Models for Determining Least-cost Investments in Electricity Supply

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This paper reviews models used in the electricity supply industry for appraising investments, and presents some extensions. Quantities demanded and the prices of inputs and outputs are assumed to be exogenous, and the models search for investments having the lowest costs. Optimization is over several time periods. Typical decision variables considered are: choice of fossil, nuclear, single- or multi-purpose hydro plant; locations of plants; directions of electrical energy transport (interconnection); timing of investments; replacement; and in all cases the optimum mode of system operation (including hydro storage policy). These variables may be analyzed by linear, non-linear, and dynamic programming as well as other methods. Both global models and optimization treatment of subproblems are reviewed.

In postwar years, the electric power industries of many high- and low-income countries have expanded at average rates of 7 percent per year to as much as 20 percent per year, requiring investments of the order of $150 billion in the U.S. and $1.5 billion in a developing country the size of Colombia; and it is expected that total investments will exceed such magnitudes in the next decade. The problems of determining optimum investment policies in the face of such rapid increases of demand, high costs, the large number and diversity of alternative investment policies, and the numerical tedium of evaluating in depth even a single policy have motivated the development of mathematical models to assist the engineer in scanning and costing alternative policies. This paper reviews these models and presents some extensions to the linear programming (LP) versions.

1. Introduction

The author trained in electrical and mechanical engineering and physics at Imperial College and Manchester University and, later, in econometrics at the London School of Economics. He worked for a number of years as an engineer and physicist for the Central Electricity Generating Board, United Kingdom (U.K.), where he also served an apprenticeship, and on industrial project analysis in the former Ministry of Technology, U.K. Mr. Anderson is now working on public utility economics for the International Bank for Reconstruction and Development (IBRD).

This paper is a condensation of a study for the IBRD [2]. Its contents are for the most part derived from the work of others and from working with and talking to others in this field. Particular thanks are due to Narong Thananart of the IBRD for writing many programs to test and apply the various models discussed, and for correcting some mistakes in the formulations in Section 6. The following people have been most generous in communicating their ideas and experience to the
The investment decision variables of the industry interact strongly at a point in time and over time. This occurs for a number of reasons, which are perhaps most easily explained through two examples. First, different energy sources have complementary functions in modern interconnected power systems. The main sources are single- and multi-purpose hydro schemes, of widely varying power and energy storage capacities; fossil fuels, mainly fuel oil, coal, gas, and lignite; thermal and (eventually) fast neutron breeder reactors; and special-purpose peaking plant, mainly gas turbines and pumped storage. Gas turbines have low capital but high generation costs; fossil, higher capital but lower generation costs; nuclear, high capital and low generation costs; and hydro, high or low capital costs (depending on the site) and near-zero generation costs, but with constraints on energy output which may stem from the multi-purpose nature, water inflows, or both. Gas turbines are thus used for peak loads; fossil, for loads of longer duration; nuclear, for base (continuous) loads; and hydro, somewhere in between, depending on the energy constraint. The optimum balance of plant in the system at any point in time will depend on the relative capital and generation costs of the alternative energy sources.

Second, the optimum balance will depend on both the inherited and the expected structure of the power system. For example, more nuclear and less fossil in future years means that the future system fuel savings of hydro schemes installed now will be less; a large nuclear power program in future years may thus shift the present balance towards more fossil and less hydro. Similarly, if the inherited structure is predominantly fossil, then the present emphasis will be on more nuclear and/or hydro to save on system fuel costs.

Because of these kinds of interaction among decision variables, models must be multi-dimensional and couched, as Turvey has said, in terms of historical dynamics. The investment decisions to be taken at the present time depend upon the past and future evolution of investments and thus upon the past and future evolution of factor prices. We shall find that the models discussed below are designed to capture this problem.

Although developed by engineers and operations researchers in the industry, and specifically concerned with investment decisions, these models are not without interest to economists. They have been

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present writer: Frank Jenkin, Ivan Whitting, George Hext, Eric Parker, and Bill Billington, on work in the U. K.; M. Stengel and M. Pouget of Electricité de France; Mr. Askerlund and his colleagues of the Statens Vattenfallverk, Sweden; Mr. van der Tak, Mr. Berrie, and Mr. Russell of the IBRD; Mr. John Rigie of AID and his colleagues in the American Institute of Electrical and Electronics Engineers (IEEE); and Professor Alan Manne and Dr. Ralph Turvey. Professor Paul MacAvoy and an anonymous reviewer also provided very helpful comments on an earlier draft. All views, mistakes, misinterpretations, etc., are of course those of the present writer.

The reference list following the text contains papers cited in the text plus other papers which also contain material of relevance for this subject. The initials PSCC refer to the Power Systems Computation Conferences organized by Queen Mary College, London. See note 55.

1 In Turkey, for example, where we are currently applying LP models, there are over 168 as yet untapped hydro sites, ranging in size from 1 MW to a proposed 4,000 MW multi-dam complex on the River Euphrates, capable of irrigating 700,000 hectares.
occasionally applied in the cost-benefit studies of single- and multi-purpose hydro schemes.  

A second and increasingly common application is the determination of the marginal cost structure of the industry for purposes of pricing policy. Bessière and Massé and Turvey in particular have demonstrated the practical value of linear and non-linear programming (LP and non-LP) global models for this purpose, and the pricing models of Littlechild, Pressman, and Williamson, for example, can be viewed as approximations of the global models discussed below.

A third, but as yet unexplored, aspect of these models is their relation to the empirical studies of the type undertaken by Nerlove, Johnston, Galat, and Dhrymes and Kurtz. These workers have attempted to estimate economies of scale and technical progress in the industry using Cobb-Douglas and CES types of empirical relations between the factor inputs and an optimizing condition for a single time period. The cost structure and the relationships among the factor inputs in the electricity supply industry are, however, defined precisely in the engineers' models without recourse to such empirical relationships. Moreover, it is the daily occupation of planning engineers in the industry to search for optimum investment programs over many time periods. Taken into consideration are economies of scale attainable from large units, external economies of scale attainable from interconnection, technical progress embodied in new equipment, substitution among factor inputs, replacement, the putty-clay nature of the investment decision, the putty-putty nature of the operating decision, the possibility of storage (hydro schemes, pumped storage), and as noted above, the past and expected future evolution of the system. It would seem therefore that the engineers' models are not without significance for econometricians who wish to study the industry.

The plan of this paper is as follows. We begin in Section 2 by formulating the investment problem in cost minimization form. We then review the various approaches used to find optimum solutions. These are three classes which we review in Sections 3, 4, and 5, respectively: marginal analysis, marginal analysis using simulation models, and global models. We shall find that while they are outwardly different in form—ranging from graphical devices and marginal analysis to dynamic, linear, and non-linear programming—this is a difference only of algorithms; they are different methods of solving the same kind of problem. We shall also find that they are often complementary approaches in the following sense. Global models can only give approximate answers in most practical situ-
ations. The reason is that the practical details of the alternative programs, particularly of the individual projects in the programs, are too numerous to be handled in one computer run. Having obtained approximate solutions from the global models, we then turn to marginal analysis using simulation models to focus on the fine details of individual project selection and design. Finally, in Section 6, we present three LP extensions to the global and simulation models reviewed. They cover (1) a fresh treatment of replacement, (2) the introduction of decision variables for hydro storage capacity and storage policy, and (3) regional decision variables, to give a fuller treatment of transmission.

Before proceeding, let us make clear a number of limitations of this paper:

(1) The quantities demanded are assumed to be exogenous, and the objective is always cost minimization. This assumption could be relaxed if required so as to maximize consumers' plus producers' surplus. The papers of Littlechild and Pressman\(^7\) would be good starting points in this respect. It is thought, however, that the most practical way to treat interactions of demand and supply when formulating an investment program is by iteration, taking demand as given (but hopefully related to some kind of rational pricing policy), searching for least-cost solutions, and then revising demand estimates on the basis of marginal costs and prices.\(^8\) In connection with formulating a pricing policy, Turvey has also argued for an iterative approach.\(^9\)

(2) Use of one or more investment models is the first of several stages of the investment decision process. Engineering analysis of solutions follows and generally requires a revision of the solutions. The investment program finally selected must satisfy a number of engineering criteria regarding system stability, short-circuit performance, the control of watts, vars, and voltage, and the reserves and reliability of supply.\(^10\) The search for an investment program which satisfies engineering and economic criteria is an iterative, multi-disciplinary process.

(3) All the formulations presented are deterministic. Allowances are of course made for uncertainties in demand, plant availability, and flows of water to hydro schemes, but in the simple form of margins of spare capacity. This is frequent practice, although people are working with stochastic counterparts to the models presented and their work is noted.

(4) There is no discussion of terminal conditions as analyzed by Hopkins\(^11\) or of the optimum breakdown of the time period of the study into discrete periods.

(5) There is no discussion of the dual variables from the LP models or of pricing policy. We thus neglect much important work of Bessière and Petcu, Turvey, Littlechild, Williamson, and many others.\(^12\)

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\(^7\) See [54] and [70].
\(^8\) See Berrie [6].
\(^9\) See [79], p. 288.
\(^10\) See Stagg and El-Abiad, for example [74].
\(^11\) In [41].
\(^12\) See [11], [77,79], [54], and [86].
(6) Finally, we do not explore the connection between these models and those customarily used in econometric research (i.e., of the type mentioned above).  

The principles of the following formulation were first enunciated in the early 1950s by Massé and Gibrat, who solved the problem using linear programming. A subsequent paper by Bessière and Massé and the book by Massé formulate the problem more generally.

The search for an optimum (least-cost) investment program also entails, for each plant program considered, the search for an optimum operating schedule. Let the power capacity of any plant in the system be defined by $X_{jv}$, $j$ denoting the type of plant (hydro, fossil, nuclear, etc.) and $v$ the vintage (year of commissioning). Also, let the power output of this plant at any instant $t$ be $U_{jv}(t)$, $0 \leq U_{jv}(t) \leq X_{jv}$. The operating costs of this plant over the interval $t = 0$ to $T$ are given by:

$$\int_{t=0}^{t=T} F_{jv}(t) \cdot U_{jv}(t) \cdot dt,$$

where $F_{jv}(t)$ are the discounted operating costs per unit of energy output.

At any instant $t$ the operator has before him $j = 1, \ldots, J$ types of plant of different vintages, comprising the initial plant composition of the system, $v = -V$ to 0, and the plant installed (at discrete intervals) between 0 and $t$. To obtain the total system operating costs in the interval $dt$, we must summate over all vintages $v = -V$ to $t$ and over all types of plant. The total future operating costs are then

$$\int_{t=0}^{t=T} \sum_{v=-V}^{t} \sum_{j=1}^{J} F_{jv}(t) \cdot U_{jv}(t) \cdot dt.$$

The investor's objective is to minimize the sum of capital and operating costs over some future time period 0 to $T$:

$$\text{Minimize } \sum_{v=1}^{T} \sum_{j=1}^{J} C_{jv} \cdot X_{jv} + \int_{t=0}^{T} \sum_{v=1}^{t} \sum_{j=1}^{J} F_{jv}(t) \cdot U_{jv}(t) \cdot dt, \quad (1)$$

where $C_{jv}$ are the capital costs per unit of capacity of plant $j$, vintage $v$. All costs of course be expressed as social opportunity costs.

The discrete approximation to (1) is often a more convenient function to use:

$$\text{Minimize } \sum_{v=1}^{T} \sum_{j=1}^{J} C_{jv} \cdot X_{jv} + \sum_{i=1}^{T} \sum_{v=1}^{i} \sum_{j=1}^{J} F_{jv}(t) \cdot U_{jv}(t) \cdot \theta_i, \quad (2)$$

where $\theta_i$ is the width of the time interval considered at time $t$.

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13 Page 269 infra.  
15 See Bessière and Massé [10] and Massé [62]. Bessière [9] outlines the development of methods used by Electricité de France during the 1950s and 1960s.
The search for optimum capacities (optimum \( X_{jv} \)) and for optimum operating schedules (optimum \( U_{jvt} \)) is of course subject to a number of important conditions. First, sufficient plant must be operating at all times to meet the instantaneous power demand, which we define by \( Q_t \). Thus

\[
\sum_{j=1}^{J} \sum_{v=-V}^{V} U_{jvt} \geq Q_t, \quad t = 1 \cdots T. \tag{3}
\]

Only at times of peak load will all of the plant be in operation. At other times the instantaneous power demand can be met with much of the plant not operating (i.e., some of the \( U_{jvt} \) are zero); this plant will in general be that with the highest operating costs.

Second, a unit of plant can be operated above its peak available capacity:

\[
0 \leq U_{jvt} \leq a_{jv} \cdot X_{jv}, \quad j = 1 \cdots J \quad v = -V \cdots V \quad t = 1 \cdots T, \tag{4}
\]

where \( a_{jv} \) is the availability of plant \( j \), vintage \( v \) (\( a_{jv} \) is usually about 0.9). Note that \( X_{jv} \) for \( v = -V \) to 0 are predefined constants and represent the capacities of the inherited capital stock.

Third, there may be constraints on the operation of hydro plant. Seasonal shortages of water inflow, or the requirements of irrigation and flood control, will impose restrictions on the amount of electricity to be generated in, e.g., a given season. There will still be a choice however on the timing of the hydro operation within the season (in general, it will be operated at times of peak demand, when fossil energy is most expensive). The simplest form the hydro constraints take is the following. Let \( H_{sv} \) be the hydro-electric energy to be delivered in season \( s \) by the hydro scheme of vintage \( v \). Then we must choose the decision variables for hydro operation, \( U_{sv}(t) \) (\( j = \text{hydro} = h \)) such that, in minimizing total system operating costs, all available hydro energy will be utilized:

\[
\int_{\text{season } s} U_{hs}(t) \cdot dt = H_{sv}. \tag{5}
\]

(At a later stage in this paper we shall be discussing methods of searching for optimum values of \( H_{sv} \)).

Fourth, there are constraints to represent what the French writers call the "guarantee conditions." These are to guarantee supply, to an unacceptable probability limit, in event of contingencies—water shortages in dry seasons, peak demand above mean expectations, or plant outage. These constraints take two forms, one to guarantee peak power supplies, and the other to guarantee energy supplies in critical periods. Let us take them in turn. Following Massé and Morlat, suppose that \( e \) is the probability that the yearly peak demand, defined by \( \bar{Q}_t \), will be met; that is, the probability that the aggregate available capacity is greater than \( \bar{Q}_t \) is \( e \), as follows:

\[
Pr(\sum_{j=1}^{J} \sum_{v=-V}^{V} a_{jv} \cdot X_{jv} - \bar{Q}_t \geq 0) = e, \quad t = 1 \cdots T, \tag{6'}
\]

\(^{16}\) An allowance for plant outages will be considered shortly.

\(^{17}\) Data on \( a_{jv} \) for different countries are discussed by Cash and Scott \cite{17}.

\(^{18}\) See \cite{60} and \cite{64}, respectively.
where \( a_{js} \) and \( \bar{Q}_t \) are stochastic variables. This is a "chance constraint" of the type discussed by Charnes and Cooper.\(^{19}\)

When reviewing the practices adopted by European countries in planning system security, Cash and Scott found that the actual choice of \( e \) varies widely between countries.\(^{20}\) They also found that, while the choice is sometimes backed up by statistical and economic calculation, it is generally determined by experience. This is partly because estimating the costs and benefits to the economy of a given level of security is exceedingly difficult and subject to large errors, and partly because the required level may rest on a number of noneconomic factors—e.g., the hostility of press and public opinion in event of supply shortages (which, almost by definition, seem to occur when least desired). Finally, they found that most countries think of the guarantee condition in terms of a "margin of available capacity" over and above what is required to meet the mean expected peak demand.

The guarantee condition (6') is therefore frequently simplified in practice. Let \( m \) be the margin of spare available capacity required to meet demands above the mean expectation; then (6') is expressed as

\[
\sum_{j=1}^{J} \sum_{t=1}^{T} a_{jv} \cdot X_{jv} \geq \bar{Q}_t (1 + m), \quad t = 1 \cdots T ,
\]

where \( a_{jv} \) and \( \bar{Q}_t \) are once again mean expected quantities.

It should be added that while condition (6) is much simpler than condition (6'), it by no means implies a loss of rigor in the planning process. On the contrary, calculating the probability distribution of available capacity in various regions of a modern interconnected system is itself a highly complex and specialized computation which may require the use of Monte Carlo techniques. For this reason it is perhaps best treated as a separate calculation, even though this may result in repetition and modification of the least-cost exercises. System planning, as we remarked in the introduction, is an iterative, multi-disciplinary operation.

A similar pattern of discussion follows when we examine the problem of guaranteeing energy supplies in dry seasons on mixed hydro-thermal systems. Let \( e \) now denote the probability that the potential energy available from both hydro and thermal plant will be greater than the energy demand in the critical period; and let \( t = t' \cdots t'' \) represent the critical period. The potential energy output from a thermal plant is limited by its available capacity, \( a_{js} \cdot X_{js} \). The potential energy output from a hydro plant is limited by constraints of type (5). The guarantee condition is therefore

\[
\Pr \left( \sum_{t=t'}^{t''} \left( \sum_{v=v'}^{v''} \sum_{j=1}^{J} a_{jv} \cdot X_{jv} + \sum_{v=v'}^{v''} \sum_{j=1}^{J} U_{jv} - Q_t \right) \theta_t \geq 0 \right) = e, \quad (7')
\]

\[
(j \neq \text{hydro}) \quad (j \neq \text{thermal})
\]

where \( a_{jv} \), \( U_{jv} \), and \( Q_t \) are stochastic.


\(^{20}\) See [17].
Again it is possible to approximate the chance constraint (7'), postponing a more rigorous study of the reserves and reliability of supply for a separate stage of the planning process. Let \( \beta_{jv} \) (with \( j = \text{hydro} \)) to be the ratio of the energy output of hydro plant \( j \), \( v \), in the critical period of a dry year, to its mean expected output in this period of an average year. Then (7') simplifies to

\[
\sum_{i=1}^{\nu} \sum_{v} \left( \sum_{j} a_{jv} \cdot X_{jv} \cdot \theta_{i} + \sum_{j} \beta_{jv} \cdot U_{jv} \cdot \theta_{i} \right) \geq \sum_{i=1}^{\nu} Q_{i} \cdot \theta_{i}, \quad (7)
\]

where \( a_{jv}, U_{jv}, \) and \( Q_{i} \) are once again mean expected quantities. Note that for hydro schemes with highly uncertain water supplies, \( \beta_{jv} \) will be low, and if such schemes are included, then other thermal or hydro capacity will be required to satisfy this constraint. Thus \( \beta_{jv} \) indirectly applies a cost penalty on schemes according to the variability of their supplies.

To complete the formulation of the investment problem, there will generally be a number of "local" constraints. Examples are constraints to limit expenditures on capital or foreign exchange, regional development constraints (e.g., a lower limit to the use of coal or hydro resources), political and social constraints, etc.

It will be evident that this formulation can be extended or contracted in many ways. Simulation models, for example, extend it in one direction and contract it in another. Essentially the \( X \)'s (i.e., the investment plans) are predefined constants in a simulation model; this leaves more computer space (core storage) available to examine the \( U \)'s (the dispatching schedules) in more detail. Another extension is to treat uncertainties in costs, water supplies, and demand forecasts through stochastic programming. Some formulations have also been adapted to optimize the operation of multi-purpose, multi-dam hydro schemes. Replacement decisions have been included in the objective function. Efforts are being made to treat both supply and demand on a regional basis. Finally, some workers are embodying the formulation for the electricity sector into larger models of the energy sector and of the economy. We shall discuss some of these and other extensions subsequently.

3. Marginal analysis

Marginal analysis was first applied to investments in electricity supply by Electricité de France in the late 1940s. Since then it has been applied regularly in many other countries. The analysis starts from an arbitrary but reasonable initial program a "reference solution" and then seeks to improve it (reduce costs) by marginal

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11 E.g., Bessière [8, 9], Electricité de France [25], Fernandez and Manne [27], Lindqvist [52], Little [53], and Gessford and Earlín [34] all discuss models which have some stochastic elements, either for demand or water supply variables.
12 E.g., Jacoby [43] and United Nations Economic Commission for Europe [83].
13 See Massé [62].
14 See Electricité de France [25]; Fernandez, Manne, and Valencia [28].
15 Forster and Whitting [30]; Manne [58].
16 See Giguet [35], Massé [60], and also Bessière’s note [9] and the bibliography contained therein.
17 E.g., United Nations [82] and van der Tak [85].
substitutions. The reference solution and the solution obtained after a marginal substitution has been made satisfy the same power and energy demands. Whenever the cost function is convex, marginal analysis should ultimately lead to a uniquely optimum investment and operating program over time.

A common application of marginal analysis has been the comparison of fossil and hydro alternatives to meet a given demand for electricity. The hydro plant may require a higher investment \( I_h \) than fossil \( I_f \), but the total system operating costs in subsequent years are less. The total, discounted system operating costs at time \( t \) are

\[
(1 + r)^{-t} \sum_{j=1}^{J} \sum_{v=V} F_{jv} U_{jv} \theta_t = (1 + r)^{-t} \phi_{th}
\]

if the hydro project is adopted and

\[
(1 + r)^{-t} \phi_{tf}
\]

if the fossil project is adopted.

The present worth of the savings if hydro is substituted for fossil is then

\[
PW = (I_f - I_h) + \sum_{t=0}^{T} (1 + r)^{-t} (\phi_{tf} - \phi_{th})
\]

and according as this value is positive or negative the hydro is or is not preferable to the fossil investment. The value of PW savings calculated in (8) is sometimes known as the relative profitability of the hydro investment, since the calculation shows whether or not the hydro investment improves upon the reference solution.

Among the advantages of this calculation are its practical simplicity and the feature that it is easy to adjust the arithmetic for many local costs and benefits of a project. For example, different locations of hydro and fossil stations will lead to different transmission costs; maintenance costs for hydro stations are lower; the fossil station (lifetime about 30 years) will be replaced before the hydro (lifetime about 50 years), so that the discounted replacement costs may have to be included; the hydro may have flood control benefits; the fossil plant may induce more employment in local coal mines; and so on. Such local features can readily be included in the arithmetic. The calculation can also be readily formulated for comparisons between nuclear and fossil plant at base load; or between fossil, pumped storage and gas turbines, at peak load.

There are, however, two difficulties with marginal analysis. First, it is tedious to calculate operating and fuel costs over a 20- or 30-year period, when the demand fluctuates rapidly by the hour, when there may be four or more types of plant on the system, 30 or more vintages of plant, and when the expansion of the system introduces new vintages and types of plant while replacing others. It is also an optimizing problem in itself, since each plant

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28 Phillips et al. [69] prove convexity for the mixed fossil-nuclear system of the U. K. Their model is described later (Section 5).
29 See, e.g., van der Tak [85], Boiteux [14], and Turvey [76].
30 See Openshaw-Taylor and Boal [68], who also consider many other kinds of decision—e.g., on transmission and distribution equipment.
must be located on the system operating schedule\textsuperscript{31} so as to minimize total fuel and operating costs. Second, the marginal substitutions to the investment plan may be many, requiring special routines to scan and cost the alternatives. It was to overcome the first difficulty that simulation models were developed, and to overcome the second that global models were developed. We discuss these in turn.

4. Simulation models

We discuss three formulations:

(1) Models which integrate the load duration curve directly.
(2) Models which use dynamic programming, and
(3) Models which use linear programming.

Non-linear programming could also be used, but the present writer did not find LP or non-LP simulation models discussed in the literature.\textsuperscript{32}

(1) Direct integration of the load duration curve.\textsuperscript{33} Simulation models which integrate the load duration curve directly are particularly suitable for power systems having thermal plant only,\textsuperscript{34} although they have been adapted for mixed fossil-hydro systems by Jacoby.\textsuperscript{35} We shall first consider the thermal power system and then look at Jacoby's model.

On an all-thermal system the cheapest way of meeting the demand at any point in time is to run the stations with the lowest operating costs. The system operator tabulates the power stations in ascending order of marginal operating costs and loads and unloads the stations sequentially as the demand rises and falls (merit-order operation). We picture this situation graphically in Figure 1. For clarity we aggregate the system into four representative power stations; in ascending order of marginal operating costs they are nuclear, new fossil, old fossil, and gas turbines. By projecting the plant capacities horizontally through the daily demand curves of frame (b), we see the times when the different plants are started up, loaded, unloaded, and shut down on different days. By continuing the projection horizontally through the load duration curve, frame (c), we can find the total operating time of each plant for the period represented by the curve. By estimating the areas sliced out of the load duration curve, we can estimate the energy delivered by each plant and thus the total system operating costs. These costs will be at a minimum under this arrangement because the plant with the highest operating costs (older vintage of fossil plant and gas turbines) will be operated the least.

This type of simulation model is used by the Central Electricity Generating Board, U. K., for the estimation of generation savings associated with different investment programs. It is reported by Berrie and Whitting and by Jonas.\textsuperscript{36} The simulation can be refined

\textsuperscript{31} Sometimes called the load dispatching schedule.
\textsuperscript{32} LP and non-LP models are used extensively for real-time scheduling [see Section 4(3)].
\textsuperscript{33} The load duration curve is defined in the Appendix.
\textsuperscript{34} The Central Electricity Generating Board in the U. K. has pursued this method in meticulous detail. See for example Jonas [45], Whitting and Berrie [7], and the review paper of Berrie [6], pp. 22, 29.
\textsuperscript{35} In [43]. See also Lieftinck, Sadove, and Creyke [51], Vol. III.
\textsuperscript{36} See [5,7] and [45], respectively.
in many ways, of which we shall mention three. First, because of maintenance schedules, the table of available plant capacities will differ between seasons. The horizontal projections will only hold, say, for a season, and not for a year as indicated in the diagrams. Thus a different load duration curve is required for each season. Second, the operating costs of each station should be adjusted for transmission losses; this introduces a small quadratic term into the operating cost of each station. Third, the transportation of the coal from several collieries to the power stations is an important element in operating costs. The model determines the operating costs of each power station as follows:

Trial (marginal operating) costs are used to obtain an initial merit order. A loading simulation study is carried out to obtain trial fuel consumptions, which are then fed into the standard (linear programming) transportation calculation to determine the minimum cost coal allocation and the corresponding station (marginal operating) costs. These are then substituted for the trial value to form a new merit order, this process being repeated until there is no significant change in the costs of generation, when generator loadings and fuel consumptions are consistent with minimum cost fuel allocations.³⁷

Monte Carlo studies using this model have also been undertaken to examine the effects on costs of uncertainties in data input, but the results remain unpublished.

We now look at Jacoby’s adaptation of this kind of model for a mixed storage hydro-thermal system. We first consider the optimum position of a single storage hydro station on the power system load dispatching schedule (i.e., the optimum position in the “merit order” table). If a system has several such stations, then the single one we
FIGURE 2
LOAD DISPATCHING ON A MIXED HYDRO–THERMAL SYSTEM (ONE HYDRO STATION)

(a) TABLE OF AVAILABLE PLANT CAPACITIES, MW (DISPATCHING SCHEDULE)
(b) WEEKDAY (NOT THE ANNUAL PEAK WEEKDAY)
(c) WEEKEND DAY
(d) DEMAND DURATION CURVE FOR PERIOD t

MW CAPACITY
GAS TURBINES, MW
OLD FOSSIL-FIRED PLANT, MW
HYDRO, MW
NEW FOSSIL-FIRED PLANT, MW
NUCLEAR, MW

DEMAND, Q, MW

TIME
7 A.M. 7 A.M. 7 A.M. 0
1 DAY 1 DAY 1 DAY

AREA = HYDRO-ELECTRIC ENERGY GENERATED

consider is taken to represent the aggregate characteristics of all hydro stations.

Suppose that in any period \( t \) the hydro energy allotted for electric power generation is \( H_t \) and that the peak power capacity of the hydro is \( X_{hi} \). If the hydro is to maximize fuel savings it must of course discharge the full amount \( H_t \). It must also be operated at times when the fuel costs on the system are most expensive; this happens at times of peak demand (during the period \( t \)) when the older and less efficient units of the thermal plant are operating. Consider a particular day [Figure 2(b)]. The hydro plant should begin operating at point A in the morning, generate full power output at B and through to C, and then reduce power to D during the night. However it will not (in the optimum) be delivering full power every day. In the example of Figure 2(c) the hydro is still operated over the peak demand periods of the weekends, but in view of the higher demands and fuel costs during the week, it is cheaper to store energy for delivery during weekdays.

In the optimum, the hydro stations will occupy the same place in the system dispatching schedule every day throughout the period. If they occupied a higher place at the weekends (i.e., were operated at lower values of total system demand) then less energy would be available for operation during the weekdays; the low efficiency thermal stations would then have to supply extra energy during the peak demand periods of the weekdays. If the hydro occupied a lower place at weekends, then extra energy would be available for use during the week, when two things could happen: (1) the hydro would be unable to discharge the extra energy, because of insufficient capacity, and there would be spillover, unless (2) it were to operate at a higher place in the dispatching schedule. If (1) were to happen

\[38\] We assume (as Jacoby does) that \( H_t \) is given. Later we consider the problem of finding the optimum value of \( H_t \) when there is an option to store hydro energy for the period \( t + 1 \) [Sections 4(2) and 4(3)].
there would simply be a waste of energy. If (2) were to happen then the more efficient thermal plant would be removed from base load operation during the week, and less efficient thermal plant would be operating in place of hydro during the weekends. To conclude: In the optimum the hydro station will occupy the same place in the table of merit order operation every day throughout the period $t$.

This conclusion enables us to use the load duration curve to determine the optimum position of the hydro station in the merit order table. The horizontal lines which represent the power capacity of the hydro (in Figure 2) must cut the load duration curve at those points (1) where the area cut out of the load duration curve exactly equals the energy to be supplied by the hydro in period $t$, and (2) where the gap between the lines represents the peak power capacity of the hydro. For two or more hydro plants the technique is the same. Each plant must deliver all the energy allotted for period $t$, and it must occupy that place on the load dispatching schedule defined above.

The areas under the load duration curve can be calculated graphically or by numerical integration, and the hydro plant located by trial and error. The approach used by Jacoby is, however, much simpler: first to integrate the load duration curve directly and plot the integral (the energy demand) against power demand. This gives us a curve known as the integrated load function shown in Figure 3(c). The energy delivered by each plant can be read directly off the abscissa. The energy delivered by nuclear plant, for example, is obtained by projecting a line vertically downwards into the abscissa from point a. Similarly, projections downwards from points (b,a), (c,b), (d,c) and (e,d) give the energy delivered by base load thermal, hydro, old thermal, and gas turbines, respectively.

These are the types of computation embodied in Jacoby's model, which can estimate operating costs in considerable detail. For example, the total system operating costs may be evaluated for each month of a 20-year period; several hydro units and over 20 to 30 thermal units may be considered. The model was used extensively in the Indus Basin project in Pakistan, and El Chocon in Argentina.39

39 See Jacoby [43] and Lieftinck, Sadove, and Creyke [51].
The model can also be formulated to calculate the operating savings associated with transmission links (although this becomes very difficult if there are many regions of generation and demand).

An assumption of the Jacoby model is that $H_i$, the hydro energy allotted for electric power generation during each month, is known in advance. For large irrigation projects, where power is often a fringe benefit, this assumption is realistic. But it is less realistic in many hydro-electric schemes, when the problem is to determine an optimal water storage policy from period to period. If the next period’s demands are high and water inflows low, how much water should be stored for the next period? This type of problem was solved by Massé, who in doing so apparently adumbrated the technique of dynamic programming as applied to this problem. Little formulated the problem explicitly in terms of dynamic programming. Since then the method has been refined and developed by many workers, in particular the Swedish engineers, who use the type of calculations undertaken in Jacoby’s model as a routine to compute the total system operating costs for a wide range of storage policies scanned by the dynamic programming (DP) algorithm. It is to this work that we now turn.

(2) Dynamic programming. DP techniques have been used by many workers to determine the optimum operating schedules for long-range storage reservoirs (fortnightly, monthly, seasonal reservoirs) on mixed hydro-thermal systems. The method can also be adapted for flood control and irrigation projects. The answers it gives can also be obtained by linear programming. The question to be answered is the following: Given that demand and water supplies fluctuate periodically, how much water should be stored for the next period and how much should be utilized in the present period? The decision process is sequential, since the next period’s decisions will depend upon how much water should be stored for the period following that, and so on.

We formulate the problem assuming water supplies are known with certainty. This assumption can be readily relaxed if required. Discrete time intervals are taken. The model below also assumes one long-range storage reservoir weighted to correspond to a whole system’s reservoirs, one hydroelectric generator (also equivalent to that of the whole system), and a number of thermal stations (fossil and or nuclear). This follows the practice of all previous writers. It is quite straightforward to extend the model and represent the

40 In [59], Vol. I.
41 In [53],
42 See Lindqvist [52].
43 It is not clear however whether they work with a load duration curve or an integrated load function.
44 Lindqvist [52] presents the model that was developed for the Swedish State Power Board, which has been used extensively for the technical and economic long-term planning of system extensions. Lindqvist’s work builds upon that of Little [53]. See also Koopmans [47] and Gessford and Karlin [34]. The subject of the optimal management of seasonal reservoirs has a long history; e.g., see Massé [59], Vol. I, and Morlat [64].
45 See for example Manne [57], Thomas and Waterneyer in Masse et al. [55] and Haismann [39].
46 See Manne [56] and Section 4(3) of this paper.
47 In the above references (notes 42, 44, and 45), all but Koopmans work with stochastic models.
system's hydro stations with two or more equivalent stations with different storage capacities and water inflow patterns, but this enlarges the dimensions of the problem considerably. Let

\[ S_t = \text{storage at the beginning of period } t \text{ (KWh)}, \]
\[ H_t = \text{hydro energy (water) discharge during period } t \text{ (KWh)}, \]
\[ W_t = \text{water inflow during period } t \text{ adjusted for losses and expressed in units of potential energy (KWh)}. \]

The storage at the end of period \( t \) is then

\[ S_{t+1} = S_t + W_t - H_t. \] (9)

Suppose we specify a value for \( H_t \). Then following the methods outlined above (e.g., for Jacoby's model), it is possible to determine the optimum system operating schedules and costs for this period. As we increase \( H_t \) the total system costs will decrease, because less fuel is burned in the thermal plant. By computing total system operating costs (denoted by \( C_t \)) for a range of \( H_t \), we can obtain the kind of curve shown in Figure 4. The shape of this curve (neglecting discontinuities) will generally be concave to the origin because (1) an increase of \( H_t \) will always reduce the energy to be delivered by fossil plant, (2) the marginal operating costs of the thermal plant, because of merit-order operation, increase with the amount of thermal plant operated, and (3) the plants with the highest operating costs are generally the oldest and smallest.

The objective is to operate the hydro scheme so as to minimize the total system operating costs over some time period \( T \). That is, we require

\[ \min \sum_{t=1}^{T} C_t(H_t) \] (10)

subject to constraints (9).

This is in fact a standard deterministic inventory problem, which is often solved by the recursive methods of dynamic programming.\(^{48}\) The principles of the method are as follows. Suppose we fix the amount of water to be stored at the beginning of period \( t \) (end of period \( t - 1 \)) at some value of \( S_t \); and suppose also that for this value of \( S_t \) we know the values of \( H_{t-1} \) to \( H_{t-1} \) which minimize total costs up to the beginning of \( t \). We define these costs by \( \phi_{t-1} \), the minimum of which will depend on the value we have chosen for \( S_t \):

\[ \phi_{t-1}(S_t) = \min \{ \sum_{j=1}^{t-1} C_j(H_j) \}. \] (11)

Now suppose that we know \( \phi_{t-1}(S_t) \) for a range of values of \( S_t \) between zero and, say, \( \bar{S}_t \). The next step we can take is to find \( \phi_t(S_{t+1}) \) for a whole range of \( S_{t+1} \) between zero and \( S_{t+1} \), as follows:

\[ \phi_t(S_{t+1}) = \min_{0 \leq S_t \leq \bar{S}_t} \{ C_t(H_t) + \phi_{t-1}(S_t) \}. \] (12)

In view of constraints (9) the hydro flows \( H_t \) are implied by \( S_{t+1} \) and \( S_t \) so that

\[ \phi_t(S_{t+1}) = \min_{0 \leq S_t \leq \bar{S}_t} \{ C_t(S_t - S_{t+1} + W_t) + \phi_{t-1}(S_t) \}. \] (13)

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48 See for example, Hadley [38], Chaps. 10 and 11.
This can be viewed as a forward recursive decision rule of dynamic programming. For each period \( t \), the system operating costs are evaluated for a range of \( S_{t+1} \) and the range \( S_t \) considered in the previous scan. For each \( S_{t+1} \), the optimum value of \( S_t \) is found from (13). Similarly, at \( t + 1 \) a range of \( S_{t+2} \) is taken, and for each \( S_{t+2} \) the optimum \( S_{t+1} \) is found; and so on. At the beginning, \( S_t \) is known so that:

\[
\phi_t(S_t) = \min_{0 \leq S_t \leq \bar{S}_t} \{ C_t(S_t - S_{t+1} + W_t) \}.
\]  

(14)

This gives us the starting point. The forward recursions are continued until the optimum decisions of interest are not influenced by extra recursions.

To solve the problem for the case when hydro supplies are treated stochastically, it is necessary to use backward recursive formulas. These formulas (but including the frequency distributions) are a mirror image of the formulas above.

DP simulation models have been applied to many problems. Lindqvist (writing in 1962)\(^4\) informs us that, in Sweden, the model has been utilized since the Spring of 1959 for several hundred calculations, e.g., for the calculation of utilization times for nuclear and thermal plants during drought years, for the optimum ratio between hydro-electric and thermal power as a function of interest rates, fuel costs and capital costs; furthermore, for the calculation of the economic consequences of errors in the long-range prediction of net consumption, and for possible secondary deliveries to neighbor countries in the future, etc.

Apparently, the model is still used for purposes such as these.

\(\Box\) (3) Linear programming. The problem as formulated in Section 2 is already in an LP form. It is convenient to alter the notation slightly and let each period \( t \) be represented by a load duration curve broken down into \( p = 1 \cdots P \) blocks each of width \( \theta_p \) (see Figure 5).

The periods \( t \) may represent months, seasons, or years, etc., according to the approximation desired. If there is to be seasonal or monthly storage hydro on the system, \( t \) will accordingly represent seasons or months. Since the capacity variables \( X_{jt} \) are predefined constants in the simulation model, the objective is to choose the operating decision variables \( U_{jtp} \) such that the total system operating costs are at a minimum. Thus [see expression (2) above]:

\[
\text{Minimize} \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{v=1}^{V} \sum_{p=1}^{P} F_{jtp} U_{jtp} \theta_p.
\]

(15)

This objective is subject to the capacity constraints

\[
0 \leq U_{jtp} \leq a_{jt} \cdot X_{jt}, \quad \text{all } j, t, v, p
\]

(16)

and to the constraints that aggregate output must be sufficient to meet the demand at all times:

\[
\sum_{v=-V}^{V} \sum_{j=1}^{J} U_{jtp} \geq Q_{tp}, \quad \text{all } t, p.
\]

(17)

Note that the "guarantee conditions" have to be handled through a separate calculation, since \( X_{jt} \) is not an endogenous decision variable in the simulation model. In addition, there are the hydro-
energy constraints. Let \( j = h \) denote hydro, and let the decision variable \( S_{hv,t} \) be the energy stored at the beginning of period \( t \) in the hydro station of vintage \( v \). If \( S_{hv,t} \) is the storage capacity of the reservoir, \( 0 \leq S_{hv,t} \leq S_{hv} \). Also let \( W_{hv,t} \) be the water inflow in period \( t \), expressed in energy units and adjusted for losses due to seepage and evaporation. Then the water stored at the end of the period plus the water used for generation during the period must be less than or equal to the water stored at the beginning plus the inflow:

\[
S_{hv,m+t-1} + \sum_{p=1}^{P} U_{hvtp} \theta_p \leq S_{hv,t} + W_{hv,t}, \quad \text{all } h, v, t, \quad (18)
\]

and these constraints will be satisfied with equality if there is no spillage.

The above linear program can be solved using standard computer programs. It can be extended in various ways, as will be shown later in this paper, to include multi-purpose schemes, regional decision variables, and transmission losses. The objective function is also separable, so that nonlinearities in the cost coefficients can be treated by separable programming.

A difficulty with LP simulation models is the large number of constraints which must be satisfied in any realistic formulation of a problem. Constraints (16) in particular can become exceedingly numerous if the load duration curve is broken down into many periods and the types and vintages of plant are many. But it is possible to overcome this difficulty. Since the capital structure is predefined and fixed, these constraints form an upper bound set and can be treated by bounded variable LP methods.\(^6\) For predominantly thermal systems the problem can be decomposed into several independent and much smaller linear programs (e.g., one for each year) since the operating decisions for one year are to a good approximation independent of those of previous years. Although LP and non-LP simulation models for planning calculations have not been reported as frequently as the other models we have discussed, it is interesting to note that they are used extensively by engineers for "real-time" load dispatching calculations.\(^6\)

One difficulty with marginal analysis is the large number of marginal changes to a basic plan that must be considered. If it were only necessary to consider investment decisions to be made at the present time, the number of marginal changes might not be too many (although this is not true if regional variables and transmission are included in the model). However, this is not the situation; investment decisions over time must be considered, and this adds enormously to the dimensions of the problem. It is the function of global models to overcome this second difficulty with marginal analysis. Specifically, they are designed to scan and cost a large number of present and "real-time" load dispatching calculations.

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\(^6\) See Hadley [37].

\(^6\) Articles are frequently published on this topic by the Institute of Electronics and Electrical Engineers (IEEE) (U. S.) and the Institute of Electrical Engineers (U. K.). See also Cory and Sasson [19], Ariatti, Grohmann, and Venturini [3], Vol. 1, Farmer, James, and Wells [26], and Tyren [81], Vol. 2. The well-known study of Kirchmayer [46] used Lagrange multipliers to solve the load dispatching problem.
future investment policies and select the optimum. For each investment policy they cost, they must of course simulate system operation and calculate optimum operating schedules and costs. Simulation models are therefore a special case of global models.

This does not mean however that simulation models are superseded by global models. To arrive at a uniquely optimum policy in one computer run is perhaps asking too much. A formulation of a global model necessarily entails approximation. But once approximate global solutions have been reached, they can be, and are, examined in more detail by a marginal analysis using simulation models. In this way, as Bessière and Petcu have argued, global models and marginal analysis using simulation models are complementary techniques.\(^2\)

The first global models to be developed were formulated as linear programs. In recent years, some workers have turned to non-LP formulations on the grounds that they are computationally more efficient. We shall consider (1) linear programming, (2) non-linear programming, and then return to (3) an LP reformulation of the global models which appears to be at least as computationally efficient as the non-LP formulations.

\(\square\) (1) Linear programming. The following formulation borrows in particular from Massé, Bessière, and Whitting and Forster.\(^3\) It is very similar to the formulation in Section 2, except that, as with the LP simulation model, we break down the load duration curve into \(p = 1 \cdots P\) discrete blocks. Adding the capital cost terms to the generation costs, the investor's objective is to choose the investments \(X_{jv}\) over \(v = 1 \cdots T\) and the associated operating decisions \(U_{jtp}\) over \(t = 1 \cdots T\) and \(p = 1 \cdots P\), so as to minimize total discounted system costs.

\[
\text{Minimize} \sum_{j=1}^{J} \sum_{v=1}^{T} C_{jv}X_{jv} + \sum_{j=1}^{J} \sum_{v=1}^{T} \sum_{v=1}^{V} \sum_{p=1}^{P} F_{jtp}U_{jtp} \theta_{p} \tag{19}
\]

subject to the following constraints:

(1) The plant in operation must be sufficient at all times to meet the instantaneous power demand:

\[
\sum_{j=1}^{J} \sum_{v=1}^{T} U_{jtp} \geq Q_{tp}, \quad t = 1 \cdots T, \quad p = 1 \cdots P. \tag{20}
\]

(2) The output of each plant must not be greater than the available capacity. In general, the available capacity is somewhat lower than actual capacity on account of planned outage (maintenance) and unplanned outage (faults). If the availability factor for plant \(X_{jv}\) in year \(t\) is \(a_{jvt}\), this constraint is then:

\[
U_{jtp} \leq a_{jvt}X_{jv}, \quad j = 1 \cdots J, \quad v = 1 \cdots V, \quad t = 1 \cdots T, \quad p = 1 \cdots P. \tag{21}
\]

\(^{2}\) See [11].

\(^{3}\) See [62], [10], and [29,30] respectively.
(3) There will be an upper limit to the hydro energy available in any period \( t \). Let this limit for each vintage be \( H_{v,t} \). Then:

\[
\sum_{p=1}^{P} U_{htv} \cdot \theta_{v} \leq H_{v,t}, \quad t = 1 \cdots T
\]

\[
\sum_{j=1}^{J} v = V \cdots t \quad j = h \text{ (hydro)}. \tag{22}
\]

(4) Equality constraints represent the initial capital stock. Let the plant initially on the system be denoted by \( \bar{X}_{jv}, j = 1 \cdots J \) and \( v = -V \) to 0. Then:

\[
X_{jv} = \bar{X}_{jv}, \quad j = 1 \cdots J
\]

\[
v = -V \cdots 0. \tag{23}
\]

(5) To guarantee peak power supplies to an acceptable probability limit, the installed capacity must be sufficient to meet the mean expected demand with a margin of reserve capacity \( m \) to allow for demands above the mean expectations:

\[
\sum_{j=1}^{J} \sum_{v} X_{jv} \geq Q_{tp}(1 + m), \quad t = 1 \cdots T \tag{24}
\]

Similarly, there may be a guarantee condition for energy supplies, requiring a constraint similar to (7). A proper study of this condition will generally require a seasonal model, which adds substantially (but not prohibitively) to the dimensions of the model. Note however that the effect of (7) is essentially to limit the ratio of hydro to thermal plant on the system. As a shortcut therefore, but one evidently involving assumptions which may sometimes be rather approximate, an annual model can still be used but with a restriction on the ratio of hydro to thermal plant on the system. This ratio can be ascertained from separate reserve and reliability studies.

(6) Finally, there are a number of "local" and other constraints. For example, the number of hydro sites may be limited, certain decisions may be political, the future investment program may have been partly determined by previous studies, and so on. Constraints to represent shortages of capital and foreign exchange may also be introduced. All decision variables are of course non-negative.

The constraint matrix has a very simple form. The coefficients are mainly zeros and ones, and fall into regular patterns. Matrix generator programs can be written to produce the constraint matrices. This cuts down on data and input preparation considerably. Since the constraint matrix is not very dense, computation can be very fast. Although this is the simplest form of the global model, much has been, and can be, accomplished with it, and the use of linear programming and its extensions by the electricity supply industry is common practice in many countries. Bessière\(^{64}\) informs us that LP formulations were first studied for Electricité de France in the mid-1950s by Massé and Gibrat, who published their results in 1957.\(^{65}\)

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\(^{64}\) In [9].

\(^{65}\) In [63]. This was soon followed by a paper by Massé and Bessière [10], a comparison of which with the following papers will show that the principles
The above model can be extended in many directions to include, for example:

1. Optimum replacement;⁶⁶
2. Optimum locations of plant and directions of bulk energy transmission;⁶⁷
3. Optimum storage capacities and storage policies for hydro-electric plant, including special constraints on the operation of multi-purpose hydro schemes involving electricity generation;
4. Integer variables to represent the large fixed-cost component of hydro and nuclear schemes and transmission equipment;⁶⁸ and
5. Nuclear fuel cycling.⁶⁹

Examples of the first three extensions are presented in Section 6. The simulation models corresponding to each of these extensions follow in a rather obvious way by pre-defining all the capacity variables and using the LP to search for optimum dispatching schedules only.

A difficulty with LP models of the above type is the large number of constraints encountered in any realistic formulation of a problem. The principal cause of this is constraint (21): we must ensure that the output of every plant on the system in every year of the study and on every interval of the load duration curve does not exceed its maximum available capacity. This is the same problem as was raised in connection with the LP simulation models; this time however, the \( X \)'s are not constants but decision variables, and the constraint cannot therefore be handled by bounded variable LP methods. If there are on the average 20 plants on the system, and we break the load duration curve into ten discrete intervals and take a 30-year period broken down into 6 by 5-year intervals, then we have about 1,200 constraints of this type and about 1,500 constraints in the problem. This is quite a large linear program, although standard computer programs are now available which can handle up to 10,000 constraints with mixed integer/continuous variable facilities;⁷⁰ and the use of matrix generators and "report writers" make data preparation and output processing quick and simple.

(2) Non-linear programming. In the early 1960s computers could not handle anything like this number of constraints. Bessière and

remain the same, whatever the country and whatever the date: Whitting and Forster [29,30]; Nitu et al. [67]; United Nations Symposia [82,84]; and papers by Eibenschutz [24], Frankowski [31], and Deonigi [20] in the 1970 Symposium held by the International Atomic Energy Authority (IAEA) in Vienna. Among the richest sources of information known to the present writer are the Power Systems Computation Conferences (PSCC) held in London (1963), Stockholm (1966), and Rome (1969), published by the Department of Electrical and Electronic Engineering, Queen Mary College, University of London.

⁶⁶ See Massé [62].
⁶⁷ See Fernandez, Manne, and Valencia [28].
⁶⁸ See Gately [33] and Fernandez, Manne, and Valencia [28].
⁶⁹ See Frankowski [31].
⁷⁰ This is the capacity of the OPHELIE II LP System for the Control Data Corporation 6600 Computer. We have made considerable use of this very powerful system at the IBRD via a remote batch terminal connected to the CDC CYBERNET nationwide computer system.
Albert and Larivaille\textsuperscript{61} report that in 1958, 180 constraints and 200 unknowns approached the maximum that computers could handle at that time; apparently the main reason why Electricit\'e de France turned to non-linear programming was to overcome this constraint problem. For the all-thermal system of the U. K., Phillips et al. also developed a non-LP model;\textsuperscript{62} it will be described here briefly in order to show how non-linear programming overcomes to a large extent the constraint problem.

The idea is to prearrange all the plants that are or may be connected to the system in any year in "merit order" in the data input. That is, the operating sequence is decided in advance by inspecting the marginal operating costs before the computer run is commenced. By this device, all the operating variables and their associated capacity constraints can be satisfied implicitly and deleted from the formulation.

To reduce notation, we shall drop the subscript \( t \) until needed. Moreover, we shall represent plant type \( j \), vintage \( v \), by a single subscript, \( w \), where \( w = 1, 2, 3 \ldots W \), and \( W \) is the total number of plant of all vintages in year \( t \). We let \( X_w \) be the available power capacity of thermal plant \( w \), \( U_w \) its power output at any instant, and \( F_w \) its operating cost. The ordinate on the demand duration curve (Figure 6) is denoted by \( x \), where \( x \) is of course in units of power demand, and we denote the duration of demand \( x \) by \( g(x) \). Now define the subscripts \( w \) such that their sequence locates the plant in merit order, as follows:

\begin{equation}
0 \leq F_1 \leq F_2 \leq F_3 \cdots \leq F_w \cdots \leq F_W, \tag{25}
\end{equation}

where \( F_w \) is the new notation for the operating costs of plant \( j \), vintage \( v \), in year \( t \).

The cost of operating plant \( w \) in merit order is then given by

\begin{equation}
\int_{U_{w-1}}^{U_w} F_w g(x) dx = F_w (G(\bar{U}_w) - G(\bar{U}_{w-1})), \tag{26}
\end{equation}

where

\[
G(\bar{U}_w) = \int_0^{\bar{U}_w} g(x) dx, \quad \text{and} \quad \bar{U}_w = \sum_{w'=1}^{w} U_{w'}. 
\]

Adding (26) over \( w = 1 \) to \( W \) gives the total operating cost (TOC) to be\textsuperscript{63}

\[
\text{(TOC)} = \sum_{w=1}^{W} (F_w - F_{w+1}) G(\bar{U}_w),
\]

which, after substitution for \( \bar{U}_w \), becomes

\[
\text{(TOC)} = \sum_{w=1}^{W} (F_w - F_{w+1}) G(\sum_{w'=1}^{w} \bar{U}_{w'}). \tag{27}
\]

A reasonable simplifying assumption can be made to further reduce the size of the problem:\textsuperscript{64} that the available plant capacity is

\begin{itemize}
\item \textsuperscript{61} In [9] and [1], respectively.
\item \textsuperscript{62} See [69].
\item \textsuperscript{63} Note that \( G(\bar{U}_0) = 0 \), and we use the convention \( F_{W+1} = 0 \).
\item \textsuperscript{64} This assumption is not made by Phillips et al. [69].
\end{itemize}
$X_w$ will be operated at full power in the interval $w$ to $w + 1$, so that $U_w = X_w$ exactly. Therefore,

$$\text{(TOC)} = \sum_{w=1}^{W} (F_w - F_{w+1})G(\sum_{w'=1}^{w} X_{w'}).$$

(28)

Reintroducing the subscripts $j$, $v$, and $t$ into the formulation, we find that the investor's objective is

Minimize $\sum_{j=1}^{j} \sum_{i=1}^{T} G_{jv} \cdot X_{jv} + \sum_{i=1}^{T} \sum_{j=1}^{j} \sum_{v=1}^{v} (F_{jv} - F_{jv+1})G_{jvt}$,  (29)

where

$$G_{jvt} = G(\sum_{i=1}^{i} \sum_{u=1}^{u} X_{iu})$$

Subject to $\sum_{j=1}^{j} \sum_{v=1}^{v} X_{jv} \geq \hat{Q}_t$,  $t = 1, \ldots T$,  (30)

where $\hat{Q}_t$ is the peak power demand in year $t$.

Thus it is possible to represent the operating cost explicitly in terms of the plant capacities of the system, their operating costs, and the shape of the load duration curve. No special algorithm is needed to schedule the plant optimally. All variables $U_{jv}$ satisfy the operating capacity constraints implicitly. Moreover, the demand constraints are satisfied implicitly except at times of peak demand. The formulation thus accomplishes an enormous reduction in the number of constraints to be satisfied—at the expense, however, of a complex, nonseparable, but convex objective function.

Apparently this model is now in constant use in the U. K. for the evaluation of investment plans. The non-linear program of Electricité de France has also been in use for several years.\(^{56}\)

The problem with turning to non-linear programming is that we lose very considerable advantages of LP computer software, which include flexibility, very versatile management processing systems, input and output processors, and integer facilities. Also not to be underestimated is the fact that the LP formulations are simpler and can be readily rewritten to cover other problems such as replacement, bulk electrical energy transmission, hydro storage policy, and multi-purpose projects.

The question arises then, Can we retain an LP form and yet reduce the constraint problem? The answer is that we can. The reason why the non-LP model reduces the number of constraints is not because it is intrinsically more efficient than linear programming; it is because in the non-LP model we include a priori information about system operating characteristics which we exclude from LP models. It is because this information is excluded that we get so many constraints in the LP model. By including it, we can reduce the problem size to virtually the same numerical proportions we encounter with non-LP forms.

\(^{56}\) See Bessière [8], Albert and Larivaille [11], and the references in Bessière's review [9].
Recovering an LP form. The way we include the information is as follows.\(^\text{66}\) We know that as we move along the time axis of the load duration curve the output of any plant will not be increased. In fact it will either remain the same as it was before, or it will be reduced. Suppose that we define new operating decision variables, \(Z\)'s, to replace the \(U\)'s, which represented the output of each plant. These \(Z\)'s (which we will call \(Z\)-substitutes) are defined to be the decrease in output of any plant as we move along the load duration curve. For any plant \(j\), \(v\):

\[
Z_{jtv} = U_{jtv} - U_{jtv,p+1} \geq 0, \quad p = 1 \ldots P - 1 \tag{31}
\]

with

\[
Z_{jtv} = U_{jtv} \geq 0. \tag{32}
\]

As we move along the load duration curve (that is, as load decreases) the power output of plant \(j\), \(v\), is never increased;\(^\text{67}\) it follows that the sum of power reductions from \(p = 1\) to \(P\) is less than the available power capacity of plant \(j\), \(v\). Hence

\[
\sum_{p=1}^{P} Z_{jtv,p} \leq a_{jst} \cdot X_{js}, \quad \text{all } j, t, v. \tag{33}
\]

This constraint, together with the non-negativity constraints on \(Z_{jtv,p}\), are necessary and sufficient conditions for constraints (21) to be satisfied. First, in view of (32), \(U_{jtv} = 0\) if \(Z_{jtv}\) is non-negative, and it follows from (31) that if \(Z_{jtv}\) is non-negative for all \(p\), so must be \(U_{jtv}\). Second, in view of (33) no combination of the values of \(Z_{jtv,p}\) can exceed \(a_{jst} \cdot X_{js}\). From (31) and (32) we derive the relation that

\[
U_{jtv} = \sum_{p=1}^{P} Z_{jtv,p} \leq a_{jst} \cdot X_{js}. \tag{34}
\]

Hence \(U_{jtv}\) cannot exceed \(a_{jst} \cdot X_{js}\) if (31), (32), and (33) are satisfied.

We can thus replace the \(U\)'s, which had to satisfy plant capacity constraints on every portion of the load duration curve, with new non-negative variables, the \(Z\)-substitutes, which satisfy one constraint for the whole curve. Approximately, the number of constraints is reduced by \(1/P\). The 1,500-constraint problem we mentioned earlier\(^\text{68}\) is now reduced to 150 constraints. It has also been our experience that, although the density of the constraint matrix is increased, computing times have been reduced by a factor of 2 or more (sometimes by a factor of 5).

This is a useful result for those who prefer to work with the simpler LP models. For those directly involved with investment planning in the industry, it means that computational problems are kept to a manageable size, without loss of generality and with many advantages in terms of computer software. If the cost coefficients are non-linear (as happens for example with studies of hydro resources),

\(^{66}\) I am grateful to Ivan Whitting of the National Gas Council, U. K., for pointing out the following device to me. The idea was apparently suggested by E. M. L. Beale during a conversation. It has apparently not been published previously.

\(^{67}\) Unless seasonal variations in hydro flows and maintenance schedules are important. In such a case it is necessary to use one load duration curve for each season, and the above statement holds once again for each load duration curve.

\(^{68}\) Section 5(1).

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the objective function is separable so that non-linearities can be treated by interpolation and the LP form recovered. For those involved with economic research it means that global models based on linear programming can provide a realistic view of the investment problem—at least from the supply side. If demand is introduced as an endogenous variable, however, we have to return to non-linear programming, particularly if peak and off-peak demands are considered interdependent since the objective function is then no longer separable.

6. Three extensions to LP models

This contraction in the size of the problem, together with the rapid expansion in the size of linear programs that can be handled on computers, also enables us to expand the detail and content of both global and simulation models. Below we look at three extensions that are being worked on at the International Bank for Reconstruction and Development: replacement, an approximate treatment of transmission, and investment and operating decisions in systems with hydro storage schemes.

☐ (1) Inclusion of replacement. Optimum replacement of a power station usually occurs when it is cheaper to expand and operate the power system without this power station. This may arise because of rising operating and maintenance costs relative to those of new plant, or because sites for new power stations are short and old ones need to be scrapped to make room for new and larger ones.

An accurate treatment of replacement requires explicit separation of fixed annual operation and maintenance costs from other costs. Usually these costs are added to the annuitized charges on capital, while the variable maintenance costs are added to the other variable operating costs. Following Massé we could define a new decision variable \( X_{jv}^* \) to represent the plant scrapped of type \( j \), vintage \( v \), and write the capital cost terms in the objective function as \( C_{jv}(X_{jv} - X_{jv}^*) \), where \( C_{jv} \) equals the fixed annual costs plus annuitized charges on capital. This can however lead to solutions which scrap plant before capital costs have been accounted for by the annuities. Since the decision to scrap new plant requires a new decision variable, we can associate this variable directly with fixed maintenance and operating costs.

We denote by \( M_{jv} \) the discounted, fixed maintenance, and operating costs of plant type \( j \), vintage \( v \), in year \( t \). The problem is to decide how much of this plant should remain in service in year \( t \). Let \( R_{jtv} \) be the amount of type \( j \), vintage \( v \), remaining in service in year \( t \). The objective and the constraints follow much the same pattern as before. Using the same notation,

Objective

Minimize

\[
\begin{align*}
& \sum_{j=1}^{J} \sum_{v=1}^{V} C_{jv} \cdot X_{jv} + \sum_{j=1}^{J} \sum_{v=1}^{V} \sum_{t=1}^{T} M_{jtv} \cdot R_{jtv} \\
& + \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{v=1}^{V} \sum_{p=1}^{P} F_{jtvp} \cdot U_{jtvp} \cdot \theta_p.
\end{align*}
\]

(35)
Constraints

Total plant remaining greater than peak demand requirements
(plus an allowance for reserves),
\[ \sum_{j=1}^{J} \sum_{v=1}^{V} R_{jv} \geq Q_{tp}(1 + n) , \quad t = 1 \cdots T, \quad p = 1 \]

Plant types \( j, v \), remaining less than plant installed of type \( j, v \),
\[ R_{jv} \leq X_{jv} \quad j = 1 \cdots J \]
\[ t = 1 \cdots T \]
\[ v = 1 \cdots T \]

Plant remaining of given type \( j, v \), never increases,
\[ R_{j,t+1,v} \leq R_{jv} \quad j = 1 \cdots J \]
\[ t = 1 \cdots T \]
\[ v = 1 \cdots T \]

Total plant operating always sufficient to meet demand,
\[ \sum_{j=1}^{J} \sum_{v=1}^{V} U_{jvp} \geq Q_{tp} \]
\[ t = 1 \cdots T \]
\[ p = 1 \cdots P \]

A plant’s output never exceeds remaining available capacity,
\[ U_{jvp} \leq a_{jv} R_{jv} \quad j = 1 \cdots J \]
\[ t = 1 \cdots T \]
\[ v = 1 \cdots T \]
\[ p = 1 \cdots P \]

Restrictions on energy available from each hydro plant,
\[ \sum_{p=1}^{P} U_{hpv} \theta_p \leq H_{vt} \]
\[ t = 1 \cdots T \]
\[ v = -V \cdots T \]

Finally, there are "local" and non-negativity constraints, and initial conditions. Z-substitutes can of course be used in this formulation. Constraints (36) are the "guarantee conditions" for peak power; constraints to guarantee energy supplies can also be introduced in the ways previously discussed.

\[ (2) \text{ Approximate inclusion of transmission.}^{70} \] Transmission systems reduce costs of supply in four ways. First, if regions are interconnected, in the event of generator failure in one region, it is possible to call upon the reserves of other regions; aggregate reserve capacity with interconnection is less than is required without interconnection. Second, if peak demands occur at different times in different regions, then interconnection permits peak power capacity to be exported and imported, and the aggregate peak demand can be met with less capacity. Third, if the transmission system is designed to transmit energy in large quantities, the markets of regions rich in energy resources (fossil or hydro) can be expanded, and regions less rich can import the cheaper energy. Fourth, with interconnection, larger units can be installed embodying considerable economies of scale. These four aspects of the transmission system—the pooling of reserve capacity, the pooling of peak capacity, the opening of markets to regions rich in low-cost resources, and economies of scale—can make large savings as compared to the costs of the transmission lines and transmission losses.

\[ ^{70} \text{I am very grateful to Mr. Narong Thananart, who corrected some mistakes in the original formulation and who wrote a matrix generator for use in case studies.} \]

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In the following formulas, regions of generation are denoted by integers \( g = 1 \ldots G \), and regions of demand by integers \( d = 1 \ldots D \). \( Y_{gde} \) denotes the increment of transmission capacity (expressed in MW) connecting \( g, d \), installed in year \( v \); \( L_{gdv} \) is the discounted cost per MW installed of this increment. The power delivered to region \( d \) by station type \( j \), vintage \( v \), in period \( t \), \( p \), from region \( g \) is denoted by \( U_{jtvpgd} \); and its total output is \( \sum_d U_{jtvpgd} \). The capital and operating costs are as before, except that they must now be summed over all regions. A cost term is also required for the transmission of capital costs.\(^{71} \) The objective function is therefore

\[
\begin{align*}
\text{Minimize} & & \sum_{g=1}^{G} \sum_{j=1}^{J} \sum_{t=1}^{T} C_{jgv} \cdot X_{jvg} \\
& & + \sum_{g=1}^{G} \sum_{d=1}^{D} \sum_{v=1}^{V} L_{gdv} \cdot Y_{gde} \\
& & + \sum_{g=1}^{G} \sum_{d=1}^{D} \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{p=1}^{P} F_{jtvpgd} \cdot U_{jtvpgd} \cdot \theta_p .
\end{align*}
\] (42)

The first constraint to be satisfied is the peak power guarantee condition that the installed capacity must be greater than the peak load presented at the power stations' terminals by a margin \( m \), to allow for demands above mean expectations.\(^{72} \) We also introduce a "diversity factor" \( c \), which is the ratio of the aggregate peak demand to the arithmetic sum of the regional peak demands. The capacity requirements are then

\[
\sum_{g=1}^{G} \sum_{j=1}^{J} \sum_{t=1}^{T} X_{jvg} \geq (1 + m)c \sum_{g=1}^{G} \sum_{d=1}^{D} \sum_{j=1}^{J} \sum_{t=1}^{T} U_{jtvpgd} ,
\] (43)

\[ t = 1 \ldots T, \quad p = 1 . \]

The diversity factor might have to be modified if the solutions suggest a different pattern of connections than the pattern used for computing the factor. Transmission capacity must also be sufficient to carry the peak load transfers:

\[
\sum_{g=1}^{G} \sum_{d=1}^{D} \sum_{t=1}^{T} Y_{gde} \geq (1 + m) \sum_{j=1}^{J} \sum_{t=1}^{T} U_{jtvpgd} ,
\] (44)

\[ t = 1 \ldots T, \quad p = 1, \quad g = 1 \ldots G \]

\[ d = 1 \ldots D . \]

Next, the plants' output must meet the demand and the transmission losses. If \( b_{gd} \) is the per-unit power attenuation between \( g, d \), then the plant must be operated in each period \( p \) such that:\(^{71} \)

\[
\sum_{u=1}^{P} \sum_{d=1}^{D} \sum_{j=1}^{J} U_{jtvpgd} (1 - b_{gd}) \geq Q_{dtp} ,
\] (45)

\[ t = 1 \ldots T, \quad p = 1 \ldots P, \quad d = 1 \ldots D . \]

\(^{71} \) We do not treat replacement in this model, although it is obviously quite possible to do so if required. Integer variables can also be introduced to represent the indivisibilities of transmission investments, but this is not done below.

\(^{72} \) Of course, there may also be a guarantee condition for energy supplies, which we do not list here.

\(^{72} \) The power attenuation will vary with the square of the load transfer between \( g, d \). This would make constraint (45) quadratic. We can only retain linearity by taking \( b_{gd} \) as a weighted average value (in fact a "mean-square" average).
Next we have the constraints that no plant can be operated above its peak available capacity:

\[
c \cdot \sum_{d=1}^{D} U_{j \nu p g d} \leq a_{j \nu g} \cdot X_{j \nu g}, \quad j = 1 \cdot J \\
y = - V \cdot t \\
t = 1 \cdot T \\
p = 1 \cdot P \\
g = 1 \cdot G.
\]

(Again, these constraints can be reduced by \(1/P\) by the use of non-negative \(Z\)-substitutes.) The hydro-energy constraint takes the same form as before:

\[
c \cdot \sum_{d=1}^{D} \sum_{p=1}^{P} U_{h \nu p g d} \cdot \theta_p \leq H_{v t g}, \quad t = 1 \cdot T \\
y = - V \cdot t \\
g = 1 \cdot G.
\]

Finally, there are "local," budget, foreign exchange, and non-negativity constraints; constraints to represent the initial conditions; and a constraint to guarantee energy supplies.

☐ (3) Inclusion of water-storage capacity and operating policy variables. We now formulate the model to search for least-cost, evolving investment programs to satisfy an exogenous demand for electricity, a planned delivery of water to irrigation, and a planned degree of flood control. The peak storage capacity and the amount stored in each season are treated as decision variables. The model can easily be couched in a regional context, as has been done elsewhere, to allow for the strong regional dependence of hydro resources and the high economies of scale which they may yield if the interconnected system is large enough to absorb them. However, to reduce notation, this is not done below; the principles are in any case identical to the ones presented in subsection (2) above.

Thermal schemes will now be denoted by subscript \(j = 1 \cdot J\) and hydro storage schemes by \(h = 1 \cdot H\). As before, maximum power capacities will be denoted by the decision variable \(X\), the instantaneous outputs by \(U\), and incremental capital and generation costs by \(C\) and \(F\), respectively. Each year will be denoted by \(t\) and divided into \(m = 1 \cdot M\) periods, which can represent months, seasons, weeks, etc., according to the accuracy desired. The demands within each period \(m\) will be represented by a load duration curve divided into \(p = 1 \cdot P\) blocks. We have the following additions to notation for the hydro schemes:

\[
\begin{align*}
\hat{S}_{h r} &= \text{decision variable representing the maximum storage capacity (expressed in energy units) of scheme } h, v; \\
K_{h r} &= \text{corresponding incremental capital cost of providing the storage capacity;} \\
S_{h t r m} &= \text{decision variable representing the actual water in storage (expressed in energy units) of scheme } h, v, \text{ at the beginning of } m \text{ in year } t; \text{ and}
\end{align*}
\]

I have benefited very much from discussions with Dr. Ralph Turvey on this model (as I also have in this paper from his writings). In particular he pointed out to me the economic significance of the Kuhn-Tucker conditions of the model.

See Anderson [2].
$W_{htvm} =$ water inflows to scheme $h, v,$ during $m$ of year $t,$ expressed in energy units and corrected for losses due to evaporation and seepage.

The objective function is then:

$$
\text{Minimize}
$$

$$
\sum_{j=1}^{J} \sum_{i=1}^{T} C_{ji} \cdot X_{ji} + \sum_{h=1}^{H} \sum_{t=1}^{T} C_{h} \cdot X_{h} + \sum_{h=1}^{H} \sum_{t=1}^{T} K_{h} \cdot S_{h} + \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{p=1}^{P} F_{jitmp} \cdot U_{jitmp} \cdot \theta_{p} + \sum_{h=1}^{H} \sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{p=1}^{P} F_{htmp} \cdot U_{htmp} \cdot \theta_{p}.
$$

Note that the last term is small for hydro schemes, and also that there is no cost term associated with actual water in storage, $S_{htvm}$.

The following constraints are exactly analogous to the ones presented above: (1) installed capacity must be greater than or equal to the annual peak demand plus a margin for reserves; (2) there must be sufficient energy reserves to meet energy demand in dry seasons; (3) the aggregate plant output must meet the instantaneous power demand at all times; (4) no plant can be operated above its peak available capacity [again, (4) can be reduced by $1 - P$ using $Z$-substitutes], and (5) “local” constraints, initial conditions, etc.

The additional constraints introduced by storage hydro are as follows. First, the water stored at the end of period $m$ (beginning of $m + 1$) plus the water used for generation is less than or equal to the initial storage plus the inflow:

$$
S_{htvm,m+1} + \sum_{p=1}^{P} U_{htmp} \cdot \theta_{p} \leq S_{htvm} + W_{htvm}, \quad (49)
$$

for $m = 1 \cdots M - 1$; and for $m = M$:

$$
S_{htvm,M+1} + \sum_{p=1}^{P} U_{htmp} \cdot \theta_{p} \leq S_{htvm} + W_{htvm}, \quad (50)
$$

where (49) and (50) must of course be satisfied for all $h, t, v,$ and $m$.

If a hydro scheme is multi-purpose, involving irrigation and flood control, there will be further restrictions on the timing and the rate of energy output. Suppose the water requirements of irrigation in period $m$ of year $t$ are $I_{htvm}$. There will still be considerable flexibility in the pattern of discharge within the period $m$ (e.g., choice between night and day discharges or between weekdays and weekends); but the aggregate amount of water discharged through turbines must at least be equal to the requirements of irrigation.$^{76}$

$$
\sum_{p=1}^{P} U_{htmp} \cdot \theta_{p} \geq I_{htvm}, \quad \text{all } h, t, v, m. \quad (51)
$$

$^{76}$ We also assume in this example that all the water, other than that lost by spillage and seepage, is discharged through turbines. If desired the assumption can be relaxed.
Flood control, on the other hand, sets an upper limit to the rate of discharge in certain periods. Let $D_{htvm}$ represent this limit in period $m$. Then the water discharged from the hydro, minus the quantity diverted to irrigation, must not exceed this limit:

$$
\sum_{p=1}^{P} U_{htvmp} \cdot \theta_p \leq I_{htvm} + D_{htvm}, \quad \text{all } h, t, v, m.
$$

(52)

This completes the present formulation. The approach is very flexible and new features can readily be introduced. If hydro schemes are large, they can be expanded in stages instead of being introduced in one period, $v$. Variable head schemes, pumped storage schemes, and multi-dam cascade schemes can also be given a full analysis. Approximations can be introduced in the representation of the costs of electric power from thermal stations so as to allow room for more detail elsewhere—for example, in the representation of multi-dam schemes.

Appendix

The difficulty of calculating optimum operating schedules and costs is presented by the high variability of power demand, which varies throughout the day and throughout the year (see Figure 7). The operating costs are the area under this curve weighted at each time interval $\theta_t$ by the fuel costs and the outputs of the plant in that interval. To simplify the calculation of operating costs it is usual to construct a curve known as the load duration curve. This curve is constructed from the above demand curve by rearranging each load for each time interval $\theta_t$ to occur in descending order of magnitude (see Figure 8). The operating costs are thus the area under the load duration curve again weighted at each time interval by the operating costs per unit energy output and the output of each plant operating in that interval. The load duration curve makes integration of costs less difficult because it can be represented by simpler functions than the curves of Figure 7. It is a convenient form to check approximations to the patterns of demand shown in Figure 7; and we can also use $Z$-substitutions if we use the load duration curve.

Use of the load duration curve for calculations of operating costs introduces one important assumption: that the costs and availability of supply depend only upon the magnitude of the load and not on the time at which the load occurs. This assumption is quite accurate for all-thermal systems (although plant availability, because of maintenance schedules, is seasonal), but it is only approximate for hydro schemes. To analyze hydro operation accurately it is often necessary to construct a separate load duration curve for each season and sometimes each month; if this is done the assumptions of the load duration curve are tenable again.

References


77 See Hufschmidt and Fiering [42] and Maass et al. [55].

78 See also Berrie [6], Openshaw-Taylor [68], and Kirchmayer [46].
FIGURE 8
LOAD DURATION CURVE
(FOR ONE YEAR'S DEMAND)

DEMAND (LOAD) X MW

DURATION OF LOADS IN EXCESS OF X

1 YEAR


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