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Annex 10

The Power System Simulation Model

Prepared by a Group of the World Bank Staff

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ANNEX 10

THE POWER SYSTEM SIMULATION MODEL

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PREFACE

Electric power systems are characterized by complex interactions among component generation and transmission facilities. The operation and performance of each piece of equipment at any particular time is heavily influenced by the composition of the rest of the supply structure. And the choice of one unit over another for installation at the present time may influence system development far into the future. Furthermore, in the West Pakistan case, there is strong interdependence between the management of electric power and the performance of other sectors of the economy, particularly agriculture. One of the primary thrusts of this study, therefore, has been in the direction of general systems analysis of investment in electric power.

The Bank Group was most fortunate in being able to secure the services of Dr. Henry D. Jacoby, a member of the economics faculty of Harvard University, who had developed a computer simulation model for use in the planning of electric power systems. 1/ Dr. Jacoby made extensive elaborations to his basic model to take account of many of the specific characteristics of the West Pakistan power systems. The result is a computer program which simulates the long-run capacity expansion and short-run operation of a major portion of what is likely to become the interconnected power grid for the Province.

The simulation model is a tool for comparing alternative programs for development of generation in each of the main power markets and EHV transmission between the markets. For purposes of the model, the Province is treated as consisting of three major markets: (1) the Northern Grid along with its sources of supply and its demand, (2) a Southern Market composed of Karachi and Hyderabad along with their loads and their existing and potential generating facilities, and (3) the Upper Sind (Mari-Sukkur area) with its potential development of thermal generation and its demand. The small Quetta system has little bearing on the questions under study, and it was not included in the analysis.

The evaluation of alternative power investment programs takes place in the context of a demand projection for each of these three markets for a twenty-year planning period. In addition to these projection data, information is required on the capabilities, heat rates, and fuel, maintenance and operation (M & O) and capital costs of all existing and potential thermal and nuclear generating facilities; the capital and M and O costs of existing and potential hydroelectric developments along with monthly patterns of capacity and energy output; the capital and M and O costs and the carrying capacities of proposed transmission lines; and economic parameters such

1/ See H. D. Jacoby, Analysis of Investment in Electric Power, Doctoral Dissertation, Harvard University, 1967.

as discount rates, foreign exchange rates and opportunity costs of capital. Alternative power programs are defined which are "equivalent" in that each will meet projected demand growth in each of the three markets with an acceptable standard of service quality as evidenced by the maintenance of a certain quantity of technical reserve. The computer model is then used to compute indicators of the relative economic attractiveness of the different programs under a variety of assumptions about the values of certain critical economic variables such as the foreign exchange rate or fuel prices.

This analysis is accomplished by means of a two-part procedure: (1) detailed simulation of system expansion and operation over the twenty-year planning period, and (2) an adjustment for the impact of different investment programs on system cost in the years beyond the twenty-year horizon through the use of a simple terminal correction. As noted above, there is strong interdependence between the various units which are found on the system at any point in time, and in order to capture the essential operating characteristics of the system as they impinge on total cost, an approximation of the results of hourly and daily load dispatching on the system is calculated for each month of the planning period. Considerable attention was devoted to the formulation of this model of short-run operation. What is desired is an aggregative technique which captures the essential characteristics of system operation without getting weighted down with excessive technical detail and computation expense. The algorithm developed here uses the integrated load function as a basis for approximating the results of the optimal dispatching of generating units to meet an ever fluctuating instantaneous power demand.

Based upon this model of system energy dispatch, the computer program calculates fuel costs incurred in each month of the planning period, and in turn combines these data with figures for the capital and M and O expenditures implied by a particular investment schedule to produce information on the present worth of total system supply cost over the plan period. This calculation is performed for a range of values of the discount rate, the foreign exchange rate, fuel prices and opportunity costs of capital in order to allow the testing of the sensitivity of results to variation in assumptions about these variables.

At the end of the plan period there will be a collection of assets which is passed on beyond the planning horizon. The form of the final asset structure will differ according to the particular pattern of development followed during the plan period, and this difference will be reflected in a variation in the cost to serve provincial electric demand in the years of the more distant future. The second part of the analysis involves the approximation of the economic impact of differing terminal conditions by means of a set of simple functions and the adjustment of the computer results to account for these effects.

The essential feature of the computer model is its ability to calculate total system fuel costs; this aspect accounts for most of its complexity, most of the time taken in the analysis of a development program, and most of the value of the model itself. The dispatching calculations are based on monthly representations of the integrated load functions for each of the major markets. This approach is described in detail in the text of this annex. During any month, the program begins by dispatching the hydro plants in such a way as to make the best use of their capacity and energy in the Northern Market. If there is unused energy in the North, the program attempts to transmit the excess to the South. This is an iterative calculation, and the program tries to minimize the sum of wasted energy in the North plus line losses. A mandatory transmission from North to South is made in the event the Southern Market would otherwise be in shortage.

Once the hydro dispatch in the North and the transmission to the South have been completed, the program begins the dispatch of thermal units. Each market is dispatched in turn in such a way as to minimize the market fuel cost for that month. If there exists generating capacity in the Upper Sind (Mari) and if a full EHV transmission system from Lyallpur to Karachi is in place, the program must consider the possibility of transmission from Mari in either direction (provided that the line from Lyallpur to Mari is not already loaded with hydro energy coming south). Since the energy cost of Mari production may be significantly cheaper than thermal energy in either the South or North, there is a problem of optimal allocation of this capacity between the two markets. Once again the computer program goes through an iterative calculation, re-allocating the Mari capacity, subject to all the relevant transmission and market demand constraints, until the particular allocation which minimizes total system fuel cost for the month is achieved.

Performance of this system dispatch, or even an approximation to it, by hand would clearly be extremely cumbersome, and the analysis would have to be limited to one or two alternative programs. On an IBM 7094 computer, on the other hand, the complete evaluation of a twenty-year development program including several separate system dispatches for each month of each year (one for each set of fuel prices) and computation of the present worth (at various different discount rates) of fuel costs, maintenance and operation costs and capital costs (with foreign exchange expenditures valued at a number of exchange rates) requires approximately four minutes; on an IBM 7090 it requires slightly longer. So long as the load forecast remains fixed, any number of alternative development programs may be simulated in a particular computer run. Thus, once the basic computer program has been prepared and data on all existing and potential system components have been drawn together, the system model may be used with reasonable ease to evaluate any number of development programs that appear to be of interest.

Besides the summary figures regarding the total costs of a development program over the plan period, the computer is also programmed to print out a large amount of additional material - about 25 pages in all - regarding each development program studied. Most of this material concerns the operation of the system in each month of the twenty-year planning period. It shows approximately how the hydro and thermal plants in each market and the transmission lines linking the markets may be used most effectively under the conditions created by the development program being studied. This information is extremely valuable because it helps to show why the total costs of any particular development program turn out as they do relative to the costs of alternatives. Much use was made of the detailed data regarding system operation in the refinement of programs and in studies of matters such as fuel requirements in different areas and the effect of transmission line capacity on the absorption of hydro energy. One major advantage accruing from the availability of detailed data on system operation was that it made possible the adoption of an approach to fuel pricing, discussed in more detail in Annex 5, which took some cognizance of the differences between programs in their requirements of thermal fuel - not only absolutely over the whole twenty-year period, but year by year as depletion of fuel reserves continued.

The experience of the Bank Group in the use of a computer model for simulating the operation of the power system convinced it that this type of approach has considerable potential for assisting in decisions regarding system development. The availability of the computer model made possible a depth and a range of computations that would otherwise have been physically infeasible; the depth was primarily due to the detail provided by the computer print-out regarding system operation under different development programs, while the range was due to the relative ease and speed with which alternative programs could be analyzed.

Nevertheless the Bank Group's studies far from exhaust the potential of the simulation model. There is a wide range of additional alternatives (such as alternative operations of the reservoirs and analyses with different load forecasts) which the Bank Group would have desired to cover had time been available. There are some questions - such as the scheduling of plant retirements or the question of the amount of generating reserves that should be maintained on the system - for which the simulation model could prove helpful but which hardly find a place in this report because the limited time available for study forced the Bank Group to set quite strict priorities. Because of the frequent changes that inevitably occur in knowledge of a country's resources and in expectations regarding loads and system developments, planning has to be a continuous process. It is because of its belief in the usefulness of the simulation model as a tool for continuous system planning that the Bank Group has included in this report a considerable amount of detail regarding the model and the way it works.

This annex, prepared in large part by Dr. Jacoby, presents this technique in considerable detail. Chapter I introduces this approach to long-run planning and discusses the different elements of the total system cost function. Chapter II describes the computer program itself - its size, data requirements, sequence of computation and part of its print-out. Chapter III is an explanation of the several adjustments to the computer results which may be required in certain cases, the principal adjustment being the terminal correction.

Chapters IV through VIII are devoted to the details of the system energy dispatching routines. Chapter IV includes a derivation of the characteristics of the integrated load function and an explanation of its use in modeling system energy dispatch. Chapters V and VI contain the details of the computer algorithms developed to handle hydro and thermal dispatching respectively, and Chapter VII presents the analysis of inter-market transmission. Finally, Chapter VIII describes the monthly system operating summary which the program provides.

GLOSSARY OF SYMBOLS *

A	- A set containing the identifying numbers, j , of one or more hydro plants, U_j .
A_1 , A_2	- Sets containing the identifying numbers, k , of one or more hydro dispatch blocks, B_k .
a_1 , a_2 , a_3 , a_4	- Parameters of the quadratic approximation to the integrated load function.
B_k	- A set containing the identifying numbers of all hydro plants to be dispatched in a single block, k .
$C_j \left[Q_j(t)\right]$	- Production function for thermal plant j representing fuel efficiency (Btu or calories per kwh) as a function of instantaneous output (mw).
$\bar{C}_j$	- Average fuel efficiency of plant j over the relevant range of $Q_j(t)$.
c	- Index identifying different transmission systems; $c = 1,\ldots,\underline{c}$.
$D(u)$	- The load-duration function.
$d_{j\lambda}$	- Domestic component of capital cost of plant j in year λ of the construction period, $1 \leq \lambda \leq \tau_j$.
E_T	- Total energy generated or demanded over the time interval T (mwh) .
E_{jT}	- Total energy produced by thermal plant j over the interval T .
$E(P)_T$	- The integrated load function; read as "the energy generated (or demanded) over the time interval T at an instantaneous system load less than or equal to P" . When it is necessary to distinguish between two markets, the function may appear as $E_1(P)_T$ and $E_2(P)_T$.

* Symbols are presented in alphabetical order; first those constructed of standard typewriter symbols, then those based on Greek letters. Because of their mnemonic value, seven characters are repeated in italic script; they are d, E, f, j, k and $\mathcal{V}$ . .

$e(p)_T$	- Unit or normalized form of the integrated load function.
E	- Surplus hydro energy after dispatch of plants into the Northern Market.
$F(\underline{x}_i, Y_c)$ or $F(\underline{x}_i)$	- Total system fuel cost in year i .
f_j	- Fuel price ($/Btu or $/calorie) for plant j .
$f_{j\lambda}$	- Foreign component of capital cost on plant j during year λ of the construction period, $1 \leq \lambda \leq \tau_j$
$G(X, Y_c)$	- The objective function of the simulation models - to be minimized.
g	- The rate of foreign exchange.
h_{jT}	- Normalized form of H_{jT} .
i	- Annual time index; $i = 1,\ldots,N$.
j	- Index identifying different plants; $j = 1,\ldots,\underline{j}$ are thermal and nuclear units; $j = \underline{j} + 1,\ldots,J$ are hydro plants.
$\mathcal{j}$	- Index identifying a dummy hydro plant used in the hydro transmission routine.
k	- Index identifying hydro dispatch blocks.
$K_i(X)$	- Total expenditure on capital investment in year i .
$k_{j\lambda}$	- Total capital expenditure on plant j during year λ of its construction period, $1 \leq \lambda \leq \tau_j$.
kw	- Kilowatts.
kwh	- Kilowatt-hours.
L_T	- System load factor over the time interval T .
l	- Index for individual consumers on an electric power system.
$M(\underline{x}_i)$	- Total system maintenance and operation cost in year i .
mw	- Megawatts.
mwh	- Megawatt-hours.
m_{ji}	- Unit maintenance and operation cost ($/kw) for plant j in year i .

N	- Length of the planning period in years.
$N+$	- Refers to the period beyond the N-year Planning horizon.
P	- Instantaneous electric power demand (mw).
$P(t)$	- Time path of instantaneous electric power demand.
p	- Instantaneous electric power demand expressed as a percentage of the maximum demand during the period T; $0 \leq p \leq 1$. This is referred to as the "normalized" form of this variable.
P_{maxT}	- Maximum value of P over the time interval T .
P_{minT}	- Minimum value of P over the time interval T .
P^*_j or P^*_k	- A set describing the load position taken by a particular hydro plant or block.
$\hat{p}_T$	- The normalized form of P_{minT} ; i.e., the ratio of P_{minT} to P_{maxT} .
$\overline{P}_j$, $\overline{P}_k$	- The upper limit of plant j or block k as dispatched on the integrated load function.
$\overline{p}_j$, $\overline{p}_k$	- Normalized form of $\overline{P}_j$ and $\overline{P}_k$.
$\underline{P}_j$, $\underline{P}_k$	- The lower limit of plant j or block k as dispatched on the integrated load function.
$\underline{p}_j$, $\underline{p}_k$	- Normalized form of $\underline{P}_j$ and $\underline{P}_k$.
Q_{jT}	- Output capacity of plant j (mw) in period T .
$Q_j(t)$	- Instantaneous load on plant j ; $Q_j(t) \leq Q_j$.
q_{jT}	- Normalized form of Q_{jT} .
R_j	- Date when plant j is retired from service $1 \leq R_j \leq N$.
r_1 , r_2	- Minimum system reserve expressed as a percentage of the amount of thermal and hydro capacity respectively.
S_j	- Date when plant j is placed in service; $1 \leq S_j \leq N$.
T	- A time interval, generally a month or a year in the calculations but often representing a day in graphical presentations.

t	- Continuous time.
U_j	- Denotes individual thermal and hydro projects.
u	- An instantaneous power level, $0 \leq u \leq P_{maxT}$, used in the derivation of the load-duration and integrated load functions.
$V_{1,2}$	- Power transmitted from Market No. 1 to Market No. 2 (mw) .
$V^*_{1,2}$	- The constraint on transmission between Markets No. 1 and No. 2.
$\Delta V_{1,2}$	- An incremental block of power added to the existing transmission between Markets No. 1 and No. 2.
v	- An instantaneous power level, $0 \leq v \leq P_{maxT}$, used in the derivation of the integrated load function.
V	- Size of incremental block of power added in each iteration of the inter-market transmission routine.
W_1 , W_2 , W_3	- Variables used to simplify the writing of certain equations in Chapter V.
X or $\|\| x_{ji} \|\|$	- Matrix of zero-one variables, x_{ji} , indicating which plants are in place at various points in time. The full matrix is called a "strategy".
$\underline{x}_i$	- A column vector from X indicating which plants are in place in year i .
$\underline{x}_N$	- The final column vector in X , indicating the asset structure passed on beyond the planning horizon.
$Y_1(\Delta V)$	- Excess hydro energy associated with capacity ΔV made available at a particular hydro plant in the Northern Market.
$Y_2(\Delta V)$	- Energy required to support a dispatch of ΔV into the Southern grid.
Z	- Total fuel cost for the interconnected system.
$\Gamma(\underline{x}_N, Y_c)$	- Total terminal correction associated with the final asset structure, $\underline{x}_N$, and transmission system, Y_c .
γ_1 , γ_2	- Percentage of Mari capacity allocated to Markets #1 and #2 respectively.

δ_T - Market excess reserve capacity in period T (mw).

θ_i - Opportunity cost of investment in year i.

Λ - Transmission loss between Market No. 1 and Market No. 2, expressed as a percentage of power transmitted.

λ - Time index for the years of the construction periods of new plants.

π - Discount rate.

τ_j - Length of construction period for plant j in months or years.

Υ_c - Denotes different transmission systems.

ψ - A set used in the derivation of the integrated load function.

ω_j - Average kw output of plant j over the hours it is actually running expressed as a percentage of the rated capacity, Q_j.

CHAPTER I

THE APPROACH TO LONG-RUN PLANNING

Equivalent Alternative Systems

Long-run planning of electric power generation involves two basic operations. First is the definition of alternative patterns of system expansion; the planner looks for alternative plans, which are "equivalent" in that each will meet forecast capacity and energy demand growth in such a way as to maintain an acceptable standard of service quality. Second is the economic evaluation of these plans. Throughout this discussion, focus is upon the planning of additions to and deletions from electric system generating capacity and the analysis of long-distance, inter-market transmissions. Intra-market distribution networks are not considered.

In order to restrict the analysis in this manner, the generation stage of supply must be defined in such a way as to yield comparable investment alternatives. For purposes of this study, generation is considered to end at the point of entry of power into the market distribution grid. If a plant is located within the geographical confines of a market, the generation stage stops at the high voltage or market side of the main generator transformer. If, on the other hand, a plant is located far from the market, then the analysis may be extended to include the operation of the transmission line, or the plant itself may be defined to include transmission to the point of entry into the grid. This normally will be at the low voltage side of the step-down transformer at the market end of the transmission line.

Analysis of the transmission of power between two distant markets requires that explicit consideration be given to the limits and losses associated with a long-distance transmission system. It also is necessary to keep track of the step-up and step-down transformers required at each terminal, and the procedures followed in this regard are described in Chapter III below.

Formulation of the planning task in these terms brings out several important assumptions which underlie most analyses of electric power systems. In working with the sub-optimization of investments in generating capacity and inter-market transmission, the existence within each market of a fully integrated grid system is implicitly assumed. Further, it is assumed that no market is so large geographically as to involve significant differences in distribution losses with supply from generating units at different locations within the market. In effect, it is as if it were possible to transmit power from one location in the grid to any other at any time at equal cost.

It should be noted that the equal cost assumption need not always hold true even for thermal plants located well within the physical boundaries of a particular market. Distribution costs often influence the allocation of production among plants and may affect the location of new generating stations in relation to centers of load concentration and fuel supplies. Furthermore, it is quite possible that the load demand in some regions of a market may go unserved while generating capacity lies idle due to the fact that distribution lines in some areas are already operating at maximum capacity. It is not often the case, however, that the characteristics of the intra-market distribution system affect the selection of investments in generation or long-distance transmission, and this does not appear to be a problem in West Pakistan. Though the Northern grid serves a rather large geographical area, the treatment of this system as a single homogeneous market is justified for purposes of long-run investment planning.

Load Forecasts and Generation Strategies

Like most approaches to electric power system planning, the simulation technique used here operates on the basis of a deterministic projection of market capacity and energy demand. System load is exogenous to the model. In effect it is assumed that investment to serve projected demand growth is always justified; the willingness of consumers to pay for electric power is assumed to be greater than the cost of supply within the relevant range of equipment and fuel costs. From the standpoint of the planning model, therefore, the load forecast is a set of constraints; all valid investment plans must provide sufficient generation and transmission capacity to serve the growing markets. Alternative assumptions about future load growth can be analyzed with the use of this model, and this has been done as part of the studies carried out for the West Pakistan system. But this extension of the analysis involves the definition of a different set of alternative power programs and separate runs of the computer model.

In order to describe how equivalent alternative plans are defined under a particular load forecast, it is convenient to begin with the case of a single isolated market. And at this point it is necessary to introduce some of the mathematical notation which will be used throughout this annex. Let the subscript i serve as an annual time index, $i = 1,\ldots,N$, where N is the length of the planning period. And let the subscript T be used as a monthly time index, $T = 1,\ldots,12.N$. Because of the extreme seasonal variations of the capabilities and energy outputs of most of the potential hydroelectric projects in West Pakistan, the analysis has been conducted on a monthly basis; the main element in the demand forecast is a projection of peak loads, P_{maxT}, in each major power market in each month of the twenty-year planning period. Alternative system plans must be defined in such a way that this forecast capacity demand is met in each and every month with a certain minimum amount of technical reserve.

Suppose there is a number of generating units, U_j, $j=1,\ldots,J$, which might be in service during some particular year where $j=1,\ldots,\underline{j}$ are existing and potential thermal and nuclear units and $j=\underline{j}+1,\ldots,J$ are the hydro possibilities. Each thermal or nuclear unit is characterized by its rated output capacity net of in-station losses and consumption, Q_j , for purposes of defining alternative system expansion plans; plant output capability is assumed constant over any year. The output characteristics of each hydro plant are represented by its capacity, Q_{jT} , and the associated energy, H_{jT} , in each month of the planning period.

For each generating unit U_j there is a scale variable, x_{ji} , which is appropriate for each year. The variable x_{ji} is limited to the values zero and one. If a particular U_j is in the system in a particular year, $x_{ji} = 1$; if not, $x_{ji} = 0$. During any year, the system supply structure will be composed of some subset of U_j . Additions to and deletions from system generating capacity are planned under a set of 12.N constraints of the following form.

$$0 \le \delta_T = \sum_{j=1}^{\underline{\underline{j}}} \frac{Q_j x_{ji}}{1 + r_1} + \sum_{j=\underline{\underline{j}}+1}^{J} \frac{Q_{jT} x_{ji}}{1 + r_2} - P_{maxT} . \qquad (1.1)$$

The term δ_T indicates the excess reserve on the system in month T , and it must not be negative for any month of the planning period else the particular plan will be ruled invalid. In these studies, the technical reserve for thermal and nuclear units, r_1 , was set at 12 percent. The reserve for hydro capacity, r_2 , was set at 5 percent of dependable capacity.

Power development programs which meet the capacity constraints of Equation 1.1 in each month of the planning period are defined as equivalent alternatives insofar as they can all meet system demand with a satisfactory level of service quality. Such a power development program is termed a "strategy" in the terminology of this simulation approach, and each strategy may be denoted by a matrix of zeros and ones, $X = \| x_{ji} \|$; $j=1,\ldots,J$; $i=1,\ldots,N$. The particular combination of plants in existence in any year, i , is indicated by the appropriate column vector, $\underline{x}_i = [x_{1i},\ldots,x_{Ji}]$.

So far as the computer analysis is concerned, a strategy is introduced in the form of two vectors which indicate the dates during the planning period when each of the planned system additions or retirements is to be made. $\underline{S}_j$ is a "start" vector; it applies to that subset of U_j composed of potential projects, those for which $x_{j0} = 0$. A potential project may be introduced at any time during the planning period; that is to say, $1 \le S_j \le N$. $\underline{R}_j$ is a "retire" or "stop" vector, and it applies to that subset of U_j which is in existence at the beginning of the planning period, i.e., $x_{j0} = 1$. Since an existing project may be retired at any time during the plan period, $1 \le R_j \le N$.

EHV Interconnection of Markets

For the purpose of the computer analysis, West Pakistan was treated as consisting of three electric power markets. The two major markets are the Northern Grid area, referred to in themmathematical notation as Market No. 1, and the Karachi-Hyderabad area, denoted as Market No. 2. A third market was identified in the Upper Sind (Mari) and denoted Market No. 3. It is much smaller than the other two in terms of demand, but it is of considerable interest due to the potential development at Mari of low cost, gas fired, thermal generation.

In any particular year of the future, one may expect the overall system to be in one of four configurations as regards interconnection between these three locations. The current situation is shown in Figure 1a; the markets are unconnected at the present time. A second possibility is that a tie will be formed between the potential Mari generating center and the Southern Market. In subsequent discussions, each transmission line will be denoted by the numbers of the source and destination of the power sent over it, and thus this connection from Mari to the South would be noted as link 3,2. The transmission capacity of the line is denoted as $\overset{*}{V}_{3,2}$ as indicated in Figure 1b. A third possibility would be to link Mari with the Northern Market as shown in Figure 1c. This would be link 3,1 with transmission capacity $\overset{*}{V}_{3,1}$.

Finally, it would be possible to form a complete interconnection between the North and South via Mari. If no power were being transmitted from the North to the South, then any capacity at Mari could be dispatched in either direction; the transmission constraints would be $\overset{*}{V}_{3,1}$ and $\overset{*}{V}_{3,2}$. If power were being shipped from North to South, then the transmission limit as far as Mari would be $\overset{*}{V}_{1,3}$; the constraint from Mari to the South would be $\overset{*}{V}_{3,2}$. If $\overset{*}{V}_{3,2}$ were greater than $\overset{*}{V}_{1,3}$, as it usually would be, then additional Mari power could be added to the power being trans-shipped to the South. Note that $\overset{*}{V}_{3,1}$ would not be equal to $\overset{*}{V}_{1,3}$; the capacity of the line would not be the same in both directions.1/

So long as the West Pakistan electric power system remains without interconnection between the major markets, the capacity constraints of Equation 1.1 must be met independently in each power market by any generating program or "strategy". When interconnection is introduced, the constraints may be met on a system-wide basis, taking into account the maximum load limits on each of the transmission lines. Each inter-market

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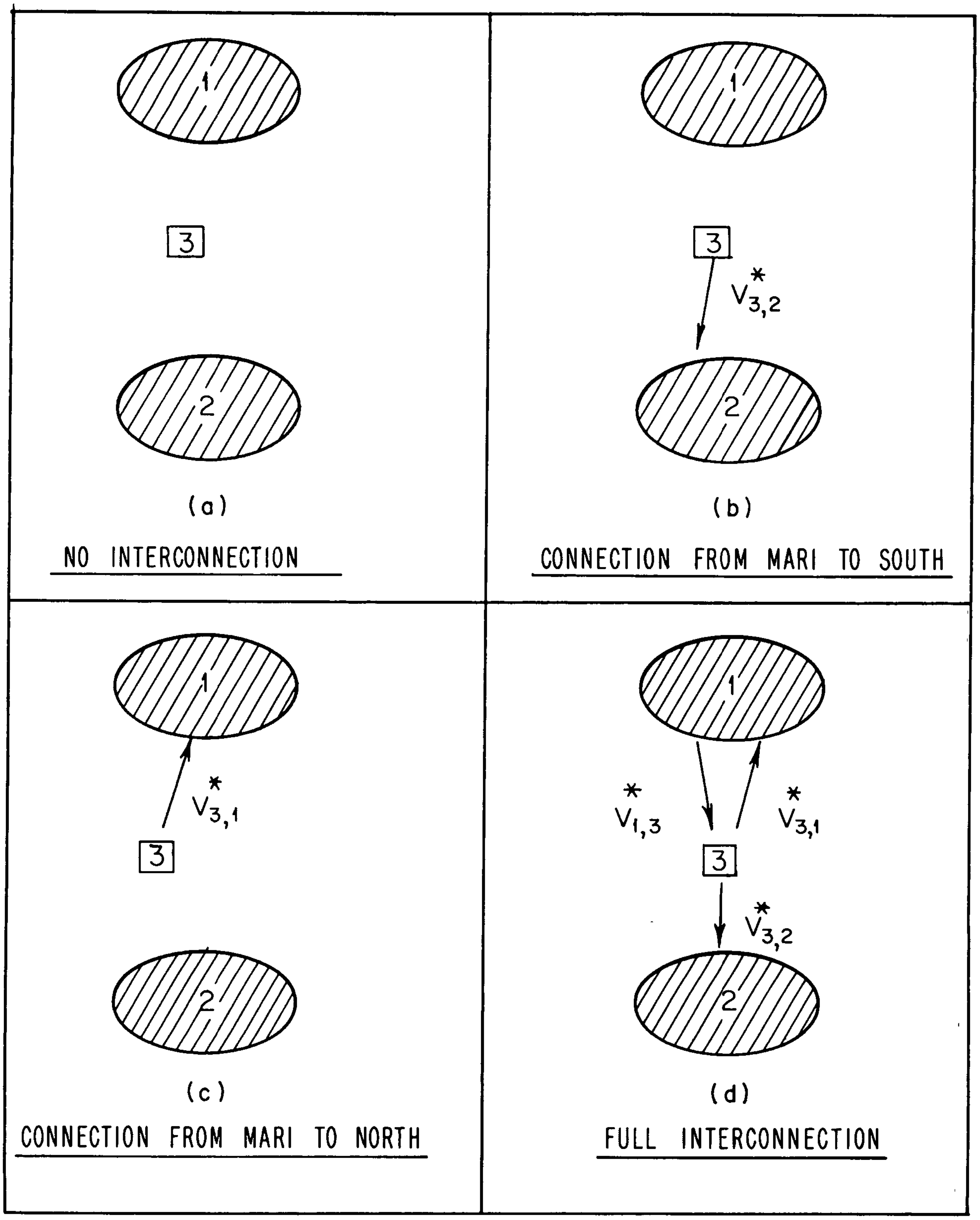
1/ The reason for this difference is a matter of technical detail and need not be elaborated here except to note that the transmission from 1 to 3 carries a greater reactive component than the shipment in the other direction. For any particular transmission line, $\overset{*}{V}_{3,1}$ is approximately 60 percent greater than $\overset{*}{V}_{1,3}$. The reason for not defining a link 2,3 to carry power from the South to Mari is that this was not an important alternative for consideration.



CONFIGURATIONS OF SYSTEM TRANSMISSION

transmission system which might be constructed during the planning period is introduced as an individual project, Y_c , $c=1,\ldots,\underline{c}$, with appropriate values of the transmission constraints noted above for each year of the plan period and the associated costs. Any number of combinations of generating strategy and inter-market transmission programs may be compared using the simulation model so long as each is able to serve the demand in all markets with the required level of technical reserve.

Total System Cost

Each power development plan which meets the constraints identified in Equation 1.1 on an individual market basis if interconnection does not exist and on a system-wide basis if interconnection has been introduced has associated with it a certain time profile of capital costs, maintenance and operating expenditures and fuel costs. The indicator which is used to compare the relative merits of alternative power programs is the present worth of these cost streams. The purpose of the planning model is to find that particular combination of generation strategy, X , and transmission program, Y_c , which has the lowest value of $G(X, Y_c)$ where

$$G(X,Y_c) = \sum_i \left[(1+\pi)^{-i}\right] \qquad (1.2)$$

$$\left[K_i(X) + M(\underline{x}_i) + F_1(\underline{x}_i,Y_c) + F_2(\underline{x}_i,Y_c) + d_{ci} + gf_{ci} + m_{ci}\right]$$

$$-(1+\pi)^{-N}\,\Gamma(\underline{x}_N,Y_c)\ , \qquad i=1,\ldots,N \qquad 1 \le c \le \underline{c}\ .$$

The term in the first set of brackets is the appropriate discount factor; this is multiplied by the sum of the terms in the second set of brackets: capital, maintenance and operation, and fuel costs associated with the particular generation strategy and transmission plan plus capital and maintenance and operation costs for the transmission system itself. The final term is a correction for terminal conditions.

The function of the computer model is to simulate the long-run capacity expansion and short-run operation of the power program described by a particular choice of X and Y_c and to calculate the costs incurred in each month and year of the planning period $i=1,\ldots,N$. By this means, the cost implications of a particular power program are evaluated in considerable detail through the end of the N-year planning period.

The differential system supply cost during the years following the planning period chosen may also be significant, however. For example, a power program composed primarily of conventional thermal units will impose higher fuel costs on the system in future years than one including heavy investment in hydroelectric facilities. The full impact of these differences among power programs during the planning period is reflected in the simulation analysis itself; the influence of different programs on succeeding years is captured by means of a set of simple functions which are combined into a single cost adjustment, referred to here as a "terminal correction". In all the computer studies of the West Pakistan system, all costs - those produced by the computer analysis and the terminal correction - were discounted back to January 1, 1965.

The cost function represented by Equation 1.2 involves consideration of market demand patterns, plant characteristics, investment decisions and short-run system operating rules. The individual elements of the function may be described in greater detail as follows.

Capital Cost

The term $K_i(X)$ represents the capital costs of new generating facilities. The capital expenditure on any particular plant j will be spread over the length of its construction period τ_j. Let the index λ denote the different years of the construction period so the vector of capital inputs for the plant might be written as $\bar{k}_{j\lambda} = (k_{j1},\ldots,k_{j\lambda},\ldots,k_{j\tau_j})$. Which of these costs is incurred in a particular year depends on the date of the start of construction; since S_j is the time of completion of the plant and entry into the system, the construction start must be at time $i=S_j - \tau_j$. In order to pick up the correct element of $\bar{k}_{j\lambda}$, the index λ for any particular new plant must be calculated as $\lambda=i-(S_j - \tau_j)$.

If θ_i denotes the opportunity cost of capital in year i, then the capital cost term of Equation 1.2 becomes

$$K_i(X) = \sum_j \theta_i k_{j\lambda} \qquad j=1,\ldots,J, \qquad (1.3)$$

where

$$\lambda = i - (S_j - \tau_j), \text{ and } k_{j\lambda} = 0 \text{ if } \lambda < 1 \text{ or } \lambda > \tau_j .$$

The second part of Equation 1.3 defines the time index, λ. The last part simply says that capital costs for plant j are zero if the particular year in question does not fall within its construction period. The capital cost, $k_{j\lambda}$ may be broken down into domestic and foreign components, d and f respectively, using an exchange rate g; this may be written as

$$k_{j\lambda} = d_{j\lambda} + g f_{j\lambda}$$

Maintenance and Operation Cost

The second term in Equation 1.2, $M(\underline{x}_i)$, represents plant maintenance and operation. This requirement is usually stated in terms of dollars per kw of installed capacity, and the unit cost, m_{ji}, varies according to plant type, size and age. For any year, M & O cost is simply summed over all plants in existence.

$$M(\underline{x}_i) = \sum_j m_{ji} Q_j x_{ji} \qquad j=1,\ldots,J \qquad (1.4)$$

Fuel Costs

Next, the fuel costs incurred in serving two different markets 1/ must be considered. $F_1(\underline{x}_i, Y_c)$ is the fuel expenditure required for Market No. 1, $F_2(\underline{x}_i, Y_c)$ is for Market No. 2. The evaluation of these two fuel cost terms requires the modeling of daily system energy dispatch; this is the origin of most of the complexity of the computer model itself, and discussion of the techniques used to estimate these functions is the subject of Chapters IV to VII of this annex.

Transmission Cost

Finally, the costs incurred during the planning period include the domestic and foreign capital inputs to transmission line construction, d_{ci} and f_{ci} and the associated annual maintenance and operating expenditures, m_{ci}.

Terminal Correction

At the end of N years planned under the constraints represented by Equation 1.1, there is a structure of assets which is passed on beyond the horizon into what might be called the "N+ period." The composition of the system at the horizon is indicated by the final column vector of the strategy matrix, $\underline{x}_N = [x_{1N},\ldots,x_{jN},\ldots,x_{JN}]$, and by the particular transmission system, Y_c^N, which has been selected. Depending upon the composition of the final asset mix - $\underline{x}_N, Y_c$ - the cost of serving electrical demand in the years beyond the end of the N-year planning period will be different. And if an appropriate correction is not made, this fixed horizon model will bias project selection against long lived

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1/ Two markets rather than three because in order to simplify the analysis the Upper Sind market was not actually dispatched although a check was always made to ensure that each strategy included sufficient capability at Mari to cover local loads as well as, where applicable, to meet part of the load in the North and/or the South. (See below Chapter VII). The demand in Upper Sind is small and, though it may grow rapidly, the load forecasts underlying these studies do not imply that it will become of comparable magnitude with the loads in the North or the South within the next twenty years. The cost of meeting the electric energy requirements of the Upper Sind would not vary greatly among strategies studied. When an allowance is needed for the Upper Sind energy cost, it is calculated in a manner described in Chapter III.

and capital intensive alternatives and in favor of retaining old generating equipment on the system.

In the comparison of a number of different strategies there are four characteristics of their final asset structures which should be included in the evaluation of an appropriate set of terminal corrections (the $\Gamma(\underline{x}_N, Y_c)$ of Equation 1.1). They are:

1. differing amounts of hydroelectric or nuclear developments as opposed to conventional thermal equipment,

2. widely varying energy utilization patterns due to differences in the availability of major inter-market transmission facilities,

3. the retirement of different combinations of old plants, and

4. the occurrence of differential quantities of over-capacity or excess reserve at the end of the plan period.

Depending upon the characteristics of the particular system, these four factors may require corrections for differential fuel costs, M & O expenditures and capital outlays as between varying equipment plans.

Although these terminal corrections can be evaluated within the computer simulation program itself, in the case of the West Pakistan analysis they were handled by a separate manual calculation and then combined with the figures produced by the simulation program. The details of these computations are presented in Chapter III below. This whole question of the terminal correction is discussed at length in the dissertation where the simulation model was originally developed.1/

1/ H. D. Jacoby, Analysis of Investment in Electric Power, Doctoral Dissertation, Harvard University, 1967.

CHAPTER II

THE COMPUTER SIMULATION PROGRAM

Program Dimensions

The simulation program prepared for the West Pakistan electric power system can handle a planning period of up to twenty years. In addition to demand data for the three markets, it accepts information on the characteristics of up to forty existing and potential hydro plants. Only six hydro units may be on the system at any time, however. It can consider up to fifty existing and potential thermal and nuclear units and ten alternative transmission systems.

For purposes of inclusion in this analysis, a thermal or nuclear generating unit can consist of an individual machine (boilers, steam turbine, generator and generator transformer), or it can be composed of several machines of similar operating characteristics within a particular plant. Combining several machines into a single unit alleviates the problem of data management and cuts computation time. Care must be taken in determining each composite unit, however, in order to insure that no important aspect of short-run operation or long-run capital planning is obscured by the aggregation. For example, an old plant may contain six machines of identical operating characteristics, but it would not be reasonable to combine them into one generating unit for purposes of the simulation analysis if there is some possibility that it might prove advisable to retire only half the machines at one time. By the same token, any potential new plant should be broken into appropriate component units in order to allow consideration of alternative patterns of staged construction.

The limitations on the number of plants and length of period which the program can handle are imposed by the size of the core storage of the IBM 7090 and 7094 computers on which this analysis was conducted. With expanded use of tape or disc storage in the course of the computation, larger systems could be analyzed even with smaller computers. The analysis of the West Pakistan system used the full capacity of the current program. Computation time per strategy was approximately four minutes on the IBM 7094 and slightly more on the IBM 7090.

Data Requirements

The data required by the program are divided into two categories - permanent data and strategies. The permanent data describe the physical and economic context within which the evaluation of various alternative strategies takes place. The information contained in the permanent data deck may be broken down as follows. First there are the characteristics of market demand:

- Monthly projections of maximum peak demand, P_{maxT} , minimum demand, P_{minT} , and load factor, L_T , over the N years of the planning period for each of the major markets.

Next there is information on each of the existing and potential thermal and nuclear generating units:

- Name of the unit.

- Number of the market in which the unit is located.

- Rated output capacity net of in-station losses and consumption, Q_{ji} . (In order to simplify the exposition, the equations in Chapter I above dealt with thermal plant capacity as a constant, Q_j . The computer program can also consider variation in unit capacity from year to year which may be useful when there is a question of staggering the installation of generators in a power plant.)

- Unit maintenance and operation cost, m_{ji} . This quantity may increase or decline over the plan period depending on the characteristics of the particular unit.

- For each conventional thermal unit, a heat rate or fuel efficiency, $\overline{C}_j$.

- A range of prices for conventional fuels, f_j .

- The percentage of fuel consumption at each plant which represents a direct foreign exchange cost.

- For each nuclear unit, a composite figure for fuel cost per kwh, $\overline{C}_j f_j$.

- For each potential new plant, domestic and imported capital cost in each year of the construction period, $d_{j\lambda}$ and $f_{j\lambda}$ (Capital costs are always used here net of duties, taxes and interest during construction, as elsewhere in these economic studies).

For each hydro development to be considered, the following data are required:

- Name of the plant.

- The monthly energy available, H_{jT} , and the monthly capacity Q_{jT} . One pair of these values is required for each month of each year of the planning period. They change over the years as more units are installed at the hydro plant. (Energy and capacity figures used in these analyses have been based on mean-year flows 1/).

- Capital and maintenance and operation data similar to that described above for thermal plants.

Since the monthly pattern of energy and capability available from a hydro-electric project depends on the size of the reservoir and dam and on the reservoir operation policy followed and also changes over the years as more units are installed, a separate project must be defined for each combination of these factors which it is desired to evaluate.

For each transmission system, Y_c , the following data are required:

- Transmission constraints (mw) for each link for each year of the planning period: $V^{*ci}_{1,3}$, $V^{*ci}_{3,1}$ and $V^{*ci}_{3,2}$, $i=1,\ldots,N$.
- Domestic and foreign capital cost over the construction period, d_{ci} and f_{ci} , $i=1,\ldots,N$.
- Maintenance and operating cost, m_{ci} , $i=1,\ldots,N$.

In addition, an estimate must be provided for the percentage loss of capacity and energy in transmission over each link, Λ . Because the distances 1,3 and 3,2 are roughly equivalent, Λ is the same for each link. It is also assumed equal for all different sizes of transmission lines. The determination of the amount of hydro capacity to be transmitted in each month requires an iterative calculation with an additional incremental block of power being added during each cycle. The size of the incremental block is an input datum, V .

Then there are data on the economic condition of the public sector:

- A range of values for the rate of time discount, π .
- A range of values for the opportunity cost of capital, θ_i .
- A range of values of the foreign exchange rate, g .

This completes the data requirements of the permanent deck. In a production run, these data are used in the evaluation of a strategy deck which may contain any number of system expansion plans. Each strategy contains four types of data.

- Name of the strategy.
- Dates when old plants are retired from the system, $\bar{R}_j$.

1/ Capacity figures would not be substantially different if they were based on critical flows because of the assumption of fixed release-patterns and unrestricted peaking - see above Annex 6, Hydroelectric Projects and Reservoir Operation.

- Dates when new units are added to the system, $\bar{S}_j$.
- Identification of the transmission system, γ_c , appropriate to the strategy.

Sequence of Computation

For each strategy to be evaluated, the program simulates system expansion and operation year-by-year over the plan period. Very briefly, the simulation proceeds as follows.

1. Read the permanent data deck and print it out again for record keeping purposes.

2. Read a strategy and proceed to the first year of the plan period.

3. Look at the retirement and construction plans anticipated by this strategy and make up a vector which represents the composition of existing generation capacity for the year under consideration, $\underline{x}_{ji}$.

4. Calculate the total system maintenance and operation cost for this year, $M(\underline{x}_i)$ of Equation 1.4.

5. Calculate the opportunity cost of capital for this year, θ_i .

6. Calculate this year's capital expenditure by summing over all plants currently in construction, applying appropriate values of θ_i and the foreign exchange rate, g . The result is $K_i(X)$ of Equation 1.3.

7. Calculate this year's fuel expenditure. Step through the year month by month simulating the operation of the transmission system and the individual hydro, thermal and nuclear generating units as they might be dispatched in practice. (This procedure is described in detail in Chapters IV through VII below).

8. Add up all fuel, maintenance, operation and capital costs incurred this year. Print out an annual summary of system operation and cost.

9. Was the year just analyzed the last year of the plan period? If so, proceed to step 10. If not, move to the next year and start again at step 3.

10. Take the total costs generated in step 8 for each of the years of the period, discount them back to the beginning of the period and sum them into a single figure for the present value of cost of market supply.

11. Print out a summary of the characteristics and costs of this strategy.

12. Was that the last strategy to be evaluated? If not, go back to step 2. If so, terminate the computer run.

Information Generated

The computer model is designed to generate the information necessary to allow comparison of alternative patterns of system capacity expansion (i.e., alternative strategies) over a fixed planning period. As each simulation run proceeds, the computer program prints-out an annual system operating summary. Included are data on the load dispatching for each month in each of the markets and on the operation of the transmission system along with an annual summary of system fuel costs, capital costs and maintenance and operation expenditures. This information on the details of monthly operation proves very valuable in revealing the internal workings of the system under the operating rules assumed. Because this particular portion of the computer print-out is so intimately related to the algorithms used to simulate system energy dispatch, full description of the annual operating summary is reserved until Chapter VIII after these computer routines have been presented.

The final result of the computer simulation of each strategy or development program is a simple two-page summary of the strategy itself. First there is a list of the generating plants and transmission lines added to the system during the planning period indicating the dates when they will be added along with annual figures for domestic and foreign capital expenditures and international financing requirements. Second there is an array of numbers representing the discounted present worth of the total cost of the program over the 20-year planning period 1966-1985. With the application of a terminal correction and certain other minor adjustments (all discussed in Chapter III below), these figures are values of $G(X,Y_c)$ as presented in Equation 1.1.

The computer program prepares this estimate of the total cost of each strategy under a range of values of fuel prices, foreign exchange rates, discount rates and opportunity costs of capital. The ranges provided make it possible to analyze the sensitivity of results to changes in assumptions regarding these economic parameters. Consistency with other aspects of the overall Indus Study required that the analysis be made on the basis of a discount rate of .08 and an opportunity cost of capital equal to 1.0, i.e., all capital inputs valued at market prices.

Table 1 gives a sample display of the portion of the computer print-out of final cost data which actually was used in these studies. All costs are given in U.S. dollars. The display basically consists of three separate columns each composed of three different total cost figures. The three columns are distinguished from one another by the fact that they are based on different assumptions with regard to the foreign exchange rate, the first being based on the current foreign exchange rate ($1=Rs. 4.76), the second on a shadow rate of 1.6 times the current rate ($1=Rs. 7.62) and the third on a shadow rate of 2 times the current rate ($1=Rs. 9.52). Each of the rows reflects a particular set of fuel prices for conventional thermal and nuclear units. The

Table 1

Final Cost Summary Prepared by the Computer Model

STRATEGY TDDN

PRESENT VALUE IN 1965 OF TOTAL SYSTEM COST

(MILLIONS OF U.S. DOLLARS)

DISCOUNT RATE = .08

OPPORTUNITY COST OF CAPITAL = 1.0

	SHADOW EXCHANGE RATE		
	1.00	1.60	2.00
FUEL PRICES 1	522.7	694.1	808.3
FUEL PRICES 2	503.8	674.7	788.5
FUEL PRICES 3	569.3	742.4	857.3

figure \$522.7 million is the present worth cost of the system when fuel is valued at 'FUEL PRICES 1' - in this computer run, the set of financial fuel prices currently ruling in the different areas of West Pakistan. \$503.8 million is the present worth cost of the system when fuel is valued at 'FUEL PRICES 2' - in this computer run the same financial prices as above for nuclear fuel and oil fuel but 14¢ per million Btu for gas at Mari/Sui and 30¢ elsewhere in the province. \$569.3 million is the present worth costs of the system when fuel is valued at 'FUEL PRICES 3' - in this computer run, the same prices as under 2 for nuclear fuel, oil fuel and gas at Mari/Sui but 70¢ per million Btu for gas elsewhere in the Province.

CHAPTER III

ADJUSTMENTS TO THE COMPUTER RESULTS

Corrections for Terminal Conditions

While direct comparison of the array of figures shown in Table 1 with a similar array for another development program can give a first indication of the relative overall merits of the two programs, it would be misleading to take these figures as a final and complete picture of the present worth of the costs of developing the system in alternative ways. One reason for this is relatively unimportant: the program does not take into account the costs of all aspects of development, such as intra-market distribution and customers' connections. These costs will be a very sizeable part of overall expenditure in the power sector but they are unimportant to the comparison of different generation and transmission programs because they will be largely the same for all programs. But there are a number of other costs and savings which will vary significantly among programs and can, therefore, affect their relative attractiveness.

The most important of these are costs that will be incurred after 1985 but which will be heavily influenced by the set of equipment installed by 1985: system fuel costs, maintenance and operation costs and costs of replacing equipment where one program foresees installation over the planning period of a greater amount of relatively short-lived equipment (e.g. thermal equipment) than another program. Besides allowance for the post-1985 fuel costs of nuclear and thermal plant in existence in 1985, adjustment also has to be made for any hydro energy which is excess to requirements in 1985, but which may be expected to be absorbed over the following years as loads grow. There are two other adjustments to the figures produced by the computer analysis which, though they are not 'terminal corrections' properly so called, are quite important for the purposes of certain comparisons. All these various adjustments and the procedures used for estimating them are described in turn below.

Fuel Costs After 1985

The stock of generation and transmission equipment which is in existence in 1985 will have an important effect on fuel expenditures after 1985. A program which includes heavy hydro investment between now and 1985 will clearly leave WAPDA with an annual fuel bill in the following years much lower than what it would face if all intermediate generating investment had been in thermal plants. The effect of this difference in the stock of equipment in existence in 1985 will gradually tail off as the system is further expanded in the following years. New hydro plants may then be built to take part of the base load. New thermal plants will be built and since they will normally have higher fuel efficiencies than those existing in 1985, they will tend to force the older units into intermittent, peaking service. On the other hand, system-wide energy requirements will also be expanding and, particularly in the first few years following 1985, an increasing amount of energy may be drawn from the units in existence in 1985. A full analysis of the effects of the stock of equipment in existence in 1985 on fuel costs in the following period would require detailed consideration of the composition of equipment at that time and of hydro opportunities not already taken up. Another important factor would be technical and economic change in the intervening period: trends in the fuel consumption of new equipment and trends in fuel prices. The likelihood is that the former will show a downward trend while the latter may well show an upward trend.

A very simple procedure has been adopted for taking care of these complex matters in most of the comparisons among programs discussed in the foregoing chapters. The simplicity is justified to some extent by the uncertainty attaching to many of these factors and to a greater extent by the heavy effect of the discount rate which makes cost differentials in 1990 or 2000 that are large in absolute terms quite small in present worth terms. The overall fuel cost effects of the stock of equipment in existence in 1985 are accounted for by carrying forward the fuel costs incurred in 1985 for a further 20 years to 2005 and discounting them back to January 1, 1965. At the 8 percent discount rate this amounts to a figure roughly equivalent to the (undiscounted) fuel costs incurred in 1985. The figure so derived is then added to the discounted present worth of system cost during the planning period which is evaluated with the computer program. An indication of the magnitude of these additions is gained from the following figures for selected programs, valued at the current foreign exchange rate.

Table 2

Terminal Correction for Fuel Costs after 1985
(Million US Dollars discounted at 8% to Jan. 1, 1965)

	Low Fuel Prices (all natural gas at 20¢/mln. Btu)	High Fuel Prices (all natural gas at 70¢/mln. Btu)
Thermal Program	79	217
Program with Kunhar in 1974	66	174
Program with Tarbela (1300) in 1975	33	70

These adjustments probably tend to overstate the effect of the stock of equipment in existence in 1985 on fuel costs in subsequent years. Carrying forward the costs for as long as 20 years means that relatively little weight is attached to the possibilities for introduction and base-load use of more economical equipment after 1985. On the other hand, an upward trend in fuel prices would tend to correct this tendency. Therefore, the adjustment may be considered to give a fair indication of the relative merits of more or less heavily hydro development programs.

This simple procedure was adequate for a large portion of the analysis because most of the comparisons among strategies were based upon the assumption that fuel prices would remain constant throughout the planning period and thereafter. This approach was not appropriate when comparisons were made on the basis of the time series of fuel prices developed in Annex 5 - particularly when comparing programs which differed substantially from one another. In the case of the analysis of the postponement of Tarbela in Chapter VI of the Main Report and in Annex 7, a separate calculation was made of the annual fuel requirements of each program from 1986, the first year after the end of the plan period, through 1996, the year when the fuel requirements of the two programs should become equal because Tarbela would have been fully absorbed. Allowance was made for a substantial further part of the fuel requirements of both programs to be met by nuclear fuel (equal nuclear development assumed in both programs) and the remaining fuel requirement was costed at the appropriate prices given in Table 3 of Annex 5.

In the case of the comparison among programs with and without interconnection on the basis of economic fuel prices a simpler procedure, more similar to the one described previously, was adopted. The fuel costs of the final year of the plan period were simply projected for fifteen years and an allowance was made for absorption of hydro energy available but unabsorbed in 1985 (see further discussion under the next item). This was done by estimating the amount that would be absorbed over the years under the different sets of conditions and valuing it at the cost per kwh of thermal generation implied by the 1985 fuel price in order to make it comparable with the fifteen-year projection of the fuel costs of the thermal and nuclear plants in existence in 1985. These procedures seem adequate for giving a rough indication of the magnitude of the differences in the post-1985 fuel costs that should be associated with different investment programs.

Eventual Absorption of Hydro Energy Excess to Requirements in 1985

The computer analyses indicate that most development programs which include 12 units at Tarbela will result in the capacity to produce in 1985 more hydro energy than can actually be absorbed by the system even if interconnexion is provided. This excess hydro energy usually is heavily concentrated in the June-August period. As system energy requirements increase after 1985 this energy will gradually be absorbed in the following years, thus saving fuel that would otherwise have had to be used to generate the energy. The rate at which this absorption will take place will depend on a number of factors, in particular the rate of growth of energy requirements, the expansion of the transmission system and the addition of further hydro development with heavy energy output in the flood months. The greater the first two factors the more rapid the absorption, while the more significant the last factor the slower will be the absorption. Table 3 shows the absolute amounts of excess hydro energy available around 1985 under various different conditions and indicates its rate of absorption on the assumption that interconnexion is available to carry a maximum of either 680 mw or 1100 mw. The table shows that the

Table 3

Excess Hydro Energy Available at the End of the Plan Period and its Rate of Absorption
(Million kwh)

Development Programs a/		Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Total
1) Tarbela 1332', without interconnection	1985	237	284	32	110	197	792	1509	1390	1099	343	93	6	6092
2) Tarbela 1300'-Kunhar, with interconnection, 680 mw max.	1984	91	79	44	5	16	261	1218	1355	952	117	1	4	4143
	1985	15	2	35	10	39	187	1141	1259	851	20	2	4	3565
3) Tarbela 1332', with inter-connection, 680 mw max.	1983	3	0	0	31	153	439	1137	1071	789	4	3	4	3634
	1984	5	0	2	60	172	376	1065	964	679	3	0	2	3328
	1985	3	0	0	0	99	316	988	868	578	4	0	2	2858
4) Tarbela 1332', with inter-connection, 1100 mw max.	1983	3	0	0	68	192	288	909	839	557	0	3	4	2863
	1984	5	0	2	99	211	236	817	713	427	0	0	2	2512
	1985	3	0	0	17	129	172	722	599	310	0	0	2	1954

a/ Tarbela here refers to 12 units and Kunhar to all four stages. Where several years are given for one program there is no intervening change in either hydro units installed or transmission lines constructed, so that the absorption shows purely the effect of growth in system energy requirements. Note the small increases in excess hydro available in January, March and May in certain years. This is related to the degree to which the North is self-sufficient in different years (i.e., has sufficient capability to meet its own load even in the critical months without drawing on Mari) and to the differences in current costs between thermal generation in the North and in the South in different years. The computer analysis shows, for instance, that with a nuclear plant in place in Karachi, more money can be saved by using a little more of the hydro capability to meet the Northern Grid load than by sending it to the South. This is the case even though the extra piece of hydro capability retained in the North will be dispatched more in peaking service in the North than what it could have been in the South so that less of the energy attached to the capability is absorbed.

maximum rate of absorption between one year and the next, with interconnexion, is about 100 million kwh per month and slightly more if the transmission line is assumed to have its full physical carrying capability (see Annex 9 Appendix 1). On the assumption that no further hydro plants will be introduced in the five to ten years following 1985, this rate of absorption will increase as system energy requirements grow by larger amounts in absolute terms. The allowance for absorption of hydro energy has been calculated with these facts in mind; the related fuel saving has been calculated by assuming that energy will be absorbed at a rate of about 100 million kwh per year in each of the months when it is available (more in the later years) and then multiplying this cumulative fuel saving by an appropriate Northern Grid fuel price and discounting the savings over the same twenty-year period as used for the fuel cost terminal correction (see above) back to January 1, 1965. The final allowance for absorption of excess hydro is significant but it is much smaller than the first terminal correction. Some representative examples are given in Table 4.

Table 4

Terminal Correction for Absorption of Excess Hydro Energy

	Low Fuel Price (all natural gas at 20¢/mln. Btu)	High Fuel Price (all natural gas at 70¢/mln. Btu)
Thermal Program	-	-
Program with Kunhar in 1974	-	-
Program with Tarbela (1300') in 1975 and interconnexion	8	229
Program with Tarbela (1332') in 1975 and interconnexion	10	34

These present-worth savings are subtracted from the total present-worth costs produced by the computer simulation model.

Operation and Maintenance Costs after 1985

The stock of equipment in existence in 1985 will affect not only fuel costs in subsequent years but also O & M costs. O & M costs tend to be somewhat lower per kw installed for larger thermal units than for smaller ones and for hydro units than for thermal units. The difference is hardly substantial enough to show up significantly in total system O & M costs, especially when they are discounted back to 1965. But for consistency with our treatment of post-1985 fuel costs, O & M costs for the plant existing in 1985 for a further twenty years have been discounted back to 1965 and added to the total costs of the programs. The additions on this account are almost all within the range of $25 - 28 million in present-worth terms. Much more precise attention would need to be given to this terminal correction if the computer studies were being made to define optimal retirement dates for existing thermal plant.

Thermal Replacement

Conventional economic lives for hydro-plant and EHV transmission equipment are about 40 to 50 years and for thermal and nuclear plants about 20 to 30 years. Whether these lives are appropriate under particular conditions as to quality of maintenance, cost and availability of hydro opportunities, technical progress in plant design and fuel prices, foreign exchange rate and discount rate is a matter that the computer model is well adjusted to assessing for each major item of generating and transmission equipment in West Pakistan. However, this study has not been attempted here, since the answers would likely not greatly affect the relative merits of the development programs under consideration. Nevertheless, some allowance was needed for the earlier replacement expenditures that would likely be involved in a program which was heavily oriented to thermal development in the next ten years. Replacement costs for hydro plant, EHV transmission equipment and thermal plant installed in the second ten years of the planning period (1975-85) would become insignificantly small in present worth terms when discounted back to January 1, 1965 at 8 percent. The procedure adopted for this correction, therefore, was to take the stock of thermal generating equipment supposed to exist by 1975 under the different programs and, in the few cases where this diverged significantly from the base level of about 1650 mw over the whole system, to add to total system costs present-worth costs of replacing the extra thermal equipment on the assumption that all replacement would be undertaken in 2000--i.e., after a thirty-year life from 1970. It was assumed for these calculations that such a replacement would take the form of large-scale thermal plants at a cost of about $125/kw installed in terms of 1965 prices. Since the amount of thermal equipment in existence in 1975 diverged from the base level in none of the programs studied by more than 300 to 400 mw this correction was of little real significance. It ranged between about $3 million and $5 million, depending on the applicable foreign exchange rate, for programs requiring even the largest allowance on this score.

Fuel Requirements for Meeting the Upper Sind Load

While the computer program makes allowance for the capability required to meet the load of the Upper Sind area, it does not explicitly dispatch energy to meet this load. The assumption was made that this relatively small load would always be met by local Mari generation or by hydro energy which would leave more of Mari capability available for serving Karachi-Hyderabad loads. Omission of explicit dispatching of the Mari market considerably simplified the computer program without doing much injustice to the facts of the situation. Therefore, that portion of the capability at Mari which was not needed to meet the load in the Upper Sind was dispatched as and when appropriate to meet the load in Karachi and the North. This gave the program flexibility for handling different potential amounts of generating capacity at Mari while ensuring that there was always sufficient capability left free at Mari to meet the local load there so that this market could be dispatched very simply by hand. Allowance for this market would make no significant difference among the strategies considered, but it was important when attention was focused on the total amount of gas that would be consumed over the years for generation of power. A simple hand calculation of the dispatch of Mari was, therefore, made for this purpose.

EHV Transmission Step-up and Step-down Transformers

The only aspect of transmission explicitly covered in this study is major inter-market EHV transmission. Thermal plants were always assumed to be located at the power market (except for Mari plants) so that no special allowance was needed for major transmission lines in connection with such plants, and the costs of transmission from the potential hydro plants to the power market were built into the costs of the hydro units themselves (See Annex 6, Appendix 2). Transmission from Tarbela to the Northern Grid (Lyallpur) was assumed for this purpose to be 380 kv. The costs attributed to the intermarket transmission systems, on the other hand, covered the lines themselves, terminals, reactors and the Moro substation proposed by the power consultant for shortening the effective length of the line between Karachi and Mari (See Annex 9, Appendix 1).

Alternative transmission systems were developed in the form of alternative schedulings of the 380 kv lines. This procedure required that an adjustment be made to all 'with interconnection' cases to cover the cost of step-up and step-down transformers which would need to be introduced to link thermal plants to the 380 kv lines and 380 kv lines to local distribution lines. The procedure adopted for handling this problem was based on the assumption that all thermal units would be tied into lines of at least 132 kv capacity, so that the cost which was of relevance to our comparisons was the differential between the costs of step-up and step-down transformers to and from 132 kv and the costs of step-up and step-down transformers between 132 kv and 380 kv. The differential between the economic costs of 380 kv and 132 kv step-up transformers included in the Power Consultant's program for thermal units average about \$0.3 m/50 kw installed capacity (total economic costs of 380 kv step-up transformers average about \$0.7 m/50 kw installed capacity). No detailed information was available on step-downs, but a generous allowance was made for them by assuming that they would cost the same as the step-ups. Total costs derived on this very simplified basis added up to the total amount which the power consultant had allowed in his program for the costs of step-up and step-down transformers between 132 kv and 380 kv. Costs were, therefore, added to programs "with interconnection" when they were being compared to programs "without interconnection" at the rate of \$0.6 m/50 kw installed thermal capacity (with an 80 percent foreign exchange component.) The additions were scheduled according to the scheduling of the relevant thermal plants in the programs in question and then discounted back to 1965. The final discounted present-worth allowance made on this account was about \$6 million at the current foreign exchange rate and \$10 million at the shadow rate of \$1 = Rs 9.52.

CHAPTER IV

THE DEFINITION OF MARKET DEMAND

Models of Electric System Operating Characteristics

The primary task of the computer model is to simulate the dispatching of system generating units as it might be carried out in practice and thereby to capture the essential operating characteristics of the West Pakistan system as they impinge upon the cost of serving a growing demand for electric power. It is the purpose of this and the next three chapters to describe in some detail how this calculation was made. This chapter describes the use of the integrated load function as an aggregate representation of the characteristics of market demand in each period. Chapter V presents the algorithms used to model the dispatching of hydroelectric units, and Chapter VI describes the procedure for approximating the dispatch of thermal and nuclear units. Finally, Chapter VII develops the algorithms used to simulate the operation of the inter-market transmission system. In order that the description of these computer routines be clear, it is useful to begin with a definition of the concepts and terminology used to define the structure of market demand.

The Demand for Electricity

In an electric power system, one with l consumers, for example, each consumer contributes an instantaneous power demand or load of $P_l(t)$. The instantaneous system load (ignoring losses) will be

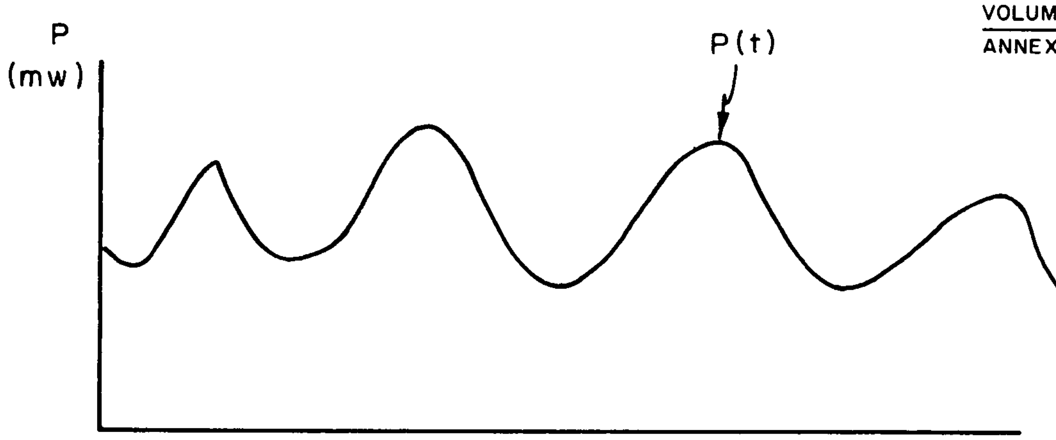
$$P(t) = \sum_{l} P_l(t) , \qquad (4.1)$$

where $P(t)$ may be expressed in kilowatts (kw) or megawatts (1 mw = 1000 kw).

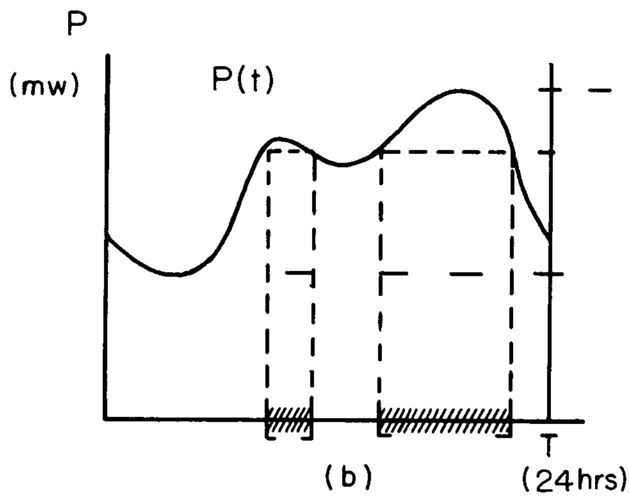
As the function $P(t)$, shown in Figure 2a, is too complex to be manipulated in practical planning calculations, the normal procedure is to break the time axis up into standard time intervals, e.g., days, months or years, and to deal with parameters or simple functions which characterize the behavior of $P(t)$ during the interval. For example, the daily system load profile for a typical diversified urban market usually looks something like Figure 2b.

The most important indicator of the demand pattern over the interval, T, is the maximum instantaneous system load, P_{maxT} . The minimum load, P_{minT} , is a significant datum as well. The total energy consumption over the interval is the area under the load curve $P(t)$,

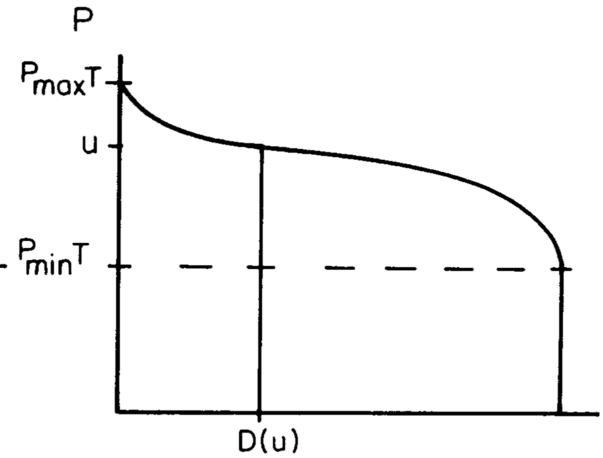
$$E_T = \int_0^T P(t)\, dt \quad , \qquad (4.2)$$



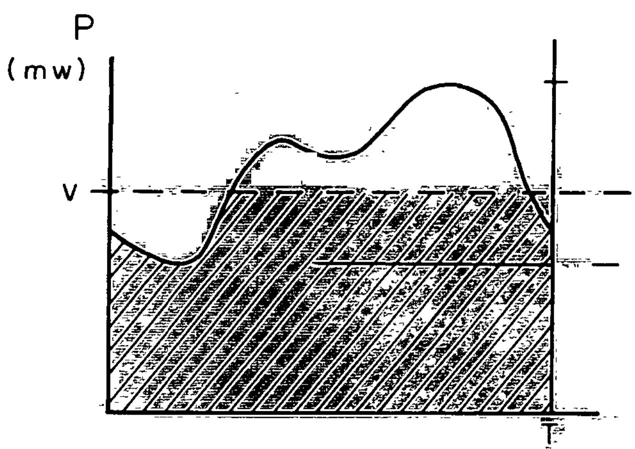
(a)
TIME PROFILE OF POWER DEMAND



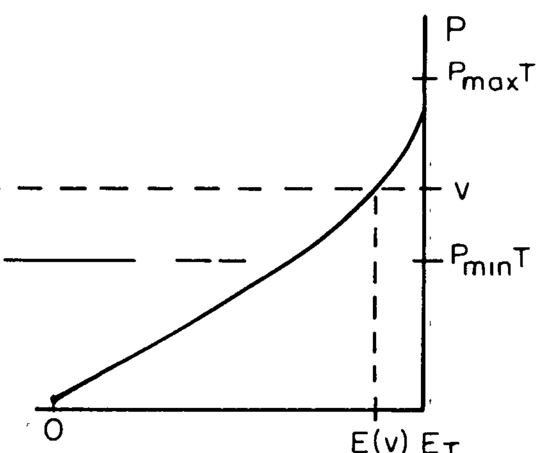
(b)
LOAD PROFILE FOR STANDARD INTERVAL (DAY)



(c)
LOAD-DURATION CURVE



(d)
LOAD PROFILE FOR STANDARD INTERVAL (DAY)



(e)
INTEGRATED LOAD FUNCTION

THE NATURE OF MARKET DEMAND

with E_T usually expressed in kilowatt-hours (kwh) or megawatt-hours (1 mwh = 1000 kwh) or, occasionally, gigawatt-hours (1 gwh = 1,000,000 kwh). And still another simple indicator of demand structure is the system load factor, L_T, where

$$L_T = \frac{E_T}{T \cdot P_{maxT}} . \tag{4.3}$$

L_T is a dimensionless parameter.

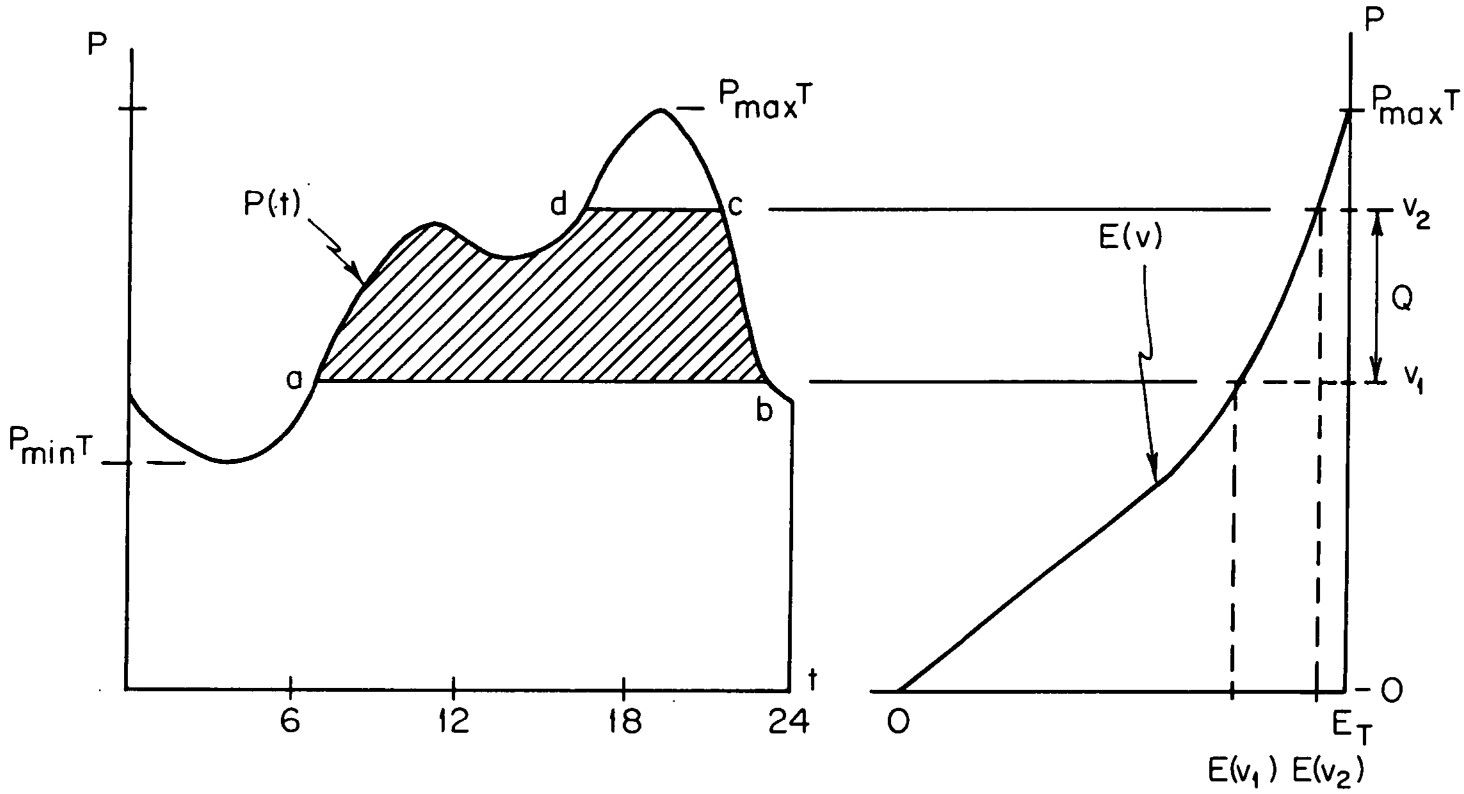
Given data on $P(t)$ it is possible to construct curves or functions which convey still more about the nature of market demand. One is a load-duration curve which indicates the portion of the interval during which instantaneous power demand exceeds some specified level. Such a curve is shown in Figure 2c. Any point on the curve is derived by selecting some power level, u, and noting the length of time during which $P(t)$ exceeds u. By repeating this procedure for many different values of u, the load-duration curve, $D(u)$, may be traced out.

Finally, there is the integrated load function, $E(v)$, which indicates the total energy which would be generated during a given interval if the system supply capacity were limited to the level v. Each point on this function is evaluated by selecting a value for v and measuring the total area under the load curve below the level v as shown in Figure 2d. Following the convention of the engineering literature, the integrated load function is plotted with the axes reversed from their normal orientation, as shown in Figure 2e.

The integrated load function is a key component of the model of system energy generation developed below. Its use in analyzing short-run system operation may be demonstrated with the use of Figure 3. The figure shows the scheduling of a particular generating unit of capacity Q over the course of a certain time interval - in this case a day. (The presentation of $E(v)$ for a time interval of one day is strictly for the purpose of clarifying the definition of the function; the time period is too short to allow an adequate representation of short-run operating policy. It is only when $E(v)$ is defined over some longer time period that it becomes useful for that task.)

Under the operating policy being followed, this particular unit begins generating when the system instantaneous load reaches v_1; on the day shown, this occurs at approximately 7:00 A.M. The unit carries the increase in load above v_1 up to the level v_2, where $v_2 = v_1 + Q$. If the plant is allowed to follow this operating rule, it must generate energy equal to the area a - b - c - d - a . This same quantity may be read directly from the integrated load function for the day shown and is equal to $E(v_2) - E(v_1)$.

The pair of values for v_1 and v_2 selected for a particular plant define a "load position" for the unit in question. By assigning such a position to each of the plants on the system, the total system



SYSTEM LOAD PROFILE
(TYPICAL DAY)

INTEGRATED LOAD FUNCTION

USE OF THE INTEGRATED LOAD FUNCTION

energy to be generated over the interval may be allocated among the individual generating units. The process of assigning plants to various portions of the load under this type of operating rule is referred to as "dispatching".

Properties of the Integrated Load Function

Because such extensive use is made of the integrated load function, it is important to have a clear statement of its mathematical properties. Precise definition of the function is facilitated if the following notation is adopted.

$$\left[P(t) - u\right]^{+} = \begin{cases} P(t) - u & \text{if} \quad P(t) - u \geq 0 \\ 0 & \text{if} \quad P(t) - u < 0 \end{cases} \tag{4.4}$$

$$\left[P(t) - u\right]^{++} = \begin{cases} 1 & \text{if} \quad P(t) - u \geq 0 \\ 0 & \text{if} \quad P(t) - u < 0 . \end{cases}$$

Note that apart from the discontinuity at $P(t) - u = 0$,

$$\frac{d}{d\{P(t) - u\}}\left[P(t) - u\right]^{+} = \left[P(t) - u\right]^{++} \tag{4.5}$$

In this notation, the load-duration curve is defined as

$$D(u) = \int_0^T \left[P(t) - u\right]^{++} dt . \tag{4.6}$$

Observation of the operating records of large, integrated power systems reveals that

$$\begin{array}{ll} P(t) & \text{may be considered to be single valued, continuous and} \\ & \text{differentiable over the interval } [0,T] \text{ and to have a} \\ & \text{finite number of relative peaks, and} \\ P'(t) = 0 & \text{may be considered to have only isolated roots, i.e.,} \\ & \text{the gradient of } P(t) \text{ is zero only at isolated points;} \\ & \text{there are no plateaus on the function.} \end{array} \tag{4.7}$$

Thus $\left[P(t) - u\right]^{++}$ is a step function with a finite number of jumps and, therefore, is integrable. The function $D(u)$ is well defined and continuous.

It is also true that $D(u)$ is non-increasing. Observe that $D(u)$ is the measure of the set

$$\Psi_u = \left\{ t : P(t) - u \geq 0 \right\}$$

(denoted by the cross-hatched segments of the abscissa on Figure 2b and that the set Ψ_u cannot increase as u increases. Also, $D(0) = T$ and $D(P_{maxT}) = 0$. Therefore, $D(u)$ cannot increase in u and must decrease on the interval $\left[0, P_{maxT}\right]$.

Now define the integrated load function as

$$E(v) = \int_0^v D(u)\, du = \int_0^v \int_0^T \left[P(t) - u\right]^{++} dt\, du \quad . \qquad (4.8)$$

In order to explore the properties of the function in greater detail, note that

$$E(v) = \int_0^T \int_0^v \left[P(t) - u\right]^{++} du\, dt \, . \qquad (4.8a)$$

Using the property expressed in Equation 4.5, the inner integral becomes

$$\int_0^v \left[P(t) - u\right]^{++} du = - \left[P(t) - u\right]^{+} \Big|_0^v \qquad (4.9)$$

and since $\left[P(t) - 0 \right]^{+} = P(t)$, (4.9) reduces to

$$\int_0^v \left[P(t) - u\right]^{+} du = P(t) - \left[P(t) - v\right]^{+} \qquad (4.9a)$$

The integrated load function then may be expressed as

$$E(v) = \int_0^T \left\{ P(t) - \left[P(t) - v\right]^{+} \right\} dt \qquad (4.10)$$

The value of the function is defined within the range $0 \leq E(v) \leq E_T$; these upper and lower limits of the function may be derived from Equation 4.10. If $v = 0$, $\left[P(t) - v\right]^{+} = P(t)$ and $E(v) = 0$. If $v = P_{maxT}$, $\left[P(t) - v\right]^{+} = 0$ and Equation 4.10 reduces to the expression for E_T stated above as Equation 4.2.

Returning to the example shown in Figure 3, the use of the function to define the total energy required of a particular generating unit may be demonstrated. The energy produced by any plant over a particular time period is the integral over the period of the instantaneous load on the unit; this quantity was expressed above as $E(v_2) - E(v_1)$, where $v_2 - v_1 = Q$, the capacity of the plant. Expressing this quantity in the notation of Equation 4.10 ,

$$E(v_2) - E(v_1) = \int_0^T \left\{ P(t) - \left[P(t) - v_2 \right]^+ \right\} dt - \int_0^T \left\{ P(t) - \left[P(t) - v_1 \right]^+ \right\} dt \qquad (4.11)$$

and this reduces to

$$E(v_2) - E(v_1) = \int_0^T \left\{ \left[P(t) - v_1 \right]^+ - \left[P(t) - v_2 \right]^+ \right\} dt \qquad (4.11a)$$

Observe that the integrand of Equation 4.11a is precisely the instantaneous load on the plant in question.

It is also important for subsequent stages of the analysis to know the slope of the integrated load function at certain points and to be assured that the function is convex. Since $E'(v) = D(v)$, one can see that the following conditions hold true.

$$\begin{aligned} &E'(0) = D(0) = T && \\ &E'(v) = D(v) = T && \text{for all } v\,, \quad v \leq P_{minT} \qquad (4.12) \\ &E'(v) = D(v) = 0 && \text{for all } v\,, \quad v \geq P_{maxT}\,. \end{aligned}$$

Thus, $E'(v)$ is positive for all $v < P_{maxT}$ but equals zero at P_{maxT} . Since it has been shown above that $D(v)$ is non-increasing, one can say that

$$E'(v_1) \leq E'(v_0) \qquad \text{for all} \quad v_1 > v_0\,. \qquad (4.13)$$

Equation 4.13 demonstrates the necessary and sufficient conditions for the convexity of the function $E(v)$. (It appears to be concave in Figure 2e only because the axes are defined in an unusual manner.)

In the discussion to follow, the integrated load function generally is stated as $E(P)_T$, where P refers to the upper or lower limit of the operating position of a particular unit over the time interval T . It is helpful to consider the function as composed of two segments. It is a straight line for all $0 < P \leq P_{minT}$. It is curvilinear for all $P_{minT} < P \leq P_{maxT}$. The slope of the straight line segment is shown in Equation 4.12 to be T . This result may be transformed so as to allow graphical interpretation by the following operation.

$$\underline{T} = T\left[\frac{E_T}{F_T}\right] = T\left[\frac{E_T}{P_{maxT} \cdot T \cdot L_T}\right] = \frac{E_T}{P_{maxT} \cdot L_T} .$$

Thus the straight line segment, if extended beyond P_{minT} , would intersect the P axis at the point $P = P_{maxT} \cdot L_T$. For some operations, the function is defined in normalized or unit form where

$$p = \frac{P}{P_{maxT}} \quad , \quad \hat{p}_T = \frac{P_{minT}}{P_{maxT}} \quad \text{and} \quad e(p)_T = \frac{E(P)_T}{E_T} .$$

In this form, the slope of the straight line segment is $1/L_T$.

These are the principal analytical tools and terminology used in this study to describe aggregate system demand for the purpose of planning system generating capacity. The combination of tools and time periods which is put together for any particular analysis depends upon the analytical method being used, the judgment of the analyst, and, most important of all, upon the characteristics of the investments under study. In the analysis of new investment, these concepts are used in the form of demand projections.

Demand Forecasts and the Approximation of the Integrated Load Function

System demand characteristics were described in Equations 4.1 through 4.13 on the basis of some given experience of $P(t)$. For purposes of system planning, $P(t)$ is unknown, and the indicators -- P_{maxT} , P_{minT} , L_T , E_T , and $E(P)_T$ -- are random variables which must be predicted.

Complete definition of $E(P)_T$ for a number of future periods is not a feasible procedure, however. Methods for forecasting the exact path of $P(t)$ are lacking, and even if such information were available, the data management problem would be quite serious, particularly in a study conducted on a monthly basis. A new method for dealing with this problem is presented here. The demand projections involve three parameters, P_{maxT} , L_T and P_{minT} . Normally, projections of minimum power demand are not incorporated into studies of this type. But it has been found that engineers and system managers have a good feel for the behavior of this indicator, particularly when it is stated as a percentage of P_{maxT} , and that it can be forecast with a degree of confidence roughly equivalent to that attached to projections of P_{maxT} and L_T .

With these three parameters, it is possible to approximate the curvilinear segment of $E(P)_T$ by means of a quadratic of the form

$$P^2 + a_1 PE + a_2 P + a_3 E + a_4 = 0 \qquad (4.14)$$

The derivation of (4.14) is presented below. The four paramenters -- a_1, a_2, a_3 and a_4 -- are completely defined by the values of P_{maxT}, L_T and P_{minT}. The algebra is a bit cumbersome, but it is the type of computation which a digital computer can handle with ease.

Comparisons with available data from the major markets of West Pakistan (monthly time interval) have shown this to be a very close approximation to the integrated load functions encountered in practice. This procedure is efficient in that it uses all the data which it seems reasonable to generate about demand in future periods. Yet it provides a function which allows the analysis of system energy generation in considerable detail.

The quadratic approximation of $E(P)_T$ may be described briefly as follows. In order to simplify the notation, the time interval subscript, T, is dropped during this derivation. It should be kept in mind, however, that the three parameters and, therefore, the shape of the function itself will, in fact, vary from one time period to the next.

Let two of the parameters which characterize market demand be put in normalized form where

$$p = \frac{P}{P_{max}} \quad \text{and} \quad \hat{p} = \frac{P_{min}}{P_{max}} ;$$

the parameter, L, is a dimensionless number as defined above. Let the unit or normalized function be denoted as

$$e(p) = \frac{E(P)}{E} , \qquad 0 \leq e \leq 1 .$$

Suppose the curved portion of the function, $e(p)$, is to be represented by a quadratic of the form

$$p^2 + a_1 pe + a_2 e^2 + a_3 p + a_4 e + a_5 = 0 .$$

It is known from the analysis of the function presented above that the following conditions hold true.

$$\begin{aligned} &\text{At } p = 1 , && e = 1 . \\ &\text{At } p = \hat{p} , && e = \frac{\hat{p}}{L} . \\ &\text{At } p = 1 , && \frac{de}{dp} = 0 . \\ &\text{At } p = \hat{p} , && \frac{de}{dp} = \frac{1}{L} . \end{aligned} \qquad (4.15)$$

Since there are five free parameters and only four limiting conditions, one of the parameters must be omitted or some other condition imposed. Experimental calculations were made on functions without the cross product term, pe , and without the e^2 term. On the basis of the information available, it was not possible to distinguish between the two approximations with regard to their accuracy in representing the demand characteristics of the markets under study. 1/ For purposes of computational convenience, therefore, the e^2 term was dropped, and the equation used throughout the analysis is

$$p^2 + a_1 pe + a_2 p + a_3 e + a_4 = 0 . \tag{4.16}$$

The integrated load function, therefore, may be expressed as

$$e(p) = \frac{p}{L} \qquad \text{for } 0 \leq p \leq \hat{p} ,$$

and

$$e(p) = \frac{-p^2 - a_2 p - a_4}{a_1 p + a_3} \qquad \text{for } \hat{p} \leq p \leq 1 . \tag{4.17}$$

The parameters of the second equation of (4.17) may be developed from Equation 4.15. The intermediate steps of the derivation are omitted here. 2/ The resultant expressions for a_1 , a_2 , a_3 and a_4 are the following.

$$a_1 = \frac{L(2L\hat{p} - 2L - \hat{p}^2 + 1)}{(L - \hat{p})^2} . \tag{4.18}$$

$$a_2 = - \frac{L(2L\hat{p} - 2L - \hat{p}^2 + 1)}{(L - \hat{p})^2} - 2 . \tag{4.19}$$

$$a_3 = \frac{L^2 + 2\hat{p}^2 L - \hat{p}^2 L^2 - 2\hat{p}L}{(L - \hat{p})^2} . \tag{4.20}$$

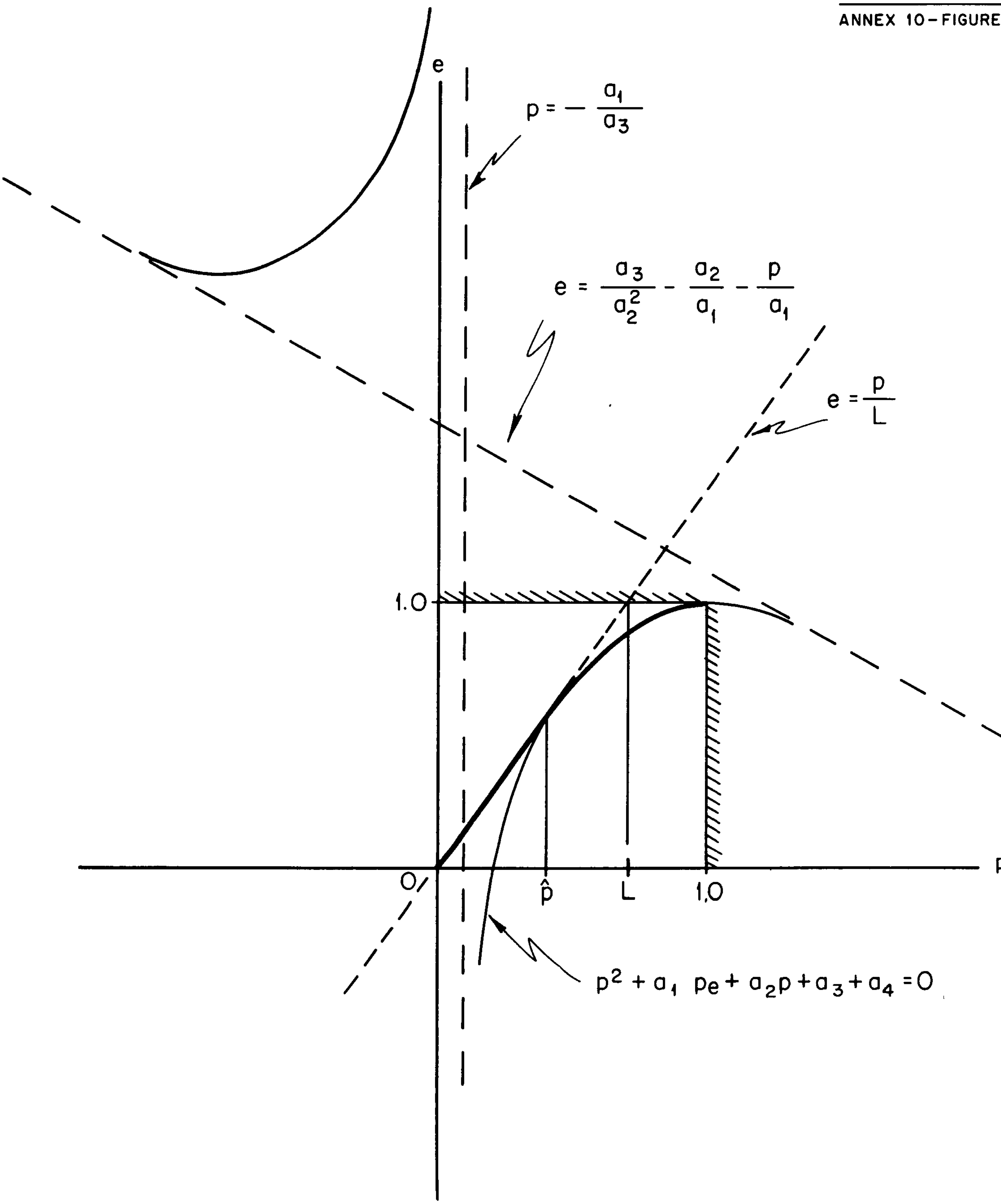
$$a_4 = \frac{\hat{p}^2 (L - 1)^2}{(L - \hat{p})^2} . \tag{4.21}$$

1/ Actual market data were limited, and this judgment was based on a comparison of graphical plots of curves produced by the different approximations and curves which could be constructed from the market records available. Were more data at hand, one would want to base this choice of which term to omit upon more explicit statistical analysis.

2/ For the full derivation, see H. D. Jacoby, op. cit.

With equations 4.18 through 4.21, the quadratic approximation to the curved portion of the normalized function, $e(p)$, as laid out in Equation 4.17 is completely defined by values of L and $\hat{p}$ for each demand period.

The resultant function is a hyperbola. The position of the hyperbola and its asymptotes is shown in Figure 4. Note the axes have been shifted to their more common orientation (as opposed to the way they are presented in Figures 2e and 3. The normalized or unit integrated load function now appears in the first quadrant for values of $0 \leq p \leq 1$, $0 \leq e \leq 1$.



QUADRATIC APPROXIMATION TO THE INTEGRATED LOAD FUNCTION

CHAPTER V

THE DISPATCHING OF HYDROELECTRIC UNITS

The integrated load function provides an aggregate representation of the structure of market demand in any period, in this case a month. This same function serves as the basis of a computational procedure which approximates the essential operating characteristics of individual system generating units as they might be dispatched in hourly and daily service.

This chapter presents a summary of the hydro dispatching calculation. The optimal dispatch of a single unit is considered first, this routine being an important element of the multi-unit procedure described below. As before, the notation is simplified by omitting the time subscript, T . Once again it should be stressed, however, that this omission is strictly for convenience of exposition; in fact, the hydro plant energy and capacity characteristics, like the market demand, change from one period to the next.

The Optimal Dispatch of a Single Hydro Unit

Assume there is a single hydro plant, U_j , to be dispatched into a market characterized by the normalized integrated load function, $e(p)$. The supply characteristics of the plant are defined over the same time interval as market demand, and these are represented by the maximum instantaneous output which the plant can attain, Q_j , and by the total energy it can produce over the time interval, H_j. 1/ Let these plant supply characteristics be expressed in normalized form as

$$q_j = \frac{Q_j}{P_{max}} \quad \text{and} \quad h_j = \frac{H_j}{E} .$$

As before, market demand characteristics are expressed in normalized form as

$$p = \frac{P}{P_{max}} , \quad \hat{p} = \frac{P_{min}}{P_{max}} , \quad \text{and} \quad e(p) = \frac{E(P)}{E} .$$

The dispatching routine must determine a load position for plant U_j . The plant dispatch or load position is defined by a range of values of market capacity demand to be met by U_j . The upper limit of the range is denoted by $\bar{p}_j$; the lower limit is $\underline{p}_j$.

1/ The multi-hydro procedure involves the dispatch of hydro blocks, B_k , which may include several individual hydro units. As each B_k is characterized by a single set of Q_k and H_k , the block dispatch procedure is exactly equivalent to that described here for an individual unit.

The first priority of the hydro dispatching routine is to schedule the operation of hydro machines so as to utilize as much as possible of available hydro energy, h_j . That is to say,

$$\max \left\{ e(\bar{p}_j) - e(\underline{p}_j) \right\} \tag{5.1}$$

subject to the constraints

$$e(\bar{p}_j) - e(\underline{p}_j) \leq h_j \quad ,$$

and (5.2)

$$\bar{p}_j - \underline{p}_j \leq q_j \quad .$$

Any energy not dispatched during the interval is lost forever while the energy actually fed to the system displaces fuel consumption by thermal plants.

A second priority objective of the routine is to utilize as much of the capacity of the hydro unit as possible. If it is possible to dispatch all of h_j at a number of feasible pairs of $\bar{p}_j$ and $\underline{p}_j$, then choose that pair or load position which will

$$\max \left\{ \bar{p}_j - \underline{p}_j \right\} \tag{5.3}$$

subject to the second constraint of (5.2). Given a certain quantity of usable hydro energy, total system fuel cost is minimized by dispatching that energy in such a way as to support as much of market capacity demand as the unit can carry. In practical terms, this means that a plant should be used in peaking service insofar as possible without wasting energy. This serves to minimize fuel cost because it minimizes the total amount of thermal capacity which must be brought into the market. Better use is made of more efficient thermal plants and fewer old, high cost, machines need be called into service.

For a particular hydro plant characterized by q_j and h_j , there is, in general, only one load position (i.e., only one pair of $\bar{p}_j$ and $\underline{p}_j$) which satisfies the objectives of the dispatching routine laid out in Equations 5.1, 5.2 and 5.3. Depending upon the relative size of the hydro plant in relation to total market demand, however, there are five distinct cases which a dispatching algorithm must be able to consider. Several tests are required to determine which case is applicable for a particular dispatch, but once this has been accomplished, it is possible to solve directly for the optimal values of $\bar{p}_j$ and $\underline{p}_j$.

Let the dispatching algorithm be represented by the following pair of functions:

$$\bar{p}_j = \bar{p}(\hat{p}, L, q_j, h_j)$$

and (5.4)

$$\underline{p}_j = \underline{p}(\hat{p}, L, q_j, h_j) \quad .$$

The exact form of these functions depends upon the characteristics of plant and market, but Equations 5.4 are completely specified by the following equations for each of the five cases:

(1) base load by Equation 5.5,

(2) base load with surplus energy by Equations 5.6,

(3) semi-peak load by Equations 5.7,

(4) peak load by Equations 5.8, and

(5) peak load with excess capacity by Equations 5.9a and 5.9b .

Each of these cases is considered in turn below.

In the discussion of multi-hydro dispatch below and in the discussion of transmission in Chapter VII, the optimal dispatch of a single plant, U_j , or block of plants, B_k , will be referred to by the shorthand of Equations 5.4 .

The Base Load Case

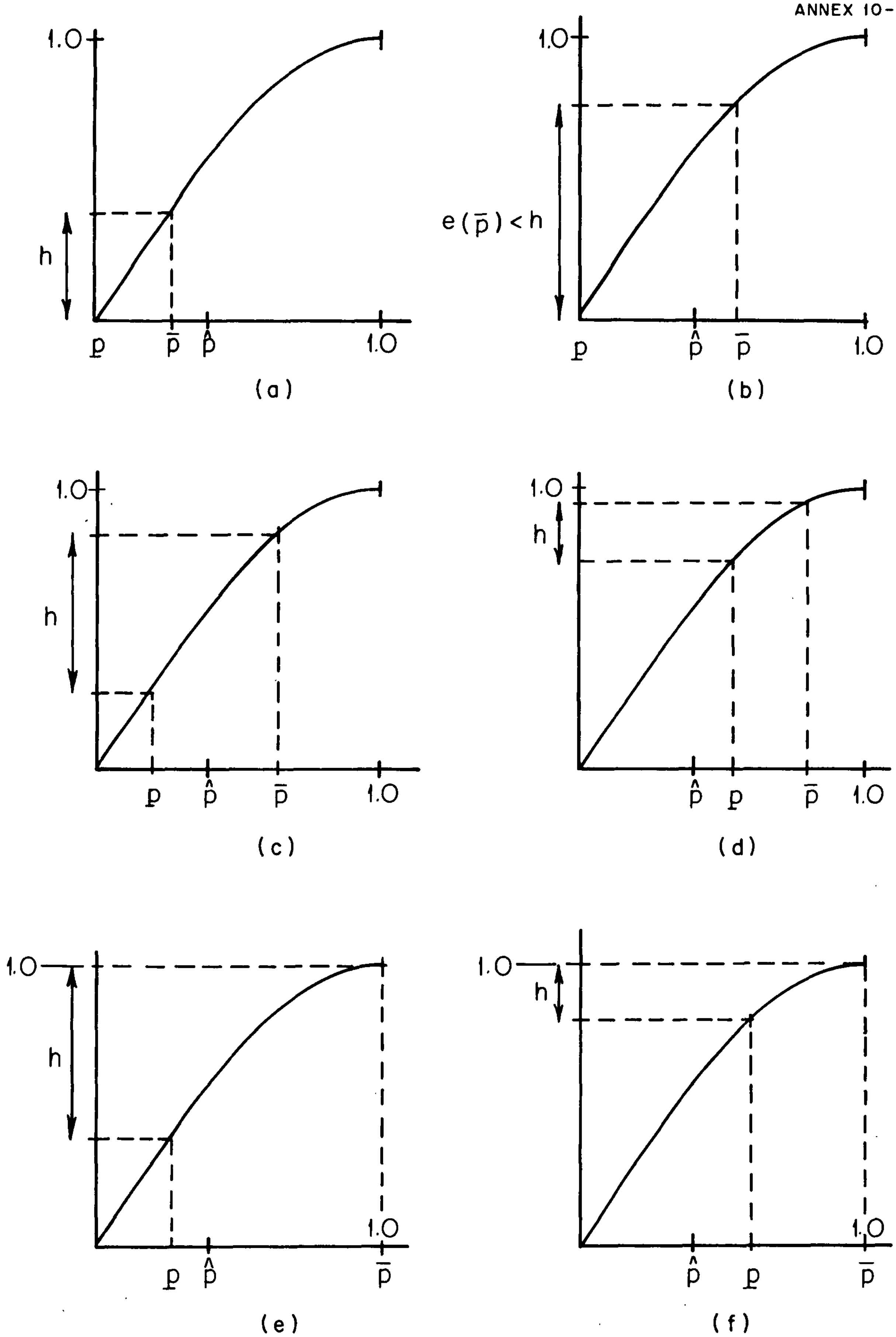
Following convention, let the term "base load" refer to those values of p such that $p \leq \hat{p}$. If

$$q_j \leq \hat{p} \qquad \text{and} \qquad h_j = \frac{q_j}{L} \, ,$$

then the entire plant capacity may be dispatched on base load. All the energy which can be generated is usable. This case is shown in Figure 5a. The optimal dispatch point is the following: 1/

$$\bar{p}_j = q_j \; ; \qquad \underline{p}_j = 0 \; . \tag{5.5}$$

1/ There is no real distinction between the dispatch resulting from Equations 5.5 and setting $\bar{p}_j = \hat{p}$ and $\underline{p}_j = \hat{p} - q_j$, although the current computer program uses the former. This is the only case for which such flexibility in selecting $\bar{p}_j$ and $\underline{p}_j$ exists.



CASES OF OPTIMAL DISPATCH

The Base Load Case with Surplus Energy

Let $\underline{p}_j = 0$ and $\bar{p}_j = q_j$. If

$$q_j > \hat{p} \qquad \text{and} \qquad h_j > e(\bar{p}_j) - e(\underline{p}_j)$$

then the hydro plant covers all the base load (i.e., the load below $\hat{p}$) and some portion of peak load. In addition, there exists surplus energy which cannot be utilized. This case is shown in Figure 5b. The optimal dispatch point is

$$\bar{p}_j = q_j \; ; \qquad \underline{p}_j = 0 \; , \tag{5.6}$$

as in the former case, but now the total energy utilized is less than h_j. This is the only case which yields surplus energy.

The Semi-peak Load Case

The distinction between the semi-peak load case and the peak load case rests upon the following test. Set $\underline{p}_j = \hat{p}$ and $\bar{p}_j = \hat{p} + q_j$. If

$$h_j \geq e(\bar{p}_j) - e(\underline{p}_j) \; ,$$

then the optimal position for the plant is at some lower point on the load function; the plant will straddle the point $\hat{p}$. This is the semi-peak load case, assuming that the first two cases have already been ruled out. If

$$h_j < e(\bar{p}_j) - e(\underline{p}_j) \; ,$$

the formulas for the peak load cases apply.

The semi-peak load case is shown in Figure 5c. The optimal load position may be derived as follows. As a result of the test shown above it is known that, at the optimal dispatch position,

$$h_j = e(\bar{p}_j) - e(\underline{p}_j) \; ,$$

where $\underline{p}_j = \bar{p}_j - q_j$, and that $\bar{p}_j$ lies on the curved portion of the function while $\underline{p}_j$ lies on the straight line segment. Using Equations 4.17 the expression for the optimal dispatch of the unit may be derived.1/ Because of the convexity of $e(p)$, there is only one load position which will allow full utilization of plant energy and capacity under these conditions. If

1/ For details of the derivation of these optimal dispatch equations, see H. D. Jacoby, *op. cit.*

$$W_1 = a_1 + L ,$$

$$W_2 = a_1 h_j L - a_1 q_j + a_2 L + a_3 ,$$

and

$$W_3 = a_3 h_j L - a_3 q_j + a_4 L ,$$

then the optimal dispatch may be calculated as

$$\bar{p}_j = \frac{-W_2 + \sqrt{W_2^2 - 4W_1W_3}}{2W_1} \quad ; \quad \underline{p}_j = \bar{p}_j - q_j . \qquad (5.7)$$

The Peak Load Case

The distinction between the peak load case and the semi-peak case has already been noted. The peak load case requires a second test for complete definition. Set $\bar{p}_j = 1$ and $\underline{p}_j = 1 - q_j$. If

$$h_j \geq e(\bar{p}_j) - e(\underline{p}_j) ,$$

then it is a peak load case; all available energy and capacity may be utilized; if

$$h_j < e(\bar{p}_j) - e(\underline{p}_j) ,$$

it is not possible to dispatch the full capacity to the market. The excess capacity case is considered in the section to follow.

The peak load case is shown in Figure 5d. The optimal load position is derived as follows. As a result of the two sets of tests, it is known that both $\bar{p}_j$ and $\underline{p}_j$ lie on the curved portion of the function and that the full capacity of the unit, q_j , may be utilized. Thus,

$$h_j = e(\bar{p}_j) - e(\underline{p}_j)$$

and setting $\underline{p}_j = \bar{p}_j - q_j$ and substituting in Equations 4.17, the best load position for this plant may be identified. If

$$W_1 = a_1 q_j + a_1^2 h_j ,$$

$$W_2 = 2a_1 a_3 h_j - a_1^2 q_j h_j - a_1 q_j^2 + 2a_3 q_j ,$$

and

$$W_3 = a_2 a_3 q_j + a_3^2 h_j - a_1 a_3 q_j h_j - a_1 a_4 q_j - a_3 q_j^2 ,$$

the optimal dispatch may be calculated as

$$\bar{p}_j = \frac{-W_2 + \sqrt{W_2^2 - 4W_1W_3}}{2W_1}$$

$$\underline{p}_j = \bar{p}_j - q_j \; . \tag{5.8}$$

The Peak Load Case With Excess Capacity

The test for this case was shown in the previous section. It is not possible to dispatch the full capacity of the plant, so as much capacity as possible is used within the limit imposed by available energy. The procedure is as follows. Set $\bar{p} = 1$. It is known that at the point of optimal dispatch,

$$h_j = e(\bar{p}_j) - e(\underline{p}_j) = 1 - e(\underline{p}_j) \; ,$$

but it is not known which segment of $e(p)$ is relevant. A separate test is required. If

$$h_j \geq 1 - e(\hat{p}) \; ,$$

then there is sufficient energy to cover all demand above $\hat{p}$; $\underline{p}_j$ falls on the straight line portion of $e(p)$. This sub-case is shown in Figure 5e. The best load position may be calculated as

$$\bar{p}_j = 1 \; ; \qquad \underline{p}_j = (1 - h_j)L \; . \tag{5.9a}$$

If, on the other hand,

$$h_j < 1 - e(\hat{p}) \; ,$$

then $\underline{p}_j$ falls on the curved portion of $e(p)$ and, using Equations 4.17 once again and setting

$$W_1 = a_1 + a_2 - a_1h_j \quad \text{and} \quad W_2 = (1 - h_j)a_3 + a_4 \; ,$$

then the dispatch position is expressed as

$$\bar{p}_j = 1 \; ; \qquad \underline{p}_j = \frac{-W_1 - \sqrt{W_1^2 - 4W_2}}{2} \tag{5.9b}$$

This last sub-case is shown in Figure 5f.

Dispatching a Multi-hydro Market

Consider now the problem of dispatching several hydro plants in the same market in a particular time period. If there are two or more hydro units, it may not be possible to dispatch each separately using the optimal dispatch rules laid out above. There may be conflict between plants for optimal positions in the market, and when this occurs it becomes necessary to conduct a joint dispatch of all conflicting units.

The purpose of the procedure developed below is to simulate multi-hydro dispatch - to approximate by means of a computer algorithm the way the system might best be operated in practice. As in the dispatch of a single unit, the first priority objective is to utilize as much of the potential hydro energy as possible. Subject to the first condition, it is desired to make best use of the capacity available at the various plants (see Equations 5.1, 5.2 and 5.3). The algorithm described below accomplishes these objectives with any number of hydro units.

Because the dispatch of several hydro plants is a complicated task, it is important that it be possible to translate the results yielded by the algorithm back into operational terms, i.e., into the types of rules or guidelines which might be used for actual system management. It is necessary to demonstrate that the dispatching procedure is feasible in practice. Otherwise the system simulation may give a biased impression of the relative worth of different combinations of hydro units. In the presentation to follow, the multi-hydro dispatching procedure is developed for a general case. And then, in the interest of demonstrating the practical implications of the computation, a three plant example is offered.

Suppose there are a number of hydro plants, U_j , $j = \underline{j} + 1,..,J$, to be dispatched into a single market during a particular time period. (Recall that $j = 1,...,\underline{j}$ are thermal plants; $j = \underline{j} + 1,...,J$ are hydro units.) Assume each of the plants is in place $(x_{ji} = 1)$ and has a positive value of available capacity, q_j , and energy, h_j . As before, all variables are stated in normalized form.

In order to simplify exposition, let the following set notation be introduced. First, define a set which contains all the identifying numbers of plants to be dispatched in a single group or "block".

$$B_k = \left\{ j : \text{plant } U_j \text{ is dispatched in block } k \right\} . \quad (5.10)$$

Next, define a set which describes the load position taken by a particular block.

$$P_k^* = \left\{ p : \underline{p}_k \leq p < \bar{p}_k \right\} \quad (5.11)$$

The dispatching of several plants involves a sequential calculation, and the description of the procedure requires the identification of a series of steps and rules for determining the moves from one step to another. Briefly, the computation proceeds by taking each plant, U_j , in turn and defining a block which contains that plant alone. The block is dispatched, and tests are made to see if there is conflict between the new block and any block dispatched earlier in the sequence. If so, the two conflicting blocks are aggregated and dispatched jointly. If no further conflict exists, the next plant is selected and the process begins again.

The particular block which is being dispatched at any point in the calculation will be denoted as $B_{\hat{k}}$. An additional set, A, is defined in the course of the computation; it is used as a "marshalling area" for those plants being drawn together to form dispatch group $B_{\hat{k}}$. Except as otherwise noted, the algorithm proceeds through the following steps in numerical order.

(1) <u>Set Initial Conditions.</u> At the outset, the plant and block indices are set to their initial values

$$j = \underline{j} + 1 ; \qquad \hat{k} = 1 .$$

(2) <u>Select the Next Hydro Plant to be Dispatched.</u> The next plant to be dispatched carries as a subscript the current value of j. Define a set, A, which contains the number of this plant.

$$A = \{ j \} .$$

(3) <u>Define a Block and Dispatch.</u> Let the current dispatch block be defined as

$$B_{\hat{k}} = A .$$

The parameters of this block may be calculated as

$$q_{\hat{k}} = \sum_{j \in B_{\hat{k}}} q_j , \qquad \text{and} \qquad h_{\hat{k}} = \sum_{j \in B_{\hat{k}}} h_j . \qquad (5.12)$$

Dispatch this block according to the single-unit procedure outlined above where

$$\bar{p}_{\hat{k}} = \bar{p}\left(\hat{p}, L, q_{\hat{k}}, h_{\hat{k}}\right) \qquad \text{and} \qquad \underline{p}_{\hat{k}} = \underline{p}\left(\hat{p}, L, q_{\hat{k}}, h_{\hat{k}}\right). \qquad (5.4a)$$

These values of $\bar{p}_{\hat{k}}$ and $\underline{p}_{\hat{k}}$ define the set $P^*_{\hat{k}}$.

(4) <u>Test for Compatibility.</u> It is possible that the load position of B_k may be incompatible with that of blocks dispatched previously. Let the null set be denoted as $\emptyset$, and conduct the following test. If

$$P^*_{\hat{k}} \cap P^*_k = \emptyset \qquad \text{for all } k = 1, \ldots, \hat{k} - 1, \qquad (5.13)$$

then the dispatch of $B_{\hat{k}}$ is compatible with all previous blocks; it can stand as it is. If $j = J$, then the multi-hydro dispatch is finished. If $j < J$, establish the conditions for dispatching the next plant by incrementing the two indices,

$$j = j + 1 \qquad \text{and} \qquad \hat{k} = \hat{k} + 1$$

and return to step (2).

If, on the other hand,

$$P^*_k \cap P^*_{\hat{k}} \neq \emptyset \qquad \text{for any } k = \tilde{k} \quad , \tag{5.14}$$

then the new dispatch is incompatible with that of block $B_{\tilde{k}}$; the two conflicting groups must be combined and dispatched jointly. Define a set, A , which is the union of the identifying numbers of all the plants in the two overlapping blocks.

$$A = \left\{ j : j \in \left(B_{\hat{k}} \cup B_{\tilde{k}} \right) \right\} \quad . \tag{5.15}$$

Then let $B_{\tilde{k}} = \emptyset$; $P_{\tilde{k}} = \emptyset$; $\bar{p}_{\tilde{k}} = 0$, and $\underline{p}_{\tilde{k}} = 0$, and return to step (3). (Note that $B_{\hat{k}}$ may conflict with more than one previously dispatched block. The compatibility test is performed sequentially for $k = 1,\ldots,\hat{k}-1$, and the first intersecting block to be encountered is denoted $B_{\tilde{k}}$ and operated upon as shown.)

This completes the multi-hydro dispatching algorithm. Note that the exit from the sequential calculation is to be found in the first part of step (4). At the end of the calculation there are $J - \underline{j}$ dispatch blocks B_k, $k = 1,\ldots,J - \underline{j}$. For each $B_k \neq \emptyset$ there is a corresponding pair of non-zero values for $\bar{p}_k$ and $\underline{p}_k$, which define the load position for that block. The blocks are mutually compatible, and as demonstrated below, each is dispatched in such a way as to make the best possible use of energy and capacity available from each component plant, U_j , $j \in B_k$. Therefore, the overall routine yields a set of pairs of $\bar{p}_k$ and $\underline{p}_k$, $k - 1,\ldots,J - \underline{j}$, which satisfies the objectives of hydro dispatch as stated in Equations 5.1, 5.2 and 5.3 .

The fact that the parameters of each dispatch block calculated by this routine are a valid representation of the optimal scheduling of the individual plants which comprise the block may be demonstrated by reviewing each of the possible dispatch cases in turn. First, consider a dispatch block comprised of a single plant, U_j . In step (3) of the calculation, the block is defined as having output parameters q_k and h_k , equal to those of the particular plant (Equations 5.12), and the dispatch position for the block (Equations 5.4a) is exactly that which would have been assigned to the plant U_j were it only hydro development in question. By virtue of the optimality of the single-plant dispatch procedure, the dispatch of any block, B_k , comprised of a single unit U_j is also optimal.

Next consider a block comprised of several individual plants as a result of the aggregation procedure described in Equation 5.15. The energy and capacity attributable to the block are the sums of the energy and capacity values of each of the units comprising the block (Equation 5.12). When the block is dispatched, the calculation follows one of the five cases described above. If the output parameters, q_k and h_k, of the block are of such magnitude in relation to market demand that the dispatch falls under cases 1, 3 or 4, then all the capacity and energy of each of the plants U_j, $j \in B_k$ are utilized. Equations 5.1 through 5.3 are satisfied, and as shown below, it is possible to translate the results of the block dispatch into operating instructions for each of the individual units comprising the block.

If the block size were such that it fell into dispatch case 2, "base load with surplus energy", then the full capacity of each of the units comprising B_k is utilized. In this case

$$\bar{p}_k = \sum_{j \in B_k} q_j \; ; \qquad \underline{p}_k = 0 \, , \tag{5.16}$$

yet there remains excess energy:

$$e(\bar{p}_k) - e(\underline{p}_k) > \sum_{j \in B_k} h_j \, . \tag{5.17}$$

Note, however, that because of the convexity of $e(p)$, there is no feasible set of values for $\bar{p}_k$ and $\underline{p}_k$ which will allow utilization of more of the available energy than that set indicated by Equations 5.16. Subject to the capacity constraint, energy utilization is maximized; the conditions stated in Equations 5.1 through 5.3 are satisfied. Once again it is possible to convert the dispatch position for the block into a set of operating rules for each of the component plants; disaggregation of this particular case is required for the transmission routine described in Chapter VII.

Finally, if a dispatch block comprised of more than one plant is dispatched under case (5), "peak load with excess capacity", then the full energy of each of the units comprising B_k is utilized. In this case

$$e(\bar{p}_k) - e(\underline{p}_k) = \sum_{j \in B_k} h_j \, , \tag{5.18}$$

but

$$\bar{p}_k - \underline{p}_k < \sum_{j \in B_k} q_j \, ,$$

where $\bar{p}_k = 1.0$. The first priority objective of the dispatch routine is met, for all the energy is utilized. The second priority objective is also met for $\bar{p}_k = 1.0$ and yet there is insufficient energy to support a capacity dispatch of greater than $\bar{p}_k - \underline{p}_k$ as shown in Equation 5.18. Because of the convexity of $e(p)$ there is no other load position which can utilize more of the available capacity at the various plants. In this case as well, the block dispatch results may be translated into operating rules for the plants which make up the block.

Thus at the end of the multi-hydro dispatch routines, there are blocks, B_k, as comprised of one or more individual units. Each block is dispatched as one of the five dispatch cases. In every case, the objectives of hydro scheduling are fulfilled.

<u>A Three Plant Example</u>

Suppose there are three hydro plants, U_j, $j = 1, 2, 3$, to be dispatched into a market characterized by an integrated load function $e(p)$ as shown in Figures 6 and 7. Following through the algorithm step by step, the dispatch would proceed as follows.

(1) <u>Initialize.</u> Let $j = 1$, $\hat{k} = 1$.

(2) <u>Select.</u> Let $A = \{1\}$.

(3) <u>Define and Dispatch.</u> Then $B_{\hat{k}} = \{1\}$ and $q_{\hat{k}} = q_1$, $h_{\hat{k}} = h_1$. Dispatch the block $B_{\hat{k}}$ to obtain the values for $\bar{p}_1$ and $\underline{p}_1$ as shown in Figure 6a.

(4) <u>Test.</u> Since this is the first block to be dispatched, there is no possibility for conflict; let $j = 2$, $\hat{k} = 2$ and return to Step 2.

(2) <u>Select.</u> Let $A = \{2\}$.

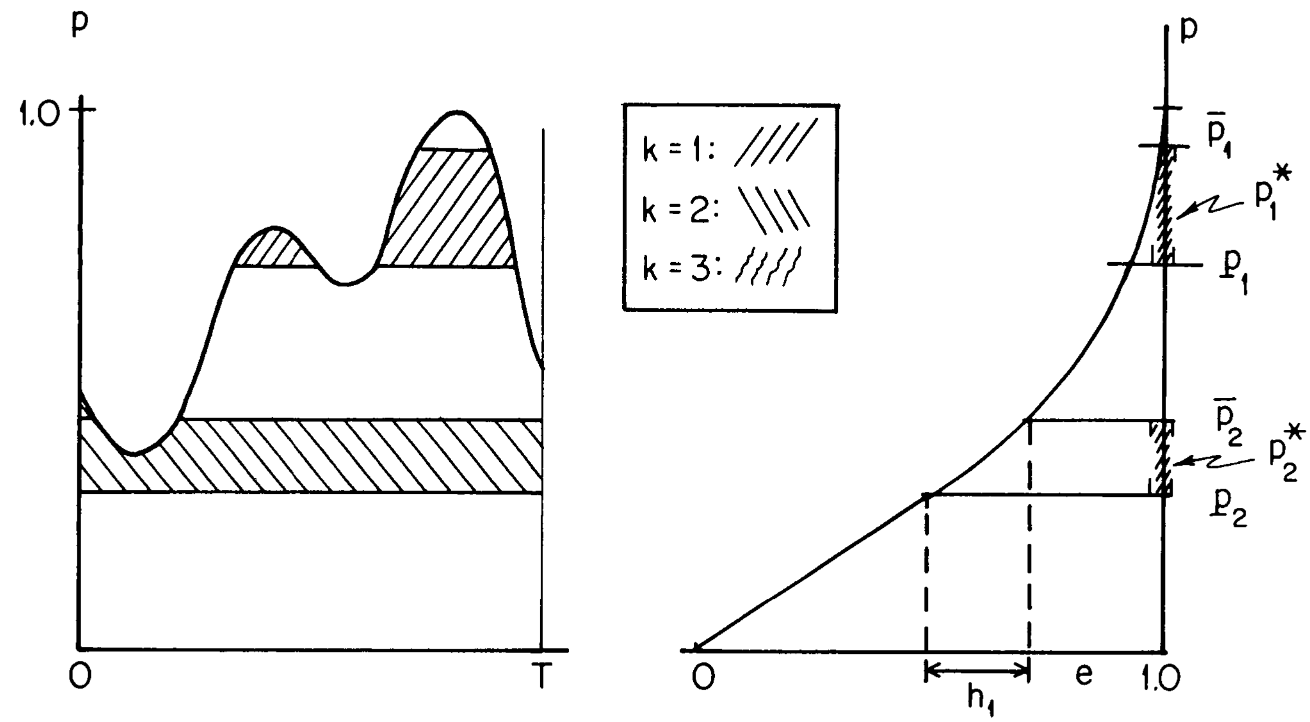
(3) <u>Define and Dispatch.</u> Then $B_{\hat{k}} = \{2\}$ and $q_{\hat{k}} = q_2$, $h_{\hat{k}} = h_2$. Dispatch the block $B_{\hat{k}}$ to obtain the values for $\bar{p}_2$ and $\underline{p}_2$ as shown in Figure 6a.

(4) <u>Test.</u> Note that $P_1^* \cap P_2^* = \emptyset$; there is no conflict. Let $j = 3$, $k = 3$ and return to step 2.

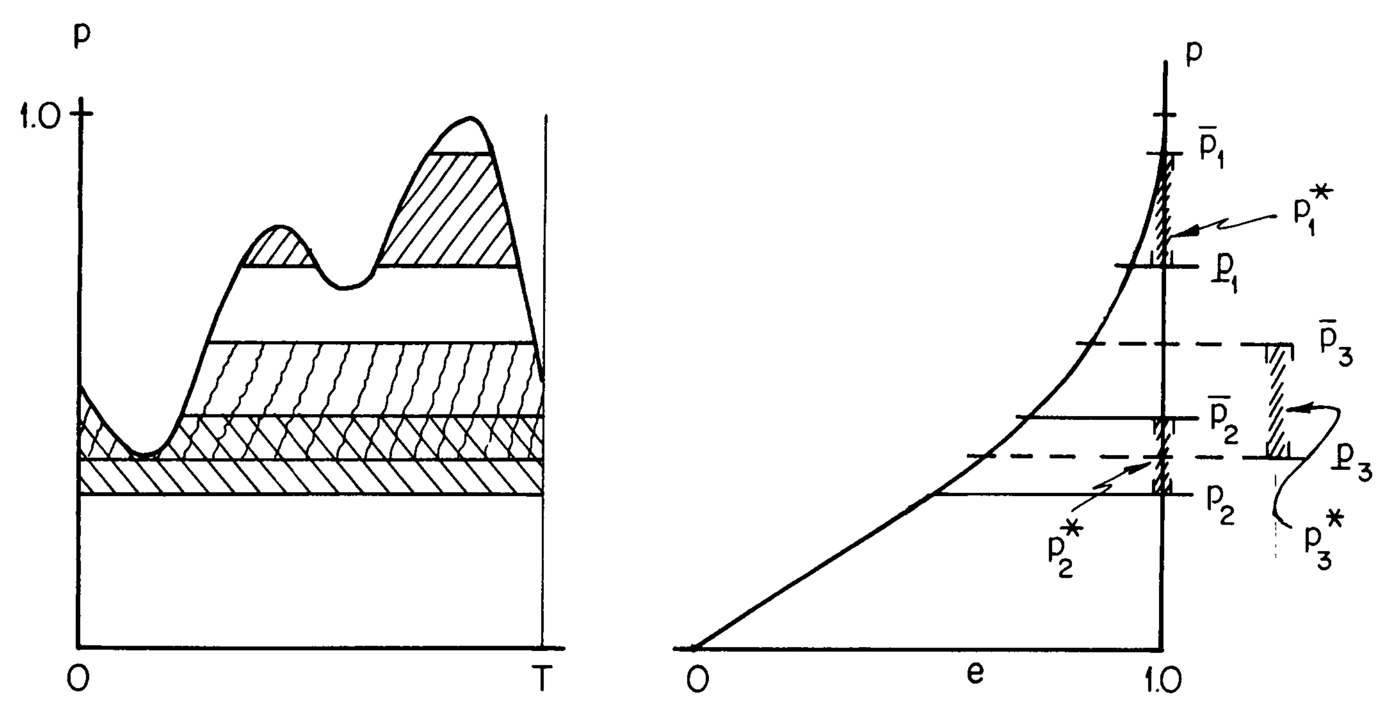
(2) <u>Select.</u> Let $A = \{3\}$.

(3) <u>Define and Dispatch.</u> Then $B_{\hat{k}} = \{3\}$ and $q_{\hat{k}} = q_3$, $h_{\hat{k}} = h_3$. Dispatch the block $B_{\hat{k}}$ to obtain the values for $\bar{p}_3$ and $\underline{p}_3$ as shown in Figure 6b.

(4) <u>Test.</u> Test first for $k = 1$ and note that $P_3^* \cap P_1^* = \emptyset$. Next test for $k = 2$; note that $P_3^* \cap P_2^* \neq \emptyset$; blocks 2 and 3 are incompatible as currently dispatched. Let $A = \{2,3\}$, and let $B_2 = \emptyset$, $P_2^* = \emptyset$, $\bar{p}_2 = 0$ and $\underline{p}_2 = 0$, and return to step 2. (Note the current



(a)



(b)

INDIVIDUAL HYDRO DISPATCH + INTERSECTION value of $\hat{k}$ is still 3.)

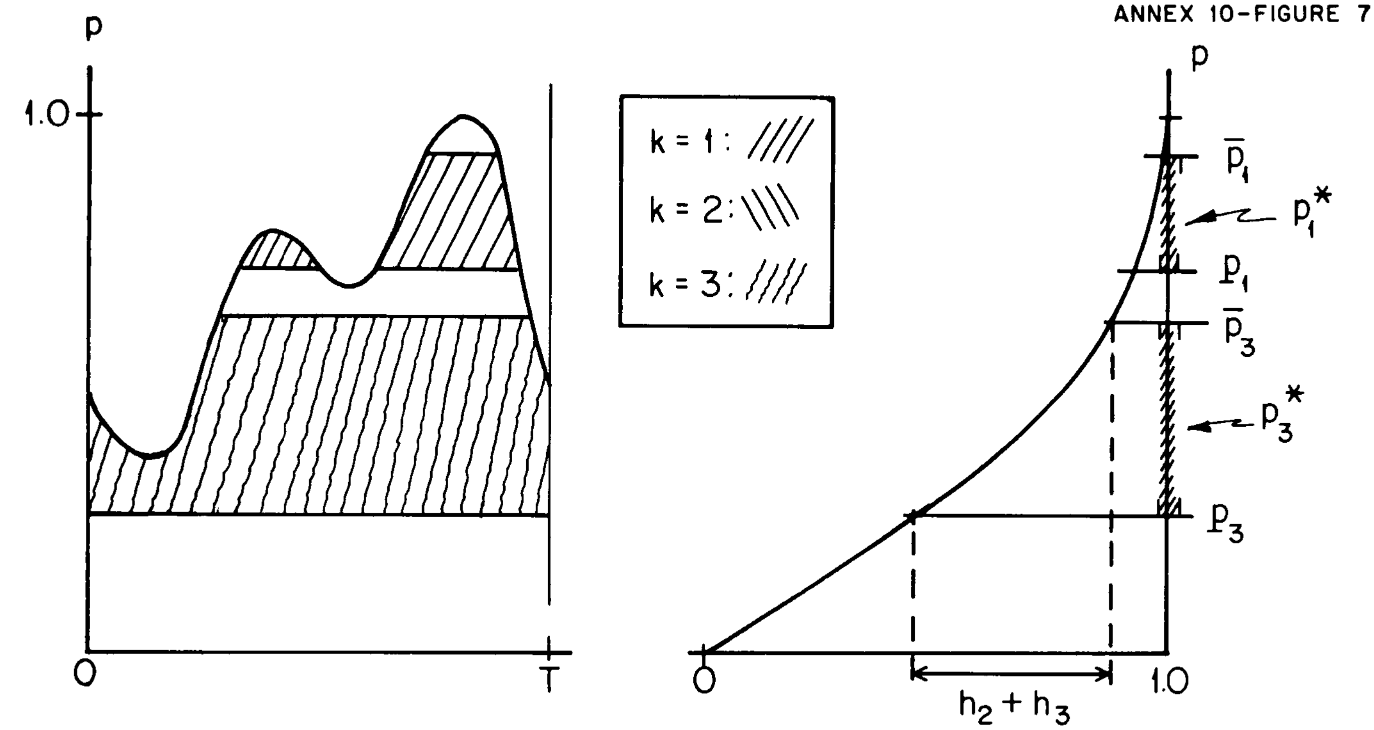
(2) Define and Dispatch. Now $B_{\hat{k}} = \{2,3\}$ and $q_{\hat{k}} = q_2 + q_3$, $h_{\hat{k}} = h_2 + h_3$. Dispatch the block $B_{\hat{k}}$ to obtain the new values of $\bar{p}_3$ and $\underline{p}_3$ as shown in Figure 7a.

(3) Test. Test first for k = 1 and note that $P^*_3 \cap P^*_1 = \emptyset$. Also, $P^*_3 \cap P^*_2 = \emptyset$ since $P^*_2 = \emptyset$. The dispatch of $B_{\hat{k}}$ is compatible with all previous blocks and j = J. Therefore, the dispatch procedure has been completed.

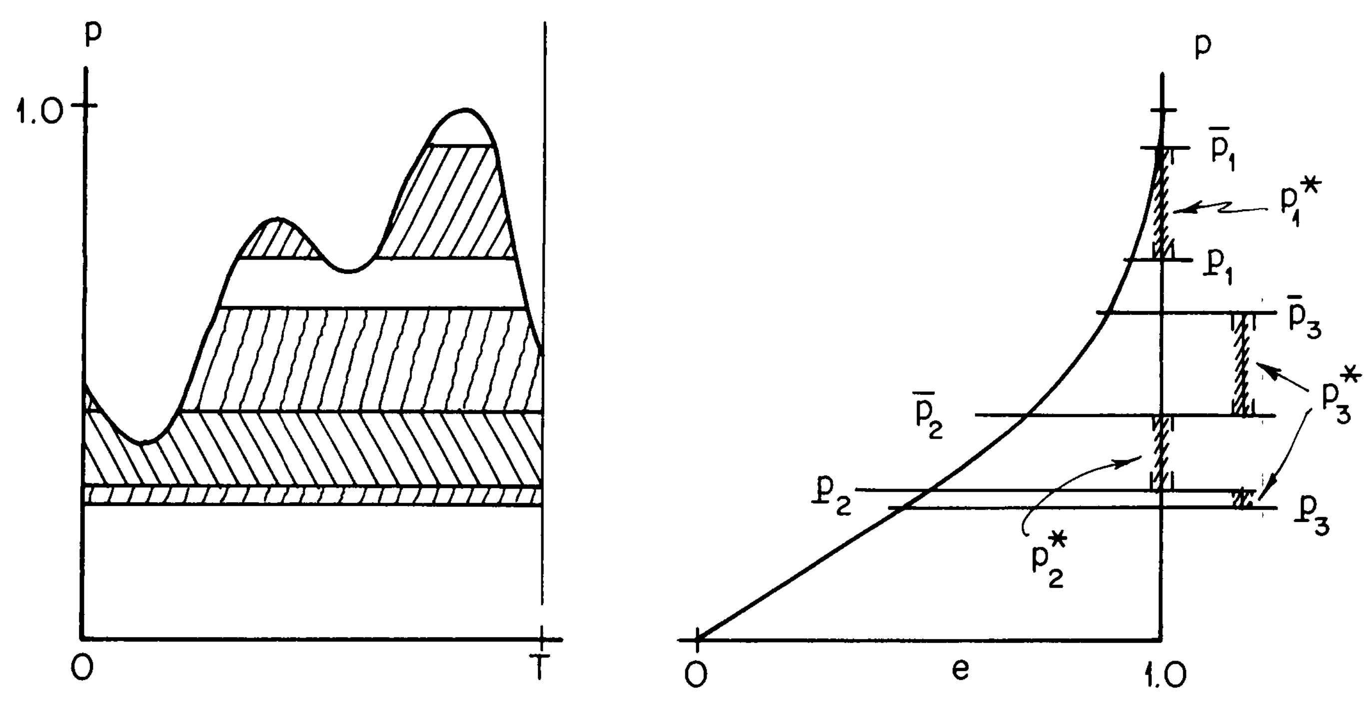
The critical step in the algorithm is the one which aggregates two or more conflicting plants or blocks and re-dispatches them jointly. The operational significance of this procedure can be seen in Figure 7b. There the dispatch of Figure 7a is repeated, only the two combined plants are separated out so one can see what type of scheduling rule is implied by the joint dispatch.

One of the plants(it does not matter which) can be considered as dispatched in its old original position. The other, however, must be split into two parts as shown. Most hydro installations have a number of turbine-generator sets, and in effect some of the machines of plant #3 are assigned to base load service while the others are put on peak load.

It might be pointed out that even with the units separated as in Figure 7b, there still exists at most one position for the plants on the load function which will satisfy the objectives of the dispatching routine. If plant #2 were dispatched first, for example, and then plant #3 split into two sub-units and dispatched around the position occupied by #2, there is only one position for #3 where all its energy and capacity may be utilized. Proof of this fact is omitted here except for the following brief statement. Note that the portion of e(p) not already occupied by plant #2 and therefore available to #3 is convex and that, therefore, there exists only one load position for #3 which will satisfy the two objectives of the routine. The resultant optimal dispatch position for plant #3 defines upper and lower limits for this plant, $\bar{p}_3$ and $\underline{p}_3$ in the figure, and there are precisely the block limits determined by the joint dispatch procedure developed above.



(a)



(b)

JOINT HYDRO DISPATCH

CHAPTER VI

THE DISPATCHING OF THERMAL AND NUCLEAR UNITS

Dispatching an All-Thermal System

The purpose of the dispatching calculation is to allocate the energy which must be generated in each period among the various system units. From this information, total system fuel cost may be calculated. The computer algorithm used to simulate the dispatching of thermal units is more easily described in the context of a system without hydroelectric units, so this case will be considered first. The procedure for modeling the dispatch of thermal plants when hydro units are in use is presented subsequently.

The scheduling of production in an electric power system is a textbook case of a multi-plant firm. The electric utility is analogous to the "firm" of economic theory, and its "plants" are generating stations or individual machines within the stations, each producing a homogeneous product. There exists a demand $P(t)$, and the firm attempts to meet this demand at minimum cost. In a system comprised of conventional thermal units, there is no interdependence of production over time (the interdependence introduced by planned maintenance being ignored here), so the firm operates to minimize fuel cost in each individual time period (hour, day, or month).

There are other elements of variable cost, but fuel is the most important. Each generator or group of generators within a station has a production function which represents fuel efficiency per unit of output or load. This usually is referred to as the "specific consumption" or the "heat rate" of the unit; with the application of a fuel price, it becomes a cost curve of the form $C_j\left[Q_j(t)\right]$ where $Q_j(t)$ is the instantaneous load on the j^{th} plant. Each of these production functions already represents a complicated optimization within the technical process of electric generation - optimization of boiler and condenser operation, steam temperatures and pressures, etc.

An integrated power system has a central office from which generation is controlled; the load dispatcher knows which plants are available for service, and he has cost functions for each. His task is to maintain

$$\sum_j Q_j(t)\, x_{ji} = P(t) \qquad j = 1,\ldots,\underline{j} \qquad (6.1)$$

in such a way as to minimize fuel consumption. In principle the minimizing solution involves the equating of marginal costs, $C'_j\left[Q_j(t)\right]$, over all plants. This is a well-worn point and need not be elaborated here.

Sometimes the optimizing calculation takes place in the mind of the dispatcher - usually a person of long experience and recognized importance within the utility. The approximation of a person who knows the system well is satisfactory so long as the system is not too large. Often in bigger systems some type of analog device is used to aid the calculations, and in a few of the very large and modern systems, dispatching is done with the help of digital computers. The process is the same; only the computation technique is different.

For the analysis of new investments, the planner needs estimates of the fuel consumption associated with the optimal dispatching of various combinations of plants, and he needs information regarding a relatively long period of time,usually several years. It would be possible to make such an estimate by analyzing system performance hour-by-hour. There are computer programs in existence which do just that, although their use is usually confined to planning over a short period of time, generally a year or less.

For long-run studies, however, these detailed calculations become quite expensive, and their accuracy is inappropriate considering the uncertainty involved in the demand projections themselves. Therefore, of the long-run planning studies which consider complexities of short-run operation, most use some aggregate technique such as the load-duration curve or the integrated load function applied here. Discussion of the use of the latter analytical device to capture the economic aspects of thermal plant operation begins with a formal definition of the fuel cost function along with a listing of the required assumptions. There follows a brief explanation of why this aggregate technique performs as well as it does in estimating system expenditure on fuel.

Assume the specific consumption of each plant, $C_j\left[Q_j(t)\right]$, can be represented by a constant, $\bar{C}_j$, over the relevant range of $Q_j(t)$. Considering only those hours when the plant is actually running, let ω_j be the average kw output expressed as a percentage of the plant's rated capacity, Q_j. Let f_j denote the fuel price in \$/Btu or \$/calorie for plant j. Plants are ranked in order of their fuel cost per kwh generated, i.e., $\bar{C}_j f_j \leq \bar{C}_{j+1} f_{j+1}$ for $j = 1,\ldots,\underline{j}-1$.

The subset of plants which is in existence in each year is indicated by the matrix of scale variables, $\| x_{ji} \|$; $j = 1,\ldots,\underline{j}$; $i = 1,\ldots,N$. (In other sections, this matrix also is denoted by the upper-case letter, X.) For any particular year, i, the plants in existence are noted by a $\underline{j}$-element column vector of zeros and ones, $\underline{x}_i = \left[x_{1i},\ldots,x_{\underline{j}i}\right]$. Total fuel cost during a particular year, $F(\underline{x}_i)$, is evaluated by adding up the fuel cost incurred in each of the individual plants as they are dispatched on the integrated load function $E(P)_T$. Let $\bar{P}_j$ be the upper limit of the load position assigned to plant j, and let $\underline{P}_j$ be the lower limit. Total system fuel cost, then, is calculated as

$$F(\underline{x}_i) = \sum_j \bar{C}_j f_j \left[E(\bar{P}_j)T - E(\underline{P}_j)_T\right] \qquad j = 1,\ldots,\underline{j}, \qquad (6.2)$$

where

$$\underline{P}_j = 0 \qquad \text{for} \quad j = 1\,,$$

$$\underline{P}_j = \overline{P}_{j-1} \qquad \text{for} \quad j = 2,\ldots,\underline{j}\,, \text{ and} \tag{6.3}$$

$$\overline{P}_j = \underline{P}_j + \omega_j Q_j x_{ji} \qquad \text{for} \quad j = 1,\ldots,\underline{j}\,,$$

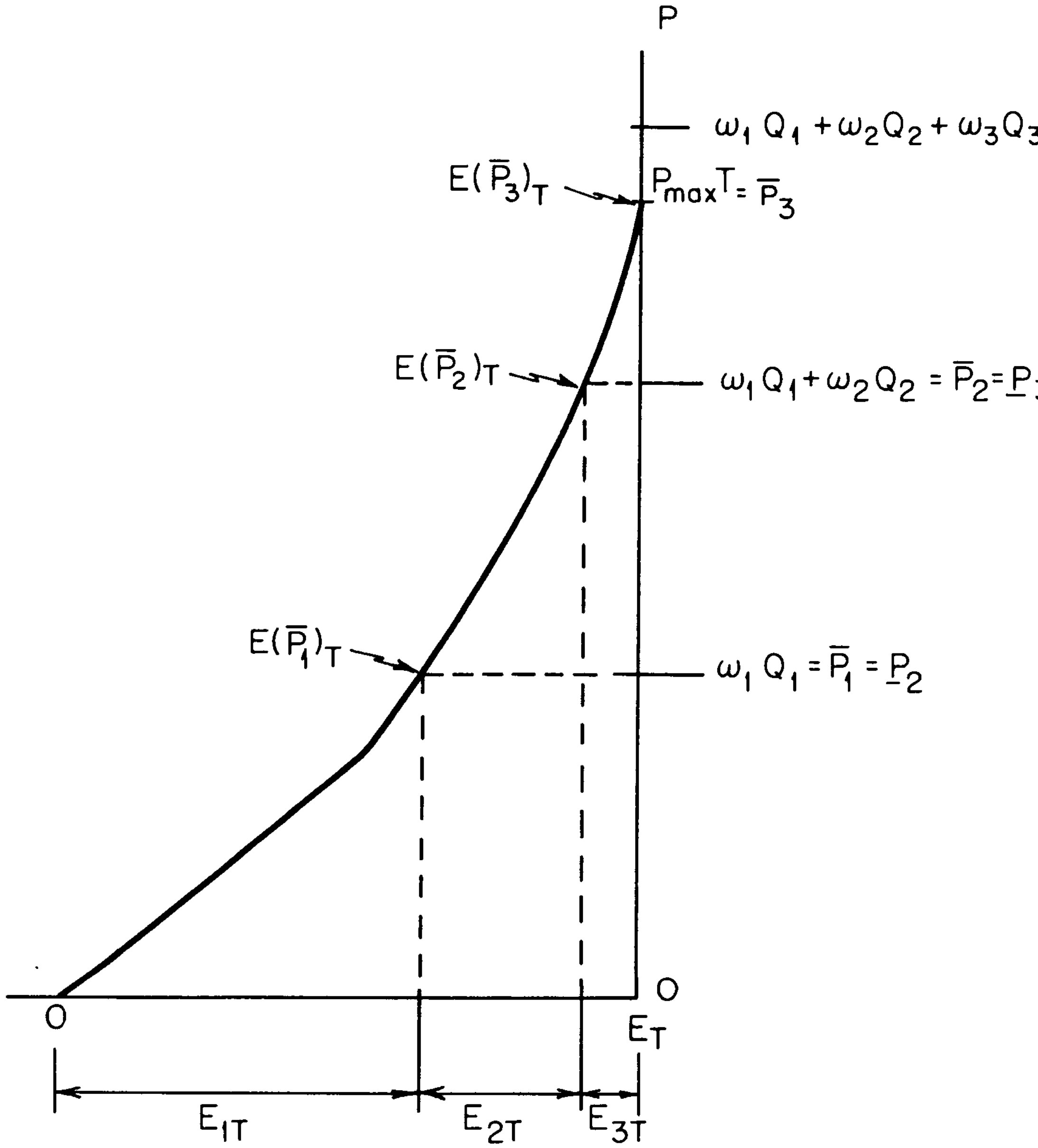
and $E(\overline{P}_j)_T$ has an upper limit of E_T , the total energy to be generated during the period.

A simple graph will help explain the content of Equations 6.2 and 6.3 . Figure 8 shows a three plant example. Plant #1, the most efficient of the three, operates as a base load unit. Its average output level, $\omega_1 Q_1$, is entered on the ordinate. The total energy produced by this plant over the year, $E(\overline{P}_1)_T$ or E_{1T} , may be read directly on the abscissa. This quantity multiplied by the average specific consumption for plant #1 and the appropriate fuel price gives the fuel cost incurred by this plant over the year.

Then plant #2 is added on the ordinate, i.e., j = 2 in Equation 6.2. The upper intercept on the ordinate becomes $\overline{P}_2 = \omega_1 Q_1 + \omega_2 Q_2$. The total energy generated by this plant, E_{2T} , is the energy associated with its upper limit, $E(\overline{P}_2)_T$, less that associated with the lower limit, $E(\underline{P}_2)_T$, where $\underline{P}_2 = \overline{P}_1$. This energy can be converted into a dollar cost as before. The process continues until all the system energy has been accounted for. Note that in this three plant example there is some reserve on the system; the sum of the capacities of the plants in service during the year shown exceeds the system peak demand.

This aggregate device yields very close estimates of fuel consumption for most systems. The reason why this short-cut procedure works so well is to be found in the performance characteristics of the various system units. First, the point of most efficient output for most large steam-electric generating units is in the region of 75 percent to 80 percent of rated output capacity. Above and below this point the specific consumption, $C_j\left[Q_j(t)\right]$, increases, but even at rated output the specific consumption is not far above that at the most efficient point. Second, technological change in electric power generation has been dramatic over recent decades; the newer plants are both larger and significantly more efficient.

The most efficient units carry the base load; P_{minT} is usually in the neighborhood of one-third of P_{maxT} . As the load builds up in the morning of a typical day, additional plants are put into service. The output levels of all plants are continually adjusted as the marginal cost of carrying an additional increment of $P(t)$ increases. But very quickly the most efficient units run up against a boundary condition; when $Q_j(t) = Q_j$, $C'_j\left[Q_j(t)\right] = \infty$. As additional higher cost units are intro-



TOTAL FUEL COST IN THIS YEAR:
$F(\underline{x}_1) = \bar{C}_1 f_1 E_{1T} + \bar{C}_2 f_2 E_{2T} + \bar{C}_3 f_3 E_{3T}$

FUEL COST CALCULATION

(3 PLANT EXAMPLE, ANNUAL AGGREGATE $E(P)_T$)

duced into the system, more of the favored units run up against output constraints.

What this means in the aggregate is that most plants operate in a relatively narrow kw range about their most efficient output points, and within this range the variation in $C_j [Q_j(t)]$ is a small relative to inter-plant differences in specific consumption. On the average, a typical plant will operate at about 85% of rated capacity; this percentage, defined as ω_j above, is referred to as the "plant factor". And, on the average, a typical plant will generate at a level of fuel consumption per kwh slightly above the minimum value of $\underline{C}_j [Q_j(t)]$. This average specific consumption was denoted earlier as $\bar{C}_j$. The accuracy of these assumptions about average kw contribution and average specific consumption for each plant and the validity of the integrated load function approach itself may all be checked against system operating records.

Thermal Dispatching in a Multi-hydro System

The scheduling of thermal units in a market where hydro plants have previously been dispatched is a straightforward extension of the procedure described by Equations 6.2 and 6.3 . Once again, the total system fuel cost in any period, i.e., month, is the sum of the costs incurred in each of the thermal plants;

$$F(\underline{x}_i) = \sum_j \bar{C}_j f_j E_{jT} \qquad j = 1,\ldots,\underline{j} , \qquad (6.4)$$

where E_{jT} is the total energy produced during the interval by thermal plant j.

The determination of E_{jT} in the multi-hydro case, however, requires a sequential calculation involving tests for the portions of market demand already assigned to hydro plants. Let B_k and P^*_k be defined as in Chapter V above and define an additional set, P^*_j which describes the load position taken by a particular thermal plant, U_j .

$$P^*_j = \left\{ p : \underline{p}_j \leq p < \bar{p}_j \right\} \qquad (6.5)$$

Two more sets, A_1 and A_2 , will be defined in the course of the computation. A_1 will contain the identifying numbers, k , of all hydro dispatch blocks, B_k , which must be "skipped over" in the course of dispatching a particular thermal plant. A_2 is similarly defined but is used to transfer certain information between different stages of the computation. Except as otherwise noted, the algorithm proceeds through the following steps in numerical order.

(1) Set Initial Conditions. Recall that thermal plants are numbered in inverse order of their fuel costs in \$/kwh, j = 1,...,$\underline{j}$. Begin by letting j = 1 .

(2) <u>Define $\underline{p}_j$</u> . If $j = 1$, $\underline{p}_j = 0$;

$$\text{If } j = 2, \ldots, \underline{j}, \quad \underline{p}_j = \overline{p}_{j-1} . \tag{6.6}$$

Let $A_2 = \emptyset$.

(3) <u>Define an Initial Trial Position.</u> Begin by dispatching plant U_j as follows. Let

$$\overline{p}_j = \underline{p}_j + \omega_j q_j x_{ji} ,$$

where q_j is the rated capacity of plant j expressed in normalized form and ω_j is the average kw output of the plant as a percentage of rated capacity. At this stage in the calculation it is not known if this dispatch position conflicts with that previously assigned to some hydro block.

(4) <u>Identify Intersection with Previous Hydro Dispatch.</u> Define a set A_1 which contains the identifying numbers of any hydro blocks, B_k , which intersect with the load position, $\overline{p}_j$ and $\underline{p}_j$, currently assigned to thermal plant j .

$$A_1 = \left\{ k : P^*_j \cap P^*_k \neq \emptyset \right\} . \tag{6.7}$$

(5) <u>Test for Intersection and, If Necessary, Re-define the Thermal Dispatch Position.</u> If $A_1 \neq \emptyset$ and $A_1 \neq A_2$, then there are one or more intersected hydro blocks which have just been identified on the last pass through step 4. Redefine $\overline{p}_j$ as follows.

$$\overline{p}_j = \underline{p}_j + \omega_j q_j x_{ji} + \sum_{k \in A_1} \left[\overline{p}_k - \underline{p}_k \right] , \tag{6.8}$$

where $\overline{p}_j \leq 1$. Next, let $A_2 = A_1$ and return to step 4.

If either $A_1 = \emptyset$ or $A_2 = A_1$, then a proper upper limit, $\overline{p}_j$, has been established for thermal plant j . Calculate E_{jT} as

$$E_{jT} = \left\{ e(\overline{p}_j) - e(\underline{p}_j) - \sum_{k \in A_1} \sum_{j \in B_k} h_j \right\} E_T . \tag{6.9}$$

(The expression in brackets is the energy generated by thermal plant j stated in normalized form; multiplication by total system energy demand for the period, E , converts this to the form used in Equation 6.4). If $\overline{p}_j < 1$ and $j < \underline{j}$, there remains more market demand to be met. Increment the plant index,

$$j = j + 1 ,$$

and return to step 3. If $\overline{p}_j = 1$ or $j = \underline{j}$, then either the demand has been met or the procedure has run through all the available plants. In either case, the thermal dispatch is completed.

CHAPTER VII

INTER-MARKET TRANSMISSION

The Multi-market Algorithm

As noted above, the computer program for the West Pakistan system must consider the dispatching of generating units in both the Northern and Southern markets, the potential development of generation facilities in the Upper Sind, and the possible transmission of power between these various locations. Any particular month may find the markets in one of four configurations with respect to interconnection as indicated in Figure 1:

(1) no interconnection at all,

(2) connection from Mari to the South only,

(3) connection from Mari to the North only, and

(4) connection of North with South via Mari.

The computer program must be able to handle all of these; all strategies start in condition 1, and most proceed to conditions 2 or 3 and then to condition 4 in the course of the planning period.

In condition 1, the markets are independent and a separate dispatch is conducted for each. In conditions 2 and 3, the situation is much the same; the available capacity at Mari is dispatched into the appropriate market like any other thermal plant, but consideration is taken of capacity and energy losses over the long-distance transmission line.

It is when the system is in condition 4 that assumptions about system operating procedures take on critical importance. The operating rules for an inter-market transmission network are strongly influenced not only by a number of technical factors characteristic of the physical system itself but also by the structure of the particular institutions and organizations responsible for the management of the two markets. The operating assumptions used in this analysis were chosen with the assistance of the Power Consultant, and they reflect the essential operating characteristics of the system as it might best be managed in practice, taking all these technical and institutional factors into account. The computer algorithm described below is specific to this set of assumptions. For a different system the details of the transmission computer algorithm would have to be revised, although the basic approach would be the same.

Subject to the appropriate transmission constraints, the operating rules selected are the following:

(1) The Northern hydro plants will be dispatched primarily into the Northern Market. Transmission of hydro power to the South will take

place only under the following two conditions :

- If the Southern Market is in a shortage condition without power from the North, then a mandatory transmission will be made so long as this shipment will not force the North into shortage.

- If there is hydro energy which cannot be utilized in the North, an attempt will be made to transmit the surplus to the South in such a way as to maximize the total quantity of hydro energy actually utilized in the two markets.

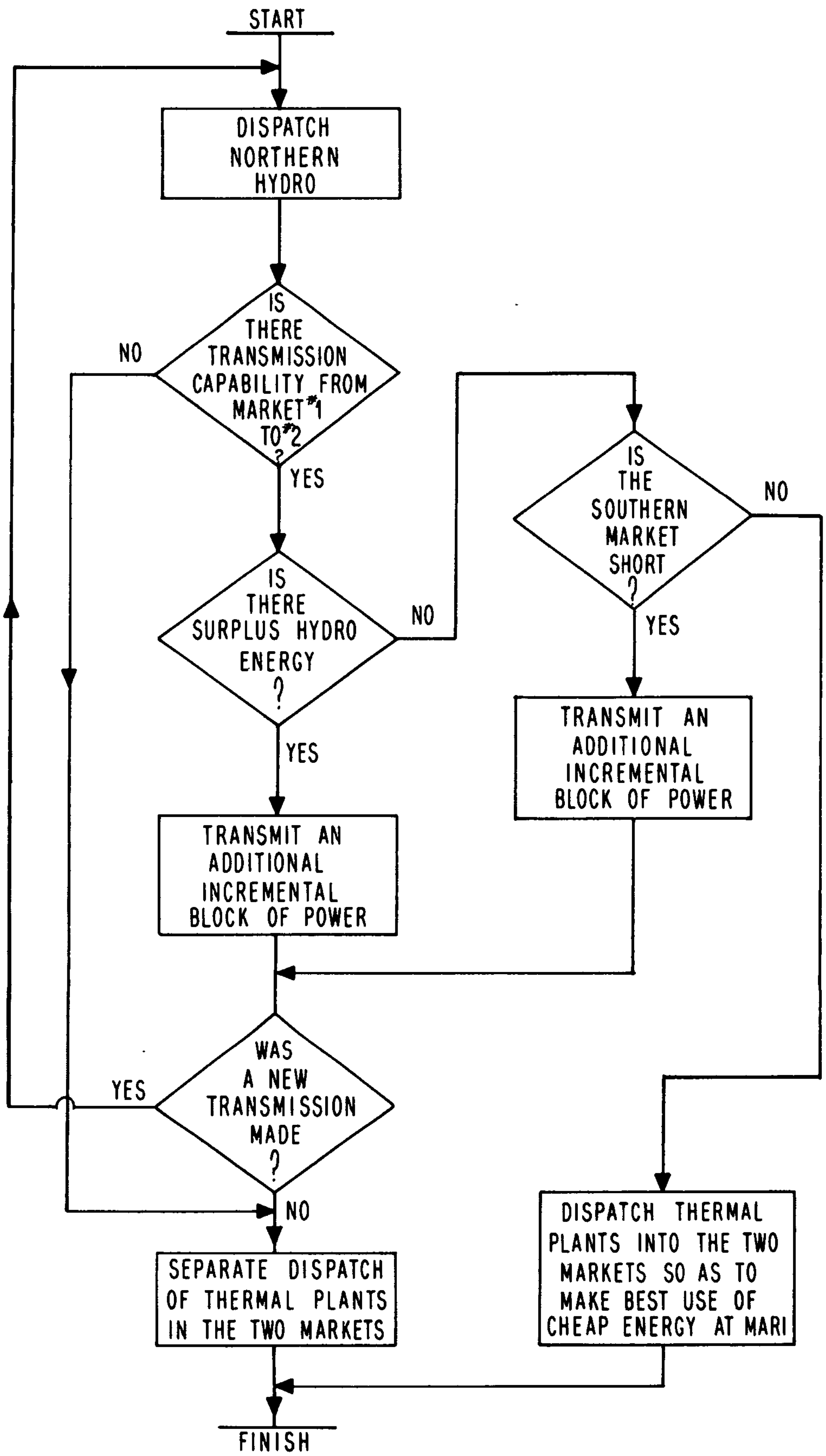
In addition, Northern hydro power will not be dispatched into pure peaking service in the South. If any transmission is made, it must first cover a portion of the Southern base load. Only if this requirement is met can the transmitted capacity and energy be dispatched in peak load service.

(2) There will be no transmission of thermal power between the North and the South. Because of this assumption, no provision need be made for studying transmission from the South to Mari.

(3) Any thermal capacity at Mari is always available for dispatch into the Southern Market. In the event that no hydro power is moving from North to South, Mari machines may also be dispatched into the Northern Market. The allocation of the Mari capacity between North and South in such a situation will be conducted so as to minimize the total fuel cost of the integrated system.

The routine designed to model monthly system energy dispatch under these conditions is summarized in the flow chart presented as Figure 9. Most of the details of the transmission calculation are discussed below; other portions of the algorithm have already been covered in Chapters V and VI.

The choice of the quantity of hydro power to be transmitted to the South is based upon an iterative calculation as shown in Figure 9. In the course of the computation, the program assigns each additional block of transmitted power to a specific Northern hydro unit. This approach of designating the units which send out the power to be transmitted proves useful for the following reasons. First, in many instances there is excess hydro energy in the North without any associated excess hydro capacity. The contribution of the hydro system to the service of the Northern capacity demand may have to be reduced in order to free the capacity (mw) necessary to dispatch a portion of the excess energy (mwh) into the Southern Market. But when machine capacity is freed in this manner, it is necessary to know how much energy is available to be transmitted, and this energy must be associated with capacity made available at a particular hydro station. Otherwise there is a danger of associating capacity available at one plant with energy available somewhere else and thus calculating a transmission which is technically infeasible. Computationally there are several ways to handle this difficulty; for the



SYSTEM ENERGY DISPATCH ROUTINE

West Pakistan system this iterative routine seems best. A second reason for keeping track of the plants feeding the transmission link is that this approach allows a rough check on the difficulties which might arise in moving large blocks of power over the Northern grid system from the plants of origin to the point where the transmission line to the South begins.

Following Figure 9 the monthly system energy dispatch proceeds as follows. After the dispatch of the Northern hydro plants, a check is made to see if the full transmission link between North and South is in place. If it has not been constructed yet, the markets are independent, and the single-market dispatch procedure is applied to each. If the transmission line is in place, a further test is needed to make sure it is not already fully loaded. If there is unused transmission capability then tests are made to determine whether a shipment of hydro power to the South is called for. If either of the conditions described above are met (excess hydro energy or Southern Market in shortage) then the program proceeds to the appropriate transmission routine. A transmission of an incremental block of power, V, is set up.

After some portion of the output of a particular unit has been assigned to the transmission line, the scheduling of hydro plants in the North must be re-evaluated. Thus each time an additional incremental shipment has been added to the overall transmission for the month, the transmitting unit is redefined as a result of having some of its capacity and energy allocated to the South, and the computation returns to the beginning to re-dispatch the Northern Market. Once the hydro transmission for the month has been established, the dispatching of thermal units proceeds as before. Since the flow of hydro power from the North preempts the capacity of the Lyallpur-Mari transmission line, the Mari thermal units may be dispatched into the Southern Market only.

If the full transmission line is in place but no hydro power is being shipped South, the Mari thermal capacity may be sent to either market. Since the variable cost of energy from these units is among the lowest on the system, there is an attempt to allocate the Mari capacity between the two markets in such a way as to minimize the overall fuel expenditure in both markets for the month.

In order to describe this algorithm, several adjustments and additions to the notation are required. Since it is necessary to keep track of two markets, it is no longer convenient to express capacity and energy variables in normalized form as was done in Chapters IV, V, and VI. Once again, the subscripts T and i are dropped from all variables or functions in order to simplify the equations. Of course, transmission line capacities, hydro and thermal plant characteristics and market demand structure all change from one period to another, but the procedure remains the same. The calculation only concerns the values of these variables for the particular interval being analyzed.

The transmission constraints - $\overset{*}{V}_{1,3}$, $\overset{*}{V}_{3,2}$ and $\overset{*}{V}_{3,1}$ - may be defined at either end of each transmission link, the difference being the capacity loss over the length of the line. In this derivation, the constraints apply to the receiving end of the particular link.

Both these hydro transmission routines concern the combination of links 1,3 and 3,2. A consistent notation would present the relevant transmission constraint as $\overset{*c}{V}_{1,2}$, thus identifying both the sending and receiving end of the line and the number of the particular transmission system under study. In order to further simplify the notation in this particular section, however, the transmission constraint for the North-South link is denoted as $\overset{*}{V}$. Likewise, the quantity of power actually transmitted is denoted as V and the incremental blocks of power evaluated in each iteration as ΔV . The value of $\overset{*}{V}$ in any particular period is determined as

$$\overset{*}{V} = \min \left\{ (1 - \Lambda) \overset{*}{V}_{1,3} , \overset{*}{V}_{3,2} \right\} , \qquad (7.1)$$

where Λ is the percentage loss in transmission over each of the two links.

Equation 7.1 states that the capacity which can be transmitted to the Southern Market is limited by the line capacity from Mari to the South, $\overset{*}{V}_{3,2}$. Transmission is also limited to the capacity which can be delivered at Mari, $\overset{*}{V}_{1,3}$, minus the transmission loss from there to the South.

Transmission of Excess Hydro Energy

Consider first the procedure followed when there is excess hydro energy in the North which might be used in the South. Since any energy not actually dispatched during some period is presumed lost, the objective of this algorithm is to maximize the total quantity of hydro energy actually utilized in the two markets, i.e., to minimize the sum of surplus energy in the North plus transmission losses. The complexity of the routine is due in large measure to the necessity of keeping track of exactly which hydro plants have excess energy and which individual units are chosen to transmit. The hydro dispatch routine described in Chapter V is greatly simplified by the aggregation of plants into dispatch blocks, but as noted above, when transmission is at issue it is useful to sort out individual plants once again.

This transmission routine operates on the Northern hydro system as dispatched under the rules laid out in Chapter V. Recall that at the completion of hydro dispatch there is a number of sets B_k , $k = 1,\ldots,J - \underline{j}$, which contain the identifying numbers, j , of all hydro plants dispatched in a single group or block. Some of these sets may be empty, others contain one or more individual hydro plants U_j , $j \in B_k$. For each $B_k \neq \emptyset$

there is a corresponding pair of non-zero values for $\bar{P}_k$ and $\underline{P}_k$ which define the load position assigned to that group.

This particular part of the transmission routine is actuated when there is surplus hydro energy, E, in the Northern Market, where

$$E = \sum_{k=1}^{J-j} \left\{ \sum_{j \in B_k} H_j - \left[E_1(\bar{P}_k) - E_1(\underline{P}_k) \right] \right\} \qquad (7.2)$$

Note that excess energy can only occur in a group dispatched under the conditions of "base load with excess energy" (see Chapter V). That is to say, when there is a group, B_k, for which $\underline{P}_k = 0$, $\bar{P}_k > P_{min,1}$. And it is the plants in this group which have the excess energy. Let this group be denoted $B_{\hat{k}}$. 1/

The task of the transmission routine is to find the plant U_j, $j \in B_{\hat{k}}$, which has the most excess energy and determine whether the shipment of an incremental block of power, ΔV, from that plant will yield an improved overall utilization of available hydro energy. The identification of the individual unit with the greatest surplus requires the disaggregation of the plants which make up hydro group $B_{\hat{k}}$, provided of course that this group is comprised of more than one plant to begin with. Each plant must be re-dispatched individually so as to yield appropriate values of $\bar{P}_j$ and $\underline{P}_j$.

Recall that in the dispatch of the "base load case with excess energy" the full capacity, Q_k, of the group is utilized. Since

$$Q_k = \sum_{j \in B_k} Q_j ,$$

this means that the full capacity of each of the plants which make up B_k also is usable. And, therefore, the re-dispatch of the individual plants requires a procedure which divides the load position of the group - $\bar{P}_{\hat{k}}$, $\underline{P}_{\hat{k}}$ -

1/ The "hat" is used in this chapter to denote this specific value of the index k, e.g., $\hat{k}$. This special identifying mark was used in Chapter V as well, but note that its meaning is re-defined for use here. Note also that the word "block" is used in two senses in this discussion. On the one hand, it is applied to a group or block of hydro plants dispatched together, B_k. On the other hand, it refers to the incremental transmission of capacity or energy between markets #1 and #2, V or ΔV. There is no connection between the two uses of this particular term.

among the contributing plants, the lower positions going to the units with the most energy per unit of capacity. Note that of all the B_k, $k = 1,\ldots,J-j$, only $B_k^\wedge$ is involved in this process. Also the contents of the set, $B_k^\wedge$ remain undisturbed through the computation.

From the viewpoint of the Southern Market, the power coming in over the transmission line is treated as if it originated at a dummy hydro plant U_j, $j = 1$. The load position assigned to the transmitted capacity and energy is defined in terms of the upper and lower limits of the range of Southern capacity demand served by this dummy unit, $\overline{P}_j$ and $\underline{P}_j$. Before the first cycle through the transmission routine, a value is assigned to $\underline{P}_j$, and $\overline{P}_j$ is set equal to $\underline{P}_j$. With each iteration, an additional block of transmitted capacity, ΔV, is added to $\overline{P}_j$. Over the course of the computation, the size of the dummy plant grows. When the full hydro transmission for the month has been established, then $(\overline{P}_j - \underline{P}_j) = V$, where V is the total hydro capacity delivered to the Southern Market. One can think of the hydro dispatch in the South as analogous to the procedure followed in scheduling thermal units; each additional block of hydro capacity, ΔV, being added on top of the one before.

In some situations, $\underline{P}_j$ might be set at zero; hydro energy brought into the receiving market would be dispatched first on base load. In the West Pakistan application, $\underline{P}_j$ was given a positive initial value in order to always reserve a certain portion of base load for a particular nuclear installation. This decision was based on the judgment that, in practice, this nuclear unit would not be shut down or put onto peaking service even if free hydro energy were available. To assume otherwise would overstate the amount of hydro energy which can actually be used.

The details of this transmission algorithm are best presented in a series of numbered steps. The routine is described without reference to the number of iterations which have already been performed. Recall, however, that before the first iteration, $\overline{P}_j = \underline{P}_j$ and $V = 0$. Unless otherwise noted, the computation proceeds through the following steps in numerical order.

(1) <u>Re-dispatch Hydro Block</u> $B_k^\wedge$. This procedure involves the definition of a set, A, which contains the identifying numbers of all plants already assigned an individual load position. The variable u is used to indicate different load levels as the re-dispatch takes place, $0 \leq u \leq P_{max,1}$. At the outset, $u = 0$, $A = \emptyset$. The re-dispatch proceeds through the following steps:

(1a) Choose that plant U_j which satisfies the following condition.

$$\max_{j \in B_k^{\wedge} \cup A'} \left\{ \frac{H_j}{Q_j} \right\} .$$

(1b) Let $\underline{P}_j = u$ and $\overline{P}_j = u + Q_j$

(1c) Then let $u = \overline{P}_j$, and define a set

$$A = \left\{ j : \text{plant } U_j \text{ has already been selected in step 1a} \right\}$$

If $A \neq B_k^{\wedge}$, return to step 1a. If $A = B_k^{\wedge}$, the re-dispatch is completed; proceed to step 2. (Once the re-dispatch is completed, no further use is made of the set, A , or the variable, u .)

(2) Pick the Plant with the Most Excess Energy. Choose that plant U_j which satisfies the following condition:

$$\max_{j \in B_k} \left\{ E_j \right\} , \tag{7.3}$$

where

$$E_j = \left\{ H_j - \left[E_1(\overline{P}_j) - E_1(\underline{P}_j) \right] \right\} .$$

Denote this plant as $U_{\hat{j}}$. It is this unit which will transmit the next incremental block of power to the South.

(3) Determine the Size of the Next Incremental Transmission. There are four restrictions on the size of the next incremental block of power, ΔV , to be transmitted. Recall that ΔV , along with other transmission variables, is defined at the receiving end of the connection. Thus the size of ΔV cannot be so large as to make the magnitude of the total transmission for the month, V , greater than the transmission line constraint, $\hat{V}$. Second, ΔV is limited to the size of the standard incremental transmission block, ν . The variable, ν , is an input datum; the selection of a value for ν involves oonsideration of the warranted accuracy of the computation and the computer time required to evaluate each strategy. In most calculations for the West Pakistan case, $\nu = 50$ mw . A third limitation is that the capacity of the incremental block of power delivered to the South cannot be greater than that of the particular hydro plant, $U_{\hat{j}}$, chosen to transmit with due account taken of losses in transmission. Finally, the incremental transmission may not be so large as to force the Northern Market into shortage.

Thus the magnitude of the next incremental block of power to be added to the transmission for the month is determined as

$$\Delta V = \min \left\{ \left[\overset{*}{V} - V \right], V, \frac{Q_j}{(1-\Lambda)^2}, \left[\sum_{k=1}^{j-1} (\overline{P}_k - \underline{P}_k) + \sum_{j=1}^{i} (Q_{j,1}) - P_{max,1} \right] \Big/ (1-\Lambda)^2 \right\} \tag{7.4}$$

(4) Determine the Amount of Energy Available. In order to evaluate an additional transmission of ΔV, information is needed regarding the amount of surplus Northern energy available at plant $U_{\hat{j}}$, $Y_1(\Delta V)$, and the amount of energy required to support a dispatch of ΔV into the Southern grid, $Y_2(\Delta V)$. The subscripts of the variable $Y(\Delta V)$ indicate whether the quantity is defined at the generating plant in market #1 or at the receiving end of the transmission line in market #2.

In order to add ΔV to the transmission into the South, the contribution of plant $U_{\hat{j}}$ to the Northern Market must be reduced by $\Delta V(1-\Lambda)^2$. Of necessity, such a cutback in capacity dispatch frees additional hydro energy, i.e. $Y_1(\Delta V) > E_{\hat{j}}$. Exactly how much additional surplus is created depends upon the manner in which the cutback is performed. Since the objective of the whole routine is to utilize as much hydro energy as possible, the desired procedure for determining the cutback is the one which frees the least amount of additional energy. Because of the convexity of the integrated load function, $E(P)$, this may be accomplished by reducing the upper limit, $\overline{P}_{\hat{j}}$, of the load position assigned to $U_{\hat{j}}$ in step 1 above. Thus if

$$\overline{P}_{\hat{j}} = \underline{P}_j + Q_{\hat{j}} - \frac{\Delta V}{(1-\Lambda)^2} \tag{7.5}$$

then the total amount of surplus energy available for the shipment to the South from this plant is

$$Y_1(\Delta V) = H_j - \left[E_1(\overline{P}_j) - E_1(\underline{P}_j) \right] \tag{7.6}$$

In other words, $Y_1(\Delta V)$ is the total energy available at the plant minus the energy dispatched into the Northern Market at the reduced capacity level.

(5) Determine the Amount of Energy Needed. As noted above, each additional transmission of ΔV is added to the top of the load position of a dummy hydro plant, U_j, which is used to represent the characteristics of the power coming in over the transmission line. A certain quantity of energy, $Y_2(\Delta V)$, is required to support the dispatch of ΔV under these conditions. It may be calculated as

$$Y_2(\Delta V) = E_2(\overline{P}_j + \Delta V) - E_2(\overline{P}_j) \, . \tag{7.7}$$

(6) Test the Feasibility of the Transmission. If

$$Y_2(\Delta V) > Y_1(\Delta V) \Big/ (1 - \Lambda)^2 ,$$

then hydro plant $U_{\hat{j}}$ cannot support a transmission of ΔV to the Southern Market under the conditions assumed. More energy would be needed than is available. Decrement ΔV and return to step 4 for another try. If $\Delta V = 0$, no further transmission can yield improved utilization of the Northern hydro energy. Return to the main program and do not attempt an additional incremental energy-saving transmission this month. If, on the other hand,

$$Y_2(\Delta V) \leq Y_1(\Delta V) \Big/ (1 - \Lambda)^2 ,$$

then the transmission of V is feasible. Proceed to step 7.

(7) Test the Productivity of the Transmission. If

$$E_{\hat{j}} - \left[Y_1(\Delta V) - Y_2(\Delta V) \right] \leq 0 ,$$

then nothing is gained by the transmission of the incremental block, ΔV. The term $E_{\hat{j}}$ is the surplus energy at plant $\hat{j}$ before the transmission; the term in brackets represents the total waste, i.e., surplus at the plant plus transmission loss, after the increment ΔV is added to the transmission for the month. In this case, an addition to the transmission for the period yields a net loss. Cancel the transmission of ΔV and return to the main program. Do not attempt another energy-saving transmission this month. Recall that plant $U_{\hat{j}}$ is the hydro plant with the most excess energy. If no additional transmission from this plant is productive, then an attempted shipment from any of the other hydro plants is likely to prove even less so. When this condition has been reached, maximum use has been made of available hydro energy.

If, on the other hand,

$$E_{\hat{j}} - \left[Y_1(\Delta V) - Y_2(\Delta V) \right] > 0 ,$$

then the transmission of this incremental block of power has produced a net improvement in the utilization of hydro energy; proceed to step 8.

(8) Re-define the System. An additional transmission of ΔV is to be made from plant $U_{\hat{j}}$. Re-define the characteristics of this plant as follows.

$$Q_{\hat{j}} = Q_{\hat{j}} - \frac{\Delta V}{(1 - \Lambda)^2} ; \qquad H_{\hat{j}} = H_{\hat{j}} - \frac{Y_2(\Delta V)}{(1 - \Lambda)^2} . \tag{7.8}$$

Next, establish the new addition to the transmission for the month and the new dispatch into the Southern Market by re-defining the relevant variables.

$$\bar{P}_j = \bar{P}_j + \Delta V ; \qquad V = V + \Delta V . \tag{7.9}$$

The transmission of an additional block of power has been completed. Return to the main program and proceed to a re-dispatch of the Northern hydro system under the revised conditions.

This completes the algorithm used to compute the inter-market transmission of excess hydro energy. As noted above, the operating assumptions for interconnected systems often must be tailored to the individual case. This proved true in the analysis of the West Pakistan grid, and the justification for this particular routine is to be found in the characteristics of the system itself.

Recall that the objective of this part of the transmission routine is to utilize as much as possible of the potential hydro energy in each period, i.e., to minimize the sum of surplus energy in the North plus transmission losses. The routine comes into play only in those months when a surplus exists at the end of the initial hydro dispatch. The relative frequency of this condition depends upon the state of the system as regards the number and size of hydro plants in place in relation to the magnitude of Northern Market demand. In order to be guaranteed of maximizing total energy utilized under all conditions, one would need an expanded version of the dispatch routine developed in Chapter V - one which could manage the simultaneous joint dispatch of hydro units into both markets. Efforts in this direction failed to produce a workable algorithm, and therefore the iterative procedure described above was adopted. It has proved a reasonable approximation of system operation. And although it can be shown that there are certain conditions under which this algorithm will not make best use of available energy, in the vast majority of months this procedure yields a transmission which does maximize total energy utilized.

There are several system characteristics which indicated this approach would prove successful. First, the relative sizes of the Southern Market demand and the planned transmission system are such that well over one half the capacity transmitted South will always be dispatched on base load. The hydro plant most likely to be able to exploit such a load position for an incremental block of its capacity is the one with the greatest surplus of energy to begin with, and therefore the selection procedure shown in Equations 7.3 was adopted. Also, the more surplus energy the transmitting plant has at its disposal, the more likely the transmission of ΔV is to be feasible (see step 6). Less computer time is spent determining how large a ΔV a particular hydro development can support in the South; the computation proceeds more rapidly.

A second characteristic of the system concerns the size of the hydro system in relation to the Northern Market demand and the capacity of the transmission line. Since the primary purpose of the market interconnection is to allow utilization of surplus hydro energy in the South, any strategy which provides for interconnection also involves a large amount of hydro capacity in the North. The Northern grid is primarily a hydro system, and the combined capacities of the hydro plants is several times that of the largest transmission line envisioned. Finally, the capacity and energy output from the various hydro units varies considerably between the wet and dry seasons. The performance of the algorithm developed above when applied to a system with these characteristics may be seen by considering those conditions which call for termination of the hydro transmission routine.

During months of high river flow, the transmission line constraint normally is binding. The first term of Equation 7.4 terminates the calculation. In this case, given the assumptions about the manner in which transmitted power is to be dispatched into the Southern Market, full utilization is made of available hydro energy.* The total energy which can be dispatched to the South is limited by $\bar{V}$, the size of the transmission line. The total energy usable in the North is limited by the amount of hydro capacity left over after transmission. All capacity is utilized and no other transmission schedule can use more energy, else the program would have stopped transmitting before it encountered this constraint. (Note that this is the case in five months of the sample year shown in Figures 11 and 12 below.)

In dry months, on the other hand, the output of the hydro plants is severely reduced. The transmission is more likely to be limited by the total capacity needed to meet Northern Market demand - the last term of Equation 7.4 terminates the transmission procedure. In this case, the maximum feasible capacity, V , has been transmitted South, and thus as much of the hydro energy as possible has been dispatched in the Southern Market. The hydro capacity remaining in the North is fully utilized but is insufficient to dispatch all energy into the Northern Market. Once again, each additional ΔV would not have been added to the transmission for the period had it not increased total energy utilization, yet no further ΔV is feasible. In this situation as in the former, the iterative procedure developed above maximizes total energy utilization subject to the relevant constraints.

A third condition which will terminate the transmission routine occurs when the additions of incremental blocks of power eventually eliminate the surplus at the Northern hydro plants. The objective of the transmission routine has been realized; the excess has been absorbed into the Southern Market. It is possible, however, that some alternative method of scheduling the transmission might attain the same result with a total shipment of less capacity and energy to the South. The difference between the two schedules would be the net difference in transmission losses. In this case the transmission calculated by the routine described above is not necessarily optimal but the error is very small.

Examples of the elimination of surplus energy by transmission can be seen in Figures 11 and 12; this occurs in February and September of the year shown. (Note that excess energy does not have to be reduced to zero. To cut computation time a tolerance limit of 10 gwh is used. If E < 10 gwh, no further transmission is considered worthwhile.)

The final way in which the transmission routine may be terminated is for the test of step 7 to reveal that the addition of another incremental block of power would yield a net loss in energy utilization. In such a situation, it is possible that there is some alternative transmission schedule which might do better, i.e., use more of the surplus by transmitting from some plant other than the one with the most excess energy. This condition occurs in very few months, however, due to system characteristics noted above, and the net difference in system cost between simulations using the operating rule developed here as opposed to some more complex one would be very small.

Transmission to Meet Southern Capacity Need

The test which calls for this routine is the following. If

$$\sum_j Q_{j,2} + V + \left(\sum_j Q_{j,3}\right) (1-\Lambda) < P_{max,2} , \quad j=1,\ldots,\underline{j} , \tag{7.10}$$

then the Southern Market is in shortage. The sum of the capacity in place in the South plus the hydro power already coming in from the North plus the additional capacity which may be shipped from Mari is not sufficient to cover peak demand. An additional mandatory transmission is needed. Since this routine is required only under unusual circumstances, no complicated routine such as that used for the surplus energy case is warranted. The Northern hydro plant with the greatest total energy available is chosen to transmit the additional block of power. Unless otherwise noted, the computation proceeds through the following steps in numerical order:

(1) Pick the Plant with the Most Total Energy. Choose that plant U_j which satisfies the following condition:

$$\max_j \left\{ H_j \right\} , \qquad j = \underline{j} + 1,\ldots,J ,$$

and denote this plant as $U_{\hat{j}}$. This unit will transmit the next incremental block to the South.

(4) Determine the Transmission Limit. The size of this incremental block, ΔV , is limited by the four conditions stated in Equation 7.4.

(5) Test for the Availability of Energy. Next, determine whether or not plant $U_{\hat{j}}$ has sufficient energy to support a dispatch of ΔV to the South. Let $Y_2(\Delta V)$ be calculated as shown in Equation 7.7. If

$$H_j < \frac{Y_2(\Delta V)}{(1 - \Lambda)^2}$$

then plant $U_j^{\wedge}$ cannot support a transmission of ΔV to the Southern Market. If plant $U_j^{\wedge}$, the greatest energy producing hydro plant on the system, cannot support this transmission, neither can any other unit. Cancel the transmission and return to the main routine. If

$$H_j > \frac{Y_2(\Delta V)}{(1 - \Lambda)^2}$$

then this transmission is feasible. Proceed to step 6.

(6) Re-define the System. The transmission of ΔV from plant $U_j^{\wedge}$ is to be made. Re-define the characteristics of this plant as shown in Equations 7.8 and re-define the system operating parameters as shown in Equations 7.9. The transmission of an incremental block of power is completed. Return to the main program to re-dispatch the Northern hydro system under the revised conditions.

Transmission of Thermal Power from Mari

If the full transmission system is in place but no hydro power is being transmitted from North to South, then any thermal units at Mari may be dispatched into either of the two markets. Because they are located directly on the gas field, the Mari machines are among the cheapest on the system as far as fuel cost is concerned. The objective of the thermal transmission algorithm is to allocate the capacity at Mari between the two markets in such a way as to minimize overall system fuel expenditure. 1/

The allocation of Mari capacity between the two markets is denoted by the variables λ_1 and λ_2, where

$$\gamma_1 = \frac{V_{3,1}}{\sum_j Q_{j,3}}, \quad \text{and} \quad \gamma_2 = \frac{V_{3,2}}{\sum_j Q_{j,3}}, \qquad j=1,\dots,i . \tag{7.11}$$

1/ Note that this is not an optimization of the overall system energy dispatch but a sub-optimization of the thermal scheduling subject to the hydro dispatch and transmission yielded by the routines described above. One of the operating assumptions chosen for the interconnected system stated that, in the absence of capacity shortage in the Southern Market, no transmission is to be made from north to south if there is no surplus hydro energy. This thermal transmission routine is designed to minimize fuel cost under these circumstances. This does not mean that some other operating assumption for the hydro transmission - for example, transmitting even when there is no surplus - might not yield an even lower overall system fuel consumption.

The overall system fuel cost function to be minimized may be expressed as

$$Z = F_1\ (\underline{x}_1, \Upsilon_c, \Upsilon_1) \ + \ F_2\ (\underline{x}_i, \Upsilon_c, \Upsilon_2). \tag{7.12}$$

The general character of the allocation problem is shown in Figure 10. Because of various transmission constraints and mandatory dispatches in the case of potential shortage, it may not be possible to send all the Mari capacity in one direction or the other. The zone of feasible combinations of Υ_1 and Υ_2 is shown between the two shaded limits on the left and right sides of the figure. Over the feasible range, the fuel costs of each market vary according to the share of Mari assigned.

The overall system fuel cost function has a single global minimum point as shown in Figure 10. The thermal transmission routine identifies a starting point at the boundary of the feasible region and then steps across the function until the minimum point is encountered. Unless otherwise noted, the computation proceeds through the following steps in numerical order.

(1) <u>Establish an Initial Boundary Point.</u> Begin by establishing an initial transmission to the South.

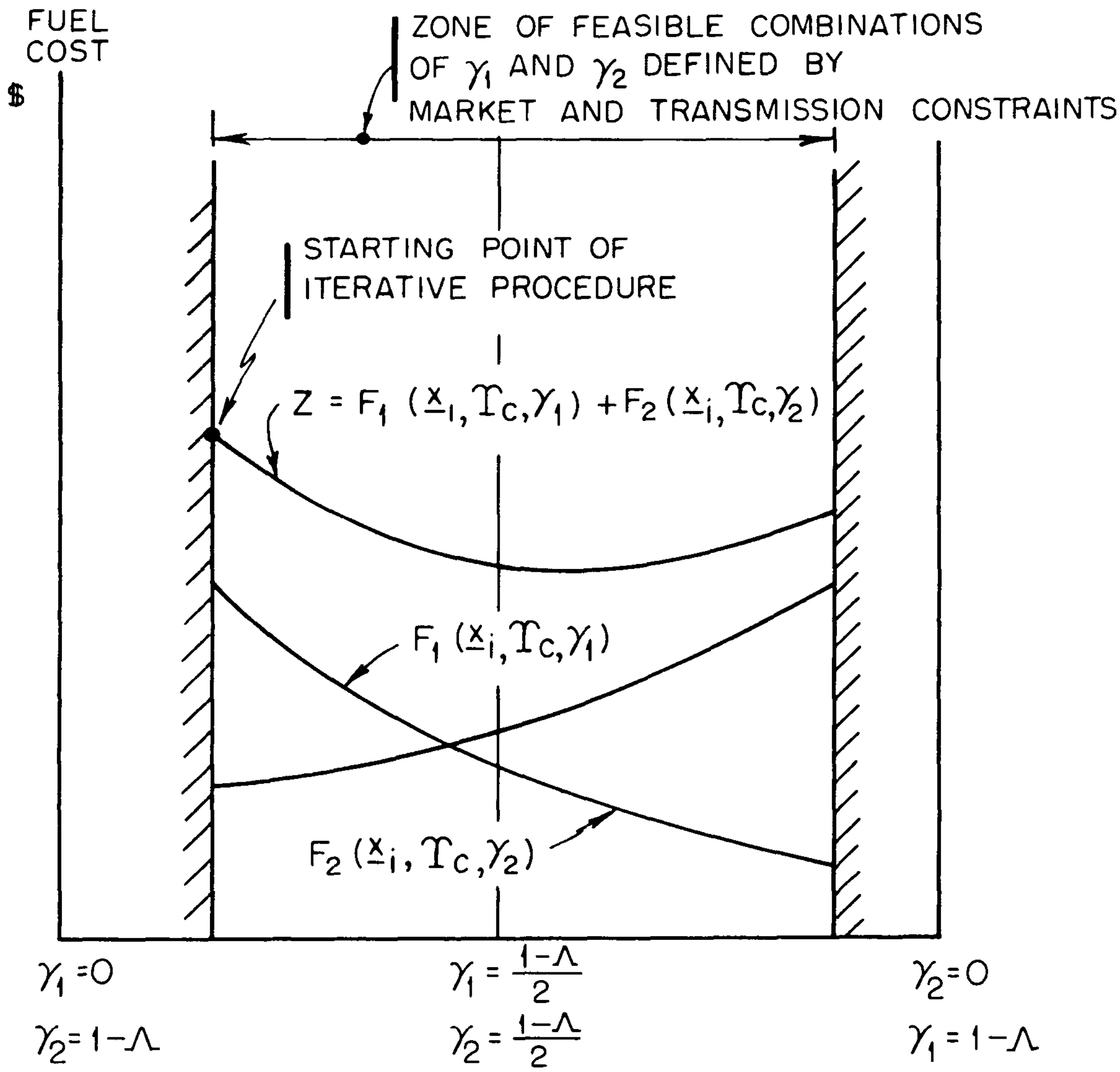
$$V_{3,2} = \min \left\{ \overset{*}{V}_{3,2}, \left(\sum_j Q_{j,3} \right) (1 - \Lambda), \right.$$
$$\left. \max \left[\left(P_{\max,2} - \sum_j Q_{j,2} \right), \left(\sum_j Q_{j,3}(1 - \Lambda) - \overset{*}{V}_{3,1} \right), 0.0 \right] \right\}$$

$$j = 1, \ldots, \underline{j}\ . \tag{7.13}$$

Stepping through term-by-term, Equation 7.13 says the following. First, the transmission to the South can be no larger than the size of the transmission line or the total capacity of the Mari units adjusted for transmission losses. Subject to these limits, however, the initial allocation south should be enough to cover any potential shortage in the Southern Market or the total amount of capacity which cannot be sent north due to transmission constraint - whichever is greater.

Whatever is left over after this initial allocation to the South is transmitted to the Northern Market subject to the appropriate line constraint.

$$V_{3,1} = \min \left\{ \left(\sum_j Q_{j,3} \right) (1-\Lambda), \overset{*}{V}_{3,1} \right\}, \qquad j=1,\ldots,\underline{j}\ . \tag{7.14}$$



RESPONSE OF TOTAL SYSTEM FUEL COST TO THE ALLOCATION OF MARI CAPACITY

The variable, $\hat{Z}$, is used to compare the net change in system fuel cost between successive iterations. As part of the initial conditions, set $\hat{Z}$ equal to some number larger than any foreseeable value of system fuel cost.

(2) <u>Calculate the Share of Mari Assigned to Each Market.</u> The appropriate values of the shares of Mari capacity dispatched north and south are calculated by Equations 7.11. Note that the shares are defined not at Mari but at the points of entry into the receiving markets.

(3) <u>Dispatch the Thermal Plants in the Two Markets.</u> Dispatch the thermal plants into the Northern and Southern markets in turn using the procedures outlined in Chapter VI. Each of the Mari units, $U_{j,3}$, is considered available for dispatch into both markets in accordance with the share allocated. In effect, the routine treats each of the Mari plants as if one portion of its capacity were dispatched north and another portion south. Since the fuel price at all Mari units is the same, this yields a good approximation of the manner in which this generating center actually would be operated. Account is taken of the energy loss in transmission by applying a specific consumption of $\bar{C}_j/(1 - \Lambda)$ to each of the Mari units.

The fuel costs incurred in each of the markets are summed to give a value for the total fuel cost for the integrated system, Z , as shown in Equation 7.12.

(4) <u>Test the Net Change in System Fuel Cost.</u> If $Z < \hat{Z}$, the fuel cost associated with the Mari shares used during this iteration is lower than that of the previous one. Let $\hat{Z} = Z$ and proceed to step 5 to try another incremental change in the allocation. (On the first pass through the routine, $\hat{Z}$ is a very large number, the computation proceeds automatically to step 5.)

If $Z \geq \bar{Z}$, the computation either has stepped past the minimum point on the total system fuel cost function or has run up against some boundary condition. Since the value of Z on the previous iteration was at least as low or lower, return to the previous values of γ_1 , γ_2 , $V_{3,2}$, $V_{3,1}$ and Z . The thermal transmission and dispatch has been completed and these parameters describe the operation of the system during this period.

(5) <u>Shift an Increment of Mari Capacity from North to South.</u> In the computer program prepared for the West Pakistan case, a step along the cost function, Z , was made by increasing the Southern share of Mari by 10 percent. A smaller step would yield greater accuracy, but since the dispatch computation required to evaluate each point on the function is time consuming, a smaller step size would require a significant increase in computer cost.

Formally, the allocation for the next iteration is defined as follows. Let the variable $\hat{V}_{3,2}$ be used to represent an intermediate value of $V_{3,2}$

in the course of the computation. First, establish a provisional value for the new transmission by letting

$$\hat{V}_{3,2} = \left(\sum_j Q_{j,3}\right)\left(\gamma_2 + .10\right), \qquad j = 1,\ldots,\underline{j}\,. \tag{7.15}$$

This new allocation may not be feasible, however, due to some operating constraint. Therefore, let

$$V_{3,2} = \min\left\{ \hat{V}_{3,2}\,,\ \overset{*}{V}_{3,2}\,,\ \left(\sum_j Q_{j,3}\right)(1 - \Lambda)\,, \left[\sum_j Q_{j,1} + \sum_k \left(\bar{P}_k - \underline{P}_k\right) + V_{3,1} - P_{max,1}\right]\right\}, \quad j=1,\ldots,\underline{j},\ k=1,\ldots,J-\underline{j} \tag{7.16}$$

Equation 7.16 states that the new transmission from Mari south is set to $\hat{V}_{3,2}$, subject to the transmission constraint and the limit on the total capacity at Mari and subject to the restriction that this shift of capacity from north to south must not put the Northern Market in shortage. The value of $V_{3,1}$ corresponding to the new value of $V_{3,2}$ is defined by Equation 7.14. Once the new transmission has been established, return to step 2.

This completes the thermal transmission procedure. Note that the exit from the routine is in step 4. 1/

1/ For this iterative procedure to converge upon the minimum value of Z it is necessary that this function be convex. A proof that this is so is presented in H. D. Jacoby, op. cit.

CHAPTER VIII

THE MONTHLY SYSTEM OPERATING SUMMARY

For each power development program (strategy) analyzed, the computer program which simulates the operation and growth of the electric power system prints out about 25 to 30 pages of detailed information. In addition to the final system cost data described in Chapter II above, it presents detailed information about the monthly operation of the overall power system. This information proved very useful in the evaluation of the performance of the system under different development programs and in assessment of why the total costs of any program turned out to be higher or lower than those of another program. It also provides the basis for evaluation of terminal conditions.

Figures 11 and 12 show the form of the system operation summary for a typical year. Where appropriate, the mathematical symbols attached to certain quantities are shown in the right-hand margins.

Figure 11 contains the information on Market No. 1, the Northern Grid. First, data on the scheduling of thermal plants are presented in the form of monthly plant factors, Ω_{jT} , where

$$\Omega_{jT} = \frac{E_{jT}}{Q_{jT} \cdot T} \quad . \tag{8.1}$$

The plant factor indicates the relative load position of each plant during the month - or, in other words, the extent to which the plant is used during the month to meet market demand. The highest plant factors occur when the unit is on base load; when this occurs, $\Omega_{jT} = \omega_j$. Recall that ω_j was defined in Chapter II as "the average kw output expressed as a percentage of the plant's rated capacity, Q_j , considering only those hours the plant is in operation". It was assumed that the maximum effective load factor that could be taken by a plant in a month was 90 percent, or in other words, $\omega_j = .90$. Therefore, when $\Omega_{jT} = .90$ it means that the plant is running continuously. The smaller the value of Ω_{jT} for an individual plant, the higher into the load peak the plant has been dispatched. When $\Omega_{jT} = 0$, the plant is not used at all during that month. It is also possible from these data to calculate the total fuel consumption by any plant over any period. This feature proved very useful in analysis of the differences among power programs in their total requirement of thermal fuel and in the need they implied for expansion of the gas-pipelines (see, for instance, Annex 9, Appendix 2).

Because the overall system operation adjusts to variation in fuel prices and the exchange rate, there is a set of Ω_{jT} for each permutation of assumptions about these prices. Before each computation is begun a decision must be made as to which set is to be printed out. The print-

	PLANT NAMES		MARKET NUMBER 1												
			JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	
THERMAL	MUL S1		.	.22	.21	.16	.24	.07	.	.	.	.08	.	.	Ω_{jT}
	MUL S2		.	.08	.07	.05	.13	.00	.	.	.	.00	.	.	
	MUL GT		.	.01	.00	.	.07	.	.	.	.	.	.	.	
	LYA D		.	.00	.	.	.07	.	.	.	.	.	.	.	
	LAHGT1		.	.	.	.	.03	.	.	.	.	.	.	.	
	LAHGT2		.	.	.	.	.01	.	.	.	.	.	.	.	
	LYA S1		.10	.36	.34	.28	.36	.19	.00	.03	.02	.21	.07	.12	
	SUKKUR		.75	.	.42	.35	.	.	.	.	.	.	.90	.69	
	LAHGT3		.	.	.	.	.05	.	.	.	.	.	.	.	
	MARI 1		.90	.	.61	.47	.	.	.	.	.	.	.90	.90	
	MARI 2		.90	.	.56	.44	.	.	.	.	.	.	.90	.88	
	MARI P		.82	.	.49	.39	.	.	.	.	.	.	.90	.75	
HYDRO	SMALL	TOP MW	346.	749.	673.	645.	708.	923.	1093.	1156.	1204.	1063.	1157.	356.	$\bar{P}_k$
		BOTTOM MW	74.	0.	0.	0.	0.	0.	0.	0.	0.	0.	81.	72.	$\underline{P}_k$
		TRANS MW	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
		TRANS GWH	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
	WARS40	TOP MW	1098.	850.	673.	645.	708.	923.	1093.	1156.	1204.	1063.	1157.	1155.	
		BOTTOM MW	418.	750.	0.	0.	0.	0.	0.	0.	0.	0.	81.	491.	
		TRANS MW	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
		TRANS GWH	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.	
	MAN10A	TOP MW	346.	749.	673.	645.	708.	923.	1093.	1156.	1204.	1063.	1157.	356.	
		BOTTOM MW	74.	0.	0.	0.	0.	0.	0.	0.	0.	0.	81.	72.	
		TRANS MW	0.	0.	0.	0.	150.	0.	0.	0.	0.	0.	0.	0.	
		TRANS GWH	0.	0.	0.	0.	109.	0.	0.	0.	0.	0.	0.	0.	
	MAN50B	TOP MW	1098.	749.	673.	645.	708.	923.	1093.	1156.	1204.	1063.	1157.	1155.	
		BOTTOM MW	418.	0.	0.	0.	0.	0.	0.	0.	0.	0.	81.	491.	
		TRANS MW	0.	50.	0.	0.	29.	179.	179.	179.	145.	150.	0.	0.	
		TRANS GWH	0.	37.	0.	0.	21.	131.	131.	131.	106.	109.	0.	0.	
	EXCESS HYDRO MW		80.	0.	0.	0.	0.	0.	0.	0.	0.	0.	55.	135.	
	EXCESS HYDRO GWH		-0.	3.	12.	50.	74.	126.	164.	204.	5.	5.	0.	-0.	
	DEMAND IN MW		1098.	1192.	1196.	1090.	1121.	1162.	1098.	1239.	1264.	1302.	1157.	1155.	$P_{max}T,1$
	SHARE OF MARI		.32	.	.46	.31	.	.	.	.	.	.	.18	.46	$Y_{T,1}$

SYSTEM OPERATION SUMMARY FOR A TYPICAL YEAR - MARKET No. 1 (NORTHERN GRID) out shown here represents system dispatch on the basis of financial fuel prices, given in detail in the Appendix Tables to Annex 4, but roughly 14¢ per million Btu for gas at Mari/Sui, 50¢ per million Btu for gas at Multan and Lyallpur and 36¢ per million Btu for gas in Karachi.

Figure 11 indicates that the Northern Grid load in this particular year was partially met in some months with supplies of power from Upper Sind. The Sukkur steam station and three plants at Mari are dispatched into the Northern Grid in January, March, April, November and December. The plant load factors given for the Mari and Sukkur units in the top block of the print-out refer to the load factor on the share of the Upper Sind capability which was made available to the North in that particular month under the dispatching procedures described in Chapter VII; this share of Mari or Upper Sind capability is shown in the last row of the print-out in Figure 11. Thus the load factors on the Upper Sind plants have to be multiplied by the share of Mari (as well as the capabilities of the plant and 730, the number of hours in the month) in order to secure an indication of the amount of energy supplied to the North from the Mari area in each month.

The next block of information in the print-out concerns the hydro plants. Each hydro plant is presented separately, except that the Eight Small Hydels were treated together as a single plant as described in Annex 6 Appendix 1 and Mangla was considered in two parts, one part on continuous base load and the other part available for peaking if necessary (see Annex 6 and its Appendix 1). TOP MW refers to the upper limit, $\overline{P}_k$, of the block of hydro-plants to which the plant in question was assigned in this particular month for dispatching purposes (see Chapter V). BOTTOM MW is the lower limit of the block, $\underline{P}_k$; both are stated in megawatts. When several plants have been dispatched in the same block, identical values of $\overline{P}_k$ and $\underline{P}_k$ appear for each. Thus, for instance, in January of the year shown in Figure 11, the Mangla A units together with the small Hydels were dispatched in one block, effectively on base load, while the remaining Mangla units (Mangla B) together with the Warsak plant were dispatched in a separate block to meet almost all the remaining load. The Mari units and the Lyallpur plant were used to fill in the gaps - a small portion of base load and a small portion of load intermediate between the parts covered by the two blocks of hydro plants.

TRANS MW denotes the total capacity in megawatts transmitted (sent out) from the particular plant to the Southern Market. TRANS GWH is the accompanying energy in gigawatt-hours (1 gwh = 1,000,000 kwh).

EXCESS HYDRO MW indicates the total hydro capacity which is left unused after the dispatch and transmission routines have been completed. EXCESS HYDRO GWH, denoted as E above, is the total amount of potential hydro energy which cannot be utilized.

DEMAND IN MW is the Northern Market capacity demand $P_{maxT,1}$.

Finally, SHARE OF MARI as discussed above indicates the percentage of the thermal capacity at the Mari generating center which has been dispatched into market #1 during a particular month, $\gamma_{T,1}$. If hydro power is being transmitted south - in May for example - no Mari power can move north, as shown by a zero share for the market and zero plant factors for the Mari units in the thermal plant listing. When the connection between Mari and the North is open for transmission of thermal power - for instance, in January - the ultimate share allocated is the result of the optimizing routine described in Chapter VII.

The print-out reproduced in Figure 12 shows similar information for the operation of the thermal plants used to meet the load of Market No. 2, the Southern Market, and also data on the performance of the transmission system, all stated in megawatts.

NORTH TO MARI is the transmission of hydro capacity as it arrives at Mari, $V_{1,3}$. One may see in June, for example, that the power arriving at Mari is exactly that sent out from the MAN50B hydro plant (as indicated in Figure 11) minus the loss in transmission.

MARI TO KARACHI indicates the power arriving in the Southern Market from Mari, $V_{3,2}$. Part of this power generally originates in the North and part at Mari itself. The capacity of the line from the North to Mari during this year is 170 mw, and that of the line from Mari to the South 250 mw. Thus, in June, for example, about 226 mw arrives in the South, about 162 mw originating in the North (i.e. 170 mw less losses) and 64 mw from the Mari thermal units.

MARI TO LYALLPUR shows the capacity dispatched from Mari to the Northern Market, $V_{3,1}$. In the year shown in the figures dispatch from Mari to the North took place in January, March, April, November and December, as discussed above in reference to the load factor placed on the Mari plants by the Northern Grid load. Again the amount of transmission is shown net of transmission losses.

The last line on Figure 12 presents the overall SYSTEM RESERVE, δ_T . If the two markets are not interconnected, a separate reserve figure is shown for each individual market.

Not reproduced here is a system cost summary which the computer prints out for each year of the planning period. It shows the capital, fuel and maintenance and operation cost incurred in that year under several assumptions about fuel prices, the shadow rate of foreign exchange and the opportunity cost of capital.

The computer print-out can of course be changed with relative ease to indicate either more or less detail, as desired. The information presented in Figures 11 and 12 really represents a small sample of the numbers generated by the computer in the process of dispatching the system. These particular items were selected for printing out because of their value in indicating the essentials of the way the system was operating under any given set of conditions.

PLANT NAMES		MARKET NUMBER 2											
	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	
THERMAL													Ω_{jT}
KAR B	.	.	.07	.	.	.00	.00	.02	.04	.07	.03	.23	
KAR BX	.11	.02	.22	.12	.09	.11	.11	.13	.16	.19	.18	.36	
KAR EL	.	.	.	.	.	.	.	.	.	.	.	.03	
KAR DF	.19	.08	.29	.19	.16	.19	.18	.20	.22	.25	.24	.42	
KAR K1	.66	.55	.75	.64	.59	.62	.60	.62	.62	.65	.65	.83	
KAR K2	.53	.41	.62	.51	.47	.50	.48	.50	.50	.53	.54	.71	
HYD S1	.	.	.01	.	.	.	.	.	.	.01	.	.17	
HYD S2	.03	.	.15	.05	.02	.04	.05	.07	.09	.12	.11	.29	
HYD GT	.	.	.	.	.	.	.	.	.	.	.	.02	
SUKKUR	.00	.	.11	.01	.00	.02	.03	.05	.07	.10	.07	.26	
KOT OF	.	.	.04	.	.	.	.	.	.01	.04	.00	.20	
KOT GT	.	.	.	.	.	.	.	.	.	.	.	.07	
HYDGT2	.	.	.	.	.	.	.	.	.	.	.	.13	
KORA 3	.33	.22	.43	.33	.29	.32	.31	.33	.34	.37	.37	.55	
KAR N1	.90	.90	.90	.90	.86	.87	.86	.87	.89	.89	.90	.90	
MARI 1	.90	.90	.90	.90	.74	.78	.76	.77	.80	.83	.90	.90	
MARI 2	.90	.83	.90	.90	.72	.75	.73	.74	.77	.80	.90	.90	
MARI P	.82	.70	.87	.80	.68	.71	.69	.70	.71	.74	.81	.90	
DEMAND IN MW	596.	588.	603.	621.	640.	652.	654.	665.	677.	692.	675.	680.	$P_{max\,T,2}$
SHARE OF MARI	.46	.44	.33	.50	.14	.14	.14	.14	.22	.21	.56	.31	$Y_{T,2}$

TRANSMISSION SYSTEM MCSPO	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	
NORTH TO MARI	C.	47.	0.	0.	170.	170.	170.	170.	138.	142.	0.	0.	$V_{1,3}$
MARI TO KARACHI	206.	243.	148.	225.	226.	226.	226.	226.	230.	230.	250.	140.	$V_{3,2}$
MARI TO LYALLPUR	146.	0.	205.	138.	0.	0.	0.	0.	0.	0.	81.	207.	$V_{3,1}$
SYSTEM RESERVE	699.	554.	317.	387.	554.	711.	943.	853.	832.	636.	682.	549.	

SYSTEM OPERATION SUMMARY FOR A TYPICAL YEAR - MARKET No. 2 (SOUTH)

WITH SUMMARY OF TRANSMMISSION SYSTEM OPERATION