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Cost-Benefit Analysis: Evaluation Criteria (Or: "Stay away from the IRR")

While only measures properly reflecting net societal benefits should be used to rank policies, several other measures also attempting to summarize the value of net flows are often presented and sometimes misused—i.e., the benefit-cost ratio, the overall rate of return, and the internal rate of return. This brief discusses why only net present value should be used to rank policy alternatives.

"It is very easy to define benefit-cost analysis: simply add all the gains from a policy alternative, substract all the losses, and choose the alternative that maximizes net benefits." (Edward Gramlich, 1990; page 8.)

1. Criteria for Project Evaluation

Let $\{b_t\}_{t=0}^{T}$ represent the stream of all benefits associated with the project under consideration, let $\{c_t\}_{t=1}^{T}$ be the stream all costs, and let r be the discount rate.[1]

Then the net present value of the project, $\mathrm{NPV}(r)$, is given by:

$$\mathrm{NPV}(r) = \sum_{t=0}^{T} \frac{b_t - c_t}{(1+r)^t} = \sum_{t=0}^{T} (1+r)^{-t} b_t - \sum_{t=0}^{T} (1+r)^{-t} c_t \qquad (1)$$

If all benefits and costs are adequately accounted for, and r is the appropriate social rate of discount, then if $\mathrm{NPV}(r) > 0$, the project is inequivocally beneficial—as its undertaking will increase the country's net worth.

Equation (1) shall be our guiding reference, and any other way to summarize a project's desirability must be consistent with it. Let's reiterate that the fundamental principle of CBA is simply: "In any choice situation, select the alternative that produces the greatest net benefit" (Stokey and Zeckhayser, 1978; page 137).

There are several other measures that are often used to summarise project characteristics. However, as it will be shown below, these measures do not necessarily rank projects in a consistent way with NPV, which is the economically correct method for ordering projects. Consequently, while these measures may appeal to intuition, they cannot generally guide economic decisions—since they can be easily mathematically manipulated.

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[1] In the context of irreversible investment under substantial uncertainty, it is important that the opportunity costs reflects the opportunity cost of exercising the option to invest, since by carrying out the project the government gives up the possibility of waiting for new information (Dixit and Pindyck, 1994).

2. Other Summary Measures

BENEFIT-COST RATIO (B/C). It follows, from (1), that if NPV is positive, discounted benefits must exceed discounted costs, so the ratio of these quantities must exceed one:

$$\text{NPV}(r) > 0 \Leftrightarrow \text{B/C} = \frac{\sum_{t=0}^{T}(1+r)^{-t}b_t}{\sum_{t=0}^{T}(1+r)^{-t}c_t} > 1 \tag{2}$$

Therefore, a benefit-cost ratio, B/C, larger than one must be equivalent to a positive NPV. Equivalently:

$$\text{NPV}(r) > 0 \Leftrightarrow \frac{\text{NPV}(r)}{\text{PV of Costs}} = (\text{B/C}) - 1 > 0 \tag{3}$$

Which gives a measure of total return on costs, all defined in present value terms.

OVERALL RATE OF RETURN (ORR). Multiply equation (2) by $(1+r)^T$, so that in the numerator we have now the flow of benefits measured at T (rather than at 0), $(1+r)^T \sum_{t=0}^{T}(1+r)^{-t}b_t$. Then we can use the ratio of future-valued benefits to present-valued costs to compute an overall rate of return:

$$\text{NPV}(r) > 0 \Leftrightarrow \underbrace{\left\{ \left(\frac{\sum_{t=0}^{T}(1+r)^{T-t}b_t}{\sum_{t=0}^{T}(1+r)^{-t}c_t} \right)^{1/T} - 1 \right\}}_{\text{ORR}} > r \tag{4}$$

As mentioned, the term in braces is called the overall rate of return of the project, ORR, and it must exceed r whenever NPV(r) is positive.

INTERNAL RATE OF RETURN (IRR). Finally, the (often misleading) internal rate of return, IRR, is calculated by equating the NPV in (1) to zero, and solving for the discount rate. Thus, solving NPV(γ) = 0 for γ:[2]

$$\sum_{t=0}^{T} \frac{b_t - c_t}{(1+\gamma)^t} = 0 \Rightarrow \text{IRR} = \gamma \tag{5}$$

If the costs are frontloaded and the benefits are backloaded, then NPV(r) > 0 $\Leftrightarrow$ IRR > r.

In summary:

$$\text{NPV}(r) > 0 \Leftrightarrow \text{B/C} > 1 \Leftrightarrow \text{ORR} > r \Leftrightarrow \text{IRR} > r. \tag{6}$$

*Unfortunately, the chain of implications in (6) does not guarantee consistent **rankings** of the different measures with* NPV*, as it will shown below. That is, the ordering of projects by* NPV *does not necessarily correspond to the ordering by any of these other criteria—*B/C*,* ORR*, and* IRR*—because these can be easily manipulated without changing* NPV*.*

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[2] This equation, however, may have more than one root. Define $x = (1+\gamma)^{-1} \in (0, 1)$, and $\alpha_i = (b_i - c_i)$, then, equation (5) can be expressed as: $\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_T x^T = 0$. The Descartes' Rule of Signs can be used to determine the maximum number of roots with positive real parts in this polynomial equation. Look at the signs of the α_i's, and count the times the sign changes from a coefficient to the next—*i.e.*, from α_i to α_{i+1}—ignoring the zeroes. This number is an upper bound for the number of roots with positive real part.

3. A Taste of the Problem

> "The IRR should be avoided as the decision criterion." (UK Green book; Item 6.9 on page 39)

> "Managers at one large industrial company approved 23 major capital projects over five years on the basis of IRRs that averaged 77 percent. Recently, however, when we conducted an analysis with the reinvestment rate adjusted to the company's cost of capital, the true average return fell to just 16 percent. The order of the most attractive projects also changed considerably. The top-ranked project based on IRR dropped to the tenth-most-attractive project. Most striking, the company's highest-rated projects—showing IRRs of 800, 150, and 130 percent—dropped to just 15, 23, and 22 percent, respectively, once a realistic reinvestment rate was considered." (Kelleher and MacCormack, 2004)

The IRR corresponds to the annual equivalent return on a given investment only when the project generates no interim cashflows. Otherwise, the implicit assumption that interim cashflows are reinvested earning γ can drive the IRR to over-optimistic territory when γ exceeds the true reinvestment rate—this problem is exacerbated the earlier the interim cashflows arise. The problem with the IRR, and the other measures in this subsection, is that they can all be manipulated easily—*i.e.*, they can be changed to any value without affecting the NPV:

Suppose that the real rate of return is r, and solving (5) we obtain IRR $> r$. Then, we can 'fabricate' NPV-equivalent projects with practically any IRR. Simply add $\$x > 0$ at t and $-\$(1+r)^s x$ at $t+s$. If $s > 0$ then the IRR goes up, while if $s < 0$, the IRR goes down. Adjusting x and s, practically any IRR can be achieved.[3] Below we show a numerical example of this.

3.1. Numerical Example

Table 1 shows hypothetical cashflows for six projects (negative entries are in parentheses). Consider first the four leftmost projects A–D. Among them, the best project is C with an NPV of $18.3. The NPV-ranking is: $C > B > D > A$.

All the other criteria are consistent with the NPV ranking except for the IRR criterion which would rank project B (with lower NPV of $15) ahead of C, and A also misleadingly ahead of D, thus: $B > C > A > D$.

Consider now the last two columns representing variants of projects C and D. The only difference is the addition of entries $\$x$ and $-\$1.06x$ one period apart (for $x = \pm 100$), which obviously have no effect on the NPV—since they cancel each other when discounted at the assumed $r = 6\%$.

However, now project D' has the largest IRR (an oustanding 269%!), while project C'—which is still the best along C—has the second-to-the-lowest IRR. The ranking is: $D' > B > C > A > C' > D$. Note that while D and D' are NPV-equivalent, they are now at the extremes of the ranking sequence!

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[3] If IRR $< r$, then simply make $\$x < 0$ instead, and the rest applies equally. Note that similar mathematical manipulations can be made to the other measures in this section. Since they are ratios, they will be altered by the addition of these two terms.

Table 1. Example 1—Evaluation Criteria ($r = 6\%$)

time	A	B	C	D	C'	D'
0	($10.0)	($10.0)	($10.0)	($20.0)	($10.0)	($20.0)
1	$5.0	$9.0	$0.0	$0.0	($100.0)	$100.0
2	($9.0)	($5.0)	$0.0	$0.0	$106.0	($106.0)
3	$25.0	$25.0	$33.7	$33.7	$33.7	$33.7
NPV(6%)	$7.7	$15.0	$18.3	$8.3	$18.3	$8.3
B/C	1.4	2.0	2.8	1.4	1.2	1.1
ORR	19%	34%	50%	19%	12%	9%
IRR	30%	58%	50%	19%	20%	269%

Note: Negative entries in parentheses.

Moreover, the rankings of neither B/C or ORR agree now with the NPV criterion—the NPV-neutral modifications have depressed both criteria, B/C and ORR, for C' and D' relative to C and D.

4. Why is the IRR so misleading?

Consider a project that has some benefits early on, at the expense of a cleanup cost that must be incurred when closing the facility at the end of its life (*e.g.*, a landfill would fit this profile). In the simplest case, assume an income of b_0 at $t = 0$, and expenditure of c_T at $t = T$. The IRR is the value of γ that solves:

$$b_0 - \frac{c_T}{(1+\gamma)^T} = 0$$

or

$$b_0(1+\gamma)^T = c_T \quad \Longrightarrow \quad \gamma = \left(\frac{c_T}{b_0}\right)^{\frac{1}{T}} - 1$$

While it is obvious that increasing cost should never make a project more attractive, here by increasing the final cleanup cost, the IRR improves! The root of the problem is that since b_0 is assumed to produce returns at a rate equal to the IRR $= \gamma$, then by simply increasing the IRR, a bad project with some early positive cashflow can turn 'good.'

In general, as shown earlier in equation (5):

$$\text{NPV}(\gamma) = 0 \quad \Rightarrow \quad \sum_{t=0}^{T} \frac{b_t - c_t}{(1+\gamma)^t} = 0$$

multiplying by $(1+\gamma)^T$, we get:

$$\sum_{t=0}^{T} (b_t - c_t)(1+\gamma)^{T-t} = 0$$

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so any positive cashflow is assumed to return γ. In general, by increasing the IRR we make any early positive cashflow look very attractive, and costs incurred later on look insignificant. Compare columns D and D' on Table 1.

4.1. Another Numerical Example

Consider a project with the cashflow shown on Table 2, and NPV(r) represented in Figure 1.

Table 2. Example 2—Computing IRR and NPV

time	Cashflow	Discounted at:	
		$r = 5\%$	$r = 15\%$
0	$10.00	$10.00	$10.00
1	($105.00)	($100.00)	($91.30)
2	$110.25	$100.00	$83.36
3	($115.76)	($100.00)	($76.12)
4	$121.55	$100.00	$69.50
5	($127.63)	($100.00)	($63.45)
6	$134.01	$100.00	$57.94
NPV(r)		$10.00	($10.08)

Note: Negative entries in parentheses.

If the appropriate discount rate is, say, 15%, the NPV is negative and the project should not be undertaken (Table 2). Solving for the IRR, we obtain two real roots, $\gamma_1 = 9\%$ and $\gamma_2 = 845\%$. The first root, $\gamma_1 < 15\%$ correctly hints to us about the negative *sign* of the NPV(15%). However, the second root is rather misleading. Figure 1 shows the graph of IRR(γ) against γ—the problem with the second root is better understood by noticing the *positive* slope of NPV(γ) at γ_2. It's quite odd that increasing the opportunity cost of capital would make a project more attractive.
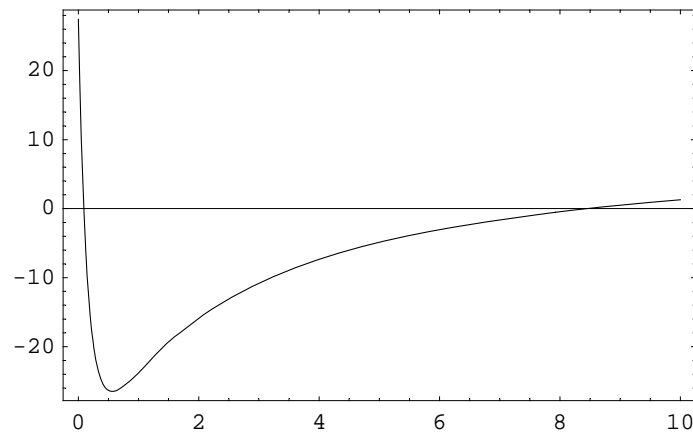

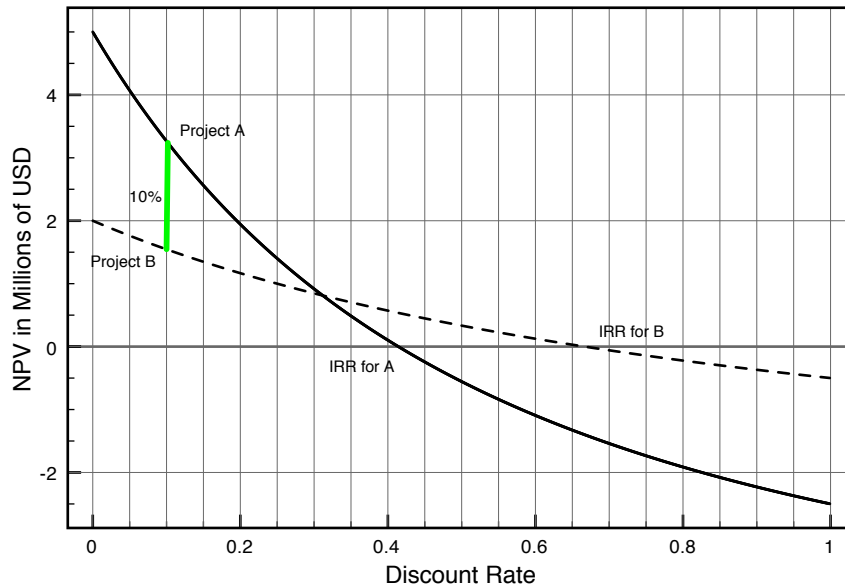**Fig. 1.** Example 2—Plot of NPV(r) of cashflow in Table 2.

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Fig. 2. Example 3—$\text{NPV}_A(r) > \text{NPV}_B(r)$ for $r < 30\%$, but $\text{IRR}_A < \text{IRR}_B$.

Note that getting a reasonable IRR which correctly identifies the *sign* of the NPV(r)—for sensible values of r—is not enough for ranking projects. Welfare depends on NPV, not IRR—which is a mathematical concept, not an economic one. Consider Figure 2 where we have two alternative projects. Project A has a substantially higher NPV for a discount rate in the relevant range (*i.e.*, for any r less than 30%), yet it crosses the zero axis to the left of project B—*i.e.*, the $\text{IRR}_A = 40\% <$ $\text{IRR}_B = 65\%$ despite $\text{NPV}_A(r)$ exceeding $\text{NPV}_B(r)$ for all $r < 30\%$—*e.g.*, by about \$1.6 million for a discount rate in a neighbourhood of 10%.

5. Conclusion

Yet again: In any choice situation, select the alternative that produces the **greatest net benefit**. That unavoidably requires computing NPV(r) for an appropriate discount rate r.

6. References

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