The Effect of Discount Rate and Substitute Technology on Depletion of Exhaustible Resources

Yeganeh Hossein Farzin

WORLD BANK STAFF WORKING PAPERS
Number 516
The Effect of Discount Rate and Substitute Technology on Depletion of Exhaustible Resources

Yeganeh Hossein Farzin

The World Bank
Washington, D.C., U.S.A.
ABSTRACT OF THE STUDY

This paper analyses the validity of several well established propositions in the theory of exhaustible resources in the presence of substitutes. By carefully modelling the technologies of resource extraction and substitute production, we show that:

(a) Despite its widespread acceptance in the literature, the basic proposition that a reduction (an increase) in the rate of discount leads to greater conservation (faster depletion) of an exhaustible resource is not generally valid. It is shown that the effect of a change in the discount rate on the rate of resource depletion depends on capital requirements for development and production of the substitute, capital requirements in resource extraction and the size of the resource stock.

(b) When there are decreasing returns in production of the substitute, the conventional proposition that a relatively high-cost substitute should not be utilized before the stock of the resource is exhausted is invalid. It is shown that in such cases the optimality requires that both the resource and substitute be produced simultaneously for some period of time.

(c) When substitute technology shows increasing returns to scale, the conventional proposition that the marginal cost of the substitute provides a ceiling for the price of the exhaustible resource does not hold. We show that, under such conditions there will be a time interval during which the price of the resource exceeds the marginal cost of the substitute and yet the substitute ought not to be introduced during that interval.

(d) When there are a number of substitutes for the resource, each having a different marginal production cost and a different fixed cost associated with its development and introduction, the conventional rule that "the substitute with the lowest marginal cost of production should be introduced first" is not generally valid. We derive a more general decision rule which requires one to develop and introduce the substitute which yields the largest flow of net social benefit per time period regardless of its effect on the value of the resource stock.

ACKNOWLEDGMENTS

I am indebted to Geoff Heal, David Newbery, Joe Stiglitz and in particular to Jim Mirrlees for their valuable comments on earlier drafts of the paper.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Exhaustible Resource Depletion and the Rate of Discount</td>
<td>5</td>
</tr>
<tr>
<td>III. A Numerical Example</td>
<td>17</td>
</tr>
<tr>
<td>IV. Resource Depletion and Returns to Scale in Production of the Substitute</td>
<td>23</td>
</tr>
<tr>
<td>V. The Optimal Policy when there are Constant or Decreasing Returns in Production of the Substitute</td>
<td>26</td>
</tr>
<tr>
<td>V-1. Constant Returns to Scale (CRS) in Production of the Substitute</td>
<td>26</td>
</tr>
<tr>
<td>V-2. Decreasing Returns to Scale (DRS) in Production of the Substitute</td>
<td>27</td>
</tr>
<tr>
<td>VI. The Optimal Policy when there are Increasing Returns to Scale (IRS) in Production of the Substitute</td>
<td>30</td>
</tr>
<tr>
<td>VII. The Choice Among Several Substitutes with Different Cost Characteristics</td>
<td>35</td>
</tr>
<tr>
<td>VIII. Summary and Conclusion</td>
<td>41</td>
</tr>
<tr>
<td>Appendix</td>
<td>44</td>
</tr>
<tr>
<td>References</td>
<td></td>
</tr>
</tbody>
</table>
I. Introduction

The succession of sharp price increases of oil in the early 1970's raised, quite naturally, the issue of (a) competition against OPEC as a supplier of oil and (b) competition against oil as a form of energy. The object of this paper is to consider the latter form of competition. Specifically, we will analyse the validity of several well established propositions in the theory of exhaustible resources in the presence of a substitute for the resource.

A key concept in many discussions of the problems of exhaustible resources is that of the "backstop technology". The concept was given prominence by Nordhaus(1973), who examined, empirically, the long run problem of switching from cheap but exhaustible sources of energy to more expensive but abundant forms. He assumed the existence of a technology which has an effectively non-exhaustible resource base, but can be used to produce a perfect substitute for the resource at possibly very high costs. The important question, then, is whether the cost of producing the backstop provides a ceiling on the price of exhaustible resource such as oil- a question which has become of some importance since the events of the early 1970's. The conventional wisdom, undoubtedly, is that it does. Thus, in an influential paper, Solow(1974,p.4) writes as follows:

"Suppose that, somewhere in the background, there is a technology capable of producing or substituting for a mineral resource at relatively high cost, but on an effectively inexhaustible resource base. Nordhaus calls this a "backstop technology". (The nearest we now have to such a thing is the breeder reactor using $^{238}$U as fuel. World reserves of $^{238}$U are thought to be enough to provide energy for over a million years at current rates of consumption. If that is not a backstop
technology, it is at least a catcher who will not allow a lot of passed balls. For a better approximation, we must wait for controlled nuclear fusion or direct use of solar energy. The sun will not last for ever, but it will last at least as long as we do, more or less by definition.) Since there is no scarcity rent to grow exponentially, the backstop technology can operate as soon as the market price rises enough to cover its extraction costs (including, of course, profit in capital equipment involved in production). And as soon as that happens, the market price of the ore or its substitute stop rising. The backstop technology provides a ceiling for the market price of the natural resource.

(Emphasis added)

It is Solow's (1974) last proposition, in the above quote, that we wish to investigate further. Despite the faith shown in it by the literature on exhaustible resources, in the analysis of this paper it is shown that, under certain conditions, this proposition does not hold. A careful modelling of the production technology of the substitute shows that the conventional wisdom can indeed be misleading if not wrong. The implications of a questioning of this basic proposition are certainly interesting, and are also discussed in the sequel.

A careful modelling of the production technology of the substitute also leads us to question one other central proposition in the economics of exhaustible resources. This is the proposition that the rate of depletion of an exhaustible resource is positively related to the rate of interest. This proposition goes back to the classic work of Hotelling (1931), and is important in the current literature, particularly in the discussion of whether the rates of extraction of exhaustible resources are "too high" from a social point of view. Thus Solow (1974, p.8), referring to Hotelling's (1931) work, writes as follows:

"Hotelling mentions, but rather pooh-poohs, the notion that market rates of interest might exceed the rate at which society would wish to discount future utilities or consumer surpluses. I think a modern economist would take that possibility more seriously. It is certainly a potentially important question, because the discount rate determines the whole tilt of the equilibrium production schedule. If it is true that the market rate of interest exceeds the social rate of time preference,

1/ For a formal proof of this proposition, see Koopmans (1974).
then scarcity rents and market prices will rise faster than they "ought to" and production will have to fall correspondingly faster along the demand curve. Thus the resource will be exploited too fast and exhausted too soon."

(Emphasis added)

Similarly, Kay and Mirrlees (1975, p. 163) have argued that

"...a general bias in the economy to consume now and leave too little to our children or our future selves... would be reflected in high rates of interest, which will lead to somewhat more rapid depletion of resources."

(Emphasis added)

An explicit statement of the comparative static proposition implicit in the above arguments is provided by Meade (1975, p. 117)

"This reduction in the market rate of interest ... would stimulate investment in oil-in-the ground to carry oil over from this year's consumption to next year's consumption, or, in other words, it would encourage the conservation in the ground of oil which would otherwise have been pumped for use this year."

(Emphasis added)

We wish to investigate the validity of the above comparative static proposition, which is without doubt a central one in the economic theory of exhaustible resources. Using a model of resource extraction in the presence of a substitute which is itself produced using capital and labour, we show that, under certain conditions, this basic proposition is not generally true— the effects of a change in the rate of interest on the costs of producing the substitute may be strong enough to invalidate the conventional wisdom.

The plan of this paper is as follows. In Section II we develop a simple resource-substitute model to argue that, in spite of a widespread belief in it, the proposition that a reduction (increase) in the rate of interest would lead to a slower (faster) depletion of exhaustible resources is not generally valid. We show that, for exhaustible resources, the allocational implication of a change in the discount rate depends on capital requirements for the development and production of the substitute, capital costs in resource extraction, and the size of the resource stock, in particular. In Section III, based on the model of Section II and data on comparative
costs of energy from alternative sources, we provide a numerical example which demonstrates that for plausible parameter values the effect of a change in the discount rate is indeed opposite to that conventionally thought.

To analyse the implications of economies or diseconomies of scale in production of the substitute, in Section IV we set the basic optimisation problem of Section II in a more general form and derive the first-order conditions for optimality. We then show in Section V that when there are decreasing returns in production of the substitute, it is no longer true (as often argued) that a relatively high-cost substitute for the resource should be utilised only after the stock of the resource is exhausted. In Section VI it will be shown that with increasing returns in production of the substitute, the marginal cost of the substitute does not (as widely believed) provide a ceiling for the price of the exhaustible resource, and therefore that the elimination of a differential between the price of the resource and the marginal cost of the substitute does not provide a sufficient reason for introducing the substitute. Section VII considers a situation where one has to choose among several technologies which are all capable of producing a perfect substitute for the resource, but that they differ in their unit production costs and in the fixed costs associated with their development and introduction. It will be shown that in such cases the conventional rule that 'the substitute with lowest marginal production cost should be introduced first' does not give the correct answer. We show that for such cases the optimal decision rule requires one to choose that technology which yields the largest flow of net benefit from use of the substitute regardless of its impact on the value of the stock of exhaustible resource. Finally, Section VIII will present a summary of the paper and some concluding remarks.
II. Exhaustible Resource Depletion and the Rate of Discount

Consider a socially planned economy which has the following characteristics:

C₁ - it is endowed with a known finite stock of an exhaustible resource, S.

C₂ - there is an extraction technology which, for the sake of simplicity, is assumed to possess constant returns to scale and to remain unchanged over time. Given this technology and the set of input prices in the economy, the unit cost of resource extraction can be described by the following cost function

\[ c = c(w, r) \]  

where \( c \) is the minimum cost of extracting a unit of the resource at any time, \( r \) is the price of capital per unit, and \( w \) is the vector of prices of all other inputs.

The cost functions used hereafter are taken to be continuous in their arguments and at least twice continuously differentiable.

C₃ - the social benefit from the use of the resource is measured, in monetary units, by \( V(p_t) \), where

\[ V(p_t) \equiv U(x(p_t)) \]  

and where \( p_t \) denotes the price of a unit of the resource at time \( t \), and \( x(p_t) \) is the market demand function which is assumed to satisfy \( x'(p_t) < 0 \) and to avoid corner problems, \( \lim_{p_t \to \infty} x(p_t) = 0 \), and \( \lim_{p_t \to 0} x(p_t) = \infty \).

Assuming that the marginal utility of income is constant, the gross social benefit (consumer surplus) is therefore evaluated from the market demand function according to

\[ U(x) \equiv \int_0^{x(p)} dx \]  

where \( p(x) \) is the inverse of the demand function \( x(p) \). Given the definition of \( U(x) \) and the assumption that \( x'(p_t) < 0 \), it is clear that \( U'(x) > 0 \) and \( U''(x) < 0 \). Moreover, differentiating (II.2) w.r.t. \( p_t \) and noting that \( U'(x) = p(x) \), we have

\[ V'(p_t) = p_t x'(p_t) \]  

We shall presently introduce a perfect substitute for the resource, so that (II.2) and (II.3) should be interpreted as referring to both the resource
and its substitute. 

Future social benefits are discounted at a constant rate which, on the assumption that capital markets are perfect, is equal to both the market rate of interest and the cost of capital.\footnote{For an analysis of the relationship between the social rate of discount, the pure rate of social time preference and the market rate of interest, see Stiglitz (1977) and (1978).}

Following Nordhaus (1973), we assume that the technology of the economy allows the production of a perfect substitute for the exhaustible resource possibly at a cost which is higher than the current price of the resource, but on an effectively inexhaustible resource base. For instance, we can think of crude oil as the exhaustible resource and the breeder reactor as an approximation to the substitute technology. We shall take it that the economy has already access to this technology and that there is no capacity constraint on production of the substitute. For expository convenience, and without loss of generality, we also assume that returns to scale in production of the substitute are constant, so that the unit cost of production of the substitute can be written as

\[ p = p(w,r) \] \hspace{1cm} (II.4)

where \( p \) is the minimum production cost per unit of the substitute which, for any given set of factor prices, exceeds the unit extraction cost of the resource, \( c \).\footnote{Needless to mention, if \( p < c \) the resource stock will never be used, and if \( p = c \) the resource and substitute can be considered to be homogeneous and thus treated as a single good.}

The problem facing the planners in such an economy is that of choosing the price path of the resource \( p_t \) and the date \( T \) at which the substitute should replace the resource so as to maximise \( W \), the present net social benefit from the depletion of the resource stock and the use of the substitute.
Formally, the planners' problem is\(^1\)

\[
\text{maximise } W = \int_0^T e^{-rt} [V(p_t) - x(p_t) c(w,r)] dt + \int_T^\infty e^{-rt} [V(p_T) - x(p_T) p(w,r)] dt
\]

subject to the constraints

\[
\int_0^T x(p_t) dt \leq S, \quad x(p_t) \geq 0, \text{ and } p_t \geq 0 \text{ for all } t \geq 0
\]

Forming the Lagrangean associated with problem (II.5), differentiating it w.r.t. \(p_t\) and \(T\), and using (11.3) we obtain the following necessary conditions for an optimum:\(^2\)

\[
\begin{align*}
\lambda - c(w,r) &= \lambda e^{rt}, \quad t \in [0,T] \\
p_t &= p(w,r), \quad t \in [T,\infty)
\end{align*}
\]

and

\[
\int_0^T x(p_t) dt = S
\]

where \(\lambda = p_0 - c(w,r)\) is the Lagrangean multiplier and measures the initial scarcity rent (price minus extraction cost) on the resource.

The optimal policy governed by conditions (II.5.a)-(II.5.c) is familiar in the literature (see, e.g. Dasgupta and Stiglitz (1976)). It requires that the scarcity rent on the exhaustible resource rises at the rate of discount until the resource price has risen sufficiently to cover the marginal cost of production of the substitute, at which time the stock of the resource is completely depleted and the substitute will be supplied at a price equal to its marginal cost of production \(p(w,r)\) and at a constant flow rate of \(x(p(w,r))\) units\(^3\).

However, to gain further insight into the properties of the optimal policy

---

\(^1\) It should be noted that in setting problem (II.5) we have assumed that the economy will use up the stock of the resource before it switches to the substitute. That this is indeed the optimum order in which the resource and substitute should be used is formally demonstrated in Section (V.1).

\(^2\) For details of the derivation of these conditions, see Appendix A.2.

\(^3\) That the optimal price path should be continuous at \(t=T\) must be obvious, for otherwise either the resource will be undercut by the substitute, or it will pay to hold back some of the resource stock, no matter how small, until time \(T\) when capital gains are huge.
and, in particular, to facilitate the subsequent analyses, it is necessary to provide some comparative static results concerning the effect of an exogenous change in the production cost of the substitute, the extraction cost of the resource, or the size of the resource stock on the optimum rate of resource depletion and the timing of the substitute. Towards this, let us use (II.5.a) and (II.5.b) to write

\[ p(w,r) - c(w,r) = \lambda e^{\lambda T} \]  

(II.5.d)

As we are concerned, for the time being, with the effects of exogenous changes in the costs (which may result, for instance, from changes in the technology of production/extraction), we can treat the unit costs as parameters and denote them respectively by \( \bar{p} \) and \( \bar{c} \). Then, for given input prices \( (w,r) \), total differentiation of (II.5.c) and (II.5.d) leads to the following information:

\[
\frac{\partial T}{\partial \bar{p}} > 0, \quad \frac{\partial p_0}{\partial \bar{p}} > 0, \quad \frac{\partial \lambda}{\partial \bar{p}} > 0
\]  

(II.5.e)

\[
\frac{\partial T}{\partial \bar{c}} > 0, \quad \frac{\partial p_0}{\partial \bar{c}} > 0, \quad \frac{\partial \lambda}{\partial \bar{c}} < 0
\]  

(II.5.f)

\[
\frac{\partial T}{\partial \bar{s}} > 0, \quad \frac{\partial p_0}{\partial \bar{s}} < 0, \quad \frac{\partial \lambda}{\partial \bar{s}} < 0
\]  

(II.5.g)

where \( p_0 \) is the optimal initial price of the resource (for the derivation of these results see Appendix A.1).

According to (II.5.e) an increase in the unit cost of the substitute delays the date at which the economy switches from the resource to the substitute, slows down the rate of resource exploitation, and benefits the resource owners by enabling them to collect larger amounts of royalties at any time. A rise in the unit cost of extraction has the same qualitative effects except that it reduces the rents accruing to resource owners. Obviously, these effects will be reversed for a reduction in the unit costs; thus, for example, a cost-reducing technological breakthrough in either the substitute or the resource sector speeds up the rate of resource extraction and advances the date of utilisation of the substitute.
The interpretation of inequalities (II.5.g) is straightforward. An addition to the existing reserves of the resource reduces the scarcity rent and hence the price of the resource. In the limiting case where the size of the resource stock becomes infinitely large the scarcity rent tends to zero, thus implying that the price of the resource in that case will be equal to its marginal cost of extraction. This is akin to economic intuition, for as the stock of the resource becomes very large the resource becomes pretty much like a conventional inexhaustible commodity, in which case the efficiency considerations dictate that the resource be priced at its marginal extraction cost. Since this statement proves to be essential in the analysis of the subsequent section, we may as well present a formal proof of it here.

This is as follows:

\[
\frac{1}{x} \ln \left( \frac{p-c}{\lambda} \right)
\]

Using (II.5.d) and (II.5.a), we can rewrite (II.5.c) as

\[
\int_0^T x(c + \lambda e^{rt}) dt = \xi
\]

which implies that for all \( S \geq 0, p-c > \lambda > 0 \). Furthermore, (from (A.1.1.e)),

\[
\frac{\partial \lambda}{\partial S} = - \frac{\lambda}{x(p)} < 0 \quad \text{for all } S \geq 0, \text{so that } \lambda \text{ is a positive decreasing function of } S.
\]

Now, the limit of \( \lambda \), as \( S \to \infty \), cannot be positive; for suppose

\[
\lim_{S \to \infty} \lambda = \hat{\lambda} > 0,
\]

then, as \( S \to \infty \), the integral on the LHS of the resource stock equation obtains a finite limit, whereas the RHS of the equation goes to infinity, thereby leading to a contradiction. Therefore, \( \lim_{S \to \infty} \lambda = 0 \), implying that \( p_c \to c \) as \( S \to \infty \).

It is worth noting that even though an addition to the reserves of the resource lowers its price and thus brings about higher rates of consumption per unit of time, it prolongs the time span over which the reserves are depleted (i.e. \( \frac{dT}{dS} > 0 \)). This is because, regardless of the size of the reserves, the price of the resource at the time of exhaustion must have risen high enough to cover the marginal cost of the substitute, thereby giving rise to a smooth transition from the resource to the substitute. But, since the initial price of the resource is lowered (due to an addition to the resource stock) it will take a longer time before the resource price reaches the level of the marginal cost of the substitute.
From a comparison of the inequalities involving the term $p_0$, it is seen that cost-reducing technological progress in the substitute and/or the resource sector is tantamount to additions to the existing reserves of the resource, so that higher levels of resource consumption can be achieved during the interval $[0,T]$ either as a consequence of cost-reducing technological progress or as a result of discoveries of new reserves of the exhaustible resource.

Let us now turn to the analysis of the effect of a change in the discount rate on the optimal rate of resource exploitation. Bearing in mind that both unit costs $p(w,r)$ and $c(w,r)$ are functions of the rate of interest, we can differentiate (II.5.c) and (II.5.d) with respect to $r$ and solve for $\frac{\partial \lambda}{\partial r}$ to get

$$\frac{\partial \lambda}{\partial r} = \lambda \left[ \frac{x(p)}{p-c} \left( \frac{\partial p}{\partial r} - \frac{\partial c}{\partial r} \right) + r \frac{3c}{2r} \int_0^T x'(p_t)dt - S \right]/x(p_0)$$  \hspace{1cm} (II.6)

where, for the simplicity of exposition, the arguments of the unit cost functions have been suppressed. Recalling that $p_0 = \lambda + c(w,r)$ and using (II.6) we have, after some manipulation

$$\frac{\partial p_0}{\partial r} = \left[ \frac{x(p)}{p-c} \frac{\partial p}{\partial r} + r \frac{3c}{2r} \int_0^T e^{-rt} x(p_t)dt - S \right]/x(p_0)$$  \hspace{1cm} (II.7)

From the theory of cost functions\textsuperscript{1} we know that $\frac{\partial p}{\partial r}$ and $\frac{\partial c}{\partial r}$ in (II.7) indicate respectively the capital requirement per unit (or capital intensity) of the substitute and resource. It is therefore evident that the capital intensities in the two sectors play an important part in determining the effect of a change in the rate of discount on the intertemporal allocation of the resource, and that their effects are such that a reduction in the rate of discount could result in a faster extraction of resources (disinvestment in resource-in-the-ground). This runs counter to the conventional proposition that a lower discount rate unambiguously leads to conservation of exhaustible resources. The intuitive belief in this proposition might have been derived

\textsuperscript{1} See, e.g., Shephard (1970).
from the following two unrealistic assumptions: (a) that there exists no potential substitute for the resource (this, in effect, leads to the exclusion of the term $\frac{3p}{3r}$ from (II.7)), and (b) that there are no costs whatsoever involved in the extraction of the resource (this has the effect of eliminating the term $\frac{3c}{3r}$ from (II.7)). Alternatively, it might have been derived from unguided intuition that the act of keeping resources in the ground is an investment activity identical to any other form of investment, so that, ceteris paribus, a "reduction in the rate of interest ... would stimulate investment in oil-in-the-ground". But, as our analysis has indicated, a reduction in the rate of discount brings about two counteracting effects: (a) in as much as the discount rate reflects the rate of time preference, a reduction acts to postpone the use of resources to the future (a conservation effect), and (b) as cost of capital, it lowers the unit costs in both the substitute and resource sectors, and hence it induces a faster rate of resource depletion (a disinvestment effect). The net effect depends, among other things, on the degree of capital intensities in the substitute and resource sectors, the cost differential between the substitute and the resource, and the size of the resource stock.

One can take the analysis further and use (II.7) to derive a general condition that unambiguously determines the qualitative role of the discount rate in the allocation of exhaustible resources. To do this, let

$$K_R = \int_0^T e^{-rt} \left[ rx(p) \frac{3c}{3r} \right] dt \quad \text{(II.7.a)}$$

and

$$K_S = \frac{\lambda x(p)}{p-c} \left. e^{-rt} \left[ rx(p) \frac{3p}{3r} \right] \right|_0^\infty dt \quad \text{(II.7.b)}$$

It may be noted that although a reduction in $r$ will reduce capital costs in the two sectors, in a general equilibrium context, other factor prices will also change, so that not all costs will necessarily fall. However, provided that the resource and substitute sectors are more capital intensive than other sectors of the economy, a reduction in $r$ will definitely reduce the production/extraction costs in the two sectors. As far as energy industry is concerned, the available statistics suggest that this proviso is indeed fulfilled. For example, Leontief's (1956) statistics on direct and indirect capital requirements suggest that the crude oil and natural gas industry's capital intensity is higher than those in the durable goods industries by a factor of 1 to 1.5. McDonald (1976) has also calculated a capital intensity of 1.43 for the U.S. oil and gas industry as compared to a capital intensity of 0.52 for manufacturing.
where $K_R$ and $K_S$ measure respectively the present value of capital requirements in the resource and substitute sectors. Also, substituting from (II.5.a) and (II.5.b) for $p_t$ in (II.5) to obtain the maximised value of $W$ (denoted by $W^*$), differentiating $W^*$ w.r.t. $S$, and making use of (II.3), (A.1.1.e) and (A.1.1.f) we have

$$\frac{\partial W^*}{\partial S} = \lambda$$  \hspace{1cm} (II.7.c)

where $\lambda$ is the shadow price of a unit of the resource stock in the ground and $\lambda S$ is the present social value of the resource stock.

Using (II.7.a)-(II.7.c), we can then rewrite (II.7) as

$$\frac{\partial p_0}{\partial S} = \frac{(K_R + K_S - \lambda S)/x(p_0)}{(11.8)}$$

which implies that

$$\frac{\partial p_0}{\partial S} \geq 0 \text{ as } (K_R + K_S) \geq \lambda S$$  \hspace{1cm} (II.9)

We can therefore state the following proposition.

**Proposition 1.** A reduction in the discount rate leads to a more rapid depletion of resources if the sum of the present values of capital requirements in the substitute and resource sectors exceeds the present value of the resource stock.

This result, which is the basic finding of this section, has not been discussed in the literature which, as we have indicated above, has suggested that the effect is unambiguous, a faster depletion of resources always being caused by a rise and never a reduction in the discount rate.

In practice, whether the required condition holds or not depends obviously on actual magnitudes of parameters which determine $K_R$, $K_S$, and $\lambda S$; namely, on the size of the stock of the exhaustible resource in question, $S$, the unit costs of production of the substitute and extraction of the resource, $p$ and $c$, the capital intensities, $\frac{\partial c}{\partial r}$ and $\frac{\partial c}{\partial r}$, the rate of discount, $r$, and the demand for the resource/substitute, $x(p_t)$. Nonetheless, from (II.7) it
is clear that the condition will be satisfied if the cost differential 
(p-c) is sufficiently small. It is also obvious from (II.7) that the 
condition will hold if the capital intensities of the substitute and/or 
the resource are sufficiently large. What is, however, less obvious is 
whether the condition will hold when the resource stock is large or when it 
is small. This is an important question because if it turns out that \( \frac{\partial \rho_0}{\partial r} > 0 \) 
when the stock of the resource is sufficiently small, then reducing the 
discount rate in the belief that it will lead to greater conservation of the 
resource (which is the effect conventionally conceived) will result in adverse 
effects which are probably far more serious than they will if \( \frac{\partial \rho_0}{\partial r} > 0 \) holds 
for sufficiently large stocks of the resource.

Let us therefore examine the values of \( S \) for which condition \( K_R + K_S - \lambda S > 0 \) 
holds. First, we study the behaviour of \( \lambda S \) as a function of \( S \). It will 
be noted that

\[
\lim_{S \to 0} \lambda(S) = p-c \tag{II.10.a}
\]

and

\[
\lim_{S \to \infty} \lambda(S) = 0 \tag{II.10.b}
\]

where the former follows directly from (II.5.c) and (II.5.d), and the latter 
was formally established above (see page 29).

Hence, from (II.10.a), we have

\[
\lim_{S \to 0} \lambda(S)S = 0 \tag{II.11}
\]

Noting that \( \lim \lambda(S) = 0 \), the limiting value of \( \lambda(S)S \) as \( S \to \infty \) can be 
\( \lambda(S) \) calculated using L'Hopital's Rule:

\[
\lim_{S \to \infty} \frac{\lambda(S)S}{S} = \lim_{S \to \infty} \frac{\frac{\partial}{\partial S}(\lambda(S)S)}{\frac{\partial}{\partial S}S} = \lim_{S \to \infty} \frac{(\lambda(S))^2}{\frac{\partial \lambda(S)}{\partial S}}
\]

But, from (A.1.1.e) of Appendix A.1,

\[
\frac{\partial \lambda(S)}{\partial S} = -r \frac{\lambda(S)}{x(p_0)} < 0 \tag{II.12}
\]

so that, by substituting (II.12) in the above expression and recalling that
\[ p_0 = c + \lambda(S) \]

we obtain

\[
\lim_{S \to \infty} \lambda(S)S = \frac{1}{\bar{r}} \lim_{S \to \infty} [\lambda(S)(c + \lambda(S))]
\]

which, using (II.10.b), gives

\[
\lim_{S \to \infty} \lambda(S)S = 0 \tag{II.13}
\]

Now, differentiating \( \lambda S \) with respect to \( S \) and using (II.12) yields

\[
\frac{\partial \lambda S}{\partial S} = S \frac{\partial \lambda}{\partial S} + \lambda = \lambda(1 - \frac{rS}{x(p_0)})
\]

so that

\[
\frac{\partial \lambda S}{\partial S} \leq 0 \quad \text{as} \quad S \leq \frac{x(p_0)}{r}
\]

Thus, the graph of \( \lambda S \) starts from the origin, rises with \( S \) until it attains its maximum for \( S = \frac{x(p_0)}{r} \), and then it declines while approaching the \( S \)-axis asymptotically (see Diagram 1.a below).

Next, we investigate the behaviour of \( K_R \) and \( K_S \) as functions of \( S \).

Noting that \( \lim_{S \to 0} T = 0 \), and \( \lim_{S \to \infty} T = \infty \) (by (II.5.c), (II.10.a) and (II.10.b)),

we can use definitions (II.7.a) and (II.7.b) to write

\[
\lim_{S \to 0} K_R = 0, \quad \lim_{S \to 0} K_S = x(p) \frac{3p}{3r}
\]

\[
\lim_{S \to \infty} K_R = x(c) \frac{3c}{3r}, \quad \text{and} \quad \lim_{S \to \infty} K_S = 0
\]

so that

\[
\lim_{S \to 0} (K_R + K_S) = x(p) \frac{3p}{3r} \tag{II.14.a}
\]

and

\[
\lim_{S \to \infty} (K_R + K_S) = x(c) \frac{3c}{3r} \tag{II.14.b}
\]

Now, differentiating \((K_R + K_S)\) with respect to \( S \), and substituting for \( \frac{\partial T}{\partial S} \)

and \( \frac{\partial \lambda}{\partial S} \) respectively from (A.11.e) and (A.11.f), we obtain

\[
\frac{\partial}{\partial S}(K_R + K_S) = \frac{re^{-rt}}{x(p_0)} \left[ (\frac{3c}{3r} - \frac{3p}{3r})x(p) - \frac{3c}{3r} e^r \int_0^T \hat{x}(p)e^{-rt} dt \right] \tag{II.15'}
\]
Also, from (II.15) we have the second-order derivative

\[
\frac{3}{2s} (K_R + K_S) = \frac{r}{x(p_0)} \left[ \frac{x'(p_0) 3s}{3s} (K_R + K_S) - \frac{3s}{3s} (K_R + K_S) \right]
\]

Since \( x'(p_0) > 0 \) and \( \frac{3s}{3s} < 0 \), it follows that

\[
\frac{3}{2s} (K_R + K_S) \leq 0 \Rightarrow \frac{3}{2s} (K_R + K_S) > 0 \quad (II.16)
\]

so that whenever \( (K_R + K_S) \) happens to be declining, it will continue to do so until it reaches a minimum at \( K_R + K_S = x(p_0) \frac{3s}{3s} \) (from (II.15)).

Finally, on taking the limit of (II.15) as \( S \to 0 \), and using (II.10.a) and (II.14.a), we obtain

\[
\lim_{S \to 0} \frac{3}{2s} (K_R + K_S) = r(\frac{3s}{3r} - \frac{3s}{3r}) \quad (II.17)
\]

We have now enough information to study the behaviour of \( (K_R + K_S) \) for various cases which may arise, depending on whether the capital intensity of the substitute is greater, equal to, or smaller than that of the resource.

**Case (a):** \( \frac{3s}{3r} > \frac{3s}{3r} \) (i.e. the substitute is more capital intensive than the resource.)

For this case, (II.17) and (II.16) together imply that the graph of \( (K_R + K_S) \), starting from \( x(p) \frac{3s}{3r} \), will decline until it attains its minimum \( (x(p_0) \frac{3s}{3r}) \); thereafter it rises with \( S \) and passes through an inflexion point before it asymptotically approaches \( x(c) \frac{3c}{3r} \) (see Diagram 1.a).  

---

**Diagram 1.a**

---

1/ In Diagram 1.a we have taken it that \( x(p) \frac{3s}{3r} > x(c) \frac{3c}{3r} \). It should however be noted that the graph of \( (K_R + K_S) \frac{3s}{3r} \) behaves as described above, regardless of whether \( x(p) \frac{3s}{3r} < x(c) \frac{3c}{3r} \).
Case (b): \( \frac{3p}{3r} < \frac{3c}{3r} \)

In this case, \( \frac{3}{3s}(K_R + K_S) > 0 \), \( \forall S > 0 \) (by (II.15')) , so that \( (K_R + K_S) \) always rises with \( S \), and passes through an inflexion point before it reaches its upper bound \( (\alpha(c) \frac{3c}{3r}) \) asymptotically (see Diagram (1.b)).

![Diagram 1.b](image)

Relationship between \( (K_R + K_S) \) and \( \lambda S \) when \( \frac{3p}{3r} < \frac{3c}{3r} \)

Thus our analysis indicates that the requirement for \( \frac{3p}{3r} > 0 \) to hold is fulfilled both when the resource stock is sufficiently small (i.e. for \( S < S_1 \) in Diagrams 1.a and 1.b) as well as when it is sufficiently large (i.e. for \( S > S_2 \)) and this independently of whether it is the substitute or resource which is more capital intensive. In either case, the effect of a change in the discount rate on the price, and hence the pace of exploitation of the resource will be opposite to that conventionally believed.

The economic intuition behind this result is quite simple. As was pointed out earlier, when the stock of resource is very large, the resource is much like an ordinary product for which the price is determined by the marginal cost of production; as such, a reduction in the rate of interest renders the resource cheaper and hence increases its rate of use. In the opposite case, when the stock of resource is very small, the resource can enjoy a scarcity

1/ Note that both Diagrams (1.a) and (1.b) have been drawn on the assumption that the point of maximum on curve \( \lambda S \) lies above the curve of \( (K_R + K_S) \). In general, this need not be the case, and the curve \( \lambda S \) may lie entirely below the curve of \( (K_R + K_S) \).
rent almost as large as the difference between the cost of producing the substitute and its own cost of extraction, implying that it will command a price roughly equal to production cost of the substitute. In that case, a reduction in the rate of interest reduces the cost of the substitute, and hence the price obtainable by the resource; leading therefore to a faster use of the resource. However, if for any reason greater conservation of the resource is deemed to be socially desirable, then reducing the discount rate to achieve this will have consequences which will be more disturbing if the economy in question is resource-poor than if it is resource-rich.

Given the result of our analysis, the question naturally arises as to the plausibility of the sizes of the resource stock for which the inequality \( \frac{\partial p}{\partial r} > 0 \) (or, equivalently, \( K_{n} + K_{s} - \lambda S > 0 \)) is satisfied. In other words, one would like to know how small \( S_{1} \) (or how large \( S_{2} \)) should actually be in order for the adverse effects of a change in the discount rate to occur. In order to have an idea of the orders of magnitude involved, we shall consider a numerical example in the following section.

III. A Numerical Example

In constructing the numerical example, we have taken the demand for the services of the resource/substitute to be iso-elastic with a long run price elasticity of unity. Normalising for the units of measurement, this can be written as

\[
x(p_{c}) = p_{c}^{-1}
\]  

(III.1)

Let \( \alpha \) and \( \beta \) denote, respectively, the percentage shares of capital cost in production costs of the resource and substitute; so that, by the assumption of constant returns to scale, we have

\[
\frac{K_{c}}{c} = \alpha , \quad 0 < \alpha < 1
\]  

(III.2)

\[
\frac{K_{r}}{r} = \beta , \quad 0 < \beta < 1
\]  

(III.3)

\[\text{noted} \]

1/ It should be noted for simplicity of exposition we have suppressed the factor price arguments of the unit cost functions \( p(w,r) \) and \( c(w,r) \).
Also, let $k$ be the factor by which the cost of the substitute (per unit of the resource equivalent) exceeds the unit extraction cost of the resource, i.e. 
\[ p = kc \quad , \quad k > 1 \] (III.4)

We assume that for all factor prices $k$ remains constant, with the implication that the capital cost shares in the two sectors will be equal, i.e. $\alpha = \beta$.

Given the demand function (III.1), we can solve the model of previous section for the optimum values of $\lambda$, $T$, and $p_0$. To do this, we use (II.5.a) to write (III.1) as
\[ x(p_t) = (c + \lambda e^{rt})^{-1} \] (III.5)

Substituting from (III.5) into (II.5.c), performing the integral and using (II.5.d) yields
\[ T = cS + \frac{1}{r} \ln\left(\frac{P_0}{P_0}\right) \]
Also, from (II.5.d),
\[ T = \frac{1}{r} \ln\left(\frac{P-C}{\lambda}\right) \]
and from (II.5.a),
\[ \lambda = p_0 - c \]

Thus, we have three equations in three unknowns $(\lambda, T, p_0)$ which, on defining $X \equiv e^{rcS}$ and using (III.4), can be solved to obtain, after manipulation
\[ \lambda = \frac{(k-1)c}{kX-(k-1)} \]
\[ T = \frac{1}{r} \ln[kX-(k-1)] \]
and
\[ p_0 = \frac{kCX}{kX-(k-1)} \] (III.6)

Now, by differentiating (III.6) w.r.t. $r$ and simplifying the resulting expression, we obtain
\[ \frac{\partial p_0}{\partial r} = \frac{ckX}{r[kX-(k-1)]^2} \left[\alpha kX - \alpha(k-1) - (1+\alpha) (k-1) \ln X\right] \]
so that,
\[ \frac{\partial p_0}{\partial r} > 0 \quad \text{iff} \quad \left[\alpha kX - \alpha(k-1) - (1+\alpha) (k-1) \ln X\right] > 0 \] (III.7)
For given values of parameters $k$ and $\alpha$, one can determine the range of values of $X$ satisfying (III.7); and then, by using these values of $X$ and specifying those of $r$ and $c$, one can determine (via $X = e^{rcS}$) the corresponding range of values of $S$. However, instead of determining the range of $S$, we calculate the range of reserve-production ratio, $S \frac{X}{x(p_0)}$, which indicates the number of years for which the stock of the resource can sustain the initial rate of production. This not only provides a more meaningful criterion for judging the plausibility of the magnitudes of $S$ involved, but also saves us from entering into speculation about the actual magnitude of $c$, for, using the definition of $X$, (III.1) and (III.6), it can readily be checked that

$$\frac{S}{x(p_0)} = \frac{kX \ln X}{r[kX-(k-1)x]}$$

so that we can calculate $\frac{S}{x(p_0)}$ without having to specify the value of $c$.

Although our analysis can be applied to all exhaustible resources, in arriving at the numerical results reported below we have had energy resources in mind, with crude oil as the exhaustible resource. Of course, as a source of energy, crude oil is so diversely used—directly, in transportation, heating and industrial uses and, indirectly, in generating electricity—that there is for it no substitute (at least not at the present state of technological knowledge) which is both inexhaustible and capable of replacing it in all its variety of uses. For instance, solar energy is inexhaustible, but can be substituted for oil mainly in its use for heating purposes. Similarly, controlled nuclear fusion (when it becomes commercially viable) can provide an inexhaustible substitute for oil in generating electricity, but not for oil used in transportation. The only perfect substitute for crude oil is synthetic oil (oil produced from shale, tar sands, and coal). This, strictly speaking, is an exhaustible substitute. However, the available reserves of the resources on which it is based are so vast, relative to reserves of conventional oil, that one can consider it as a reasonable approximation to
We shall now present the numerical values used for the parameters involved. To specify values for $k$, we need to have information about production costs of oil from different energy sources. In a study by Shell Oil Company\(^1\), the following figures are given as the estimated cost of synthetic oil:

<table>
<thead>
<tr>
<th>Source</th>
<th>Cost (1979 $) per barrel of oil equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid from oil sands</td>
<td>15-25</td>
</tr>
<tr>
<td>Liquid from shale</td>
<td>15-35</td>
</tr>
<tr>
<td>Liquid from coal (U.S.)</td>
<td>30-37</td>
</tr>
</tbody>
</table>

* Excluding taxation, refining, and distribution costs.

These figures should be compared with production cost of oil, estimated by the same study at 7-11 (1979 $) for 'medium-cost' fields such as those in the United States and North Sea. Taking the midpoint of each range as the average cost and calculating the ratios of the average costs of synthetic oil to that of conventional oil, we get 2.22 for oil sands, 2.77 for shale, and 3.72 for coal. To allow for possible improvements or drawbacks in synthetic oil:

---

\(^1\) For instance, Oil and Gas Journal (1977) puts the estimated U.S. shale oil resources at 1.8 trillion (10\(^{12}\)) barrels, of which it is estimated that 610 billion barrels could be produced by means of current technology. This alone would be more than 22 times greater than U.S. proven reserves of crude oil (as 1 Jan. 1980) and would be equivalent to a 206 year supply at the U.S. 1979 crude oil production rate.

The figures for coal are even more impressive; the U.S. Geological Survey (1975) has estimated total coal resources at almost 4 trillion tons, of which at least 218 billion tons (or about 1000 billion barrels of oil equivalent) has been estimated as recoverable reserves. This is 37 times greater than U.S. proven crude oil reserves and 180 times greater than its crude oil consumption (domestic production plus imports) in 1979. There are also massive reserves of oil sands concentrated mainly in Venezuela and Canada. The Canadian Society of Petroleum Geologists (1974) has put the estimated reserves of oil sands in these two countries at, respectively, 2050 and 800 billion barrels; so that with a recovery factor of only 20%, each country's recoverable reserves of oil sands would be 23 times greater than its proven reserves of crude oil (as of 1 Jan. 1980).

---

oil technology, we have let parameter \( k \) take values in the range 1.5-4.0 by steps of 0.5. Also in calculating \( \frac{S}{x(p_0)} \), whenever this has been necessary, we have used two alternative discount rates of 5\% and 10\%.

The numerical results are presented in Table 1 which shows the lower and upper ranges of the reserve-production ratios (in number of years) satisfying \( \frac{3P_0}{\partial r} > 0 \). Interestingly enough, from the table we see that for a large range of values of \( k \) and for any plausible value of the capital cost share, \( \alpha \) the relationship \( \frac{3P_0}{\partial r} > 0 \) holds for all sizes of the resource stock. For example, with \( k=1.5 \) and a capital cost share of greater than only 16.44\% the relationship holds independently of the stock size. For \( k=1.5 \), it will hold for all sizes of the stock if the capital cost share is greater than nearly 63\%, a condition which is by no means stringent for an industry as capital intensive as the oil industry in the United States. Even for a substitute whose production cost is 4 times higher than that of the resource, the reserve-production ratios required to satisfy \( \frac{3P_0}{\partial r} > 0 \) are seen to be neither unreasonably small nor unduly large. For instance, with a capital cost share of 60\% and a discount rate of 5\%, one requires a reserve-production ratio of either less than 13.5 or greater than 28.36 years (at 1979 production levels, the crude oil reserve-production ratios for the United States and Canada were, respectively, 8.6 and nearly 12 years). At the discount rate of 10\% the lower range becomes narrower while the upper range becomes wider; however, as the capital cost share increases the two ranges get closer to each other, so that for \( \alpha \geq 72.41\% \) the inequality \( \frac{3P_0}{\partial r} > 0 \) will hold independently of the size of the reserves.

To summarize so far, we have argued that when allowance is made for capital costs in production of the substitute and in extraction of the resource, a change in the discount rate can affect the rate of resource depletion in a direction opposite to that conventionally believed. In particular, it was shown that a decrease (an increase) in the discount rate will lead to a faster (slower) depletion rate initially if the sum of discounted present values of
Table 1. The ranges of the reserve-production ratio for which $\frac{\partial p_0}{\partial r} > 0$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\alpha$</th>
<th>$\frac{\partial p_0}{\partial r} &gt; 0$ for all $S &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.5$</td>
<td>$\alpha \geq 16.44%$</td>
<td>$\frac{\partial p_0}{\partial r} &gt; 0$ for all $S &gt; 0$</td>
</tr>
<tr>
<td>$2.0$</td>
<td>$\alpha \geq 30.20%$</td>
<td>$\frac{\partial p_0}{\partial r} &gt; 0$ for all $S &gt; 0$</td>
</tr>
<tr>
<td>$2.5$</td>
<td>$\alpha \geq 42.26%$</td>
<td>$\frac{\partial p_0}{\partial r} &gt; 0$ for all $S &gt; 0$</td>
</tr>
<tr>
<td>$3.0$</td>
<td>$\alpha = 50%$, $r=5%$: $0.0-15.68$, $24.96+$, $r=10%$: $0.0-7.84$, $12.48+$</td>
<td>$\alpha = 50%$, $r=5%$: $0.0-15.68$, $24.96+$, $r=10%$: $0.0-7.84$, $12.48+$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 60%$, $r=5%$: $0.0-13.32$, $24.18+$, $r=10%$: $0.0-8.16$, $12.09+$</td>
<td>$\alpha = 60%$, $r=5%$: $0.0-13.32$, $24.18+$, $r=10%$: $0.0-8.16$, $12.09+$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 70%$, $r=5%$: $0.0-11.10$, $28.74+$, $r=10%$: $0.0-5.55$, $16.37+$</td>
<td>$\alpha = 70%$, $r=5%$: $0.0-11.10$, $28.74+$, $r=10%$: $0.0-5.55$, $16.37+$</td>
</tr>
<tr>
<td>$3.5$</td>
<td>$\alpha = 50%$, $r=5%$: $0.0-14.06$, $30.06+$, $r=10%$: $0.0-7.03$, $15.03+$</td>
<td>$\alpha = 50%$, $r=5%$: $0.0-14.06$, $30.06+$, $r=10%$: $0.0-7.03$, $15.03+$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 60%$, $r=5%$: $0.0-13.50$, $28.36+$, $r=10%$: $0.0-6.75$, $14.18+$</td>
<td>$\alpha = 60%$, $r=5%$: $0.0-13.50$, $28.36+$, $r=10%$: $0.0-6.75$, $14.18+$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 70%$, $r=5%$: $0.0-12.94$, $23.16+$, $r=10%$: $0.0-5.55$, $11.58+$</td>
<td>$\alpha = 70%$, $r=5%$: $0.0-12.94$, $23.16+$, $r=10%$: $0.0-5.55$, $11.58+$</td>
</tr>
<tr>
<td>$4.0$</td>
<td>$\alpha = 50%$, $r=5%$: $0.0-11.10$, $32.74+$, $r=10%$: $0.0-5.55$, $16.37+$</td>
<td>$\alpha = 50%$, $r=5%$: $0.0-11.10$, $32.74+$, $r=10%$: $0.0-5.55$, $16.37+$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 60%$, $r=5%$: $0.0-13.50$, $28.36+$, $r=10%$: $0.0-6.75$, $14.18+$</td>
<td>$\alpha = 60%$, $r=5%$: $0.0-13.50$, $28.36+$, $r=10%$: $0.0-6.75$, $14.18+$</td>
</tr>
<tr>
<td></td>
<td>$\alpha = 70%$, $r=5%$: $0.0-17.14$, $23.16+$, $r=10%$: $0.0-8.57$, $11.58+$</td>
<td>$\alpha = 70%$, $r=5%$: $0.0-17.14$, $23.16+$, $r=10%$: $0.0-8.57$, $11.58+$</td>
</tr>
</tbody>
</table>

$k=\frac{P}{c}$: The ratio of the production cost of the substitute (per unit of the resource equivalent) to production cost of the resource.

$\alpha$: The percentage share of capital cost in production cost of the resource/substitute.
capital requirements for resource extraction and production of the substitute exceeds the present value of the stock of the resource.

Our theoretical analysis showed that this condition will be satisfied not only for small sizes of the resource stock but also for large stocks. Indeed, the results of our numerical example for energy resources indicated that for a wide range of plausible values of the parameters involved, the condition will hold for all sizes of the resource stock.

IV. Resource Depletion and Returns to Scale in Production of the Substitute

In the previous section, we noted that capital costs in production of the substitute play a significant part in determining the impact of a change in the discount rate on the pace of resource depletion. Another aspect of the substitute production which can have strong implications for both the optimal pricing (depletion) of the resource as well as the timing of the substitute is the possibility of economies or diseconomies of scale. The study of these implications, which is the object of this section, is of importance, because both the theoretical and empirical investigations of the optimal resource depletion (or pricing) policy in presence of a substitute have usually been undertaken on the assumption that the substitute technology shows constant returns to scale.

Hence, with the exception of the assumption of constant returns to scale (CRS) in production of the substitute, we shall retain all other assumptions made previously. Also, since factor prices \((w,r)\) will remain unchanged throughout the analysis, in writing the cost functions we shall suppress them so that \(\bar{c}\) and \(C(y_t)\) now denote, respectively, the unit (marginal) extraction cost of the resource and the total production cost of the substitute. Finally, for the ease of exposition, in this section we shall consider quantities, rather than prices, as the planners' decision variables.

Thus, written in its general form, the problem confronting the planners is now to choose the paths of resource extraction \(\{x_t\}\) and substitute produc-
tion \{y_t\} so as to

\[
\text{maximise} \quad \int_0^\infty e^{-rt}[U(x_t + y_t) - \bar{c} - C(y_t)]dt \\
\text{subject to the constraints} \\
\int_0^\infty x_t dt \leq S \quad \text{and} \quad x_t, y_t \geq 0 \quad \text{for all} \quad t \geq 0
\]  

(IV.1)

The necessary conditions for optimality are routinely established as

\[
e^{-rt}[U'(x_t + y_t) - \bar{c}] \leq \lambda', \quad x_t > 0
\]

(IV.2)

\[
U'(x_t + y_t) - C'(y_t) \leq 0, \quad y_t > 0
\]

(IV.3)

where \(\lambda\) is the Lagrangean multiplier.

The inequalities in (IV.2) and (IV.3) hold with complemantary slackness and, together, imply three possible production regimes characterised by

\[
x_t > 0, \quad y_t = 0 : \quad U'(x_t) = \bar{c} + \lambda e^{rt} < C'(0) \quad \text{(IV.4)}
\]

\[
x_t > 0, \quad y_t > 0 : \quad U'(x_t + y_t) = \bar{c} + \lambda e^{rt} = C'(y_t) \quad \text{(IV.5)}
\]

\[
x_t = 0, \quad y_t > 0 : \quad U'(y_t) = C'(y_t) < \bar{c} + \lambda e^{rt} \quad \text{(IV.6)}
\]

(In what follows, we shall refer to these regimes as \(R_1, R_2\) and \(R_3\), respectively.)

The first question which comes to mind is: In what order should the utilisation of the resource and the substitute take place under different conditions of returns to scale in production of the substitute? One approach to this question could be to examine all possible production policies which can arise from various orderings of the regimes \(R_1, R_2\) and \(R_3\), and then identify the optimum policy for various cases of returns to scale. However, that will be tedious and unnecessary because, even without specifying the conditions of returns to scale, one can at the outset discard a number of

\[1/ \text{ Notice that our earlier assumption about the demand function (see Section II, page 5) imply that } \lim_{z \to 0} U'(z) = \infty \text{ which automatically rules out the possibility of } x_t = 0 \text{ and } y_t = 0.\]
policies as being non-optimal. These will be all policies which do not consist of a final phase during which the regime $R_3$ is active. This is best seen by noting from (IV.6) that as long as the regime $R_3$ is in action the substitute will be produced at a constant flow rate $y_t = \bar{y}$ (where $\bar{y}$ is the solution to $U'(y_t) = C'(y_t)$ ) and supplied at a constant price which equals its marginal cost of production $C'(y_t)$, so that the flow of the net social benefits accruing under the regime $R_3$ will be constant $(U(\bar{y}) - C(\bar{y}))$ over time. Now, consider any policy which consists of a final phase during which a regime other than $R_3$ (i.e. $R_1$ or $R_2$) is operative. From (IV.4) and (IV.5) it is then seen that along such a policy the supply price of the resource/substitute $c + \lambda e^{rt}$ will be rising indefinitely with time, so that the consumption of the resource $x_t$ (or the aggregate consumption $x_t + y_t$) and hence the flow of social benefits will be declining over time correspondingly. This implies the existence of a sufficiently large $T$ such that for all $t > T$ the net social benefits from the use of the resource (or the resource and substitute) will fall short of $[U(\bar{y}) - C(\bar{y})]$ which would be available if during $(T, \infty)$ the regime $R_3$ were in force. Therefore, in order for a policy to be optimal it is necessary that it involves a final phase during which only the substitute is produced.

This condition reduces the number of candidates for an optimum to only two policies, namely those specified by the orderings $R_R R_3$ and $R_2 R_1 R_3$. However, the policy characterised by the ordering $R_2 R_1 R_3$ can not be optimal. To show this, let $(0, T_1)$ be the time interval during which the regime $R_2$ is in operation. Then, at time $T_1$ when $R_2$ terminates, it should be the case that either $y_{T_1} = 0$ or $x_{T_1} = 0$. In the former case, one would have (from (IV.4)) $U'(x_{T_1}) = c + \lambda e^{T_1} = C'(0)$, in which case for all $t > T_1$ $U'(x_t) = c + \lambda e^{rt} > C'(0)$. But this contradicts condition (IV.4) which must be satisfied whenever the regime $R_1$ is operative. So, 'R$_2$-followed-by R$_1$' is not an optimal sequence.

In the latter case where $x_{T_1} = 0$, one has (from (IV.5)), $U'(y_{T_1}) = c + \lambda e^{T_1} = C'(y_{T_1})$
in which case \( U'(y_t) = C'(y_t) < c + \lambda e^{\tau t} \) for all \( t > T_1 \). But this is the condition which must hold whenever the regime \( R_2 \), and not \( R_1 \), is active. Thus, again, '\( R_2 \)-followed-by \( R_1 \)' can not be an optimal sequence.

Having ruled out the sequence \( R_2R_1R_3 \) as non-optimal, we are left with \( R_1R_2R_3 \) as the general order in which different regimes should appear in an optimal production/extraction policy.

This result can now be used to investigate in detail the behaviour of the optimum policy under different conditions of returns to scale in production of the substitute. This shall be done in the following two sections with the next section devoted to the cases of constant and decreasing returns.

V. The Optimal Policy when there are Constant or Decreasing Returns in Production of the Substitute

V.1 Constant Returns to Scale (CRS) in Production of the Substitute

This is the case which has been customarily assumed in all studies which have incorporated a substitute for the resource in their analyses and, in fact, the optimal policy for it has been already discussed in Section II.1/ Nevertheless, as the benchmark with which the other cases of returns to scale are compared, we present a brief discussion of it here.

The condition of CRS implies that the marginal cost of the substitute is constant, so that, together with the assumption that the substitute is more costly than the resource, we have

\[
C'(y_t) = \bar{p} > \bar{c} \quad \text{for all} \quad y_t \geq 0 \quad (V.1)
\]

where \( \bar{p} \) denotes the marginal cost of the substitute for this case.2/

Bearing condition (IV.5) in mind, an immediate implication of (V.1) is to eliminate the regime \( R_2 \) from the optimum ordering \( R_1R_2R_3 \); (i.e. the possibility of simultaneous production of the resource and substitute).

1/ Of course, there we simply assumed that the substitute will be introduced after the resource is exhausted. Here, this will be demonstrated rigorously.

2/ It should be perhaps mentioned that throughout we shall be assuming that returns to scale are uniform, i.e. that they will be decreasing, constant, or increasing at all production levels.
So that the optimal policy for this case is specified by the sequence $R_1R_3$. Also, recalling that $U'(z) = p(z)$, and using conditions (IV.4) and (IV.6) describing regimes $R_1$ and $R_3$, respectively, it can readily be verified that the conditions governing the optimal policy are indeed identical to those given by (II.5.a)-(II.5.c) in Section II. The features of the optimal policy are also illustrated in Diagram 2.

**Diagram 2**

![Diagram 2](attachment:image)

The optimal policy under CRS in production of the substitute

**V.2 Decreasing Returns to Scale (DRS) in Production of the Substitute**

Under the conditions of DRS, the marginal cost of production will be an increasing function of the production rate, i.e.

\[ C''(y_t) > 0 \quad \text{for all} \quad y_t \geq 0 \quad (V.2) \]

Moreover, since it is assumed that the production cost of the substitute exceeds the extraction cost of the resource, we have

\[ C'(0) > \bar{c} \quad (V.3) \]

Given this information and our previous result that the ordering of different regimes along an optimal policy is $R_1R_2R_3$, it can readily be checked that in the present case the optimal policy is governed by conditions (IV.4)-(IV.6). More specifically, the optimal policy will consist of three phases. During an initial phase $[0,T_1]$, only the resource will be extracted,
with the rate of extraction so determined that the net marginal benefit from its use rises at the rate of discount (see condition (IV.4)). This phase will be in force until such a date $T_1$ at which the price of the resource has risen high enough to cover the marginal cost of producing the first unit of the substitute, i.e., $U' (x_{T_1}) = \bar{c} + \lambda e^{T_1} = C'(0)$. Following this phase, will be a second phase, $[T_1, T_2]$, during which both the resource and the substitute will be utilised simultaneously. As seen from condition (IV.5), during this interval the price of the substitute will be rising; so that, to ensure that it is always supplied at a price equal to its marginal cost of production, it will be produced at rates which will be increasing over the interval. The rate of resource extraction will, on the other hand, be falling continuously with time just to ensure that the net marginal benefit from the aggregate services of the resource and substitute rises at the rate of discount. This second phase will continue to be effective until such a time, $T_2$, at which the stock of the resource is exhausted and the rate of extraction falls to zero. From then onwards, only the substitute will be in use, with its rate of production and price remaining constant throughout (as determined by the condition $U'(\bar{y}) = C'(\bar{y})$).

The optimal policy is lucidly portrayed in Diagram 3.

---

1/ At time $T_2$, one has $U'(y_{T_2}) = \bar{c} + \lambda e^{T_2} = C'(y_{T_2})$.

2/ I am grateful to Jim Mirrlees for suggesting this diagramatical presentation of the optimal policy.
That the optimal policy involves a phase during which both the resource and the substitute are produced simultaneously is quite interesting, for it has been widely believed in the literature that optimality requires that the relatively high-cost substitute be introduced only after the resource is exhausted. For instance, Dasgupta and Heal (1979, p. 191) have argued that "As long as the production cost of the substitute exceeds the extraction cost of the exhaustible resource the substitute ought not make its appearance initially... The resource and the substitute will not be utilised simultaneously, but sequentially". However, as our analysis shows, for the case where the substitute is produced under the conditions of DRS this proposition is not valid, with the implication that under those conditions it would be wrong, for example, to postpone the utilisation of nuclear fusion or solar energy until the existing reserves of relatively low-cost oil are used up. Furthermore, since our model can be viewed as a limiting case of a situation where there are several deposits of the same resource with different marginal extraction costs, it follows that if extraction from some of these deposits is subject to DRS then simultaneous exploitation of deposits will indeed be the case.

Having analysed the main features of the optimal policy under the conditions of DRS, we wish now to compare the paths of resource depletion and the dates of introduction of the substitute under the conditions of constant and decreasing returns to scale. Depending on whether $\frac{p}{c'y} > c'$, three different cases may arise. For each case, the comparison can easily be carried out by using the optimality conditions (IV.4)-(IV.6) and the fact that the stock of the resource is the same under both CRS and DRS. Consequently, we do not present the details, but directly summarise the results as follows. Letting $\lambda_c$ and $\lambda_d$ denote the present discounted value of the scarcity rent under, respectively, the conditions of constant and decreasing returns, $T_c$ and $T_d$ the dates of

---

1/ In fact, when the stock of the resource is sufficiently small and/or the marginal cost of the initial units of the substitute is sufficiently low (so as to support the condition $c+\lambda > C'(0)$) it will be optimal to produce the substitute at all points of time.
resource exhaustion, and \( T_s \) the date of utilisation of the substitute, it can be shown that if \( \tilde{p} \geq C'(\tilde{y}) \) one has: (a) \( \lambda_d < \lambda_c \) implying that the pace of resource depletion will be faster if the substitute is produced under DRS than under CRS, and (b) \( T_s < T_c \), so that the substitute will be introduced more quickly if its technology shows DRS than if it exhibits CRS. For the case where \( \tilde{p} < C'(\tilde{y}) \) one has \( T_d > T_c \), indicating that the resource stock will last longer if production of the substitute is governed by the condition of DRS than if it is subject to CRS (although the rate of depletion can still be initially faster under the former condition than under the latter). Also, as in the previous case, we must have \( T_s < T_c \), which enables us to conclude that the substitute will generally be utilised sooner if returns to its production are decreasing than if they are constant.

VI. The Optimal Policy when there are Increasing Returns to Scale (IRS) in Production of the Substitute

Increasing returns to scale implies that the marginal cost of the substitute decreases with the scale of production, i.e.

\[
C''(y_t) < 0 \quad \text{for all } y_t > 0
\]  

(VI.1)

With this condition, the possibility of simultaneous production of the resource and the substitute is immediately ruled out, so that, in the light of our discussion in Section IV, the optimal policy for this case will be characterised by the ordering \( R_1 R_3 \). That is, it consists of two phases:

---

1/ An alternative interpretation of these results could be as follows. Suppose the planners are faced with a choice between two substitutes available for the resource, where one of the substitutes is produced under the conditions of CRS with a marginal production cost of \( \tilde{p} \), while the other is subject to DRS and that, if utilised according to an optimal plan, it will in the long-run (i.e., after the resource is exhausted) be supplied at a constant flow rate \( \tilde{y} \) (where \( U'(\tilde{y}) = C'(\tilde{y}) \)). Suppose further, that \( \tilde{p} \geq C'(\tilde{y}) \). Then, the results stated above, under (a) and (b), imply that the optimal choice would, \textit{ceteris paribus}, be the substitute with DRS in production.
during an initial phase, [0,T) the resource will be exploited while from T onwards only the substitute will be in use.

Although this sequence is the same as that obtained for the case of CRS, the properties of the optimal policy in the present case are distinctively different from those described there. To bring out these properties in a clear fashion, it will be more convenient to return to the notation of Section II and again consider prices, rather than production levels, as decision variables.

Since we have already shown that during the second phase of the optimal policy only the substitute is going to be available, the equilibrium in that market entails \( y_t = x(p_t) \) for all \( t > T \) (where as before \( x(p_t) \) denotes the market demand function). Hence, the production cost of the substitute can now be written as \( C(x(p_t)) \), and the problem facing the planners as

\[
\begin{align*}
\max_{(p_t,T)} & \int_T^\infty e^{-rt} [V(p_t) - C(x(p_t))] dt + \int_T^\infty e^{-rt} [V(p_t) - C(x(p_t))] dt \\
\text{subject to constraints} & \\
\int_T^\infty x(p_t) dt & \leq S, \quad \text{and } p_t > 0 \text{ for all } t \geq 0
\end{align*}
\]

The necessary conditions for optimality are

\[
\begin{align*}
p_t &= \hat{c} + \lambda e^{rt} \quad \text{for all } t \in [0,T) \quad \text{(VI.3.a)} \\
p_T &= \hat{p} = C'(x(\hat{p})) \quad \text{for all } t \in (T, \infty) \quad \text{(VI.3.b)} \\
V(p_T) - p_T x(p_T) &= V(\hat{p}) - C(x(\hat{p})) \quad \text{(VI.3.c)} \\
\int_T^\infty x(p_t) dt &= S \quad \text{(VI.3.d)}
\end{align*}
\]

where \( p_T \) presents the resource price at the date of exhaustion (i.e. \( p_T = \lim_{t \to T^-} p_t = \hat{c} + \lambda e^{rt} \)) and \( \hat{p} \) is the supply price of the substitute which must be such that it complies with the efficient rule of marginal cost pricing.\(^2\)

Conditions (VI.3.a) and (VI.3.b) are similar to require further

\(1/\) For the derivation of these conditions, Appendix A.2.

\(2/\) It can be shown that a sufficient condition of \( \hat{p} \) satisfying (VI.3.b) is \( C''(x) > \), i.e. that the marginal utility be diminishing more rapidly than the marginal cost of the substitute.
remarks. Condition (VI.3.c), on the other hand, explains all the specific
features of the optimal policy under IRS, and is rich enough to enable us
to base a number of arguments upon it. It states that at time \( T \), when
the resource is exhausted and the substitute is to make its appearance, the
price of the resource must have risen to such a level that the consumers'
net benefit from the use of the resource is just equal to the net benefit
from the services of the substitute, \textit{i.e.} to such a level which renders
consumers indifferent between using the resource or having the substitute
available to them. That level will then constitute the ceiling price for
the resource.

An immediate implication of this condition can be stated in the form
of the following proposition.

**Proposition 2.** When there are economies of scale in production of the
substitute, the price of the resource will \textit{exceed} the marginal production
cost of the substitute, so that the latter \textit{does not} provide a ceiling for
for the former.

**Proof.** From the assumption of economies of scale, we have

\[
C(x(p_T)) > C'(x(p_T))x(p_T) \quad \forall \quad x(p_T) > 0
\]

In particular, for \( x(p_T) = x(\hat{p}) \)

\[
C(x(\hat{p})) > C'(x(\hat{p}))x(\hat{p})
\]

or, using condition (VI.3.b)

\[
C(x(\hat{p})) > \hat{p}x(\hat{p}) \quad \text{(VI.4)}
\]

Now, (VI.4) together with (VI.3.c) implies that

\[
V(p_T) - p_T x(p_T) < V(\hat{p}) - \hat{p} x(\hat{p}) \quad \text{(VI.5)}
\]

On the other hand, differentiating \( [V(p_T) - p_T x(p_T)] \) \textit{w.r.t.} \( p_T \) and using
(II.3), we have

\[
\frac{d}{dp_T} [V(p_T) - p_T x(p_T)] = -x(p_T) < 0 \quad \text{(VI.6)}
\]
Therefore, from (VI.5) and (VI.6) it follows that

\[ P_T > \hat{p} \]  \quad (VI.7)

as required. ||

Thus, contrary to the cases of diminishing and constant returns to scale where the optimal price path is continuous and the economy makes a smooth transition from the resource to the substitute, in the present case, the transition will not be smooth because at the date of resource exhaustion there will be a discontinuous fall in price from \( P_T \) to \( \hat{p} \), and hence a discontinuous rise in consumption from \( x(P_T) \) to \( x(\hat{p}) \) (see Diagram 4).

Diagram 4

\[ \text{The optimum price path under IRS in production of the substitute} \]

Furthermore, since \( P_T > \hat{p} \) and since the resource price rises continuously over the interval \([0, T)\), there will be always an interval of time \([T, T_0)\) in Diagram 4 during which the price of the resource exceeds the marginal cost of the substitute and yet the substitute will not be introduced in that interval\(^1\). From this it follows that when the substitute is produced under the condition of IRS, the erosion of a differential between the marginal cost of the substitute and the price of the resource does not provide a

\(^1\) In fact, it can readily be checked that if the stock of the resource is sufficiently small, then throughout the resource life the price of the resource will be above the marginal cost of the substitute without the resource ever being undercut by the substitute.
sufficient cause for introducing the substitute.

The reason for not introducing the substitute during \((\bar{T}, T)\) is simple: during this interval the price of the resource, although higher than the marginal cost of the substitute, will still be lower than the price which renders consumers indifferent between the use of the resource and the substitute. Accordingly, consumers will prefer to continue using the resource until such a time \(T\) at which the resource price has risen to the critical level \(P_T\).

As an alternative interpretation of the same event we may consider the following. Clearly, with increasing returns in production, pricing the substitute at its marginal cost means that a portion of its total cost, amounting to \(C(x(\hat{p})) - \hat{p} x(\hat{p})\), can not be covered through its sales. In the absence of any other source to finance these costs, one can imagine that the planners turn to the resource sector and raise the initial price of the resource above the level which would prevail if the substitute were produced under the condition of CRS with a marginal production cost of \(C'(x(\hat{p})) = \hat{p}\) (the optimal price path which would prevail under that condition is shown in Diagram 4 by the dashed curve). Of course, the price of the resource must be raised optimally, i.e. in such a way that the required sum becomes available just at the time when the stock of the resource is exhausted. Once the price of the resource is raised in this way, the date of exhaustion and, hence, introduction of the substitute will be postponed to such a time, \(T\), by which the needed will have been acquired to make production of the substitute viable.

The foregoing interpretation brings about the question of whether the optimal policy can be attained as the outcome of a competitive equilibrium. As is well known, for the cases of diminishing and constant returns in production of the substitute, the second theorem of welfare economics ensures that possibility. However, when there are increasing returns in production of the substitute the matter becomes complicated, for pricing the substitute
at its marginal cost entails firms to incur losses. The general problem of attaining efficiency in the presence of increasing returns to scale has been recently analysed in a general equilibrium framework by Brown and Heal (1980), (1981). Interestingly enough, Brown and Heal (1980) show that any Pareto-efficient equilibrium can be supported by a system of marginal cost pricing with two-parts tariffs whereby consumers pay a fixed charge for entering the market for the product produced under conditions of increasing returns, and then purchase the product at a price equal to marginal cost\(^1\).

We shall end this section by comparing the paths of resource depletion and the dates of utilisation of the substitute under the conditions of constant and increasing returns to scale. As before, let \(\bar{p}\) denote the marginal (unit) cost of the substitute under conditions of CRS. To facilitate the comparison, we note that as far as the path of resource depletion and the date of introduction of the substitute are concerned, a substitute which is produced with IRS and is supplied at an optimal price \(\hat{p} = C'(x(\hat{p}))\) can be regarded as equivalent to a substitute with CRS but with a marginal (unit) cost of \(p_T\), where \(p_T\) is such that it satisfies condition (VI.3.c). Bearing this in mind and recalling our earlier comparative-static results that \(\frac{\partial l}{\partial p} > 0\), \(\frac{\partial m}{\partial p} > 0\) (see Section II, page 28), it is immediate that as long as \(\bar{p} < p_T\), increasing returns in production of the substitute leads to a more conservative resource depletion policy and delays the utilisation of the substitute.

VII. The Choice Among Several Substitutes with Different Cost Characteristics

The previous section analysed the features of the optimal extraction (pricing) policy for an exhaustible resource in the presence of a substitute which is produced under the conditions of IRS. In the present section, we

---

\(^1\) As Brown and Heal (1980) have shown, unless one is prepared to make restrictive assumptions about consumers' preferences, the fixed charge in general varies from one person to another, depending on individuals' preferences. This clearly leads to problems of preference revelation and free-riding, and hence reduces the attractiveness of the system.
use the results of that analysis to provide an answer to the following basic question; namely, when there are several technologies capable of producing substitutes for the resource, what should be the criterion for selecting one as the optimal technology? The importance of this question hardly ever needs emphasizing. In fact, following the "oil crisis" of the early 1970's, there has been a great deal of debate among energy policy makers in the oil-importing industrialised countries as to which one of various alternative sources of energy should be developed in order to ease the constraint of dependency on foreign oil. For instance, in the United States during 1974-1977 the Administration's energy policy insisted on the accelerated development of the synthetic fuel technologies (production of oil from coal, shale, and tar sands), but this was strongly criticised by the Treasury on the grounds of their requiring massive amount of capital expenditure; also, whereas the Administration's policy supported the breeder reactor, the Congress opposed it on the environmental safety grounds; on the other hand, the Democratic Presidential candidate (Jimmy Carter) pointed to solar energy as the answer.

Clearly, there are many respects in which energy substitute technologies may differ from one another; among these are the degree of environmental acceptability, the extent to which the economy’s existing capital stock is to be adjusted to the use of substitute, the lead times needed for commercial development, etc. However, in the analysis which follows we shall assume that all these differences can be translated into cost differences. More specifically, we shall characterise each substitute technology, which is taken for simplicity to be of a 'backstop' type, by a constant unit variable cost, \( p \), and a minimum fixed cost, \( Z \), which is interpreted broadly enough to include fixed development costs, the capital-adjustment costs involved in switching from the resource to the substitute (or alternatively, the costs of bringing the substitute into perfect competition with the resource), and the costs required for meeting certain environmental quality standards (in
what follows we shall refer to \( Z \) briefly as the development and introduction cost. Furthermore, we assume that \( Z \) does not depend on the date of introduction of the substitute.

If it is decided to introduce a substitute at time \( T \), the present value of its associated fixed costs will be \( e^{-rT}Z \) which can equivalently be written as
\[
e^{-rT}Z = \int_{T}^{\infty} e^{-rt}rZ \, dt
\]
i.e. as the value, discounted back to time \( T \), of a hypothetical constant stream \( rZ \) per unit of time of costs to be incurred during \((T, \infty)\). Accordingly, the substitute's cost function in problem (VI.2) is now replaced by
\[
C(x(p_t')) = p\bar{x}(p_t') + rZ
\]
which, in turn, leads to replacement of the optimality conditions (VI.3.b) and (VI.3.c) by, respectively
\[
p_t = \bar{p} \quad \text{for all} \quad t \in (T, \infty) \quad (\text{VII.1})
\]
and
\[
V(p_t') - p_t \bar{x}(p_t') = V(\bar{p}) - \bar{p}\bar{x}(\bar{p}) - rZ \quad (\text{VII.2})
\]
(with other necessary conditions remaining unaltered).

Thus, corresponding to each substitute technology characterised by \((\bar{p}, Z)\), there will be an optimal policy indicating the price and hence the extraction path of the resource, the date of introduction of the substitute, and the price (and hence the rate) at which the substitute will be supplied. The question is: Of the several substitute technologies specified by \((\bar{p}_1, Z_1)\), \((\bar{p}_2, Z_2)\), ..., \((\bar{p}_n, Z_n)\) which one ought to be developed and introduced? Of course, a broad answer would be to choose that technology for which the present value of the net social benefits from services of both the exhaustible resource and substitute is the largest; but one is no doubt interested in a more specific and perhaps practical criterion. To provide such a criterion, let us note from (VII.2) that a substitute technology specified by \((\bar{p}, Z)\) would yield, along an optimal policy, a constant flow of net benefits (from the use of the substitute) given by
\[ I(\bar{p},Z) = \bar{V}(\bar{p}) - \bar{p} \bar{x}(\bar{p}) - rZ \]  

(VII.3)

so that we may as well characterise each technology by its associated stream of net benefits \( I(\bar{p},Z) \). Let us also recall from the analysis of the previous section that as far as the path of resource depletion and the date of introduction of the substitute are concerned, a substitute technology \((\bar{p},Z)\) can be regarded as equivalent to a hypothetical technology with CRS and a unit (marginal) production cost of \( p_T \), where \( p_T \) is such that

\[ \bar{V}(p_T) - p_T \bar{x}(p_T) = I(\bar{p},Z) \]  

(VII.4)

This interpretation enables us to use the comparative-static results obtained under the condition of CRS in production of the substitute1/ in order to examine the implications that the choice of substitute technology would have for the present social value of the resource stock, which is

\[ V_R = \int_0^T e^{-rt} [V(p_T) - c(x(p_T))] dt, \]

the present social value of the substitute

\[ V_S = \int_0^T e^{-rt} [V(p_T) - \bar{p} \bar{x}(\bar{p}) - rZ] dt = \frac{1}{x} e^{-rT} I(\bar{p},Z) \]

and hence the present net social benefits from use of the resource and substitute \( W_R = W_R + W_S \). Thus, bearing in mind that both \( \bar{T} \) and \( \lambda \) depend on \( p_T \) (via (VI.3.a)) which, in turn, depends on \( I(\bar{p},Z) \) (via (VII.4)), we can differentiate \( W_R \) w.r.t. \( I(\bar{p},Z) \) to obtain2/ after manipulation

\[ \frac{dW_R}{dI} = \frac{dW_R}{dp_T} \frac{dp_T}{dI} \]

\[ = \frac{x(p_T)}{x(p_T)(p_T - c)} \frac{dp_T}{I} \]

which, using (VII.4) and noting that

\[ \frac{dp_T}{dI} = -\frac{1}{x(p_T)} < 0 \]  

(VII.5)

we have

\[ \frac{dW_R}{dI} = \frac{x(p_T) - x(p)}{x(p_T) x(p_T)(p_T - c)} I < 0 \]  

(VII.6)

1/ In derivation of the formulae which follow we shall make use of comparative-static results (A.1.1.a) and (A.1.1.b) presented in Appendix A.1. It should however be noted that \( p_T \) now replaces \( \bar{p} \).

2/ For expositional convenience and without causing any misunderstanding, we shall write \( I(\bar{p},Z) \) simply as \( I \).
Therefore, as would be expected, the choice of a substitute technology with a larger $I$ would lead to a reduction in the value of the stock of exhaustible resource.

Next, we differentiate $W_S$ w.r.t. $I$ to obtain

$$\frac{dW_S}{dI} = \frac{1}{r} e^{-rT} - e^{-rT} \frac{dT}{dI}$$

Noting that $\frac{dT}{dI} = \frac{dI}{dP_T} \frac{dT}{dP_T}$ and using (A.I.I.b) and (VII.5), we have

$$\frac{dT}{dI} = \left[ \frac{x(p_0) - x(p_T)}{r x(p_0) x(p_T) (p_T - c)} \right] < 0$$

(VII.7)

Substituting from (VII.7) into $\frac{dW_S}{dI}$ yields

$$\frac{dW_S}{dI} = \frac{1}{x(p_0) x(p_T) (p_T - c)} \left[ x(p_0) - x(p_T) \right] e^{-rT} I + \frac{1}{r} e^{-rT}$$

(VII.8)

which reflects the obvious fact that the present value of the net benefits from services of the substitute will be greater for a substitute technology with a larger $I$ than for one with a smaller $I$.

What is interesting to note is that although a substitute technology with a larger $I$ reduces the asset value of the exhaustible resource, it increases the present value of the net benefits from the substitute by an amount larger than required to compensate the loss in the value of the resource stock, and therefore leads to a larger present value of the aggregate net benefits from the use of the resource and substitute. More precisely, from (VII.6) and (VII.8) we have

$$\frac{dW}{dI} = \frac{dW_R}{dI} + \frac{dW_S}{dI} = \frac{1}{r} e^{-rT} > 0$$

(VII.9)

This in fact establishes the criterion for choosing among several substitutes; namely, when there are a number of substitute technologies with different unit production costs, $p$, and different fixed development and introduction costs, $Z$, one should select that technology for which the flow of net benefits from use of the substitute, i.e. $I(p, Z) = V(p) - \bar{p} x(p) - rZ$,
is the largest. There are several aspects of this criterion that are worth drawing attention to.

(1) A rather striking feature of this criterion is that the optimal choice of the substitute technology is totally independent of the impact of the substitute on the value of the stock of exhaustible resource. In other words, the optimal rule is such that as if there were no exhaustible resource in the economy and one were to make an independent choice of investment in substitutes. In this sense, the optimal decision rule may be viewed as myopic.

(2) The fact that the optimal choice between various substitute technologies is independent of their impacts on the value of the resource stock, makes our decision rule operationally attractive, for in comparing the net flow of benefits, \( I(\vec{p}, \vec{Z}) = V(\vec{p}) - \vec{p} x(\vec{p}) - r \vec{Z} \), associated with different substitutes the only information we need in addition to costs are the market demand function and the rate of discount.

(3) It is a direct implication of our criterion that when there are fixed costs associated with the development and introduction of substitutes, it is not generally true, as has been widely believed in the literature, that the substitute with lowest marginal cost of production should be introduced first. As our criterion indicates, of the two substitutes characterised by \((\vec{p}_i, Z_i)\) and \((\vec{p}_j, Z_j)\) with \(\vec{p}_i < \vec{p}_j\) and \(Z_i > Z_j\), provided that \(I(\vec{p}_j, Z_j) > I(\vec{p}_i, Z_i)\), it is the substitute with the higher marginal cost of production which ought to be developed and introduced. In fact, our criterion is a generalisation of the marginalist rule to cases where there are fixed costs associated with the development and introduction of substitutes.\(^1\)

We end the analysis of this section by looking at the impact of fixed development and introduction cost of the substitute on the path of resource depletion and the date of introduction of the substitute.

---

\(^1\) It seems worth mentioning that Weitzman (1976) has generalised the marginalist rule of sequential development to cases where substitutes take the form of several pools of the same exhaustible resource and resource pools have arbitrary extraction cost structures. He assumes that the demand for the resource is fixed over time and converts an arbitrary cost stream to an "equivalent stationary cost" in order to find the cheapest alternative.
Noting from (VII.3) that $\frac{\partial I(p,z)}{\partial z} = -r < 0$, it is immediate from (VII.5) and (VII.7) that $\frac{\partial p}{\partial z} > 0$ and $\frac{\partial T}{\partial z} > 0$. Accordingly, the larger the costs required for the development and introduction of a substitute, the higher will be the ceiling price obtainable for the exhaustible resource (which, in turn, implies a more conservative depletion policy) and the later will be the date at which the substitute is introduced. It should not then be surprising to observe that some of the available substitute sources of energy whose production costs are even lower than current prices of crude oil have not yet been able to fully replace the demand for crude oil.

VIII. Summary and Conclusion

This paper has been concerned with the validity of some basic propositions in the economics of exhaustible resources in the presence of substitutes. We have shown that, despite its widespread acceptance in the literature, the proposition that a decrease (increase) in the rate of discount leads to a slower (faster) depletion of exhaustible resources is not generally valid. Using a simple resource-substitute model in which the technologies of resource extraction and substitute production are explicitly formulated in the form of cost functions, we have shown that a decrease in the rate of discount has two distinct and countervailing effects. The first of these is the straightforward "conservation effect" of a lower discount rate—consumption in the future is now more attractive. The second of these is the less straightforward "disinvestment effect"—the lower discount rate reduces the unit costs of extraction of the resource and production of the substitute, and hence encourages faster rates of resource extraction. This latter effect has been completely neglected in the literature and yet our analysis has shown that for plausible parameter values it is in fact this effect that dominates.

Of course, to maintain the simplicity of our model, we abstracted from resource exploration activities which no doubt play an important role in making decisions on the rates of resource depletion. However, to the extent that a lower discount rate encourages investments in exploration activities and
hence increases the long-run resource availability, the inclusion of such activities in the model would in fact reinforce the result of our analysis.

We also analysed the implications of economies or diseconomies of scale in production of the substitute for the optimal resource depletion policy. It was shown that when the substitute is produced under the conditions of decreasing returns to scale, the conventional proposition that the utilisation of the relatively high-cost substitute should not begin before the exhaustion of the resource is not valid. Our analysis indicated that under such conditions the optimal policy requires that the resource and substitute be utilised simultaneously for some period of time. We also noted that in general decreasing returns in production of the substitute provide an argument for a faster depletion policy and quicker introduction of the substitute. Furthermore, we showed that when there are increasing returns in production of the substitute, it is no longer true (as has been widely accepted in the literature) that the marginal production cost of the substitute provides a ceiling for the price of the resource. It was shown that under such conditions there will be an interval of time during which the price of the resource exceeds the marginal cost of the substitute without the resource being undercut by the substitute. We therefore argued that in such cases the observation that the price of the resource has risen high enough to cover the marginal cost of the substitute is not a sufficient reason to introduce the substitute.

As a special case of increasing returns in production of the substitute, we considered the case where a substitute technology has a constant unit production cost and a fixed development and introduction cost. The latter cost component was defined broadly to include not only costs required for the development of the substitute, but also costs involved in adjusting the modes of production and consumption to the use of the substitute, and environmental costs. The question asked was: Given the availability of several such substitute technologies, how should the society make a decision as to which technology to choose as the substitute in the future? We noted that
in such cases the conventional marginalist rule that 'the substitute with lowest marginal cost should be introduced first' does not in general give the correct answer. It was shown that the optimal decision rule is to choose that technology for which the flow of per period net social benefits from use of the substitute is the largest. An interesting implication of this criterion is that the choice among substitutes should be made independently of their impacts on the value of the stock of exhaustible resource. This suggests that in our model the uncertainty about the size of the stock of resource is irrelevant to the choice of substitute—although it would be interesting to allow explicitly for such uncertainties in the model and confirm this suggestion formally.

We also noted that the existence of fixed development and introduction costs of the substitute leads to higher prices obtainable for the resource (and hence to a more conservative resource depletion policy) and to a later introduction of the substitute. Given this result, it would be interesting to see how Nordhaus' (1973) calculation of a low efficient supply price for crude oil would be modified if one allowed for the fixed development and introduction costs of alternative sources of energy.

In the light of the results mentioned above, it seems right to conclude that the optimal decisions concerning the intertemporal allocation (pricing) of an exhaustible resource depends critically on the specific economic characteristics of substitutes available for it.
Appendix

A.1 Derivation of Comparative-Static Results

Total differentiation of (II.5.c) and (II.5.d) gives

\[
\begin{bmatrix}
\frac{x(\bar{p})-x(p_0)}{x(\bar{p})-x(p_0)}/x'(p) \\
\frac{x(\bar{p})-x(p_0)}{x(\bar{p})-x(p_0)}
\end{bmatrix}
\begin{bmatrix}
d\lambda \\
dT
\end{bmatrix}
= 
\begin{bmatrix}
-T \int_0^T x'(p(t))dt & 0 & d\lambda \\
-rT e^{-rT} & e^{-rT} & 0 & d\epsilon
\end{bmatrix}
\begin{bmatrix}
d\epsilon \\
dp
\end{bmatrix}
\]
\tag{A.1.1}

Solving (A.1.1) for \(d\lambda\) and \(dT\), we obtain the following partial derivatives:

\[
\frac{3\lambda}{dp} = e^{-rT} x(\bar{p})/x(p_0) > 0 \quad \tag{A.1.1.a}
\]
\[
\frac{3T}{dp} = -e^{-rT} \frac{[x(\bar{p})-x(p_0)]}{x(\bar{p})} > 0 \quad \text{(since} \quad \bar{p} > p_0 \quad \text{and} \quad x'(p_0) < 0) \quad \tag{A.1.1.b}
\]
\[
\frac{3\lambda}{dc} = \frac{[r \int_0^T x'(p(t))dt - e^{-rT} x(\bar{p})]}{x(\bar{p})} < 0 \quad \tag{A.1.1.c}
\]
\[
\frac{3T}{dc} = \frac{[-\int_0^T \lambda x'(p(t)) + e^{-rT} x(\bar{p}) - x(p_0)]}{x(\bar{p})} \quad \tag{A.1.1.d}
\]
\[
\frac{3\lambda}{ds} = -\frac{r\lambda}{x(p_0)} < 0 \quad \tag{A.1.1.e}
\]
\[
\frac{3T}{ds} = \frac{1}{x(p_0)} > 0 \quad \tag{A.1.1.f}
\]

To determine the sign of \(\frac{3T}{dc}\), we note from (II.5.a) that \(\dot{p}_t = r\lambda e^{-rt}\), or

\[
\lambda = \frac{r}{\dot{p}_t} e^{-rt} \quad \tag{A.1.2}
\]

Substituting for \(\lambda\) from (A.1.2) and noting that \(\dot{x}(p_0) = x'(p_0)\dot{p}_t\), we can rewrite (A.1.1.d) as

\[
\frac{3T}{dc} = \frac{e^{-rT}}{r} (x(\bar{p}) - x(p_0)) - \frac{1}{r} \int_0^T x(p(t)) e^{-rT} dt] / \lambda x(p_0).
\]

which upon integration by parts becomes

\[
\frac{3T}{dc} = -\int_0^T e^{-rT} [x(p(t)) - x(p_0)] dt / \lambda x(p_0) > 0 \quad \tag{A.1.3}
\]

Moreover, from (II.5.a) we have \(p_0 = \lambda + c\) so that

\[
\frac{3p_0}{dp} = \frac{3\lambda}{dp} > 0, \quad \frac{3p_0}{dc} = \frac{3\lambda}{dc} < 0, \quad \text{and} \quad \frac{3p_0}{dc} = \frac{3\lambda}{dc} + 1
\]
To determine the sign of \( \frac{\partial p_o}{\partial \xi} \), we use (A.1.1.c) and (A.1.2) to write

\[
\frac{\partial p_o}{\partial \xi} = 1 + \left[ \int_0^T x(p_T)e^{-rt} \, dt - e^{-rt} x(p) \right] / x(p_o)
\]

which, upon integration by parts, reduces to

\[
\frac{\partial p_o}{\partial \xi} = r \int_0^T e^{-rt} x(p_T) \, dt / x(p_o) > 0 \quad (A.1.4)
\]

### A.2 Derivation of the Necessary Conditions for Optimality

To derive the necessary conditions for the optimal solution, we form the Lagrangean associated with problem (VI.2) and solve

\[
\max_{\{p_T,T\}} L = \int_0^T e^{-rt} \phi(p_T,t) \, dt + \int_0^\infty e^{-rt} \psi(p_T) \, dt
\]

where

\[
\phi(p_T,t) \equiv v(p_T) - x(p_T) (c + \lambda e^{rt})
\]

and

\[
\psi(p_T) \equiv v(p_T) - C(x(p_T))
\]

Rewrite \( L \) as

\[
L = \int_0^T e^{-rt} \left[ \phi(p_T,t) - \psi(p_T) \right] \, dt + \int_0^\infty e^{-rt} \psi(p_T) \, dt - \int_0^T e^{-rt} \psi(p_T) \, dt
\]

\[
= \int_0^T e^{-rt} \left[ \phi(p_T,t) - \psi(p_T) \right] \, dt + \int_0^\infty e^{-rt} \psi(p_T) \, dt \quad (A.2.2)
\]

Now define \( \theta_T(t) \) as

\[
\theta_T(t) \equiv \begin{cases} +1, & t \in [0,T) \\ 0, & t \in (T,\infty) \end{cases}
\]

Using (A.2.3), we can write (A.2.2) as

\[
L = \int_0^\infty e^{-rt} \left\{ [\phi(p_T,t) - \psi(p_T)] \theta_T(t) + \psi(p_T) \right\} \, dt + \int_0^\infty e^{-rt} \psi(p_T) \, dt = \int_0^\infty \lambda \, dt
\]

where

\[
\lambda = e^{-rt} \left\{ [\phi(p_T,t) - \psi(p_T)] \theta_T(t) + \psi(p_T) \right\} \quad (A.2.4)
\]

Applying the Euler-Lagrange equation and noting that \( \frac{\partial \lambda}{\partial p_T} = 0 \), we require for a maximum that \( \frac{\partial \lambda}{\partial p_T} = 0 \) for any given \( T \).

Therefore

\[
\theta_T(t) \left\{ x'(p_T) C'(x(p_T)) - x'(p_T) (c + \lambda e^{rt}) \right\} + \left\{ v'(p_T) - x'(p_T) C'(x(p_T)) \right\} = 0
\]

\[
(A.2.5)
\]
From (A.2.5) and (A.2.3) it follows that

\[ V'(p_t) - (c + \lambda e^{rt})x'(p_t) = 0, \quad t \in [0,T) \]  
(A.2.6)

\[ V'(p_t) - x'(p_t)c'(x(p_t)) = 0, \quad t \in (T,\infty) \]  
(A.2.6')

Using (II.3) and the fact that \( x'(p_t) < 0 \), we can write (A.2.6) as

\[ p_t - c = \lambda e^{rt}, \quad t \in [0,T) \]  
(A.2.7)

and

\[ p_t = c'(x(p_t)), \quad t \in (T,\infty) \]  
(A.2.7')

(A.2.7) and (A.2.7') characterise the optimal price trajectory for any given \( T \). To find the optimal \( T \), we substitute from (A.2.7) and (A.2.7') into (A.2.1) to write

\[ L = \int_0^T e^{-rt} [V(p_t) - p_t x(p_t)] dt + \int_T^\infty e^{-rt} [V(p_t) - C(x(p_t))] dt \]  
(A.2.8)

Differentiating (A.2.8) w.r.t. \( T \) and setting \( \frac{dT}{dT} = 0 \), gives

\[ V(p_T) - p_T x(p_T) = V(p) - C(x(p)) \]  
(A.2.9)

where \( p_T \) is the limit of \( p_t \) as \( t \to T^- \) in (A.2.7), and \( \hat{p} \) is the solution to equation (A.2.7').
REFERENCES


---


---


World Bank Publications of Related Interest

Electricity Pricing: Theory and Case Studies
Mohan Munasinghe and Jeremy J. Warford

Describes the underlying theory and practical application of power-pricing policies that maximize the net economic benefits to society of electricity consumption. The methodology provides an explicit framework for analyzing system costs and setting tariffs, and allows the tariff to be revised on a continual basis. Case studies of electricity pricing exercises in Indonesia, Pakistan, the Philippines, Sri Lanka, and Thailand describe the application of the methodology to real systems.

LC 81-47613. ISBN 0-8018-2703-5, $22.50 (£15.75) hardcover.

Global Energy Prospects
Boum Jone Choe, Adrain Lambertini, and Peter K. Pollak

A background study for World Development Report, 1981. Examines adjustments in the use of energy that have taken place in five major country groups since the 1973–74 oil price increase. Based on these trends, energy conservation and efficiency, and assumptions about income levels and of a 3 percent annual increase in real energy prices, the paper provides energy projections by major fuels for these country groups in the 1980s.

Stock No. WP-0489. $5.00.

Interrelationships in Energy Planning: The Case of the Tobacco-Curing Industry in Thailand
Gunter Schramm and Mohan Munasinghe


Mobilizing Renewable Energy Technology in Developing Countries: Strengthening Local Capabilities and Research

Focuses on the research required to develop renewable energy resources in the developing countries and on the need to strengthen the developing countries' own technological capabilities for using renewable energy. (One of three publications dealing with renewable energy resources and issues in developing countries. See Alcohol Production from Biomass in the Developing Countries and Renewable Energy Resources in the Developing Countries.)

July 1981. iv +52 pages.
Stock No. EN-8101. $5.00.

Principles of Modern Electricity Pricing
Mohan Munasinghe


World Bank Series in Water Supply and Sanitation

The United Nations has designated the 1980s as the International Drinking Water Supply and Sanitation Decade. Its goal is to provide two of the most fundamental human needs— safe water and sanitary disposal of human wastes—to all people. To help usher in this important period of international research and cooperation, the World Bank is publishing three volumes on appropriate technology for water supply and waste disposal systems in developing countries. Since the technology for supplying water is better understood, the emphasis in these volumes is on sanitation and waste reclamation technologies, their contributions to better health, and how they are affected by water service levels and the ability and willingness of communities to pay for the systems.

Number 1: Appropriate Sanitation Alternatives: A Technical and Economic Appraisal
John M. Kalbermatten, DeAnne S. Julius, and Charles G. Gunnerson

This volume summarizes the technical, economic, environmental, health, and sociocultural findings of the World Bank's research program on appropriate sanitation alternatives and discusses the aspects of program planning that are necessary to implement these findings. It is directed primarily toward planning officials and sector policy advisers for developing countries.


Number 2: Appropriate Sanitation Alternatives: A Planning and Design Manual
John M. Kalbermatten, DeAnne S. Julius, Charles G. Gunnerson, and D. Duncan Mara

This manual presents the latest field results of the research, summarizes selected portions of other publications on sanitation program planning, and describes the engineering details of alternative sanitation technologies and how they can be upgraded.

LC 80-8963. ISBN 0-8018-2584-9, $15.00 (£10.50) paperback.
**Low-Cost Technology Options for Sanitation—a State-of-the-Art Review and Annotated Bibliography**

Towid Rybczynski, Chongrak Polprasert, and Michael McGarry

A comprehensive bibliography that describes alternative approaches to the collection, treatment, reuse, and disposal of wastes.

A joint World Bank/International Development Research Centre publication, 1978. Available from International Development Research Centre (IDRC), P.O. Box 8500, Ottawa K1G 3H9, Ontario (Canada).


**Low-Cost Water Distribution—a Field Manual**

Charles Spangler

Provides nonengineers with the information necessary to design and implement simple water distribution systems for small communities.

Available in 1982.


**Technical and Economic Options**

John M. Kalbermatten, DeAnne S. Julius, and Charles G. Gunnerson

Reports technical, economic, health, and social findings of the research project on "appropriate technology" and discusses the program planning necessary to implement technologies available to provide socially and environmentally acceptable low-cost water supply and waste disposal.

December 1980.


**World Bank Research in Water Supply and Sanitation—Summary of Selected Publications**

A bibliography summarizing the papers in the Water Supply and Sanitation Series, as well as the World Bank studies in Water Supply and Sanitation published for the World Bank by The Johns Hopkins University Press.

November 1980.


**Alcohol Production from Biomass in the Developing Countries**

Explains the techniques for manufacturing ethyl alcohol from biomass raw materials: analyzes the economics of and prospects for production and government policies needed to accommodate conflicting needs of various sectors of the economy in promoting production; and discusses the role the World Bank can play in assisting developing countries in designing national alcohol programs. (One of three publications dealing with renewable energy resources in developing countries. See Mobilizing Renewable Energy Technology in Developing Countries: Strengthening Local Capabilities and Research and Renewable Energy Resources in the Developing Countries.)

September 1980. ix + 69 pages (including 12 annex figures).

English, French, and Spanish.

Portuguese (forthcoming).


**Costs Incurred by Residential Electricity Consumers Due to Power Failures**

Mohan Munasinghe


**The Economic Choice between Hydroelectric and Thermal Power Developments**

Herman G. van der Tak

A logically correct method for handling the economic comparison of alternative systems.


LC 66-28053. ISBN 0-8018-0646-1, $5.00 (£3.00) paperback.

**Economic Criteria for Optimizing Power System Reliability Levels**

Mohan Munasinghe and Mark Gellerson


**Electricity Economics: Essays and Case Studies**

Ralph Turvey and Dennis Anderson

Argues the merits of relating the price of electricity to the marginal or incremental cost of supply and deals with interactions between pricing and investment decisions, income distribution, and distortions in the pricing system of the economy.


ISBN 2-7178-0165-0, 58 francs.


ISBN 84-309-0822-6, 710 pesetas.
Integrated National Energy Planning in Developing Countries
Mohan Munasinghe


Municipal Water Supply Project Analysis: Case Studies
Frank H. Lamson-Scribner, Jr., and John Huang, editors

Eight case studies and fourteen exercises dealing with the water and wastewater disposal sector.

$8.50 paperback.

A New Approach to Power System Planning
Mohan Munasinghe


The Oil Price Revolution of 1973–74
Salah El Serafy


Planning for Electrical Power: Costs and Technologies
Mohan Munasinghe


Prospects for Traditional and Non-Conventional Energy Sources in Developing Countries
David P. Hughart

Stock No. WP-0346. $5.00.


Farzin, Yeganeh Hossein, 1950-
The effect of discount rate and substitute technology on depletion of exhaustible