THE OPTIMUM DEPTH OF WATER IN A PORT

February 18, 1970

This paper discusses the problems of cost benefit analyses of port deepening projects. It defines the relationship between the concepts of optimum vessel size and optimum shipment size and discusses how this relationship is affected by depth limitations in the port. It analyses the nature of the benefits that may be expected from a port deepening project, emphasizing the distinction between international and domestic trade oriented ports. A general formula for the measurement of benefits is developed.

The paper points out the major practical difficulties of employing a satisfactory theoretical approach, the nature of the simplifications often made to overcome these difficulties and the biases implicit in them. The paper also presents some data on depth requirements for various vessel sizes and on vessel operating costs.

The author is grateful to Mr. J. de Weille for his constructive criticism and comments during the study; to Mr. Y. Abe for permitting him to reproduce in the annexes some material collected by him; to Messrs. T. Husain, T. M. Malkani and A. P. Israel for their comments on an earlier draft of the paper; to Miss Suzanne M. Snell for editing the paper; and to Miss Audrey Mayne-Nicholls for typing it.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. TRANSPORT COST AND MARKET EQUILIBRIUM</td>
<td>6</td>
</tr>
<tr>
<td>A. Introduction</td>
<td>6</td>
</tr>
<tr>
<td>B. Transport Demand and Supply Curves</td>
<td>7</td>
</tr>
<tr>
<td>1. The Two-Country Case</td>
<td>7</td>
</tr>
<tr>
<td>2. Transport Costs Disaggregated</td>
<td>9</td>
</tr>
<tr>
<td>3. The Three or More-Country Case</td>
<td>10</td>
</tr>
<tr>
<td>4. Trade Duties</td>
<td>11</td>
</tr>
<tr>
<td>C. Incidence of Transport Costs</td>
<td>12</td>
</tr>
<tr>
<td>D. Summary</td>
<td>17</td>
</tr>
<tr>
<td>III. THE TRANSPORT SUPPLY FUNCTION AND THE DEPTH CONSTRAINT</td>
<td>18</td>
</tr>
<tr>
<td>A. Introduction</td>
<td>18</td>
</tr>
<tr>
<td>B. Components of Transport Cost</td>
<td>18</td>
</tr>
<tr>
<td>C. The Unit On-Sea Cost Curve</td>
<td>19</td>
</tr>
<tr>
<td>D. Unit Inland Costs</td>
<td>22</td>
</tr>
<tr>
<td>E. Total Unit Cost of Transport</td>
<td>24</td>
</tr>
<tr>
<td>F. Summary</td>
<td>26</td>
</tr>
<tr>
<td>IV. THE MEASUREMENT OF BENEFITS</td>
<td>27</td>
</tr>
<tr>
<td>A. Introduction</td>
<td>27</td>
</tr>
<tr>
<td>B. Benefits to Producers and Consumers, No Duties</td>
<td>27</td>
</tr>
<tr>
<td>C. Benefits in the Presence of Duties</td>
<td>35</td>
</tr>
<tr>
<td>D. Benefits to Transport-related Industries and Pricing Policy</td>
<td>35</td>
</tr>
<tr>
<td>E. Summary</td>
<td>37</td>
</tr>
<tr>
<td>V. PROJECT COSTS</td>
<td>38</td>
</tr>
<tr>
<td>VI. THE PROBLEM IN PRACTICE</td>
<td>41</td>
</tr>
<tr>
<td>A. Introduction</td>
<td>41</td>
</tr>
<tr>
<td>B. Commodity Classes and the Measurement of Benefits</td>
<td>41</td>
</tr>
<tr>
<td>C. Unit Transport Costs</td>
<td>44</td>
</tr>
<tr>
<td>D. Two Additional Points</td>
<td>45</td>
</tr>
<tr>
<td>E. Summary</td>
<td>46</td>
</tr>
</tbody>
</table>

- ANNEX I  Vessel Operating Costs
- ANNEX II  Draft and Vessel Size
- ANNEX III  Incidence Formulas

Bibliography
THE OPTIMUM DEPTH OF WATER IN A PORT

I. INTRODUCTION

1. A previous paper has analyzed the problem of how to determine the optimum service capacity of a port, given the number of arrivals of ships in the port over time.¹ The issue examined in this paper is the determination of the optimum capacity of a port in terms of the size of the largest ship (in terms of its maximum laden draft, length or breadth) that it can accommodate. The question is: what is the maximum size of ships that a port should be designed to accommodate?²

2. The deeper the water in access channels and alongside berths, the larger the ship owners and shippers can employ.³ Unless a port is so deep that all the shippers who use it can employ whatever ship sizes are most economical from their point of view, some benefits will be generated by deepening a port. A comparison of the present values of benefits and costs at the appropriate discount rate will be necessary to decide on the increment in depth to be provided. This is illustrated in Fig. 1. The optimum depth is that depth at which the marginal increase in the present value of benefits equals the marginal increase in the present value of costs, viz., 40' in Fig. 1.

¹/ J. de Weille and A. Ray (5).

²/ For the sake of brevity, the maximum size of ships that a port can accommodate will be referred to as its "maximum size capacity" or simply, its "size capacity". While this term is meaningless for a pure lighterage port it becomes relevant when berthing facilities are being planned for such ports.

³/ The commercially relevant concept of size is the ship's cargo carrying capacity. The appropriate measures of a ship's cargo carrying capacity are d.w.t. (deadweight cargo carrying capacity) and c.c.g. or c.c.b. (cubic capacity in terms of grain or bale equivalents). For a discussion of these and other measures, see C.F.A. Cufley (2). Note that the laden draft tends to increase (though less than proportionately) with increases in the cargo carrying capacity. See Annex II.
3. It is to be noted, however, that the benefits generated by each additional foot of depth can very rapidly fall to zero unless complementary investments are undertaken in other areas of the port. After a point, the size capacity of the port may be limited not by depth but by the width of locks or the construction design of berths. Also, shipowners and shippers may not find it profitable to employ large ships unless the service facilities in the port, e.g., cargo-handling, storage, inland transportation links, etc., are adequate for large shipments. If a project included improvements besides deepening of the port then the cost of the whole "package" program is the relevant cost.

4. When comparing the present values of costs and benefits a decision has to be taken on the timing of investments. For example, if the benefits generated during the first three years after dredging 10' are no greater or only slightly greater than those generated by dredging only 5', then it may well be desirable to dredge only 5' initially, and dredge another 5' after three years.1/

1/ See J. de Weille (6).
5. The purpose of this paper is to present a logical framework for the analysis of port deepening projects, with primary emphasis on the nature of the benefits. The technique of measuring benefits suggested in this paper (Section IV) is simple and will often be more convenient than other methods. However, the paper does not give "rules of thumb" for port project appraisals. General rules of this kind are unreliable because benefits and costs of a deepening project will vary widely from case to case. Considerable empirical research may be necessary before benefits and costs can be reliably quantified for any individual case.

6. Outline of the paper. The analysis in the next chapter (and the accompanying Annex III) is general and applies to all port projects, not just port deepening projects. In Chapter III, the transport cost functions are discussed, treating the depth of a port as a constraint on the decisions of shippers and shipowners. Chapter IV discusses the generation and measurement of benefits and is, like Chapter II, quite general in scope. Chapter V briefly discusses the costs of a port deepening project. Finally, Chapter VI discusses some of the practical problems of appraising a port deepening project. Each of these chapters is outlined and briefly summarized below.

7. Chapter II. Transport Cost and Market Equilibrium
   (i) The relationship between changes in the prices of traded commodities and changes in unit transport costs with and without trade restrictions is discussed. The concept of "incidence" which measures this relationship is defined.
   (ii) It is demonstrated that, generally, the impact on import and export prices of a fall in unit transport cost will depend on (a) the price elasticities of demand and supply, (b) the number of trading countries, (c) the transport supply functions, and (d) export and import duties.

8. Chapter III. The Transport Supply Function and the Depth Constraint
   (i) The general nature of the transport supply functions or the shippers' cost functions is discussed, distinguishing between unit on-sea costs and unit inland costs. The effect on these functions of limited port depth is also discussed.
   (ii) It is demonstrated that each point on the unit total transport cost curve, i.e., the transport supply function, will be the result of a cost minimization decision by the shipper. The particular shape and position of the curve will be different for different commodities. Depth limitation will affect the transport supply functions by
interfering with the cost minimization decisions of port users.

9. Chapter IV. The Measurement of Benefits

(i) The generation and measurement of benefits is discussed, with emphasis on the producers and consumers of traded commodities. The benefits accruing to the producers of transport and transport-related industries, including the port authority, are also considered.

(ii) A general formula for the measurement of benefits to the producers and consumers of the traded commodities is presented, using the concepts of consumers' and producers' surpluses. It is shown that both increases in revenue from trade duties and increased profits of transport industries should be included in total benefits. The role of pricing policies is discussed.

10. Chapter V. Project Costs

This chapter comments briefly on:

(i) the nature of a port deepening project as distinct from a project designed to increase service efficiency;

(ii) the capital costs of initial dredging;

(iii) secondary effects of incorrect input pricing.

(iv) secondary benefits arising from externalities, and

(v) the alternative of lighterage.

11. Chapter VI. The Problem in Practice

(i) Some of the practical difficulties of measuring benefits and changes in unit transport costs are considered. Fluctuations of port depth (due to tidal conditions, etc.) and the importance of a port's location on the trade routes are commented on.

(ii) It is demonstrated that by disregarding some trades and by making other simplifying assumptions, the measurement problems can be substantially reduced.
12. **Annexes I, II and III**

(i) **Annex I. Vessel Operating Costs**

The variation of vessel operating costs by the size of vessel is often taken as an index of the variation in tramp market rates. This annex discusses these costs and discusses in particular the influence which voyage length and time spent in ports have on the optimum vessel size. It presents some data on the relationship of unit costs to vessel size.

(ii) **Annex II. Draft and Vessel Size**

The draft requirements of different vessel sizes are discussed.

(iii) **Annex III. Incidence Formulas**

In this annex, the incidence formulas on which Chapter II is based are derived from a general model. The model given can easily be adapted to accommodate various assumptions.
II. TRANSPORT COST AND MARKET EQUILIBRIUM

A. Introduction

13. The total direct benefits of investments in the transport modes that are expected to service only domestic traffic (such as domestic ports, roads, etc.) can be estimated solely as a function of the reduction in unit transport cost. It is not necessary to estimate separately the incidence of the transport cost reduction on the prices of commodities in different domestic markets unless one is interested in the regional distribution of benefits. However, the benefits from an investment in an international trade oriented port will be distributed among the various trading countries. A country undertaking such a port investment may be expected to be interested in only those benefits that accrue to the nationals of the country. There is no justification for departing from a purely nationalistic criterion of benefits, aside from altruism or special circumstances such as when the foreign beneficiaries can be expected to pay compensation. It is thus vitally important to estimate the effects of a port investment on the prices received by domestic exporters and the prices paid by domestic importers.

14. The primary purpose of this chapter is to discuss the ways of measuring price changes brought about by changes in the unit transport cost. The following description of the remaining sections in this chapter may be helpful to the reader.

Section B: Demand and Supply Curves for Transport

1. The Two-Country Case

Introduces two equivalent ways of analyzing an international market equilibrium situation and shows how these use the demand and supply functions of transport.

2. Transport Costs Disaggregated

Provides a convenient breakdown of the total unit transport cost into three separate elements.

3. The Three or More-Country Case

Generalizes the equilibrium situation presented in (1).

4. Trade Duties

Generalizes (1) to take account of trade duties.

1/ See: P. Samelson (19), and H. G. van der Tak and A. Ray (24).
Section C: Incidence of Transport Costs

Discusses the impact of changes in unit transport cost on the export and import prices.

Section D: Summary

15. The mathematical justification for the analysis in this chapter is contained in the more general framework given in Annex III.

B. Transport Demand and Supply Curves

1. The Two-Country Case

16. The market equilibrium for a particular good will be illustrated, first under the simplifying assumptions that (i) there are only two trading countries and (ii) there are no trade duties or quotas. Suppose, for example, that the U.S.A. exports wheat to Guatemala. The wheat is sent from, say, Chicago via New York and Santo Tomas (on Guatemala's Atlantic Coast) to Guatemala City. In the absence of trade duties, etc. the unit price of wheat in Guatemala will be above the U.S. local price by the amount of the unit transport cost. In the Fig. 2 (a) below the curve S represents the U.S. exporter's export supply curve and the curve D represents the Guatemalan importer's import demand curve. When trade equilibrium is attained, the amount the U.S. exporter wishes to export at some export price $P_x$ will equal the amount the Guatemalan importer wants to buy at some import price $P_m$, the difference between these prices being equal to the unit transport cost of transporting that volume of wheat from Chicago to Guatemala City, $C_t$. The equilibrium volume of trade will be different for each level of unit transport cost. The relationship between the equilibrium volume of trade and the unit transport cost when the latter is parametrically varied (regarded as independent of the volume of trade), that is, the derived demand curve for transport, is shown in Fig. 2 (b) by $D_t$.

Figure 2

Derivation of the Transport Demand Curve

![Figure 2a](image)

![Figure 2b](image)
17. The intersection of the transport demand curve with the transport supply curve (giving the variation in the unit transport cost with the volume of trade) represents an equilibrium situation. This is shown in Fig. 3 (a) below. An equivalent way of showing the same equilibrium is to add vertically the transport supply function ($S_t$) to the U.S. export supply curve $S$. This is shown in Fig. 3 (b) where $S_{1}$ is the transport cost inclusive export supply curve, or in other words, the export supply curve in terms of the import price in Guatemala City, i.e. $P_m$. To indicate this $m$ is put in parenthesis after $S_{1}$ i.e. $S_{1}(m)$. If the transport supply function is horizontal then $S_{1}$ will be parallel to $S$, i.e. $S$ will be shifted upwards by the amount of the constant unit transport cost. If the transport supply function is rising then the distance between $S$ and $S$ will increase as the volume of trade increases (as in Fig. 3 (b) below).

**Figure 3**

*Two-country Equilibria in the Trade and Transport Markets.*

**Figure 3a**

**Figure 3b**
18. In this paper the representation in Fig. 3 (b) will be used: the transport demand curve will not explicitly enter into analysis. It is much more convenient to use this representation as the values of export and import prices explicitly appear. Nevertheless, it is important to bear in mind that the situation depicted in Fig. 3 (b) contains all the information necessary to derive a transport demand curve, and that the change in the area under the transport demand curve in Fig. 3 (a) represents the total direct project benefits when the supply curve S shifts downward due to a port investment. This important proposition will not be used here since in this paper we are interested not in total world benefits but in the benefits to the country investing in transportation.

2. Transport Costs Disaggregated

19. There are many possible ways of breaking up the total unit transport cost into separate elements. The relevant separation is the one that is convenient for the purpose at hand. In this paper the unit transport cost will be broken down into three components, unless otherwise mentioned. This is illustrated with reference to the example of transporting wheat from Chicago to Guatemala City as follows:

<table>
<thead>
<tr>
<th>Transport stages:</th>
<th>Chicago</th>
<th>New York, St. Thomas, Guatemala</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>on board on board</td>
<td>City</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Unit transport costs:</th>
<th>← Cr → ← Cs → ← Cu →</th>
</tr>
</thead>
</table>

| Price of good being shipped: | Px | Pf | Pc | Pm |

20. The price definitions can be algebraically written:

(i) $P_x$ = export price, ex-transport cost (U.S. local price)

(ii) $P_f = P_x + C_r$ = f.o.b. export price; all port charges and inland transport costs in U.S.A. are included in $C_r$.

(iii) $P_c = P_x + C_r + C_s$ = c.i.f. price in Guatemala. $C_s$ is the unit on-sea cost, including insurance.

(iv) $P_m = P_c + C_u$ = import price (Guatemalan local price). $C_u$ includes all port charges and inland transport costs in Guatemala.

\[\text{Note: (iv) This theorem is proved in van der Tak and Ray, ibid.}\]
21. It may be noted that in order to represent diagrammatically the equilibrium situation in the way shown in Fig. 3(b) one has to adopt price definitions which are consistent with each other. If the import demand curve is drawn in terms of $P_m$, the export supply curve should also be drawn in terms of $P_m$ (i.e., shifted upwards by the amount of the unit transport cost). Similarly if the export supply curve is drawn in terms of $P_x$ then the import demand curve should also be drawn in terms of $P_x$ (i.e., shifted downward by the amount of the unit transport cost).

3. The Three or More-Country Case

22. The case when the importing country buys the same commodity from more than one country is shown in Fig. 4 below. $S_1(m)$ is one of the export supply curves in terms of price $P_m$ (the local price is in the importing country) and $S_2(m)$ is the other export supply curve, also in terms of $P_m$. Their sum, $S(m)$, is the total export supply curve. $D$ is the import demand curve. The equilibrium export prices in the first country ($P_{x_1}$) and in the second country ($P_{x_2}$) as well as their export volumes, $OB$ and $OC$ respectively, are shown. Note that competitive equilibrium requires the import price be the same regardless of the commodity's origin. The unit transport costs from country 1 ($C_{t_1}$) and from country 2 ($C_{t_2}$) will be:

$$C_{t_1} = C_u + C_{s_1} + C_{r_1}$$
$$C_{t_2} = C_u + C_{s_2} + C_{r_2}$$

Figure 4

Three-Country Market Equilibrium
4. Trade Duties

23. With export and import duties, the price difference between any two countries will no longer equal the unit transport costs between the countries. Usually, the ad valorem import duties are levied on the c.i.f. price ($P_c$), as defined in paragraph 8. If this is the case and there is also an ad valorem export duty on the export price ($P_x$), then the equilibrium price difference will be

$$P_m - P_x = C_t + \psi P_x + (C_r + C_s) q;$$

where $\psi = (e + q + eq)$ and $e = \text{export duty}$, and $q = \text{import duty}$.

24. With no export duty, the price difference will simply be

$$P_m - P_x = C_t + P_c q;$$

$P_c$ being the c.i.f. price, i.e. the base for duty $q$ or,

$$= C_t + P_c q + (C_r + C_s) q.$$  1/

Fig. 5 below illustrates the situation. $S_1$ is the export supply curve in terms of the c.i.f. price. $S_2$ is the same curve but duty inclusive. $S_3$ is the duty and transport cost inclusive supply curve. The shaded area is the import duty revenue generated.

Figure 5

Market Equilibrium in the Presence of Duties

1/ This implies that the normal practice of using the c.i.f. price as the base for an import duty penalizes the import sources with higher transport costs more than the differences in transport costs warrant, since such a duty compounds the differences in the transport costs.
25. With specific import or export duties the difference between import and export prices will simply be the sum of the unit transport cost and the duties. After all, the unit transport cost is identical, in principle, with a specific duty on the export price.

C. Incidence of Transport Costs

26. The total transport bill may be paid entirely by the consignor or the consignee or shared between them. Many different financial arrangements may be entered into by consignors and consignees. Although the actual financial arrangement may be relevant from the point of view of the balance of payments of the countries concerned it will not usually reflect the incidence of transport costs. From the importer's (or exporter's) point of view the incidence of a change in the transport cost may be measured by the percentage change of the price he actually pays (receives) over what the price would have been if the transport cost had not changed.

27. An alternative way of defining the incidence is to express the change in the import (or export) price as a ratio of the change in the transport cost. This ratio may be zero, indicating that the import (or export) price does not change at all when the transport cost changes. It may be unity, indicating that the import (or export) price changes by the full amount of the change in transport cost. The ratio will be non-negative for an importer, since the import price will not increase when the transport cost falls. It will be non-positive for an exporter as the export price will not decrease when the transport cost falls.

28. The incidence of a change in the transport cost will depend, in general, on (i) the price elasticities of demand and supply for the commodity in trade, (ii) the number of trading countries, (iii) the supply curves of the separate elements of transport and (iv) export and import duties. The general formula is given in equation (13) in Annex III. Only three simple, but not necessarily unrealistic, cases will be illustrated here. All three cases assume trade between only two countries.

29. Case 1. Assume there are no export or import duties and that all the components of the unit transport cost are constant. A port deepening project reduces the unit total transport cost. The incidence ratio for an importer is given by the following formula:
Let $dP_m$ = change in import price

$\frac{dC_t}{dC_t}$ = change in unit total transport cost

$e_x$ = elasticity of the supply curve in terms of the import price. That is, it is the elasticity of the transport cost inclusive supply curve.

$e_m$ = elasticity (negative) of the import demand curve.

$I_m = \frac{dP_m}{dC_t}$ = incidence ratio for the importer.

Then

$$I_m = \frac{dP_m}{dC_t} = -\frac{e_x}{e_m - e_x}$$

$I_m$ varies between 0 and 1.

30. This incidence ratio will be unity (the import price decreases by the amount of the decrease in unit transport cost) if either $e_m = 0$ or $e_x \rightarrow \infty$. It will be zero (import price will not change) if either $e_m \rightarrow \infty$ or $e_x = 0$. The incidence ratio will be 0.5 (the import price will fall by half the decrease in the unit transport cost) if $e_x = \frac{1}{2}e_m$. These cases are illustrated in Table 1 below.

<table>
<thead>
<tr>
<th>(1) Export Unit Price (x)</th>
<th>(2) (Tons)</th>
<th>(3) Import Volume (t)</th>
<th>(4) Import Price cost = 1+2</th>
<th>(5) I_m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initial situation</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2. New situation</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>A. $e_m = -1$, $e_x = 1$</td>
<td>8.5</td>
<td>1</td>
<td>9.5</td>
<td>105,000</td>
</tr>
<tr>
<td>B. $e_m = 0$</td>
<td>8</td>
<td>1</td>
<td>9</td>
<td>100,000</td>
</tr>
<tr>
<td>C. $e_m \rightarrow \infty$</td>
<td>9</td>
<td>1</td>
<td>10</td>
<td>112,500</td>
</tr>
<tr>
<td>D. $e_m = 1$</td>
<td>9</td>
<td>1</td>
<td>10</td>
<td>100,000</td>
</tr>
<tr>
<td>E. $e_m = -1$</td>
<td>8</td>
<td>1</td>
<td>9</td>
<td>112,500</td>
</tr>
</tbody>
</table>
31. The incidence ratio for an export good is given, in a similar fashion, by
\[ I_x = \frac{dP_x}{dC_t} = -\frac{e_m}{e_m - e_x} \]

\[ I_x \] varies between 0 and -1.

32. From the formulas and Table 1 we see that:

(i) Importers' price will decrease by the full amount of the reduction in transport cost if a) import demand is fully price inelastic or if b) the supply curve is infinitely elastic.

(ii) The price received by exporters will rise by the full amount of the transport cost if a) the import demand is infinitely elastic or if b) the export supply is fully inelastic.

(iii) If the import demand and export supply curves have equal elasticities (though opposite in sign) then the import price and the export price will change by equal amounts (though in opposite directions).

33. The first two cases are diagrammatically illustrated in Fig. 6 and 7. These simple cases may approximate reality in quite a few cases. In Fig. 6a and 6b, the foreign export supply curves are expressed in terms of the import price, \( P_m \). When the unit transport cost falls by \( dC_t \), the export supply curve (in terms of \( P_m \)) shifts down and the import price falls from \( P_{m0} \) to \( P_{m} \). In Fig. 6c and 6d, the foreign import demand curves are expressed in terms of the export price, \( P_x \). When the unit transport cost falls by \( dC_t \), the demand curve (D) shifts upward by the amount \( dC_t \). The equilibrium export price increases to \( P_{x1} \) from \( P_{x0} \), by the full amount of the transport cost reduction.
Figure 6

Incidence of transport costs on the trade market

Figure 6a

Figure 6b

Figure 6c

Figure 6d
Case 2. The same assumptions are made as in Case 1, except that an ad valorem export duty, $e$, is added to the export price and an ad valorem import duty, $q$, is added to the c.i.f. price. Then the formulas are (same notation):

(i) For an import good:
$$ I_m = -\frac{e_x}{(1 + e)(1 + q)} \frac{e_m - e_x}{e_m} ; \quad 0 \leq I_m \leq 1 $$

(ii) For an export good:
$$ I_x = -\frac{e_m}{(1 + e)(1 + q)} \frac{e - e_m}{e_m} ; \quad (1 + e + q + eq) \leq I_x \leq 0 $$

Note that in this case there will be an asymmetry between $I_m$ and $I_x$.

If $e_x = 0$ or if $e_m \to \infty$, $I_x$ will have the value of $\frac{1}{(1 + e)(1 + q)}$ and not $-1$.

Case 3. No trade duties are assumed for this case but the foreign unit inland transport cost ($C_r$), the unit on-sea transport cost ($C_s$) and the domestic unit inland transport cost ($C_u$) are each assumed to increase with the volume of trade. The incidence ratio for an import good now can be written as follows:

Let
$$ \left\{ \begin{array}{c} \frac{E}{C_r} = \frac{\partial C_r}{\partial E} \text{ the elasticity of the unit foreign inland cost with respect to the volume of trade, where } E \text{ is the volume of export;} \\
\frac{E}{C_s} = \frac{\partial C_s}{\partial E} \\
\frac{E}{C_u} = \frac{\partial C_u}{\partial E} \end{array} \right. $$

and
$$ \left\{ \begin{array}{c} \frac{M}{C_u} = \frac{\partial C_u}{\partial M} ; (M = \text{volume of import}) \\
\frac{M}{C_t} = \frac{\partial C_t}{\partial M} \end{array} \right. $$

and Let
$$ T = \frac{C_r}{P_m} \int r + \frac{C_s}{P_m} \int s + \frac{C_u}{P_m} \int u $$

be the trade elasticity of the total unit transport cost, i.e. $\frac{M}{C_t} \frac{\partial C_t}{\partial M}$, multiplied by the share of total unit transport cost in the import price, i.e. by $\frac{C_t}{C_m}$.

Then,
$$ I_m = -\frac{e_x}{e_x e_m. T + e_m - e_x} $$
D. Summary

36. In this chapter we have examined the nature of a market equilibrium for an internationally traded good (B-2, B-3, B-4), classified the total unit transport costs into three separate components (B-2), and defined the incidence ratios and illustrated the incidence formulas for simple cases (B).

37. The incidence for the import (export) price is defined as the ratio of the change in import (export) price to the change in the unit transport cost. The primary justification for the introduction of this concept is to pave the way for the use of a nationalistic criterion of benefits. Generally, the impact of a fall in the unit transport cost on the import and export prices will depend on (i) price elasticities of supply and demand, (ii) number of trading countries (iii) the transport supply functions, (iv) export and import duties (Annex III).

38. If unit transport costs do not change with the volume of trade in a two-country case, the following conclusions hold.

(i) Importer's price will fall by the full amount of the reduction in unit transport cost if a) import demand is fully inelastic or b) the foreign supply curve is infinitely elastic (paragraph 20).

(ii) The price received by exporters will rise by the full amount of the transport cost if a) the foreign import demand curve is infinitely elastic or b) the export supply curve is fully inelastic (paragraph 20).

(iii) The conclusion under (i) above remains unchanged even in the presence of duties (paragraph 22).

(iv) In the presence of duties, the price received by exporters will rise by the amount \( \frac{1}{(1 + e)(1 + q)} \), where e and q are ad valorem export and import duties respectively, if a) the foreign import demand curve is infinitely elastic or b) the export supply curve is fully inelastic (paragraph 22).
III. THE TRANSPORT SUPPLY FUNCTION AND THE DEPTH CONSTRAINT

A. Introduction

39. In the previous chapter the role of the transport supply function in the general context of market equilibrium was discussed. This chapter concentrates on the transport supply function and discusses how the depth of water in a port may be expected to affect it.

40. The basic assumption underlying the discussion of the transport supply functions, also called shippers' cost functions, is that the shipper always tries to minimize the unit transport cost for any given amount of a commodity to be transported. In other words, the shipper will choose rail rather than road if rail is cheaper, other things being equal: if he has the choice, he will choose a 30,000 d.w.t. ship rather than a 10,000 d.w.t. ship if the unit transport cost is less with the former than with the latter.

41. Section B of this chapter discusses in more detail the definitions of the components of the unit transport cost given earlier (A-2, Chapter II). The nature of the on-sea cost function is dealt with in Section C and the nature of the inland cost functions in Section D. Section E develops the unit total transport cost function and Section F provides a summary.

B. Components of Transport Cost

42. We reconsider the components of the unit transport cost in order to emphasize the logic on which the separation given in the previous chapter is based. Thus, the definition of the unit on-sea costs, $C_s$, excludes all port charges, even those payable by the ships. It also excludes the cost of loading and unloading cargo. Logically, these costs are not related to the ocean voyage but to the efficiency and the policies of the ports visited. In reality, the unit cost, $C_s$, is not readily identifiable, as freight rates often include different elements. Nevertheless, this definition of $C_s$ should be borne in mind; for example, if harbor dues are reduced, a freight rate which included harbor dues should also fall.

43. As defined earlier, the unit inland transport costs, $C_r$ and $C_u$, are much more comprehensive than $C_s$. For instance, both $C_r$ and $C_u$ include unit storage and inventory costs, inland rail, or road transport costs, as well as unit port charges. The unit inland costs should therefore, be further disaggregated for a more refined analysis. For the present purpose, however, further disaggregation is not necessary.
C. The Unit On-Sea Cost Curve

44. A shipper using a particular port may use liner ships or tramp ships. Given the volume of his trade he may vary either the number of shipments or the volume per shipment, i.e., the cargo size, in order to minimize the unit on-sea costs. In order to isolate the on-sea options open to the shipper, it is convenient to temporarily assume that the unit inland costs are constant and that the storage cost is zero. Variations in these costs will be considered in the next section, Unit Inland Costs.

45. Liner Market. If a shipper chooses to use liner ships, his individual decision will affect neither the liner rates available to him nor the vessel sizes employed by the liner companies, although shippers' aggregate demand for liner space may. There is no satisfactory explanation of the existing liner rates structure, and rates are therefore extremely difficult to predict. The location of the port, its importance in the liner route, the time the ships normally spend in the port - these and other factors will tend to affect the liner rates. One significant characteristic of the liner rates is that these do not depend only on the peculiarities of the port concerned. The vessel size employed as well as the rates will depend also on the other ports of call in the liner routes. Frequently therefore the most efficient port along a liner route will be implicitly subsidizing the less efficient ports.

46. Moreover, the conferences to which liner companies belong normally set rates which discriminate among different commodities. Liner rates for commodities which are shipped in sufficiently large volumes will often be kept "open," i.e., negotiable, to allow the liner companies to compete with tramp ships. It will normally be cheaper for the shipper to use liner ships below a certain minimum cargo size.

---

1/ Bulk carriers and oil tankers are here considered as tramp ships, because, like small and medium-sized non-specialized tramps, they belong to a competitive market. The effect on shippers' decisions of the regularity of liner relative to tramp services operates through storage costs and is therefore considered in Section D (Unit Inland Costs).

2/ In the case of unexpected congestion, the liner conferences may impose a special congestion "surcharge" which is, by definition, of a temporary nature. If the congestion becomes routine, the surcharge tends to be incorporated in the quoted liner rates.

3/ Quoted freight rates of a conference are usually the same for all ports along the route except for imposts, surcharges, etc. Sometimes, but not always, a separate lighterage fee is quoted for lighterage ports.
47. Under the assumption of constant inland costs and zero storage costs, the shipper will attempt to maximize his cargo size if the liner rates tend to fall as cargo size increases. The maximum cargo size in the liner market is determined by the sizes of the available vessels. The liner market freight curve for a given commodity may be drawn as in Fig. 8.

**Figure 8**

*Unit Cost of Liner Space*

<table>
<thead>
<tr>
<th>Liner Rate</th>
<th>Total liner capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$V_O$ represents total liner capacity. This capacity need not be constant, as in the figure, since liner companies can always increase capacity by increasing the number of ships, the frequency of service, or the vessel sizes (if permitted by the size capacity of ports on the route). It is unlikely, however, that a single commodity will utilize all the liner space available. But as the volume of trade in a particular commodity increases, total demand for liner ships will exceed available capacity at a volume less than $V_O$. Furthermore, the point at which it becomes more economical to use tramp vessels for transporting a particular commodity will tend to be reached at much lower volumes than $V_O$.

48. *Tramps.* The term tramp market has been used here in a general sense to cover small tramps as well as tankers and bulk carriers. The basic characteristic of this market is its competitiveness: a shipper can bid for the vessel size, vessel type, and the type of contract he desires. Supply prices will therefore tend to reflect the structure of vessel operating costs as well as the technological economies of scale internal to the shipbuilding industry. If the rates of profit earned by shipowners
do not differ much by vessel size and type, the market price will tend to reflect
the supply prices and unit on-sea costs to the shipper will tend to fall with
larger vessel sizes. For any given route, however, on-sea costs will begin
increasing beyond a certain vessel size. The longer the route and the shorter
the fraction of total time spent idle in ports, the larger the optimum vessel
size will tend to be.\(^1\) The most economic or the optimum vessel size will also
tend to increase with the volume of trade. If, for instance, 100,000 tons is
the initial volume of trade between two given ports during a year, and the
corresponding optimum cargo size is 20,000, then there will be only five ship-
ments (assuming the shipper is indifferent to the timing and number of shipments).
If, however, the volume of trade for the same journey and period is 1 million
tons, the number of shipments will increase to 50, creating congestion costs
which will increase the most economic vessel size. The back-haulage problem
will also tend to keep the most economic size below what it otherwise would have
been. The costs of liner and tramp space is shown in Fig. 9.

\(^1\) See Annex I, Vessel Operating Costs.
D. Unit Inland Costs

51. In the previous section the unit inland cost was assumed constant and storage costs were assumed zero in order to isolate clearly the options open to the shipper in the liner and tramp markets. In this section the unit on-sea cost will be assumed constant to isolate more clearly the impact of inland costs on shippers' behavior. While inland cost structure may be different in different trading countries, the nature of the costs is the same and we make no distinction among countries.

52. It is assumed that for a particular volume of trade the unit inland cost curve related to the cargo size will be U-shaped. For each level of trade there will be a corresponding U-shaped cost curve. In Fig. 10, two such curves are shown (C_0 and C_1), each corresponding to a level of trade (T_0 and T_1, T_1 > T_0). The minimum point on C_0 gives the optimum cargo size V_0, for the level of trade T_0. Similarly, the optimum cargo size is V_1 when the volume of trade is T_1. The C_1 and the C_0 curves become price inelastic at V_{max}, demonstrating that the port's cargo size capacity is reached for that commodity at V_{max}.

**Figure 10**

*Unit Inland Cost Curve*
To see what this U-shape curve implies, suppose the cargo size is initially $V_2$ (on $C_0$). As the cargo size is increased, unit costs fall. On a per ton basis, the inland transportation costs will fall as cargo size increases because of lower bulk rates by rail or truck and lower port charges on ships, agent's commissions, lower storage costs, etc. The loading/unloading costs per ton will also tend to fall. Beyond the point $V_0$ unit costs rise as diseconomies of scale set in.

The minimum points on all the cost curves define a locus curve $C$ which shows the optimum cargo size for any level of trade. The shape of the curve implies that the unit inland cost at any given cargo size will be higher, the higher the trade volume. For example, at the cargo size $V_0$, the unit cost is higher when the trade volume is $T_1$ than when it is $T_0$ (see Fig. 10).

Unit storage costs will influence the shape of the cost curves. Each commodity production pattern in the exporting country will be associated with an optimum inventory level which minimizes the unit storage costs. Similarly, in the importing country, storage costs will depend on the rate and distribution of consumption over time. The unit storage costs will tend to discourage large cargo sizes when the production and the consumption rates are evenly spread over time. Unit storage costs will tend to encourage large cargo sizes when the production and consumption rates are "bunched up" over time.1/

The restrictive impact of the depth of water in a port can be seen in Fig. 11. Fig. 11-a is a repetition of Fig. 10 above and the same explanation applies (paragraph 52). A vertical line through $V_2$ is drawn, representing the maximum cargo size at a depth of 20', to show the effect of increases in trade volumes at a given depth. If the volume of trade is $T_0$, $C_{S0}$ will be the unit cost at the depth of 20'. If the volume of trade is $T_1$, $C_{S3}$ will be the unit cost at the same depth. The optimum cargo size, $V_0$, for the volume of trade $T_0$ will be reached at a bigger depth, and the corresponding unit cost will be $C_{S1}$. The optimum cargo size, $V_1$, for the volume of trade $T_1$ will be reached when the depth is increased to 30'. The unit cost will be $C_{S2}$. The variations in the unit cost with changes in the volume of trade is shown in Fig. 11-b. $R_0$, $R_1$ and $R$ correspond to depths of 20', 30' and no depth constraint respectively. These cost curves are derived from the ones in Fig. 11-a.

1/ The regularity of liner services and the non-regularity of tramp services affect the shippers' decision via the impact on unit storage costs. Thus, if a shipper is very sensitive to storage costs, he is likely to prefer liner services over tramp services. This is less true with long-term charters, however. Long-term chartering of tankers and bulk carriers can eliminate the non-regularity feature of the non-liner market.
Figure 11
Effect of Depth Constraint on Unit Inland Cost Curve

Figure 11-a

Figure 11-b

E. Total Unit Cost of Transport

57. The general shapes of both on-sea and inland cost curves in relation to the volume of trade have been discussed. The vertical sum of these two cost curves will yield the total unit cost curve for transport. This total cost curve is shown in Fig. 12, with and without a depth constraint. The curve without any depth constraint is $C_t$. The curves with depth constraints are $C_1$ and $C_2$ (lower depth).
58. The unit transport cost curve can now be added vertically to the export supply curve to obtain the foreign export supply in terms of the import price in the importing country. In Fig. 13 below the curve $C_2$ from Fig. 12 (port depth 20') is added to the export supply curve expressed in terms of the export price in the exporting country to obtain curve $S_3$. The $S_2$ curve corresponds to the transport cost curve $C_1$ (port depth 30'). The $S_1$ curve represents the situation with no depth constraint. With a port 20' deep, the domestic price for the importers will be $P_{m3}$, with one 30' deep, the price will be $P_{m2}$. If the initial depth is 20' and an additional 10' are provided, the cost of the initial volume of import will fall by $(P_{m3} - P_{m2})$ and additional imports ($V_2 - V_3$) will be induced.1/

1/ The same analysis applies even if the importer (or the exporter) buys (or sells) the commodity from (to) many different countries. The analysis needs modification only when the optimum cargo size on one route depends on the cargo size on another route. In such cases the export supply curves from the different sources will be interdependent.
Summary

59. For a given volume of trade in a commodity, the unit total transport cost will reflect the optimization decisions of the shipper with respect to cargo size and frequency of shipment. The cargo size and the frequency of shipment will be optimized using information about the variation of port charges, rail or road costs, storage costs, etc. with cargo size. The optimum cargo size and frequency of shipment will minimize the unit transport cost for a given volume of trade in the commodity.

60. Thus, each point on the unit total transport cost curve, i.e. the transport supply function, will be the result of a cost minimization decision by the shipper. For a given commodity and a given trade route, the unit total transport cost function will be U-shaped. The particular shape and position of the curve will, of course, be different for different commodities.

61. Limitation of the size capacity of a port in depth or other terms will interfere with the shipper's cost minimization decisions if they prevent him from using the cargo and vessel sizes which he considers optimal. In such an event, the transport supply function will lie above the one which would have obtained with unlimited size capacity (after the size capacity is fully utilized).
IV. THE MEASUREMENT OF BENEFITS

A. Introduction

62. This chapter will discuss the generation and measurement of benefits. As indicated in the introduction to Chapter II, we have chosen to use a nationalistic definition of benefits. Benefits which might accrue to non-nationals of the investing country are ignored. From the point of view of the country concerned, the beneficiaries to be considered are:

(i) shipowners of that country;

(ii) the port authority;

(iii) producers of inland transport and transport-related services, and

(iv) the domestic producers and consumers of traded commodities. 1/

63. Beneficiaries (i), (ii) and (iii) will be discussed together in section D below. The emphasis of this chapter is on the effects of a port investment on domestic producers and consumers of traded commodities, discussed in the following two sections. In section B, duties are ignored; their effects are dealt with in C. Section E summarizes the chapter.

64. Indirect benefits are ignored. Much of these indirect benefits will be accounted for if all inputs are valued at their shadow prices.

B. Benefits to Producers and Consumers, No Duties.

65. Cost savings. If either the export supply or the import demand curve is fully inelastic, the measurement of benefits accruing to exporters or importers will be entirely in terms of cost savings. If the export supply curve is fully inelastic, the incidence ratio will be -1, i.e. the price the exporters receive will increase by the full amount of the reduction in transport cost. Similarly, if the import demand curve is fully inelastic, the incidence ratio will be 1, and the price the importers pay will fall by the full amount of the fall in the transport cost. In both cases, the benefits will be given by the product of the quantity of export/import and the decrease in the transport cost.

1/ Benefits and costs to producers and consumers of non-traded products should also be considered. These are ignored here owing to the complexity of measuring such benefits and costs in most practical cases. See paragraph 72 below.
66. Totally price inelastic import demand or totally price inelastic export supply do not, however, imply that the quantity of imports or exports cannot change over time. If, for instance, the import demand increases at a rate of 5 per cent per annum, because of population growth, the cost savings will have to be estimated separately for each year. Fig. 14-a shows the export supply curves in terms of domestic price in the importing country corresponding to depths of 20' (S (20')) and 30' (S (30')) and Fig. 14-b shows the time path of the total import cost and cost savings with the 30-foot port depth.

Figure 14

Savings in Import Cost over Time from Increased Port Depth

Fig. 14-a

Fig. 14-b

Cost savings of 30' vs. 20'

port depth

Years

imports in years 1, 2, 3, etc.
Induced trade. The occurrence of induced increases in the volume of trade implies that the import demand curves or the export supply curves are not fully inelastic in the trading countries. Hence, the absolute value of the incidence ratio for exporters and importers will be less than unity. In other words, the domestic price for an importer or exporter will not change by the full amount of the fall in transport cost. Since, from the point of view of measuring benefits, it is the change in the price the importers pay or the price the exporters receive that is relevant, we need to know the change in transport costs to find out, via the incidence formulas, what the changes in these prices will be. A simple way of measuring the benefits is to use the concepts of consumers' and producers' surpluses. 1/ Fig. 15 illustrates these concepts for the importer. In Fig. 15 a, \( D_m \) is the excess demand curve, i.e. the import demand curve. If the domestic price falls from \( A \) to \( E \) because of a reduction in unit transport cost, the increment in consumer's surplus is \( ADHE \) and the loss in producers surplus (to domestic producers of the imported commodity) is \( ACFE \), which is less than \( ADHE \). The net gain is \( CDHF \), which equals the area \( ABGE \), the increment in consumers' surplus under the import demand curve. Similarly, in Fig. 15 b, when the export price (received by domestic producers) rises to \( I \) from \( M \), net gain (all to domestic producers) is \( JLPO \), which is the same as the increased rent (profits) accruing to exporters measured with the export supply curve, i.e. \( IKNM \).

---

1/ For expositions and uses of these concepts the reader is referred to the bibliography contained in A. Ray (16); see also H. G. Johnson (14) and H. G. van der Tak and A. Ray (24).
68. The benefits to importers due to a change in transport cost can now be measured as follows:

\[(P_{m1} - P_{m2}) M_1 + \frac{1}{2} (P_{m1} - P_{m2}) (M_1 - M_2)\]

where \(P_{m1}\) = initial price

\(M_1\) = initial import

\(P_{m2}\) = new price (lower)

\(M_2\) = new import volume
This is illustrated with respect to an increase of port depth from 20' to 30' in Fig. 16. The measurement of benefits to exporters is exactly analogous. Note that this benefit is the benefit to the consumers of the imported product after taking into account the losses suffered by domestic producers of the importable good.

Figure 16
Net Benefits from Trade Induced by Deepening of Port

69. To measure the total benefit of the change in transport cost, these measures have to be separately applied to each import and export good of the domestic country. Thus, if there are five import goods and five export goods, the total welfare increase is given by:

\[
W = \sum_{i=1}^{5} \left( P_{m_i} dM_i + \frac{1}{2} dM_i dP_{m_i} \right) + \sum_{i=1}^{5} \left( X_i dP_{x_i} + \frac{1}{2} dX_i dP_{x_i} \right)
\]

where
- \( M_i \) = the volumes of imports
- \( P_{m_i} \) = import prices
- \( X_i \) = the volumes of exports
- \( P_{x_i} \) = export prices
- \( dM_i \), etc. = changes

A numerical illustration is given in Table 2.

1/ See A. Ray (16) for proof.
### Table 2

<table>
<thead>
<tr>
<th>Foreign Price</th>
<th>Unit Transport Cost</th>
<th>Domestic Price</th>
<th>Volume of Trade</th>
<th>Changes in:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>- (-) (dollars)</td>
<td>- (-) (tons)</td>
<td>- (-) (dollars)</td>
<td></td>
</tr>
</tbody>
</table>

**For an Importer**

1. Initial Situation
   - Foreign Transport Price: 8
   - Domestic Volume of Trade: 10
   - Domestic Price: 2
   - Volume of Trade: 100,000

**New Situations:**

2. $e_d = -1$, $e_s = 1$
   - Foreign Transport Price: 8.5
   - Domestic Volume of Trade: 10
   - Domestic Price: 9.5
   - Volume of Trade: 105,000
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: -0.50
      - Foreign Transport Price: 8.5
      - Domestic Price: 9.5
      - Volume of Trade: 105,000
      - Welfare Gain: 5,000
      - 51,250

3. $e_d = 0$, $e_s = 1$
   - Foreign Transport Price: 8
   - Domestic Volume of Trade: 10
   - Domestic Price: 9
   - Volume of Trade: 100,000
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: -1
      - Foreign Transport Price: 8
      - Domestic Price: 9
      - Volume of Trade: 100,000
      - Welfare Gain: 0
      - 100,000

4. $e_d \to \infty$, $e_s = 1$
   - Foreign Transport Price: 9
   - Domestic Volume of Trade: 10
   - Domestic Price: 10
   - Volume of Trade: 112,500
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: 0
      - Foreign Transport Price: 9
      - Domestic Price: 10
      - Volume of Trade: 112,500
      - Welfare Gain: 12,500
      - 0

5. $e_d = -1$, $e_s = 0$
   - Foreign Transport Price: 9
   - Domestic Volume of Trade: 10
   - Domestic Price: 10
   - Volume of Trade: 100,000
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: 0
      - Foreign Transport Price: 9
      - Domestic Price: 10
      - Volume of Trade: 100,000
      - Welfare Gain: 0
      - 0

6. $e_d = -1$, $e_s \to \infty$
   - Foreign Transport Price: 8
   - Domestic Volume of Trade: 10
   - Domestic Price: 9
   - Volume of Trade: 112,500
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: -1
      - Foreign Transport Price: 8
      - Domestic Price: 9
      - Volume of Trade: 112,500
      - Welfare Gain: 12,500
      - 106,250

**For an Exporter**

1. Initial Situation
   - Foreign Transport Price: 10
   - Domestic Volume of Trade: 8
   - Domestic Price: 2
   - Volume of Trade: 100,000

**New Situations:**

2. $e_s = 1$, $e_d = -1$
   - Foreign Transport Price: 9.5
   - Domestic Volume of Trade: 10
   - Domestic Price: 8.5
   - Volume of Trade: 105,000
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: 0.5
      - Foreign Transport Price: 9.5
      - Domestic Price: 8.5
      - Volume of Trade: 105,000
      - Welfare Gain: 5,000
      - 51,250

3. $e_s = 1$, $e_d = 0$
   - Foreign Transport Price: 9
   - Domestic Volume of Trade: 10
   - Domestic Price: 8
   - Volume of Trade: 100,000
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: 0
      - Foreign Transport Price: 9
      - Domestic Price: 8
      - Volume of Trade: 100,000
      - Welfare Gain: 0
      - 0

4. $e_s = 1$, $e_d \to \infty$
   - Foreign Transport Price: 10
   - Domestic Volume of Trade: 9
   - Domestic Price: 9
   - Volume of Trade: 112,500
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: 12,500
      - Foreign Transport Price: 10
      - Domestic Price: 9
      - Volume of Trade: 112,500
      - Welfare Gain: 12,500
      - 106,250

5. $e_s = 0$, $e_d = -1$
   - Foreign Transport Price: 10
   - Domestic Volume of Trade: 9
   - Domestic Price: 10
   - Volume of Trade: 100,000
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: 0
      - Foreign Transport Price: 10
      - Domestic Price: 9
      - Volume of Trade: 100,000
      - Welfare Gain: 0
      - 100,000

6. $e_s \to \infty$, $e_d = -1$
   - Foreign Transport Price: 9
   - Domestic Volume of Trade: 10
   - Domestic Price: 8
   - Volume of Trade: 112,500
   - New Situation
      - Unit Transport Cost: 1
      - Domestic Price: 1
      - Volume of Trade: 1
      - Welfare Gain: 12,500
      - Foreign Transport Price: 9
      - Domestic Price: 8
      - Volume of Trade: 112,500
      - Welfare Gain: 12,500
      - 0
70. There are several important points which should be borne in mind when applying this measure.

(i) The gain in consumers' surplus to importers measured here is not the total gain to all domestic consumers of the product, but rather that part of the gain to consumers which is not offset by a fall in producers' surplus, or rent, earned by the domestic producers of the importable goods.

(ii) Similarly, the gain (in terms of rent) to producers of the exportable goods is not the total gain accruing to them, but only that part which is not offset by losses to domestic consumers of the exportable goods in terms of consumers' surplus.

(iii) The measure of the gain to importers may have to be amended if domestic production of the importable goods ceases after a decrease in transport cost. The measure does not have to be amended, however, if products not previously imported are imported after the reduction in the transport cost.

(iv) The measure of the gain to exporters may also have to be amended if domestic consumption of the exportable goods ceases. The measure needs no change if some goods which were not previously exported are now exported.

(v) If the relative prices of goods which are not traded change as a result of the fall in transport cost, the gains and losses to producers and consumers of these untraded goods must be accounted for.

(vi) The measure has to be corrected for market imperfections that may exist in the economy.

71. The amendments necessary in the cases (iii) and (iv) above are as follows:

(a) If the domestic production of an import good ceases completely due to a reduction in unit transport cost, the measure of the benefits for that good becomes:

\[ \text{MdP}_m + \frac{1}{2}\text{dP}_m \text{dC} + \frac{1}{2} (1+b) \text{SdP}_m \]

where

\[ P_m = \text{initial domestic price} = \text{import price.} \]

\[ S = \text{initial domestic production of the product.} \]
C = initial domestic consumption of the product.

M = import volume = C - S

\[ b = \frac{Z - P_{m1}}{dP} \], Z being the "elimination price",

i.e. the price at which domestic production ceases. $P_{m1}$ is the new domestic price.

(b) If the domestic consumption of an export good ceases completely, the measure for that export good becomes:

\[ XdP_x + \frac{1}{2} SdP_x + \frac{1}{2} (1 + a) CdP_x \]

where S is the initial domestic production

C is the initial domestic consumption

$P_x$ is the initial domestic price = export price

X = export volume = S - C

a = $x_1 - Z$, where $x_1$ is the new export price and Z

is the "elimination price" at which domestic consumption ceases.

When a good is exported for the first time, the measure is simply $1/2dXdP_x$. Similarly, when a good is imported for the first time, the measure is simply $1/2dMdP_m$.

72. We do not consider the amendments to the measure necessary in cases (v) and (vi) because it would take the discussion too far afield. Furthermore, the measurement of welfare changes due to the existence of market imperfections and to changes in the relative prices of untraded goods presents serious problems.1/

73. This benefit measure can take account of all commodities traded affected by the port improvement in the country. Thus, the benefits to diverted traffic, if any, will automatically be taken account of if this measure is properly applied.

1/ The interested reader may find suggestions on how to amend the measure to take account of these factors in H. G. Johnson (14).
C. Benefits in the presence of duties

74. Since the incidence formulas are different in the presence of duties, the measurement of benefits must be preceded by the estimation of export and import price changes with the help of the different incidence formulas. Changes in import and export revenues must also be taken into account. In order to compute the changes in trade duty revenue, one needs to know a) the base of the duties and b) which of the elements of unit total transport cost change as a result of port deepening. In fact, the inland costs, both abroad and at home, and the unit on-sea costs may change following the deepening of a port. As the consideration of all the possibilities would require excessive space, the reader is once again referred to Annex III for the general framework in terms of which all alternatives can be analysed.

75. As an illustration, one might consider the "cost saving" cases discussed in paragraphs 65 and 66. Suppose that only the unit domestic inland transport cost for a commodity is affected by the port project and that export and import duties are imposed on the price bases assumed earlier. Let e be the domestic country's export duty levied on the export price $x$ of a commodity, $q$ being the foreign country's import duty on the commodity levied on the c.i.f. price. Then, if the export supply curve is fully inelastic, the incidence ratio will be

$$1 - \frac{1}{(1 + \gamma)}$$

where $\gamma = (e + q + eq)$ (paragraph 22). If, say,

e = 10 per cent and $q = 10$ per cent, then the incidence ratio will be 0.82 (approx.). That is, if the unit inland (domestic) cost falls by $10.00 a ton, the export price will increase by $8.26. The increase in export duty revenue will be 0.826 cents multiplied by the total tonnage. This increase will then be part of the benefits attributable to the project.

D. Benefits to Transport-related Industries and Pricing Policy

76. Apart from the benefits, if any, in terms of increased profits to shipowners who are nationals of the country (which will be zero or negligible for most developing countries), the increased profits to other transport intermediaries should also be taken into account. These intermediaries are the sellers of inland transport services, agents handling the trade, and the port authority.

77. The pricing policies of port authorities and trucking and railway companies are thus pertinent to the generation of benefits. Suppose, for instance, that the unit port costs fall by $5.00 a ton for a particular commodity, all other costs remaining constant. If this decrease is not reflected in the charges set by the port authority, the benefits of induced trade will be foregone entirely. If no trade is induced (as in the cost saving cases discussed in paragraphs 65 and 66), any increase...
in port authority's profits will fully offset the foregone cost savings that otherwise would have accrued to the producers and consumers of the commodity. No benefits will be foregone for the society as a whole; the benefits that would have accrued to the domestic traders will in this case accrue to the port authority alone. However, if a decrease in unit port charges would induce trade increases, the society as a whole will forego a part of the benefits from the project if the port authority chooses not to lower charges by the full amount of the cost reduction. The same reasoning applies to the price policies followed by trucking and other branches of the transport industry.

78. These considerations are illustrated in Fig. 17. In Fig. 17-a the port supply function for the commodity concerned (assumed to be an important commodity) is assumed constant. In this case, the foregone benefits of not decreasing port charges are BDE (horizontally shaded). BDE will be zero if the import demand curve (MM) is perfectly inelastic. The increase in the port authority's profits with price maintenance is ABDG (shaded vertically). In Fig. 17-b the marginal cost curves are parallel before and after the project; each is an increasing linear function. If the initial price were OA, then the benefits foregone as a result of maintaining the price OA (calculated with a marginal cost pricing policy) would be BCD+CDE; BDC would have accrued to the importers, and CDE to the port authority. IBEF, of course, is the increase in profit that the port authority would have gained by price maintenance. Society will be better off the lower the price, up to the price level OG at which social welfare is maximized.

Figure 17

Benefits Foregone if Port Charges Fail to Reflect Decreased Transport Costs.

Figure 17 a

Figure 17 b
79. It follows from the above that a port authority will not be justified in maintaining the initial price after a reduction in its costs unless

(i) trade would not be induced by lower port charges, and

(ii) all the benefits from the cost reduction would accrue solely to foreign countries.

80. The comments on pricing policy above are not inconsistent with the suggestions by some economists that port pricing policy should be designed to counteract the effects of range-rate pricing practiced by conference liners. Such a policy is fully justified when the deepening of a port benefits the conference liner companies without compensation. In such a case the port authorities can recoup some of the port improvement benefits which accrue to liner companies without penalizing the shippers using the port. Shippers using other ports in the conference range will, however, be penalized.

E. Summary

81. This chapter has discussed the ways in which the benefits to the domestic producers and consumers of traded commodities may be measured. The formula that may be used is (see paragraph 57):

\[ \sum_{i=1}^{I} \left( M_i dP_m + 1/2 dM_i dP_m \right) + \sum_{j=1}^{J} \left( X_j dP_x + 1/2 dX_j dP_x \right) \]

Modifications are necessary in several cases, as indicated in paragraphs 70 and 71 of Section B.

82. The increases (decreases) in export and import duty revenues should be added (subtracted) from the total of benefits as measured above (Section C).

83. The increased profits of the port authority and other transport-related domestic industries should also be treated as part of the total benefits (Section D). In this context, a proper pricing policy followed by the port authority and also other transport industries is important. Improper pricing policy may prevent the society from obtaining the full benefits from the port investment (Section D).

1/ See Shoup, D. S., pp. 144-54, (20). These suggestions refer to charges on ships not on shippers.
V. PROJECT COSTS

84. A port deepening project merely removes a physical bottleneck to the optimization decisions of shippers. Thus, a project to widen locks or strengthen berths is identical in intention to a port deepening project. In many cases, more than one physical bottleneck should be removed simultaneously. In terms of the unit transport cost curves relating to cargo size, the removal of these bottlenecks allows a shipper to move along a curve rather than being blocked at a non-optimal point.

85. A project to increase the cargo handling efficiency in the port is not of the same nature as the physical bottlenecks. If the port becomes more efficient, the unit cost curve (C) will shift down and may also change shape, as Fig. 18 illustrates. The transport cost reduction realized from a port deepening project alone may very rapidly fall to zero if the C curve is steep. In such cases, a program to improve port efficiency as well should be considered as part of the port investment program.

Figure 18

Increase in depth with and without improved efficiency.
86. If a port deepening project alone is considered, the primary capital cost will be the dredging cost. The dredging cost will depend not only on the cubic area to be dredged, the characteristics of the bottom soil, the wave regime in the area, and other factors but also on the mobilization cost of the dredging and accessory equipment. The mobilization cost will be higher, the further the equipment must be brought to the port. The geographic location of many developing countries is such that equipment must usually be brought great distances for the initial and subsequent maintenance dredging. It may be more economical in the long run for the port to purchase the necessary equipment, bringing the additional benefit of insurance against unforeseen emergencies, such as the sinking of a ship in the entrance channel. The cost of the hydrological survey of the port area which normally precedes dredging can also amount to a substantial fraction of the total project cost, primarily because of the mobilization cost.

87. A large proportion of the so-called "secondary" benefits and costs of projects are accounted for if the inputs are valued at their shadow prices. Also, certain externalities must be accounted for if project costs and benefits are to be properly quantified. The main externalities arise when a shipper or a shipowner can affect the costs of others without being appropriately penalized (or rewarded). In other words, unit cost curves may be interdependent. For a port deepening project the following observations may be made.

(i) Since larger cargo sizes imply less frequent shipments (for a given volume of trade), the provision of additional depth will tend to reduce congestion costs for ships (less port time necessary for all ships).

(ii) If ships which were previously using other ports start using the deepened port, congestion costs in terms of port time for all ships will tend to increase. This increase may not be fully offset by reduced costs in other ports.

(iii) Since ships with high drafts are also longer, the effective number of berths may be reduced, creating new waiting costs. Thus the increase in the size capacity may reduce the effective service capacity of the port.

(iv) Similar congestion may arise in the port's transit sheds and in the inland transportation links.

1/ When valuing imported goods and services, the shadow foreign exchange rate is to be used. Apart from this, care is required when adjusting for the import duty revenue generated by the additional imports. Unless the foreign supply curve is fully elastic or the import demand curve is fully inelastic, the no-duty equilibrium price of an imported good will differ from the with-duty equilibrium price by less than the full amount of the duty. In such cases the shadow price will be underestimated if all the import duty revenue is substracted. On this point, the discussions in H. A. Adler (1) and U.N. (23) are open to misinterpretation.
An alternative to the deepening of a port is the installation or expansion of lighterage facilities. Practically all ports maintain some lighterage facilities. These facilities may be used for two purposes:

(i) to relieve congestion in the port when all available wharfage is occupied, and

(ii) to service ships which cannot be accommodated in the port because of a limitation of its size capacity (e.g. insufficient depth).

The relative importance of the latter purpose varies from port to port but normally it is less important than the first. Nevertheless, the expansion of lighterage is certainly an alternative to the provision of increased depth in the port. However, the basic disadvantage of lighterage is its tendency to involve higher cargo-handling time and higher unit cargo handling costs. Cargo has to be loaded or unloaded twice instead of once; some efficient cargo handling equipment and processes cannot be utilized. Thus it will be difficult to justify reliance on lighterage facilities as a viable economic alternative to increases in the depth of the port unless the expected number of calls by oversized vessels is sufficiently small.
VI. THE PROBLEM IN PRACTICE

A. Introduction

89. So far, the analysis has been deliberately general and may therefore appear too complex for practical use. In a sense, this is true, because time and the data limitations may well preclude the attainment of the refinements discussed in the paper. The advantage of a detailed analysis is that it makes us aware of the theoretical possibilities to bear in mind when attempting approximations.

90. Section B discusses some of the problems connected with the estimation of benefits. Section C discusses some of the problems connected with the estimation of costs. Section D discusses a few additional points regarding the variability of depth availability and draft requirements. Section E provides a summary.

B. Commodity Classes and the Measurement of Benefits

91. The general method of measuring benefits discussed in Chapter IV involves three key steps. For a particular commodity one must first estimate the decrease in the unit transport cost. Then, one must estimate the change in the appropriate trading price of the commodity. When moving from the shift in the transport supply function to the price change, one needs to know the foreign and domestic elasticities, duties, and the number of countries. Knowledge of the elasticities will also help in estimating the quantity changes. Then, the benefit formulas can be applied.

92. Since the information required to carry out each of these steps in a satisfactory way is often very difficult to find and/or requires a great deal of time to collect, one is usually forced to simplify the problem by assuming that (i) the unit transport cost functions are constant and (ii) the domestic import demand and export supply functions are totally inelastic. Furthermore, the unit transport costs usually considered are (i) ocean freight rates and (ii) unit port costs, all other transport costs being assumed to be invariant with respect to the port investment. The benefit measures thus derived represent "cost savings", due to the inelasticity assumption.

93. If these assumptions are not valid, the consequences will be as follows:

(i) The assumption of constant unit transport cost will overestimate the benefits, if this cost is in fact increasing.

(ii) The inelasticity assumption will (a) overestimate the benefits to the extent that the change in price resulting
from the investment is overestimated and (b) underestimate
the benefits to the extent that induced benefits are not
accounted for.

(iii) The estimate may be positively or negatively biased from
ignoring the unit transport costs abroad or the non-port
domestic unit transport costs, depending on the direction
of change of these costs.

94. Since it may be difficult to avoid making such simplifying
assumptions in most cases, it is advisable to use a computerized sensitivi-
ty model to test different "reasonable" values of the variables. The
appropriate incidence formulas should be incorporated in such a model.

95. The information and estimation problem will be less bothersome
the less the number of commodities affected by the port investment. Thus
the problem will be less difficult the narrower the range of commodities
flowing through the port and the more isolated from the rest of the
country it is (less traffic diversion).

96. Even for a port that services a wide range of commodities, the
problem is not necessarily complex. The number of traded commodities that
may be expected to be affected by a port deepening project will depend on
the initial depth in the port and may be quite small. For this reason it
is advisable to classify all trades using the port into two groups:

(i) those currently employing ships much smaller than the
maximum permitted size and unlikely to switch to larger
ships in future\(^1\) (for instance, coastal trades\(^2\)).

(ii) those currently employing ships close to the maximum
permitted and those currently using smaller ships but
likely to use larger ships in future.

97. The percentage breakdown in the first and second groups depends,
of course, on the initial depth alongside the berths and in the entrance
channel. For example, considering only the depth alongside berths,\(^3\)

(i) if the depth is increased to 60' from 50', very few trades
will be affected (possibilities are oil and iron ore); all
but the very large oil tankers and bulk carriers can be
accommodated;

---

\(^1\) Within the time horizon chosen for the cost benefit analysis.
\(^2\) The trades in this group may be deleted from consideration except for
measurable external effects as indicated in paragraph 75 above.
\(^3\) As a "thumb" rule large vessels require about 3' clearance from the
ocean bottom alongside berths. For small vessels the clearance needed
will be less, say 2.5'. The clearance needed depends, of course, on
the soil at the bottom. The clearance needed near the entrance channel
is, however, larger because of wave action and squat. The squat
itself may be 1' to 2' or more.
(ii) if the depth is increased from 40' to 50', only those trades which can be expected to use ship sizes between say, 45,000 d.w.t. and 60,000 d.w.t., will be affected; again, the number of such trades is likely to be very small;

(iii) if the depth is increased from say, 15' to 20', then only the trades which employ vessel sizes ranging from 1200 d.w.t. to 2500 d.w.t. (for freighter) will be affected, that is, only large coastal vessels and small freighters;

(iv) for ports which are designed to serve general purpose liner vessels, the depth requirement will depend on the depths on other ports along the liner routes. Generally speaking a depth of about 35' alongside berths is likely to be quite adequate for liner vessels. Such a depth will also be adequate for most container vessels. Insufficient depth will mean reliance on lighterage facilities for servicing liner vessels.

76. Thus, since the problem of estimating benefits in any practical case will depend on the number of trades affected by the port investment it appears that:

(i) With an initial depth alongside berths of say, 35' to 40', the number of commodities that need to be considered will be very small (say, two or three) and are likely to be dry bulk goods.\textsuperscript{1}

(ii) If the initial depth is low (say 15' or 20' or less) and additional depth will involve switching from lighterage to service alongside berths of break bulk ships, the number of trades affected may be quite large. Note, however, that even if the port is currently servicing ships alongside berths, additional depth may avoid the necessity of lighterage in the future.

\textsuperscript{1} The largest vessels are used in the oil trade. One need not, however, be overly concerned with this trade for the following reasons: (i) oil ports often do not service other trades and are owned and operated by oil companies: (ii) the investment problem in the oil trade is that of providing oil storage and/or refining facilities near the port, because oil tankers need not be accommodated alongside berths but can be serviced by submarine pipelines. The same applies to other bulk liquid cargoes.
For ports which service a wide range of commodities, the procedure is:

(i) eliminate the irrelevant trades (paragraph 82),

(ii) identify the major items in the remaining trades,

(iii) group the remaining relevant trades which are individually not important into a small number of groups.

Since most developing countries have a limited range of exports (mostly primary commodities such as cotton, sugar, coffee, iron ore, copper, etc.), category (iii) in the previous paragraph is likely to be a small fraction of total export trade flowing through the port. On the other hand, the same category for imports is likely to be quite large. Most of the errors of estimation will probably arise in the import category.

The grouping or the classification of commodities most convenient for estimating transport cost functions is not necessarily the most convenient grouping for calculating the incidence formulas, estimating trade elasticities, or making future projections of traffic. For instance, the classification system used in port statistics is not always convenient for making projections. One must either arbitrarily avoid the problem or define a conversion rule from one classification system to another. And though no available commodity classification (SITC, etc.) is perfect for estimating trade elasticities, it is still possible to arrive at meaningful results.

C. Unit Transport Costs

There is very little that can be generally said regarding the quantitative behavior of unit on-sea costs for individual commodities. No detailed empirical study has been undertaken regarding the behavior of tramp market rates in relation to vessel size, except for oil. For tankships carrying oil there is a significant negative relationship between freight rates and vessel size. Vessel size is, in fact, the single most important variable in explaining time charter rates. The greater the size, the lower the rate because some of the economies of size are passed on to charterers. Presumably, the same proposition will apply to time charters for other commodities as well. But it is preferable to make quantitative estimates regarding freight rates for each port separately than to rely on the "rules of thumb" sometimes provided. However, if certain assumptions

1/ See e.g., the excellent article by Houthakker, H. S. and Magee, S. P. (12).

2/ From the point of view of economic theory, a group of commodities can be treated as a single commodity only if the relative prices within the group remain constant. See Hicks, J. R. (13).

3/ Z. S. Zannetos (25, 26).

C. F. Cufley (3).
can be made regarding the competitive character of the tramp market, one can presume that freight rates will reflect vessel operating costs.\(^1\)

103. Changes in liner rates are also notoriously difficult to predict. There is a wide range of commodities for which liner companies can set discriminatory freight rates without worrying about potential competition from tramps. There has been speculation in the literature about the determinants of the price policies followed by conferences. These theories have not yet been empirically tested and verified to yield quantitative rules for estimating the magnitude of changes in the liner rates, even though they may yield qualitative results.

104. Finally, the unit inland costs abroad are very difficult to account for. It is therefore advisable to use f.o.b. prices for imports from abroad and c.i.f. prices for exports. Unit domestic inland costs are not difficult to measure for some commodities (e.g., the primary commodities exported). The domestic origins and destinations as well as trucking and railway rates can be obtained for some commodities. However, for the majority of commodities, the estimation of unit domestic inland costs other than port costs will be difficult.

D. Two Additional Points

105. It should be clearly understood that the depths alongside berths and in the entrance channel may vary substantially from day to day,\(^2\) and available depth figures are thus rather ambiguous. If the depth available is at least 30' throughout the year, then the available depth may be significantly higher during certain times of the day or the year. While this variability in the available depth does not significantly affect the analysis of the cost functions, it is important to follow a definition consistently (whether the "available depth" is defined as the maximum, minimum or the average depth).

106. Also, if a port holds the terminal position in the routes of the trades using it, ships will tend to be fully laden or nearly so. On the other hand, if it is at the midpoint of a liner route with many calls, this will not be so, as far as liners are concerned. The depth requirement depends, of course, on the load factors of the ships using it. Thus if vessel sizes employed by liner companies along a route increase, a particular port will have less concern with depth if the vessels are expected to arrive and leave with excess capacity. The excess capacity required to reduce the required depth by 1' varies a great deal. For Liberty type ships it may be about 600-700 tons and for large bulk carriers it may be more than a 1000 tons or even 2000 tons.

\(^1\)/ See Annex I for a further discussion of this point.
\(^2\)/ See Annex II, Draft and Vessel Size.
E. Summary

107. This chapter has commented on some of the difficulties of measuring benefits in actual cases. Other things being equal, the difficulties will be greater, the greater the number of traded commodities affected by a port deepening project. The number of commodities affected will depend on the nature of the port, its relation to the economy, and the initial depth of water in the port. In many cases, the number affected is likely to be small and the measurement problem simple. In other cases there may not be any feasible alternative to making simplifying assumptions. The biases implicit in some of the common simplifications have been indicated (paragraph 81). In paragraphs 84 and 85 a procedure is outlined to eliminate irrelevant factors. For most developing countries, the estimation errors are likely to be greater for imports than for exports (paragraph 88).

108. Concerning the measurement of changes in unit transport costs, the difficulties stem from the lack of an empirically tested and verified theory of liner rates. The same is true to a lesser extent for tramp market rates. Also, the estimation of changes in unit inland costs, except unit port costs, may be difficult for many commodities.

109. As long as a consistent definition of the available depth is used, its daily or seasonal variations do not affect the analysis. And even if liner vessel sizes along a route increase, any single port on the route will be less concerned with its depth adequacy if the larger vessels are expected to arrive and leave with unutilized capacity.
Vessel Operating Costs

1. Knowledge of the variation in freight rates \(^1\) by vessel size is not in itself sufficient for a proper evaluation of a port deepening project. Nevertheless, freight rate variations may be important; it is therefore regrettable that no quantitative guidelines are readily available for estimating such variation. The variation in average costs to shipowners by vessel size is often used as a substitute. The justification given for such a substitution is that, since the tramp market is competitive, a 10 percent reduction in average cost of sea-carriage implies a 10 percent reduction in the freight rate. Thus, when a shipper wishes to charter a tramp to transport his cargo from port A to B, competition among shipowners will ensure that the shipowner who can transport the cargo at the least cost per ton will be forced to give a proportionate reduction in the freight rate. This will indeed be the case if there are sufficient number of shipowners who can transport the cargo at the same cost per ton. Otherwise, the most efficient shipowner will be able to make a "super-normal" profit, i.e., will reduce the freight rate just enough to prevent his nearest rival from getting the contract.

2. Whether or not the foregoing assumption is justified, it is still useful to study the variation in cost per ton of sea-carriage; at least it might give an indication of the variation in freight rates to be expected. Two general propositions can be made regarding the variation in sea transport cost per ton by vessel size.

(i) Given the level of cargo (in ton miles) to be transported, it is cheaper to use big vessels rather than small ones, provided the time taken to transport the cargo (in ton-miles) is the same for both and that the ships can be operated with full loads.

(ii) The relative advantage of a big ship is greater, the greater the amount of on-sea time as a percentage of total voyage time.

3. The first proposition follows from the existence of economies of scale in shipping. Thanks to the economies of scale and technological progress in the shipbuilding industry, the larger the vessel size, the less the building cost per ton of cargo carrying capacity. Operating costs can be split into two parts: (i) non-voyage, i.e., the operating

\(^1\) Freight rates exclude port charges and cargo handling costs.
costs which are fixed for a period of time regardless of the voyages undertaken (such as crew's wages, insurance, etc.) and (ii) voyage, i.e., costs which depend on the voyages undertaken (mainly bunker costs). Economies are possible in both these categories if the vessels are operated with full loads. If the vessels do not operate with full loads, the economies of scale may be lost entirely; in fact, diseconomies of scale may be realized.

4. An example may be helpful to illustrate the argument. Suppose a shipowner has accepted an offer to transport 40,000 tons of cargo between two ports (20,000 tons each way). He can do so in, say, three ways: (i) use two 10,000 tonners, (ii) use a 20,000 tonner, or (iii) use a 40,000 tonner. In each case, the total time taken will be 340 days, half on sea and half in port. The two 10,000 tonners and the 20,000 tonner will voyage with full loads and the 40,000 tonner will voyage half empty. The following table gives the cost data on the basis of which the shipowner will make his choice. The influence of economies of scale is seen in the less than proportionate increases in capital costs (for simplicity, the building cost) and in operating costs as the vessel size increases. The best choice will be the 20,000 tonner. The 40,000 tonner, despite the unrealistically favorable assumption that its bunker cost is the same as that for the 20,000 tonner, will be the worst choice. This is due to the fact that the 40,000 tonner will be voyaging half-empty. Note that the supply of tonnage over the 340-day period just matches the demand with the first two alternatives but is twice the demand in the case of the 40,000 tonner.

5. The assumption that total in-port time is the same is, however, very unrealistic. It implies that the cargo handling rate is doubled when the 20,000 tonner calls on the ports. That is, a 10,000 tonner loads/unloads 20,000 tons in 170 days while the 20,000 tonner loads/unloads 40,000 tons during the same period. If this is not the case, then the relative advantage of the 20,000 tonner will be greater the greater the amount of time on sea (the longer the voyage) as a percentage of total voyage time. This can easily be seen by comparing two alternative voyages, one with 200 days on sea and the other with only 50 days on sea. If the cargo handling rate in both ports is 1,000 tons per day for both the 10,000 tonner and the 20,000 tonner, then the 10,000 tonner will spend 20 days in port and the 20,000 tonner will spend 40 days in port during the round trip voyage. Thus the total voyage time will be 20 days longer for the 20,000 tonner. The capital charges, non-voyage costs, and voyage costs are assumed to be the same as before (Table 1) on a 340-day period basis. The costs given in Table 1 are shown on a daily basis in Table 2.

1/ In terms of cargo carrying capacity.
2/ The 40,000 tonner is left out of consideration as it will spend the same time in port as the 20,000 tonner.
### Table 1

**Cost Alternatives**

*(Total Voyage Time: 340 days)*

<table>
<thead>
<tr>
<th>Vessel size</th>
<th>Capital cost</th>
<th>Non-voyage costs</th>
<th>On-sea (170 days)</th>
<th>In-port (170 days)</th>
<th>Total (10% of charges)</th>
<th>Cost Carried</th>
<th>Cargo Average (tons) (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7) (8) (9)</td>
</tr>
<tr>
<td>1. 10,000 tons (each ship)</td>
<td>1,000</td>
<td>100</td>
<td>34</td>
<td>17</td>
<td>51</td>
<td>100</td>
<td>251</td>
</tr>
<tr>
<td>2. 10,000 tons (both ships)</td>
<td>2,000</td>
<td>200</td>
<td>68</td>
<td>34</td>
<td>102</td>
<td>200</td>
<td>502</td>
</tr>
<tr>
<td>3. 20,000 tons</td>
<td>1,800</td>
<td>160</td>
<td>60</td>
<td>30</td>
<td>90</td>
<td>180</td>
<td>430</td>
</tr>
<tr>
<td>4. 40,000 tons</td>
<td>3,000</td>
<td>250</td>
<td>60</td>
<td>30</td>
<td>90</td>
<td>300</td>
<td>740</td>
</tr>
</tbody>
</table>

*(in $ '000)*
<table>
<thead>
<tr>
<th>Vessel size</th>
<th>Interest</th>
<th>Depreciation</th>
<th>Non-voyage costs (in dollars)</th>
<th>Voyage costs</th>
<th>Total costs plus</th>
<th>Total fixed costs plus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 10,000 tonner</td>
<td>294.11</td>
<td>294.11</td>
<td>200.00</td>
<td>100.00</td>
<td>588.22</td>
<td>788.22</td>
</tr>
<tr>
<td>2. 20,000 tonner</td>
<td>529.41</td>
<td>470.58</td>
<td>352.94</td>
<td>176.47</td>
<td>999.99</td>
<td>1,352.93</td>
</tr>
</tbody>
</table>
6. The costs for each ship for the two voyages are shown below in Table 3. The costs are based on the daily costs given in Table 2.

Table 3

<table>
<thead>
<tr>
<th>Vessel size (tons)</th>
<th>Time on sea (days)</th>
<th>Total Port Costs (dollars)</th>
<th>Voyage Costs (tons)</th>
<th>Total Port Costs (dollars)</th>
<th>Cargo Carried (tons)</th>
<th>Average Cost (dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Time on sea: 50 days</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. 10,000 (each ship)</td>
<td>20</td>
<td>61,175</td>
<td>10,000</td>
<td>2,000</td>
<td>12,000</td>
<td>53,175</td>
</tr>
<tr>
<td>2. 10,000 (both ships)</td>
<td>40</td>
<td>82,350</td>
<td>20,000</td>
<td>4,000</td>
<td>24,000</td>
<td>106,350</td>
</tr>
<tr>
<td>3. 20,000 (each ship)</td>
<td>40</td>
<td>89,999</td>
<td>17,647</td>
<td>7,059</td>
<td>24,706</td>
<td>114,705</td>
</tr>
<tr>
<td>B. Time on sea: 200 days</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. 10,000 (each ship)</td>
<td>20</td>
<td>129,408</td>
<td>40,000</td>
<td>2,000</td>
<td>42,000</td>
<td>171,408</td>
</tr>
<tr>
<td>2. 10,000 (both ships)</td>
<td>40</td>
<td>258,816</td>
<td>80,000</td>
<td>4,000</td>
<td>84,000</td>
<td>342,816</td>
</tr>
<tr>
<td>3. 20,000 (each ship)</td>
<td>40</td>
<td>239,998</td>
<td>70,588</td>
<td>7,059</td>
<td>77,647</td>
<td>317,645</td>
</tr>
</tbody>
</table>
7. The relationship shown in Table 3 holds as well if the cargo handling rate is higher than 1,000 tons a day, for any given time spent on sea. Thus, the longer the voyage and/or the more efficient the ports of call, the higher the reduction in sea transport cost that can be realized by increasing the vessel size. A diagrammatic representation makes this even clearer. The total cost of a voyage for a ship can be calculated as follows:

\[
\text{Total cost} = (\text{Time on Sea}) \times (\text{Total daily fixed costs} + \text{daily costs on sea}) + (\text{Time in port}) \times (\text{Total daily fixed costs} + \text{daily costs in port}).
\]

or, dividing through by the volume of cargo carried,

\[
\text{Average total cost} = (\text{Time on Sea}) \times (\text{Daily fixed costs per ton} + \text{daily costs on sea per ton}) + (\text{Time in port}) \times (\text{Daily fixed costs per ton} + \text{daily costs in port per ton}).
\]

Using the figures in columns 6 and 7 of Table 2, we find:

Average total cost with 10,000 tonner = (Time in Sea) x (3.4 cents) + (20 days) x (3.4 cents). This is shown by the curve A in Figure 1.

And:

Average total cost with 20,000 tonner = (Time in Sea) x (3.4 cents) + (40 days)x (2.9 cents). (See curve B).

Curve A is steeper than B. They cross when the on-sea time is 96 days. That is, if the on-sea time is 96 days, then cost equality is attained. As the time on sea is increased beyond 96 days, the 20,000 tonner becomes more and more advantageous.

Using the same equations, Figure 2 shows that as the cargo handling rate improves (hence, the in-port time decreases), the 20,000 tonner becomes more advantageous, cost equality being reached with a cargo handling rate of 2,000 tons per day in Figure 2 (average for the two ports). Beyond the rate of 2,000 tons per day, the 20,000 tonner's superiority increases.

8. Some data on costs of employing vessels of different sizes from the shipowners' point of view are given in Tables 4, 5 and 6 and in Chart I. The "Daily total cost" in Table 4 takes into account the daily depreciation and interest charges and the daily operating costs, as well as the daily bunker costs at sea or in port. Conceptually, it corresponds to the columns 6 (for costs on sea) and 7 (for costs in port) of Table 2. In the last two columns of Table 4, the daily total costs are divided by the cargo carrying capacity. Hence these unit costs are only relevant when the vessels operate with full loads.
Figure 1

Average cost of transporting 40,000 tons, $  

Given cargo handling rate of 1,000 tons per day.

Figure 2

Average cost of transporting 40,000 tons, $  

Given on-sea time of 50 days.  

Time in port, for 10,000 tonner, in days, per voyage.  
(Cargo handling rate per day in brackets)
<table>
<thead>
<tr>
<th>Kind of Ships</th>
<th>Deadweight Tonnage (1)</th>
<th>Daily Total Cost At Sea (2)</th>
<th>Daily Total Cost In Port (3)</th>
<th>Daily Unit Cost per 1,000 ton of Cargo At Sea (4)</th>
<th>Daily Unit Cost per 1,000 ton of Cargo In Port (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tramp</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7,445</td>
<td>1,118</td>
<td></td>
<td>907</td>
<td>166.68</td>
<td>131.71</td>
</tr>
<tr>
<td>13,600</td>
<td>1,596</td>
<td></td>
<td>1,489</td>
<td>145.54</td>
<td>118.38</td>
</tr>
<tr>
<td>16,070</td>
<td>2,072</td>
<td></td>
<td>1,610</td>
<td>136.00</td>
<td>104.30</td>
</tr>
<tr>
<td>Ore Carrier</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15,341</td>
<td>2,550</td>
<td></td>
<td>2,150</td>
<td>143.19</td>
<td>111.82</td>
</tr>
<tr>
<td>18,555</td>
<td>2,658</td>
<td></td>
<td>1,756</td>
<td>121.18</td>
<td>90.29</td>
</tr>
<tr>
<td>21,588</td>
<td>2,958</td>
<td></td>
<td>1,916</td>
<td>98.70</td>
<td>78.12</td>
</tr>
<tr>
<td>26,707</td>
<td>3,500</td>
<td></td>
<td>2,308</td>
<td>76.05</td>
<td>56.98</td>
</tr>
<tr>
<td>34,997</td>
<td>3,792</td>
<td></td>
<td>3,181</td>
<td>71.01</td>
<td>51.38</td>
</tr>
<tr>
<td>48,564</td>
<td>4,194</td>
<td></td>
<td>3,665</td>
<td>67.80</td>
<td>51.18</td>
</tr>
<tr>
<td>A. Westinform Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53,285</td>
<td>3,792</td>
<td></td>
<td>3,181</td>
<td>71.01</td>
<td>51.38</td>
</tr>
<tr>
<td>66,662</td>
<td>4,194</td>
<td></td>
<td>3,665</td>
<td>67.80</td>
<td>51.18</td>
</tr>
<tr>
<td>80,815</td>
<td>4,588</td>
<td></td>
<td>4,163</td>
<td>64.68</td>
<td>50.34</td>
</tr>
<tr>
<td>89,674</td>
<td>4,980</td>
<td></td>
<td>4,765</td>
<td>62.30</td>
<td>48.18</td>
</tr>
<tr>
<td>B. Department of the Army Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ore Carrier</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20,000</td>
<td>2,880</td>
<td></td>
<td>2,474</td>
<td>126.52</td>
<td>110.52</td>
</tr>
<tr>
<td>25,000</td>
<td>2,920</td>
<td></td>
<td>2,592</td>
<td>123.65</td>
<td>117.79</td>
</tr>
<tr>
<td>30,000</td>
<td>3,008</td>
<td></td>
<td>2,690</td>
<td>120.00</td>
<td>107.43</td>
</tr>
<tr>
<td>35,000</td>
<td>3,096</td>
<td></td>
<td>2,800</td>
<td>117.17</td>
<td>102.61</td>
</tr>
<tr>
<td>40,000</td>
<td>3,184</td>
<td></td>
<td>2,916</td>
<td>114.27</td>
<td>98.70</td>
</tr>
<tr>
<td>45,000</td>
<td>3,288</td>
<td></td>
<td>3,032</td>
<td>111.43</td>
<td>95.80</td>
</tr>
<tr>
<td>50,000</td>
<td>3,392</td>
<td></td>
<td>3,150</td>
<td>108.60</td>
<td>93.49</td>
</tr>
<tr>
<td>55,000</td>
<td>3,496</td>
<td></td>
<td>3,276</td>
<td>105.77</td>
<td>91.00</td>
</tr>
<tr>
<td>60,000</td>
<td>3,600</td>
<td></td>
<td>3,392</td>
<td>102.93</td>
<td>88.54</td>
</tr>
<tr>
<td>70,000</td>
<td>3,708</td>
<td></td>
<td>3,524</td>
<td>100.17</td>
<td>87.00</td>
</tr>
<tr>
<td>80,000</td>
<td>3,888</td>
<td></td>
<td>3,656</td>
<td>97.32</td>
<td>85.11</td>
</tr>
<tr>
<td>C. Civil Works Data</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bulk Carrier</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25,000</td>
<td>3,384</td>
<td></td>
<td>2,760</td>
<td>116.33</td>
<td>112.35</td>
</tr>
<tr>
<td>30,000</td>
<td>3,624</td>
<td></td>
<td>2,880</td>
<td>130.59</td>
<td>126.78</td>
</tr>
<tr>
<td>35,000</td>
<td>3,792</td>
<td></td>
<td>2,976</td>
<td>117.13</td>
<td>113.22</td>
</tr>
<tr>
<td>40,000</td>
<td>3,984</td>
<td></td>
<td>3,120</td>
<td>107.67</td>
<td>102.30</td>
</tr>
<tr>
<td>45,000</td>
<td>4,176</td>
<td></td>
<td>3,240</td>
<td>100.32</td>
<td>94.64</td>
</tr>
<tr>
<td>50,000</td>
<td>4,372</td>
<td></td>
<td>3,364</td>
<td>93.92</td>
<td>88.13</td>
</tr>
<tr>
<td>55,000</td>
<td>4,568</td>
<td></td>
<td>3,492</td>
<td>88.73</td>
<td>83.04</td>
</tr>
<tr>
<td>60,000</td>
<td>4,664</td>
<td></td>
<td>3,728</td>
<td>83.14</td>
<td>77.84</td>
</tr>
<tr>
<td>70,000</td>
<td>4,864</td>
<td></td>
<td>3,960</td>
<td>78.61</td>
<td>74.36</td>
</tr>
<tr>
<td>80,000</td>
<td>5,060</td>
<td></td>
<td>4,192</td>
<td>74.36</td>
<td>70.89</td>
</tr>
<tr>
<td>90,000</td>
<td>5,259</td>
<td></td>
<td>4,428</td>
<td>70.89</td>
<td>67.36</td>
</tr>
<tr>
<td>100,000</td>
<td>5,454</td>
<td></td>
<td>4,664</td>
<td>67.36</td>
<td>64.36</td>
</tr>
</tbody>
</table>

Note: Westinform also provides cost data on dry bulk carriers. These are, however, not significantly different from the cost data for ore carriers (for comparable sizes).

Table 4
Ship Size and Unit Costs (dollars)

Board of Engineers for Rivers and Harbors, "Preliminary Economic Analysis for Further Deepening of Delaware River, Delaware, New Jersey and Pennsylvania", April 1967. Table 12, indicating operating costs of non-American ore carriers. In calculating columns (4) and (5), the approximate average tonnage of "DWT" cargo is assumed to be 92% of deadweight tonnage of ships since it is based on Westinform data.

### Table 5 - Structure of Unit Costs (on the sea)

<table>
<thead>
<tr>
<th>Kind of Ships</th>
<th>Deadweight Tonnage</th>
<th>Capital Cost (%)</th>
<th>Operating Cost (%)</th>
<th>Fuel Cost (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Westinform Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tramp</td>
<td>7,445</td>
<td>33.9</td>
<td>42.7</td>
<td>23.3</td>
</tr>
<tr>
<td></td>
<td>13,600</td>
<td>38.6</td>
<td>41.2</td>
<td>20.2</td>
</tr>
<tr>
<td></td>
<td>16,070</td>
<td>36.7</td>
<td>40.9</td>
<td>22.4</td>
</tr>
<tr>
<td>Ore Carriers</td>
<td>15,341</td>
<td>35.3</td>
<td>43.1</td>
<td>21.6</td>
</tr>
<tr>
<td></td>
<td>18,655</td>
<td>34.9</td>
<td>42.4</td>
<td>22.7</td>
</tr>
<tr>
<td></td>
<td>21,588</td>
<td>35.4</td>
<td>39.5</td>
<td>25.1</td>
</tr>
<tr>
<td></td>
<td>26,707</td>
<td>37.7</td>
<td>39.1</td>
<td>25.4</td>
</tr>
<tr>
<td></td>
<td>34,997</td>
<td>37.8</td>
<td>38.3</td>
<td>22.6</td>
</tr>
<tr>
<td></td>
<td>48,564</td>
<td>35.6</td>
<td>39.3</td>
<td>25.1</td>
</tr>
<tr>
<td></td>
<td>53,385</td>
<td>37.0</td>
<td>35.4</td>
<td>20.1</td>
</tr>
<tr>
<td></td>
<td>66,662</td>
<td>36.0</td>
<td>35.4</td>
<td>28.7</td>
</tr>
<tr>
<td></td>
<td>80,815</td>
<td>37.3</td>
<td>34.1</td>
<td>28.3</td>
</tr>
<tr>
<td></td>
<td>89,674</td>
<td>37.1</td>
<td>31.7</td>
<td>31.1</td>
</tr>
<tr>
<td><strong>B. Civil Works, U.S. Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bulk Cargo</td>
<td>25,000</td>
<td>37.6</td>
<td>33.8</td>
<td>23.6</td>
</tr>
<tr>
<td></td>
<td>30,000</td>
<td>37.2</td>
<td>37.9</td>
<td>24.0</td>
</tr>
<tr>
<td></td>
<td>35,000</td>
<td>37.1</td>
<td>37.0</td>
<td>25.3</td>
</tr>
<tr>
<td></td>
<td>40,000</td>
<td>37.4</td>
<td>36.0</td>
<td>26.6</td>
</tr>
<tr>
<td></td>
<td>45,000</td>
<td>37.5</td>
<td>35.4</td>
<td>27.1</td>
</tr>
<tr>
<td></td>
<td>50,000</td>
<td>37.7</td>
<td>34.7</td>
<td>27.6</td>
</tr>
<tr>
<td></td>
<td>55,000</td>
<td>37.8</td>
<td>34.1</td>
<td>28.1</td>
</tr>
<tr>
<td></td>
<td>60,000</td>
<td>37.9</td>
<td>33.6</td>
<td>28.5</td>
</tr>
<tr>
<td></td>
<td>70,000</td>
<td>38.4</td>
<td>32.8</td>
<td>28.3</td>
</tr>
<tr>
<td></td>
<td>80,000</td>
<td>39.0</td>
<td>31.8</td>
<td>29.1</td>
</tr>
<tr>
<td></td>
<td>90,000</td>
<td>39.4</td>
<td>31.1</td>
<td>29.4</td>
</tr>
</tbody>
</table>
Table 6

Comparative Costs of Westinform Standard Liner Types
Expressed as Cost Per Measurement Ton Per Day

<table>
<thead>
<tr>
<th>Type</th>
<th>Gross Tons</th>
<th>Cargo (measurement tons)</th>
<th>Cost Per Ton Per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>In Port</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(U.S. Dollars)</td>
</tr>
<tr>
<td>General Cargo Liners</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GL 1</td>
<td>3,775</td>
<td>3,812</td>
<td>.4111</td>
</tr>
<tr>
<td>GL 2</td>
<td>5,700</td>
<td>9,677</td>
<td>.2147</td>
</tr>
<tr>
<td>GL 3</td>
<td>9,469</td>
<td>13,401</td>
<td>.1707</td>
</tr>
<tr>
<td>GL 4</td>
<td>11,157</td>
<td>15,388</td>
<td>.1831</td>
</tr>
<tr>
<td>GL 5</td>
<td>13,057</td>
<td>15,585</td>
<td>.2020</td>
</tr>
<tr>
<td>Reefer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RL 1</td>
<td>3,129</td>
<td>3,241</td>
<td>.5819</td>
</tr>
<tr>
<td>RL 2</td>
<td>6,790</td>
<td>9,677</td>
<td>.4911</td>
</tr>
<tr>
<td>RL 3</td>
<td>9,634</td>
<td>9,331</td>
<td>.3713</td>
</tr>
<tr>
<td>RL 4</td>
<td>12,591</td>
<td>14,309</td>
<td>.2523</td>
</tr>
<tr>
<td>General Cargo Liner with Reefer Capacity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RGL</td>
<td>11,455</td>
<td>13,260</td>
<td>.2456</td>
</tr>
</tbody>
</table>

Types: Liners, August 1968.
Chart I: Ship Sizes and the Unit Total Cost on the Sea

(1) Ore Carriers (Westinfram)
(2) Ore Carriers (US Department of the Arny)
(3) Bulk Carriers (Civil Works)

Source: See Table 4.
Draft and Vessel Size

1. Draft is defined as the vertical distance from the lowest point of the underside of the keel to the line formed by the water in which the ship floats. The draft usually noted in shipping statistics and the one used here is the draft on summer marks, that is, the draft needed to reach the summer load waterline. A ship cannot be loaded beyond the point at which the waterline reaches the summer load line. Deeper draft is permitted in the tropical zones. The permissible draft is less for Winter Zones and Winter North Atlantic. The fresh water draft is higher than the draft in salt-water.2

2. The available depth of water in a port may vary daily and seasonally. The available depth depends on factors such as the tidal cycle, effect of wind, wave regime, siltation, and rainfall. In other words, a ship with a draft just below the maximum permitted (on an average) may not be able to enter the port at all times of the day and the year.

3. The drafts given in the Tables 7, 8 and 9 and the Charts 2 and 3 below are the drafts which the ships will require when fully laden (up the summer marks). If the ships are not fully laden then the draft required will be lower. For Liberty-type ships, the required draft decreases by about 1 foot per 600 tons. For bulk carriers, the required draft decreases by about 1 foot per 1000-1200 tons.

4. Chart 2 indicates the general relationship between vessel size and draft. The draft requirements tend to increase with vessel size but less than proportionately. It is apparent, however, from Tables 7, 8 and 9 that larger vessels can have lower draft than smaller vessels, for reasons:

   (i) Modern ships tend to economize on draft to keep the operational flexibility as high as possible. That is, if a 20,000 DWT requires less draft than a 15,000 DWT, it may be because the 20,000 DWT ship is much younger.

   (ii) The statistics cover ships of different designs. Ships are often designed for specific cargoes or cargo type.

### Table 7: Drafts and Ship Sizes

| Draft (feet) | Kinds of ships | All Freighters | Under 2,000 | 2,001-3,999 | 4,000-6,999 | 7,000-8,999 | 9,000-10,000 | 10,001-11,999 | 12,000-13,999 | 14,000-15,000 | Over 15,000 | All Bulk Carriers | Under 10,000 | 10,001-19,999 | 20,000-29,999 | 30,000-39,999 | 40,000-49,999 | 50,000-59,999 | 60,000-69,999 | 70,000-79,999 | Over 79,999 |
|-------------|----------------|----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-----------|-----------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 15          | 5%             | 65%            | 1%          | 2%          | 2%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%        | 10%               | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 20          | 25%            | 99%            | 87%         | 15%         | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%        | 10%               | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 25          | 48%            | 100%           | 99%         | 95%         | 60%         | 16%         | 1%          | 1%          | 1%          | 1%          | 1%        | 10%               | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 30          | 96%            | 100%           | 100%        | 99%         | 99%         | 99%         | 99%         | 99%         | 99%         | 99%         | 99%        | 10%               | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 35          | 100%           |                | 100%        | 100%        | 100%        | 100%        | 100%        | 100%        | 100%        | 100%        | 100%       | 99%                 | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 40          |                |                |             |             |             |             |             |             |             |             |             | 99%                  |             | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 45          |                |                |             |             |             |             |             |             |             |             |             | 100%                 |             | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 50          |                |                |             |             |             |             |             |             |             |             |             | 100%                 |             | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |
| 55          |                |                |             |             |             |             |             |             |             |             |             | 100%                 |             |             | 100%        | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          | 1%          |

Source: Derived from Tables 8 and 9.
<table>
<thead>
<tr>
<th>Draft Feet</th>
<th>All Sizes</th>
<th>Under 2,000 DWT</th>
<th>2,000-3,999 DWT</th>
<th>4,000-5,999 DWT</th>
<th>6,000-8,999 DWT</th>
<th>9,000-9,999 DWT</th>
<th>10,000-10,999 DWT</th>
<th>11,000-11,999 DWT</th>
<th>12,000-12,999 DWT</th>
<th>15,000 &amp; Over</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Number</td>
<td>Number (%)</td>
<td>Number (%)</td>
<td>Number (%)</td>
<td>Number (%)</td>
<td>Number (%)</td>
<td>Number (%)</td>
<td>Number (%)</td>
<td>Number (%)</td>
<td>Number (%)</td>
</tr>
<tr>
<td>1</td>
<td>598</td>
<td>56 (56)</td>
<td>79 (14)</td>
<td>35 (2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>863</td>
<td>56 (56)</td>
<td>193 (17)</td>
<td>44 (2)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>1,793</td>
<td>102 (102)</td>
<td>390 (23)</td>
<td>50 (3)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>2,197</td>
<td>121 (121)</td>
<td>1,566 (73)</td>
<td>112 (6)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>2,000</td>
<td>25 (25)</td>
<td>1,850 (87)</td>
<td>313 (15)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>1,796</td>
<td>30 (30)</td>
<td>2,066 (95)</td>
<td>766 (37)</td>
<td>14 (0)</td>
<td>-</td>
<td>2 (0)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>1,036</td>
<td>35 (35)</td>
<td>1,073 (98)</td>
<td>1,189 (59)</td>
<td>31 (3)</td>
<td>2 (0)</td>
<td>-</td>
<td>2 (0)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>1,533</td>
<td>39 (39)</td>
<td>2,090 (98)</td>
<td>1,596 (79)</td>
<td>93 (9)</td>
<td>3 (0)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>5,007</td>
<td>23 (23)</td>
<td>2,102 (99)</td>
<td>1,809 (89)</td>
<td>334 (16)</td>
<td>10 (0)</td>
<td>3 (0)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>5,581</td>
<td>65 (65)</td>
<td>2,106 (99)</td>
<td>1,933 (95)</td>
<td>627 (60)</td>
<td>134 (16)</td>
<td>3 (1)</td>
<td>19 (1)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>6,489</td>
<td>56 (56)</td>
<td>2,148 (99)</td>
<td>1,995 (95)</td>
<td>894 (56)</td>
<td>157 (53)</td>
<td>83 (8)</td>
<td>5 (2)</td>
<td>5 (2)</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>7,366</td>
<td>53 (53)</td>
<td>2,106 (99)</td>
<td>2,014 (99)</td>
<td>966 (94)</td>
<td>612 (71)</td>
<td>670 (65)</td>
<td>196 (14)</td>
<td>13 (5)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>9,110</td>
<td>81 (81)</td>
<td>2,100 (100)</td>
<td>2,050 (100)</td>
<td>1,021 (99)</td>
<td>806 (89)</td>
<td>2,176 (83)</td>
<td>42 (10)</td>
<td>21 (9)</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>10,415</td>
<td>90 (90)</td>
<td>2,029 (100)</td>
<td>1,030 (99)</td>
<td>867 (98)</td>
<td>2,587 (98)</td>
<td>892 (65)</td>
<td>128 (74)</td>
<td>10 (10)</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>11,127</td>
<td>96 (96)</td>
<td>2,030 (100)</td>
<td>1,032 (100)</td>
<td>862 (100)</td>
<td>2,630 (100)</td>
<td>1,896 (91)</td>
<td>317 (66)</td>
<td>97 (11)</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>11,646</td>
<td>99 (99)</td>
<td>1,034 (100)</td>
<td>863 (100)</td>
<td>2,632 (100)</td>
<td>1,393 (98)</td>
<td>336 (82)</td>
<td>187 (89)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>11,688</td>
<td>99.6 (99.6)</td>
<td>1,034 (100)</td>
<td>864 (100)</td>
<td>2,632 (100)</td>
<td>1,419 (100)</td>
<td>551 (97)</td>
<td>213 (90)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>11,601</td>
<td>99.9 (99.9)</td>
<td>1,035 (100)</td>
<td>861 (100)</td>
<td>2,632 (100)</td>
<td>1,419 (100)</td>
<td>551 (97)</td>
<td>213 (90)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>11,609</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>11,610</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>11,610</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>11,612</td>
<td>100 (100)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Chart II: Ship Size and Draft

Ore Carrier:  
Tramp:  

Chart III: Drafts and Cumulative Number of Ships
(as of December 1966)

Incidence Formulas

A. The General Model (Components)

1. The components of a general model of incidence for an import good are described below. Once the formulas for an import good are derived, the corresponding formulas for export goods can easily be calculated.

2. Price Definitions

Let \( i, i = 1, \ldots, N \) stand for \( N \) countries exporting a commodity to the importing country. Then for each \( i \),

\[
\begin{align*}
x_i & \equiv \text{exporter's price} \\
r_i & \equiv \text{unit inland cost in country } i \\
s_i & \equiv \text{unit on-sea cost from the } i^{th} \text{ country to the importing country} \\
u & \equiv \text{unit inland cost in the importing country} \\
t_i & \equiv r_i + s_i + u \\
m & \equiv \text{importer's price} \\
p_i & \equiv m - x_i \text{ the price difference} \\
e_i & \equiv i^{th} \text{ country's ad valorem export tax, on price } x_i \\
q_i & \equiv \text{import duty, ad valorem, levied on imports from } i^{th} \text{ country, levied on c.i.f. price, i.e., on } (1 + e_i) + r_i + s_i
\end{align*}
\]

Uniform duty will mean

\[
q_i = q_i + 1, \text{ all } i.
\]

1/ In the interest of simplification, the \( F \) and \( C \) are omitted and only the subscripts used.
3. **Transport Supply Functions**

These functions are defined to depend not only on the volume of trade but also on other factors. For ports (subsumed in $u$), one of these factors is the depth of water. Depth of water might also enter into $r$ and $s$.

$$
\begin{align*}
  r_i &= R_i \left( a_i, E_i \right) \\
  s_i &= S_i \left( b_i, E_i \right) \\
  u &= U \left( \kappa, M \right)
\end{align*}
$$

4. **The Export and Import Functions**

The export supply functions are

$$
E_i = E_i \left( x_i, \bar{K} \right), \text{ where } \bar{K} \text{ is a vector of factors other than own price; the import demand is}
$$

$$
M = M \left( m, \bar{L} \right), \text{ where } \bar{L} \text{ is similar to } \bar{K} \text{ above.}
$$

5. **Equilibrium Conditions**

In equilibrium the following equations must hold:

$$
\begin{align*}
  P_i &= m - x_i - t_i + \psi x_i + (r_i + s_i) q_i, \\
  \text{where } \psi &= \left( e_i + q_i + e_i q_i \right), \\
  \text{and } \sum_{i=1}^{N} E_i &= M \text{ (trade balance)}
\end{align*}
$$

B. **The General Model, Complete**

6. The purpose is to calculate the change in the import price when the parameter $\omega$ changes (see para. 3 above). The change in this parameter represents the effect of port deepening. All other parameters are held constant.
7. When the effect of a change in $\xi$ on the import price is known, the effects on all export prices, $x_i$, will also be known by the equilibrium relations of the model.

8. Define:

$$
\frac{E_i}{x_i} = \frac{\partial E_i}{\partial x_i} > 0 \quad \frac{E_i}{R_i} = \frac{\partial E_i}{\partial R_i}
$$

$$
M = \frac{\partial M}{\partial m} < 0 \quad M = \frac{\partial M}{\partial L}
$$

$$
R_i = \frac{\partial R_i}{\partial E_i} > 0 \quad R_i = \frac{\partial R_i}{\partial a_i}
$$

$$
S_i = \frac{\partial S_i}{\partial E_i} > 0 \quad S_i = \frac{\partial S_i}{\partial b_i}
$$

$$
U = \frac{\partial U}{\partial M} > 0 \quad U = \frac{\partial U}{\partial \xi} < 0.
$$

9. The complete model is now given in terms of the derivatives with respect to $\xi$. Assume $dR = dL = da_1 = db_1 = d\psi_1 = dq_1 = 0$.

$$
\frac{dE_i}{d\xi} = \frac{E_i}{x_i} \frac{dx_i}{d\xi}; \text{ all } i \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)
$$

$$
\frac{dM}{d\xi} = M \frac{dm}{d\xi} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2)
$$

$$
\sum_{i=1}^{N} \frac{dE_i}{d\xi} = M \frac{dm}{d\xi} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3)
$$

$$
\frac{dp_i}{d\xi} = \frac{dm}{d\xi} - \frac{dx_i}{d\xi}; \text{ all } i \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4)
$$

$$
\frac{dm}{d\xi} = \frac{dx_i}{d\xi} = \frac{dt_i}{d\xi} + \frac{dr_i}{d\xi} + \frac{ds_i}{d\xi} q_i; \text{ all } i \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5)
$$

1/ This model has $6N + 3$ variables and $6N + 3$ equations.
\[
\frac{dt_i}{d\lambda} = \frac{dr_i}{d\lambda} + \frac{ds_i}{d\lambda} + \frac{du}{d\lambda}, \text{ all } i \ldots \ldots \ldots \ldots \ldots \ldots \text{(6)}
\]
\[
\frac{dr_i}{d\lambda} = R_i \frac{dE_i}{d\lambda}; \text{ all } i \ldots \ldots \ldots \ldots \ldots \ldots \text{(7)}
\]
\[
\frac{ds_i}{d\lambda} = S_i \frac{dE_i}{d\lambda}; \text{ all } i \ldots \ldots \ldots \ldots \ldots \ldots \text{(8)}
\]
\[
\frac{du}{d\lambda} = U^l + U^l \frac{dM}{d\lambda}; \ldots \ldots \ldots \ldots \ldots \ldots \text{(9)}
\]

10. The steps are (to derive the incidence formulas):

(a) Using equations 4 to 9,

\[
\frac{dp_i}{d\lambda} = \frac{dm}{d\lambda} - \frac{dx_i}{d\lambda} = \frac{dE_i}{d\lambda} + U^l + U^l \frac{dM}{d\lambda} + \psi_i \frac{dx_i}{d\lambda},
\]

all \( i \ldots \ldots \ldots \ldots \ldots \) \( \text{(10)} \)

where \( \lambda_i = (R_i^l + S_i^l) (1 + q_i) \)

(b) Now using equations 1, 2 and 10,

\[
\frac{dp_i}{d\lambda} = \Pi_i \frac{dx_i}{d\lambda} + U^l + U^l \frac{dm}{d\lambda}, \text{ all } i \ldots \ldots \text{(11)}
\]

(c) From 4 and 11, the equilibrium relation between the export prices and the import price is now obtained

\[
\frac{dx_i}{d\lambda} = \frac{(1-\theta) \frac{dm}{d\lambda} - U^l}{\Pi_i + 1}, \text{ all } i \ldots \ldots \ldots \text{(12)}
\]

where \( \theta = U^l M^l \).

(d) Multiply (12) by \( E_i^1 \). Sum over \( N \). Then use (3).

\[
E_i^1 \frac{dx_i}{d\lambda} = E_i^1 \left\{ (1 - \theta) \frac{dm}{d\lambda} - U^l \right\}, \text{ all } i \frac{1}{\Pi_i + 1}
\]
Equation (13) is the general incidence formula for the import price.

ll. All incidence formulas can be derived from equation (13). The ones illustrated below are the ones used in the text. The incidence formulas for the import price are given, since the incidence formulas for the export price or prices can be simply obtained by using equation (12).

(a) Case 1 (paragraphs 29-33 in the text):

Assumptions: \( N = 1; R^1 = R^{11} = S^1 = S^{11} = Y^1 = Y^{11} = \psi^1 = 0; \)

In this case, \( \Psi^1 = 0 = \theta. \) The subscript \( i \) is dropped.

Equation (13) becomes,

\[
\frac{dm}{d\chi} = - \frac{E}{M^1 - E^1} = - \frac{e_x}{e_m - e_x} \quad \text{(as } E = M, \text{ trade balance)}.
\]

where \( e_x \) = elasticity of export supply in terms of price \( m = \frac{m}{E}, E^1 \)

\( e_m \) = elasticity of import demand = \( \frac{m}{M}, M^1 \)
(b) Case 2 (paragraph 34 in the text)

Assumptions: same as in case 1, but $\Psi \neq 0$.

But $d\Psi = 0$ (duties do not change).

In this case $\lambda = 0$, $\Theta = 0$, but $P = \Psi$

Equation (13) becomes

$$\frac{dm}{d\theta} = -\frac{E}{(1 + \Psi)} \cdot \left( \frac{M}{(1 + \Psi)} - \frac{E}{(1 + \Psi)} \right)$$

$$= -\frac{E}{(1 + \Psi) M^2 - E}$$

$$= -\frac{e_x}{(1 + e)(1 + q) e_m - e_x}$$

as $(1 + \Psi) = (1 + e)(1 + q)$.

(c) Case 3 (paragraph 35 in the text)

Assumptions: $N = 1$, $R^1 = S^1 = U^1 > 0$, $\Psi = 0$, $U^{11} = 1$

In this case $P = \Psi^1$

$\lambda = R^1 + S^1$

$\Theta = U^1 - M^1$

$\therefore P = (R^1 + S^1)E^1$

$$= \frac{\partial R}{\partial E} \cdot \frac{\partial E}{\partial x} + \frac{\partial S}{\partial E} \cdot \frac{\partial E}{\partial x}$$

$$= \frac{\partial R}{\partial E} \cdot \frac{E}{m} e_x + \frac{\partial S}{\partial E} \cdot \frac{E}{m} e_x$$

$$= \left( \frac{E}{m} r_{r} + \frac{S}{m} r_{s} \right) e_x$$

and $\Theta = \frac{\partial U}{\partial m} \cdot \frac{\partial M}{\partial m} = \frac{\partial U}{\partial m} \cdot \frac{\partial M}{\partial m} e_m$
Then

Equation (13) implies

\[
\frac{dm}{d\zeta} = - \frac{E}{(T + 1) M + (Q - 1) E}
\]

\[
= - \frac{e_x}{(T + 1) e_m (Q - 1) e_x}
\]

\[
= - \frac{e_x}{e_x e_m (T + e_m - e_x)}
\]

where \( T \) is as defined in the text (paragraph 35).

12. Changes in duty revenue. It is not enough to know the value of \( E \) to know the change in the import duty revenue, unless, of course, the price \( m \) is the duty base. If the export duty, \( e \), is on price \( x \) and the import duty is on the c.i.f. price, i.e., on \( (x(1 + e) + r + s) \), then the calculation of the changes in revenue can be illustrated with reference to case 2 above (paragraph 10 b).

We have

\[
I_m = \frac{dm}{d\zeta} = - \frac{e_x}{(1 + \psi) e_m - e_x}, \quad \psi = (1 + e)(1 + q)
\]

and, using equation (12),

\[
I_x = \frac{dx}{d\zeta} = - \frac{e_m}{(1 + \psi) e_m - e_x}
\]

Let \( c = x(1 + e) + r + s \).

then \( dc = (1 + e) \, dx \), as \( r \) and \( s \) are constant.

\[
dx = I_x, \quad d\zeta
\]

\[
dc = (1 + e) \cdot I_x \, d\zeta.
\]
The import duty revenue is $cq$. The change in revenue is $qdc$. Therefore, the change in import duty revenue is:

$$q(1 + e) \cdot Ix \cdot \frac{d\lambda}{(1 + \psi) \cdot \frac{e_m}{e_x}} . \frac{d\lambda}{d\lambda} .$$

The change in export duty revenue is simply:

$$e \cdot dx = - \frac{e \cdot e_m}{(1 + \psi) \cdot \frac{e_m}{e_x}} . \frac{d\lambda}{d\lambda} .$$
Bibliography

This list includes some references consulted but not referred to in the text.