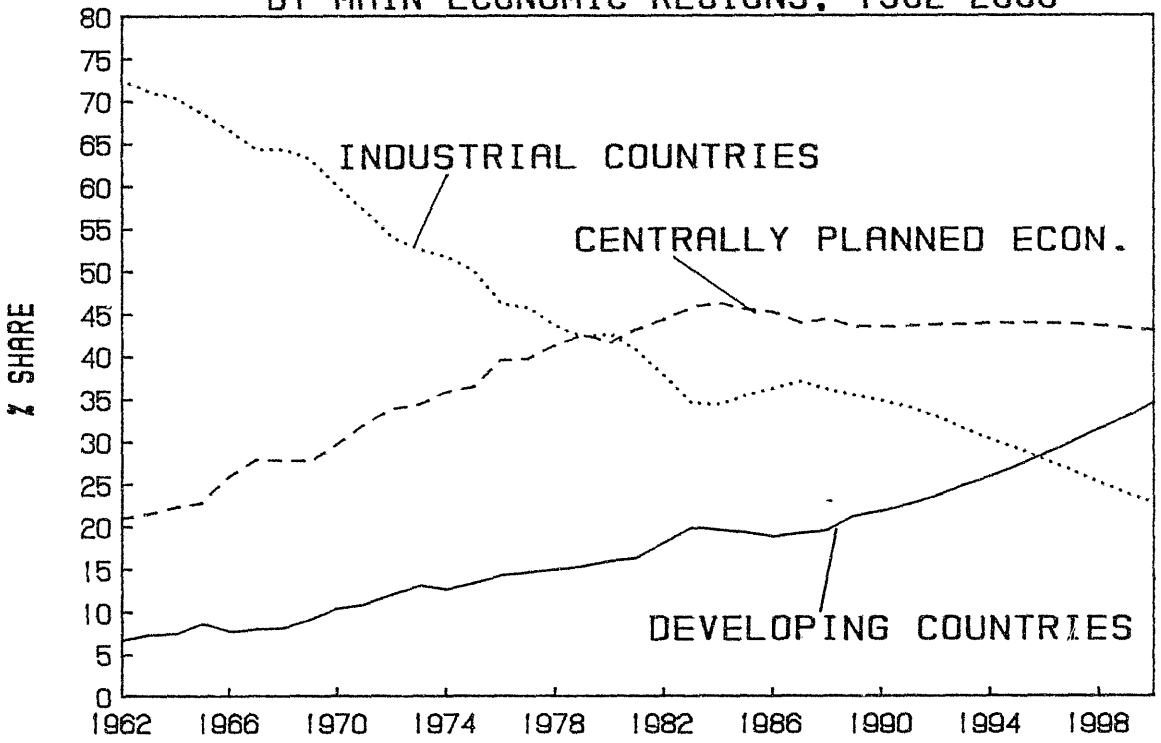


FIGURE 7.5: PHOSPHATE PRODUCTION SHARE OF WORLD TOTAL BY MAIN ECONOMIC REGIONS, 1962-2000

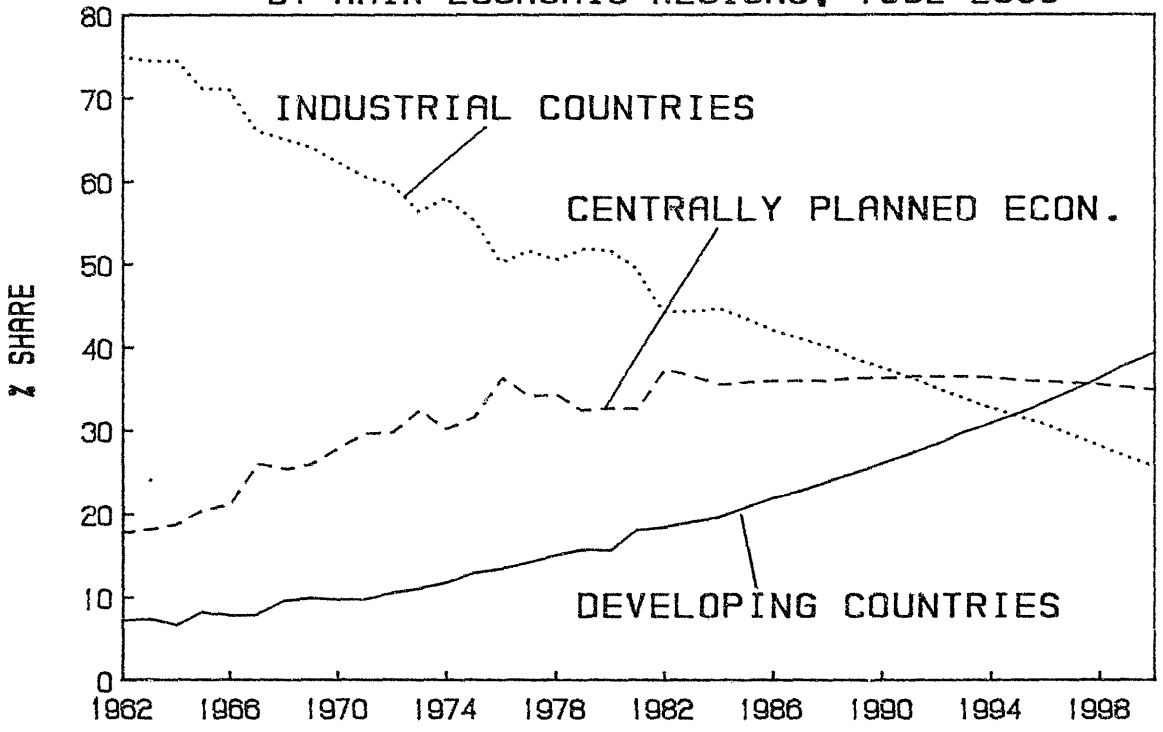


FIGURE 7.6: NITROGEN PRICE FORECAST
(ACTUAL: 1960-85, FORECAST: 1986-2000)

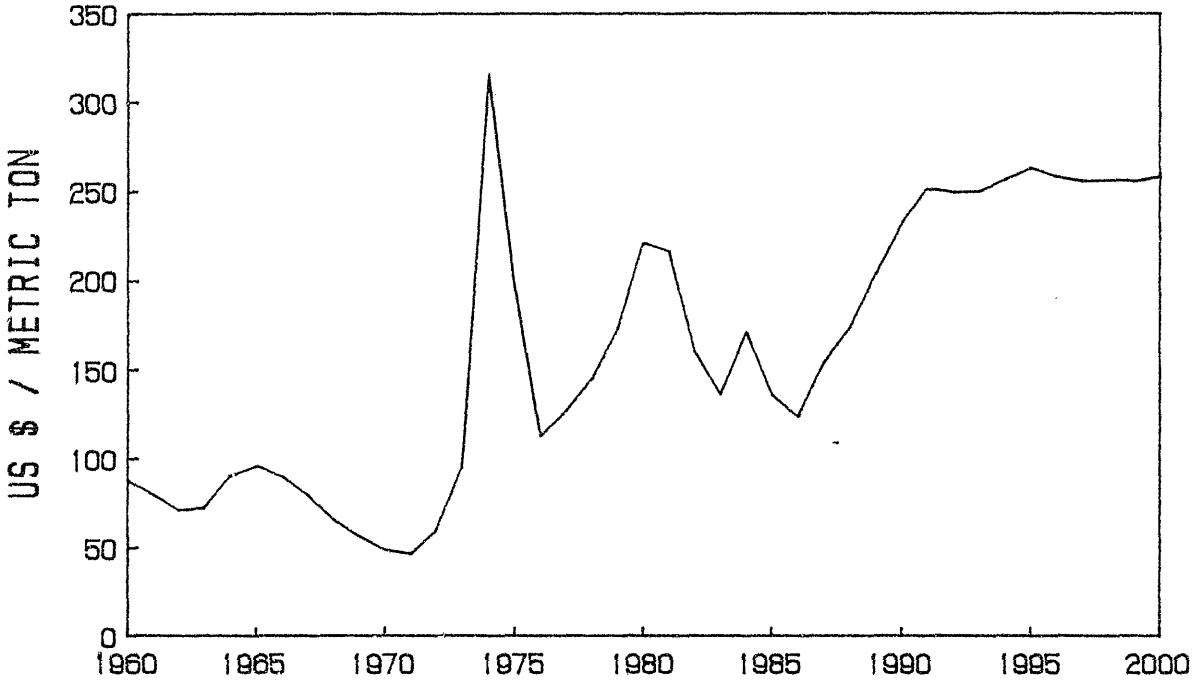


FIGURE 7.7: PHOSPHATE PRICE FORECAST
(ACTUAL: 1960-85, FORECAST: 1986-2000)

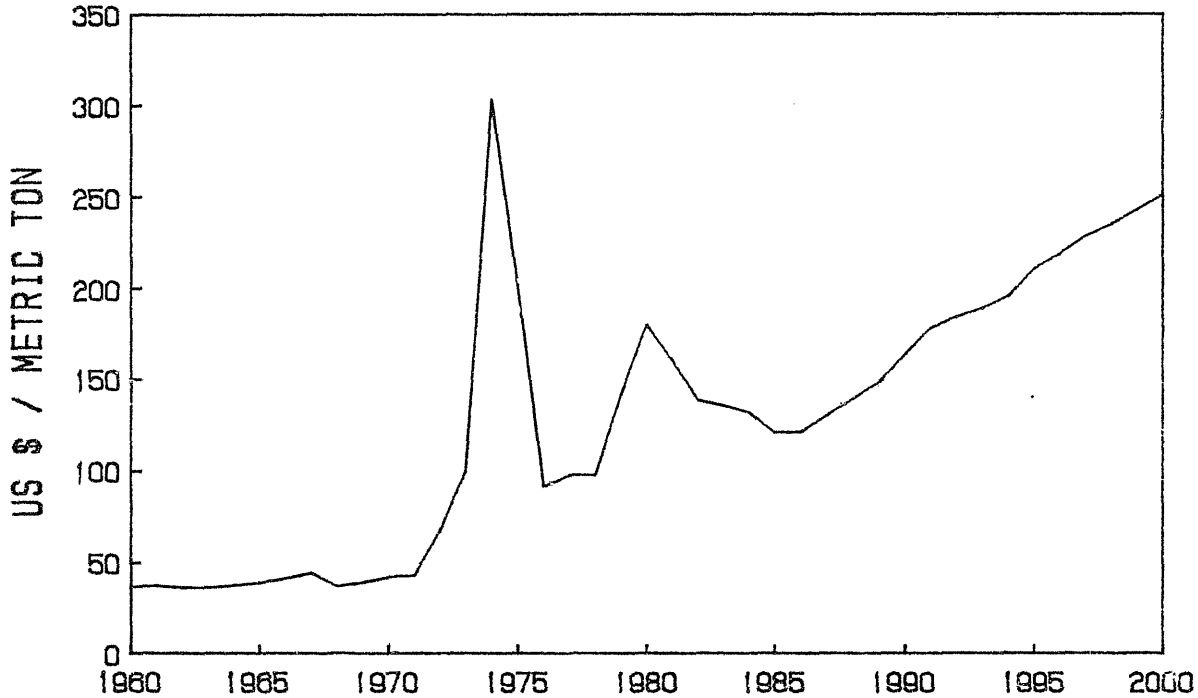
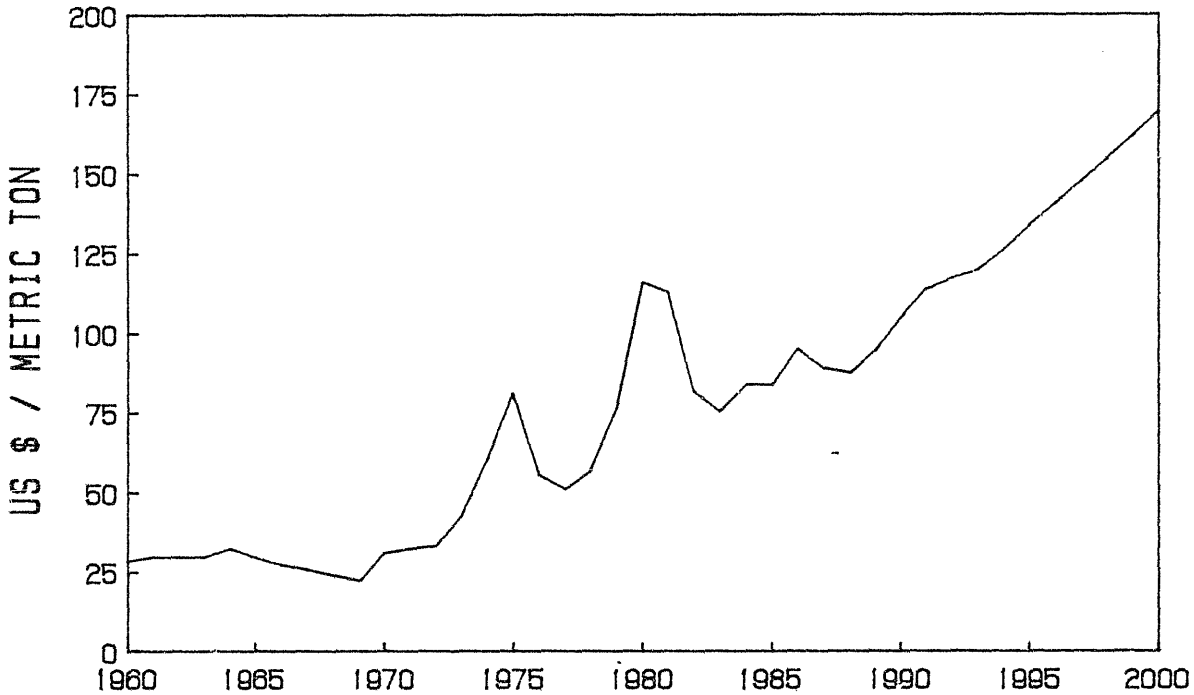


FIGURE 7.8: POTASH PRICE FORECAST
(ACTUAL: 1960-85, FORECAST: 1986-2000)



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VIII. CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

57. This paper summarizes the work that has been done to date on the global fertilizer model. It describes the structure of the model and the estimated equations, as well as providing statistical measures which allows evaluation of the performance of each equation individually and the model as a whole.

58. The model performs well. However, additional work would provide improved analysis of the nature and characteristics of the fertilizer markets. For example, one improvement would be to estimate the consumption equations on a per acre basis and allow the yield and price of both fertilizers and commodities produced determine the level of fertilizer use per acre.

59. Another refinement would be to disaggregate the supply side further and with greater detail for individual countries, especially for the large producers. The model could also be expanded to include trade, and perhaps also stocks in addition to consumption and production. The degree of expansion will be largely determined by data availability.

60. Much additional work would be needed to reach a stage where the fertilizer model could be solved simultaneously with the grains model. Such a model would enable us to examine the interaction between products and fertilizers in a more comprehensive way. This exercise would be very difficult to carry out due to its size, however.

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APPENDIX A

FORECAST ERROR STATISTICS

1. Root mean squared error (RMSE)

Let Y_t be the actual value of variable Y at time t, and let $\hat{Y}_t$ be the simulated value of that same variable at time t, then

$$\text{(1)} \qquad RMSE = \left[\frac{\sum_{t=1}^{T} (Y_t - \hat{Y}_t)^2}{T}\right]^{\frac{1}{2}}$$

2. Average absolute percentage error (AAPE):

Under the same definition of Y_t and $\hat{Y}_t$

$$\text{(2)} \qquad AAPE = \left[\frac{\sum_{t=1}^{T} (Y_t - \hat{Y}_t)/Y_t}{T}\right] \cdot 100$$

and

3. Root mean squared percentage error (RMSPE):

will be defined as:

$$\text{(3)} \qquad RMSPE = \left\{\frac{\sum_{t=1}^{T} [(Y_t - \hat{Y}_t)/Y_t]^2}{T}\right\}^{\frac{1}{2}} \cdot 100$$

A very useful statistic related to the root mean square (RMS) simulation error and applied to the evaluation of historical simulations (ex-post forecasts) is Theil's inequality coefficient defined as:

$$(4) \qquad U = \frac{[\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_t - Y_t)^2]^{\frac{1}{2}}}{[\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_t)^2]^{\frac{1}{2}} + [\frac{1}{T}\sum_{t=1}^{T}(Y_t)^2]^{\frac{1}{2}}}$$

the numerator is exactly RMSE but it is scaled by such a denominator that allows U to always fall between zero and one. As U approaches one the forecast may be considered better.

This Theil inequality coefficient can be decomposed into three different elements.

It can be shown that the following identity holds

$$(5) \qquad [\frac{1}{T}\sum (\hat{Y}_t - Y_t)^2 = (\bar{\hat{Y}} - \bar{Y})^2 + (\sigma_s - \sigma_a)^2 - 2(1-\rho)\,\sigma_s\sigma_a$$

where σ_s and σ_a are the standard deviation of the simulated variable and the actual variable respectively, and ρ is their correlation coefficient. The proportions of inequality can then be defined as:

The bias proportion $$U^{\mu} = \frac{(\bar{\hat{Y}} - \bar{Y})^2}{\frac{1}{T}\sum_{t=1}^{T}(\hat{Y}_t - Y_t)^2}$$

The variance proportion $$U^{s} = \frac{(\sigma_s - \sigma_a)^2}{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}_t - Y_t)^2}$$

and the covariance proportion $$U^{c} = \frac{2(1-\rho)\,\sigma_s\sigma_a}{\frac{1}{T}\sum_{t=1}^{T}(\hat{y}_t - Y_t)^2}$$

These three statistics are essential in determining the source of a problem if there is one.

U^m gives an indication of a systematic error since it measures the extent to which the average values of the simulated and actual series deviate from each other.

U^s indicates the ability of the model to replicate the degree of variability in the variable simulated. Therefore, a large U^s indicates that the variability of the actual series is different from the variability of the simulated series.

U^c measures the degree of correlation between the actual and simulated series. It represents the remaining error after deviations from average value and average variabilities have been accounted for.

In the case of this model the inequality Theil statistic is calculated based on log-relative changes, which can be explained as follows. Let $\hat{y}_t$ be defined as $\hat{y}_t = \ln(\hat{Y}_t/\hat{Y}_{t-1})$ and similarly let y_t be $y_t = \ln(Y_t/Y_{t-1})$.

Let S_a and S_p be the standard deviation of the series $\hat{y}_t$ and y_t respectively, and let ρ_{ap} be the correlation coefficient of these two series; then the inequality coefficient U* will be determined by:

$$U^* = \frac{\left[\frac{1}{T-1}\sum_{t=2}^{T}(\hat{y}_t - y_t)^2\right]^{\frac{1}{2}}}{\left[\frac{1}{T-1}\sum_{t=2}^{T}(\hat{y}_2)^2\right]^{\frac{1}{2}} + \left[\frac{1}{T-1}\sum_{t=2}^{T}(y_t)^2\right]^{\frac{1}{2}}}$$

Hence the proportions of inequality U_M^*, U_s^* and U_c^* will be defined as:

$$U_m^* = \frac{(\bar{\hat{y}} - \bar{y})^2}{[\frac{1}{T-1} \sum_{t=2}^{T} (\hat{y}_t - y_t)]^2}$$

where $\bar{y} = \frac{1}{T-1} \sum_{t=2}^{T} y_t$

and $\bar{\hat{y}} = \frac{1}{T-1} \sum_{t=2}^{T} \hat{y}_t$

$$U_s^* = \frac{(S_p - S_a)^2}{\frac{1}{T-1} \sum_{t=2}^{T} (\hat{y}_t - y_t)^2}$$

$$U_c^* \frac{2(1-\rho_{ap}) S_p S_a}{\frac{1}{T-1} \sum_{t=2}^{T} (\hat{y}_t - y_t)^2}$$

When a variable changes sign from positive to negative (or vice versa) the ratio y_t/y_{t-1} is negative and therefore y_t (and/or $\hat{y}_t$) is not defined. That is what has happened when we see the number 999.999 in the table. In this case the Theil statistic and hence the inequality proportion are completely irrelevant.

APPENDIX B

DATA SOURCES AND DEFINITIONS

The primary historical data sources are the World Bank, FAO and the International Monetary Fund.

The world prices that are used are as follows:

Nitrogen - Urea, fob Europe bagged.
Phosphate - Triple superphosphate, fob US Gulf ports.
Potash - Muriate of potash (potassium chloride) fob Vancouver.
Phosphate Rock - 72%, fas Casablanca.
Rice - US No. 2 long-grained, milled, fob Houston, Aug-July year.
Corn - (proxy for all coarse grains) US No. 2 yellow, fob Gulf ports, Oct-Sept year.
Wheat - US No. 1. HRW, ordinary protein, fob Gulf ports, July-June year.

The macro economic data is taken from the World Bank and International Financial Statistics (International Monetary Fund). Income is defined in terms of gross domestic product measured in billions of local currency. The exchange rate is measured in units of local currency per US dollar and the consumer price index (CPI) is expressed as an index with the base year in 1980.

The data for the regions was created in a special way. It was aggregated from the country data for those variables with common units (hectares, tons, etc.). GDP for a region was obtained by first converting the GDP of all of the countries in a region into dollars using the average exchange rate of 1971-80, and then aggregating it across countries in a region. The regional exchange rate and CPI were obtained also in a two-stage process. First, every exchange rate was converted to an index and then the regional exchange rate was created as a weighted average where the weight was the GDP share of a country of the regional total.

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