EXCHANGE RATE RULES, BLACK MARKET PREMIA AND FISCAL DEFICITS
THE BOLIVIAN HYPERINFLATION

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CPD Discussion Paper No. 1986-35
July 1986 (Revised August 1986)

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by

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* We thank Willem Buiter, Daniel Cohen and Sweder van Wijnbergen for helpful comments. Errors and omissions are our own. The views herein are those of the authors, and not necessarily shared by the World Bank or affiliated institutions.
Abstract

Standard explanations of hyperinflation have focused on the impact of a rapid expansion of the money supply required to finance a given real fiscal deficit. There are, however, a continuum of initial price levels and dynamic price paths that would lead to the hyperinflation stationary state, which is completely locally stable. This non-uniqueness property poses conceptual and empirical problems. This paper presents a scenario under which there is a unique saddle-path for prices that leads to the hyperinflationary steady-state. This results from our consideration of an open economy with a dual exchange system: an official system in which foreign exchange is rationed, and a black market rate which freely floats. The latter is supplied by smuggled exports and finances foreign asset accumulation and imports which are rationed out of the official market.

Hyperinflation can arise if the authorities attempt to integrate the black and official foreign exchange markets by systematically devaluing the official rate towards the black market rate. The interaction between the official rate of devaluation and the black market premium determines the dynamics of internal price movements. The model is applied to Bolivia with some success in simultaneously explaining the pattern of inflation, black market rates and domestic money holdings.
I. INTRODUCTION

Recent high inflations in several less developed countries have stimulated renewed interest in the properties of policy rules or structural features that propagate such phenomena. One common factor in Argentina, Brazil and Israel was a high degree of indexation which may have been responsible for removing any "nominal anchor," causing self-generating price increases (Bruno and Fischer (1984), Bruno (1986)). Indexation was not, however, as prevalent in Bolivia which has suffered the worst inflation in recent times. Alternatively, increases in inflation are associated with real shocks such as terms of trade deterioration or increases in the real fiscal deficit. By contrast, the movement from high inflation to hyperinflation in Bolivia in the last quarter of 1984 was not associated with any discernible real shocks. This paper presents an alternative explanation for the Bolivian inflation, based on a policy rule to devalue the official nominal exchange rate according to movements in the black market rate. Surprisingly, this nominal change is shown analytically to have substantial real effects.

Standard explanations of high inflation have focused on the expansion of the money supply required to finance an exogenously given real fiscal deficit (Cagan (1956); Dornbusch and Fischer (1985); Sargent and Wallace (1985)). These studies point to two stationary equilibria at which the deficit is balanced by seignorage. The high inflation stationary equilibrium point, which is on the downward sloping side of the seignorage Laffer curve, is completely locally stable, resulting in a continuum of initial conditions and of dynamic paths to the stationary-state. This poses conceptual and econometric problems. Sargent and Wallace (1985) assume that the dynamic process observed in hyperinflations is a drift towards the high inflation,
stable equilibrium. They show that it is possible to identify and estimate the behavioral parameters of the system, but not the initial price level. Others, such as Dornbusch and Fischer (1985) and van Wijnbergen (1985), focus on the low-inflation stationary equilibrium, which is saddle-point stable under rational or adaptive expectations, and dismiss the other stationary point on the grounds of non-uniqueness of paths leading to it. Hyperinflations, however, eventually far exceed the seignorage-maximizing inflation rate. Our model shows one scenario under which the high-inflation equilibrium may, indeed, be saddle-point stable, resolving the conundrum of uniqueness in high inflation conditions.

The model presented in this paper follows the public finance approach to inflation, in the sense that the fiscal deficit drives the process of money creation. The main innovation is to switch from a closed to an open economy specification. This has several important implications. First, given the institutional climate in LDCs featuring capital controls and official exchange rates that do not duplicate a free float, black markets arise. Foreign exchange is supplied to these markets by export smuggling. Domestic residents can, therefore, hold foreign fiat money, despite administrative controls. These basic ideas have been developed by Lizondo (1984) and Pinto (1985). They follow another strand in the literature, that of currency substitution (for example, Calvo and Rodriguez (1977); Obstfeld (1981); Ramirez-Rojas (1985)), which emphasizes important differences that arise in the specification of domestic money demand and the impact on domestic prices of money supply innovations if individuals can hold foreign currency.

Second, the black market premium gives rise to distortions and rent-seeking activities. It is shown analytically that as the premium rises, real income falls. The determination of the premium is integrated with the infla-
tionary process. The authorities are motivated to trade-off higher inflation against a lower premium through their choice of an official exchange rate crawl. We show that the interaction of the exchange-rate rule with the black market premium critically effects the dynamics of inflation.

Third, the assumption that the official foreign exchange market is rationed implies that there are rents associated with the allocation of foreign exchange. We assume that the government appropriates part of these rents by importing goods at the official exchange rate. This is an implicit tax on exporters which must be properly integrated into the fiscal accounts.

Section II presents the basic model in which the steady-state black market premium, domestic and foreign money balances are determined. In contrast to closed economy models, we find a continuum of stationary inflation rates under the assumption of an exogenous official rate of crawl. The comparative statics and dynamics of this system are derived to demonstrate the uniqueness of the black market premium given rational expectations. The last part of the section considers alternative exchange rate rules motivated by a desire to reduce the black market premium. It shows that exactly two equilibria emerge if the official rate is constantly devalued in proportion to the difference between the black market and official exchange rates. In this system, the high inflation stationary equilibrium could be saddle-point stable, ensuring a unique path for nominal prices.

Section III applies the basic model to the Bolivian high inflation of 1982-1985. Bolivia is an obvious candidate for illustrating the model because of its high inflation experience, the considerable dollarization of the economy that was observed, the presence of "black market" trades which were either legal (between February and November of 1982) or semi-legal (thereafter), the ease and persistence of smuggling across its vast land borders and the substantial internally financed fiscal deficit from 1982 to 1985.
II. THE MODEL

Our basic goal is to examine the inflationary and stability properties of different nominal exchange rate rules in the presence of parallel exchange markets. This is linked to the question of exchange rate unification, an important issue in countries where black market premia are high. There is a presumption in such countries, owing to the relative fixity of the official exchange rate and its lack of responsiveness to domestic-foreign inflation differentials, that the real exchange rate is excessively appreciated. To consider these phenomena, we need at a minimum two goods: a traded good and a non-traded good.

The idea of an official exchange rate can be motivated by the inessential simplification that the government spends only on imported goods, buying foreign exchange from the private sector for the purpose at an arbitrary rate. The existence of a fiscal deficit then enables us to examine simultaneously the inflation tax and the tax implicit in the official exchange rate.

Official net foreign assets do not play a role in our story: the countries to which this model would be applicable have depleted reserves and protect them through foreign exchange rationing. We assume quite simply that there is no official net foreign asset accumulation. This "hand-to-mouth" assumption is justified by realism as well as the short-run macro-stabilization—two to three years—focus of this paper.

The model below emphasizes the asset view of exchange rates in a dual market context. There is currency substitution, with residents holding two assets, domestic money and foreign money. The dynamics of the former are governed by the financing of the fiscal deficit, and the latter by the current account.
Consider a small country producing two goods: an exported good with given international price $p_X$, and a home (nontraded) good with price $p_H$. The home good is produced using domestic labor and imported inputs. Expenditure by domestic private residents is entirely on the home good, while government expenditure is exclusively on imported goods. Government imports are at the official exchange rate $e$ (domestic price of foreign currency). Private citizens are expected to surrender their export earnings to the Central Bank for domestic currency at the official rate and are not permitted to hold foreign exchange, but do so illegally. There is an unofficial parallel market where the exchange rate is $b > e$, with smuggled exports feeding into the current account for illegal asset accumulation. The parallel exchange market is a "freely" floating asset-type market admitting both capital and current transactions.

The official foreign exchange market is rationed as follows: a certain amount of surrendered export earnings is set aside for government imports and the rest is administratively allocated to the private sector, being sold at the official exchange rate, $e$. Official reserves $R$ (in foreign currency terms) are thus protected and remain constant, i.e., $R = 0$. This form of administrative allocation is equivalent to a set of taxes and offsetting subsidies within the private sector, or from the private sector to the government. All imports are consequently valued at the parallel market rate, $b$, which is the true marginal cost of foreign exchange. It is assumed that $e$ depreciates at an exogenously given rate $e/e = \hat{e} > 0$.

**Goods and Labor Market**

There is a fixed endowment of labor, $\bar{L}$, and no capital. Home goods $(H)$ are produced according to a Cobb-Douglas technology using imported inputs $(I)$ and labor $(L_1)$. The price of imports $p_I$ is normalized to unity. Exports
are produced according to a simple constant-returns-to-scale technology with $X_2$ being smuggled out and $X_3$ being declared, i.e., going through official channels. $L_2$ and $L_3$ are the amounts of labor devoted to $X_2$ and $X_3$ respectively. The private sector maximizes profits subject to technological, endowment and non-negativity constraints. It solves the following problem, where $w$ is the (domestic currency) wage and $C(X_2)$ is a strictly convex function representing the costs of smuggling exhibiting $C(0) = 0$, and $C', C'' > 0$:

\[
\begin{align*}
(1) \quad \text{max} & \quad p_H H + b p_x X_2 + ep_x X_3 - C(X_2) - w (L_1 + L_2 + L_3) - b I \\
\{I, L_1, L_2, L_3\} \\
\text{s.t.} & \quad \Lambda \leq 1 - \alpha \Lambda_1 \\
& \quad X_2 \leq L_2 \\
& \quad X_3 \leq L_3 \\
& \quad L_1 + L_2 + L_3 \leq \bar{L} \\
& \quad I \geq 0 \\
& \quad L_i \geq 0 \quad (i=1, 2, 3).
\end{align*}
\]

The first-order conditions for an interior solution are:

\[
\begin{align*}
(2) & \quad (1-\alpha) p_H H = b I \\
(3) & \quad \alpha p_H H = w L_1 \\
(4) & \quad b p_x = C'(X_2) = w \\
(5) & \quad ep_x = w.
\end{align*}
\]

**Balance of Payments and Real Income**

The balance of payments is given by:

\[
(6) \quad p_x (X_2 + X_3) = I + g + F,
\]

where $g$ is government imports (fixed in terms of imported goods) and $F$ is net foreign asset accumulation by the private sector, since by construction that
by the Central Bank is zero. Using $X_2 = L_2$, $X_3 = L_3$ and substituting (2) and (3) into (6) gives:

$$L_1 = \frac{\alpha \phi (p_x \bar{L} - \bar{F} - g)}{p_x (1 - \alpha + \alpha \phi)} \text{ and}$$

$$I = \frac{(1 - \alpha) (p_x \bar{L} - \bar{F} - g)}{1 - \alpha + \alpha \phi}$$

where $\phi \equiv b/e$ is the black market premium (actually the premium plus 1.0).

Using (7) and (8) with either (2) or (3) now yields one of our central equations:

$$p_H = \frac{1}{\alpha^2 (1 - \alpha)^{1-\alpha}} e^{\phi^{1-\alpha} - p_x^{1-\alpha}},$$

or the domestic inflation equation (for given $p_x$):

$$\pi = \hat{\pi} = \hat{e} + (1 - \alpha) \hat{\phi} = \alpha \hat{e} + (1 - \alpha) \hat{b},$$

where "\(^\wedge\)" denotes proportionate change. Thus, domestic inflation is equal to a weighted average of the official and unofficial rates of depreciation with the weights dependent upon the share of imports in home goods.

**PROPOSITION 0:** In the steady-state, real income (income expressed in foreign currency at the black market rate) falls as the premium rises.

**Proof:** Let $y$ represent real income. Since income is the sum of factor payments, we get: $y = wL_1/b + p_x(X_2 + X_3)$. Substituting for $w$ and $L_1$ from (5) and (7) respectively, the steady-state expression for $y$ can be reduced to $y = p_x \bar{L} - (\phi - 1) \alpha (p_x \bar{L} - g)/(1-\alpha+\alpha\phi)$. Straightforward differentiation yields $dy/d\phi < 0$. Q.E.D.
Portfolio Balance and Foreign Asset Accumulation

Domestic residents hold two non-interest bearing assets: domestic money, M, and foreign money, F, the latter illegally as mentioned above. Continuous asset-market clearing and perfect foresight give the portfolio balance condition:

\[ M = \frac{\lambda(b/b)}{1 - \lambda(b/b)} \cdot bF, \quad \lambda'(\ast) < 0, \]

where \( \lambda \) is the fraction of total wealth \( W = M + bF \) held as domestic money and \( (b/b) \), the rate of depreciation in the black market, is the differential rate of return. Private nominal spending is a fixed fraction of nominal wealth:

\[ P_{HH} = aW = a(M + bF). \]

Define \( m = M/e \). Using (2), (8) and (12) gives the dynamic equation

\[ F = \frac{pL}{1 - g - a(m + F)(1 - \alpha + \phi)}. \]

Deficit Finance and the Money Supply

In contrast to the closed-economy literature, which fixes the deficit in terms of home goods, we assume that government expenditure \( g \) and taxes \( t \) are fixed in terms of imported goods. The deficit, \( (g - t) \), is financed by domestic credit, \( D \). This gives the dynamic equation for domestic money, \( M \):

\[ \dot{M} = (eR) + D = e(g - t). \]

In accordance with standard practice, it is implicitly assumed in (14) that changes in the domestic currency value of official reserves are not monetized. Equation (14) also reflects the rationing of foreign exchange whereby \( R = 0 \).

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Dynamics of the System

Equations (11), (13) and (14) are the three dynamic equations of the system. Recalling that \( m \equiv M/e \) and \( \phi \equiv b/e \), they can be rewritten as:

\[
(15) \quad m = \frac{\lambda(\hat{\phi} + \hat{a})}{1 - \lambda(\hat{\phi} + \hat{e})} \cdot \phi F
\]

\[
(16) \quad F = \frac{p x L - g - a(m + F)(1 - \alpha + a\phi)}{\phi}
\]

\[
(17) \quad m = (g - t) - \hat{m}
\]

where equation (13) has been reproduced as equation (16) for convenience.

**PROPOSITION 1:** For any given \( \hat{e} > 0 \), the system represented by (15) - (17) has a unique and saddle-point stable steady-state solution.

**Proof:** See Appendix I.

The steady-state solution (corresponding to \( (\phi, F, m) = (0, 0, 0) \)) based on (15) - (17) is:

\[
(18) \quad \phi^* = \frac{a(1-a) \cdot (g-t)}{\lambda(\hat{e}) \cdot \hat{e} (p x L - g) - a\alpha(g-t)}
\]

\[
(19) \quad F^* = \frac{(1 - \lambda \cdot \hat{e})}{a(1-a) \lambda(\hat{e}) \cdot \hat{e}} \cdot \left[ \lambda(\hat{e}) \cdot \hat{e} (p x L - g) - a\alpha(g-t) \right]
\]

\[
(20) \quad m^* = \frac{(g-t)/\hat{e}}{\hat{e}}
\]

Clearly, in the steady-state \( \pi \equiv \hat{p}_H = \hat{e} = \hat{b} \) (see (10)), i.e., domestic inflation equals the official rate of depreciation for given terms of trade, i.e., constant \( p x^* \). It is immediate from (20) that in the steady state, the
deficit \((g-t)\) is financed by the inflation tax, \(m^* \cdot \hat{e}\).

It is clear from (17) that \(\hat{e} = 0\) will always have a positive solution as long as \(\hat{e} > 0\). We define an inconsistency between fiscal and exchange rate policy as a situation characterized by \((g-t) > 0\) and \(\hat{e} = 0\), i.e., the government runs a deficit but refuses to devalue (crawl). Since by (14), domestic money is increasing, the black market rate will continue to depreciate to maintain portfolio balance so that domestic inflation 
\[ \pi = (1-a) \hat{b} \] (see (10)). Further, since \(e\) is fixed, it follows that \(\hat{\phi} = \hat{b} > 0\), i.e., there is no steady-state solution for the premium or \(m\). As the premium rises without bound, the incentive for avoiding official channels increases. Government imports as well as the administrative allocation regime rely on officially surrendered export receipts. With \(\hat{\phi}\) increasing without bound, such a situation is unsustainable. Ultimately government spending will have to be cut, the exchange regime changed, or both. We shall have no more to say about this case except that the end result is likely to be chaotic. 2/

Henceforth, we rule out the case \(\hat{e} = 0\) and assume \(\hat{e} > 0\).

Comparative Statics

Consider the following comparative static expressions:

\[
(21) \quad \frac{d\phi^*}{dg} = a(1-a) \cdot \frac{D + (g-t)(\lambda(\hat{e}) \hat{e} + a\hat{a})}{D^2} > 0
\]

\[
(22) \quad \frac{d\phi^*}{dp_x} = -a(1-a)(g-t) \cdot \frac{\lambda(\hat{e}) \cdot \hat{e} \hat{L}}{D^2} < 0
\]

\[
(23) \quad \frac{d\phi^*}{d\hat{e}} = a(1-a)(g-t) \cdot \frac{\lambda(\hat{e})(p_x \hat{L} - g)(n-1)}{D^2} > 0.
\]

In (21) – (23), \(D\) stands for the denominator on the RHS of eq. (18). The
signs of the comparative static expressions \( \frac{d\phi^*}{dg} \) and \( \frac{d\phi^*}{dp_x} \) are intuitively obvious: according to (21), an increase in \( g \) increases the premium. This is because the dynamics of \( M \) are affected. With \( m^* \) going up, a rise in \( \phi \) is necessary to restore portfolio balance. By (22), a TOT improvement, through its positive effect on the dynamics of foreign asset accumulation, lowers the steady-state premium.

In contrast, \( \frac{d\phi^*}{de} \) is of ambiguous sign. By (23), \( \text{sgn} \frac{d\phi^*}{de} = \text{sgn} \left( \eta - 1 \right) \), where \( \eta \equiv \lambda'(\hat{e}) \cdot \hat{e} / \lambda(\hat{e}) \) is the partial elasticity of domestic money balances \( M \) with respect to \( \hat{e} \). If \( \eta > 1 \), an increase in the rate of crawl actually raises the value of the steady-state premium instead of creating a unifying trend. If \( \eta < 1 \), however, the opposite is true, i.e., an increase in \( \hat{e} \) lowers \( \phi^* \). The intuition is as follows: when \( \eta > 1 \), a rise in \( \hat{e} \) on impact lowers the revenue from the inflation tax, which since portfolio balance must always hold, is given by \( \lambda(\hat{e})(m^* + \phi^*) \cdot \hat{e} \). In steady-state, however, the inflation tax always equals \( (g - t) \). Since \( m^* \) is going to be lower in the new steady-state, one way of raising the inflation tax to \( (g - t) \) is through a rise in \( \phi \) and \( F \).

Lastly, a once-and-for-all devaluation of \( \hat{e} \), in keeping with the standard results in the literature, e.g., Dornbusch et al. (1983), Lizondo (1984), Macedo (1982), has only temporary effects and does not affect the steady-state values of \( (\phi, F, m^*) \).

**Dynamics of the \((2 \times 2)\) system in \((\phi, F)\)**

The \((2 \times 2)\) system formed by (15) and (16) can be used to analyze the dynamics of adjustment to a terms of trade shock (change in \( p_x \)) effectively treating \( m \) as exogenous, i.e., for given \( (g-t) \) and \( \hat{e} \). This is valuable for gaining insights into the dynamic behavior of the premium and
domestic inflation following, say, a deterioration in the terms of trade.

**PROPOSITION 2:** (a) In $\phi - F$ space, the curve ($F = 0$) is either positively sloped or negatively sloped. When negatively sloped, it is steeper than the ($\phi = 0$) curve, which is unambiguously negatively sloped.

(b) The (2 x 2) system formed by (15) and (16) is saddle-point stable. The saddle path is negatively sloped and flatter than the ($\phi = 0$) curve.

**Proof:** See Appendix I.

The steady-state solution ($\phi^*, F^*$) continues to be unique by **PROPOSITION 1.** The ($F = 0$) curve is negatively (positively) sloped according as $\lambda(\hat{e})/(1-\lambda(\hat{e})) \cdot \phi$ is less than (greater than) $\phi'(1-\alpha)$. This condition amounts to comparing the relative shares of domestic and foreign money in domestic private wealth with those of labor and imports in home goods. The presumption is that at high inflation rates, ($F = 0$) will be negatively sloped, since both $\phi$ and the share of $F$ in nominal wealth are likely to be high. The dynamics we are interested in, however, are not influenced by whether ($F = 0$) is negatively or positively sloped. Fig. 1 summarizes the dynamics, exploiting **PROPOSITION 2(b).**

In Fig. 1, $E$ is the original equilibrium, and $SS$ the saddle path. The solid line $\phi = 0$ consists of pairs of $\phi$ and $F$ that ensure current account balance. The line $\phi = 0$ represents portfolio balance and is a hyperbola because from (15) it is clear that in steady state, $\phi \cdot F$ is a constant.

Consider a TOT deterioration, i.e., a once-and-for-all fall in $p_x$ that is unanticipated. The curve $\phi = 0$ shifts upwards to the dotted line $\phi = 0$, with $S'S'$ representing the new saddle path. The premium immediately jumps from
Fig. 1: Dynamics in \((\phi-F)\) Space
\( \phi^* \) to \( \phi_1 \) and then adjusts along \( S'S' \) to the new steady state, \( \phi^{**} \) corresponding to \( E_1 \). From (9), it is evident that there will be a once-and-for-all change in the CPI, \( p_H \) corresponding to the reduction in \( p_x \) and jump of \( \phi \) from \( \phi^* \) to \( \phi_1 \). As \( \phi \) increases from \( \phi_1 \) to \( \phi^{**} \) along \( S'S' \), however, it is clear that \( b > \hat{\epsilon} \). Eq. (10) then implies that during this adjustment, \( \hat{\epsilon} < \pi < b \), i.e., domestic inflation exceeds official exchange depreciation.

This would be interpreted in common parlance as an official real exchange rate appreciation.

**Stability of Exchange Rate Rules and Inflation**

It has been shown above that for any exogenously chosen rate of crawl \( \hat{\epsilon} > 0 \), a unique and saddle-point stable solution exists for the model under consideration. It was also demonstrated in PROPOSITION 0 that as the black market premium rises, real GDP falls. This suggests the possibility of a tradeoff between the choice of the rate of inflation (\( \hat{\epsilon} \)) and the level of the premium, implicit in eq. (23). According to eq. (23), an increase in the rate of crawl when the demand for domestic money is elastic raises the steady-state premium. Assuming that the inflation elasticity of domestic money balances \( \eta \) rises with the rate of inflation yields an inflation tax Laffer curve, i.e., the unit inflation tax \( \lambda(\hat{\epsilon}) \cdot \hat{\epsilon} \) is a concave function of \( \hat{\epsilon} \) with a (global) maximum when \( \eta = 1 \). This in turn implies that the steady-state premium \( \phi^* \) given in eq. (18) is a convex function of \( \hat{\epsilon} \) with a (global) minimum, which is shown as the u-shaped graph labelled \( \phi^*(\hat{\epsilon}) \) in Fig. 2.

The nature of the tradeoff is now evident: to the left of \( B \) in Fig. 2, a rise in \( \hat{\epsilon} \) lowers \( \phi^* \) and raises real GDP. The price paid for more real income is higher inflation. To the right of \( B \), however, the relationship between \( \hat{\epsilon} \) and \( \phi^* \) is monotonically increasing.

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Fig. 2: Equilibria with premium-based rule
The magnitude of the premium is usually taken as a rough indicator of the degree of overvaluation of the official nominal exchange rate. In countries where the black market rate is much higher than the official rate, discussion often centers around the notion of an "equilibrium" rate, which it is generally argued is in the interval \((e, b)\). Given this reasoning plus PROPOSITION 0, standard advice is to devalue towards the black market rate in the hope of reducing the premium and unifying the exchange rate. Suppose the equilibrium exchange rate \(e^*\) is considered to be given by

\[ e^* = \theta b + (1-\theta)e, \theta \in (0, 1) \]

Then the above is tantamount to saying

\[ \dot{e} = e^* - e = \theta(b - e) \]

Since \(e^*\) is clearly variable, continuous updating is equivalent to the rule:

\[ \dot{e} = \theta(\phi - 1). \]

To examine the dynamics and steady-state properties of the new system, we plug (24) into the system formed by (15), (16) and (17), effectively substituting out \(e\), which is now endogenous. It is shown graphically in Fig. 2 that there are at most two equilibria of which at least one is a high inflation-high elasticity-high premium equilibrium. The line RR with intercept of 1 and slope \((1/e) > 1\) graphs the exchange rate rule in eq. (24) inverted to give

\[ \phi = 1 + \hat{e}/\theta \]

Fig. 2 has been drawn for the case where there are two equilibria, a low inflation equilibrium corresponding to A and a high inflation equilibrium corresponding to C. Between 0 and B on the horizontal axis, \(0 < \eta \leq 1\), while beyond B, \(\eta \geq 1\). At B, \(\eta = 1\). The value of \(\hat{e}\) corresponding to B maximizes the revenue from the inflation tax. From Fig. 2, it is easy to imagine that there are no equilibria (\(\phi(\hat{e})\) is completely above RR), that there is a unique high inflation equilibrium (RR is tangent to \(\phi(\hat{e})\) at some point beyond B) or that there are two high inflation equilibria (RR...
intersects $\phi^*(\hat{e})$ twice to the right of $B$. Abstracting from existence
problems, we have:

**PROPOSITION 3:** Consider the system formed by eqs. (15) - (17) and the policy
rule (24). When $\eta_{e}(1, 1 + \delta_1(\phi))$, where $0 < \delta_1(\phi) < 1$, high-inflation is a
saddle-point stable equilibrium. For $\eta$ to the right of this interval, high
inflation is either completely (locally) stable or unstable. When
$\eta_{e}(0, \delta_2(\phi))$, where $0 < \delta_2(\phi) < 1$, low inflation is either saddle-point
stable or unstable. For $\eta_{e}(\delta_2(\phi), 1)$, low inflation is saddle-point stable.

**Proof:** See Appendix I.

Essentially (see Fig. 2), choosing $\theta$ fixes the two equilibria of
interest, $E_1$ and $E_2$. Their stability properties are determined by computing
$\eta$ at $E_1$ and $E_2$ and comparing these with the cutoff values $\delta_2(\phi)$ and
$(1 + \delta_1(\phi))$, respectively. What is interesting is that the switch to a
plausible and seemingly innocuous exchange rate rule eq. (24) affects the
nature of the equilibrium (multiple equilibria) and its stability properties
in a fundamental way: a purely nominal change has large real effects.

The discussion hereafter is motivated by a desire to capture the
actual situation as it seems to have evolved in Bolivia. We have so far
assumed that the real deficit $(g - t)$ is fixed. In times of high inflation,
which is the focus of this paper, it is usually easier to control nominal
variables. Typically, people postpone paying taxes so that the real deficit
grows. One would expect the real deficit to be positively associated with the
premium. We therefore consider an alternative scenario: together with the
switch to the nominal exchange rate rule in eq. (24), the government attempts
to keep domestic money constant in terms of home goods, effectively targeting
seignorage. This implies that $\hat{M} = p_H = \hat{e} + (1 - \alpha)\hat{\phi}$ by (10), i.e., nominal

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money grows at the rate of inflation. \( \frac{5}{t} \) Since \( \dot{M} = e(g - t) \), it follows that
the real deficit must adjust passively. Under the new regime, therefore, we
have \( e(g - t) = \hat{p}_H \cdot M \), so that \( (g - t) = \hat{\pi} + (1 - \alpha) \hat{\phi} \), where \( \pi \equiv \hat{p}_H \). The
growth of the real deficit is thus related to the growth in premium and
inflation. Since \( m = M/e \), we get:
\( \dot{m} = (1 - \alpha) \hat{\phi} \cdot m \).

Our new dynamic system, which we denote S, consists of (15), (16), (25) and
the policy rule (24) which can be used to substitute out \( \hat{e} \).

**PROPOSITION 4:** When the demand for domestic money is elastic, i.e., \( \eta > 1 \),
the system formed by S has the property that high inflation is a saddle-point
stable equilibrium.

**Proof:** See Appendix I.
III. THE BOLIVIAN HYPERINFLATION

The model is applied to Bolivia over the period August 1980 to June 1985. We restrict ourselves to post-August 1980 because the Central Bank suspended convertibility and introduced foreign exchange rationing in that month. In August 1985, a stabilization package was implemented that has successfully eliminated high inflation. The main thrust of the argument is that the high inflation during 1982-85 was a consequence of a large fiscal deficit exacerbated by a nominal exchange rate rule that sought to catch up with the black market rate. The economy reached the inefficient, high inflation equilibrium depicted in Fig. 2 as point C.

The argument is developed below in three stages. First, we present an overview of the data and argue that it does not conform to the patterns expected under a standard, monetary, closed economy approach. Second, we estimate the domestic portfolio balance equation (15). This indicates that the economy may have entered the elastic money demand region and was kept there by exchange rate policy. Third, we estimate the reduced form expression for the black market premium (18) to demonstrate the importance of the key exogenous variables—namely government spending, the terms of trade and real income—in determining the evolution of the black market premium and inflation.

We should emphasize at the outset, however, that this paper is not on the econometrics of hyperinflation. There are several problems associated with multiple equilibria (see Sargent and Wallace (1985)), with specification of the dynamic lag structure and with the seasonality of the data, particularly the expectation of large increases in government spending in December as
year-end bonuses fall due (see Saracoglu and Sargent (1978)) that are left for future work.

Overview of the Data

Table 1 summarizes key fiscal and monetary indicators over 1980-1985.2. The rapid increase in net domestic credit to the government coincided with a sharp decline in external funds for financing the fiscal deficits (lines 1. and 2.). After improving vastly in 1981 and 1982, the terms of trade stayed relatively flat (line 3.). Real GDP declined monotonically, while the black market premium rose (lines 4. and 5.). Official exchange depreciation, which was modest prior to 1981, picked up in 1982 and accelerated rapidly thereafter. Nevertheless (line 6.) it tended to be erratic, unlike the continuous float of the black market (line 7.), where depreciation grew rapidly. Official depreciation appeared to be playing "catch up," as suggested by the quarterly numbers for 1984 and 1985. Inflation (line 8.) appeared to mimic depreciation in the black market, while base money (line 9.) grew at a rapid rate, undoubtedly influencing black market depreciation via portfolio balance.

During the early 1980s, consequently, Bolivia typified an inflationary environment caused by monetary emission to accommodate high fiscal deficits. Quarterly inflation rates rose from levels around 6 percent through 1981 to a new plateau of 50 percent between 1982 and 1984 before reaching record levels in 1985. First-quarter price increases of 496 percent in 1985 correspond to an annualized inflation rate of 126,000 percent. There did not seem to have been, however, any steady tendency for inflation to accelerate over the period, as one would expect on the basis of standard monetary arguments (Fig. 3). Rather, inflation seems to have settled at equilibrium plateaus over several quarters, jumping from one plateau to the next almost instant
Table 1: SELECTED FISCAL AND MONETARY INDICATORS, 1980-1985.2

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. A Net Dom. Credit to Gov't./GDP(%)</td>
<td>4.1</td>
<td>2.2</td>
<td>17.6</td>
<td>12.5</td>
<td>13.2</td>
<td>0.24</td>
<td>-2.34</td>
<td>1.70</td>
<td>13.57</td>
</tr>
<tr>
<td>2. Portion of Deficit Externally Financed (%)</td>
<td>53.7</td>
<td>55.9</td>
<td>5.0</td>
<td>0.4</td>
<td>0.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Terms of Trade</td>
<td>102.5</td>
<td>120.1</td>
<td>134.7</td>
<td>145.4</td>
<td>131.4</td>
<td>131.21</td>
<td>129.82</td>
<td>130.48</td>
<td>131.39</td>
</tr>
<tr>
<td>4. Real GDP (billion 1980 pesos)</td>
<td>128.6</td>
<td>127.4</td>
<td>116.3</td>
<td>107.4</td>
<td>103.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Black Market Premium</td>
<td>1.20</td>
<td>1.38</td>
<td>2.26</td>
<td>2.81</td>
<td>3.58</td>
<td>5.60</td>
<td>1.63</td>
<td>6.50</td>
<td>2.60</td>
</tr>
<tr>
<td>6. Official Exchange Depreciation (%)</td>
<td>20.19</td>
<td>0.00</td>
<td>160.34</td>
<td>260.10</td>
<td>907.00</td>
<td>0.00</td>
<td>300.00</td>
<td>0.00</td>
<td>350.00</td>
</tr>
<tr>
<td>7. Black Market Exchange Depreciation (%)</td>
<td>25.45</td>
<td>15.08</td>
<td>326.00</td>
<td>348.55</td>
<td>1,181.21</td>
<td>125.10</td>
<td>16.07</td>
<td>299.91</td>
<td>79.90</td>
</tr>
<tr>
<td>8. Inflation</td>
<td>47.23</td>
<td>32.14</td>
<td>123.54</td>
<td>275.59</td>
<td>1,281.40</td>
<td>63.41</td>
<td>148.88</td>
<td>66.12</td>
<td>236.91</td>
</tr>
<tr>
<td>9. Growth Rate of Base Money (%)</td>
<td>40.20</td>
<td>17.83</td>
<td>293.64</td>
<td>205.73</td>
<td>1,616.59</td>
<td>28.12</td>
<td>80.02</td>
<td>107.99</td>
<td>257.83</td>
</tr>
</tbody>
</table>

Sources and Notes: International Financial Statistics (IFS), IMF; World Currency Yearbook (black market rates up to 1981); and World Bank Staff estimates in conjunction with the Central Bank of Bolivia. Net Domestic Credit to the Government is line 32 on IFS; Base Money is line 14, IFS (Reserve Money); and (5) is defined as the ratio of the black market to the official exchange rate.
Fig. 3: Quarterly Inflation, 1980-1985

Fig. 4: Black Market Exchange Premium, 1980-1985

Fig. 5: Real Monetary Base, 1980-1985 (billion 1980 pesos)

Fig. 6: Quarterly Seigniorage, 1980-1985 (billion 1980 pesos)
taneously. This has also been observed in other countries (e.g. Bruno (1986)).

Another characteristic of the period 1982-85 is the substantial black market premium defined as the ratio of the black market to official exchange rate (Fig. 4). There was considerable volatility in the premium reflecting the fact that the official exchange rate was only periodically devalued. Nevertheless, it is clear that the average premium rose after the first quarter of 1982 when the civilian government of Siles Zuazo took power. The premium rose further during the three hyperinflationary quarters in 1985.

The real money base (defined as Reserve Money, line 14, IFS, deflated by the CPI) shows a marked decline after the second quarter of 1983 (Fig. 5). By 1985, the real money stock was only about one-fifth of its peak level. This was not, however, associated with any decline in seignorage (defined as the change in Reserve Money deflated by the CPI) (Fig. 6). In fact, seignorage fluctuates around a fairly constant level from late 1982 to the middle of 1984, growing somewhat during the hyperinflation. This suggests that there were no systematic increases in the real government deficit, and that the rapid growth in base money remarked upon in Table 1 was to compensate for the decline in the base of the inflation tax.

This overview suggests that there are two important changes in the stationary inflation equilibrium to be accounted for. The first is the jump after 1981; the second is the hyperinflation beginning in the last quarter of 1984. We now turn to empirical estimates that bear on these developments.

Estimation Results

Prior to November 1982, Bolivians were permitted to hold dollar-denominated time deposits in domestic banks. Accordingly, statistics are available on a monthly basis for domestic and foreign assets, as well as other
pertinent variables, whose definitions and source are shown in Appendix II. These data permit estimation of portfolio choice behavior (eq. 15). We write
the share of money in wealth,

\[ \lambda = \lambda_0 e^{-\gamma \hat{b}} \]

recalling that \( \hat{b} \equiv \dot{b}/b \) is the rate of depreciation of the peso in the black market. Combining (27) with (15), we derive an estimating equation

\[ \text{log} \left( \frac{m}{m + 4F} \right) = \text{log} \left( \frac{\lambda_0}{\lambda} \right) - \gamma \hat{b}_t + u_t \]

The left-hand side of (28), PORT, is the logarithm of domestic money balances in Bolivian pesos divided by wealth in pesos. The conversion of foreign currency accounts into pesos is at the black market exchange rate. In moving from continuous time to discrete time, we posit that individuals base their portfolio decisions on current and future expected changes in the black market rate, since this is the relevant differential rate of return. Eq. (28) is then run with a partial adjustment mechanism. The results are shown in Table 2 as Regression 1. Changes in the black market exchange rate (BHAT) clearly have a strong negative impact on domestic money holdings. From the point estimates, the partial elasticity of money demand with respect to domestic inflation can be calculated: \( \eta = 3.68 \) times the inflation rate. 6/

Thus, the steady-state inflation rate at which \( \eta = 1 \) is 27 percent per month. Extrapolating these coefficients to the later period (for which \( \eta \) cannot be computed as data on dollar asset holdings do not exist post-1982), we conclude that the economy was at an inefficient equilibrium after the third quarter of 1984, when monthly inflation rates averaged 44 percent. 7/
Table 2: REGRESSION RESULTS
(Standard errors in parentheses)

<table>
<thead>
<tr>
<th>Regression</th>
<th>Dep. Var.</th>
<th>Independent Variables</th>
<th>Sample period</th>
<th>$R^2$</th>
<th>D.W.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Portfolio</td>
<td>PORT</td>
<td>C, BHAT (+1), BHAT, PORT(-1)</td>
<td>80.08-82.06</td>
<td>0.980</td>
<td>2.36</td>
</tr>
<tr>
<td>choice</td>
<td></td>
<td></td>
<td></td>
<td>(-0.034* -0.253* -0.302* 0.849*)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(.013) (.039) (.045) (.048)</td>
<td></td>
</tr>
<tr>
<td>2. Premium</td>
<td>PHI</td>
<td>C, EHAT, EHAT2, Y, G(1), TOT, LAMBDA, $\rho_1$</td>
<td>80.08-82.06</td>
<td>0.900</td>
<td>1.05</td>
</tr>
<tr>
<td>(a) OLS</td>
<td></td>
<td></td>
<td></td>
<td>4.387* -9.131* 17.014* -0.010 -1.260 0.207 -2.581* 0.548</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.630) (1.449) (2.899) (.015) (1.559) (0.373) (0.940) (0.441)</td>
<td></td>
</tr>
<tr>
<td>(b) TSLS</td>
<td></td>
<td></td>
<td></td>
<td>-2.476 -9.374* 17.464* 0.025 -2.009 2.283 -2.562 0.605</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(22.429) (2.694) (5.348) (0.119) (3.897) (6.971) (1.614) (0.575)</td>
<td></td>
</tr>
</tbody>
</table>

* Indicates significance at 95% level.

a/ Estimated first-order serial correlation (Cochrane-Orcutt procedure).
Table 3 presents computed values of the inflation elasticity of domestic money demand ($\eta$) and the share of domestic money in total wealth ($\lambda$) based on the estimated relationship in Regression 1, Table 2. The actual inflation rate is also reported. The time period is broken up into three subperiods, 1980-81; 1982-84.09 and 1984.10-85.06, corresponding to the inflation plateaus mentioned earlier. Interestingly, $\eta$ grew steadily from about 0.07 to 1.64, while $\lambda$ decreased from 76 percent of total wealth to a mere 24 percent by the second quarter of 1985. Not surprisingly, this coincided with an increase in monthly inflation from 1.83 percent in 1980-81 to 14.37 percent before reaching 44.40 percent in the third quarter of 1984.

Table 3: COMPUTED ELASTICITY AND PORTFOLIO SHARES OF DOMESTIC MONEY DEMAND, 1980-1985.06 (Monthly Averages)

<table>
<thead>
<tr>
<th></th>
<th>1980-81</th>
<th>1982-84.09</th>
<th>1984.10-85.06</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Elasticity ($\eta$)</td>
<td>0.0673</td>
<td>0.5293</td>
<td>1.6353</td>
</tr>
<tr>
<td>2. Portfolio Share ($\lambda$)</td>
<td>0.7581</td>
<td>0.4881</td>
<td>0.2436</td>
</tr>
<tr>
<td>Actual Inflation (%)</td>
<td>1.83</td>
<td>14.37</td>
<td>44.40</td>
</tr>
</tbody>
</table>

Notes: $\eta$ and $\lambda$ are forecast from the estimated equation for PORT presented in Table 2 as Regression 1.
The model predicts that the equilibrium value of the premium (PHI) will be affected by changes in the official exchange rate (EHAT), real income (Y), the government deficit (G), the terms of trade (TOT) and domestic money demand share in portfolio preferences (LAMBDA) (eq. 18). One problem in estimating this equation is that observed values for the premium may not correspond to equilibrium values. This suggests that the error term is likely to be serially correlated. Accordingly, the model using monthly data is estimated with a Cochrane-Orchutt correction for first-order serial correlation. The sample period stops in mid-1982 because of the absence of data for the domestic money demand portfolio share preference, λ, in later periods. The change in the official exchange rate is included as a quadratic to capture the shape of the function depicted in Fig. 1. OLS regression results are shown in Table 2 as Regression 2.(a). The premium is negatively affected by a change in the official exchange rate at low levels, but positively affected by high devaluations. Higher demand for domestic money is also significantly associated with a lower premium as expected. The other variables are insignificant, but they display little monthly variance. For our purposes, the premium-minimizing devaluation rate is crucial to establish at which of the two possible stationary equilibria the economy found itself. This also corresponds to the devaluation rate at which \( \eta = 1 \) (see eq. (23)). This can be computed from the coefficients on EHAT and EHAT2: it takes a value equal to 27 percent, exactly the same as that estimated from the portfolio balance equation shown in Regression 1. 9/

One potential source of bias in the EHAT coefficients is simultaneity with the premium. Accordingly, the model was re-estimated using instrumental variables, with the lagged values of the dependent and independent variables as instruments. Although the standard errors of the coefficients
change markedly, the point estimates do not. Importantly, the coefficients on E\(HAT\) and E\(HAT^2\) remain significant and do not change in magnitude, as can be seen from Regression 2.(b) in Table 2.

Based on these results, we may interpret the Bolivian experience as follows. The economy in 1980 and 1981 was experiencing moderate inflation, required to finance the government deficit. This performance was assisted by favorable external factors: substantial loans from the Argentine government to Garcia Meza which helped finance the budget deficit; improving terms of trade as a result of the expansion in natural gas exports which more than offset sluggish tin prices; and stable levels of real output. Each of these factors was to change for the worse in 1982, with the exception of the terms of trade, which stayed relatively flat. Foreign loans dried up and real output fell by about 10 percent. In terms of Fig. 2, the \(*$\) curve shifted up and the authorities devalued the nominal exchange rate more rapidly to reduce the adverse impact on exports. Because the economy was initially at the low inflation equilibrium point (less than 27 percent monthly inflation) the new stationary equilibrium was characterized by higher inflation, a larger black market premium and higher seignorage to fill the hole in the government deficit caused by the cutoff of external finance.

Inspection of the change from this equilibrium to hyperinflation in the fourth quarter of 1984, however, is not associated with similar changes in these exogenous variables. The terms of trade, real government spending and output were all essentially flat. We surmise that at the end of 1984, the conditions underlying PROPOSITION 4 were fulfilled in Bolivia, creating a favorable environment for the jump to hyperinflation.

Table 4 presents data on official nominal exchange depreciation, the black market premium, seignorage and the growth rate of nominal money over
Table 4: EXCHANGE DEPRECIATION, SEIGNORAGE AND THE PREMIUM, 1980-1985.06 (Monthly Averages)

<table>
<thead>
<tr>
<th>Period</th>
<th>Exchange Depreciation (%)</th>
<th>Seignorage (billion 1980 pesos)</th>
<th>Base Money Growth (%)</th>
<th>Premium (%)</th>
<th>Inflation (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980-81</td>
<td>0.00</td>
<td>0.2320</td>
<td>2.08</td>
<td>1.29</td>
<td>1.83</td>
</tr>
<tr>
<td>1982-84.09</td>
<td>13.34</td>
<td>0.9808</td>
<td>12.29</td>
<td>2.95</td>
<td>14.37</td>
</tr>
<tr>
<td>1984.09-85.06</td>
<td>40.27</td>
<td>1.3341</td>
<td>37.99</td>
<td>4.53</td>
<td>44.40</td>
</tr>
<tr>
<td>1985 July</td>
<td>0.00</td>
<td>164.5045</td>
<td>442.39</td>
<td>10.68</td>
<td>50.86</td>
</tr>
</tbody>
</table>

Notes: Seignorage is the change in Base Money, line 14, IFS (Reserve Money) deflated by the CPI.

1980-85.06. Monthly averages are presented for the three sub-periods as demarcated earlier in Table 3. Exchange depreciation increased dramatically, as did the premium. So did the growth rate of nominal money, from 12 percent over 1982-84.09, to 38 percent over 1984.10-85.06. Inflation kept pace, attaining levels greater than the estimated seignorage maximizing rate of 27 percent. Surprisingly, seignorage grew, suggesting that the deficit in terms of home goods grew. While the jump in seignorage from 1980-81 to 1982-84.09 can be attributed to the cutoff of external finance, the increase from 1982-84.09 to 1984.10-85.06 is consistent with the assumptions underlying PROPOSITION 4. 10/

Changes in the nominal exchange rate rule appeared to have clinched the jump to hyperinflation. Towards the end of 1984, Siles Zuazo announced early elections. A new economic team was also installed, which actively sought to arrive at a debt rescheduling agreement with commercial creditors. Part of the reform package presented to creditors was a restructuring of the
exchange rate. With the premium having more than doubled between 1980-81 and 1982-84.09, there was considerable pressure on the government to move towards exchange rate unification. Policy discussions revolved around moving the official exchange rate, $e$, towards an equilibrium exchange rate, $e^*$, calculated as a weighted average of the black market and official rate, with weights equal to the share of exports in each market. Letting $\theta$ denote the parallel market share, this gives $e^* - e = \theta b + (1 - \theta)e - e = \theta (b - e)$, which is reminiscent of the exchange rate rule in eq. (24). As the dynamics in PROPOSITION 4 suggest, with this system in place, the economy may have jumped to the saddle-point stable high inflation equilibrium, where it remained until the stabilization program of September 1985 was introduced.

Examination of the monthly time series re-inforces this point of view. Monthly inflation jumped abruptly from 14 percent in August 1984 to 32 percent in September, despite the fact that the nominal exchange rate was fixed from June to November 1984 and nominal base money was growing at modest levels. Thereafter, average inflation remained in the elastic money demand region. This large upward shift in inflation (and the black market premium) can be rationalized as higher inflation now owing to higher anticipated inflation: with the pressure on the government to unify exchange rates, additional seignorage would be required to compensate for the lost "tax" revenues implicit in the overvalued official rate unless government spending were credibly reduced.

It is worth mentioning that July 1985 appears to signal a disillusionment with the new policies: with the exchange rate fixed, nominal base money was increased by 442 percent, raising seignorage to 164.50 billion 1980 pesos and causing the premium to exceed 1000 percent!
IV. CONCLUDING REMARKS

This paper explores the Bolivian high inflation of the 1980s. In common with other work on hyperinflation, it suggests that the Bolivian economy had moved into an inefficient, high inflation equilibrium. In contrast to other work, however, we show that there could be a unique dynamic path that leads to this stationary equilibrium. This finding reflects the open-economy specification of our model. We introduce a black market for foreign exchange which is driven by individuals' demands for foreign assets to hedge against domestic inflation. The authorities, however, are unwilling to allow too great a premium to develop between the black market and official exchange rates because of the subsequent distortions and real income losses. They assign nominal exchange rate policy to adjust towards the black market rate. Such advice is also commonly proffered by international institutions and external creditors. But nominal exchange rate devaluations also have a role to play in determining the evolution of domestic inflation and, hence, the seigniorage required to finance real government deficits. It is this dual function that leads to the possibility of two stationary equilibria, with the inefficient, high inflation stationary equilibrium being saddle-point stable.

At the high inflation equilibrium, the black market premium and inflation are positively correlated. We find evidence that increasing the rate of devaluation of the exchange rate, in an effort to narrow the premium perhaps, has the perverse effect of widening the gap between the black market and official rates and accelerating inflation. This appears to be the major explanatory factor behind the jump in inflation in late 1984 and early 1985.

There are important lessons for policy coordination and sequencing that follow from this analysis. First, despite the fact that inflation is
driven by the fiscal deficit, reductions in the real deficit will not bring down inflation unless coordinated with changes in exchange rate policy. Second, exchange rate policy on its own (as in a pre-announced "tablita" for example) cannot bring down inflation beyond a certain point (27 percent per month for Bolivia) without having harmful effects on the real economy, inducing smuggling and other side-effects. This suggests interestingly enough that the achievement of exchange rate unification (minimizing the black market premium) through discretionary rules is a tricky matter. To a certain point, the premium can be reduced by arbitrarily slowing down official exchange depreciation; but as Fig. 2 clearly shows, beyond this point, the premium rises once again owing to the u-shaped relationship. This raises the issue, left for future research, about whether the only way to achieve unification is by moving overnight to a float.
Footnotes

1. When $\alpha = 0$, (18) - (20) reduce to the expressions for $\phi^*$, $F^*$ and $m^*$ in Pinto (1985), eqs. (8) - (10). Notice that $\alpha = 0$ amounts to assuming that like the government, expenditure by domestic residents is exclusively on imports, which is the assumption in Pinto (1985).

2. It is argued in Pinto (1985) that this was the case in Ghana.

3. This is essentially Lizondo's (1984) result. Note that since $M$ has unitary elasticity w.r.t. $W$, $\eta$ is also the elasticity of $m$ w.r.t. $e$.

4. $p_H$ could fall or rise depending upon $\alpha$ and the relative magnitudes of the change in $p_X$ and the initial jump of $\phi$ from $\phi^*$ to $\phi_1$ in Fig. 3.

5. Such a rule would be suggested by a standard money demand function with real income constant.

6. This formula yields a steady-state inflation elasticity for domestic money balances based on Regression 1, Table 2 ($3.68 = \frac{0.253 + 0.302}{1 - 0.84}$). Note that $\beta$ equals inflation in the steady-state.

7. This high inflation equilibrium is inefficient precisely because the inflation elasticity of domestic money exceeds unity, implying that the seignorage maximizing rate is exceeded. A lower inflation rate could support the same deficit.

8. Suppose the demand for domestic money shifted downwards as a result of the fall in real income associated with the rising premium. Then the estimates of $\eta$ presented in Table 3 would underreport the inflation elasticity of domestic money demand, erring on the side of conservatism.

9. Writing $\phi = \alpha e + \beta e^2 + \ldots$, yields as FOC, $d\phi/de = \alpha + 2\beta e = 0$. Solving, we get $e^* = -\alpha/2\beta$, which is the formula used.

[pintro10bh]
10. In terms of the analytical model, seignorage in home goods,

\[ H \equiv \frac{\hat{p}}{p_h} = e \left( q - t \right) / p_h. \]

So \[ H = e + (g - t) - p_h = (g - t) - (1 - \alpha) \phi. \]

Since \( \hat{H} > 0 \) and \( \phi > 0 \), \( (g - t) > 0 \), i.e., the deficit in terms of imported goods grew even faster.
Appendix I

Proof of PROPOSITION 1

Consider the system formed by equations (15) - (17). We first establish uniqueness of the steady-state solution, i.e., the solution corresponding to $(\dot{\phi}, \dot{F}, \dot{m}) = (0,0,0)$. Equation (17) immediately yields a solution for $m^*$. Substituting out $\phi$ in (16) using (15) gives a linear equation in and hence unique solution for $F$. Since by (15), $\phi \cdot F$ must be constant in the steady state, the uniqueness of $\phi$ follows. The steady-state solution $(\phi^*, F^*, m^*)$ is thus unique. To establish saddle-point stability of the steady-state solution, consider the linearization around the steady state:

\[
\begin{align*}
\dot{\phi} &= \phi_{\phi} \dot{\phi} + \phi_{F} \dot{F} + \phi_{m} \dot{m} \\
\dot{F} &= \phi_{\phi} \dot{\phi} + \phi_{F} \dot{F} + \phi_{m} \dot{m} \\
\dot{m} &= \phi_{\phi} \dot{\phi} + \phi_{F} \dot{F} + \phi_{m} \dot{m} \\
\end{align*}
\]

or $x = A \cdot y$, where $x' = (\dot{\phi}, \dot{F}, \dot{m})$, etc., and the partial derivatives in $A$ are evaluated at $(\phi^*, F^*, m^*)$. Since there are two pre-determined variables, $m$ and $F$, saddle-point stability requires that $A$ have two negative eigen values and one positive one. The expressions for the partial derivatives in $A$ are as follows (the argument of $\lambda$ has been omitted for convenience):

\[
\begin{align*}
\phi_{\phi} &= \frac{-\lambda(1 - \lambda)}{\lambda'}, \\
\phi_{F} &= \frac{-\lambda(1 - \lambda)}{\lambda'} \cdot \frac{\phi}{F}, \\
\phi_{m} &= \frac{(1 - \lambda)^2}{\lambda'} \\
\end{align*}
\]

\[
\begin{align*}
\hat{\phi} &= aF(1-\alpha)[\frac{\lambda}{(1 - \lambda)} \phi - \frac{\alpha}{1 - \alpha}] \\
\hat{F} &= -a(1 - \alpha + \alpha\phi) \\
\hat{m} &= -\frac{a}{\phi} (1 - \alpha + \alpha\phi) \\
\end{align*}
\]

\[
\begin{align*}
\dot{m} &= 0, \\
\dot{F} &= 0, \\
\dot{m} &= -e. \\
\end{align*}
\]
Note that $|A| = -\hat{e} (\phi_F \cdot F_F - \phi_F \cdot F_F)$, and that one eigen value of $A$ is $-\hat{e} < 0$. The product of the other two eigen values is the $(2 \times 2)$ determinant $$(\phi_F \cdot F_F - \phi_F \cdot F_F) \equiv \Delta$. It can be shown that $\Delta = \lambda a(1 - \alpha)/\lambda' < 0$. Therefore, exactly one of the remaining eigen values is negative, and one positive.

Proof of PROPOSITION 2

First, note from (A2) that $F_F > 0$ as $\lambda/(1 - \lambda) \cdot \phi > \alpha/(1 - \alpha)$.

Since $\frac{d\phi}{dF} | \hat{f} = 0 = -\frac{F_F}{F} \hat{\phi}$ and $\frac{\hat{F}}{F} < 0$ it follows that sgn $\frac{d\phi}{dF} | \hat{f} = 0 = \text{sgn} \frac{\hat{F}}{F}$.

Now $\frac{d\phi}{dF} | \hat{f} = 0 = -\frac{\hat{F}}{F}$, we shall show that when $\hat{F} < 0$, i.e., the curve $(\hat{F} = 0)$ is negatively sloped in $(\phi - F)$ space, it is steeper than the curve $(\hat{F} = 0)$. Since

$$(A3) \quad x_1 \equiv \frac{d\phi}{dF} | \hat{f} = 0 = \frac{1 - \alpha + \alpha \hat{F}}{F(1 - \alpha) \left[ \frac{\lambda}{(1 - \lambda)\hat{F}} - \frac{\alpha}{1 - \alpha} \right]}$$

and

$$(A4) \quad x_2 \equiv \frac{d\phi}{dF} | \hat{f} = 0 = -\frac{\hat{F}}{F},$$

we need to show that when $0 > x_1$, $|x_1| > |x_2|$. Directly,

$$|x_1| - |x_2| = \frac{1}{F} \cdot \frac{1}{(1 - \lambda) \left[ \frac{\alpha}{1 - \alpha} - \frac{\lambda}{(1 - \lambda)\hat{F}} \right]} > 0. \text{ Q.E.D.}$$
We now show that the saddle path in \((\phi - F)\) space is negatively sloped and flatter than the \((\phi = 0)\) curve. Consider the \((2 \times 2)\) system (linearized around the steady state):

\[
\begin{align*}
\frac{\dot{\phi}}{\check{\phi}} &= \check{\phi} \cdot \frac{F}{\check{F}} - \frac{\phi}{\check{F}} \cdot \check{F}, \text{ or} \\
Z &= B \cdot q,
\end{align*}
\]

\(Z' = (\check{\phi}, \check{F})\), etc., and the partial derivatives in \(B\) are given in equations (A2). Since \(\text{det. } B = \Delta = (\check{\phi} \cdot \check{F} - \check{F} \cdot \check{\phi}) = \lambda a(1 - \alpha)/\lambda' < 0\), it follows that the \((2 \times 2)\) system is saddle-point stable. Let \(\mu\) be the negative eigen value of \(B\) and \((r, 1)\)' the (normalized) eigen vector corresponding to \(\mu\). Along the saddle path, \(\phi = r F\). We consequently need to show that 

\[
|r| < |x_2|,
\]

where \(x_2\) is defined in (A4). Define \(\theta = \lambda(1-\lambda)/\lambda' < 0\). It is straightforward to show that

\[
(A6) \quad r = -\frac{\theta}{\theta + \mu} \cdot \frac{\phi}{F} < 0
\]

Consequently, \(|r| < \phi/F = |x_2|\). Q.E.D.

Proof of Proposition 3

Consider the system formed by eqs. (15) - (17) and the policy rule (24). Linearizing around the steady-state solution yields a system analogous to (A1). With the exception of \(\check{\phi}, \check{m}_\phi\) and \(\check{m}_m\), the partial derivatives in (A2) remain unchanged.
\[
\dot{\phi} = -\frac{\lambda (1 - \phi)}{\lambda} - \theta \phi
\]

(A7) \[\dot{m} = - m \theta\]

\[\dot{\hat{e}} = - \hat{e} = - \theta (\phi - 1)\]

Using the definition of \(\eta = -\lambda \hat{e}/\lambda\) and noting that \(\hat{e} = \theta (\phi - 1)\), it is straightforward to show that \(\eta > 1\) implies that \(\dot{\phi} < 0\) in (A7). By obtaining expressions for \(|A|\) and Trace (A), it can be shown that \(|A|<0\rightarrow \eta>1+\delta_1(\phi)\) and that Trace (A) \(<0\rightarrow \delta_2(\phi)\), with:

\[\delta_1(\phi) = \frac{(1-\alpha)(\phi-1)/((1-\alpha+\alpha)\phi)}{\phi}\]

(A8)

\[\delta_2(\phi) = \frac{(1-\lambda)\theta(\phi-1)/(\theta(\phi-1)+\theta\phi+\alpha(1-\alpha+\alpha))}{\phi}\]

From (A8), it is clear that \(0 < \delta_1(\phi), \delta_2(\phi) < 1\). The proof is completed by noting that saddle-point stability requires 2 negative eigenvalues and 1 positive one and noting from Fig. 2 that \(\eta > (\eta)1\) corresponds to a high (low) inflation equilibrium.

**Proof of PROPOSITION 4**

Consider the system represented by S, namely, eqs. (15), (16) and (25), and the policy rule (24). Linearizing around the steady-state solution yields a set of equations analogous to (A1). The partial derivatives in (A2) remain unchanged, with the exception of \(\dot{\phi}, \dot{m}, \) and \(\dot{\hat{e}}\):

[pinto10bh]
\[ \dot{\phi} = -\frac{\lambda (1 - \lambda)}{\lambda'} - \theta \phi, \quad < 0 \text{ when } \eta > 1 \]

\[(A9) \quad \dot{m} = (1 - \alpha) \frac{m}{\phi}, \quad < 0 \text{ when } \eta > 1 \]

\[ \dot{m} = (1 - \alpha) \dot{\phi} = 0 \text{ in steady state.} \]

Substituting (A9) for the relevant expressions in (A2), it is straightforward to show that \( |A| = \frac{m}{\phi} (1 - \lambda) \alpha (1 - \alpha + \alpha \phi)/\lambda' F > 0 \) when \( \eta > 1 \). In other words, there is at least one positive eigenvalue, and the other two have the same sign when \( \eta > 1 \). The trace of \( A \) is given by

\[ (-\lambda (1 - \lambda)/\lambda' - \theta \phi - \alpha (1 - \alpha + \alpha \phi)). \]

Using the definition of \( \hat{\epsilon} = -\lambda' \hat{\epsilon}/\lambda \), and noting that \( \hat{\epsilon} = \theta (\phi - 1) \), this can be reduced to:

\[(A10) \quad \text{Trace } (A) = \theta (\phi - 1) \left\{ \frac{1 - \lambda}{\eta} - \frac{\theta \phi}{\theta(\phi - 1)} - \frac{\alpha (1 - \alpha + \alpha \phi)}{\theta(\phi - 1)} \right\}. \]

Since \( \lambda \in (0, 1) \), it follows from (A10) that \( \eta > 1 \) is sufficient for trace \((A) < 0 \). Therefore, since \( |A| > 0 \) and Trace \((A) < 0 \), \( A \) has exactly one positive eigenvalue and two negative ones when \( \eta > 1 \). Since there are two predetermined variables \( m \) and \( F \) and one jump variable \( \phi \), \( \eta > 1 \) consequently guarantees saddle-point stability. All we need to complete the proof is to note from Fig. 2 that \( \eta > 1 \) corresponds to a high-inflation equilibrium. Q.E.D.
### Appendix II

#### Data Definitions and Sources

<table>
<thead>
<tr>
<th>Basic Data</th>
<th>Symbol</th>
<th>Last Period Available</th>
<th>Frequency</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross domestic product in current prices</td>
<td>GDP</td>
<td>1984</td>
<td>Annual</td>
<td>IFS line 99B</td>
</tr>
<tr>
<td>Gross domestic product in constant 1980 prices</td>
<td>Y</td>
<td>1984</td>
<td>Annual</td>
<td>IFS line 99 Bp</td>
</tr>
<tr>
<td>Net claims on government (current pesos, end of period)</td>
<td>DCG</td>
<td>85.06</td>
<td>Monthly</td>
<td>IFS line 32 an</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>CPI</td>
<td>85.12</td>
<td>Monthly</td>
<td>IFS</td>
</tr>
<tr>
<td>Official exchange rate</td>
<td>XR</td>
<td>85.12</td>
<td>Monthly</td>
<td>Pre-1982 World Currency Yearbook</td>
</tr>
<tr>
<td>Black market exchange rate</td>
<td>B</td>
<td>85.12</td>
<td>Monthly</td>
<td>1982-1985 Banco Central de Bolivia</td>
</tr>
<tr>
<td>Reserve money</td>
<td>RM</td>
<td>85.12</td>
<td>Monthly</td>
<td>IFS line 14</td>
</tr>
<tr>
<td>Foreign currency deposits in commercial banks 1/</td>
<td>F</td>
<td>82.10</td>
<td>Monthly</td>
<td>IFS line 25 b divided by XR.</td>
</tr>
<tr>
<td>(pesos converted to dollars at the official exchange rate)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terms of trade 2/</td>
<td>TOT</td>
<td>85.06</td>
<td>Monthly</td>
<td>Banco Central de Bolivia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Calculated Variables</th>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent change in official exchange rate</td>
<td>EHAT</td>
<td>log (XR/XR(-1))</td>
</tr>
<tr>
<td>Percent change squared</td>
<td>EHAT2</td>
<td>EHAT^2</td>
</tr>
<tr>
<td>Percent change in black market rate</td>
<td>BHAT</td>
<td>log (B/B(-1))</td>
</tr>
<tr>
<td>Real government deficit</td>
<td>G</td>
<td>(DCG-DCG(-1))/GDP</td>
</tr>
<tr>
<td>Black market premium</td>
<td>PHI</td>
<td>B/XR</td>
</tr>
<tr>
<td>Log of portfolio share of domestic money</td>
<td>PORT</td>
<td>log (RM/(RM + B.F))</td>
</tr>
</tbody>
</table>

1/ For obtaining foreign currency held in dollars F in commercial banks, the pesos are converted at the official rate; but for expressing wealth in domestic currency terms, F is converted at the (relevant) black market rate.

2/ The terms of trade is computed as a monthly weighted average of the price of tin and gas exports taken from Central Bank Monthly Bulletins, with weights equal to the relative value shares in exports for each year. This is deflated by the monthly U.S. wholesale Price Index.
References


