Do Intensity Targets Control Uncertainty Better than Quotas?

Conditions, Calibrations, and Caveats

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Abstract

Among policy instruments to control future greenhouse gases emissions, well-calibrated general intensity targets are known to lead to lower uncertainty on the amount of abatement than emissions quotas (Jotzo and Pezzey, 2004). This paper tests whether this result holds in a broader framework, and whether it applies to other policy-relevant variables as well. To do so, we provide a general representation of the uncertainty on future GDP, future business-as-usual emissions and future abatement costs, and derive the variances of four variables, namely (effective) emissions, abatement effort, marginal abatement costs and total abatement costs over GDP under a quota, a linear (LIT) and a general intensity target (GIT)—where for the latter the emissions ceiling is a power-law function of GDP. We confirm that GITs can always yield a lower variance than a quota for marginal costs, but find that this is not true for total costs over GDP. Using economic and emissions scenarios and forecast errors of past projections, we estimate ranges of values for our model parameters. We find that quotas dominate LITs over most of this range, that calibrating GITs over this wide range is difficult, and that GITs would yield only modest reductions in uncertainty relative to quotas.

JEL Classification: Q28, Q54, D81
Keywords: climate change, intensity target, post-Kyoto, quota, uncertainty


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1 Introduction

With the entry into force of the Kyoto Protocol, the design of the climate regime beyond 2012 is now a central issue on the international negotiation agenda. A lively debate has emerged on the form that the future regime should take to facilitate the participation of the U.S. and, in the medium term, of major emitters among developing countries (see, e.g., Bodansky, 2004, and Aldy et al., 2003, for recent reviews).

The ‘acceptability’ of a regime obviously depends on the global mitigation effort that is requested, and on the way this effort is shared among parties. In the case of climate change, however, this problem is compounded by the fact that decisions are made “in a sea of uncertainty” (Lave, 1991): future business-as-usual (BAU) emissions are very uncertain, and so are current—let alone future—costs of mitigating greenhouse gas (GHG) emissions.

No regime will eliminate uncertainty altogether. However, different instruments (e.g., caps, coordinated taxes or intensity targets) will allocate uncertainty very differently among the key variables that parties focus on when negotiating future climate policies. For example, a quota system implies with near certainty that a predetermined level of emissions is not exceeded, but the costs of abatement (whether measured at the margin, globally, or as a share of GDP) are very uncertain. Conversely, a coordinated tax system would provide certainty as to the marginal cost of abatement, but would leave ex post emissions or total costs uncertain.

The objective of this paper is to examine how different policy instruments distribute uncertainty to key variables for decision-makers, including marginal costs of abatement (the price of carbon), total costs of abatement, and effective emissions, when future GDP, BAU emissions and marginal abatement costs are uncertain. We focus on two main instruments, absolute quotas and intensity targets, i.e., emissions quotas indexed on economic output. This choice is motivated by the fact that intensity targets have often been proposed as an alternative to the continuation of the current absolute quota approach embedded in the Kyoto Protocol, notably on the grounds that they would reduce uncertainty.

Since various forms of intensity targets have been proposed in the literature and in policy

1Barring non-compliance of course.
circles—e.g., linear dependence of the emissions ceiling on GDP by the US administration, or square-root dependence by Argentina—, we consider in this paper both a linear intensity target, in which the quota depends linearly on GDP, and a ‘general’ intensity target with a power-law indexing.

This article is organized as follows. After reviewing the literature (Section 2), we build a simple but general model of the uncertainties associated with BAU emissions, future GDP and future abatement costs (Section 3). On this basis, we derive explicit conditions under which intensity targets reduce uncertainty on key policy variables—namely effective emissions, reduction effort, marginal costs, and total costs relative to GDP—with respect to quotas (Section 4). We then estimate ranges of values for the parameters of our model (Section 5). On this basis, we discuss how, in practice, different instruments compare with regard to uncertainty (Section 6). Section 7 concludes.

2 Literature Review

Two related strands of literature compare the performances of economic instruments under uncertainty. One stems from Martin Weitzman’s (1974) paper on “Prices vs Quantities”. Here, ‘performance’ is measured in terms of the welfare implications of each instrument, given assumptions about the marginal costs of abatement, about the marginal benefits of depollution, and about the uncertainty surrounding these costs and benefits. Pizer (1999) and Newell and Pizer (2003) apply this approach to the problem of climate change and show that, in the short-run at least, a tax dominates a cap approach because the slope of the marginal damage curve is likely to be flat relative to the slope of the marginal abatement cost curve. Using the same approach, Quirion (2003) finds that (linear) intensity targets are dominated by either tax or fixed quota approaches for a wide range of parameters—even though in his model there is no uncertainty on future GDP. Quirion also points out that the result of the comparison may

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2 In February 2002, the U.S. government announced a plan to reduce national greenhouse gas emissions relative to GDP by 18% by 2012 compared with 2002 (http://www.whitehouse.gov/news/releases/2002/02/climatechange.html).

3 The Government of Argentina proposed in 1998 to adopt an intensity target such that the quota would be a function of the square root of GDP (Barros and Grand, 2002).
depend on whether abatement costs depend on the absolute amount of abatement, or on the percentage of abatement relative to the baseline.

The above approach is comprehensive because the performance indicator is welfare. However, it requires detailed knowledge about the shape of the damage function, which remains highly controversial (Ambrosi et al., 2003). In addition, it does not provide decision-makers with information about the way different economic instruments reduce or increase the uncertainty on key decision variables, such as effective \textit{(ex post)} emissions, marginal abatement costs (the price of carbon), or total costs. For this reason a second—and more recent—strand of literature analyzes how the choice of a tax, a quota, or an intensity target impacts on the variance of these variables. The papers in this group differ in the way they model uncertainty on input variables (future GDP, future baseline emissions, etc.), and in the policy variables they analyze.

The idea that intensity targets would reduce uncertainty on abatement costs relative to a quota system seems rather intuitive, and is often mentioned among the arguments in support of the adoption of such an instrument. For example, Frankel (1999) argues that intensity targets will, among others, “moderate the effects of uncertainty”. Kim and Baumert (2002) suggest that intensity targets could “reduce economic uncertainty”. Similarly, Strachan (in press) finds that “using GHG intensities reduces baseline uncertainty”. The first detailed discussion of this argument is, to our knowledge, provided by Höhne and Harnisch (2002), who compare intensity targets and caps with regard to the amount of emissions that is abated relative to the baseline—a proxy for abatement costs. They find that a general intensity target dominates a cap when the elasticity of emissions with regard to GDP is high enough, and superior to 0.5 in the case of a linear intensity target. They note, however, that the elasticity of emissions with regard to GDP is difficult to estimate from historical data.

In a broad paper aimed at exploring the negotiation spaces provided by various policy instruments in a multi-region model, Jotzo and Pezzey (2004) provide—in passing—the first analytical treatment of the performance of a general intensity target under uncertainty, and derive an optimal calibration that depends on the stringency of the reduction target and the
strength of the GDP-emissions nexus. In their model, the marginal abatement cost functions are assumed certain and linear in the abatement level. The level of future business-as-usual GDP and the emissions intensity of GDP are independent random variables. In a subsequent paper (Jotzo and Pezzey, 2005), the authors derive an explicit expression for the variance of the required abatement effort and show that, with a set of parameters calibrated on historical time series, a linear intensity target does not necessarily dominate a quota, but that an optimally indexed general intensity target can always reduce the variance of marginal abatement costs relative to a cap.

Sue Wing et al. (2006) derive an explicit analytical condition under which a general intensity target yields a lower variance of the expected abatement effort than a cap. In their model, both BAU emissions and future GDP are random variables. They find that a linear intensity target is preferred to a cap unless the correlation between emissions and GDP is very low, or the uncertainty about future GDP is much larger than the uncertainty about future emissions. They also find that a partial intensity target, defined as a weighted average of a cap and an intensity target, can always outperform a cap when the weights are set correctly. Testing their results empirically, they suggest that intensity targets are clearly preferred to caps for developing countries, while the result is more ambiguous for developed countries.

Finally, Kolstadt (2005) develops a model where both abatement costs and output are uncertain. Within his framework, he shows that under a linear intensity target, total costs of abatement relative to GDP are only sensitive to the uncertainty on abatement costs, and not to the uncertainty on output. This result, however, rests on the assumption made in the paper that abatement costs depend only on output and on emissions intensity, and not on the level of emissions per se.

The present paper is attached to the second strand of literature. It adds to this literature in three ways. First, it models abatement costs in a very general way, and explicitly represents the uncertainty on abatement costs as a separate source of uncertainty. Since abating greenhouse gas emissions has virtually never been experienced before, uncertainty on abatement costs is very large (Hourcade et al., 2001), and plays a prominent role in the public debate on
climate policies. From an analytical perspective, introducing uncertainty on abatement costs allows us to address two questions raised in the literature: (i) whether uncertainty on marginal abatement costs can indeed be proxied by the uncertainty on the reduction effort, and (ii), following Quirion (2003), whether the fact that abatement costs depend on the absolute level of abatement, on the relative level of abatement, or on some combination thereof, matters for the relative performances of intensity targets and quotas.

Second, our paper provides an explicit comparison of the ‘performance’ of a cap and a general intensity target for four policy relevant variables: effective \((ex\ post)\) level of emissions, abatement effort, marginal cost of abatement (the price of carbon), and total costs of abatement relative to GDP. This point is important because different stakeholders will likely relate differently to these variables. For example, environmental NGOs might be more sensitive to the effective level of emissions, industries might look carefully at the price of carbon, while governments may be particularly interested in the total costs of abatement relative to GDP. Finding common ground on climate policies between stakeholders thus requires to consider all these variables.

Third, our paper is the first to explicitly take into account the risk that BAU emissions may be below the emissions ceiling if the target (quota or intensity) is not too strict. This risk is not purely theoretical, as exemplified by the amount of ‘hot air’ in Russia and other transition economies under the Kyoto Protocol. In addition, a compromise with developing countries might well involve targets that are not too far off projected BAU emissions, at least initially. In the analytical part, we need to make the assumption that the target is stringent enough so that this risk can be considered negligible. But in the numerical computations, we explicitly go back to the ‘full’ model, and assess the validity of our analytical results.
3 Model Description and Key Assumptions

3.1 Definitions: Quota, Linear Intensity Target, and General Intensity Target

We assume a unique region, and compare three possible climate policy instruments for a future ‘commitment period’ of arbitrary but fixed and finite length: an emissions quota, a linear intensity target, and a general intensity target. Let $E$ be the expected emissions of the region in the BAU scenario, and $\bar{E}$ be the effective (i.e., after abatement) emissions during the same period. If the region adopts an emissions quota $Q$, its effective emissions $\bar{E}$ are constrained as follows:

$$\bar{E} = \begin{cases} Q & \text{if } Q \leq E \\ E & \text{otherwise} \end{cases}$$ (1)

Let $Y$ be the economic output during the commitment period. An intensity target is defined as an emissions quota indexed on $Y$. As noted above, several indexation methods have been proposed. In this paper, we consider two variants. First, we consider a linear intensity target (LIT) such that, if $q$ is the target GHG intensity (in volumes of emissions per unit of output), effective emissions $\bar{E}$ are constrained as follows:

$$\bar{E} = \begin{cases} qY & \text{if } q \leq \frac{E}{Y} \\ E & \text{otherwise} \end{cases}$$ (2)

In addition, we consider a general intensity target (GIT) in which the relationship between the emissions quota and GDP is given by a power-law:

$$\bar{E} = \begin{cases} qY^m & \text{if } q \leq \frac{E}{Y^m} \\ E & \text{otherwise} \end{cases}$$ (3)

A GIT with $m = 1$ is equivalent to a linear intensity target, and a GIT with $m = 0$ is equivalent to a quota.\footnote{Ellerman and Sue Wing (2003), and subsequently Jotzo and Pezzey (2004) and Sue Wing et al. (2006) consider a slightly different but essentially equivalent form of general intensity target, where the emissions target...}
If future BAU emissions ($E$), future output ($Y$), and future costs of abatement were known with certainty, the implications of each instrument for all policy variables could be perfectly predicted \textit{ex ante}. In particular, for any given emissions target $\bar{E}^*$, there would be a unique level of quota ($Q^* = \bar{E}^*$), and a unique level of intensity target ($q^* = \bar{E}^*/Y^m$) that would guarantee that the target is reached. In reality, future BAU emissions ($E$), future output ($Y$), and future costs of abatement are uncertain. But $Q^*$ and $q^*$ will be used as a benchmark throughout the text. Precisely, when comparing intensity targets with quotas, we will use an objective $q^* = Q^*/\langle Y \rangle^m$, so that the two instruments would lead to the same level of emissions in the certainty case ($\langle \cdot \rangle$ denotes the expected value operator).

\section*{3.2 Modeling Uncertainty on GDP and Emissions}

Future BAU emissions and future output are not known with certainty, and there is no agreed-upon probability distribution for these variables. Many sets of projections are available in the literature for both variables. But in its Special Report on Emissions Scenarios (SRES) (Nakićenović and Swart, 2000), the Intergovernmental Panel on Climate Change (IPCC) has resisted to attach probabilities to these scenarios. To our knowledge, only Wigley and Raper (2001) have constructed a probability distribution function for cumulative emissions from 1990 to 2100, by considering each of the scenarios from Nakićenović and Swart (2000) as equiprobable.

In this paper, we assume that there are probability distribution functions that represent the possible values of $E$ and $Y$, respectively, but we do not make specific assumptions about their functional forms. We simply assume that $Y$ is a random variable of mean 1 (the central forecast value), and of variance $\sigma_Y^2$ (the normalized mean square error). Similarly, we assume that $E$ is a random variable of mean 1, and of variance $\sigma_E^2$. We denote by $\iota = Y - 1$ the random perturbation of $Y$ around its mean, and by $\epsilon = E - 1$ the random perturbation of $E$ around its mean. By construction, $\iota$ has a mean of zero and a variance of $\sigma_Y^2$, and $\epsilon$ has a mean of zero and a variance of $\sigma_E^2$.

Future BAU emissions and future GDP are closely related, at least when considering CO$_2$ has a fixed part and a variable part that depends on future GDP (using our notations, $\bar{E} = (1-m)Q + mqY$.)
emissions from fossil-fuel combustion—a major component of total GHG emissions—and they usually move in the same direction. Cross-country panel data show a robust relationship between the two variables (Heil and Selden, 2001, Ravallion et al., 2000), and country-level panel data tend to confirm this finding (Höhne and Harnisch, 2002, Kim and Baumert, 2002). However, since GDP and emissions time series are usually non-stationary (and often increasing), linear regressions over panel data capture only how the underlying trends correlate over time. They do not capture how, at each point in time, a deviation of emissions with regard to its forecasted level is correlated with a deviation of GDP from its forecasted level.\footnote{High linear correlation between time series does not necessarily imply a high correlation between residuals.} In our model, it is the latter indicator that matters for comparing absolute and intensity targets. This indicator can be measured by the linear correlation coefficient $\rho$.

\[ \rho = \frac{<EY> - <E><Y>}{\sigma_E \sigma_Y} = \frac{<\epsilon_1>}{\sigma_E \sigma_Y} \]  

(4)

### 3.3 Modeling Uncertainty on Marginal Abatement Costs

Marginal costs of abatement are modeled in many different ways in the literature. First, the argument of the cost function is sometimes the percentage of BAU emissions that has been abated (e.g., Nordhaus, 1992), and sometimes the absolute amount of emissions abated (e.g., Ellerman and Decaux, 1998). Second, marginal abatement costs are usually represented as an increasing and convex function of the level of abatement, but several functional forms have been used, including quadratic (Ellerman and Decaux, 1998), exponential (such as GTEM curves in the CERT model by Grütter Consulting, 2003) or general power-law functions (Ghersi, 2003).

In this paper, we adopt a general representation of costs. Marginal abatement costs are assumed to be a continuous, increasing and convex function of the abatement effort $R$, such that marginal abatement costs are zero when the effort is zero, and such that marginal costs remain finite for any finite value of $R \geq 0$. The abatement effort itself is expressed as a combination of a ‘relative’ and an ‘absolute’ effort, in the form

\[ R = E^{\alpha - 1}(E - \bar{E}) \quad \text{where} \quad 0 \leq \alpha \leq 1 \]  

(5)
The index $\alpha$ characterizes the elasticity of the effort with respect to baseline emissions. When $\alpha = 0$, the marginal costs depend on the amount of abatement relative to BAU emissions, i.e. the reduction percentage. For example, consider a fleet of known size of identical cars. The marginal costs of reducing their emissions by a given percentage depend only on the unit cost of more fuel-efficient cars, and remain the same even if the emissions per car are higher or lower than expected. Conversely, when $\alpha = 1$, marginal costs depend on the absolute amount of emissions that is abated. For example, the marginal costs of sequestering carbon through plantations will depend on the total amount that is sequestered, rather than on the fraction of total baseline emissions that this amount represents. Economy-wide marginal abatement costs are likely to fall somewhere in between these two extremes.

To model a general but simple uncertainty on marginal abatement costs we postulate that they take the form $a C(R)$, where $a$ is a random variable of mean $<a> = 1$ and of variance $\sigma_C^2$. Again, it is useful to define $\kappa = a - 1$, the random perturbation of $a$ around its mean. $\kappa$ has mean zero and a variance of $\sigma_C^2$. We assume throughout the text that the variations of $a$ are independent from the variations of $E$ and $Y$, i.e. that $\kappa$ is not correlated with $\epsilon$ or with $\iota$.

### 3.4 Additional Assumptions: Tight Regime and Small Variances

We now have the mathematical framework in place to analyze the variances of key output variables such as the effective amount of emissions or the price of carbon under a quota, a LIT, and a GIT. But before we can proceed, two additional assumptions must be made.

First, a forthright analytical treatment is hampered by the fact that the constraint on the effective emissions of each instrument, as represented by Eqs.(1)-(3), is non-differentiable at the point where the BAU emissions are equal to emissions target. To avoid lengthy case differentiations (e.g., to avoid considering separately the cases $E < Q$ and $E > Q$ in the analysis of a quota), we restrict ourselves to quotas $Q$ that are sufficiently small relative to $<E> - \sigma_E$ so that the probability of having $E < Q$ is very small. Similarly, we consider intensity targets $q$ small enough so that the probability that $\frac{E}{Y} < q$ is negligible. In other words, our first assumption is to restrict our analysis to ‘tight’ regimes, in which the probability
of ‘compliance by chance’ can be neglected.

Our second assumption is that the variances of $E$ and $Y$ are small enough so that the variations of output variables that depend on $E$ and $Y$ can be well approximated by a second-order Taylor expansion. Note that we do not need to make the same assumption for the variance of marginal abatement costs, since all policy variables that we analyze are linear in the random variable $a$.

Based on the two assumptions above, we can write the variance $\sigma_F$ of a generic function $F$ of random variables $E$, $Y$, and $a$. Detailed calculations can be found in Appendix A.

$$\sigma_F^2 \approx \sigma_C^2 \left( \frac{\partial F}{\partial a} \right)^2 + \sigma_E^2 \left( \frac{\partial F}{\partial E} \right)^2 + \sigma_Y^2 \left( \frac{\partial F}{\partial Y} \right)^2 + 2\rho \sigma_E \sigma_Y \frac{\partial F}{\partial E} \frac{\partial F}{\partial Y} \quad (6)$$

4 Intensity vs. Quota: Analytical Approach

In this section, we successively provide analytical expressions for the variances of four policy variables—effective emissions, abatement effort, marginal abatement costs and total abatement costs relative to GDP—under a quota, a LIT and GIT. We then compare these expressions to determine which instrument dominates the other, i.e., leads to the lowest variance—and hence standard deviation—for the variable in question. Relying solely on variances is of course not sufficient to fully characterize the underlying probability density function. For example, the variance does not provide any indication on how symmetric or asymmetric a distribution is. But variance and standard deviation are sufficient to provide some valuable insights into the relative performance of quota, LIT and GIT vis-à-vis uncertainty.

4.1 Effective Emissions

The first policy variable we examine are the effective (i.e. after abatement) GHG emissions $\bar{E}$. Table 1 lists the variances of $\bar{E}$ under a quota, a LIT and a GIT. These expressions are obtained by applying Eq.(6) to Eqs.(1), (2), and (3), respectively.

The results are intuitive. Since we assume that the quota is tight enough so that the probability of ‘compliance by chance’ is negligible, future emissions are equal to the quota
with certainty and the variance is zero. Under a LIT or a GIT, on the other hand, the emissions ceiling and thus the effective emissions are uncertain because the ceiling is indexed on future GDP, which is itself uncertain. In fact, uncertainty on future GDP ($\sigma_Y$) is mapped one to one onto effective emissions in the LIT case (second line of Table 1). In other words, if the standard error of GDP forecasts is 10%, the corresponding standard error for effective emissions under a LIT is also 10%. This effect is attenuated under a GIT when $m < 1$.

The comparison between instruments is straightforward: an intensity target (LIT or GIT) always increases the uncertainty on effective emissions relative to a quota.

### 4.2 Emissions Reduction Effort

The second policy variable we examine is the general abatement effort $R$. We compute the variance $\sigma_R^2$ of $R$ under the three instruments by applying Eq.(6) to $R$ as defined in Eq.(5), where $\bar{E}$ is substituted by its values from Eqs.(1), (2), and (3), respectively. As discussed in Section 3.1, we compare the variances by setting $q < Y >= q = Q$ (i.e., in the certainty case all three instruments would yield the same outcome).

Table 2, where for convenience we have defined

$$q_\alpha := \alpha + q(1 - \alpha) \quad ,$$

(7)

gives the expression of $\sigma_R^2$ under each of the three instruments. The reader will easily check that in general all variances are positive. This is an obvious but important point that we will find again throughout the paper: a given instrument can control at most one output variable (effective emissions for a quota, emissions intensity of GDP for an intensity target, etc.), but

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Emissions Variance ($\sigma_E^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quota</td>
<td>0</td>
</tr>
<tr>
<td>Linear Intensity Target</td>
<td>$q^2 \sigma_Y^2$</td>
</tr>
<tr>
<td>General Intensity Target</td>
<td>$q^2 \sigma_Y^2 m^2$</td>
</tr>
</tbody>
</table>

Table 1: Variance of effective future emissions $\bar{E}$ under a quota, a linear intensity target and a general intensity target.
it will in general leave all the other output variables uncertain. A second point worth noting is that all the variances depend on \( \alpha \), and thus on the type of abatement effort that is measured (absolute, relative, or in between the two). This translates the fact that, for any given objective \( Q \) or \( q \), a 10% increase in baseline emissions \( E \) leads to a higher increase in absolute effort \( E - Q \) than in relative effort \( 1 - \frac{Q}{E} \). As a result, variations of \( E \) around its mean result in higher variations of \( R \) around its mean for absolute than for relative efforts. The difference between the relative and the absolute case might be quite significant for stringent targets. In the quota case for example, if the target is to reduce emissions by half relative to the baseline (namely \( Q = q = 0.5 \)), the standard deviation of the absolute effort is twice as high as the standard deviation of the relative effort.

Third, whereas in the quota case uncertainty on \( R \) stems only from uncertainty on BAU emissions \( E \), two sources of uncertainty combine to make \( R \) uncertain under a LIT or a GIT: uncertainty on \( Y \) and uncertainty on \( E \). If these random variables are uncorrelated (\( \rho = 0 \)), the two uncertainties just add up. But if \( Y \) and \( E \) are positively correlated, the two uncertainties partially cancel out. In fact, if \( Y \) and \( E \) are fully correlated (\( \rho = 1 \)), the two uncertainties might even completely cancel out.

The discussion above suggests that, if \( Y \) and \( E \) are sufficiently correlated, uncertainty on the abatement level might be lower in the intensity case than in the quota case. Precisely,

**Proposition 1** The variance of the general abatement effort \( R \) under a general intensity target with \( q > 0 \) and \( m > 0 \) is lower than the variance of the general abatement effort under a quota with \( Q = q \) if and only if:

\[
\rho > \frac{m}{2} \frac{q}{q_0} \frac{\sigma_Y}{\sigma_E}
\]

Table 2: Variance of general abatement effort \( R \) under a quota, a linear intensity target, and a general intensity target.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Variance of Abatement Effort ( \sigma_R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quota</td>
<td>( q_0^2 \sigma_E^2 )</td>
</tr>
<tr>
<td>Linear Intensity</td>
<td>( q_0^2 \sigma_E^2 + q^2 \alpha^2 - 2q q_0 \rho \sigma_E \sigma_Y )</td>
</tr>
<tr>
<td>Gen. Intensity</td>
<td>( q_0^2 \sigma_E^2 + q^2 \alpha^2 m^2 - 2q q_0 \rho \sigma_E \sigma_Y m )</td>
</tr>
</tbody>
</table>
**Proof:** Equation 8 is obtained by setting the third line of Table 2 to be greater than the first and solving for $\rho$.

Condition (8) is a generalized version of the dominance condition obtained by Sue Wing et al. (2006) for the absolute abatement effort ($\alpha = 1$). We denote by $\rho_{\text{minR}} := \frac{m}{q} \frac{q}{q_{\alpha}} \frac{\sigma_Y}{\sigma_E}$ the minimum value of $\rho$ necessary for condition (8) to be met.

For a LIT ($m = 1$), the condition cannot always be met, since $\rho_{\text{minR}}$ becomes greater than unity if the standard deviation of $Y$ is sufficiently larger than that of $E$. On the other hand, if $\sigma_E = \sigma_Y$, the condition becomes $\rho > 0.5$ in the relative case ($\alpha = 1$), and $\rho > q/2$ in the absolute case ($\alpha = 0$).

With a GIT, on the other hand, regardless of the relative values of $\sigma_Y$ and $\sigma_E$, and for any given value of $\rho$ and $q$, condition (8) can always be met by choosing an exponent $m$ that is small enough. In other words, by adequately choosing $m$, one can always ensure that the uncertainty on the reduction effort under a GIT is lower than the uncertainty on the reduction effort under a quota. And the reduction in variance can be maximized as follows:

**Proposition 2** The variance of the abatement effort under a GIT can be minimized by setting the parameter $m$ to

$$m_{\text{R}}^* = \rho \frac{q_{\alpha} \sigma_E}{q_{\alpha} \sigma_Y}$$

(9)

Condition (8) is then automatically met, and the remaining variance of the abatement effort is

$$\sigma_{R}^2 = q_{\alpha}^2 \sigma_E^2 \left(1 - \rho^2\right)$$

(10)

**Proof:** $m_{\text{R}}^*$ is the zero of the derivative of $\sigma_R$ (third line of Table 2) with regard to $m$. Since $\sigma_R$ is a degree-2 polynomial function of $m$ with a positive coefficient in $m^2$, $m_{\text{R}}^*$ is unique and corresponds to a minimum. Eq.(10) gives the value of the extremum.

Eq.(10) shows that when $m$ is set to its optimal value $m_{\text{R}}^*$, the uncertainty on future GDP $\sigma_Y$ is completely eliminated from the variance of the abatement effort. Relative to the quota case, the variance of the abatement effort is then reduced by $\rho^2$ percent. The reduction of uncertainty is thus large when $E$ and $Y$ are well-correlated. In fact, uncertainty can even be
eliminated completely if \( \rho = 1 \). But the reduction of uncertainty diminishes rapidly as the degree of correlation between \( E \) and \( Y \) decreases. For example, if \( \rho = 0.5 \), using an ‘optimal’ GIT reduces the standard deviation of the abatement effort relative to the quota case by 13\%, a figure that becomes a mere 2\% if \( \rho = 0.2 \). In fact, when the degree of correlation between \( Y \) and \( E \) diminishes, Eq.(9) shows that \( m^*_R \) also diminishes. In other words, the optimal general intensity target gives less and less weight to GDP, and thus becomes closer and closer to an absolute target.

General intensity targets, however, are more difficult to apprehend and might be more difficult to negotiate. So what if a linear intensity target \( (m = 1) \) is selected instead? If the correlation between \( Y \) and \( E \) is high, condition (8) is still likely to be met, at least as long as \( \sigma_Y \) is no more than twice as large as \( \sigma_E \) (this limit can even be relaxed when \( \alpha = 1 \)). Whether the gain in uncertainty is maximal or not is determined by whether \( m^*_R \) is close to 1 or not. On the other hand, if \( \rho \) is small, then condition (8) is likely not to be met, and the uncertainty on the abatement effort under a LIT becomes higher than under a quota. The standard deviation of \( R \) might increase by a significant amount, for example by 63\% if \( \alpha = 1 \), \( \sigma_Y = 2\sigma_E \), \( q = 0.75 \) and \( \rho = 0.2 \).

### 4.3 Price of Carbon

The third variable we consider is the marginal cost of abatement \( aC(R) \), which can also be thought of as the price of carbon. Applying Eq.(6) to this variable, we can write the variance of the marginal cost of abatement \( \sigma^2_{MAC} \) as a function of the variances of \( a \) and \( R \). The expression below is valid for all instruments.

\[
\sigma^2_{MAC} \approx C_0^2 \left( \sigma^2_C + \left( \frac{C'_0}{C_0} \right)^2 \sigma^2_R \right)
\] (11)

where \( C_0 := C(1 - q) \) (12)

and \( C'_0 := C'(1 - q) \) (13)
The variance of the marginal cost of abatement can thus be expressed as the sum of two terms: one related to the variance of marginal abatement costs, and the other related to the variance of the abatement effort $R$. Since the former does not depend on the instrument, the relative performances of a quota, a LIT or a GIT with regard to uncertainty on the price of carbon are the same as the relative performances of a quota, a LIT or a GIT with regard to the uncertainty on the abatement effort. The analytical condition follows from the previous section.

**Proposition 3** Let $R$ be the abatement effort that the marginal cost function $C(R)$ takes as argument. A linear or general intensity target reduces the variance of marginal abatement costs relative to a quota, if and only if it also reduces the variance of the abatement effort $R$ relative to a quota, and thus if and only if condition (8) is verified.

In addition, the coefficient $m$ of a general intensity target can be set in such a way that (i) condition (8) is met, and that (ii) the reduction of variance relative to the quota is maximized. This optimum coincides with $m^*_R$ from Eq.(9).

**Proof:** See proofs of Propositions 1 and 2.

Much of the discussion regarding Eqs.(8) and (9) has already been conducted in the previous section. We simply make two additional remarks here. First, when $\rho$ is between $1/2 \sigma_Y/\sigma_E m q$ and $1/2 \sigma_Y/\sigma_E m$, the relative performances of quota and GIT with regard to uncertainty on marginal abatement costs are determined by the value of $\alpha$. We come back to this point in more detail in Section 6.5.

A second remark is that the reduction of uncertainty that one can achieve by selecting an optimal GIT (i.e., by setting $m = m^*_R$) is lower for marginal abatement costs than for the abatement effort. This is because the variance of marginal abatement costs is now given by the sum of two terms, one of which is unrelated to the uncertainty on $R$, and thus irreducible for all instruments.

4.4 Total Costs Relative to GDP

The fourth policy variable we examine is total costs of abatement relative to GDP (hereafter ‘relative costs’ or $RC$). We consider total costs relative to GDP as opposed to total costs
because the intensity target, which is indexed on GDP, might presumably do a better job at controlling that particular variable. Also, total costs expressed as a fraction of GDP represent a better indicator for the effective impact of climate mitigation on a country’s economy than absolute costs. Total costs relative to GDP are defined as

\[ RC(R) = \frac{1}{Y} \int_{qY}^{E} de a C(E^\alpha - e E^{\alpha-1}) = \frac{a E^{1-\alpha}}{Y} \int_0^R C(r)dr \quad (14) \]

The term \( E^{1-\alpha} \) in Eq.(14) translates the fact that the additional costs of abatement caused by an increase of the effort \( R \) by \( dR \) is the marginal cost of abatement at effort \( R \), \( C_0(R) \), times the additional amount of carbon that is abated by increasing the effort by \( dR \). Given the definition of \( R \) (Eq.5), that additional amount is equal to \( E^{1-\alpha} dR \).

The variance of \( RC \) is given below. The equation is written for a GIT, but the quota case is easily obtained by setting \( m = 0 \) and the LIT case by setting \( m = 1 \). Detailed calculations can be found in Appendix B.

\[ \sigma^2_{RC} \approx RC_0^2 \left( \sigma_C^2 + \sigma_E^2 \Omega^2 + \sigma_Y^2 \left( 1 + qm \frac{C_0}{RC_0} \right)^2 - 2\rho \sigma_E \sigma_Y \Omega \left( 1 + qm \frac{C_0}{RC_0} \right) \right) \quad (15) \]

where \[ \Omega := (1 - \alpha) + \frac{C_0}{RC_0} q \alpha \quad (16) \]

and \[ RC_0 = \int_0^R C(r)dr \quad (17) \]

From Eq.(15), the condition under which an intensity target reduces uncertainty on total relative costs vis-à-vis a quota follows.

**Proposition 4** A general intensity target \( qY^m \) with \( m > 0 \) and \( q > 0 \) leads to a lower variance of total abatement costs relative to GDP than a cap if and only if

\[ \rho > \frac{1}{2 \Omega} \frac{\sigma_Y}{\sigma_E} \left( 2 + qm \frac{C_0(R)}{RC(R)} \right) =: \rho_{minRC} \quad (18) \]

**Proof:** See Appendix B.

Condition (18) reveals three major differences between marginal abatement costs and total
relative costs when it comes to uncertainty. First, unlike in the marginal abatement costs case, it is not always possible to find a positive value of \( m \) that will make the variance of total relative costs under a GIT lower than under a quota. In fact, \( \rho \) needs to fulfill condition (19) below to guarantee that such a value exists:

\[
\rho > \frac{1}{\Omega} \frac{\sigma_Y}{\sigma_E}.
\] (19)

For example, if function \( C \) is quadratic, if we consider the relative case \( (\alpha = 0) \), and if the target is \( q = 0.5 \), a quota always dominates a general intensity target with regard to uncertainty on total relative costs, regardless of the value of \( m \), as soon as \( \rho \) is lower than 0.25 \( \sigma_Y/\sigma_E \).

The second difference between marginal abatement costs and total relative costs vis-à-vis uncertainty is that \( \rho_{\min RC} > \rho_{\min R} \) whenever \( m < 2 \). In other words, a higher degree of correlation between emissions and GDP is required for an intensity target to reduce uncertainty on relative costs relative to a quota.

Third, the optimal calibration of the intensity target is generally different:

**Proposition 5** The value \( m^*_{RC} \) that maximizes the mitigation of uncertainty relative to the quota case for a given degree of correlation between \( E \) and \( Y \) is, when it exists, given by:

\[
m^*_{RC} = m^*_{R} + \frac{RC_0}{qC_0} \left( \rho \frac{\sigma_E}{\sigma_Y} (1 - \alpha) - 1 \right)
\] (20)

**Proof:** The result is obtained by finding the minimum of Eq.(15), and solving for \( m \).

Thus, in most cases, \( m^*_{RC} \) is smaller than \( m^*_{R} \). This is true, in particular, when abatement costs depend on the absolute amount of emission reductions \( (\alpha = 1) \), and in a wide range of cases—e.g. for \( \sigma_E \leq \sigma_Y \)—when \( \alpha = 0 \). When \( m \) is set to \( m^*_{RC} \), the uncertainty on GDP is completely eliminated (Eq.21) and, if the correlation \( \rho \) is very high, \( \sigma^2_{RC} \) can even be reduced to an irreducible minimum \( RC_0^2 \sigma^2_C \). However, the reduction in variance—and even more so in standard deviation—will again by meager as long as \( \rho \) remains moderate.

\[
\sigma^2_{RC}(m^*_{RC}) \approx RC_0^2 \left[ \sigma^2_C + \sigma^2_E \Omega^2 (1 - \rho^2) \right]
\] (21)
On the other hand, choosing a linear intensity target in absence of an appreciable positive correlation between $E$ and $Y$ can lead to large uncertainties on total relative costs. For instance, for the absolute case and with a quadratic MAC function, a target reduction of $q = 0.75$ under a linear intensity target with $\rho = 0$ yields a normalized variance, i.e. the variance divided by the square of the deterministic value of $RC$, $RC_0$, that is higher than the normalized variance for a quota by a margin of 99 times $\sigma_Y^2$.\footnote{Precisely, the normalized variance $\sigma_{RC}^2/RC_0^2$ under a LIT is $\sigma_C^2 + 144 \sigma_E^2 + 100 \sigma_Y^2$, while it is $\sigma_C^2 + 144 \sigma_E^2 + \sigma_Y^2$ under a quota.}

In sum, total relative costs are well-controlled neither by the cap nor by the intensity target. Since the effort is indexed on GDP, the intensity target might have been expected to perform better than the quota vis-à-vis uncertainty on relative costs. But the analysis demonstrates that this is not the case. In fact, intensity targets perform better than quotas vis-à-vis relative costs less often than they do vis-à-vis marginal abatement costs. And unlike in the marginal abatement cost case, general intensity targets can no longer automatically be calibrated to perform better than quotas if the correlation between $E$ and $Y$ is not large enough.\footnote{This result is not consistent with Kolstadt (2005), who shows that total relative costs are subject solely to cost function uncertainty under an intensity target. This is because, in his model, total abatement costs depend on emissions per unit of GDP ($E/Y$). As a result, setting an intensity target, which is precisely setting a level of emissions per unit of GDP, automatically fixes total abatement costs, up to the uncertainty on the functional form itself.}

5 Estimation of Model Parameters

In this section, we estimate the parameters $\sigma_E$, $\sigma_Y$, $\sigma_C$ and $\rho$. The commitment period we consider here is 2013-2017, because much of the current debate focuses on the post-Kyoto period. Ideally, one would like to estimate these parameters for individual countries because future targets, like those in the Kyoto Protocol, are likely to be adopted by individual countries. However, many of the GDP, emissions, and abatement cost projections on which we base our estimates are available only for regions, and not for individual countries. As a result, the uncertainties that we obtain with these aggregated data are likely to be smaller than the uncertainties that we would have obtained had we had country-level data.
5.1 Estimation of $\sigma_E$, $\sigma_Y$ and $\rho$

Let us first recall that $E$ and $Y$ are uncertain because there is no single model that would accurately predict their value based on observables. The existing models that project future output and future emissions are themselves based on parameters that are unobservable and uncertain, such as the rate of autonomous technical change. In addition, there are competing models that project future output and future emissions.

To estimate $\sigma_E$, $\sigma_Y$ and $\rho$, three main techniques are available. First, one can take sets of projections generated by one model (e.g., the scenarios of the US Energy Information Administration), and use the difference between the high and the low scenario as a proxy for uncertainty. Data points in this method are often too few to allow for the estimation of $\rho$. Second, one can compute the variance of projections originating from different models, as listed for example in the IPCC Special Report on Emissions Scenarios (Nakićenović and Swart, 2000). The internal consistency of the scenarios is lost because different models are involved, but larger data sets allow for the estimation of $\rho$. Finally, one can compute historical forecast errors, and take them as a proxy for the accuracy of today’s forecasts for emissions and GDP during a post-Kyoto period.

Each approach has limitations. Using scenarios may lead to an overestimation of the actual variance because scenarios are often built to explore a wide range of plausible futures, and thus are not intended to be interpreted probabilistically. Historical forecast errors, on the other hand, suffer from data scarcity, especially when considering long-range forecasts. In this method therefore, linear correlation coefficients can often be computed only from forecasts for the same time horizon, but for different countries (assuming forecasts as independent). This approach provides a cross-country average value for $\rho$, but individual countries may have higher or lower coefficients depending on their particular relationships between emissions and GDP.

In this paper, we pursue all three approaches to make our estimates of the uncertainties as robust as possible. The upper section of Table 3 shows our estimates for the normalized standard deviation (also called coefficient of variation) of GDP and BAU CO$_2$ emissions in 2015, as inferred from the US Energy Information Administration’s (EIA, 2005) low, mid and
high scenarios, assuming all three as equiprobable. We find relatively small values for both $\sigma_Y$ (between 0.06 and 0.16) and $\sigma_E$ (between 0.03 and 0.10). Uncertainty is in general higher for GDP than for emissions.

The bottom part of Table 3 presents estimates for the normalized standard deviation of cumulative GDP and BAU CO$_2$ emissions for the period 2013 to 2017, based on 25 scenarios from multiple sources (IPCC, IIASA and US EPA) harmonized by Lecocq and Crassous (2003) and assumed equiprobable. We find values of $\sigma_Y$ between 0.06 for WEU and 0.25 for India, while values for $\sigma_E$ lie in a slightly narrower band ranging from 0.10 (US) to 0.22 (SAFR). Uncertainty about $E$ and $Y$, here, are of similar magnitude. Interestingly, parameter $\rho$ can take a very wide range, from a negative $-0.33$ for LAM to a strongly positive value of 0.89 for MENA, with most values, however, taking on positive values above or equal to 0.4.

We also carried out a limited assessment of the accuracy of past emissions and GDP forecasts, similar to Lutter (2000). To this end, we compared the reference 7-year forecasts of the 1995 issue of the International Energy Outlook (EIA, 1995) with actual data (EIA, 2005), and likewise the 1994 8-year ahead WEO forecasts with current data (IEA, 1994, 2002, 2004). Relative forecast errors are reported in Table 4.

As can be observed, forecast errors can be quite large (up to 67% for 8-year forecasts of East

<table>
<thead>
<tr>
<th></th>
<th>World</th>
<th>US</th>
<th>Chn</th>
<th>Ind</th>
<th>Jap</th>
<th>WEU</th>
<th>EEU</th>
<th>FSU</th>
<th>LAM</th>
<th>MENA</th>
<th>SAFR</th>
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<td>$\sigma_Y$</td>
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<td>0.06</td>
<td>0.10</td>
<td>0.10</td>
<td>0.06</td>
<td>0.06</td>
<td>0.11</td>
<td>0.16</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
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<td>0.07</td>
<td>0.06</td>
<td>0.03</td>
<td>0.04</td>
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</tr>
<tr>
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<td>0.07</td>
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<td>0.12</td>
<td>0.12</td>
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<td>0.11</td>
<td>0.11</td>
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<td>0.16</td>
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<td>0.22</td>
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<td>$\rho$</td>
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<td>0.78</td>
<td>0.87</td>
<td>0.50</td>
<td>0.42</td>
<td>0.25</td>
<td>-0.12</td>
<td>-0.33</td>
<td>0.89</td>
<td>0.67</td>
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Table 3: Normalized standard deviations of the EIA (2005) scenarios for 2015 (top part), and normalized standard deviations and liner correlation coefficients of $E$ and $Y$ for 25 scenarios from various sources harmonized by Lecocq and Crassous (2003) (bottom part). Data refer to World, USA, China, India, Japan, Western Europe (WEU), Eastern Europe (EEU), Former Soviet Union (FSU), Latin America (LAM), Middle East and North Africa (MENA), and Sub-Saharan Africa (SAFR). For the EIA (2005), the latter two actually correspond to Middle East and Africa as a whole, respectively.
<table>
<thead>
<tr>
<th>Country or Region</th>
<th>IEO CO₂ 7 years</th>
<th>IEO GDP 7 years</th>
<th>WEO CO₂ 8 years</th>
<th>WEO GDP 8 years</th>
<th>Country or Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>World</td>
<td>4.4%</td>
<td>-6.6%</td>
<td>3.4%</td>
<td>-6.1%</td>
<td>World</td>
</tr>
<tr>
<td>Industrialized C.</td>
<td>3.1%</td>
<td>-1.3%</td>
<td>0.9%</td>
<td>-1.4%</td>
<td>Developing C.</td>
</tr>
<tr>
<td>Non-OECD Asia</td>
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<td>-0.5%</td>
<td>-0.4%</td>
<td>-16.8%</td>
<td>US and Canada</td>
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<td>2.1%</td>
<td>38.3%</td>
<td>OECD Pacific</td>
</tr>
<tr>
<td>Canada</td>
<td>-1.4%</td>
<td>-7.4%</td>
<td>6.5%</td>
<td>-18.1%</td>
<td>China</td>
</tr>
<tr>
<td>Western Europe</td>
<td>14.0%</td>
<td>-1.4%</td>
<td>4.5%</td>
<td>66.9%</td>
<td>East Asia</td>
</tr>
<tr>
<td>Japan</td>
<td>4.1%</td>
<td>19.3%</td>
<td>-5.1%</td>
<td>-7.3%</td>
<td>South Asia</td>
</tr>
<tr>
<td>Former SU</td>
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<td>1.9%</td>
<td>-14.5%</td>
<td>-1.7%</td>
<td>Middle East</td>
</tr>
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<td>Eastern Europe</td>
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<td>-0.5%</td>
<td>-0.2%</td>
<td>19.6%</td>
<td>Latin America</td>
</tr>
<tr>
<td>Middle East</td>
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<td>5.1%</td>
<td>0.2%</td>
<td>13.4%</td>
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<tr>
<td>Mexico</td>
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<tr>
<td>China</td>
<td>4.8%</td>
<td>-13.3%</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Africa</td>
<td>1.1%</td>
<td>2.6%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Latin America</td>
<td>-0.8%</td>
<td>12.7%</td>
<td></td>
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</tr>
</tbody>
</table>

Table 4: Historical forecast errors for CO₂ emissions and GDP of the International Energy Outlook 1995 (EIA, 1995) and the World Energy Outlook 1994 (IEA, 1994). The WEO regions OECD and OECD Europe could not be considered, as their composition changed significantly during the 1990s. Likewise, we excluded South Korea from the OECD Pacific region.

Asia’s GDP). Secondly, while for the IEO the average absolute error for emissions is larger than the one for GDP, the opposite is true for the WEO. Thus, as with the data in Table 3, it cannot be asserted that either type of uncertainty (emissions or output) is necessarily lower than the other. Third, when pooled over the different regions listed in Table 4, forecast errors of emissions and GDP do not show a strong correlation: the ρ corresponding to the two ‘pairs’ of forecasts from Table 4 is -0.04 and 0.31, respectively.

Table 5 summarizes this section’s findings. Four key conclusions can be drawn. First, σₐ and σᵧ are almost always found to be below 20%. Second, there is no obvious difference in patterns of uncertainties between developing and industrialized countries, except for an apparently lower emissions uncertainty for industrialized countries. Third, no robust statement

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8Our results are comparable to those of Lutter (2000), who finds that historical ten-year ahead emissions forecasts for the United States are subject to a 4.2% absolute error, and that ‘simulated’ forecasts for cumulative emissions of five-year periods are subject to errors between 5% and 25%, or even up to 37% (India), if the five-year period starts six or more years ahead in the future.

9To avoid double-counting, these linear correlation coefficients are computed by using only those individual regions and countries that do not overlap.
can be made on the basis of the results above as to which uncertainty—emissions or output—is higher, even though for the limited data considered here, the average uncertainty on future GDP is somewhat higher than the average uncertainty on future emissions. This finding is significant because the ratio between the two variances plays a crucial role in the equations obtained in Section 4. Fourth, the most difficult parameter to estimate remains the correlation of forecast errors \( \rho \). While the evaluation of the long-term scenarios led to a remarkably large range of values, the analysis of historical forecasts produced lower values not too far from zero. On the aggregated level values for \( \rho \) tend to be positive, around 0.25, with a tendency to be higher in industrialized countries than in developing countries.

### 5.2 Estimating Uncertainty about Marginal Abatement Costs

To estimate \( \sigma_C \), one would need several estimates of the marginal abatement cost curve for the period 2013-2017, all with the same functional form. Such a set, however, is not readily available. Most of the marginal abatement cost curves currently available apply to the first commitment period only. In addition, available surveys of marginal abatement cost curves...
Table 6: Values for the MAC uncertainty $\sigma_C$, as derived from Gheresi (2003). Taking for each country/region the lower estimate leads to an overall average of 66%, and a range of $49\% - 81\%$, valid for both the absolute and relative case.

(e.g. Metz et al., 2001, Gheresi, 2003) report only data points and not functional forms. Third, most available studies of abatement costs focus on developed countries only.

To get some insights into $\sigma_C$, we use a compilation of modeling results from Gheresi (2003), who reports two-point estimates for marginal abatement costs of 13 different models, both in terms of absolute and relative reductions. The marginal abatement costs are valid in 2010 for decisions made in 2000. For our study, we consider all these to represent random draws of the true cost function, and perform a least-square fit with an exponential and power-law function. Our estimate of $\sigma_C$ is then the normalized standard deviation of the residuals, as reported in Table 6. Typical values are around 0.5 for the US, but significantly higher, between $0.6 - 0.9$, for other industrialized countries. Because of insufficient data, we cannot provide estimates for developing countries, but it seems reasonable—also for the very fact that no data is available—to assume even higher values for these less researched economies.

6 Intensity vs. Quota in a Real-World Setting

We now test whether an intensity target performs better than a cap vis-à-vis uncertainty on key policy variables, using the analytical conditions derived in Section 4 and the empirical values identified in Section 5.
<table>
<thead>
<tr>
<th>Country or Region</th>
<th>1990 Emissions (btCO₂)</th>
<th>2010 Forecast (btCO₂)</th>
<th>Kyoto Target (% of 1990 emissions)</th>
<th>Kyoto Forecast Target (% of 2010)</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>4.90</td>
<td>6.31</td>
<td>93%</td>
<td>72%</td>
</tr>
<tr>
<td>Canada</td>
<td>0.50</td>
<td>0.67</td>
<td>94%</td>
<td>71%</td>
</tr>
<tr>
<td>Western Europe</td>
<td>3.73</td>
<td>4.43</td>
<td>92%</td>
<td>77%</td>
</tr>
<tr>
<td>Japan</td>
<td>1.13</td>
<td>1.71</td>
<td>94%</td>
<td>62%</td>
</tr>
<tr>
<td>Former Soviet Union</td>
<td>3.77</td>
<td>3.20</td>
<td>100%</td>
<td>118%</td>
</tr>
<tr>
<td>Eastern Europe</td>
<td>1.13</td>
<td>1.12</td>
<td>92%</td>
<td>93%</td>
</tr>
<tr>
<td>Annex B Total</td>
<td>15.2</td>
<td>17.4</td>
<td>94.5%</td>
<td>82%</td>
</tr>
</tbody>
</table>

Table 7: Kyoto targets expressed as a fraction of BAU emissions in 2010, as projected in 1997. Only CO₂ emissions from fossil-fuel combustion are considered. Source: EIA (1997)

6.1 Relative Performances of Quota, LIT and GIT with Regard to Abatement Effort and Price of Carbon

The condition under which a GIT reduces uncertainty on the abatement effort and the marginal abatement costs relative to a quota is given by Eq.(8). The key parameters in this equation are $\rho$, the ratio $\sigma_Y/\sigma_E$, $q$ and $\alpha$.

Section 5.1 provides a range of plausible values for $\rho$ and $\sigma_Y/\sigma_E$. In Table 5, $\rho$ ranges between 0.27 and 0.44, and the ratio $\sigma_Y/\sigma_E$ between 0.8 and 22 for developed countries (note that Table 5 shows rounded values). We limit the analysis to developed countries because there is less data on developing countries, and because the ‘tight regime’ assumption is more likely to be valid for industrialized countries, at least for the period 2013-2017.

We first discuss the relative performances of a quota and a linear intensity target. Fig.1 shows the area in the $(\log(\sigma_Y/\sigma_E), \rho)$ plane where a quota dominates a LIT in terms of uncertainty on abatement effort and marginal abatement costs. The box represents the range of plausible values as extracted from Table 5. In Fig.1, we set $q = 0.75$. As shown in Table 7, this value is comparable to the Kyoto targets, under which the whole of Annex B committed to limit emissions to 82% of the BAU level as it was projected in 1997. The value 0.75 is also in the range of targets that have been proposed in the literature for the second commitment period (see, e.g., Brouns and Ott, 2005), at least for developed countries.
Fig. 1 (Left) shows that over most of the plausible values for $\rho$ and $\sigma_Y/\sigma_E$, a quota dominates a LIT with regard to uncertainty on effort and marginal abatement costs. This result is valid regardless of the value of $\alpha$ since the frontier between the areas where LIT and quota dominate does not move much when $\alpha$ goes from 0 to 1.

With a more stringent target—i.e. a lower $q$—, the difference between the relative and the absolute cost functions becomes more acute. The frontier in the relative case ($\alpha = 0$) remains unchanged for any value of $q$ because, when $\alpha = 0$, the term $q/q_\alpha$ in Eq.(8) is equal to one. But the frontier in the absolute case ($\alpha = 1$) becomes flatter and flatter as $q$ diminishes because the term $q/q_\alpha$ is in this case equal to $q$. In other words, a more stringent target increases the ‘gray area’, where the relative performances of quota and LIT depend on $\alpha$ (Fig.1, Right).

Contrary to a linear intensity target, a general intensity target will always dominate a quota in terms of the uncertainty on abatement effort and marginal abatement costs, as long as $m$ is well-chosen. However, when the ratio $\sigma_Y/\sigma_E$ is large, a small value of $m$ is necessary.

For example, Fig.2 (Left) shows that for $m = 0.3$, a GIT dominates a quota for no more than half of the box of plausible parameter values. One would have to set $m = 0.03$ for a GIT to dominate over the entire range of plausible values. At this level, the GIT is very close to a cap, each additional point of GDP leading to a 0.05% increase in the emissions quota. And
the gains in terms of the variance of the reduction effort $\sigma_R$ are rather modest. In this case, numerical calculations show that the lowering of the standard deviation is 10% at most over the range of plausible values listed in Table 5.

Finally, it is interesting to note that the optimal parameter $m^*$ that minimizes the variance takes a very wide range of values over the set of plausible values for $\rho$ and $\sigma_Y/\sigma_E$: from 1 to 0.05 according to Eq.(9). This suggests that setting an optimal GIT cannot be done properly without first reducing the uncertainty on the values of $\rho$ and $\sigma_Y/\sigma_E$.

### 6.2 Relative Performances of Quota, LIT and GIT with Regard to Total Relative Costs

The condition under which a GIT reduces uncertainty on the total costs per GDP relative to a quota is given in Eq.(18). We test this condition using the same ranges of parameter values as above. We use a power-law function for the marginal abatement cost function $C(R) = R^\gamma$, with $\gamma > 1$. Under this assumption,

$$\frac{C_0}{RC_0} = \frac{\gamma + 1}{1 - q} \quad (22)$$
For exponents $\gamma$ between 1 and 2, and for an effort $q = 0.75$, $C_0/RC_0$ is thus between 8 and 12. This coefficient increases rapidly when the abatement effort becomes less stringent.

Fig.2 (Right) shows the area in the $(\log(\sigma_Y/\sigma_E), \rho)$ plane where a quota dominates a LIT in terms of the uncertainty on total relative costs, with $q = 0.75$ and $\gamma = 1.5$. Except for a small area in the upper left corner of the box, a quota dominates a linear intensity target for all of the plausible values for $\rho$ and $\sigma_Y/\sigma_E$.

More stringent targets again increase the discrepancy between the relative and the absolute cost functions, and result in a larger area where the dominance of quota or LIT could be determined by $\alpha$. Both frontiers move, but whereas the frontier for $\alpha = 1$ moves downward slightly, the frontier for $\alpha = 0$ moves upward significantly. This is because, in the relative case, $q$ appears in the denominator of $\rho_{\min}RC$ in Eq.(18), whereas it appears only in the numerator in the absolute case. As a result, most of the box of plausible values remains dominated by the quota even when $q$ is small (Fig.3, Left).

Unlike in the case of marginal abatement costs, a general intensity target will not always dominate a quota with regard to the uncertainty on total abatement costs relative to GDP. Fig.3 (Right) shows the maximum area over which a GIT can dominate a quota. For $q = 0.75$ and $\gamma = 1.5$, this maximal area covers only about half of the range of plausible parameters.
reported in Table 5. It is important to note that in order to secure the dominance of the GIT across the entire rectangular area, parameter $m$ must be very small, making the GIT very similar to an absolute quota.

Additionally, the figure indicates that the value of $m$ needed to ensure that a GIT dominates a quota for relative costs is always lower than the one required for the marginal abatement costs. If we take again the previous example—$\sigma_Y/\sigma_E = 2$ and $\rho = 0.3$—, $m$ needs to be lower than 0.075 for a GIT to dominate a quota vis-à-vis the uncertainty on total relative costs, to be compared with 0.3 when uncertainty on marginal abatement costs is considered.

Similarly, the optimal parameter $m^*$ that minimizes the variance, when it exists, takes a very wide range of values over the set of plausible values for $\rho$ and $\sigma_Y/\sigma_E$: from 1 to nearly 0 according to Eq.(20). Thus, we find again that setting an optimal GIT cannot be done properly without first reducing the uncertainty on the values of $\rho$ and $\sigma_Y/\sigma_E$.

6.3 Quantifying Uncertainty

In this section, we present numerical values for the absolute level of uncertainty on output parameters over our set of policy variables. To do so, we build three representative cases, the characteristics of which are summarized in Table 8. The first case ('average') is based on the average of the parameters for industrialized countries reported in Table 5. By contrast, the 'pro-intensity' case is constructed to be the most favorable for intensity targets, i.e., with the highest value of $\sigma_E$ and $\rho$ for industrialized countries from Table 5, the highest value for $\alpha$ and the lowest value for $\sigma_Y$. The 'pro-cap' case is the exact opposite. As in the previous section, we use $Q = q = 0.75$.

Table 9 presents the normalized standard deviations for each output variable under each of the instruments—subject to the parameter values of one of the three cases. It shows that an intensity target (LIT or GIT) leads to non-negligible uncertainty on emissions, especially—by construction—in the pro-cap case (0.18). The uncertainty on the abatement effort is larger, giving rise to very high uncertainties on marginal abatement costs, ranging from 68% (pro-cap case, GIT) to more than 100% (pro-cap case, LIT). Uncertainty on total relative costs is higher
still, with normalized standard deviations always higher than 72%.

A robust pattern emerges: While as the LIT outperforms the quota by a relatively small margin in the pro-intensity case (except of course on emissions), it leads to a medium-to-large increase of uncertainty in the other two cases. Therefore, Table 9 suggests again that adopting a LIT could introduce significant uncertainty into the system.

As expected, an optimal GIT dominates all other instruments on all policy indicators save emissions. Still, because the empirically found linear correlations $\rho$ are not high, in particular always below $1/2$, the impact of the GIT remains limited, and its performance is on the whole comparable to that of the cap. In addition, given that $\rho$ and $\sigma_E/\sigma_Y$ are uncertain, it is more realistic to assume that the GIT is calibrated for the central ‘average’ case (fourth line for each instrument), and not with the optimal $m^*$ of each representative case. In that case, the GIT no longer outperforms the cap.

### 6.4 Validity of the ‘Tight Regime’ Assumption

Table 9 also provides some insights on the validity of the assumptions made in Section 3.4 in order to derive the analytical conditions, namely ‘tight regime’ and ‘limited uncertainty’ on future emissions and GDP. In each cell of the table, the first value is computed based on the approximate analytical formulae derived in Section 4, while the second (between parenthesis)
Table 9: Normalized standard deviation for each policy variable and instrument, for each of the three representative cases, as derived from the analytical formulae, and computed by using a fully general numerical model (values in parenthesis). (1) \( m = 0.5 \) for all cases. (2) GIT calibrated using optimal values for \( m \) for each representative case. (3) GIT calibrated using only one value for \( m \), which corresponds to the optimal value for the ‘average’ case.

shows the actual value, as computed numerically with a bivariate normal distribution for \( E \) and \( Y \), fully taking into account the possibility of ‘compliance by chance’.

Table 9 confirms that our assumptions lead to acceptable results for a reduction target of

\(-25\% \) w.r.t. baseline emissions/intensity, at least for the three representative cases that we have selected. In fact, there are generally only modest deviations between the analytically approximated and rigorous numerical values, except for total relative costs, where analytical formulae systematically underestimate real uncertainty, sometimes by a wide margin.

Since the main purpose of the paper is to examine dominance conditions, we also test the validity of the formulae for \( \rho_{\text{minR}} \) and \( \rho_{\text{minRC}} \), the threshold values of \( \rho \) for marginal abatement costs and total relative costs, respectively. Table 10 shows that the approximations made at the beginning of the paper do not lead to significant errors on the values of these parameters: the errors made on \( \rho_{\text{minR}} \) and \( \rho_{\text{minRC}} \) remain small compared with the range of uncertainty on the actual value of \( \rho \).
Table 10: Validity of ‘tight regime’ assumption made for analytical calculation. We confront the analytically approximated and actual (numerical computations) threshold values for the linear correlation $\rho$ above which a linear intensity target dominates the cap.

6.5 Sensitivity to the MAC Function

In this final section, we come back to the uncertainty surrounding the argument (value of $\alpha$) and the functional form of the MAC function. We have seen that there are realistic parameter configurations in which the choice between quota and LIT depends on $\alpha$. However, numerical calculations (not shown here) suggest that the stakes are not high, since the costs of an error in terms of additional uncertainty are relatively low.

The same applies for the curvature of the MAC function—which plays a role both through $C_0/C'_0$ and through $RC_0/C_0$. $C_0/C'_0$ influences the absolute amount of uncertainty on the marginal costs, but plays no role for the relative performances of the various instruments with regard to price uncertainty (Section 4.3). $RC_0/C_0$, on the other hand, influences both the level of uncertainty on total relative costs and the relative performance of cap and intensity target. However, numerical calculations (not shown) suggest again that the ‘wrong’ choice leads to very modest increases of uncertainty.
7 Conclusion

In this paper, we have examined the relative performances of a quota, a linear intensity target, and a general intensity target with regard to uncertainty on four key variables for decision-makers: emissions, abatement effort, price of carbon, and total costs of abatement relative to GDP. Assuming that the overall constraint on carbon is tight enough, and that the uncertainties surrounding future GDP and future business-as-usual emissions are not too large, we have derived analytical conditions of dominance for each instrument and for each output variable.

We have derived ranges of plausible values for the uncertainties on future GDP, future BAU emissions, and the linear correlation coefficient between the two, as well as for the uncertainty on future abatement costs. On this basis, we have examined which instrument is likely to dominate in practice. The range of plausible values that we have derived—even for developed countries where uncertainties appear lower—is so large that the result is ambiguous. However, a quota seems to dominate a linear intensity target over most of the plausible area of parameter values. A general intensity target can be constructed to dominate the quota, but in practice an optimal calibration of the GIT would most likely lead to a target that is only weakly dependent on GDP, and thus very similar to a quota. Therefore, the potential reduction of uncertainty on key output variables that could be achieved remains modest.

Three concluding remarks can be made based on these results. First, we find little evidence to support the adoption of a linear intensity target over a quota, at least on uncertainty grounds. There are clearly areas where a LIT dominates, but the overlap with the range of plausible values for the key parameters $\rho$ and $\sigma_Y/\sigma_E$ appears rather limited. More ambitious emission targets improve the performances of a LIT relative to those of a cap, but very stringent targets would be necessary—50% below BAU emissions or more—for the LIT to dominate a quota. Such levels appear beyond the range of plausible climate agreements, at least for the second commitment period.

Second, we confirm the finding of Jotzo and Pezzey (2004) and Sue Wing et al. (2006) that a well-calibrated general intensity target can always dominate a quota with regard to the uncertainty on marginal abatement costs. This result, however, is no longer valid for total costs
relative to GDP. In addition, even when an optimal GIT can theoretically be constructed, given
the wide range of plausible values for the key parameters $\rho$ and $\sigma_Y/\sigma_E$, a very small value for $m$
has to be selected to limit the risk of error, in which case the GIT becomes basically equivalent
to a cap. In other words, we do find support for a GIT, but only when it is calibrated to be
close to a quota.

The two previous remarks stem from the fact that the range of plausible values that we
have found in the paper for $\rho$ and $\sigma_Y/\sigma_E$ is very large. Ultimately, these values translate
beliefs about how the economy of a given country or group of country will behave over the
next decade or so. If a policy-maker or an expert has a more precise view of those parameters,
his or her selection of instruments might be different. But further analysis is necessary to
provide hard data that could support such intuitions.

Finally, let us note that the ‘tight regime’ assumption we make in this paper is not necessar-
ily valid in practice, as countries may negotiate targets that are close to their projected BAU
emissions. In this case, the possibility that BAU emissions be spontaneously below the target
can no longer be sidestepped. Examining the relative performance of various instruments when
the ‘tight regime’ assumption is relaxed is a subject for future research.

Appendix

A Approximation of Mean and Variance

Let $F$ be a function of future BAU emissions $E$, of future output $Y$, and of the slope of the
marginal cost curve $a$. Since $E$, $Y$, and $a$ are random variables, $F(E,Y,a)$ is also a random
variable. Assuming that the fluctuations of $E$, $Y$, and $a$ around their mean are small, we can
approximate $F(E,Y,a) = F(<E> + \epsilon, <Y> + \iota, <a> + \kappa)$ by a Taylor expansion around
the deterministic value $F(<E>,<Y>,<a>) = F_0$. Precisely,

\[
F(E,Y,a) \approx F_0 + \left( \kappa \frac{\partial}{\partial a} + \epsilon \frac{\partial}{\partial E} + \iota \frac{\partial}{\partial Y} \right) F + \frac{1}{2} \left( \kappa \frac{\partial}{\partial a} + \epsilon \frac{\partial}{\partial E} + \iota \frac{\partial}{\partial Y} \right)^2 F
\]

\[
= F_0 + \kappa \frac{\partial F}{\partial a} + \epsilon \frac{\partial F}{\partial E} + \iota \frac{\partial F}{\partial Y}
\]
\[ + \frac{\kappa^2}{2} \frac{\partial^2 F}{\partial a^2} + \frac{\epsilon^2}{2} \frac{\partial^2 F}{\partial E^2} + \frac{\left(\alpha \frac{\partial^2 F}{\partial Y^2} + \epsilon \frac{\partial^2 F}{\partial Y \partial E}\right)}{2}, \]  
\tag{23}

where all mixed derivatives except \( \partial Y \partial E \) vanish because \( \kappa \) is independent from both \( \epsilon \) and \( \epsilon \).

The expected value \(< F >\) then follows:

\[
<F(E,Y,a) > \approx F_0 + \frac{\sigma_C^2}{2} \frac{\partial^2 F}{\partial a^2} + \frac{\sigma_E^2}{2} \frac{\partial^2 F}{\partial E^2} + \frac{\sigma_Y^2}{2} \frac{\partial^2 F}{\partial Y^2} + \rho \sigma_E \sigma_Y \frac{\partial^2 F}{\partial Y \partial E}, \tag{24}
\]

where we have used the fact that, by definition, \(< \kappa^2 > = \sigma_C^2\), and so on. Finally, we obtain the variance by rewriting Eq.(23) for the function \( F^2\):

\[
F^2(\epsilon, \kappa, \tau) \approx F_0^2 + \left(\kappa \frac{\partial}{\partial a} + \epsilon \frac{\partial}{\partial E} + \tau \frac{\partial}{\partial Y}\right) F^2 + \frac{1}{2} \left(\kappa \frac{\partial}{\partial a} + \epsilon \frac{\partial}{\partial E} + \tau \frac{\partial}{\partial Y}\right)^2 F^2
- \quad F_0^2 + 2F_0 \left(\kappa \frac{\partial}{\partial a} + \epsilon \frac{\partial}{\partial E} + \tau \frac{\partial}{\partial Y}\right) F
+ \quad F_0 \left(\kappa \frac{\partial}{\partial a} + \epsilon \frac{\partial}{\partial E} + \tau \frac{\partial}{\partial Y}\right)^2 F + \left(\kappa \frac{\partial F}{\partial a} + \epsilon \frac{\partial F}{\partial E} + \tau \frac{\partial F}{\partial Y}\right) F\right)^2, \tag{25}
\]

which yields an expected value of

\[
<F^2> \approx F_0^2 + 2F_0 \left(\kappa \frac{\partial}{\partial a} + \epsilon \frac{\partial}{\partial E} + \tau \frac{\partial}{\partial Y}\right) F^2 + \sigma_C^2 \left(\frac{\partial F}{\partial a}\right)^2 \frac{\partial^2 F}{\partial a^2} + \sigma_E^2 \left(\frac{\partial F}{\partial E}\right)^2 \frac{\partial^2 F}{\partial E^2} + \sigma_Y^2 \left(\frac{\partial F}{\partial Y}\right)^2 \frac{\partial^2 F}{\partial Y^2} + 2\rho \sigma_E \sigma_Y \frac{\partial F}{\partial E} \frac{\partial F}{\partial Y} + 2 \rho \sigma_E \sigma_Y \frac{\partial F}{\partial E} \frac{\partial F}{\partial Y}. \tag{26}
\]

Subtracting Eq.(26) from the square of Eq.(24) finally yields the variance of \( F \)

\[
\sigma_F^2 \approx \sigma_C^2 \left(\frac{\partial F}{\partial a}\right)^2 + \sigma_E^2 \left(\frac{\partial F}{\partial E}\right)^2 + \sigma_Y^2 \left(\frac{\partial F}{\partial Y}\right)^2 + 2 \rho \sigma_E \sigma_Y \frac{\partial F}{\partial E} \frac{\partial F}{\partial Y}. \tag{27}
\]

**B Calculation of Variance of Total Relative Costs**

Total relative costs of abatement are defined by Eq.(14)

\[
RC(R) = \frac{1}{Y} \int_{qY^m}^{E} a \ C(E^\alpha - e \ E^{\alpha-1}) \ dr = \frac{a E^{1-\alpha}}{Y} \int_{0}^{R} C(r)dr, \tag{28}
\]
The variance is obtained by applying Eq.(6) to $RC$. Partial derivatives of $RC$ are as follows:

\[
\left( \frac{\partial RC}{\partial a} \right)^2 = RC_0^2
\]  
(29)

\[
\left( \frac{\partial RC}{\partial E} \right)^2 = RC_0^2 \left( (1 - \alpha) + \frac{C_0}{RC_0} \frac{\partial R}{\partial E} \right)^2
\]  
(30)

\[
\left( \frac{\partial RC}{\partial Y} \right)^2 = RC_0^2 \left( \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} - 1 \right)^2
\]  
(31)

\[
\left( \frac{\partial RC}{\partial Y} \right) \left( \frac{\partial RC}{\partial E} \right) = RC_0^2 \left( (1 - \alpha) + \frac{C_0}{RC_0} \frac{\partial R}{\partial E} \right) \left( \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} - 1 \right)
\]  
(32)

\[
= RC_0^2 \left( \frac{C_0^2}{RC_0^2} \frac{\partial R}{\partial E} \frac{\partial R}{\partial Y} + \frac{C_0}{RC_0} \left( (1 - \alpha) \frac{\partial R}{\partial Y} - \frac{\partial R}{\partial E} \right) - (1 - \alpha) \right).
\]

The variance is thus:

\[
\sigma_{RC}^2 \approx RC_0^2 \left( \sigma_C^2 + \sigma_E^2 \left( (1 - \alpha) + \frac{C_0}{RC_0} \frac{\partial R}{\partial E} \right)^2 + \sigma_Y^2 \left( 1 - \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} \right)^2 \right.
\]

\[
+ \ 2 \rho \sigma_E \sigma_Y \left( (1 - \alpha) + \frac{C_0}{RC_0} \frac{\partial R}{\partial E} \right) \left( \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} - 1 \right) \left( \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} - 1 \right) \right).
\]  
(33)

For a quota, $\partial R/\partial Y = 0$, and thus:

\[
\sigma_{RC,cap}^2 \approx RC_0^2 \left( \sigma_C^2 + \sigma_E^2 \left( (1 - \alpha) + \frac{C_0}{RC_0} \frac{\partial R}{\partial E} \right)^2 + \sigma_Y^2 \right.
\]

\[
- \ 2 \rho \sigma_E \sigma_Y \left( (1 - \alpha) + \frac{C_0}{RC_0} \frac{\partial R}{\partial E} \right) \left( \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} - 1 \right) \right).
\]  
(34)

The condition under which the variance is lower under an intensity target than under a cap then becomes

\[
\Leftrightarrow \ \sigma_Y^2 \left( 1 - \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} \right)^2 + 2 \rho \sigma_E \sigma_Y \left( (1 - \alpha) + \frac{C_0}{RC_0} \frac{\partial R}{\partial E} \right) \left( \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} - 1 \right) < \sigma_Y^2
\]  
(35)

\[
\Leftrightarrow \ \frac{1}{2} \frac{\sigma_Y}{\sigma_E} \left( 2 - \frac{C_0}{RC_0} \frac{\partial R}{\partial Y} \right) < \rho
\]  
(36)

\[
\Leftrightarrow \ \frac{1}{2} \frac{\sigma_Y}{\sigma_E} \left( 2 + q \frac{C_0}{RC_0} \right) < \rho.
\]  
(38)
References


