A New View of Economic Growth

Four Lectures

Maurice FG. Scott
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(Continued on the inside back cover.)
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Maurice FG. Scott
FOREWORD

Maurice FG. Scott was a Visiting Scholar in the International Economics Department of the World Bank from October-December 1990. During this period, he presented a series of lectures to World Bank staff based on his recent book, *A New View of Economic Growth*. These lectures contributed to the World Bank's ongoing research efforts to understand the determinants of long run growth in developing countries. In particular, Mr. Scott's research emphasized the importance of investment, instead of disembodied technical progress, in generating growth. In addition, his work illustrates the importance of correct measurements of both human and physical capital.

The four lectures in this volume summarize Mr. Scott's theory of economic growth. The first lecture presents a critique of conventional growth theory and the second explores the origins of investment opportunities. The third lecture presents Scott's theory of growth based on material investment and quality-adjusted employment, including the effects of human investment. The final lecture presents empirical results which support Scott's new view of economic growth.

Ronald Duncan
Acting Director
International Economics Department
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INTRODUCTION AND SUMMARY

The following four lectures were given at the World Bank in October 1990. In them, I have tried to explain some of the main ideas and findings in my recently published book *A New View of Economic Growth* (Oxford, Clarendon Press, 1989).

The first lecture starts with a parable which will, I hope, enable readers to see a fundamental mistake that has been made, and still goes on being made, in the analysis of economic growth. This is to neglect the fact that changes in the values of capital assets occur as a result of changes in relative prices as well as a result of physical changes. Only the latter can directly explain changes in output, but growth theorists have implicitly argued as if the former could as well. Because relative price changes have, in aggregate, reduced capital values, the effect has been to understate, by a very large amount, the contribution of investment to growth. This has resulted in the famous residual in growth accounting studies which has been attributed to "technical progress", which is independent of investment. There are some other radical implications. Depreciation of material capital is offset by appreciation of human capital as real wage rates rise in progressive economies, and so gross national product is a better measure of national income than is net national product. Total factor productivity cannot be measured in the conventional way. I am doubtful whether the production function has a role to play in the analysis of economic growth. It seems better to abandon the attempt to explain both the level and the change in the level of output. Instead, one should take some starting point as a datum and only seek to explain subsequent changes -- the approach that any historian would adopt. These are controversial conclusions which imply that much analysis in this field is flawed, not just marginally but substantially. I do not expect them to be swallowed easily, but I have yet to see them controverted.

The second lecture is rather more constructive than the first. Four different theories of the origin of investment opportunities are discussed, and one is favored. The view that opportunities must steadily deteriorate, as we undertake the best first and then move on to worse and worse ones, is quickly dismissed, since it does not allow for discovery and invention. The orthodox view that "technical progress", which is quite independent of investment, is responsible is challenged by Schmookler's evidence that inventions respond to investment, as well as by Hirschman's exposition of how one thing leads to another, and Arrow's idea of learning by doing. The favored conclusion is that investment, by changing the world, both creates and reveals new investment opportunities. There is neither reason nor evidence to suppose that there are diminishing returns to the stock of cumulative investment. "Technical progress" can then be made endogenous rather simply, and there seems to be no need to posit some additional input in the production function, such as accumulated knowledge, as has recently been suggested by several writers. Not only is that vulnerable to the objections made in the first lecture to the production function, but it multiplies entities unnecessarily.

The third lecture sketches out my own growth theory. Growth is proximately caused by only two things: material investment and the growth of quality-adjusted employment, which includes the effects of human investment. The analysis is of a steady growth equilibrium in which the typical firm invests at a constant rate and finds its investment opportunities continually renewed. It is never in static equilibrium, as change is always profitable, but the rate of change converges to a steady state. Material investment may be more or less labor intensive, and it is shown that a firm in steady growth will determine the labor intensity by reference both to the share of wages in value-added and to the share of investment. The greater is the latter, *cet. par.*, the lower will be the wage
in relation to the marginal product of labor. This provides a new theory of the distributive shares of wages and profits. The efficiency of investment, measured by a parameter $\rho$, is a key concept which features in the fourth lecture. $\rho$ may decline as the share of investment, $s$, increases, but diminishing returns to the rate of investment are not the same as diminishing returns to the capital stock in orthodox growth theory. In the latter, the long-run rate of growth is the same whether $s$ is 10, 20 or 50 percent of output. In my theory, by contrast, long run growth increases with $s$, though possibly at a diminishing rate. The theory can be applied to individual sectors as well as to a whole economy, and then the role of relative prices becomes important, and it is shown how $P$, the relative price, may substitute for $\rho$.

The final lecture looks at two empirical applications. In the first, a linear version of the basic growth equation is fitted to observations of trend rates of growth of output and quality-adjusted employment, and of average investment shares, in different periods and countries (all developed). The fit is good, and the most striking point to emerge is the tendency for $\rho$ to increase, especially after the second world war. Reasons for this are discussed. The second application concerns the large gap between average social returns and marginal private returns to investment in the USA and the UK in the "golden" post-war years. It is argued that this gap can be only partly explained by taxation, and is mainly evidence of a learning externality to investment. There is also reason to believe that imperfect markets result in a "market" externality, but that this was offset in these years by strong "animal spirits" which drove businessmen to expand output faster than strict present value maximization required. These externalities imply that investment is sub-optimal, and should, perhaps, be lightly taxed or not at all. As well, governments should save more themselves. But measures to increase the efficiency of investment may be still more important, and it is all kinds of investment which matter, and not just those sometimes conjured up by the words "technical progress".

While the four lectures are inevitably somewhat compressed and selective compared with the book, they are also shorter and so may reach a wider audience. I am grateful for the opportunity to address such an audience provided by the World Bank's visiting scholar programme, and for comments and encouragement received in particular from Paul Armington, Shahrokh Fardoust, Nemat Shafik and other members of the International Economic Analysis and Prospects Division, none of whom bears responsibility for mistakes that remain. I am also grateful for much assistance and typing undertaken by Jacquelyn Queen.
I. WHAT IS WRONG WITH ORTHODOX GROWTH THEORY?

By orthodox growth theory I mean the theory which appears in economics textbooks and forms the basis for innumerable empirical studies and which characterizes and explains economic growth as follows. At any one time producers have available to them a set of inputs, which can be subdivided into land, labor and capital, and also a given state of knowledge about how to combine these inputs so as to produce useful output. As time passes, there are changes in the quantities of inputs available. Generally one can expect that there will be more labor and capital, but little if any more land. These increased inputs make it possible to produce more and more output, and are an important part of the explanation for the actual growth in output. But an additional very important part of the explanation is that knowledge of how to combine inputs increases, so that more output can be produced from a given set of inputs, or more output per unit of input. Thus an increase in what is called total factor productivity, as well as an increase in the quantities of inputs, is important in explaining why growth in output occurs.

There are, naturally, many qualifications to this story, relating for example to changes in the ratio of actual output to capacity output, or to possible distinctions between the efficiency with which inputs are combined within the existing technology and changes in that technology. However, none of these is essential to the main argument I want to put before you. If this argument is right, then I think orthodox growth theory is clearly wrong, and ought to be not merely modified but simply abandoned. That is a pretty sweeping claim, made in the hope of provoking a response. Thus far my experience has been that no-one has shown me that the argument is wrong, but few are prepared to accept what seem to me to be the inevitable consequences of its being right. I hope I can persuade at least some of you to come down on to my side of the fence.

In order to do this, I have set out a simple arithmetical example in Table 1. Let me, without further ado, plunge straight into this.

We are observing what could be a firm, an industry or a whole economy, at two points in time, \( t_0 \) and \( t_1 \), separated by one year. Only workers are involved in production for the firm, industry or economy (I shall call it "industry" for short in what follows), and there is no land or capital. There are, however, two kinds of workers with different skills whom I have chosen to label by two convenient letters of the alphabet, namely, K-workers and L-workers, five of each kind at \( t_0 \). At \( t_1 \), the number of K-workers has increased to 5.1 (you can think of the units as being thousands or millions of workers if you like) but the number of L-workers is still just 5. At both \( t_0 \) and \( t_1 \), each worker in this simple example produces one unit of their type of output, thereby, so it seems, ruling out by assumption any changes in total factor productivity. I will return to that later on. The price of each unit of output is £10, both for K-type and L-type workers at \( t_0 \). However, the increase in the number of K-workers, resulting in an increase in their output, drives down its relative price from £10 to £9.8. Meanwhile, the average price level being stable (this is merely a convenient simplifying assumption), the price of L-type output rises to £10.2. These are the data.

The questions to be answered are two: what has been the industry's increase in output from \( t_0 \) to \( t_1 \)? How can it be explained in terms of changes in input and factor productivity?
TABLE 1.

<table>
<thead>
<tr>
<th></th>
<th>Number of workers, N, and their output since each worker produces one unit of output</th>
<th>Wage per worker, W, and also price of output</th>
<th>Value of output, NW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>at t₀ prices</td>
</tr>
<tr>
<td>At time t₀</td>
<td></td>
<td></td>
<td>£</td>
</tr>
<tr>
<td>K-type workers</td>
<td>5</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>L-type workers</td>
<td>5</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>At time t₁</td>
<td></td>
<td></td>
<td>£</td>
</tr>
<tr>
<td>K-type workers</td>
<td>5.1</td>
<td>9.8</td>
<td>51</td>
</tr>
<tr>
<td>L-type workers</td>
<td>5</td>
<td>10.2</td>
<td>50</td>
</tr>
<tr>
<td>Total</td>
<td>101</td>
<td>100.98</td>
<td></td>
</tr>
</tbody>
</table>

As usual, one can measure the change in output either at t₀ prices or at t₁ prices, or possibly at some third set of prices. As is clear from the table, whether one uses t₀ or t₁ prices makes little difference: the increase in output is 1 percent, or close to it.

One way of explaining this increase in terms of changes in input is as follows. The only change in the quantity of input is the increase of 2 percent in the number of K-workers, which increases their output by 2 percent. Since, whether at t₀ or t₁, they account for exactly half or close to a half of total output, they contribute the whole of the 1 percent increase in total output. L-workers do not change in numbers and so contribute nothing. There are no changes in factor productivity.

I may be mistaken, but I think I can claim that the above analysis of the data is uncontroversial. However, let me now introduce two characters to stir up a bit of controversy. The first of these is a prudent accountant. This fellow approaches the K-workers and points out that their wages are falling from t₀ to t₁ from £10 to 9.8. Furthermore, let us suppose, there are good grounds to expect similar increases in the number of this type of worker in future years and, consequently, still further falls in their (real) wages. It would be very imprudent, the accountant says, for K-workers to spend all their wages. They could not hope to maintain the standard of living to which they would become accustomed if, at t₀, they spent at the rate of £10 each per annum. What they ought to do, he suggests, is to subscribe to a special fund he happens to know about
which will pay them a very useful 10 percent per annum. If they subscribe at the rate of £2 per annum, over the course of the coming year they will have added £0.2 to their income from this fund, and that will precisely offset the drop of £0.2 in their earnings from production. This will leave them with £8 p.a. to spend, and that will be a sustainable, and so prudent, level of expenditure. The K-workers, let us suppose, fall for this sales-talk, and who am I to blame them? We may note, however, that the accountant ignores the L-workers completely. That does seem to me rather reprehensible.

Now enters the second character, who happens to be an economist enamored of the concept of human capital. He approves of the accountant’s mission to the K-workers, and points out that their subscription to the fund could be regarded as a payment corresponding to the depreciation of their human capital. Had K-workers expected no change in their earnings, those earnings could have been capitalized at £100, using the 10 percent rate of return of the fund as an appropriate capitalization factor. The drop of two percent in earnings over the year to \( t_1 \) would reduce this capitalization to £98, and this is nicely offset by the £2 subscribed to the fund.

There seems nothing wrong with this, but now he allows his enthusiasm to get out of control. The correct way to measure the change in the input of K-workers, he says, is by the change in the quantity of human capital that they put into the industry. There are two influences at work. On the one hand, each worker’s productive capital is falling by 2 percent. On the other, the number of workers is increasing by 2 percent (the subscription of £2 per worker to the fund is irrelevant since that is outside the industry). These two influences offset each other, so that the input of K-workers is, in sum, unchanged.

"What about L-workers?" one might reasonably ask. Unfortunately, the economist is here misled by the accountant. L-workers were ignored by the latter, and so they are by the former.

How, then, does the economist explain the one percent rise in the industry’s output? If the input of K-workers has not changed, as he argues is the case, and if he leaves the input of L-workers to be measured by the change in numbers, as indeed he does, there appear to have been no changes in input at all. However, that is no way disconcerts the fellow. What has happened, he explains, is that there has been an increase of 1 percent in total factor productivity, due to an increase in knowledge. That, then, explains the whole increase in the industry’s output.

Everyone must surely agree that this explanation is totally unacceptable. Not to mince words, it is rubbish. Our economist has thoroughly confused prices and quantities. The explanation I first provided of the change in the quantity of output was entirely in terms of a change in quantity of input, of K-workers. Our economist has brought the change in the price of their output into his reckoning, and that should be no part of an explanation of the change in the quantity of input although he has made it so. Furthermore, and even on his own terms, he can be accused of inconsistency in his treatment of K-workers and L-workers. If K-workers should subscribe £2 p.a. to a depreciation fund to offset the fall in their earnings, L-workers could equally borrow £2 p.a. from the fund to offset the rise in their earnings. Their human capital is appreciating by £2 p.a., i.e. by 2 percent. Were one to follow our economist’s procedure consistently one would have to attribute the whole 1 percent increase in output to this 2 percent increased input (on his reckoning) of L-workers. There would be nothing left for increased factor productivity. For my own part, it
seems to be more meaningful to attribute the extra output to the K-workers rather than the L-workers, and I suspect most will agree with that.

So far, then, have I said anything controversial? I hope not. But now I will. Although many will accuse me of grossly caricaturing the reality, I would like to point out the analogy between my imaginary economist’s analysis and that provided by orthodox growth theory. Let us call K-workers capital and L-workers labor (and forget about land, as growth theory often does). In measuring the change in capital input no-one that I know of actually counts machines, buildings, stocks of goods, etc. and weights each by its quasi-rent at constant prices. Instead, the procedure adopted is to equate the relevant change in capital input to gross investment minus depreciation (there is the alternative of gross investment minus scrapping, but this is open to objections at least as severe as those I am about to make, and which I discuss later). I have no quarrel with gross investment as a workable proxy for the increase in numbers of inputs, suitably weighted. This corresponds, in my parable, to the increase in the number of K-workers. But then depreciation is subtracted, just as our reprehensible economist subtracted it. Furthermore, the same inconsistency of treatment between capital and labor is followed by ignoring appreciation of workers. The same conclusion is erroneously drawn that there is a large increase in output unexplained by increases in input, and it is attributed to an increase in factor productivity due to increased knowledge.

Not so! I can imagine readers saying to themselves. The analogy does not hold. In my example depreciation occurs essentially and only because of a fall in relative prices or wage-rates. In the real world depreciation corresponds to the physical deterioration of capital goods. That being so, it is quite correct to ignore the first, the effect of price changes, while not ignoring the second --- which is similar to a decline in number.

This is, indeed, a key question. Is depreciation wholly, or largely, or not at all due to relative price changes, or, on the contrary, to physical changes of the assets concerned? I will admit that it could be due to either. However, I believe that, at least in progressive economies such as those in Europe, North America and the Far East, it is mostly due to relative price changes, and that it should be defined so as to be entirely due to them. Let us first consider how depreciation is in fact defined and the case for defining it in my way. Then I will consider the evidence for believing that in reality relative price changes account for most or all of depreciation as conventionally measured. If I succeed in this task, my parable should, I think, be taken very seriously indeed.

In conventional systems of national accounts, depreciation is termed "capital consumption", and covers both foreseen obsolescence and normal wear and tear. Obsolescence is due essentially to relative price changes, whereas wear and tear is a physical change. Since maintenance expenditures, which are a current cost of production, also counteract or offset wear and tear, the distinction between depreciation and maintenance is unclear. I think, however, that in the minds of many orthodox growth theorists depreciation is regarded as being essentially the same as maintenance. It covers the cost of replacing lumpy items of capital equipment, whereas maintenance deals with all the other, generally smaller, items. Depreciation is then a current cost of production on all fours with maintenance.
This view, however, does not square with that of some experts who, one would think, should have known what they were talking about. Tibor Barna, reporting on his empirical investigations into the stock of capital in UK manufacturing industries, remarked that, "it is obsolescence rather than wear-and-tear which is the dominant cause of mortality - homicide to make room for a new favorite, rather than natural death." (Barna, 1961, 85). He also stated that the physical efficiency of plant tended, very often, to increase rather than decrease with age, as a result of maintenance expenditures. "Increased costs of maintenance can rarely be encountered, since plant is generally scrapped before this occurs". He also pointed out that "Any improvement in the technical efficiency of the plant may, however, be offset by economic factors, generally a relative fall in the price of output. In fact it is the dominance of economic over the technical factors which makes it so difficult to obtain quantitative results" (concerning depreciation curves) (Barna, 1961, 91).

Simon Kuznets took the view that maintenance and repair covers physical deterioration while "capital consumption" is due largely to obsolescence. (Kuznets with Jenks 1961, 396). "We know that the physical life of equipment is far longer than is assumed in depreciation charges; that maintenance cannot be long postponed without impairing operation; and that the rise in operating expenses within the ordinarily assumed lifetime of capital goods is relatively moderate. Consequently, the life period used in depreciation charges is cut short largely by considerations of obsolescence and the latter must account for a substantial part of depreciation charges". (Kuznets 1951, 65).

It is possible to estimate total depreciation on the assumption that it is due entirely to relative price changes, and to compare the resulting estimates with those derived by the conventional perpetual inventory method. You may recall from my arithmetical example that the depreciation suffered by K-workers was matched by appreciation accruing to L-workers. This was no accident. Let us take aggregate consumption as our numeraire, and so measure all gains and losses in terms of consumption. Let us also confine attention to the non-residential business sector of the economy (which accounts for most of its output, capital and employment) and assume that its terms of trade with the rest of the world do not change. Three groups in that sector can be distinguished: capitalists, workers and consumers, all overlapping to some extent. The capitalists own all the capital, and in that role they suffer depreciation at the rate of £D p.a. If it were due entirely and only to relative price changes, then the other groups must, by symmetry, enjoy appreciation at the rate of £D p.a. Any relative price change, considered in isolation, benefits some people by as much as it harms others. The group of consumers, however, can neither gain nor lose since a change in relative prices does not in itself affect aggregate consumption. There remain, therefore, only the workers who must be enjoying appreciation at the rate of £D p.a. If, then, we can estimate that rate of appreciation, we have an estimate of the depreciation being suffered by capitalists, in so far as that is due to relative price changes, which can then be compared with the conventional estimates.

I provide such estimates, together with details of their derivation, in my book on economic growth (Scott 1989, 210). In the post-second world war years up to 1973, they actually exceed conventional estimates based on the perpetual inventory method for the USA and the UK (the only two countries for which the comparison was made). This supports my claim that very little
depreciation is due to wear and tear, and most is due to relative price changes. In fact, over the period and in the countries mentioned, more than all conventionally estimated capital consumption could be accounted for by such price changes. It seems likely, indeed, that the conventional estimates were too small (and allowing for changes in the terms of trade would not change this conclusion).

My case against orthodox growth theory thus far may be summed up as follows. The growth of output consists of changes in quantities, and has to be explained in the first place by changes in quantities. Price levels and changes may, and very likely will, help to explain in turn why these changes have occurred, but they must not be confused with the primary causes of growth. To regard the change in the net capital stock as if it were due entirely to changes in quantities is very much mistaken. The change in the net capital stock equals gross investment minus depreciation, and depreciation in recent years has been two-thirds as large as gross investment. Yet depreciation is certainly mainly, and should be defined in my view as being entirely, due to relative price changes. It is only then that we can distinguish it clearly from maintenance, which is concerned with making good physical wear and tear. Relative price changes are symmetrical in as much as they result in appreciation which equals and offsets depreciation. Depreciation does not, therefore, subtract from the growth of output. It is, instead, a transfer of income from capitalists to workers, whose human capital appreciates. It is therefore gross investment, not gross investment minus depreciation, which is the right way to measure the changes in capital input, and which is most closely analogous to the way in which the change in labor input is measured. If this right way is chosen, there is no reason to expect any unexplained residual, due to technical progress, greater knowledge, increased total factor productivity or what have you.

Perhaps I should dwell a little longer on this last point. My parable ruled out changes in factor productivity by assumption. Why should these not occur in practice? While I think there are different ways in which labor productivity can increase, I believe that the only important sustainable source of such increases in the long run is investment, whether material or human. There can be important fluctuations in productivity due to changes in the ratio of output to capacity. There can be once-for-all increases or decreases in productivity due to what has been called x-efficiency. There can be very large catastrophes such as wars or earthquakes which affect productivity temporarily or permanently. There are also gestation lags in many investments, as new processes and methods of working are thoroughly mastered and learned. These are analogous to the maturing of wine or the ripening of corn - they are not a source of continuing productivity growth. Finally, one might mention Barna's point that maintenance expenditures often in fact steadily improve plant in countless minor ways. Such expenditures are then really partly investment and partly maintenance, and form no exception to the rule that the only important sustainable source of productivity growth is investment.

This point emerges more clearly if one defines investment as I believe it should be defined, that is, as the cost in terms of consumption foregone of changing, hopefully of improving, economic arrangements. Growth in labor productivity results only from such changes in the long run, and so only from investment.

You may have noticed that I have carefully avoided reference to total factor productivity in what I have just said. Conventionally, this is measured by the difference between the growth of
output and the growth of all inputs, the latter being a weighted sum of proportionate growth rates of capital and labor. I do not see how one can use the proportionate growth rate of capital if one accepts my argument. One can measure existing inputs of capital by the net capital stock, but, for the reasons stated, the change in the net capital stock is not the right measure of the change in capital inputs to explain the change in output. The change in the net capital stock is gross investment minus depreciation, and depreciation, I have argued, should not be subtracted, since it is the result of relative price changes.

If gross investment without any deduction is the right way to measure the change in capital input, is cumulative gross investment then the right way to measure the capital stock? I would agree that the amount of such investment over any period is relevant to explaining the change in output over that period. However, I do not see what meaning can be attached to the proportionate increase in the stock of cumulative gross investment. The different items in the stock will not be contributing to current output in proportion to their contributions to the value of the stock. A machine bought 20 years ago, and perhaps relegated to occasional use, has a quasi-rent which is far less than that of a new machine which costs the same. Both will be included at the same value in the gross stock, but their contributions to current output will differ greatly. Indeed, the cumulative gross stock will include machines and buildings bought, not just 20 years ago, but 200 years ago, or even, in principle, 2000 years ago, few of which will still exist. Cumulative gross investment cannot, therefore, be used as an argument in a production function explaining both the level and the change in the level of output. Such a function seeks to relate current output to current input, but there is no sense in which a machine or building scrapped years ago is still a current input.

There are some growth theorists who favor, not cumulative gross investment, but cumulative gross investment minus those investments scrapped, as the best measure of the stock whose proportionate increase should be measured. However, that is open to the objection I have just made. The different elements in the stock do not contribute to output in proportion to their value in the stock. The objection applies with particular force to the deduction made for scrapping. In competitive conditions, the contribution of scrapped assets will be zero, so that their being scrapped has no effect on total output. Hence, in measuring the change in capital input, it cannot be right to deduct the original cost (suitably updated for inflation) of scrapped assets from gross investment. Once again, it is gross investment without deductions which is the better relevant measure of the change in capital input.

As some may find this puzzling, let me take an example to clarify the point. Imagine a perfectly good house situated near where I live in North Oxford which is demolished to make way for flats. The house will have had a substantial rentable value beforehand, and clearly none once it has been pulled down. How, then, can I say that this scrapped asset contributed nothing to total output? Let me suppose a developer bought the house and paid for its demolition and the subsequent construction of the flats, which he sold, making a normal profit. What he paid for was the land plus the house plus its demolition plus the construction of the flats. All he sold was the land plus the flats. It follows that the value of the house plus its demolition was zero. Consequently, the house itself was worth minus the cost of the demolition. Had there been no house, the land would have sold for that much more. Hence the substantial rentable value was attributable all (more than all) to the land, and this was not lost when the house was pulled down.
This example, incidentally, also illustrates some of the other points made already. The value of the house has declined, not because of any physical changes in it, but because of a rise in land prices in North Oxford. Its loss in value has been matched by appreciation in the value of the site. The house’s contribution to output, that is, the rentable value attributable to it, and not to the site, has steadily declined to less than zero, so that its value in the gross capital stock at different times in the past 50 or 100 years has borne very different ratios to its true rentable value.

I am therefore at a loss to see how any meaning can be attached to the proportionate increase in any of the capital stocks I have mentioned as an explanation of the growth of output occurring at the same time. The only exception to this would be if there were no relative price changes accompanying such growth, but I doubt that this is a very interesting exception. If I am right, we should abandon both the production function and the measurement of total factor productivity as currently undertaken, if our object is to explain economic growth.

Before concluding, let me mention some comparisons of outputs and inputs to which these objections do not apply. One might, for example, explain differences in output per worker in different manufacturing industries in the UK by differences in capital per worker. This is, indeed, what Barna did in the study I have already referred to (Barna 1961). The point is that in such a study one can reasonably assume that prices confronted by all the industries are approximately the same. One needs to measure capital, in my belief, in such a way that the ratio of capital value to quasi-rent is at least approximately uniform, which seems to point to the use of the net capital stock (Barna used replacement cost new, as obtained from fire insurance valuations, which does not appear to be the same). The result will not enable one to say anything much about production functions, however, since each observation represents only one point on each industry’s production function, and what one observes is the envelope of these functions, as Barna pointed out.

If one wishes to extend the comparison to different countries, one has somehow to deal with differences in relative prices, and then problems akin to those found in intertemporal comparisons will occur. In that case, the right way to proceed, so far as I can see, is to multiply all quantities of inputs and outputs by the same set of weights. In my example, one compares \( t_0 \) and \( t_1 \), either using \( t_0 \) prices or \( t_1 \) prices. \( t_0 \) and \( t_1 \) could be different countries, or could be the same country at different periods, but, whichever they are, one has to use the same set of weights for \( t_0 \) as for \( t_1 \). If one does this, one may be able to devise some suitable production function which will serve to generalize the results from different observations. I do not know any study which has actually done this, and it certainly would be a difficult undertaking.

In my own study, I have sought to explain only changes in output between different periods, and have done this using gross investment and quality-adjusted labor input. This does not provide a production function which explains levels of output, only an investment function which explains changes in levels. That is akin to any historical explanation which tells how things changed over some period in history, but takes the starting point as a datum. This is a more modest objective than that of explaining levels, and perhaps a more easily attainable one.
II. WHAT CREATES INVESTMENT-OPPORTUNITIES?

I will begin by stating four contrasting views about the creation of investment opportunities, which I call the stagnationist, the orthodox, the new look, and my own views. The first of these can be quickly dismissed, but the others require longer consideration. No prizes are offered for guessing which one, in the end, I favor.

(1) The stagnationist view is that, at a given moment in time, say now, there exists a set of investment opportunities of varying quality. This set remains fixed, except that, as investment proceeds, the better projects are undertaken and get removed from the set. Hence, gradually the quality of the projects undertaken must deteriorate and the rate of return to investment must fall. Keynes remarked in the General Theory that, assuming full employment and a "not disproportionate" rate of investment, "I should guess that a properly run community equipped with modern technical resources, of which the population is not increasing rapidly, ought to be able to bring down the marginal efficiency of capital in equilibrium approximately to zero within a single generation." Now, two generations since he wrote that, I doubt he would have wanted to stand by it. The view seems to imply that we are, at each point in time, aware of all possible future investment projects, so that we can select and undertake the best first, and then the next best, and so on. This rules out the possibility of discovery and invention, and credits us with omniscience which we clearly do not possess. It seems scarcely worthwhile considering this view further. Nevertheless, I will dwell a little longer on it, because it exemplifies a dilemma from which the other points of view to be discussed seek an escape. Now-a-days the dilemma arises because people think in terms of a constant returns to scale production function, \( Q = f(K, L) \). If \( K \) grows faster than \( L \), as seems to have been the case for a century and more in the developed countries, and if the production function has diminishing returns to \( K \) as \( K/L \) rises, the return to investment should have steadily fallen over these years, and could be expected to continue to fall in the future. Yet this does not seem to have happened. Indeed, I would argue that the evidence shows the opposite to have happened in as much as rates of return in both the U.K. and the U.S.A., and probably also Japan, were somewhat higher at least in the "golden" years of post 2nd world war economic growth than they were in the late 19th century and earlier 20th century. The solution to this dilemma has then been proposed along two different lines, which represent, successively, the second and third of the views I shall consider.

(2) The orthodox view, as expounded in the textbooks, is that exogenous technical progress shifts the production function outwards (or inwards - depending on the diagram) as time passes. The marginal product of capital (i.e. the rate of return) is thereby increased, but investment itself, by increasing the ratio of capital to labor, \( K/L \), drives down the rate of return. In steady growth, those two opposite influences offset each other, so that the rate of return remains constant. The implication is, however, that a faster rate of investment would gradually depress the rate of return, since it would outrun the supply of new investment opportunities coming from the exogenously given rate of technical progress. Vintage theories with exogenous technical progress are basically similar to this. In both, the key factor is the rate of technical progress (plus, it should be added, labor force growth, since a certain amount of investment is needed merely to keep \( K/L \)
constant if \( L \) is increasing). It is not always clear what determines that rate, but it has to be independent of investment for the theory to work. Presumably the idea is that inventions and, underlying them, scientific discoveries, proceed at a pace which is independent of investment, and it is this which determine the rate of technical progress.

**3** The new-look view has several variants, but I think there is enough common ground to consider them together. The main idea seems to be, in one way, similar to the orthodox view. The key factor is knowledge, and it is the increase in the stock of knowledge which keeps up the return to physical capital. However, an important difference from the orthodox view is that the stock of knowledge can be increased in a similar way to the stock of physical capital, that is, by sacrificing consumption in order to invest in it. Consequently there is an additional input in the production function, besides \( K \) and \( L \), which is the stock of knowledge, and the function itself need not shift through time. The function now exhibits increasing, rather than constant, returns to scale in all its inputs, and this implies the existence of imperfect markets and/or externalities.

**4** My view escapes from the above-mentioned dilemma by abandoning the production function. This is not done just to make this escape, but rather for the reasons given in my last lecture. In my view, investment, by changing the world, both creates and reveals new investment opportunities. Hence, unlike the orthodox view, which says that technical progress is quite independent of investment, I say that it is entirely due to investment. Unlike the new look view, which says that it is one special kind of investment, namely, investment in knowledge, which does the trick, I do not seek to single out any particular kind of investment as being special. Doubtless there will be some investments which create more and better investment opportunities than others. However, I am skeptical that anyone - in government, business, universities, research institutions or anywhere else - can spot the winners in this competition much in advance. Indeed, if my theory is true it is impossible that they should, since, according to the theory, it is not until today's investments are undertaken that tomorrow's opportunities can be seen.

Having outlined briefly these four different views about the creation of investment-opportunities, and abandoning the first, stagnationist, view without further ado, let me now explain what I think is wrong with the orthodox and new look views, and why I prefer my own. It is helpful to start by summarizing some of Schmookler's findings in his book on *Invention and Economic Growth* (1966).

In this book, he examines and lists 934 important inventions made in all countries in four industries between the years 1800 and 1957: 235 in agriculture, 284 in petroleum refining, 185 in paper-making, and 230 in railroading. He instructed his research workers "to record any suggestion in the literature that a particular scientific discovery (or any other event) led to the making of an important invention". While for most of the inventions no initiating stimulus could be identified, 'for a significant minority of cases, the stimulus is identified, and for almost all of these that stimulus is a technical problem or opportunity conceived by the inventor largely in economic terms ... In those few instances where the literature identifies a stimulus which is not an economically significant problem per se, the stimulus is an accident ... In contrast to the many accounts identifying economic problems as the immediate stimulus,
in no single instance is a scientific discovery specified as the factor initiating an important invention in any of these four industries." (Schmookler, 1966, 66-7, emphasis in the original).

He does not deny that, in some science-based industries (e.g. electrical, electronic, nuclear, chemical and pharmaceutical industries) invention and research is heavily dependent on scientific knowledge, so that in these industries there were many instances of important inventions being directly induced by scientific discoveries. However, he argues that, even in these industries, 'economically evaluated technical problems and opportunities arising in the normal conduct of business are dominant,' (p.68). And even if the invention is stimulated by scientific discovery, most of its social and economic significance derives from a host of other inventions which improve and adopt the basic one, and these depend on the economic value perceived to result from them.

Just as inventions were not stimulated directly by scientific discoveries in his four industries, nor were they in his view stimulated by other inventions (with exceptions).

In Schmookler's view, inventions are motivated and caused by similar factors to those which cause investment, that is, by their expected profitability. This, in turn, depends on economic factors. He supports this view by three pieces of evidence which I will mention here (for more, you should refer to his book). In all of these he measures inventive activity by the number of patents granted for capital goods used by the particular industry or activity with which he is concerned. He is aware of the objections which can be made against this particular measure of inventive activity, but, in default of something better, he argues that it is better than nothing. His general conclusion is that the number of patents, and so inventive activity, in capital goods for industry X varies directly with sales of capital goods to industry X, and that this shows the dominance of economic factors in determining inventive activity. The three pieces of evidence I shall cite are as follows:

(i) The number of patents for railroad track followed the same broad sweep of rise and subsequent decline as did the number of patents for all other railroad fields thus suggesting that there was some common factor explaining them all.

(ii) Investment in railroading and numbers of patents of capital goods used in railroading fluctuated in the same way in the century or more ending in 1950. This is true both of very long swings and of deviations of seven- and nine-year moving averages from trend as given by 17 year moving averages. Furthermore, the less adequate statistics for investment and patents in petroleum refining and building show a similar behavior, and for all of these Schmookler argues that, at the turning points, patents usually lag behind investment, thus suggesting that investment causes patents rather than the other way around.

(iii) The relation between the number of patents and the amount of investment in about 20 industries is examined. The logarithm of the number of patents is regressed on the logarithm of investment in an earlier year, and it is found that $r^2 = 0.9$ and the coefficient of the investment term is not significantly different from 1, implying that if you double investment in an industry, you double the number of patents on capital goods for that industry.
'Necessity is the mother of invention' is an old saying which sums up Schmookler. He has been interpreted as arguing that only demand matters, and it is not difficult to show that there must be supply factors too. If only demand mattered, why have we not invented Aladdin's wonderful lamp, a cloak of invisibility, seven-league boots, a time-machine, and many other such apparently desirable objects which storytellers have imagined? Clearly invention needs a father as well as a mother, and I don't believe Schmookler would have denied it. But even if we can all agree on that, it is enough to recognize the importance of the mother to force one to abandon the orthodox view. If the volume of investment does play a significant part in determining the volume of inventive activity, then one cannot proceed as if investment merely gobbles up investment opportunities whose number depends on an exogenously determined rate of inventive activity. Investment does not just gobble them up, it also, on the evidence provided by Schmookler, gives birth to new investment opportunities.

Those who stress the supply-side factors have sometimes argued that there is an exogenously-determined major invention or discovery, and that this is succeeded by a fairly long period in which this major invention is exploited in innumerable ways of steadily declining importance until, eventually, the original stimulus has exerted its full effect and nothing more remains. Thus, for example, Kuznets says:

"The stimulus for technical changes in other processes of industry is thus present from the moment the first major invention is introduced ..."

While the stimulus for further inventions appears early, the number of operations to be improved is limited and gradually becomes exhausted" (Kuznets, 1930, 31-3, quoted by Schmookler 1966, 87).

Salter (1960) argued in a similar way that there is a major stimulus followed by technological exhaustion.

The phenomenon of technological exhaustion is, however, denied by Schmookler. If the rate of invention does tail away in some field, it is, he argues, for economic and not for technical reasons, and he gives a lovely example to prove his point which I must mention.

It relates to the horseshoe which, since it was introduced in the 2nd Century B.C., should, according to the view just described, be a field in which the technical possibilities for further improvement were exhausted long ago. Yet, as he shows, the annual number of U.S. patents for horseshoes rose continually until the close of the 19th century, and then declined. 'Once the steam traction engine and, later, the internal combustion engine began to displace the horse, inventive activity in the field began to decline - because of a decline not in the technical possibilities of the field but in its economic payoff' (Schmookler, 1966-93).

The idea that investment itself creates further investment opportunities has, in varied forms, been put forward by others besides Schmookler, although I think it is he that has given the best direct evidence of the link between investment and inventive activity. Two other writers I should like to refer to are Hirschman and Arrow.
In his "The Strategy of Economic Development" (1958) Hirschman concluded that 'development depends not so much on finding optimal combinations for given resources and factors of production as on calling forth and enlisting for development purposes resources and abilities that are hidden, scattered, or badly utilized' (Hirschman 1958. 5). He thus sets out to look for the 'inducement mechanisms' that will do this. 'If backwardness is due to insufficient number and speed of developmental tasks, then the fundamental problem of development consists in generating and energizing human action in a certain direction ' (p. 25); 'development is held back primarily by the difficulties of channeling existing or potentially existing savings into available productive investment opportunities i.e. by a shortage of the ability to make and carry out development decisions' (p.36). Hirschman calls this a shortage of the 'ability to invest.'

Hirschman next discusses 'the complementarity effect of investment', by which he means not merely that investment increases income and then also increases savings, but that in addition it induces further investment decisions; it makes them easier to take, because, for example, a new market is created for the output of the B industry when the output of the A industry is expanded, and, likewise, the new output from A may lowers costs of production in C. Hence investment in A has made it easier to decide to invest more in B and C. Hirschman makes use of the illuminating analogy of a jigsaw puzzle (pp.81-2). One is trying to complete the puzzle as quickly as possible. It is easier to fit in a particular piece the more of its neighbors are already in place, while the hardest pieces to join on are those with only one neighbor in place. Development, however, unlike a puzzle, does not come to an obvious end. In a real puzzle, the task of fitting on new pieces become progressively easier as the stock of unfitted pieces diminishes, so that one has fewer and fewer possibilities to search among. To preserve the analogy, we must therefore imagine that the loose pieces are being added to as fast as they are being diminished by fitting them on to the existing picture. That picture then becomes like history, stretching back for a very long time and growing indefinitely.

This view of development is contrasted with the 'balanced growth' doctrine, of which Hirschman strongly disapproves.

'It is argued that a new venture - say, a shoe factory - which gets underway by itself in an underdeveloped country is likely to turn into a failure: the workers, employees, and owners of the shoe factory will obviously not buy all of its output, while the other citizens of the country are caught in an 'underdevelopment equilibrium' where they are just able jointly to afford their own meager output. Therefore, it is argued, to make development possible it is necessary to start, at one and the same time, a large number of new industries which will be each others' clients through the purchases of their workers, employers, and owners. For this reason, the theory has now also been annexed to the 'theory of the big push'. (Hirschman 1958, 51).

Balanced growth is attacked because 'The theory fails as a theory of development. Development presumably means the process of change of one type of economy, into some other more advanced type' (pp.51-2).

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1 Rosentstein-Rodan (1957).
The adherents of balanced growth, according to Hirschman, are escapist and also pessimistic. They give up the existing economy as a hopeless case, and wishfully think that a beautiful new economy, complete in itself, can be constructed all at once inside it. 'As Singer writes: 'The advantages of multiple development may make interesting reading for economists, but they are gloomy news indeed for the underdeveloped countries. The initial resources for simultaneous developments on many fronts are generally lacking', (p.53).

Balanced growth leads naturally to a desire for centralized investment decision-taking. This is to ensure that the externalities resulting from individual investments are internalized so that, to return to a previous example, the benefits of investment in A for B and C are fully taken into account when investment in A is considered. However, as Hirschman points out, there will not, in general, just be external benefits. There will also be external costs. Because of these costs, it is by no means clear that centralization of investment decisions will speed development. On the contrary, the success of capitalism may have owed as much to the ruthlessness with which such costs could be, and were, ignored. Centralization would have permitted the groups that lost out to mobilize resistance to change, which might have slowed down the pace of development substantially. Hirschman instances the guild system as an example of this.

In contrast to balanced growth, Hirschman envisages development as a chain of disequilibria:

Therefore, the sequence that 'leads away from equilibrium' is precisely an ideal pattern of development from our point of view: for each move in the sequence is induced by a previous disequilibrium and in turn creates a new disequilibrium that requires a further move. This is achieved by the fact that the expansion of industry A leads to economies external to A but appropriable by B, while the consequent expansion of B brings with it economies external to B but subsequently internal to A (or C for that matter), and so on. At each step, an industry takes advantage of external economies created by previous expansion, and at the same time creates new external economies to be exploited by other operators. (Hirschman 1958, 66-7).

There are many other fascinating insights provided in Hirschman's book. He has interesting things to say on the relationship between social overhead capital and 'directly productive activities', making the point that inducements to investment must, and do, operate on governments as well as on private businesses. His analysis of forward and backward linkages is well known, and exemplifies the ideas described above, as well as showing how economies of scale come into the picture.

In a famous and much quoted article (Arrow 1962), Arrow devised a growth model to explore 'The Economic Implications of Learning by Doing'. After expressing his dissatisfaction with orthodox growth theory on the grounds that it leaves unexplained such a large part of the growth in productivity, which he agrees is due to a growth of knowledge, Arrow proposes 'an endogenous theory of the changes in knowledge which underlie intertemporal and international shifts in production functions' (1962, 155). He offers two generalizations about learning gleaned from the work of psychologists. First, 'Learning is the product of experience. Learning can only take place through the attempt to solve a problem and therefore only takes place during activity'. Second,
'learning associated with repetition of essentially the same problem is subject to sharply diminishing returns' (p. 155).

In the light of these generalizations, he sets out a clay-clay vintage model of growth in which the quantity of labor required to man machines of successive vintages (each machine having the same fixed output capacity) declines. Unlike orthodox vintage models, however, Arrow does not assume that this fall in labor requirements per unit of capacity output depends only on time. Instead, he makes it depend on the amount of cumulative gross investment, $G$, which has occurred from the beginning of time. He takes this, rather than cumulative output, as an index of experience because 'Each new machine produced and put into use is capable of changing the environment in which production takes place, so that learning is taking place with continually new stimuli' (p.157). Total output would be less satisfactory as an index of experience, since it would, for example, continue to grow in a static society in which nothing changed, so that learning could hardly be supposed to occur.

Arrow's model, however, suffers from two severe disadvantages. First, being a vintage model, it is very complicated and difficult to apply empirically. Secondly, and more fundamentally, it produces a quite unacceptable result. The steady-state rate of growth of output in the model is a constant (bigger than one) times the rate of growth of the labor force. This might just pass muster when the rate of growth of the labor forces is positive, but what if it is zero or negative, as it has at times been in my own country? Must output and labor productivity then decline? I'm afraid that we cannot adopt Arrow's model (and I don't think anyone has for any substantial piece of empirical work), even though the basic idea of learning from investment is a good one.

Let me now sum up to see where I think the argument has led us thus far. I think it has led us to my position, which I listed as number four in the views about the creation of investment opportunities. As I have not properly considered view number three, the new look, I will return to that afterwards.

Investment opportunities undoubtedly come from inventions, but inventions do not come like manna from heaven (as orthodox theory holds), but are in turn a response to economic factors similar to investment themselves. In fact, is it not true to say that every investment has a component of invention within it? The component may be large or small, but it is there because every investment is to some degree unique. Investments are changes in economic arrangements. In order to improve economic arrangements one has to devise, ie. invent, a way to do it. A great deal of what one does is suggested by what others have already done in similar circumstances before. Nevertheless, there will always be some new features of the problem with which one is confronted. The place and time must be different, other things will have changed, and perhaps the market one is serving, or the people one is employing, or the experience elsewhere which one can draw on will all be different from the last time some similar thing was done. It is true that the element of novelty may be so small on occasion that one can safely neglect it for many purposes. The $n + 1^{th}$ car off the assembly line way be identical to the $n^{th}$, and one may ignore the fact that it will be used in a different location by a different driver. The manufacturer will not take out separate patents for the $n^{th}$ and $n + 1^{th}$ cars. Nevertheless, there are frequent model changes, and those may be patented, or contain new components separately patented. It is hardly surprising, then, that patents
and investment activity move together since they are different aspects of the same thing, that is, making changes in economic arrangements.

Furthermore, undertaking investment, i.e., changing economic arrangements, causes new investment opportunities to come into existence. As with Hirschman's jig-saw puzzle, each new piece fitted into the others makes it possible to fit the next one on to it. My own preferred analogy is that of climbing up a hill (which I enjoy rather more than solving jig-saw puzzles). The higher you go, the better the view - and that corresponds to the higher level of consumption as GNP rises. However, in order to improve the view further, one must climb further, and that involves sacrificing one's enjoyment of the view for a time. Climbing thus equals investment. There are no paths up the hill, and one can only see a little way ahead, since the hill is convex and the ground is broken. However, each advance enables you to see further. At each stage, therefore, you have to decide how to advance in the light of what you can see ahead, and your choice may be crucial both to your immediate speed of advance and to your speed later on - since you always start from where you have reached, and that may turn out to be a better or worse way up when you come to look back.

No analogy is perfect, but I hope I have said enough to make you understand my view about the generation of investment opportunities. Let me now turn to what I called the new look view that it is the accumulation of knowledge which counteracts what would otherwise be a tendency for the rate of return on investment to fall as capital accumulates.

One version of the new look is that of Lucas (1988) whose emphasis is placed on human capital. He argues that different workers have different amounts of human capital, \( h \), embodied in them, so that the quantity of the labor force has to be a weighted sum of workers, each worker being weighted by his \( h \). If marginal products are directly proportionate to \( h \), as he assumes, then I go along with this. My quality-adjusted employment is intended to be precisely such a weighted sum of workers. If worker A's marginal product is twice worker B's, then he counts for the same as two worker B's (no pun intended). The only gloss I would add to Lucas in this respect is that the location of the worker may be very important, and not just the amount of education he has received. Workers in the town may have had the same education as workers in the countryside and yet receive wages that are twice as high in some countries.

However, Lucas does not merely quality-adjust his labor force - there would be nothing particularly novel in that - he assumes that the average skill level of the work force exerts a double effect on output. There is the effect already noted in which better quality workers are like having more workers. Additional to that, a higher average skill level increases output from both capital and labor.

It is not clear to me just how he thinks this double effect works. In comparison to the space devoted to developing the mathematics of the model, the space devoted to explaining and defending this key point seems inadequate. He remarks that people benefit from interacting with each other, and benefit more (in their work) the more skilled are the people with whom they interact. I believe this is true, but that people do not just interact with people. They learn, and get ideas from, the way in which other people do things which are similar to what they are doing, or which have demand or supply implications for what they are doing, and that depends on other people's systems of work, the equipment they have at their disposal, and, in short, on their economic
arrangements which have both material and human aspects. I am prepared to believe that both material and human investment create further investment opportunities, but not that only human investment does - if that is what Lucas implies.

One way in which human capital might affect output is through its effect on the efficiency of investment. More skilled managers, for example, should both perceive and select better investments from the opportunities available than less skilled ones. To test this, one would, so it seems to me, need to measure the efficiency of investment and see if it was related to a measure of human capital. While this looks like a good idea, it is not one that Lucas pursues.

Lucas' human capital, according to his own statement, has to be something which does not die with its possessor, but is somehow handed on, at least in part, to later generations. That makes it akin to the stock of accumulated knowledge which Romer and others have used as the additional input in the production function which maintains the supply of investment opportunities. In what follows, and in order to keep things manageable, I will concentrate attention on this version of the new look, which can then be characterized by the production function:

\[ Q = f (K, L, \kappa) \]

where \( Q \) is the level of output, \( K \) is the conventional 'physical' capital stock, \( L \) is employment (which I shall assume is quality adjusted) and \( \kappa \) is the stock of accumulated knowledge. \( \kappa \) can be increased by devoting resources to it which otherwise could have been directly used to produce consumption or physical capital. If you double \( K \) and \( L \) you double \( Q \), and if you increase \( \kappa \), with \( K \) and \( L \) constant, you again increase \( Q \). Hence, taking \( K, L \) and \( \kappa \) together, there are increasing returns to scale. I am not, however, here concerned with whether or not there is a possibility of a competitive equilibrium. My objections apply whether that is true or not.

My first objection is the one I gave in my last lecture. One cannot use \( K \) in the above way as an argument in a production function explaining both the level of output and its growth over time. I think this is a very important objection, since much of the motivation behind the new look is to find a way of preserving the production function approach while at the same time overcoming objections to it which arise from its implausibility as a way of explaining difference in levels and growth rates between different countries in the world. If I am right, the production function cannot be used to explain levels and growth rates even of one country in the world, and the new look has not addressed my reasons for that conclusion.

A second objection is that the above functional relationship assumes that knowledge could accumulate even when \( K \) and \( L \) are held constant, and that the result would be a rise in the level of output. There seem to be two mistakes here. First, if I am right, the proximate causes of growth are material investment and changes in quality-adjusted employment (which includes the effects of human investment). These are both necessary and sufficient causes of growth, and I do not see how there can be increases in output if \( L \) does not change and if there is no investment and so no changes in economic arrangements. Regardless of increasing knowledge, we would then, I believe, have a static economy.
The second mistake is that I do not believe that knowledge which was useful for economic output could accumulate independently of changes in $K$ and $L$. To put my point very briefly and starkly: if you had some research institute in existence for, say, 100 years during which it was never allowed to try out any of its ideas in practice, and during which the economy in which it was embedded was entirely static, so that no changes in economic arrangements took place, then I predict that at the end of the 100 years the institute would be capable of contributing scarcely any more to useful knowledge than at the beginning. In fact, it would be impossible to confine the attention of the researchers to useful knowledge at all, and any that they did produce would be a chance by-product of fascinating inquiries such as, for example, determining the number of angels that can stand on the point of a pin. It is worth recalling Arrow's proposition which I quoted earlier that learning is the product of experience. Learning can only take place through the attempt to solve a problem and therefore only takes place during activity. Denied activity, the researchers would turn to purely intellectual pursuits.

A third objection is to the operational feasibility of the new look. How is the accumulated stock of knowledge to be measured? It cannot, so it seems to me, be measured by the amount of education embodied in the population or labor force. It is an essential part of the new look that knowledge accumulates from one generation to the next, so that even if successive generations all received the same amount of education, the stock of knowledge would still increase from generation to generation. Arrow's suggestion was that it should be measured by cumulative gross investment. While I am in sympathy with that, since it virtually amounts to my own approach, it leads me to my fourth, and final, objection.

This is that I believe that an adequate theory of growth, and of the generation of investment opportunities, can be formulated without bringing in this further entity, the accumulated stock of knowledge. 'Entities should not be multiplied unnecessarily' wrote William of Occam (in Latin) in the 14th Century. If accumulated knowledge is measured by cumulative gross investment, then there is no reason to stray beyond investment in explaining growth and the generation of new investment opportunities. Why add a fifth wheel to the coach?
III. A MODEL OF ECONOMIC GROWTH

There are three essential features of the model of economic growth which I am going to outline. First, it takes on board the points made in the last two lectures. It makes no use of a production function explaining the level of output by, *inter alia*, the capital stock; and it makes investment opportunities depend on investment. Secondly, it is operational. All the relevant magnitudes can be measured or estimated, at least for the main developed countries. Whether suitable proxies exist for many less developed countries is, however, something which needs further investigation. Finally, it is as simple as I can make it while still, so it seems to me, enabling one to tackle many interesting questions.

Despite these brave claims, I am aware that the theory is subject to a good many limitations, and that on first acquaintance it may seem strange and quite complicated. I welcome suggestions for widening its scope and simplifying it, although I suspect that these two objectives may conflict. For this lecture I cannot hope to do more than give you the barest outline, and I will retain several simplifying assumptions which are relaxed in the book.

The first, and crucial, simplification, which is retained for most of the book, is the assumption of steady growth. I will attempt to justify this later. Let me first explain what it involves. It means that the output of the typical enterprise (and the theory applies only to the business sector of the economy) is growing at a constant exponential rate, \( g \), with its employment of workers (quality adjusted) growing at a constant rate \( g_L \), so that labor productivity is also growing at a constant rate, \( g - g_L \). In a progressive economy one would expect \( g - g_L \) to be positive, but that is not essential to the theory. Apart from these constant rates of growth, two important ratios are also constant, namely, the ratio of gross investment to output (i.e. value-added, the sum of gross profits and wages), which is \( s \), and the ratio of wages to output, which is \( \lambda \). The fact that \( \lambda \) is constant means that the rate of growth of wage rates, \( g_w \), must equal the rate of growth of labor productivity, so

\[
g_w = g - g_L.
\]

The rate of growth of the wage bill is then \( g_w + g_L = g \), the rate of growth of output.

At a given point in time the typical enterprise is producing a rate of output \( Q \), whose price is unity (since this is the typical enterprise, and the price of average output is the numeraire) with employment \( L \), wage rate \( W \), and rate of investment \( S \). For simplicity I assume here that the enterprise is both selling its output and buying its labor in perfect markets, so that the determination of its prices and wage rates are out of its hands. I also assume that it is entirely self-financed, there being no capital market. Both of these assumptions can be relaxed, but will not be in this lecture. Throughout, I ignore purchases of materials, etc. by the enterprise, so that implicitly they bear a fixed proportion to output.
There are then, in this simplified model, only two decisions which the managers of the enterprise must make. They must decide how much out of their gross profits to invest, and they must decide on the nature or characteristics (as I shall call them) of this investment. There are only two characteristics with which I am concerned: the extent to which the investment increases output, which I label the $q$ characteristic, and the extent to which it increases employment, which I label the $l$ characteristic. I have not yet defined what $q$ and $l$ are precisely, but will do so presently.

In the full model, the enterprise chooses both the rate of investment $s$ and the characteristics, $q$ and $l$, so as to maximize the present value of the firm to its owners. In this lecture I shall content myself with a simpler situation. The enterprise will be given a rate of investment $s$ determined exogenously, and will choose the characteristics, $q$ and $l$, so as to maximize the rate of growth of what I shall call 'take-out'. Take-out is gross profits less those reinvested, and so is similar to dividends, or cashflow to the owners. The present value of the firm to its owners must be the discounted sum of all future take-out. I do not claim that my assumptions about the behavior of the firm necessarily imply maximization of this present value. To demonstrate that that is occurring, one needs to specify how the rate of discount is determined, and $s$ must be endogenous, and not exogenous as in my example. Nevertheless, the steady growth behavior of the firm in my simplified example does sufficiently resemble that of a firm which is maximizing present value for it to enable me to illustrate some important points about the model.

I want, now, to introduce the main growth diagram of the model, and to do so I must first define $q$ and $l$, as they are going to be measured along the $y$- and $x$-axis respectively.

$q$ measures the amount of extra output per unit of the investment that causes it. However, instead of measuring absolute amounts of each, I shall measure the proportionate increase in the rate of output per unit share of investment. Hence

\[
q \equiv \frac{1}{Q} \cdot \frac{dQ}{dt} \cdot \frac{S}{Q} \\
\equiv \frac{g}{s}
\]

Likewise I measure the amount of extra employment per unit of investment that causes it and, again, I take the proportionate increase in the rate of employment per unit share of investment. Hence

\[
l \equiv \frac{1}{L} \cdot \frac{dL}{dt} \cdot \frac{S}{Q} \\
\equiv \frac{g_L}{s}
\]

Let me illustrate with figures (see also the arithmetical example in Appendix 1). Suppose

\[
Q = \$100 \text{ p.a.} \\
S = \$20 \text{ p.a.}
\]

so

\[
s = 0.2
\]
Suppose that, with this rate of investment, output increases by $4 p.a. That is a proportionate increase of 0.04 p.a. and so \( q = 0.04/0.2 = 0.2 \). Likewise, if employment were growing proportionately at 0.02 p.a.

\[
l = 0.02/0.2 = 0.1
\]

Strictly speaking, these have to be \textit{exponential} rates of increase so that, for example, the actual increase in output if \( q = 0.2 \) would be \((e^{q.t})Q = 4(e^{0.04}) \times 100 = 4.08\) With exponential rates of growth we can simply add and subtract without error, so that in this example the rate of growth of labor productivity is \( g - g_L = 0.04 - 0.02 = 0.02 \) which would also have to be \( g_w \) in steady growth.

I can now show you the diagram illustrating the firm's steady growth position. As already remarked, \( q \) and \( l \), and exponential rates of change of output and employment, are measured along the \( y \)- and \( x \)-axis respectively. The curve \( CC' \) is what I call the Investment Program Contour, or IPC, for a given rate of investment, \( s \). For that rate of investment, it shows the menu of choices of combinations of \( q \) & \( l \) confronting the firm at a given point in time, say now. \( q \) increases with \( l \), for obvious reasons, and there are also diminishing returns to \( l \), which I hope you will agree is plausible. So

\[
q = f(l) \quad f' > 0, \quad f'' < 0.
\]

The crucial assumption underlying the whole model is that the IPC remains constant from one period to the next. This is based on the idea that investment creates and reveals investment opportunities, discussed in my last lecture. I will come back to this again later.

Let us consider a particular point \( C \) where, in conformity with my arithmetical example, \( q = 0.2 \) and \( l = 0.1 \). These would then be the rates of growth of output and employment if \( s = l \) (so long as the IPC is the same - more on this below). However, we know that \( s \) is only 0.2, and so the rates of growth are only one-fifth as much. Consequently, we can draw a shrunken version of the IPC which is only one-fifth as far out from the origin. Then, in the diagram, \( E \) is on this shrunken IPC and \( OE/OC = 0.2 \).

To see which point on either the IPC or its shrunken version is consistent with steady growth we need to know at what rate wage rates are growing. This rate of growth is exogenous to the firm, and is 0.02 in my example. So we lay off \( OG = 0.02 \) and draw a 45° line up from \( G \) to cut the shrunken IPC at \( E \). We project \( OE \) to cut the IPC itself at \( C \). \( E \) and \( C \) are then consistent with steady growth because the following conditions are all satisfied:

(i) If the firm continues to invest at rate \( s \) with characteristics as given by \( C \), then it will continue to grow steadily at \( g = 0.04 \) and \( g_L = 0.02 \) because the IPC remains unchanged through time -- the crucial assumption I mentioned earlier.

(ii) These rates of growth imply that \( g - g_L = 0.02 = g_w \), i.e. that labor productivity grows as fast as wage-rates. Consequently the wage-bill grows at \( g_L + g_w = 0.04 \)
which is the same rate as output. The share of wages in output, \( \lambda \), is then constant.

It should now be clear why I chose to define \( q \) and \( l \) as I did. If the conditions for steady growth are satisfied (i.e. constancy in \( g, g_L, s \), and \( \lambda \)) then the points (C and E in this case) which refer to a particular set of values for these variables remain stationary in the diagram. They are feasible points if the IPC, defined in terms of \( q, l \), and \( s \) (i.e. \( q = f(l) \) for a given \( s \), or, if you like \( q = f(l, s) \)) also remains stationary. I interpret the proposition that investment creates investment opportunities as leading to this situation, the justification for that being the fact that steady growth is a reasonably good way to summarize the growth experience of the USA (especially), the UK (to a lesser, but still substantial, extent) and Japan (to a much lesser extent -- only really well in 1961-73). The evidence for this is provided in Ch.2 of my book. One has to abstract from cyclical changes in output, which the theory does not explain, and which are, of course, an enormous and important subject in themselves. Like Solow, I think that steady growth is a good starting point for growth theory, in the sense that any such theory should be able to explain it. Like him, I also believe that we need somehow to integrate cyclical and long-term growth theory, but have not attempted to do that myself.

This is all very fine, you may say, but why should the firm want to behave in this way? Why should it want to invest with characteristics at C now, and to go on investing in subsequent periods with the same characteristics?

It is helpful to take the simple case of a one-shot investment in a static economy first. Suppose that a firm in such an economy is contemplating investing a small amount with characteristics as at C (the base case) or, as an alternative, investing the same amount with characteristics such that \( q \) is \( \Delta q \) bigger than at C and \( l \) is \( \Delta l \) bigger. The small amount invested is \( \delta s \) (expressed as a share of output) and this takes place over a short period of length \( \delta t \). Then it can easily be shown that, to a first approximation, the level of output in the alternative case, after the end of this short period, will be higher than in the base case by \( \delta Q \), and employment will be higher by \( \delta L \), where

\[
\frac{\delta Q}{Q} = \delta s \cdot \Delta q \cdot \delta t
\]

\[ (1) \]

\[
\frac{\delta L}{L} = \delta s \cdot \Delta l \cdot \delta t
\]

\[ (2) \]

The firm's rate of take-out, where it relapses back into a static state after \( \delta t \), will then have increased in the alternative case compared with the base case by \( \delta Q - W \delta L \), where \( W \) is the wage rate. This wage is constant, of course, in a static economy. If, then, the firm is to be indifferent between the alternative and C, it must be the case that

\[
\delta Q - W \delta L = 0
\]

\[ (3) \]
Equation (3) is simply stating the familiar condition that the marginal product of labor, \( \frac{\delta Q}{\delta L} \), equals the wage. Substituting from (1) and (2) into (3), and dividing through by \( \delta s \cdot \delta t \), gives

\[
Q \cdot \Delta q - W \cdot L \cdot \Delta l = 0
\]

or

\[
\frac{\Delta q}{\Delta l} = \frac{WL}{Q} = \lambda
\]

What this shows, then, is that the firm is indifferent between investing at \( C \) and investing, instead, anywhere along the straight line through \( C \) with slope \( \lambda \). I say anywhere along the line because \( \Delta q \) and \( \Delta l \) can be as large as you please (and can both be positive or both negative) without affecting the argument. Since \( L \) and \( Q \) at the given moment in time we are considering are themselves given, the slope of the line depends on \( W \), the wage. Along this line we satisfy the condition that the marginal product equals the wage. So I label the line \( WW \) and call it the EPC, or equal profits contour. For a static firm wishing to maximize profits (= take-out, since investment is zero in a static economy) the firm will clearly choose to invest where the EPC which is furthest out from the origin, \( O \), touches the IPC. In the diagram, the firm wants to choose investments which increase output as much as possible while saving on employment as much as possible. It therefore seeks investments in the North-West direction (North West is best). As I have drawn it, point \( C \) is the best this firm can do for itself with its one-shot investment.

Now I move on to consider a growing firm in a growing economy. The firm is going to be in steady growth to begin with, investing at point \( C \) (i.e. with characteristics \( q \) and \( l \) as at \( C \)), and we will then see whether or not it pays the firm to make a small deviation away from steady growth or not. If not, then the firm will continue in steady growth, and we will have located its dynamic equilibrium position.

Consider, then, a firm at time \( t \) investing at rate \( s \) with characteristics \( q \) and \( l \) as at \( C \). Let it now contemplate diverting \( \delta s \) of its investment during a short interval \( \delta t \) from investment at \( C \) to investment at a point with characteristic \( q \) which is \( \Delta q \) bigger than at \( C \) and characteristic \( l \) which is \( \Delta l \) bigger than at \( C \). At the end of time \( \delta t \) then equations (1) and (2) will still hold for the differences this deviation in its program will have made to the level of its output, which will be \( \delta Q \) higher, and the level of its employment, which will be \( \delta L \) higher. What difference will this have made to the rate of take-out? Since the firm is in steady growth, investing a share \( s \) of its output, the higher level of output will require a higher rate of investment \( \delta S \) to keep it growing as fast as before, and we must have, to keep the share \( s \) unchanged

\[
\delta S = s\delta Q
\]

Then the increase in the rate of take-out will be

\[
\delta Q (1-s) - W\delta L
\]

Now if this increase in the rate of take-out is, in fact, zero, the small deviation will have left take-out unchanged. It will also have left the rate of growth of take-out unchanged since, with \( s \)
Diminishing returns to the rate of investment

In my model, there is nothing that corresponds to diminishing returns to the capital stock. At each instant in time, so long as the IPC remains fixed, the same rate of investment will earn the same average return, regardless of how much investment has gone before. In my belief, that accords better with experience than the assumption of diminishing returns to the capital stock, and it is a natural corollary of the idea that investment creates further investment opportunities. It is true that the assumption of constancy in the IPC is merely a working hypothesis. The IPC could shift in or shift out for all sorts of reasons. But there is no particular reason I know of to suppose that it will always and only shift inwards.

So much for diminishing returns to the stock, but how about diminishing returns to the rate of investment? There are surely many reasons why these should diminish for the individual firm. Some of these were described by Edith Penrose in her "The Theory of the Growth of the Firm" (1959) and concern the management costs and problems of trying to grow faster. Some of them were mentioned by Schumpeter when he described how the entrepreneur had to overcome resistance to change by various different groups. There are studies which show that R&D costs rise as one tries to shorten the lag between initiating a program and completing it. One would also suppose that, at any given time, a businessmen would be aware of projects of varying quality, and would do the best first, so that increasing the number of projects done would pull in more inferior ones.

In terms of the model, what this all implies is that \( \rho \) declines as \( s \) increases for the individual firm. In the diagram, there is one IPC which corresponds to each level of \( s \), with the IPC closer to the origin for higher levels of \( s \). There is, then, a difference between the firm's average rate of return and its marginal rate of return. The average rate of return is given by \( r_a = q - \lambda l \), where \( q \) and \( l \) refer to the relevant IPC as before. However, a marginal unit of investment will not have characteristics as good as those of that IPC. \( q_m \) and \( l_m \), as we may call them, may be much smaller than \( q \) and \( l \), and so \( r_m \) (or \( r \) without any suffixes, as I refer to it) will be well below \( r_a \).

What about the economy as a whole, however? If all firms together invest at a faster rate, must that reduce their average rate of return? Please note that I am talking about a difference between two steady states, and not a sudden investment boom, when bottlenecks of various kinds could be expected. The answer seems to be that there are factors working in opposite directions, so that the net outcome is uncertain. If all firms invest at a faster rate, the flow of new opportunities for further investment will be speeded up, and this should tend to increase \( \rho \). This case is quite different from that of the individual firm, whose opportunities are very largely determined by the rate of investment elsewhere, so that, at any given time, its opportunities are limited and more means worse. Dynamic economies of scale have been alleged by various writers, and these, if they exist, should raise \( \rho \) as all firms invest more together. However, it is still true that the Penrose effects, and the Schumpeterian resistance to change, and the wasteful R&D effects would all be in operation. So the net outcome to a collective increase in \( s \) might perhaps be little change in \( \rho \). This is a topic to which I return in my last lecture, where it is linked to the question of externalities to investment.
The role of relative prices

Thus far I have considered only a whole economy and a representative firm within it. Relative prices have played no role because the numeraire is the price of average output, and that cannot change with respect to itself. The only relative price that has been allowed to change has been the price of labor. Now, however, I want to consider the position of a firm or industry producing a non-representative bundle of goods whose price, $P$, can then change in relation to that of the representative bundle, whose price I shall assume is always 1.

It becomes important now to distinguish between the ratio of the value of investment to the value of output (both in terms of the numeraire), which I have labelled $s$, and the ratio of the value of investment to the quantity of output, which I now label $\sigma$. Thus

$$s = \frac{S}{PQ}$$

$$\sigma = \frac{S}{Q}$$

and so

$$\sigma = PS$$

$\sigma$ is a ratio of two quantities, that of the quantity of the numeraire sacrificed to undertake investment to the quantity of output of the particular industry which concerns us. It seems reasonable to suppose that it is this real ratio which determines the real rates of growth of output and employment in the industry. Hence, I replace the equations $g = sq$ and $gL = sl$ by equations $g = \sigma q$ and $gL = \sigma l$. Where I was dealing with the whole economy, or its representative firm, the distinction between $\sigma$ and $s$ did not matter because then $P = 1$ and we could forget it. Now, however, $P$ may vary and, as we shall see, it has an important role to play.

In steady growth, you will remember, $s$ has to be constant. It is also true that $\sigma$ must be constant, and so it follows that $P$ must also be constant.

If $P$ is constant, the rate of growth of the industry has to equal the rate at which the demand curve for its output is shifting to the right. We can consider that to be given by the rate of growth of real income in the whole economy multiplied by the income elasticity of demand. So far as the industry is concerned, all this is exogenously given. It is this exogenously given growth rate which determines the equilibrium level of $P$.

Let me illustrate this by an example which serves to make another important point as well. Suppose we happen to start off with the industry in equilibrium growth matching the rate of growth of demand for its output so that $P$ is indeed constant. For simplicity assume that $P$ also happens to be 1, as this enables me to use the same diagram as before. Then the IPC is $C'C$ as before, and its shrunken version with $s = 0.2$ is $c'c$ as before, and the equilibrium growth position is at $E$ as before. So the rate of growth of demand is $g = 0.04$ as before.
Now, however, imagine that for some exogenous reason, such as a great discovery, $\rho$ doubles. As a result, the IPC, C'C, shifts out of so that, for example, OC now is double its former distance. There is no immediate effect on price, output or employment, but is clear that the rate of return, OR, to investment in this industry has approximately doubled. It is then difficult to remain with the simple assumption which I have clung to thus far that $s$ remains fixed. In practice, $s$ will increase and so will the rates of growth of output and employment in the industry. However, I assume that all of this has a negligible effect on the rate of growth of the whole economy, and so the demand curve for the industry's output continues to shift rightwards at rate $g$. The result must be that the price of output will fall as actual output grows faster than $g$. How far will $P$ fall?

The answer turns out to be very simple, $P$ must halve, and that will restore the original equilibrium with the original $s$ as well as the original $g$, $g_L$, $\lambda$ and $r_a$. I shall show this first in terms of the equations of the system, and then on the diagram.

First, since $\sigma = Ps$, if $s$ is unchanged while $P$ halves, then $\sigma$ must halve.

Secondly, since $q = f\left(\frac{l}{\rho}\right)$, if $\rho$ doubles $q$ and $l$ must double along any particular ray from the origin. If we stick to the original ray OEC, then the original $q$ and $l$ will double.

Thirdly, since $g = \sigma q$, $g_L = \sigma l$, if $\sigma$ halves and $q$ and $l$ double, $g$ and $gL$ are each unchanged.

Fourth, since we are on the original ray OEC, the slope of the IPC is unchanged, i.e. $\mu$ is unchanged. Then, since $\mu = \lambda/(1-s)$, with $\mu$ and $s$ unchanged, $\lambda$ is unchanged.

Finally, we come to the rate of return. The equation for this must now be written

$$r_a = P (q - \lambda l)$$

to allow for the possibility that $P \neq l$. It is then clear that $r_a$ is unchanged because $\lambda$ is unchanged, and, while $P$ has halved, $q$ and $l$ have doubled.

I have thus shown that a doubling of $\rho$ and halving of $P$ exactly offset each other. In terms of their effects on growth these two magnitudes are perfect substitutes. In the diagram, a doubling of $\rho$ pushes out the IPC, but a halving of $P$ shrinks back to its original position what one could call the 'financial' IPC, which is simply $P$ times $q$ or $l$. It is the position of the 'financial' IPC which is relevant to the values of $s$ and $r$.

Let me conclude by drawing attention to three further points. First, $P$ is the price of value-added output in the industry. Hence, $P$ may vary both because of a change in the price of the industry's gross output and because of a change in the price of its material or fuel inputs. A rise in the price of fuel, say, which cannot all be passed on, will have similar effects to a fall in the price of the industry's output.\(^5\)

\(^5\) However, a rise in the price of a single input could be mitigated by the substitution of other inputs.
Secondly, it is the level of $P$, not its rate of change, which determines the rate of growth of supply. This is very different from the usual notion of movement along a fixed supply curve where what would matter would be the rate of growth of $P$.

Finally, I believe that the kind of analysis I have briefly sketched here can be used to explain a pattern of behavior which has been revealed as being common to many countries and periods. This pattern emerges when one reviews the changes in outputs, prices and productivity of a group of industries over a given period in a given country, and is described in Ch. 13 of my book.
IV. SOME EMPIRICAL RESULTS USING THE MODEL

(a) Why growth rates differ

In my last lecture I derived a linear equation to explain the rate of growth, $g$, in terms of the share of gross investment in output, $s$, and the rate of growth of quality-adjusted employment, $g_L$:

$$g = aps + \mu g_L$$

This equation, modified in a way to be explained, was fitted to 26 observations relating to different periods and countries. The observations were of trend rates of $g$ and $g_L$ within each period, and of the unweighted averages of the annual investment ratios for each period. In computing the trend rates of growth, I omitted years in which output was below its previous peak, or in which there was a war on. The countries included the USA, Japan and the UK, for which I had data back to the latter part of the 19th century but ending in 1973, and 7 continental European countries for which I had data only for the years 1955-62. All the observations related to the non-residential business sector, thus excluding output, employment and investment in general government, health, education and housing.

The regressions were fitted to weighted observations. More weight was given in direct proportion to the length of the period, the size of the country (measured by its population) and the statistical reliability of the data. Assuming that the goodness of fit for any observation is likely to increase with each of these three factors, as seems reasonable, the effect of weighting is to reduce heteroscedasticity and so to improve the estimates of confidence intervals.

Apart from this, the econometrics was elementary, and I expect it could be improved by proper econometricians. In one respect, however, I do not see that improvement is required although I have heard criticisms being voiced. It has been said that there is a need to control for possible simultaneity. To my way of thinking, one would need to control for simultaneity only if there was some important and systematic factor explaining growth other than the variables I have allowed for. While I can accept that my attempts to excluded cyclical factors have not been entirely successful, I am not aware of any other important systematic factor I have omitted.

Let me now go on to list the factors I did consider. The term $\rho$ in the above equation measures the efficiency of investment. As you may remember from the last lecture, it measures the distance of the IPC out from the origin which I call its radius. There are several different factors which could influence it.

First, as I mentioned in my last lecture, the radius, $\rho$, could depend on $s$, the rate of investment. If there are diminishing returns to the rate of investment, then $\rho$ will fall as $s$ increases. We are here considering changes in aggregate investment within a country, and there is less reason to expect there to be diminishing returns to aggregate investment than to a particular firm's investment, for reasons already discussed.
Secondly, $\rho$ could differ between periods for a whole set of reasons which I will discuss presently.

Thirdly, $\rho$ could differ between countries. Thus some might expect $\rho$ to be bigger in Japan than in, say, the UK.

Finally, $\rho$ could be influenced by what has been called "catch-up". Countries behind the leading country (the USA in my sample) could invest to earn a higher return than it by imitating it in certain respects. They could save on R&D expenditures and they could make fewer mistakes. The scope for catch-up would be greater the further behind the leader you were, and I have measured this by the ratio of output per quality-adjusted man in the non-residential business sector in the country covered relative to that in the USA in the relevant period. However, I excluded agriculture from this comparison on the grounds that output per man there was likely to be a bad indicator of the scope for catch-up because of the importance of land.

To sum up, the above suggests, using a linear approximation, that

$$\rho = 1 + a_1 s + a_2 D_t + a_3 D_c + a_4 \ln cu$$

where $a_1 s$ allows for diminishing returns to the rate of investment, $a_2 D_t$ allows for period dummies as $\rho$ varies between periods, $a_3 D_c$ allows for country dummies, and $a_4 \ln cu$ is the catch-up variable (since $cu = 1$ for the USA, its logarithm is zero for it and increasingly negative for countries whose productivity is further and further below it).

I tried out numerous equations, but will report only the following one:

$$g = 0.9034 g_L + 0.05228 s + 0.03329 D_n s$$

$$+ 0.08480 D_p s - 0.05428 D_p s \ln cu$$

The figures in brackets are t-statistics.

$\bar{R}^2$ (weighted residuals) = 0.956; $\bar{R}^2$ (unweighted residuals) = 0.889

standard error (" ") = 0.00482; standard error (" ") = 0.00786

Apart from the term in $D_n s$, all the other terms are satisfactorily significant. The term in $D_n s$ is significant at a 0.2 probability level on a two-tailed test, and this equation containing the term fits rather better than the one omitting it when I consider (in my book) a curved rather than a linear IPC, as here. Dropping the term does not greatly alter the other coefficients. I choose to retain it as it does help in the analysis which follows.

An idea of the goodness of fit of the equation is given by the scatter diagram (reproduced from my book). In this I have plotted, not $g$ and $g_L$, but $q/r$ and $l/r$, with $q = g/s$ and $l = g_l/s$ and $\rho$ obtained from the regression equation using the linear expression for $\rho$ given earlier. The straight line of best fit is then the IPC when $\rho = 1$. The curved line is another estimate of the
IPC which is described in Ch.11 of the book. The periods and countries are as in Table 2, but only the first year of each period is shown, and a comma before this denotes the 19th century. Thus J'87 means Japan 1887-1899, while J28 means Japan 1928-1936. Many of the observations which deviate most from the line of best fit refer to the 19th century, and are statistically less reliable. Some refer to periods in which it seems probable that output was growing faster than capacity (e.g. Japan 1928-1936 and UK 1937-1951), where the observations lie above the line, or the reverse (e.g. Japan 1911-1928 and UK 1913-1924) where they lie below it. The exceptionally slow rate of growth of productivity in the UK in the earliest years of this century (UK 1901-1913) has been much discussed in the literature. It seems to have been shared by Japan (1899-1911) and, to a lesser extent, by the USA (1900-1913).

Let me now comment on each term in the regression equation in turn. The first term is very much what my theory would predict. It is $\mu$ which, you may remember, is given by

$$\mu = \frac{\lambda}{1-s}.$$  

Since the weighted mean value of $\lambda = 0.697$ and that for $s = 0.154$, this would suggest $\mu = 0.824$. However, that assumes perfect markets, no taxation of investment (or savings) and no borrowing. Allowing for these should raise the value of $\mu$, so that 0.9034 is perfectly reasonable.

The dummy term $D_n$ refers to the inter-war years, and $D_p$ to the post-second world war years. The coefficient $\alpha$ in my original linear equation shows what $q$ would be if $g_L = 0$. Since $r_a = q - \lambda l$

and since if $g_L = 0$, $l = 0$, it follows that $\alpha$ gives the rate of return to investment if $g_L = 0$. Hence the second term in the regression equation shows that the average real rate of return, on that assumption, before the first world war was about 5% p.a. In the inter-war years we have to add together the second and third terms, and that gives a rate of return (with $g_L$ zero) of about 9% p.a. Post second world war we have to add the second and fourth terms and that gives us a rate of return in the USA (but setting $g_L$ zero) of not quite 15% p.a. I say in the USA because for other countries one needs to add in the catch-up term, the last one in the equation. How much one adds there depends on how far behind the USA the relevant country is in the period selected. In the case of Japan in 1961-73, for example, the ratio of output per quality adjusted man to that in the USA is put at 0.4537. Since $\ln 0.4537 = -0.7903$, and $-0.0542 x (-0.7903) = 0.0429$, the effect of catch-up in Japan in 1961-73 is estimated to have raised the rate of return to investment by rather more than 4% p.a., which is quite substantial. However, the presence of the $D_p$ dummy shows that this effect only occurs in the post second world war years, and I will come back to that later.

I have now explained all the terms in the equation, and will next comment on some absentees. As Sherlock Holmes remarked, the dog that did not bark in the night can be significant.
First, I tried inserting a constant term to represent some sort of independent, costless, technical progress. However, it turned out to be negative, very small, and not significantly different from zero.

Secondly, neither different country dummies, nor a term in $s^2$, were significant. The first implied that the IPC was the same in all countries (apart from catch-up after the second world war), and the second that there were no diminishing returns to the rate of aggregate investment. However, it is possible that one finding explained the other. The investment ratio $s$ tended to be highest in Japan, and lowest in the UK. Hence the failure of $p$ to show up as being higher in Japan than in the UK could have been due to there really being diminishing returns to $s$. I tried to test for this with an equation including both country dummies and $s^2$, and this gave some rather weak support to the hypothesis. That is, $p$ in Japan did seem better than average, and worse than average in the UK, and the term in $s^2$ had a negative coefficient. However, as the relevant t-statistics were not particularly large I did not pursue the matter further, although I hope others will.

Table 2 lists the countries and periods and the contributions of $s$, $g_L$ and catch-up to growth in each. It also shows the residual unexplained growth. I should perhaps point out that this residual is not at all the same as the residual in conventional growth accounting. The expected value of my residual is zero, while that in conventional growth accounting is, I suppose, positive. Whereas the size of my residual provides a test of the ability of my model to explain growth, the residuals in conventional growth accounting do not provide any such test. Conventional growth accounting is in no sense a test of orthodox growth theory, and, indeed, I do not know of any tests of that theory comparable to mine.

You will see from the means at the foot of the table that, on average, investment (including catch-up) contributed rather more to growth than did the growth of quality adjusted employment. However, that was by no means true in some periods, and in fact there was very little relationship between the two contributions. For the countries and periods to which it was relevant, catch-up was quite important.

The growth of employment can itself be analyzed into three components: the growth in full-time equivalent numbers, $g_N$, the growth in quality-adjusted hours, $g_H$, and the final index of quality-adjusted employment taken, $g_L$, which differs from $g_H$ only by making an allowance for an

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6 In fact, this trial was made with the statistically preferable version of the equation from which the term in $D_s s$ was dropped.

7 The relevant coefficients and t-statistics were:

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<tr>
<th></th>
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<th>t-statistic</th>
</tr>
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<td>-1.32</td>
</tr>
<tr>
<td>$D_{s} s$</td>
<td>0.02488</td>
<td>0.78</td>
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<tr>
<td>$s^2$</td>
<td>-0.2269</td>
<td>-1.07</td>
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</table>
Table 2. The proximate causes of growth

<table>
<thead>
<tr>
<th>Country</th>
<th>Period</th>
<th>Investment</th>
<th>Growth of labor force</th>
<th>Catch-up</th>
<th>Unexplained</th>
<th>Total</th>
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<td></td>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
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<td>(5)</td>
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<td>Efficiency of an hour's work $g_L - g_H$</td>
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</table>

Note: see text for meaning of $g_N$, $g_H$, and $g_L$.
increase in labor efficiency as hours of work per worker per annum fell. These three components are shown in Table 3, and the main point to which I should like to draw your attention is that there is a substantial difference between \( g_{N} \), the crudest index, and \( g_{L} \), the latter being, on average, twice as big as the former, with this difference ranging quite a bit from one period or country to another. Quality-adjustment of employment is, I believe, important in the analysis of different causes of growth. The adjustments included allowances for changes in average annual hours worked, changes in age- and sex-composition, changes in education, and, very important, changes in the distribution of employment between agriculture and unincorporated enterprises and the rest of non-residential business.

Let me now return to Table 2. Perhaps the most striking feature in it is the very big increase in the contribution of investment to growth comparing the years after the second world war with those before. For the USA, for example, this contribution was about doubled. For Japan and the UK, including catch-up, it was multiplied about five times, or even more depending on the periods compared. This remarkable increase was due to increases both in the share of investment, \( s \), and in its efficiency, \( \rho \). No doubt the increase in \( \rho \) is one reason why \( s \) increased. But what explains the increase in \( \rho \)?

One suggested explanation is the high level of demand in the post-war years, and I think there is something in this. If the ratio of output to capacity is higher on average, that will increase the efficiency of investment, since a given unit of consumption sacrificed will result, for example, in twice as much incremental output if the average ratio of output to capacity is doubled. I also believe that confident demand expectations can result in greater efficiency by counteracting monopolistic tendencies. In a situation of pervasive imperfect competition, which I believe is the reality, we do not want firms to maximize profits (or present values) but to give more weight to increasing output than that would imply. Confident demand expectations may have this effect, and it could be the sapping of confidence since 1973 that partly explains why productivity growth has slowed since then. However, while all this may be true, it can hardly explain the whole of the increase in \( \rho \). We know that the inter-war years were ones of low demand on average and of pretty weak business confidence after 1929, yet my regression equation shows no decline in \( \rho \) from before 1914-18 to afterwards, and, indeed, there is some evidence that \( \rho \) actually increased then, as shown by the term \( D_{s} s \).

Some writers (e.g. Maddison) have emphasized the importance of catch-up. My equation supports the view that that was important, but it still leaves a sizeable increase in \( \rho \) unexplained.

Another possible explanation is the liberalization of trade. While I think that must be true to some extent, estimates of its importance vary, and some are quite small. Furthermore, we again face the difficulty posed by the inter-war years. \( \rho \) probably increased then compared with before the first-world war, yet trade was heavily restricted in the 1930's.

I believe that a considerable part of the explanation is the great improvement and cheapening of communications, together with the greater professionalism of management and, possibly, higher levels of education generally (perhaps this is what Lucas (1988) was thinking of). These factors will have increased the speed with which businessman and others have learnt from each other, and have improved the quality of the investment opportunities perceived and
undertaken. Furthermore, this explanation is consistent with some other facts. It can explain why \( \rho \) increased both from the pre-first world war years to the inter-war years and from the inter-war years to the post-second world war years. It can also explain why catch-up shows up as significant only after the second world war. It is borne out, too, by the shortening of the lag between inventions (defined as "the first idea, sketch, or contrivance of a new product, process or system, which may or may not be patented") and the relevant subsequent innovations ("the first introduction of a new product, process or system into the ordinary commercial or social activity of a country") over the last century (see Freeman, Clark and Soete 1982, Table 3.1(a), (b) and (c), and p.201), and by an increase in the rate of diffusion of mature technologies (Ray 1984).

To some extent the increase in \( \rho \) may have been a statistical artifact. In fitting my equation I was compelled to use the available estimates of gross investment. These exclude R&D expenditures as well as management costs, whereas I would have liked to include at least part of both. It is possible that they increased faster than the rest of gross investment over the period of observation and, if so, the true increase in \( \rho \) was less than I have estimated it to have been. Nevertheless, I very much doubt that this can account for more than a small part of the increase in \( \rho \) observed.

\[(b)\] Externalities to investment and the gap between social and private rates of return

I now want to turn to a very different set of questions which relate to the optimality of investment and growth. Does my model shed any light on the question of whether investment and growth are too low or too high? I say "shed any light" because I do not want to claim that I can provide answers to these questions, which must depend on important considerations outside the limited purview of the model.

Let me first present some estimates of average rates of return earned in non-residential business in the USA and the UK in the "golden" post-war years:

<table>
<thead>
<tr>
<th></th>
<th>Average exponential real rates of return on investment in NRB</th>
<th>Average real returns received by shareholders before deducting income tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>U.S.A. 1948-73</td>
<td>0.154</td>
<td>0.063</td>
</tr>
<tr>
<td>U.K. 1951-73</td>
<td>0.153</td>
<td>0.064</td>
</tr>
</tbody>
</table>
You may remember from my last lecture that the formula for the private average rate of return is

\[ r_a = q - \lambda l = \frac{g - \lambda g_L}{s} \]  

This formula would be the same as the following one

\[ r_{as} = q - \mu (1-s) l = \frac{g - \mu (1-s) g_L}{s} \]  

if \( \mu - \frac{\lambda}{1-s} \), as it should if the firm is maximizing present value in perfect markets with no taxation. However, in reality none of these conditions may hold, and so (2) need not be the same as (1). It can be shown, however, that (2) is the average social return to investment even in the absence of fulfillment of these conditions, provided \( g \) correctly measures the social rate of growth of output, and \( \mu g_L \) is the contribution of employment growth to \( g \) (which is derived from the regression analysis described earlier\(^8\)). Consequently I have labelled this rate of return \( r_{as} \), to show that it is both average and social.

The figures in col(1) of Table 4 are \( r_{as} \), but they happen to be virtually the same as \( r_a \) for the particular periods and countries shown.

The figures in col(2) are average pre-tax returns on a wide sample of equities. They are the sum of average dividend yields and the trend rates of growth of real dividends over the periods in question.\(^9\) They therefore represent both average and marginal pre-tax real returns to shareholders. It is at once clear that these returns are very much less than those in col(1), and in what follows I seek to explain the differences and draw out their implications.

Taxation is one rather obvious part of the difference. However, it by no means appears to explain it all. Good estimates of effective tax rates on investment and savings are hard to come by, and I was able to make estimates only for 1960, 1970 and 1980 using King and Fullerton’s (1984) study. Of these, 1960 seems the best year to take, as it is about in the middle of the period considered. Excluding personal income tax, the rates of tax on investment then were estimated to be 0.268 for the USA and 0.208 for the UK. These rates allow for investment-grants, accelerated depreciation allowances, and also assume particular sources of finance for the investment (mainly retained earnings, but also some debt and new share issues). One can then ask what rate of return

---

\(^8\) However, in deriving \( \mu \) for each country allowance was made for curvature of the IPC, so the equation used was not quite the same as that given earlier.

\(^9\) Instead of the rate of growth of real dividends, many people use the rate of growth of real share prices. However, I believe my method is preferable, although I doubt that the two kinds of estimates would differ much in this case.
a company would need to earn, pre-tax, on investment in order to be able to pay out the returns, before deduction of personal income tax, shown in col(2) above. For the USA the answer is 0.063/(1 - 0.268) = 0.086, and for the UK it is 0.064/(1 - 0.208) = 0.081. These rates, which we might call the equity cost of capital to companies, are still well below the private or social average rates of return of 0.154 or 0.153.

They may not, however, be below the marginal private rates of return earned by companies (before tax). As you may recall from my last lecture, I distinguished between average and marginal returns to investment. There are two reasons I want to consider now why marginal private returns may be less than average private returns (all before tax). Both do not apply to marginal social returns, and so they each imply a discrepancy between marginal private and marginal social returns, and so the existence of positive externalities to investment.

Marginal private returns may be less than average private returns, in the first place, because of what I call the learning externality. At a given moment in time, a businessman is aware of a set of investment opportunities, some better and some worse. Given his chosen rate of investment, he will try to select the best of the bunch, and that implies that the marginal ones he selects are worse than the average, and that if he were to choose to invest a bit faster he would add still worse projects to those he already has selected. It may, of course, not be so much a question of adding discrete projects as of spending a bit more on each of the projects already selected. So, his marginal returns are below his average returns.

However, as time passes, and because everything is changing around him, both because of his own investment but perhaps mainly because of others' investment, he becomes aware of new investment opportunities, and, as I have argued, there is no reason why their average quality should deteriorate (or improve, for that matter). So, if he keeps his rate of investment constant he will typically find his average rate of return constant.

All the above applies to marginal versus average private returns. What about social returns? As I pointed out earlier in this lecture, there is no strong evidence that countries with higher rates of investment have lower \( p \) than countries with lower rates of investment. There was, admittedly, some weak evidence for this, but it was only weak. Furthermore, many writers have argued that there are dynamic increasing returns of various kinds. If all firms invest at a faster rate they generate new investment opportunities at a faster rate for each other. Consequently, even if the marginal social return is a bit below the average social return (and it may not be at all below it) it is unlikely to be anywhere near as far below as the marginal private return. It follows that much of the gap between the average social (or private return) and the marginal private return can be attributed to what I call the learning externality. In symbols:

\[
r_a \approx r_{as} \approx r_s > r
\]

A measure of the externality is then the gap between \( r_s \) and \( r \).
Now let me turn to the second reason why marginal social and private returns may diverge. I call this the *market externality* because its existence depends on the existence of imperfect markets.

You will recall my formula for the private (or social) rate of return in the simple situation beloved of economists in which markets are perfect and there is no taxation and no diminishing returns to investment and so no learning externality. It is:

\[ r = r_a = q - \lambda t = \frac{g - \lambda g_L}{s} \]

In this formula, \( q \) is the proportionate increase in output from a unit share of investment, i.e. \( q = \frac{1}{Q} \frac{dQ}{dt} = \frac{dQ}{S} \). In fact, then, \( q \) is the marginal output to (gross) capital ratio. For the formula to give the private return, \( dQ \) must measure the increment in the *value* of output resulting from the investment \( S \). I assumed, if you remember, that this was a typical firm, the price of whose output was one (say $1). Since it was selling in a perfect market, this price was unaffected by the quantity of its own sales. Now, however, I want to allow for the possibility that it is selling in an imperfect market in which marginal revenue is less than price. Suppose, then, that each extra $1 of sales only nets the firm $\eta$, with $\eta < 1$. For example, if the price elasticity of demand were only 2 we would have $\eta = \frac{1}{2}$. We could have $\eta < 1$ because of selling costs of various kinds, as well as because of price cuts. I shall make no corresponding assumption on the labor market side, continuing to assume that the firm has no market power over wages.

It will, I hope, be intuitively clear that the formula for the *marginal* rate of return must then be modified so that it becomes:

\[ r = \eta q - \lambda t. \]

The previous formula still gives the *average* rate of return, \( r_a \), and it is then clear that the marginal return is below the average by an amount which is bigger the smaller is \( \eta \), i.e., the more imperfect is the market.

The marginal *social* return, however, is not affected by \( \eta \). If the price of a unit of output is $1, and if (as I assume) that also measures its social marginal value, then the marginal social return in the circumstances postulated is given by \( r_a \) and not by \( r \). Consequently, the existence of market imperfections is another reason why the marginal social return exceeds the marginal private return.

Let us now perform the same thought-experiment as we did with the learning externality. Let us compare two otherwise identical economies one of which, called Hare, is investing at a faster rate than the other, called Tortoise, so that output is growing faster in Hare. The typical firm in Hare is, let us assume, investing \( \Delta S \) more than his opposite number in Tortoise. On that extra investment, what is his rate of return? On my present assumptions (no learning externality and no diminishing returns other than those due to market imperfections) he will be getting \( r_a \) if he is conscious of the demand curves for the products he sells all shifting to the right faster than in Tortoise. He has to invest more merely, so to speak, to remain on his demand curve at the same
price. Although his marginal return is the same as in Tortoise, a meager \( \eta q - \lambda l \), the faster rate of investment by all the other firms in Hare has enabled him, as compared with his opposite number in Tortoise, to invest \( \Delta \alpha \) more on which he gets a generous \( q - \lambda l \).

Some of you may be conscious of a similarity between this argument for collective expansion and the so-called "locomotive effect". However, in so far as the locomotive effect envisages faster expansion in output than in capacity, so that the ratio of output to capacity rises, then that is different from my market externality. If the output to capacity ratio rises, that tends to increase inflationary pressure, but that does not happen in my story. There need be no greater inflationary pressure in Hare than in Tortoise. In both, output is growing at the same rate as capacity, and the ratios, which are constant in each, could be the same in each. I am not talking about a Keynesian-type expansion in which slack is taken up and unemployment falls.

There is one more concept I should like to introduce in this lecture which is relevant to these various rates of return, and which is more in the spirit of Keynes, and that is animal spirits, which I label \( an \). Several other writers, such as Baumol, Marris and Williamson, have pointed out that managers of firms will not always seek to maximize the present values of their firms for their shareholders. Managers' rewards may be more dependent on the size of the firms they manage than on its profits, and they may value output and its growth more than shareholders do, not merely because of its effects on their pay but also because they take pride in, and obtain power by, increasing the size of the firm. While managers' motivation is complex, my model has to remain simple, and so I have dealt with this set of issues by allowing for the possibility that managers attach a premium, \( an \), to each unit of output, over and above its value to shareholders. Consequently, while the marginal revenue from a unit of output whose prices is $1 is \( \eta \), managers treat it, I assume, as if it were \( \eta + an \).

I claimed in my last lecture that all the variables in the model could be estimated. The way I have sought to estimate \( an \) is as follows. You will recall the equation

\[
\mu = \frac{\lambda}{1-s}
\]

which relates \( \mu \), the proportionate marginal product of labor, to the shares of wages and of investment. This formula requires modification in the presence of imperfect markets, animal spirits and taxation of investment. Instead of getting 1 from additional output, the firm now gets (at managers' valuation) \( \eta + an \). Instead of a unit of investment involving a sacrifice of a unit of consumption, it now involves a sacrifice of \( 1/(1-T_s) \), where \( T_s \) is the proportionate tax rate on investment. Consequently, the formula now becomes:

\[
\mu = \frac{\lambda}{\eta + an - \frac{s}{1-T_s}}
\]

If we now insert into this equation an estimate of \( \mu \) from a suitable regression equation, plus direct estimates of \( \lambda, s \) and \( T_s \), we find that, for the USA and the UK in the post-second world war period
This is rather remarkable, as it implies that in that period and those countries, animal spirits offset monopoly. Firms on average behaved, it seems, just as if price equalled marginal revenue. I believe that the circumstances of prolonged growth and low unemployment, with only very mild recessions, probably strengthened business confidence in those years, and resulted in a high average value for \( \alpha \). After 1973, and probably before 1939, I believe \( \alpha \) was almost certainly smaller, and firms behaved with much closer attention to the requirements of present value maximization - the pressures in that direction were more severe. The consequences, however, were probably bad in various respects for society as a whole, since it is surely better that firms behave as if price equals marginal revenue than if there is a big gap between them.

If I am right about this, the implication is that in the post second world war years the whole of the gap between the marginal social return and the marginal private return, apart from taxation, was due to the learning externality. There was also a market externality, as I believe that \( \eta \) is significantly below 1 for the typical producer. However, that was offset by the strong animal spirits which made managers prepared to accept low marginal private returns because the accompanying faster rate of investment boosted their firms' growth rates and expanded output.

The figures I gave earlier suggest that the learning externality is very substantial. If one adds to that the taxation of investment one reaches the conclusion that investment and growth are well below their optimal rates, and that policy should be changed to stimulate investment more. I am doubtful of the wisdom of outright subsidies to investment, but at least one should consider taxing it less, or not at all. As well, governments might aim to save more themselves, thereby, I believe, encouraging both national savings and investments.

**CONCLUSION**

In these four lectures I have tried to show why orthodox growth theories which have dominated both economics textbooks and empirical work in the last 30 years are mistaken, and I have put forward a theory which, I believe, avoids the mistakes and also fits the facts reasonably well. One implication for policy is that the volume of investment is sub-optimal, but perhaps the main implication is that both the volume and the efficiency of investment are key determinants of the rate of growth in the long run. It makes a difference, I think, to the emphasis given to policy reform whether one believes that it is investments of all kinds which matter, or only that rather narrow type of investment conjured up by the words "technical progress."
Scatter diagram relating $q/p$ to $l/p$

For key to periods see e.g., Table 2. Each period's first year is shown above, and if this is in the nineteenth century it is shown with an apostrophe; thus, USA '89 = USA 1889-1900.

The equations (10.3) and (11.3) are given in Scott (1989).
APPENDIX 1

An arithmetical example

Exogenous variables:

\[ s = 0.2, \quad g_w = 0.02, \quad \text{and three constants in } q = f(l), \text{ see below.} \]

Endogenous variables:

\[ q, \quad l, \quad g, \quad g_L, \quad \lambda, \quad \mu, \quad r_a \]

Equations:

\[
\begin{align*}
g & = sq \\
g_L & = sl \\
\mu & = \frac{\lambda}{1-s} \\
g_w & = g \cdot g_L \\
q & = 0.1085 + 0.955 l - 0.4 l^2 \\
\mu & = \frac{dq}{dl} = 0.955 - 0.8 l \\
r_a & = q - \lambda l
\end{align*}
\]

Solution:

\[
\begin{align*}
q & = 0.2 \\
l & = 0.1 \\
g & = 0.04 \\
g_w & = 0.02 \\
g_L & = 0.02 \\
\lambda & = 0.7 \\
\mu & = 0.875 \\
r_a & = 0.13
\end{align*}
\]
APPENDIX 2: The Rate of Return

In the lecture I gave a formula for the average rate of return to investment for a firm in steady growth, which, allowing for the case in which $P$, the price of output (i.e. value-added) is not 1, is as follows:

$$r_a = P \left( q - \lambda I \right)$$

In words, this formula states that the average rate of return (for a price-taking firm) is the initial extra gross profit per unit of investment that causes it. The formula assumes that all the effects of investment on output and employment occur instantaneously, no allowance being made for lags which will inevitably occur in practice. Even so, it will be surprising to many that the private rate of return (which we are considering here) should make no allowance for depreciation and, indeed, should be quite independent of the rate at which wage rates are rising (or falling).

In my book I provide three proofs of the formula, the second of which perhaps most clearly explains why it departs from conventional formulae. The essential reason is that, unlike such formulae, mine allows for the effect of extra investment now on future investment opportunities. No such allowance is made conventionally, because future investment opportunities are assumed to be independent of current investment. In my view this is not the case, and the interaction between them is, indeed, a key part of the theory as I sought to show both in Lecture 2 and Lecture 3.

In this Appendix I present only the third formula, which has the advantage of being mathematically simpler, as well as enabling one to consider how the short period rate of return should vary out of steady growth.

While the above formula gives the average rate of return to investment, it would also give the marginal return, $r$, if there were no diminishing returns to the rate of investment. I start by making that assumption, and the discussion is in terms of the marginal rate of return. The proof follows the idea put forward first (so far as I know) by Solow (1963), and repeated by King (1977, ch.8). The firm has an investment, output, and input plan for future periods which it believes is optimal. This plan is made to undergo a small perturbation, so that in period 1 the firm invests one unit more and in period 2 it cuts back investment by whatever amount is necessary to restore the position in period 3 and all succeeding periods to the same as it would have been without the perturbation. Comparing the situations with and without the perturbation, there is a sacrifice of take-out of one unit in period 1. In period 2 there is an increase in take-out, for two reasons: first, there is the yield of the extra investment in period 1; second, there is the cut-back in investment. There are no other differences. The marginal rate of return is then that rate of discount that makes the $PV$ of these differences, discounted back to period 1 (say), zero.

In conventional theory, the argument runs as follows. Let the yield per unit of extra investment, before deducting depreciation, be MRR. Let depreciation per unit of capital be at the rate of $\delta$. Then in period 2, since the capital stock is $1 - \delta$ higher than it would otherwise be, we must cut back investment by $I - \delta$ to restore it to its original position in period 3 and thereafter.
Take-out in period 2 is then $MRR + 1 - \delta$ higher than it would otherwise have been. If $r$ is the required rate of discount, the value of the extra take-out in period 2, discounted back to period 1, must equal the unit sacrificed in that period, or

$$1 = \frac{MRR + 1 - \delta}{1 + r}$$

Hence

$$r = MRR - \delta.$$ 

This argument needs to be modified in two important respects before it can be accepted. First, as conventionally presented, $\delta$ is essentially physical deterioration of the capital stock. In my terminology it corresponds, then, to required maintenance, and not to depreciation. I certainly accept that the rate of return must be measured net of required maintenance, but would argue that gross profit, as conventionally measured, is approximately so measured. In any case, my disagreement with conventional theory is not about the subtraction of maintenance expenditures in calculating gross profit, but about subtracting allowances for depreciation arising from obsolescence (or relative price changes). Although King, for example, starts by making $\delta$ refer to physical deterioration, he subsequently uses data that are undoubtedly dominated by obsolescence. The confusion of the two must, as I have argued, be avoided. If one therefore adopts the conventions followed here of measuring profits net of maintenance, and if $MRR$ is so measured, then $\delta$ must be eliminated, and this then leaves us with

$$r = MRR;$$

i.e., the marginal rate of return equals the initial profit gross of depreciation.

This proof rests on an assumption that I have not yet exposed, and this is the second important modification to the argument. The crucial assumption is that, if the firm enters period 3 with the same capital stock with as without the perturbation, then it is in all other respects the same, so that the future course of events is unchanged. Under what conditions is this assumption

---

10 This is explicit in King (1977, 230) and in Solow (1963, 29). Subsequently (see p.61 et seq.) Solow also introduces the effect of obsolescence in the context of a vintage model with costless exogenous capital-augmenting technical progress. Objections to vintage models are given in Ch.3 of my book.

11 Readers may ask what has become of depreciation? Since we are here concerned with the private, and not the social, rate of return, the loss to capitalists that results when wages rise should, it seems, be allowed for. However, in the period analysis in the text, there is no such loss on the assets resulting from the perturbation. There is none in period 2, when $MRR$ is earned, and in period 3 and later periods, when there would have been a loss had the investment remained, there is none because the original investment in period 1 has been offset by disinvestment in period 2.

12 Again, this assumption is explicitly stated in King (1977, 229).
likely to be fulfilled? It turns out that these conditions are a constant map of IPCs and steady growth by the firm, at least as long as the formula is to hold.

The argument assumes that a unit less of investment in period 2 undoes the effect of a unit more of investment in period 1. Unless the characteristics of the investment in each period are the same, i.e. unless both $Pq$ and $Pl$ are the same, this will not be so. That $q$ and $l$ may differ is obvious if the map of IPC is changing through time, but even if it is constant, $q$ and $l$ could differ. Thus, if there are diminishing returns to the rate of investment, and if the rate of investment without is either increasing or decreasing from period 1 to period 2, then $Pq$ and $Pl$ will not be the same. For example, if the firm is increasing its rate of investment, a marginal investment in period 2 will be less productive than in period 1. In that case, the cut-back in investment in period 2 required to restore the without position in period 3 will be more than one unit. Consequently, $r$ will be greater than the MRR.

Nor is this all. Even if the rate of investment is the same in periods 1 and 2, that is not sufficient to ensure that the firm will choose investments in each period which have the same $q$ and $l$ as its marginal investments. For the same $q$ and $l$ to be optimal in both periods, not only must the map of IPC be the same, and not only must $s$ be the same, but also $\lambda$ must be the same. But the satisfaction of all these conditions amounts to specifying steady growth for periods 1 and 2. So the formula holds for period 1 as long as there is steady growth for periods 1 and 2. To make it hold for period 2 requires steady growth for periods 2 and 3. And so on.

One benefit we can gain from the above argument is the light it sheds on what may be called supernormal (or subnormal) depreciation (or appreciation). I have argued that, under steady growth conditions, the marginal rate of return should make no allowance for depreciation, despite the fact that depreciation that is due to rising real wage rates may (and usually will) be occurring. This depreciation probably accounts for most, or indeed all, of total depreciation in a closed economy,\textsuperscript{13} and so may be called normal. However, there will be industries and firms that are not in (approximately) steady growth, and they will be subject to supernormal or subnormal depreciation or appreciation. An example may help to clarify this.

Suppose that there is some important invention. This may be regarded as shifting out the IPC of the affected firms or industry, and so increasing the rate of return, volume of investment, and rate of growth of the industry. As time passes, competition and the rapid growth of output force down prices of output.

While the IPC are shifting outwards, $Pq$ and $Pl$ are tending to increase (assuming no change in $P$). While this is happening, the preceding argument shows that $r$ will tend to be less than MRR. This is because the cut-back in investment in period 2 required to restore the without situation in period 3 and thereafter is less than one unit. If $q$ and $l$ increase in the proportion $\delta$, which is to say if $\rho$ increases in the proportion $\delta$, between periods 1 and 2, then the cut-back in investment in period 2 is $1 - \delta$, and the conventional result that $r = MRR - \delta$ holds. This case

\textsuperscript{13} In an open economy there will be, in addition, depreciation (or appreciation) arising from adverse (or beneficial) changes in the terms of trade.
might be one in which the price of the relevant capital goods in terms of consumption was falling at rate \( \delta \) per period, or in which their quality was improving at that rate.

The case when the price of output, \( P \), is falling is different. The falling price means that \( MRR \) in successive periods declines, but it does not affect the result that each (short-period) \( MRR = r \). It is still the case that, so long as \( q \) and \( I \) remain constant, the cut-back in investment needed to compensate for the initial unit of extra investment is one unit. However, this result does suggest that the long-period rate of return when \( P \) is falling is below the short-period one, and so below the initial gross profit per unit of investment. Out of steady growth, therefore, with \( P \) falling, one must subtract an allowance for depreciation from the initial gross profit; and one must add appreciation if \( P \) is rising. A similar point applies if, out of steady growth, the share of wages is rising or falling, thus causing short-period rates of return to fall or rise.

In conclusion, let us revert to a point made earlier to explain the difference between the rate of return formula given here and conventional rate of return formulae. I pointed out that the formula allows for the effect of current investment on future investment. It does this through shifting it forward in time into periods in which wages are lower (if wages are rising -- higher if they are falling). This effect occurs for the marginal investment, which is our concern here. We have been considering the effect of investing and saving a little more, and so consuming a little less. In cost-benefit analysis this may well not be the relevant comparison. Instead, one is comparing a situation with a certain project and one without it. In the without situation, investment could be just as high as with, the difference being that it takes a different form. One is then selecting between alternative investment opportunities, rather than deciding how much to invest. In that case, the point about shifting forward investments does not apply, and it is certainly the case that, in comparing the alternatives, future relative price changes (including real wage changes) must be allowed for.
APPENDIX 3: Convergence to Steady Growth

This Appendix provides a demonstration that a firm whose share of investment in output, \( s \), is fixed at some exogenously given constant amount, and which chooses its investments so as to maximize the rate of increase of take-out, will converge on a steady rate of growth. In conformity with the assumptions made in the lecture, the firm is a price taker, and is confronted by investment opportunities represented by a constant IPC. All investment is financed by retained profits. As the firm is representative, \( P=I \) throughout. I do not pretend that this is a satisfactory set of assumptions. Amongst theoreticians it would be more acceptable to endow the firm with certain intertemporal preferences and allow \( s \) to be determined endogenously. However, the mathematics required to handle such a case are beyond me and so, faute de mieux, I present the following demonstration. While unsatisfactory, it may nevertheless have some interest. Reality is much more complex than any model, and those which assume stable intertemporal preferences, although perhaps intellectually more satisfying, may be little nearer the mark than those which assume a stable value for \( s \). One should not, in my view, be unduly purist in these matters, as theoreticians are prone to be. We are all still groping in the dark.

The demonstration proceeds as follows. First, I set out the essential equations of the model and show that a firm with given \( s \) which maximizes the rate of growth of take-out, \( C \), will always choose the characteristics of its investment so as to satisfy:

\[
\mu = \frac{\lambda}{1 - s} \tag{1}
\]

where \( \mu \) is the slope of the IPC at the point which has the characteristics of investment, \( q \) and \( l \), chosen by the firm, and \( \lambda \) is the current share of wages in output. Then I show that this behavior will lead to convergence on a steady growth path.

The equations of the model are

\[
q = f(l), \quad \mu = f' > 0, \quad f'' < 0 \tag{2}
\]
\[
g = sq \tag{3}
\]
\[
g_L = sl \tag{4}
\]
\[
C = Q(1 - s - \lambda) \tag{5}
\]
\[
\lambda = \frac{WL}{Q} \tag{6}
\]

At a given point in time, \( C, Q, W \) and \( L \) are all given, and so therefore is \( \lambda \). \( s \) is also given, and then equations (1) to (4) suffice to determine \( \mu, q, l, g \) and \( g_L \). These then determine \( C, Q \) and \( L \).
in the next period, and \( W \) is determined exogenously, so a new value for \( \lambda \) is determined, and so on.

Differentiating (5) and (6) with respect to time, and denoting time derivatives by a dot:

\[
\dot{C} = \dot{Q}(1-s) - Q\lambda - \lambda \dot{Q} \tag{7}
\]

\[
\lambda = \frac{-\lambda}{Q} \dot{Q} + \frac{W}{Q} \dot{L} + \frac{L}{Q} \dot{W} \tag{8}
\]

Also, from (3) and (4)

\[
\dot{Q} = Q \, sq \tag{9}
\]

\[
\dot{L} = L \, sl \tag{10}
\]

and, if \( g_w \) is the exogenous exponential rate of growth of wage rates,

\[
\dot{W} = W \, g_w \tag{11}
\]

Substituting from (9), (10) and (11) into (8) gives

\[
\lambda = \lambda \left( -sq + sl + g_w \right)
\]

and so (7) becomes

\[
\dot{C} = Q \left( sq(1-s) - \lambda \left( -sq + sl + g_w \right) - \lambda \, sq \right)
\]

If the firm now selects \( \mu \) so that \( \dot{C} \) is maximized, we must have

\[
\frac{\partial \dot{C}}{\partial \mu} = 0 = Q \left( s \left( 1-s \right) \mu - \lambda s \right)
\]

whence equation (1) follows.

To show that this will lead to convergence on steady growth, suppose that the values of \( \lambda, \mu, q, l, g \) and \( g_w \) consistent with steady growth are denoted by asterisks. Let us suppose that, initially, \( \lambda > \lambda^* \) (the argument is symmetrical if the inequality is reversed). Since \( s \) is given, it follows from (1) that \( \mu > \mu^* \). It then follows from (2) that

\[
q - l > q^* - l^*
\]

given that \( \mu < l \). As was shown in the lecture, we must have \( \mu < l \) for take-out to be positive as long as (1) holds. Multiplying through by \( s \) then gives
the exogenously given rate of increase of wage rates. It then follows that, since labor productivity is growing faster than wage rates, \( \lambda \) must be falling, and so approaching \( \lambda_* \). All the other magnitudes must then likewise be converging towards their steady growth equilibrium values.
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